Disorder in a classical spin liquid: topological spin glass, cluster algorithm and dynamical Griffiths phase

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Novel Phases of Quantum Matter, ICTS

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also thanks to
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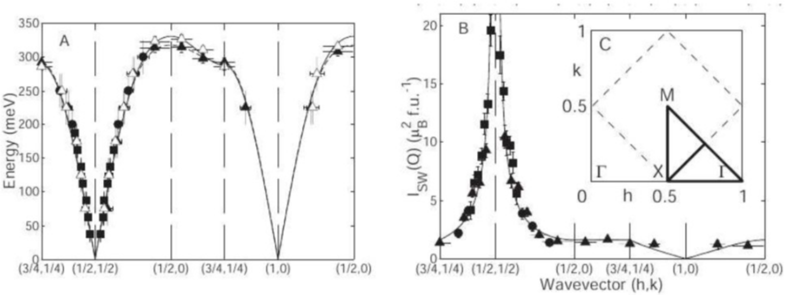
Reference:

Bose, Moessner and Sen, Phys. Rev. B **100**, 064425 (2019),
Sen and Moessner, Phys. Rev. Lett. **114**, 247207 (2015),
and ongoing work

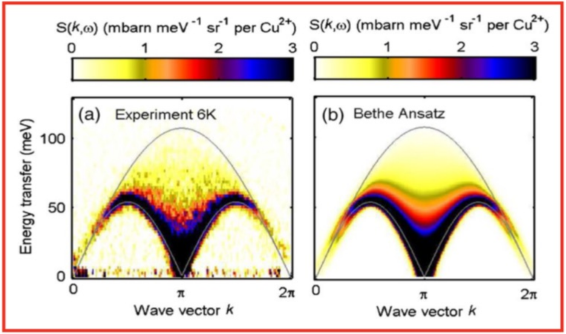
What is a spin liquid?

- Spin solid — e.g. A ferromagnet or an antiferromagnet



- Fig shows dispersion in La_2CuO_4 [from Coldea et. al, PRL 86, 5377 (2000)] –shows existence of magnons
- Spin gas — e.g. an uncorrelated paramagnet
- Spin liquid—strongly “correlated” paramagnet

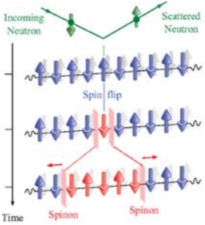
Phases of matter with fractionalized excitations



B. Lake et. al. (2010)

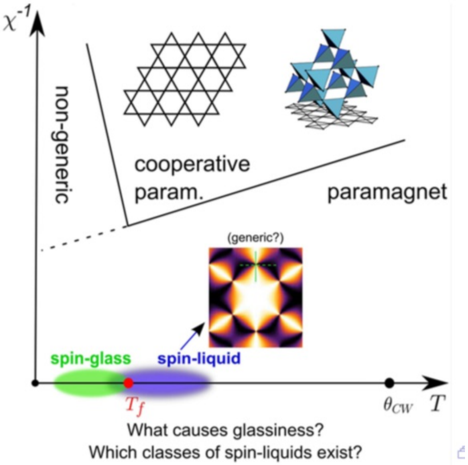
Experiments confirm existence of spinons (S=1/2) in 1D

[Matches Bethe Ansatz]



- These break no symmetries of the underlying Hamiltonian but possess *completely different* excitations from conventional ordered states.
- $H = J\sum_{\langle i,j\rangle} \vec{S}_i \cdot \vec{S}_j$ with $S = 1/2$ spins in 1D
- Excitations not magnons ($S = 1$) but spinons ($S = 1/2$)

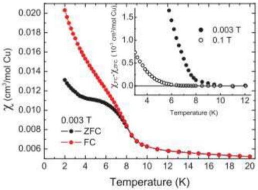
(Weak) Diagnostics



- No magnetic ordering even well below $|\Theta_{CW}|$ unlike unfrustrated magnets
- $T_f \ll T \ll |\Theta_{CW}|$—spin liquid regime (Ramirez)

Glassiness in spin liquids?

