Superpolynomials And Super-A-polynomial for Twist Knots

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Based on work done with Satoshi Nawata, P. Ramadevi, Xinyu Sun

[JHEP11(2012)157, arXiv:1209.1409]

ADVANCED SCHOOL AND DISCUSSION MEETING ON KNOT THEORY AND ITS APPLICATIONS

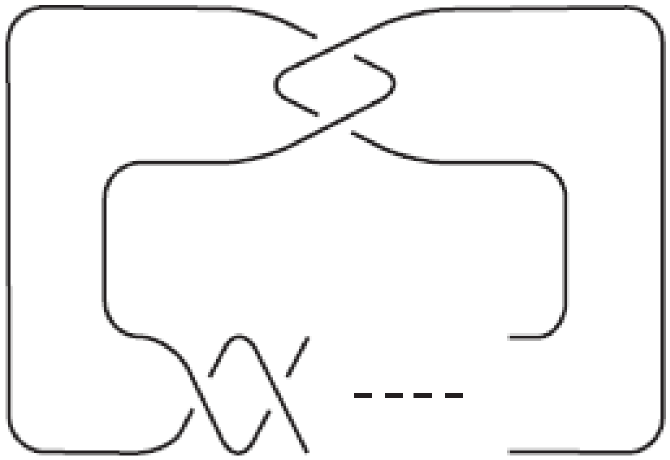
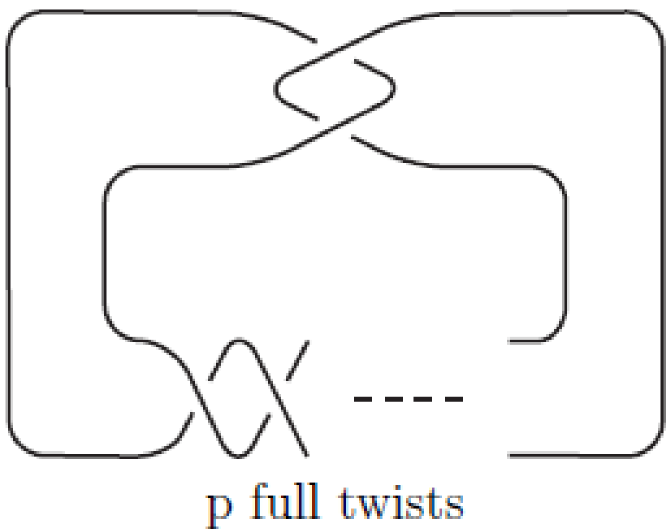
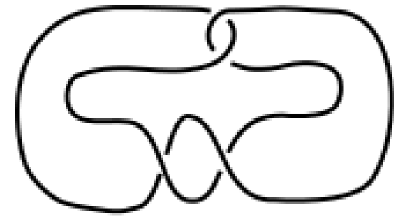


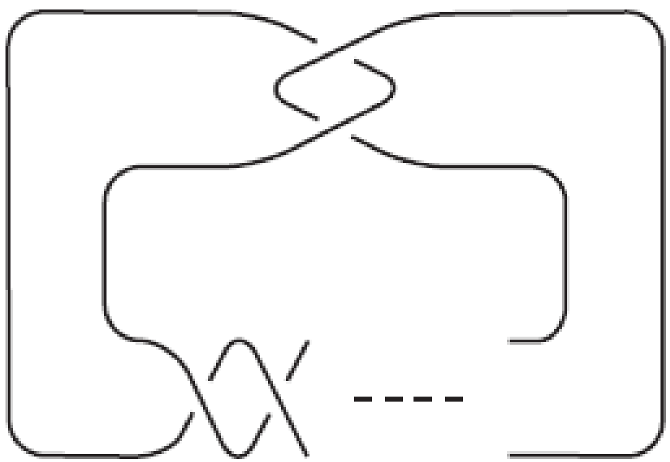
Figure: Twist knot.





P=1

Figure: Twist knot.


p full twists

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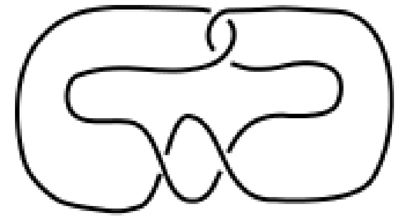
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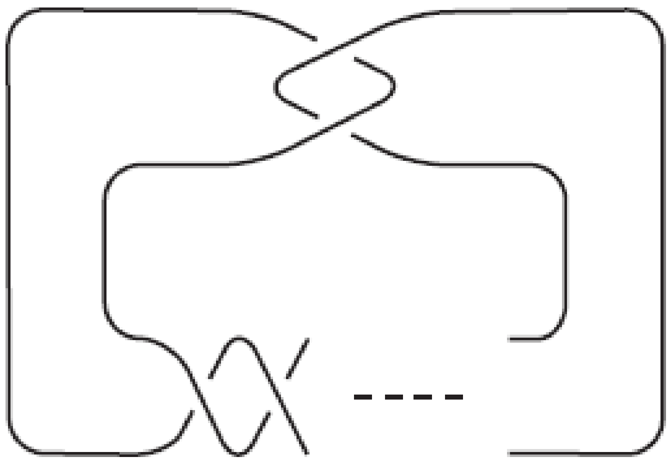
3_1



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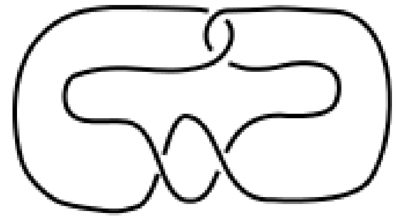

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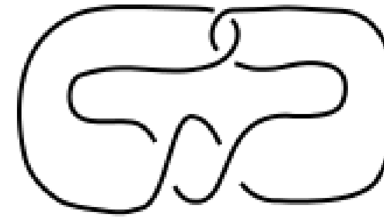

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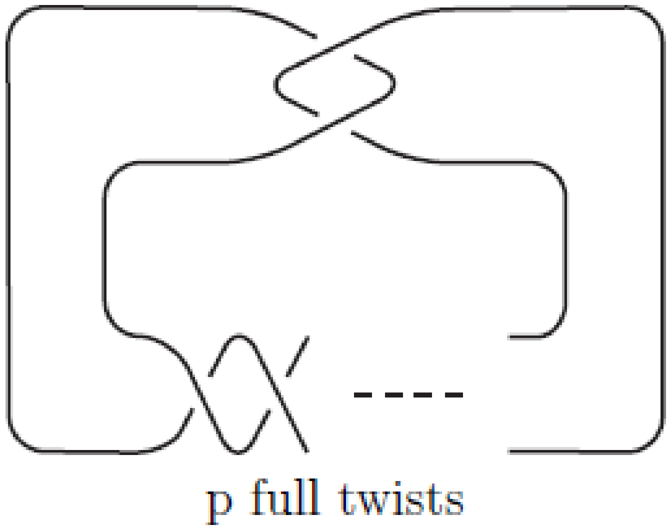

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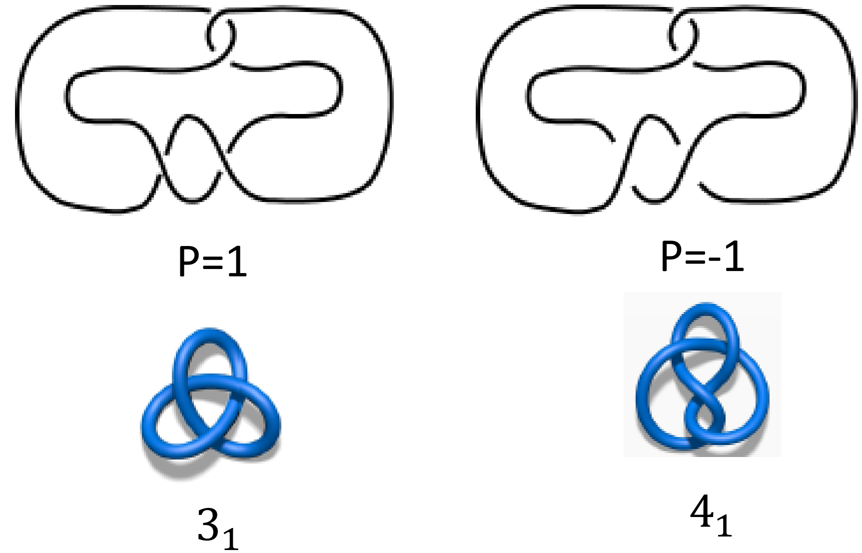
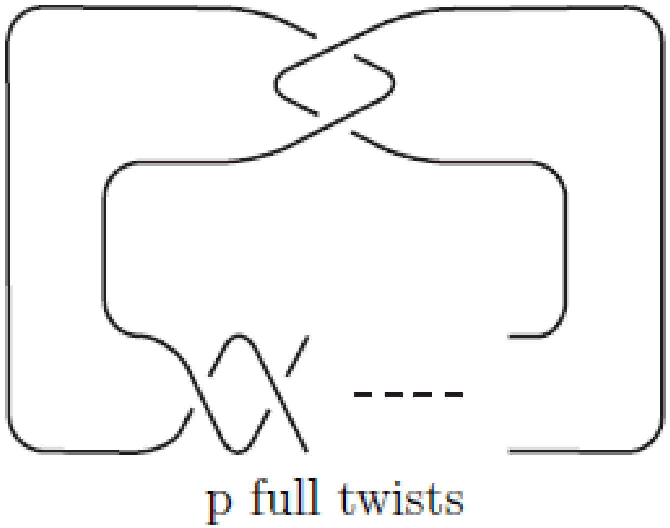
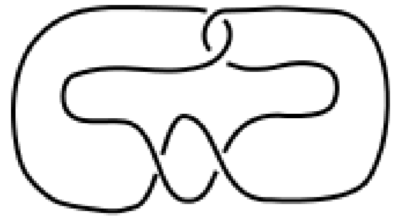