SPIN DYNAMICS IN THE $S = \frac{1}{2}$ QUANTUM KAGOME . . .



PHYSICAL REVIEW B **83**, 180416(R) (2011)

Spin dynamics in the $S = \frac{1}{2}$ quantum kagome compound vesignieite, $Cu_3Ba(VO_5H)_2$

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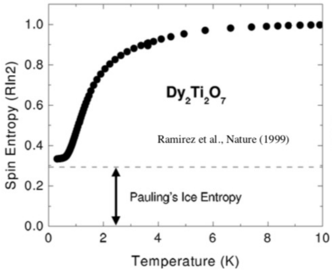
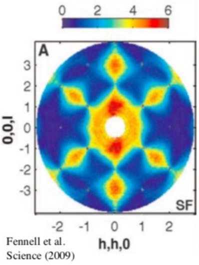
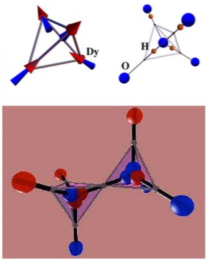
We report the study of high-quality samples of the frustrated $S = \frac{1}{2}$ kagome antiferromagnet vesignieite, $Cu_3Ba(VO_5H)_2$. Neutron powder diffraction measurements evidence the excellence of the kagome lattice and show no sign of a transition to magnetic long-range order. A kink in the susceptibility below $T = 9$ K is matched to a reduction in paramagnetic-like correlations in the diffraction data and a slowing of the spin dynamics observed by μSR. Our results point to an exotic quantum state below 9 K with coexistence of both dynamical and small frozen moments $\sim 0.1\mu_B$. We propose that this novel quantum ground state is stabilized by a large Dzyaloshinsky-Moriya interaction.

- Exp in $Cu_3Ba(VO_5H)_2$ which is $S = 1/2$ kagome antiferromagnet.
- $SrCr_{9p}Ga_{12-9p}O_{19}$ (SCGO) has a spin-glass transition at $T_g \approx 3.5$ K but persistent spin dynamics down to 100 mK (from μSR probes)
- No sign of transition to magnetic long range order
- Coexistence of both dynamical (for majority of spins) and small frozen moments, no phase seperation

Spin ice



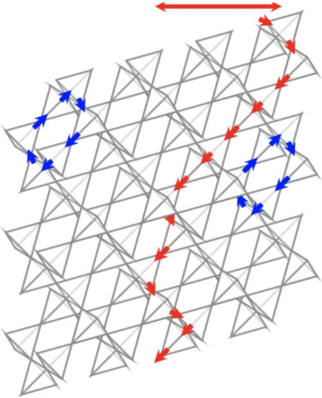




$$H = \frac{J}{3}\sum_{\langle ij\rangle} S_i S_j + Da^3 \sum_{(ij)} \left(\frac{\hat{e}_i \cdot \hat{e}_j}{|r_{ij}|^3} - \frac{3(\hat{e}_i \cdot \mathbf{r}_{ij})(\hat{e}_j \cdot \mathbf{r}_{ij})}{|r_{ij}|^5} \right) S_i S_j$$

- Experimentally relevant: $Dy_2Ti_2O_7$, $Ho_2Ti_2O_7$, $Er_2Ti_2O_7$, $SrCr_8Ga_4O_{19}$

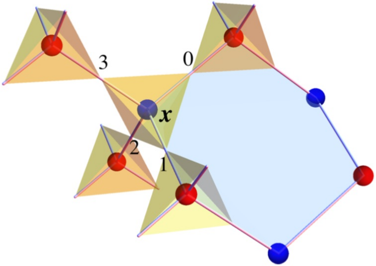
Short and long loops

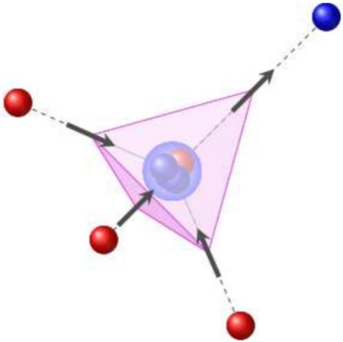


- $S(T=0) \approx k_B \ln\left(2^N \left(\frac{6}{16}\right)^{N/2}\right) = \frac{N}{2} k_B \ln\left(\frac{3}{2}\right)$ [Pauling, 1935]
- Power-law spin correlations gives rise to pinch points in momentum space [Isakov, Gregor, Moessner, Sondhi (2004), Henley (2005)]