P=1

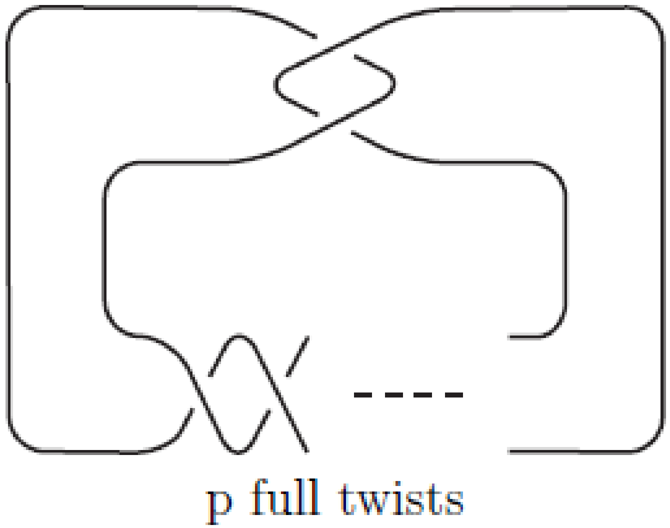
P=-1

3_1

4_1

Figure: Twist knot.

p	-4	-3	-2	-1	0	1	2	3	4
knots	$\mathbf{10_1}$	$\mathbf{8_1}$	$\mathbf{6_1}$	$\mathbf{4_1}$	$\mathbf{0_1}$	$\mathbf{3_1}$	$\mathbf{5_2}$	$\mathbf{7_2}$	$\mathbf{9_2}$



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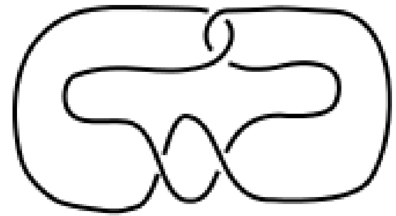

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K_p → Twist knot with p full twist

p	-4	-3	-2	-1	0	1	2	3	4
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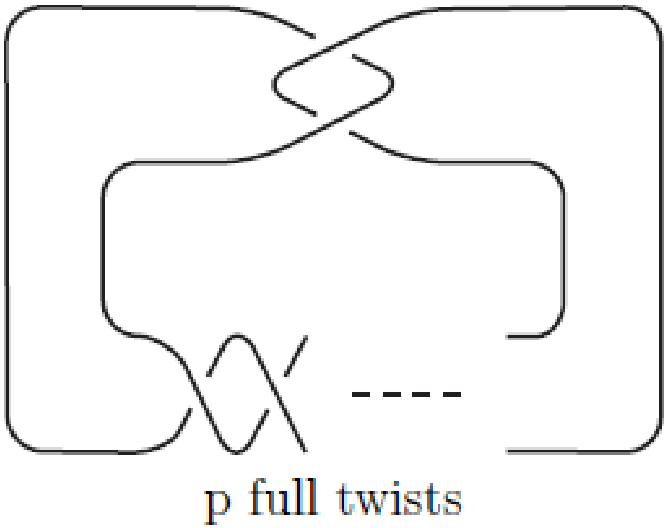

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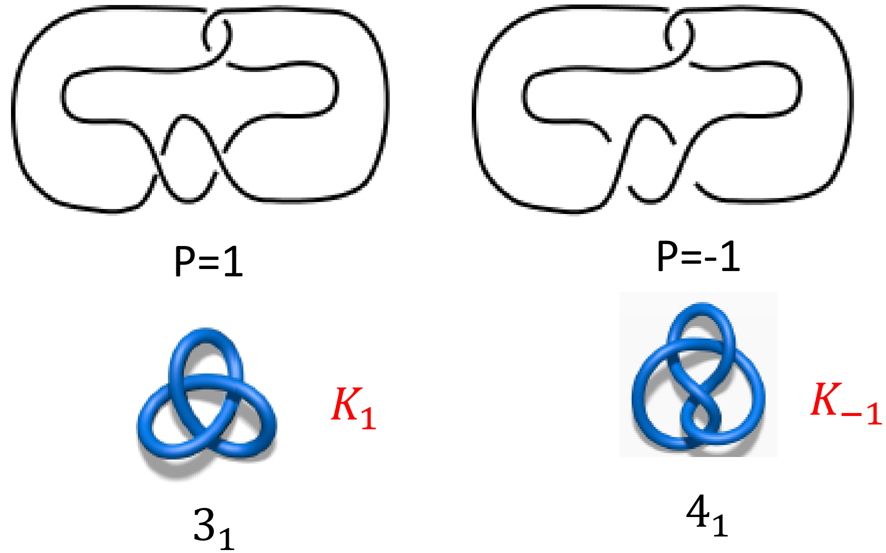

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Colored HOMFLYPT polynomial $\Rightarrow$ $P_R(K; a, q)$

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$$a^{1/2}P_{\Box}\left(\diagup\!\!\!\!\diagdown\right) - a^{-1/2}\,P_{\Box}\left(\diagdown\!\!\!\!\diagup\right) = (q^{1/2}-q^{-1/2})P_{\Box}\left(\,)\,(\,\right)$$

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$$R=\overbrace{\Box\Box\Box\Box\Box\Box\Box\Box}^{n-1}\equiv n$$

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$P_n(\mathrm{K})$

Knot homologies

sl(2) homology

Khovanov homology

→ $\mathcal{H}_{i,j}^{\mathfrak{sl}_2}$ [Khovanov]

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Euler characteristics

$$\sum_{i,j}(-1)^i q^j \dim \mathcal{H}_{i,j}^{\mathfrak{sl}_2}(K) = J_2\,(K;q)$$

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⇒ $\mathcal{H}^{\mathfrak{sl}_2}_{i,j}$ [Khovanov]

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Poincare polynomial

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$$\mathcal{P}^{\mathfrak{sl}_2}_2(K;q,t) = \sum_{i,j} t^i q^j \dim \mathcal{H}^{\mathfrak{sl}_2}_{i,j}(K)$$

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Colored sl(2) homology

Categorify colored Jones polynomial $\Rightarrow$ $\mathcal{H}^{\mathfrak{sl}_2,R}_{i,j}$

[Cooper-Krushkal] [Frenkel-Stroppel-Sussan][Webster]

Knot homologies

sl(N) homology $\Rightarrow$ $\mathcal{H}_{i,j}^{\mathfrak{sl}_N}$ [Khovanov-Rozansky '04]

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Triply graded homology which categorify HOMFLYPT polynomial

[Khovanov-Rozansky '05]

⇒ $H_{i,j,k}$

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[Dunfield-Gukov-Rasmussen '05]

A triply graded homology $\mathcal{H}_{i,j,k}$ which is equipped with a families differentials which relates it to Sl(N) homology.

Colored HOMFLY homology → $\mathcal{H}^{R}_{i,j,k}$

[Gukov-Stosic '12]

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Colored HOMFLYPT

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Colored superpolynomials

$$\mathcal{P}_R(K;a,q) = \sum_{i,j,k} t^i q^j a^k \dim \mathcal{H}^R_{i,j,k}(K)$$

Example:

$$\mathcal{P}_{\square}(\mathbf{3_1}) = aq^{-1}t^0 + aqt^2 + a^2q^0t^3$$

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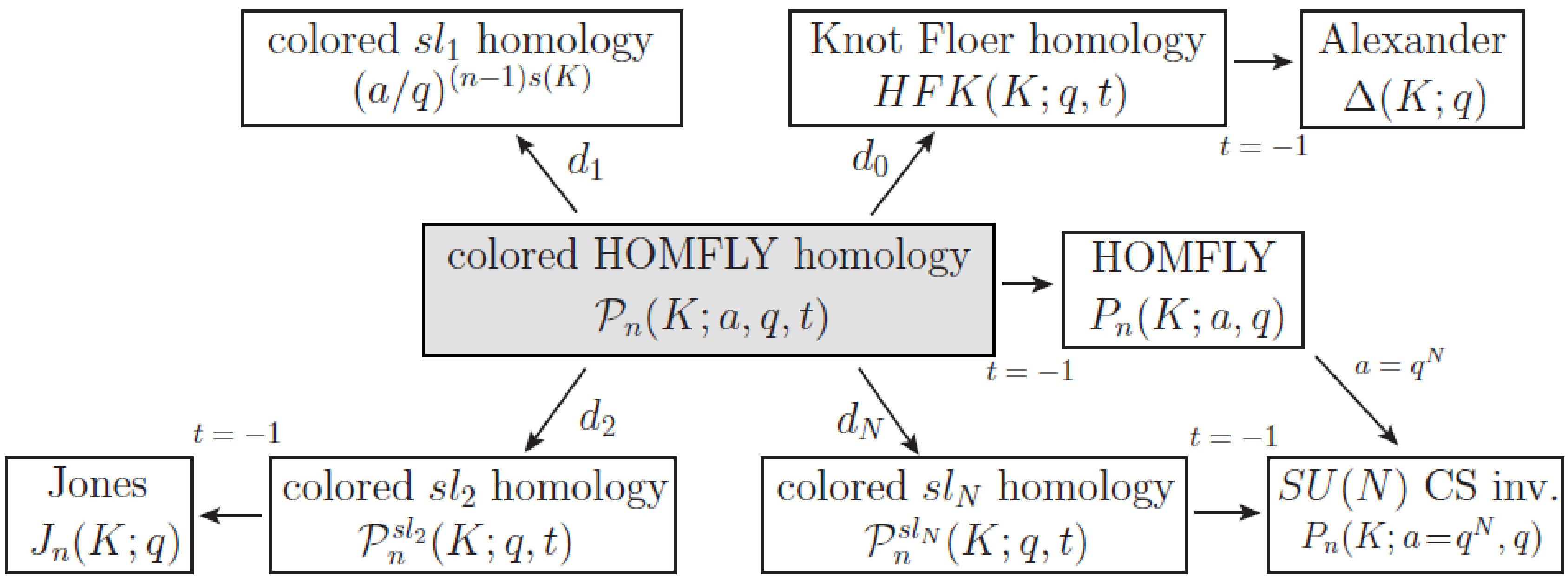
colored sl_1 homology
$(a/q)^{(n-1)s(K)}$
Knot Floer homology
$HFK(K;q,t)$
Alexander
$\Delta(K;q)$
$t=-1$
d_1
d_0
colored HOMFLY homology
$\mathcal{P}_n(K;a,q,t)$
HOMFLY
$P_n(K;a,q)$
$t=-1$
$a=q^N$
d_2
d_N
$t=-1$
$t=-1$
Jones
$J_n(K;q)$
colored sl_2 homology
$\mathcal{P}_n^{sl_2}(K;q,t)$
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$\mathcal{P}_n^{sl_N}(K;q,t)$
$SU(N)$ CS inv.
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Differentials

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differentials

$S^r \equiv \overbrace{\square\square\square\square\square}^{r}$

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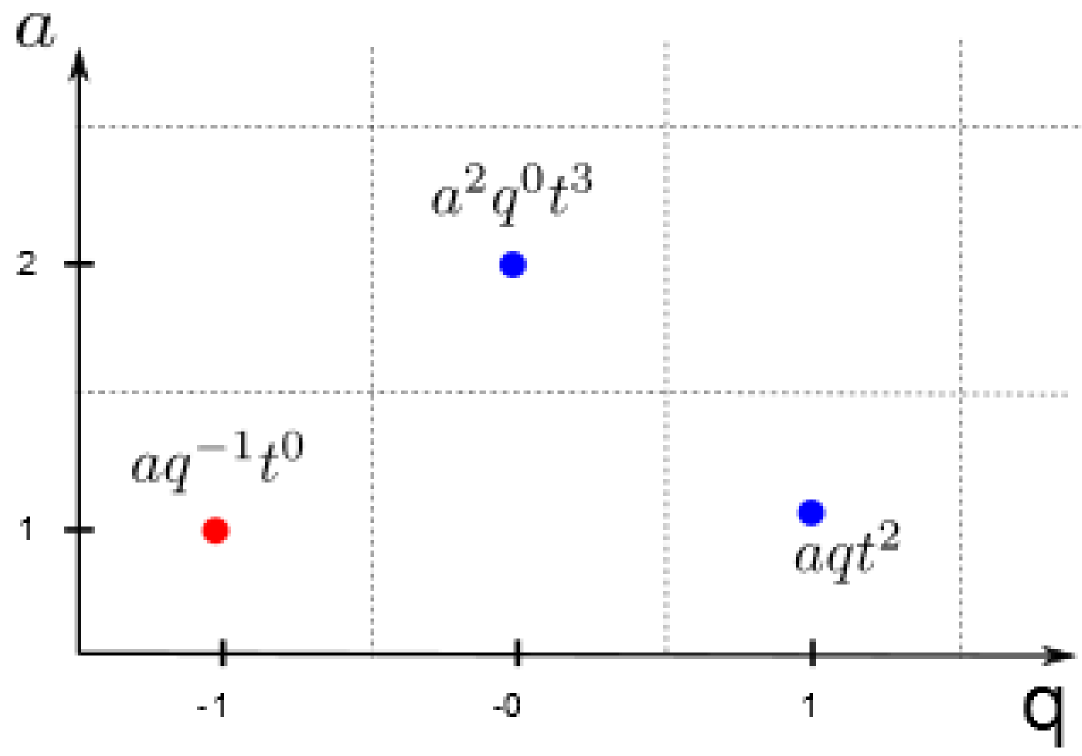

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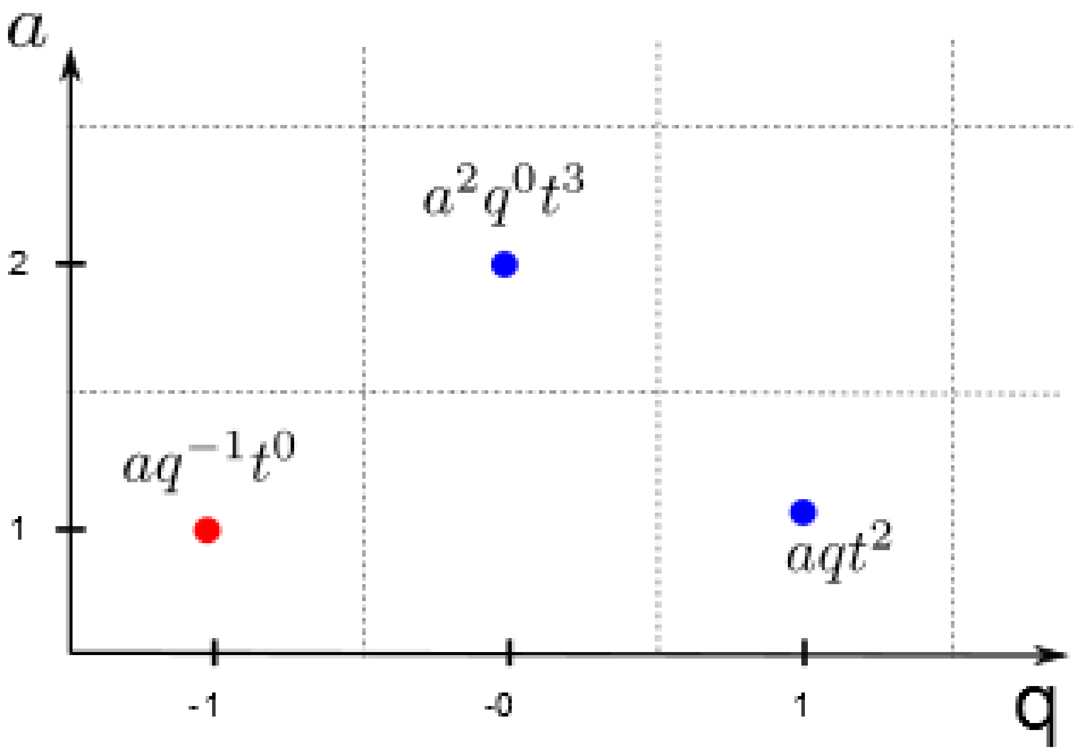

$$d_1^R : \mathcal{H}^R_{i,j,k}(K) \longrightarrow \mathcal{H}^R_{i-1,j+1,k-1}(K) \qquad a^i q^j t^k \longrightarrow a^{i-1}q^{j+1}t^{k-1}$$

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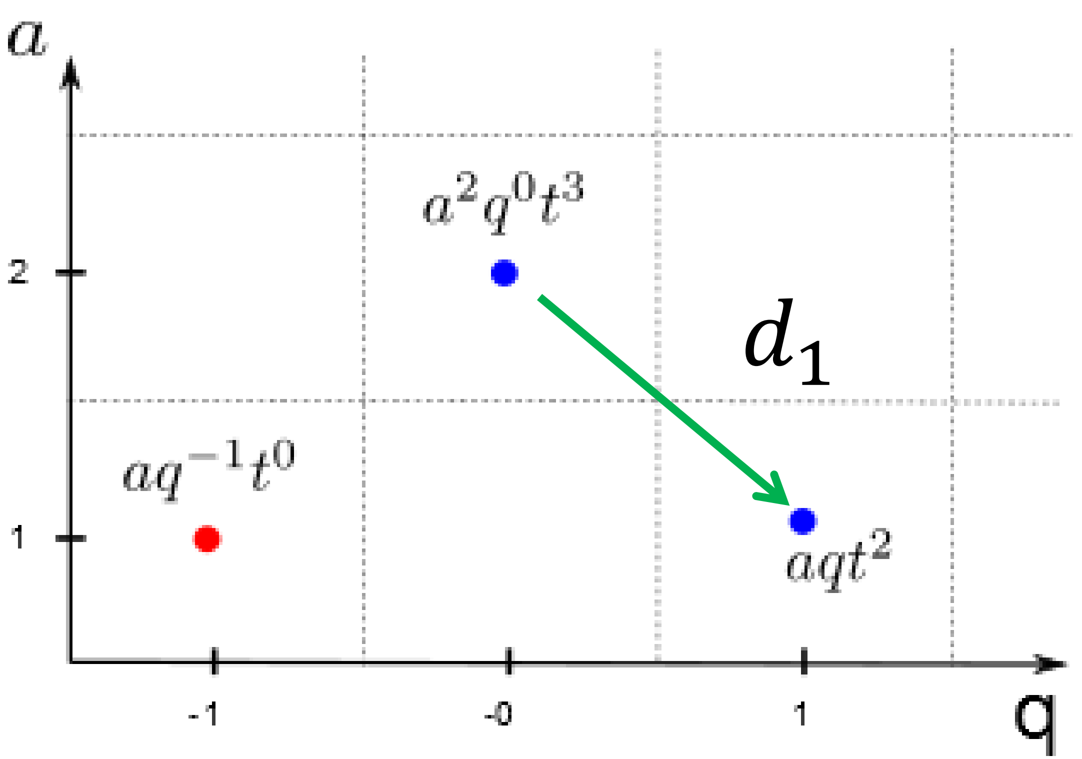