Spin ice and Dumbbell model

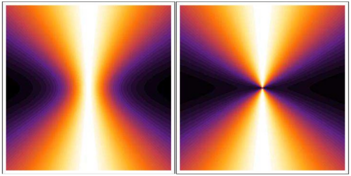
- Represent the “point” dipoles by dumbbells that live on the ends of the dual diamond lattice.
- $H_d = \frac{2\sqrt{2}}{3\sqrt{3}} D a_d \sum_{i>j} \frac{Q_i Q_j}{r_{ij}} + \Delta \sum_i (Q_i/2)^2$ where $Q_i = \eta_i (S_{\boxtimes})_i$ and $\Delta = \frac{2J}{3} + \frac{8}{3}\left(1 + \sqrt{23}\right) D$
  (Castelnovo, Moessner and Sondhi (2008))

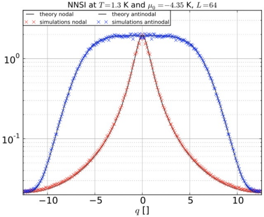


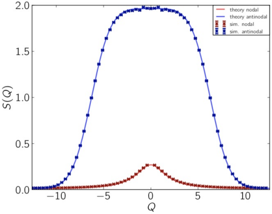


Pinch points at low T

Arnab Sen, R. Moessner, S. L. Sondhi, Phys. Rev. Lett. **110**, 107202 (2013)+ unpublished work with V. Kaiser



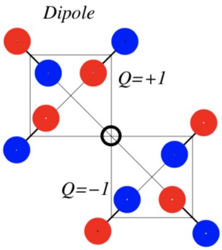


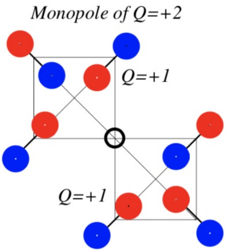


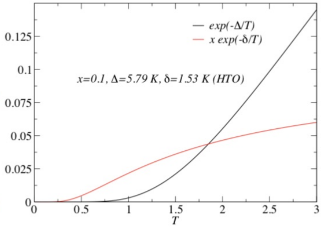
- HWHM of scattering in nodal dir. $\sqrt{\left(1+\frac{4\sqrt{2}\pi D}{3T}\right)\sqrt{\rho}}$ and discontinuity at pinch point $1-\left(\frac{1}{1+\frac{4\sqrt{2}\pi D}{3T}}\right)$

Introducing disorder

E.g., $Dy_{2-x}Y_xTi_2O_7/Ho_{2-x}Y_xTi_2O_7$



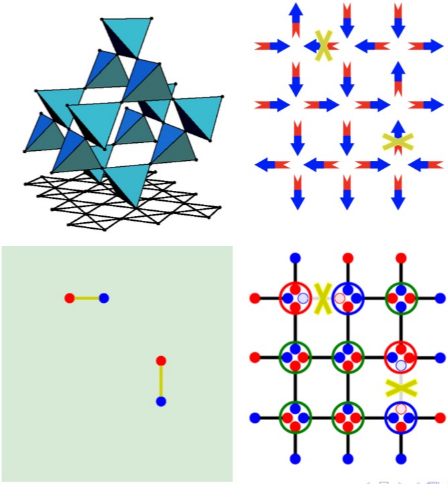




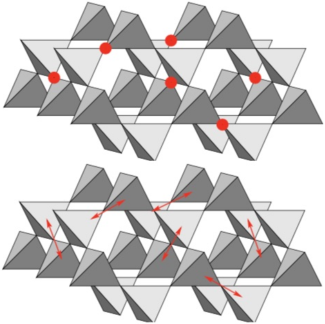
- At defective tetrahedra, $S_{\boxtimes} \neq 0$ even at $T = 0$.
- Cost of bulk $Q = \pm 2$ monopoles (Δ) higher than impurity $Q = \pm 2$ monopoles ($\delta = \frac{4\sqrt{2}D}{3\sqrt{3}}$)
- Impurity monopoles dominate below $T_\delta \sim \frac{\delta - \Delta}{\ln x}$

Disorder effects

Arnab Sen, R. Moessner, Phys. Rev. Lett. **114**, 247207 (2015).



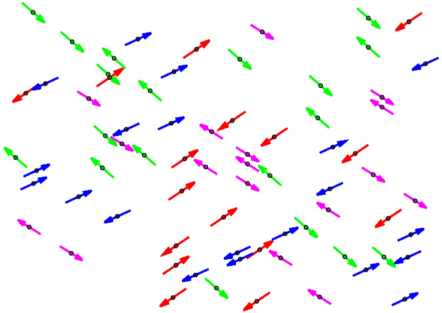
Particle-hole transformation



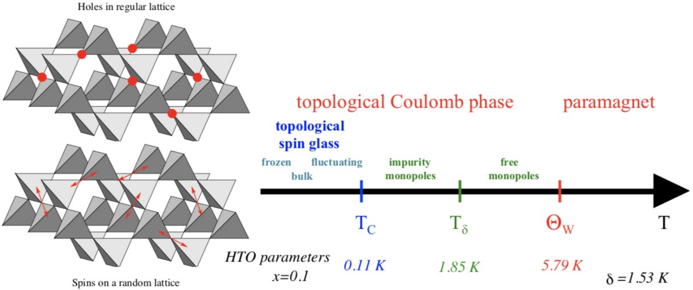
- Interaction between the “ghost spins” have two parts:
- $\left(D + \frac{3T}{\sqrt{2\pi}}\right)\left(\frac{a}{r_{ij}}\right)^3 (\hat{e}_i \cdot \hat{e}_j - 3(\hat{e}_i \cdot \hat{r}_{ij})(\hat{e}_j \cdot \hat{r}_{ij}))$
- The T dependent renormalization is because of the background spin liquid.

Spin glass of ghost spins

- High temperature : $\langle S_i \rangle = 0$
- Low temperature: $\langle S_i \rangle \neq 0$ but $\frac{1}{N}\sum\langle S_i \rangle = 0$;
  $q_{EA} = \frac{1}{N}\sum\langle S_i \rangle^2 \neq 0$
- We simulate the dilute system of ghost spins and "confirm a glass transition" [cluster algorithm needed].
- Dense dipoles with random orientations and dilute but collinear dipoles both studied earlier in 3D

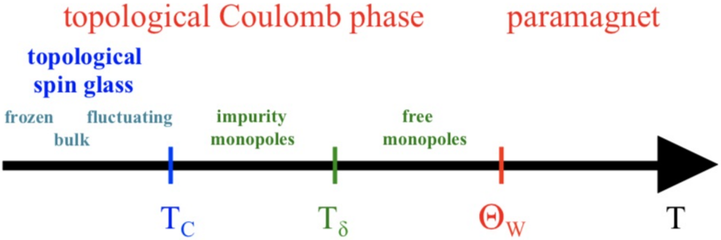


Phase diagram of diluted spin ice



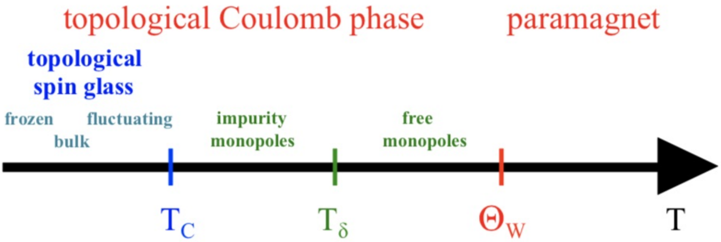
$$\Theta_W \sim \Delta = \frac{2J}{3} + \frac{8}{3}\left(1+\sqrt{\frac{2}{3}}\right)D;\ \delta = \frac{4\sqrt{2}D}{3\sqrt{3}};\ T_\delta = \frac{\delta-\Delta}{\ln x};\ T_c \sim 0.78Dx$$