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$$(\mathcal{H}^{\square}(\mathbf{3_1}), d_1)$$

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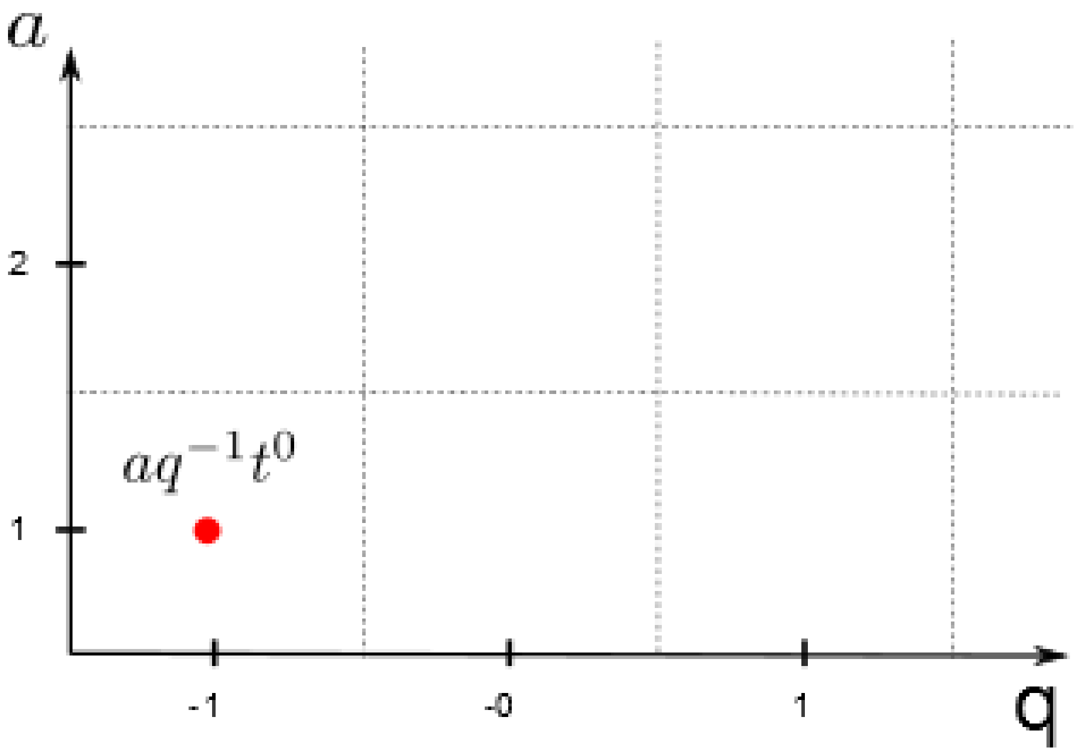

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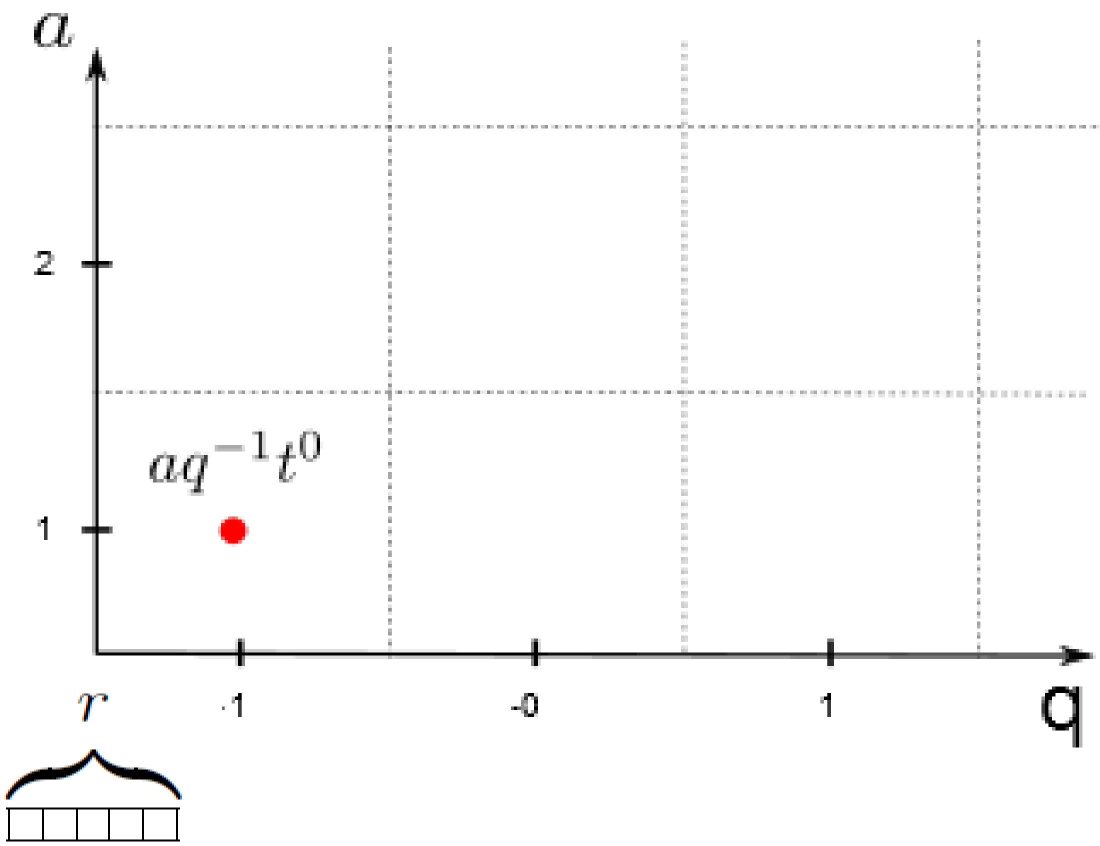

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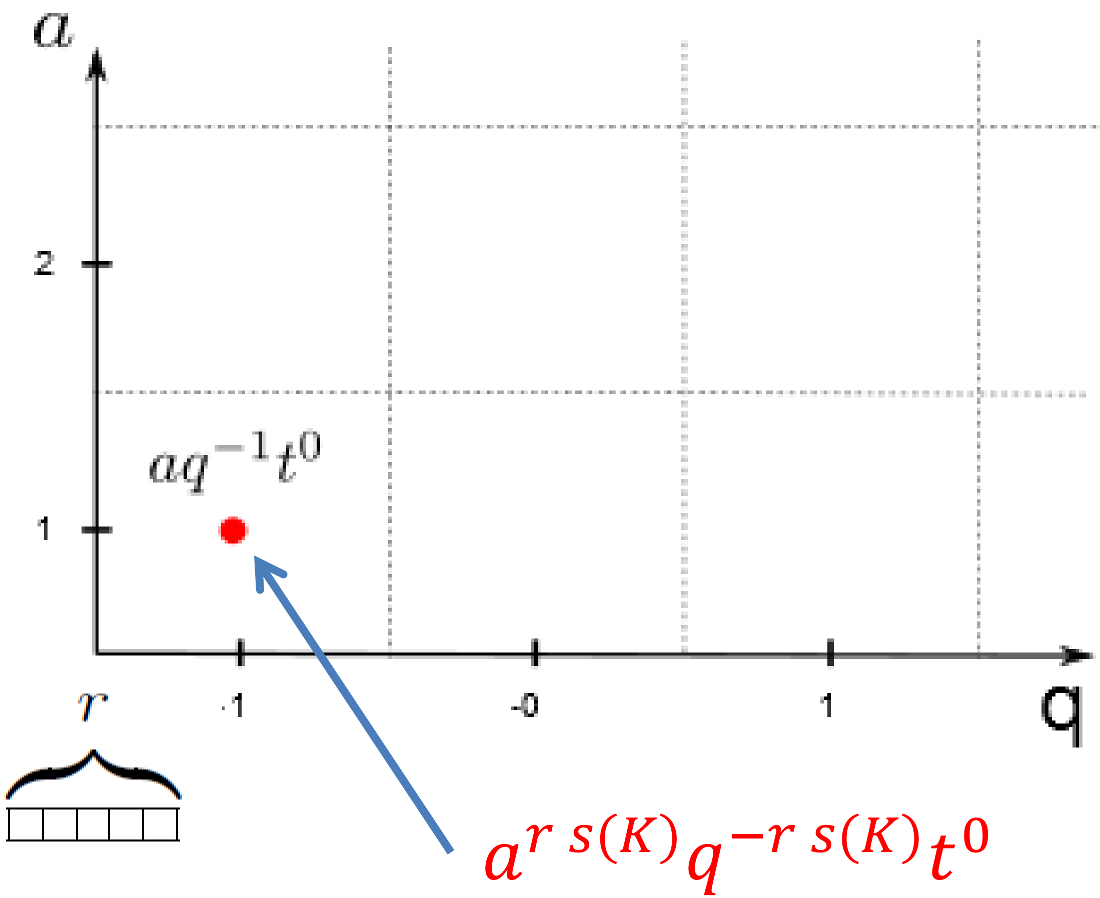