Below glass transition



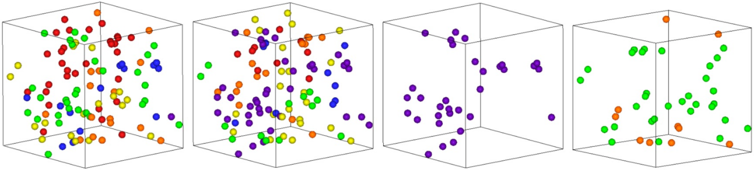
- A small frozen moment q_{EA} develops continuously with T below transition
- Sets up local fields. Resulting Zeeman energy $\sim D\sqrt{q_{EA}}x$ attempts to pin bulk spins along the local fields
- Competition with Pauling entropy of $\frac{1}{2}\ln\frac{3}{2}$

Signatures?



- Glass: Freezing of ghost spins below $T_c(x)$. Probes like nonlinear susceptibility, history dependent mag.
- Liquid: Pinch points persist below $T_c(x)$. Neutron scattering
- Interplay shows up in gradual but complete loss of Pauling entropy as T is lowered below $T_c(x)$. Probe specific heat

Cluster algorithm



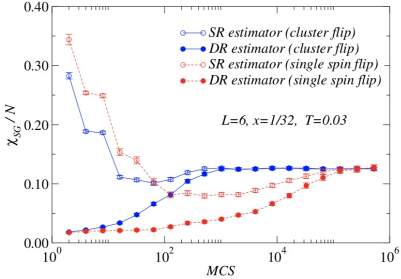
(a) $T = 0.072 \approx 2T_x$ (b) $T = 0.047 \approx 1.3T_x$ (c) C_2 (d) C_1, C_0

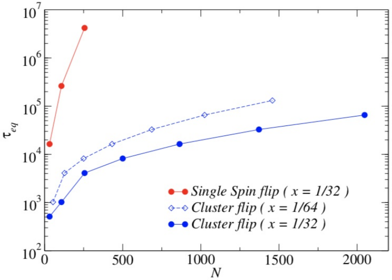
FIG. 1. A particular disorder realization for $L = 6$ and $x = 1/32$. Colors at different sites in (a) and (b) represent different acceptance ratios of spin flips (R_i) using a conventional single spin-flip Metropolis algorithm. Sites with ($0 < R < 10^{-4}$) are denoted by violet, ($10^{-4} < R < 10^{-3}$) by blue, ($10^{-3} < R < 10^{-2}$) by green, ($10^{-2} < R < 0.1$) by yellow, ($0.1 < R < 0.25$) by orange and ($0.25 < R < 1$) by red. With the chosen cluster parameters ($a_s = 1.3125$, $b_s = 0.75$ and $C_L = N/5$), three cluster sets C_0, C_1 and C_2 are obtained for this disorder realization. (c) shows the member sites of the clusters that belong to the set C_2 (in violet). (d) shows the additional member sites of the clusters in C_1 that are already not part of C_2 (in green) and the additional member sites of the clusters in C_0 that are already not part of C_2, C_1(in orange). The figures were generated using the graphics software QMGA [41].

with Tushar Kanti Bose (IACS)

- Problem with single spin flips –rare clusters of spins with $|J_{ij}| \gg |J_{avg}|$ frozen
- Can simulate much larger number of dipoles reliably

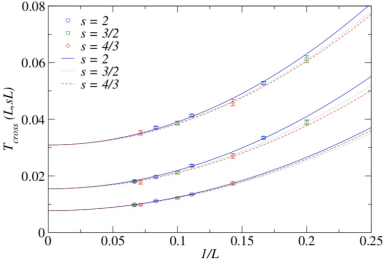
Equilibration tests

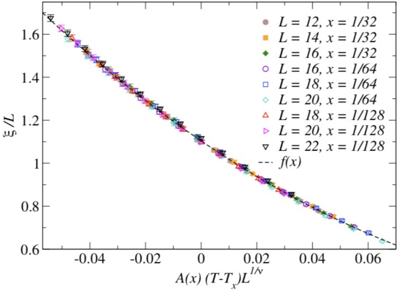




- $q_{EA}^{\alpha\beta}(\mathbf{k}) = \frac{1}{N}\sum_i \mu_i^{\alpha(1)}\mu_i^{\beta(2)}\exp(i\mathbf{k}\cdot\mathbf{r}_i)$ where $N = 16L^3x$, $\alpha,\beta = x, y, z$
- $\chi_{SG}(\mathbf{k}) = N\sum_{\alpha,\beta}[\langle|q_{EA}^{\alpha\beta}|^2\rangle]$

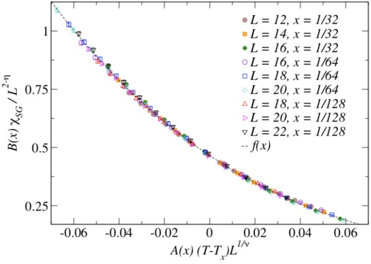
Universality (I)

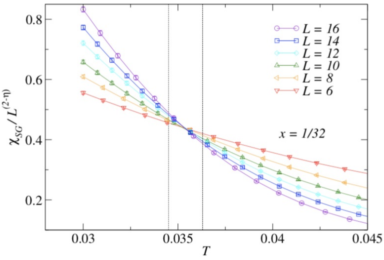




- $\xi = \frac{1}{2\sin\left(\frac{|k_{min}}{2}\right)}\left(\frac{\chi_{SG}(0)}{\chi_{SG}(k_{min}} - 1\right)^{1/2}$ where $k_{min} = \frac{2\pi}{L}(1,0,0)$

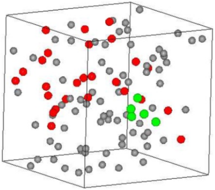
Universality (II)

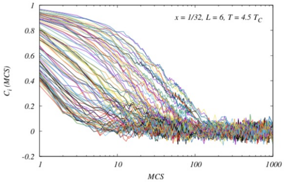


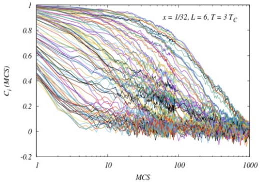


- Thermal exponent $\nu = 1.27(8)$ and anomalous exponent $\eta = 0.228(35)$ extracted from the finite size scaling analysis

Dynamical Griffiths effects



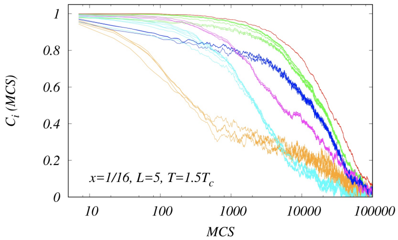


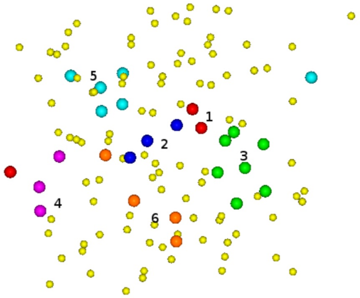


with Tushar Kanti Bose (IACS)

- Spins in different clusters have different timescales for fluctuations
- Dynamical heterogeneity remain even for $T \sim 4T_c$

Clusters from dynamics





- Spins with slow dynamics ($R < 0.01$) clearly show clustering
- Inter-cluster correlations are also significant between certain clusters

Conclusions: disorder in spin liquids

- Plays many roles: may be a nuisance in some cases but may be really interesting in others
- Topological glass in spin ice
- Identification of glass-like and liquid-like degrees of freedom
- Novel cluster algorithm to simulate this spin glass
- Dynamical Griffiths effects for local dynamics

Thanks for your attention