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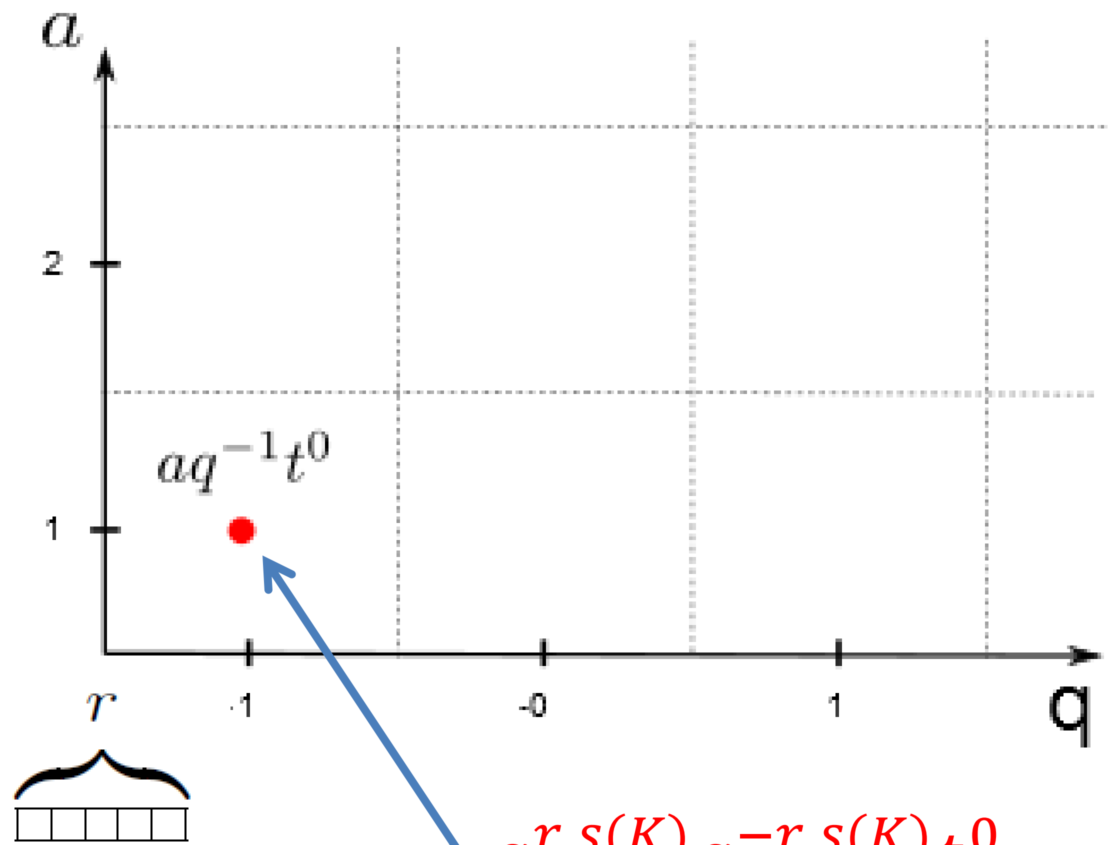
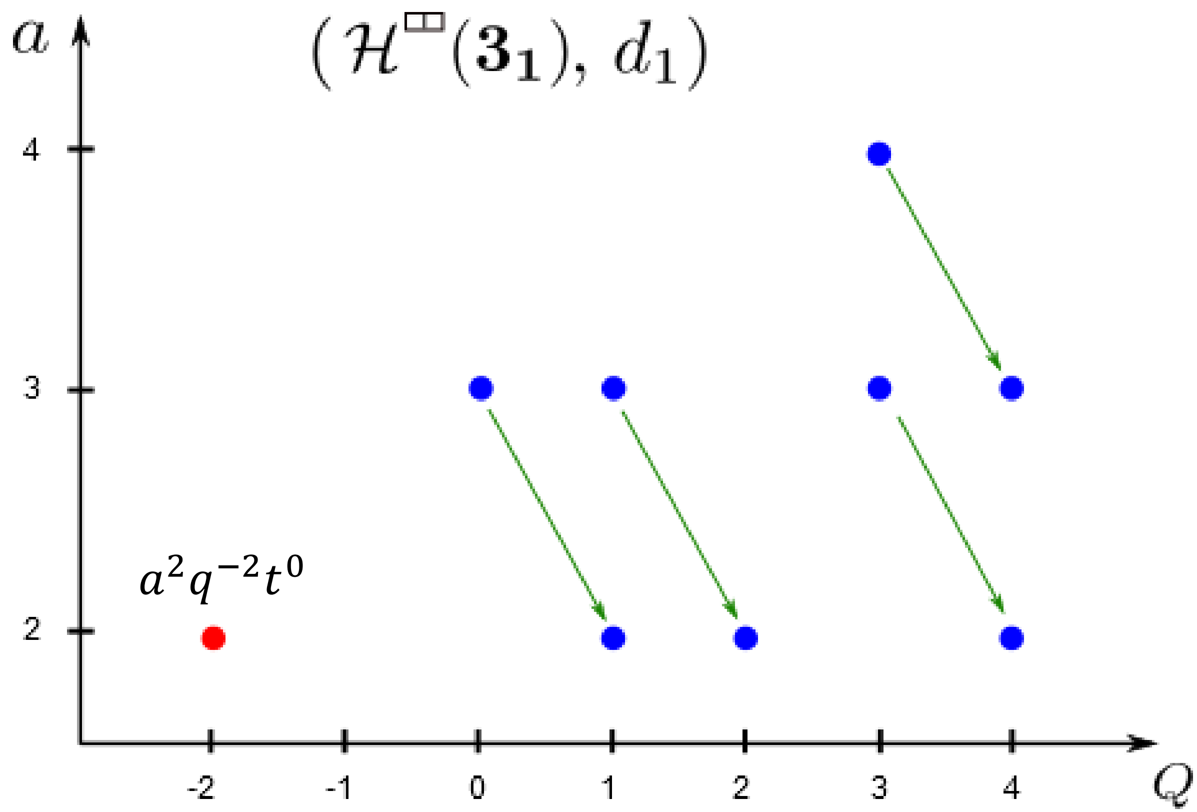


$$a^{r\,s(K)}q^{-r\,s(K)}t^0$$

S(K): Rasmussen invariant

$$
\mathcal{P}_{\square\square}(\mathbf{3}_1) = a^2(q^{-2} + qt^2 + q^2t^2 + q^4t^4) + a^3(t^3 + qt^3 + q^3t^5 + q^4t^5) + a^4q^3t^6
$$



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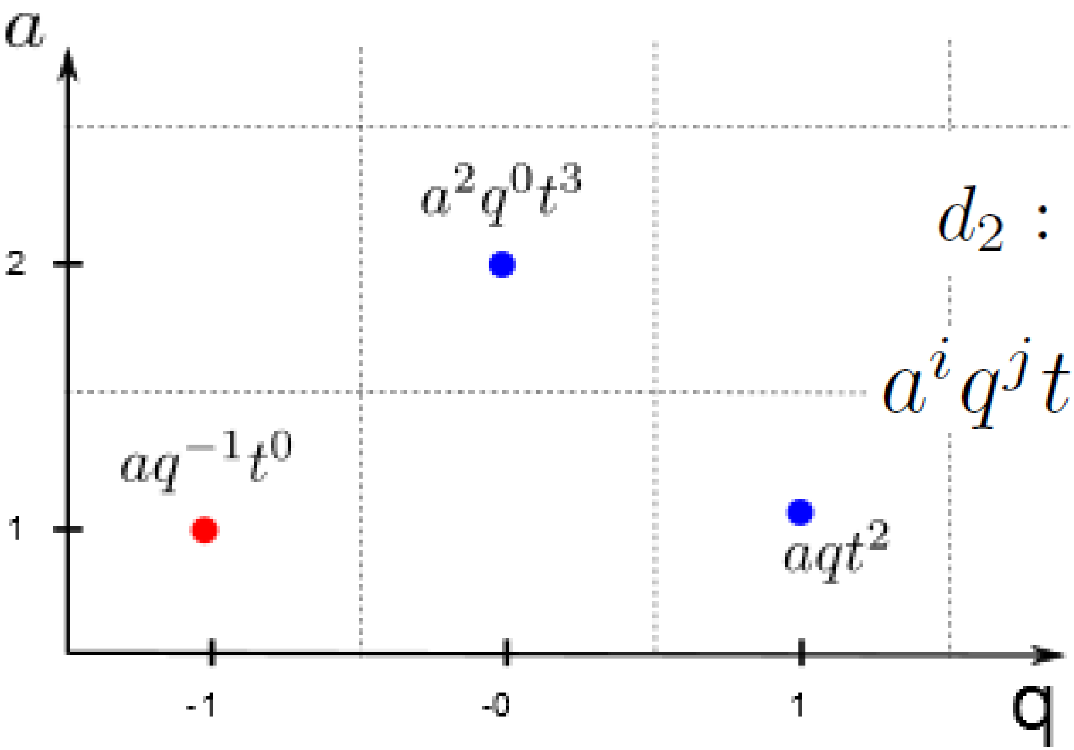

$$d_2 : \mathcal{H}^{\square}_{i,j,k} \longrightarrow \mathcal{H}^{\square}_{i-1,j+2,k-1}$$

$$a^i q^j t^k \longrightarrow a^{i-1} q^{j+2} t^{k-1}$$

Put a $= q^2$

Sl(2) homological invariant

Colored Jones polynomials for Twist Knots

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Trefoil $J_n(K_1;q) = \sum_{k=0}^{\infty} q^k \left(q^{1-n};q\right)_k (q^{1+n};q)_k$ [Habiro '00]

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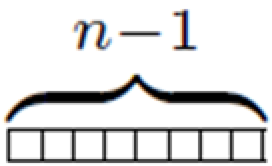
$n \equiv$ 

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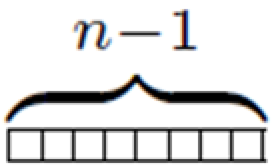
$n \equiv$ 

$$(a,q)_k = \prod_{i=0}^{k-1}(1-aq^k)$$

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Trefoil $$J_n(K_1;q) = \sum_{k=0}^{\infty} q^k \underbrace{(q^{1-n};q)_k (q^{1+n};q)_k}_{f_{n,k}}$$ [Habiro '00]

$$n \equiv \overbrace{\square\square\square\square\square\square\square\square}^{n-1}$$

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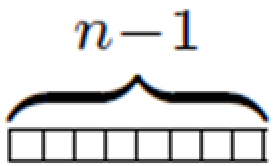
$n \equiv$ 

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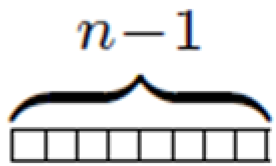
$n \equiv$ 

For K_p, p>0

$$J_n(K_{p>0};q) = \sum_{s_p \geq \cdots \geq s_1 \geq 0}^{\infty} q^{s_p} (q^{1-n};q)_{s_p} (q^{1+n};q)_{s_p} \prod_{i=1}^{p-1} q^{s_i(s_i+1)} \begin{bmatrix} s_{i+1} \\ s_i \end{bmatrix}_q$$

[Masbaum '03]

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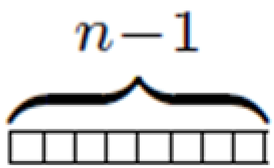
$n \equiv$ 

For K_p, p>0

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[Masbaum '03]

Colored Jones polynomials for Twist Knots

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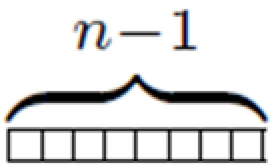
$n \equiv$ 

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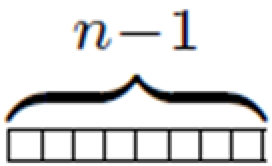
$n \equiv$ 

$$\begin{bmatrix} n \\ k \end{bmatrix}_q \equiv \frac{(q;q)_n}{(q;q)_k (q;q)_{n-k}}$$

For K_p, p>0

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[Masbaum '03]

Figure of eight knot

$$J_n(K_{-1};q) = \sum_{k=0}^{\infty} \underbrace{(-1)^k q^{-\frac{k(k+1)}{2}} (q^{1-n};q)_k (q^{1+n};q)_k}_{g_{n,k}}$$

[Habiro '00]

For K_p, p<0

$$J_n(K_{p<0};q) = \sum_{s_{|p|}\geq\cdots\geq s_1\geq 0}^{\infty} \overbrace{(-1)^{s_{|p|}} q^{-\frac{s_{|p|}(s_{|p|}+1)}{2}} (q^{1-n};q)_{s_{|p|}} (q^{1+n};q)_{s_{|p|}}}^{g_{n,s_p}}$$

$$\times \underbrace{\prod_{i=1}^{|p|-1} q^{-s_i(s_{i+1}+1)} \begin{bmatrix} s_{i+1} \\ s_i \end{bmatrix}_q}_{\text{Twisting factor}}$$

[Masbaum '03]

Colored superpolynomials for Twist Knots

Trefoil

Colored superpolynomials for Twist Knots

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[Fuji-Gukov-SulKowski '12]

$$\mathcal{P}_n(K_1; a, q, t) = (-t)^{-n+1} \sum_{k=0}^{\infty} q^k \frac{(-atq^{-1}; q)_k}{(q; q)_k} (q^{1-n}; q)_k (-at^3 q^{n-1}; q)_k$$

Colored superpolynomials for Twist Knots

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[Fuji-Gukov-SulKowski '12]

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[Itoyama-Mironov-Morozov-Morozov '12]

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Colored superpolynomials for Twist Knots

$\mathcal{P}_2(\mathbf{5_2};a,q,t)$	$aq^{-1}+at+aqt^2+a^2q^{-1}t^2+a^2t^3+a^2qt^4+a^3t^5$
$\mathcal{P}_3(\mathbf{5_2};a,q,t)$	$a^2q^{-2}+(a^2q^{-1}+a^2)t+(2a^2q+a^2q^2+a^3q^{-2}+a^3q^{-1})t^2$ $+(a^2q^2+a^2q^3+2a^3+2a^3q)t^3+(a^2q^4+2a^3q+3a^3q^2+a^3q^3+a^4)t^4$ $+(2a^3q^3+2a^3q^4+a^4+2a^4q+a^4q^2)t^5$ $+(a^3q^4+a^3q^5+a^4q^2+3a^4q^3+a^4q^4)t^6$ $+(a^4q^3+2a^4q^4+a^4q^5+a^5q^2+a^5q^3)t^7$ $+(a^4q^6+a^5q^3+a^5q^4)t^8+(a^5q^5+a^5q^6)t^9+a^6q^5t^{10}$
$\mathcal{P}_2(\mathbf{6_1};a,q,t)$	$a^{-2}t^{-4}+a^{-1}q^{-1}t^{-3}+a^{-1}t^{-2}+q^{-1}t^{-1}+a^{-1}qt^{-1}+2+qt+at^2$
$\mathcal{P}_3(\mathbf{6_1};a,q,t)$	$a^{-4}q^{-4}t^{-8}+(a^{-3}q^{-5}+a^{-3}q^{-4})t^{-7}+(a^{-2}q^{-5}+a^{-3}q^{-3}+a^{-3}q^{-2})t^{-6}$ $+(a^{-2}q^{-4}+2a^{-2}q^{-3}+a^{-3}q^{-2}+a^{-2}q^{-2}+a^{-3}q^{-1})t^{-5}$ $+(a^{-2}+a^{-1}q^{-4}+a^{-2}q^{-3}+a^{-1}q^{-3}+3a^{-2}q^{-2}+2a^{-2}q^{-1})t^{-4}$ $+(2a^{-2}+2a^{-1}q^{-3}+3a^{-1}q^{-2}+a^{-2}q^{-1}+a^{-1}q^{-1}+qa^{-2})t^{-3}$ $+(4a^{-1}+q^{-3}+a^{-1}q^{-2}+4a^{-1}q^{-1}+qa^{-2}+qa^{-1})t^{-2}$ $+(1+2a^{-1}+2q^{-2}+3q^{-1}+3qa^{-1}+q^2a^{-1})t^{-1}$ $+\left(5+q^{-1}+2q+a^{-1}q^2+a^{-1}q^3\right)+\left(a+q^{-1}a+2q+3q^2+q^3\right)t$ $+\left(a+2aq+aq^2+q^3\right)t^2+\left(aq^2+aq^3\right)t^3+a^2q^2t^4$

[Dunfield-Gukov-Rasmussen '05]

[Gukov-Stosic '12]

$5_2 knot \equiv K_2$ $6_1 knot \equiv K_{-2}$ $n=2,3\equiv \square, \square\square$

Colored superpolynomials for Twist Knots

$$\mathcal{P}_2(\mathbf{5_2}; a, q, t) = F_{2,0}(a, q, t) + (1 + at^2) F_{2,1}(a, q, t)$$

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$$\mathcal{P}_n(\mathbf{5_2}; a, q, t) = \sum_{s_2 \geq s_1 \geq 0}^{n-1} F_{n,s_2}(a,q,t)\ (at^2)^{s_1} q^{s_1(s_1-1)} \begin{bmatrix} s_2 \\ s_1 \end{bmatrix}_q$$

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Twisting factor for K_p, p>0 ⇒ $\prod_{i=1}^{p-1} (at^2)^{s_i} q^{s_i(s_i-1)} \begin{bmatrix} s_{i+1} \\ s_i \end{bmatrix}_q$

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Twisting factor for K_p, p<0 $\Rightarrow$ $\prod_{i=1}^{|p|-1}(at^2)^{-s_i}q^{-s_i(s_{i+1}-1)}\begin{bmatrix} s_{i+1} \\ s_i \end{bmatrix}_q$

Our Conjecture

For K_P, p>0

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For K_p, p<0

$$\mathcal{P}_n(K_{p<0};a,q,t) = \sum_{s_{|p|} \geq \cdots \geq s_1 \geq 0}^{\infty} (-at^2)^{-s_{|p|}} q^{-s_{|p|}(s_{|p|}-3)/2} \frac{(-atq^{-1};q)_{s_{|p|}}}{(q;q)_{s_{|p|}}} (q^{1-n};q)_{s_{|p|}} (-at^3q^{n-1};q)_{s_{|p|}} \times \prod_{i=1}^{|p|-1} (at^2)^{-s_i} q^{-s_i(s_{i+1}-1)} \begin{bmatrix} s_{i+1} \\ s_i \end{bmatrix}_q$$

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Convert multiple summation to double summation using Bailey chains

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$$\mathcal{P}_n(K_{p<0};a,q,t) = \sum_{k=0}^{\infty}\sum_{\ell=0}^{k} q^k \frac{(-atq^{-1};q)_k}{(q;q)_k}(q^{1-n};q)_k(-at^3q^{n-1};q)_k \times(-1)^\ell (at^2)^{p\ell} q^{(p+1/2)\ell(\ell-1)} \frac{1-at^2q^{2\ell-1}}{(at^2q^{\ell-1};q)_{k+1}} \begin{bmatrix} k \\ \ell \end{bmatrix}_q$$

Checks for the conjecture

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$SU(N)$ Chern-Simons theory → colored HOMFLY polynomials
Twist knots upto 10 crossings for n=2,3,4

Checks for the conjecture

Kawagoe [arXiv:1210.7574 **[math.GT]**]

linear skein relation → colored HOMFLYPT polynomials for twist knots colored by symmetric representation

This matches with the $t=-1$ limit of our formula.

Checks for the conjecture

Action of d_1

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$$\mathcal{P}_{n+1}(K_{p>0};a,q,t) = a^n q^{-n} + (1+a^{-1}qt^{-1})Q^{\mathfrak{sl}_1}_{n+1}(K_{p>0};a,q,t)$$

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$$a^{n\,s(K_{p<0})}q^{-n\,s(K_{p<0})}t^0 \qquad s(K_{p<0}) = 0$$

Checks for the conjecture

$$\mathcal{P}_n(K; a = q^2, t) = \mathcal{P}_n^{\mathfrak{sl}_2}(K; q, t)$$

Knot	$\mathcal{P}_{n=2}^{\mathfrak{sl}_2}(K; q, t)$
4_1	$q^2t^2 + \frac{1}{q^2t^2} + qt + \frac{1}{qt} + 1$
5_2	$q + q^2t + 2q^3t^2 + q^4t^3 + q^5t^4 + q^6t^5$
6_1	$\frac{1}{q^4t^4} + \frac{1}{q^3t^3} + q^2t^2 + \frac{1}{q^2t^2} + qt + \frac{2}{qt} + 2$
7_2	$q + q^2t + 2q^3t^2 + 2q^4t^3 + 2q^5t^4 + q^6t^5 + q^7t^6 + q^8t^7$
8_1	$\frac{1}{q^6t^6} + \frac{1}{q^5t^5} + \frac{1}{q^4t^4} + \frac{2}{q^3t^3} + q^2t^2 + \frac{2}{q^2t^2} + qt + \frac{2}{qt} + 2$
9_2	$q + q^2t + 2q^3t^2 + 2q^4t^3 + 2q^5t^4 + 2q^6t^5 + 2q^7t^6 + q^8t^7 + q^9t^8 + q^{10}t^9$
10_1	$\frac{1}{q^8t^8} + \frac{1}{q^7t^7} + \frac{1}{q^6t^6} + \frac{2}{q^5t^5} + \frac{2}{q^4t^4} + \frac{2}{q^3t^3} + q^2t^2 + \frac{2}{q^2t^2} + qt + \frac{2}{qt} + 2$

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$\mathbf{6_1}$	$\frac{1}{q^4t^4} + \frac{1}{q^3t^3} + q^2t^2 + \frac{1}{q^2t^2} + qt + \frac{2}{qt} + 2$
$\mathbf{7_2}$	$q + q^2t + 2q^3t^2 + 2q^4t^3 + 2q^5t^4 + q^6t^5 + q^7t^6 + q^8t^7$
$\mathbf{8_1}$	$\frac{1}{q^6t^6} + \frac{1}{q^5t^5} + \frac{1}{q^4t^4} + \frac{2}{q^3t^3} + q^2t^2 + \frac{2}{q^2t^2} + qt + \frac{2}{qt} + 2$
$\mathbf{9_2}$	$q + q^2t + 2q^3t^2 + 2q^4t^3 + 2q^5t^4 + 2q^6t^5 + 2q^7t^6 + q^8t^7 + q^9t^8 + q^{10}t^9$
$\mathbf{10_1}$	$\frac{1}{q^8t^8} + \frac{1}{q^7t^7} + \frac{1}{q^6t^6} + \frac{2}{q^5t^5} + \frac{2}{q^4t^4} + \frac{2}{q^3t^3} + q^2t^2 + \frac{2}{q^2t^2} + qt + \frac{2}{qt} + 2$

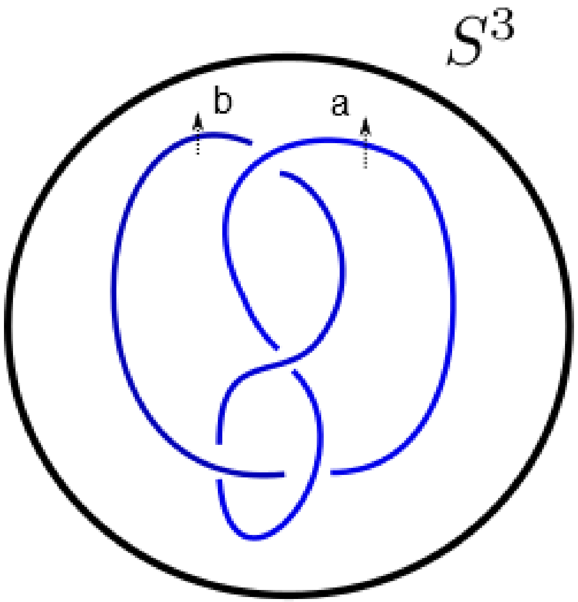
Match with results obtained by the Mathematica Package `KnotTheory``

Classical A-polynomial

Knot complement ⟹ $S^3\backslash \mathrm{K}$

Knot group $\pi_1(K)$ ⟹ fundamental group of $S^3\backslash \mathrm{K}$

Classical A-polynomial



Knot complement ⟹ $S^3 \backslash \mathrm{K}$

Knot group $\pi_1(K)$ ⟹ fundamental group of $S^3 \backslash \mathrm{K}$

Example: Figure-eight knot

$$\pi_1(K) =< a, b; bab^{-1}a^{-1}ba^{-1}b^{-1}aba^{-1} = 1 >$$

Classical A-polynomial

Representation of the fundamental group in $SL(2,\mathbb{C})$

$$\rho: \pi_1(K) \to SL(2,\mathbb{C})$$

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$$bab^{-1}a^{-1}ba^{-1}b^{-1}aba^{-1} = 1$$

$$d^2 + d(3 - x^2 - x^{-2}) + 3 - x^2 - x^{-2} = 0$$

$$d = \frac{1}{2}(x^2 + x^{-2} - 3 \pm \sqrt{(x^2 + x^{-2} + 1)(x^2 + x^{-2} - 3)}$$

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Longitude l $$l = ab^{-1}aba^{-2}bab^{-1}a^{-1}$$

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Classical A-polynomial

$$A(\mathbf{4_1};x,y) = y^2 + (x^2 - x - 2 - x^{-1} + x^{-2})y + 1$$

Volume conjecture

[Kashaev '95]
[H. Murakami, J. Murakami '01]

$$\lim_{n\to\infty} \frac{2\pi}{n} \log \left| J_n(K; q = e^{\frac{2\pi i}{n}}) \right| = \mathrm{Vol}(S^3 \backslash K)$$

Generalized volume conjecture

[Gukov '05]
[Murakami]

$$n \longrightarrow \infty$$

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Integration done the zero locus of the A-polynomial: A(K;x,y)=0

Generalized volume conjecture [Gukov '05] [Murakami]

$$\log y = -x \frac{d}{dx} \left[\lim_{\substack{n \to \infty, q = e^{\hbar} \to 1 \\ x = e^{n\hbar} = \text{fixed}}} \log J_n(K; q = e^{\hbar}) \right]$$

⇓

A(K;x,y)=0

Super-A-polynomials

[Fuji-Gukov-Sulkowski –Awata '12]

Asymptotic behavior
of $\mathcal{P}_n(K;a,q,t)$

Super-A-polynomials

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Asymptotic behavior of $\mathcal{P}_n(K;a,q,t)$ ⟹ Classical-super-A-polynomial $A^{\text{super}}(K;x,y;a,t)$

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$$n \longrightarrow \infty \qquad q = e^{\hbar} \longrightarrow 1 \qquad x, a, t = \text{fixed}$$

$$\lim_{\substack{n \longrightarrow \infty \\ \hbar \longrightarrow 0}} \mathcal{P}_n(K;a,q=e^{\hbar},t) = \exp\left(\frac{1}{\hbar}\int_1^x \frac{dx}{x}\log y + \ldots\right)$$

$$A^{\text{super}}(K;x,y;a,t) = 0$$

Classical super-A-polynomials

$$\mathcal{P}_n(K_{p>0};a,q,t) = (-t)^{-n+1}\sum_{k=0}^{\infty}\sum_{\ell=0}^{k} q^k \frac{(-atq^{-1};q)_k}{(q;q)_k}(q^{1-n};q)_k(-at^3q^{n-1};q)_k$$

$$\times(-1)^{\ell}(at^2)^{p\ell}q^{(p+1/2)\ell(\ell-1)}\frac{1-at^2q^{2\ell-1}}{(at^2q^{\ell-1};q)_{k+1}}\begin{bmatrix} k \\ \ell \end{bmatrix}_q ,$$

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$$z = e^{\hbar k} \qquad w = e^{\hbar \ell}$$

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$$z = e^{\hbar k} \qquad w = e^{\hbar \ell}$$

$$(\lambda;q)_k \sim e^{\frac{1}{\hbar}\left(\mathrm{Li}_2(\lambda) - \mathrm{Li}_2(\lambda q^k)\right)}$$

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$$\mathcal{P}_n(K_p;a,q,t) \sim \int dz dw \; e^{\frac{1}{\hbar}\left(\widetilde{\mathcal{W}}(K_p;z,w)+\mathcal{O}(\hbar)\right)}$$

Classical super-A-polynomials

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$$\begin{aligned}\widetilde{\mathcal{W}}(K_{p>0};z,w) \;=\; & -\log x\log(-t)-\frac{\pi^2}{3}+i\pi\log w+p(\log a+2\log t)\log w+\left(p+\frac{1}{2}\right)(\log w)^2\\ & +\mathrm{Li}_2(x^{-1})-\mathrm{Li}_2(x^{-1}z)+\mathrm{Li}_2(-at)-\mathrm{Li}_2(-atz)+\mathrm{Li}_2(-at^3x)\\ & -\mathrm{Li}_2(-at^3xz)-\mathrm{Li}_2(at^2w)+\mathrm{Li}_2(at^2wz)+\mathrm{Li}_2(w)+\mathrm{Li}_2(zw^{-1})\end{aligned}$$

Classical super-A-polynomials

$$\widetilde{\mathcal{W}}(K_{p<0};z,w) = -\frac{\pi^2}{3} + i\pi \log w + p(\log a + 2\log t)\log w + \left(p+\frac{1}{2}\right)(\log w)^2$$
$$+\mathrm{Li}_2(x^{-1}) - \mathrm{Li}_2(x^{-1}z) + \mathrm{Li}_2(-at) - \mathrm{Li}_2(-atz) + \mathrm{Li}_2(-at^3x)$$
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$$
\begin{aligned}
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\end{aligned}
$$

The leading behavior with respect to $\hbar$ comes from the saddle points

Classical super-A-polynomials

$$
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\end{aligned}
$$

The leading behavior with respect to $\hbar$ comes from the saddle points

$$
\left.\frac{\partial \widetilde{\mathcal{W}}(K_p;z,w,x)}{\partial z}\right|_{(z,w)=(z_0,w_0)} = 0 = \left.\frac{\partial \widetilde{\mathcal{W}}(K_p;z,w,x)}{\partial w}\right|_{(z,w)=(z_0,w_0)}
$$

Classical super-A-polynomials

$$\widetilde{\mathcal{W}}(K_{p<0}; z, w) = -\frac{\pi^2}{3} + i\pi \log w + p(\log a + 2\log t)\log w + \left(p + \frac{1}{2}\right)(\log w)^2$$
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The leading behavior with respect to $\hbar$ comes from the saddle points

$$\left.\frac{\partial\widetilde{\mathcal{W}}(K_p; z, w, x)}{\partial z}\right|_{(z,w)=(z_0,w_0)} = 0 = \left.\frac{\partial\widetilde{\mathcal{W}}(K_p; z, w, x)}{\partial w}\right|_{(z,w)=(z_0,w_0)}$$

super-A-polynomials $\quad y = \exp\left(x\frac{\partial\widetilde{\mathcal{W}}(K_p; z_0, w_0, x)}{\partial x}\right)$

Classical super-A-polynomials

For p>0

$$1 = \frac{w_0(x - z_0)(1 + atz_0)(1 + at^3xz_0)}{x(w_0 - z_0)(1 - at^2w_0z_0)} ,$$

$$1 = -\frac{a^p t^{2p} w_0^{2p}(w_0 - z_0)(1 - at^2w_0)}{(1 - w_0)(1 - at^2w_0z_0)} ,$$

$$y = -\frac{(x - 1)(1 + at^3xz_0)}{t(x - z_0)(1 + at^3x)} ,$$

For p<0

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Classical super-A-polynomials

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$$y = \frac{(x-1)(1+at^3xz_0)}{(x-z_0)(1+at^3x)}$$

Eliminate z_0 and w_0 to find classical super-A-polynomial

Classical super-A-polynomials

Knot	$A^{\text{super}}(K;x,y;a,t)$
5_2	y^4 $-\frac{a}{1+at^3x}(2-x+tx-2t^2x+3t^2x^2+at^2x^2+4at^3x^2-2at^3x^3+2at^4x^3+2at^5x^3-at^5x^4+2a^2t^5x^4+2a^2t^6x^4-a^2t^6x^5+a^2t^7x^5+a^3t^8x^6)y^3$ $-\frac{a^2(-1+x)}{(1+at^3x)^2y^4}(1+tx-2t^2x+2t^2x^2-2t^3x^2+4at^3x^2+t^4x^2-3t^4x^3+at^4x^3-2at^5x^3+4at^5x^4-4at^6x^4+6a^2t^6x^4-4at^7x^4+3at^7x^5-a^2t^7x^5+2a^2t^8x^5+2a^2t^8x^6-2a^2t^9x^6+4a^3t^9x^6+a^2t^{10}x^6-a^3t^{10}x^7+2a^3t^{11}x^7+a^4t^{12}x^8)y^2$ $+\frac{a^3t^3(-1+x)^2x^2}{(1+at^3x)^3}(1+tx-t^2x-t^3x^2+2at^3x^2+2at^4x^2+2at^4x^3-2at^5x^3-2at^6x^3+3at^6x^4+a^2t^6x^4+4a^2t^7x^4+a^2t^7x^5-a^2t^8x^5+2a^2t^9x^5+2a^3t^{10}x^6)y$ $-\frac{a^5t^{11}(-1+x)^3x^7}{(1+at^3x)^3}$

At t=-1 a=1, it reduces to the classical A-polynomial

Classical super-A-polynomials

Knot	$A^{\text{super}}(K;x,y;a,t)$
$\mathbf{5_2}$	y^4 $-\frac{a}{1+at^3x}(2-x+tx-2t^2x+3t^2x^2+at^2x^2+4at^3x^2-2at^3x^3+2at^4x^3+2at^5x^3-at^5x^4+2a^2t^5x^4+2a^2t^6x^4-a^2t^6x^5+a^2t^7x^5+a^3t^8x^6)y^3$ $-\frac{a^2(-1+x)}{(1+at^3x)^2y^4}(1+tx-2t^2x+2t^2x^2-2t^3x^2+4at^3x^2+t^4x^2-3t^4x^3+at^4x^3-2at^5x^3+4at^5x^4-4at^6x^4+6a^2t^6x^4-4at^7x^4+3at^7x^5-a^2t^7x^5+2a^2t^8x^5+2a^2t^8x^6-2a^2t^9x^6+4a^3t^9x^6+a^2t^{10}x^6-a^3t^{10}x^7+2a^3t^{11}x^7+a^4t^{12}x^8)y^2$ $+\frac{a^3t^3(-1+x)^2x^2}{(1+at^3x)^3}(1+tx-t^2x-t^3x^2+2at^3x^2+2at^4x^2+2at^4x^3-2at^5x^3-2at^6x^3+3at^6x^4+a^2t^6x^4+4a^2t^7x^4+a^2t^7x^5-a^2t^8x^5+2a^2t^9x^5+2a^3t^{10}x^6)y$ $-\frac{a^5t^{11}(-1+x)^3x^7}{(1+at^3x)^3}$

At t=-1 a=1, it reduces to the classical A-polynomial

$$A^{super}(K;x,y;t=-1,a=1)=A(K;x,y)$$

Quantum A-polynomials

[Stavros '04][Gukov '04]

$$\hat{x} J_n(K;q) = q^n J_n(K;q) \qquad \hat{y} J_n(K;q) = J_{n+1}(K;q)$$

Quantum A-polynomials

[Stavros '04][Gukov '04]

$$\hat{x} J_n(K;q) = q^n J_n(K;q) \qquad \hat{y} J_n(K;q) = J_{n+1}(K;q)$$

Quantum A-polynomials ⇒

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Quantum A-polynomials $\Rightarrow$ $\widehat{A}(K;\hat{x},\hat{y};q) J_n(K;q) = 0$

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$$b_k(\hat{x},q) J_{n+k}(K;q) + \cdots + b_0(\hat{x},q) J_n(K;q) = 0$$

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$$b_k(\hat{x},q) J_{n+k}(K;q) + \cdots + b_0(\hat{x},q) J_n(K;q) = 0$$

$$\widehat{A}(K;\hat{x},\hat{y};q) = \sum_{j=0}^{k} b_j(\hat{x},q) \hat{y}^j$$

Quantum A-polynomials

[Stavros '04][Gukov '04]

$$\hat{x} J_n(K;q) = q^n J_n(K;q) \qquad \hat{y} J_n(K;q) = J_{n+1}(K;q)$$

Quantum A-polynomials $\Rightarrow$ $\widehat{A}(K;\hat{x},\hat{y};q) J_n(K;q) = 0$

$$b_k(\hat{x},q) J_{n+k}(K;q) + \cdots + b_0(\hat{x},q) J_n(K;q) = 0$$

$$\widehat{A}(K;\hat{x},\hat{y};q) = \sum_{j=0}^{k} b_j(\hat{x},q)\hat{y}^j$$

$$\widehat{A}(K;\hat{x},\hat{y};q=1) = A(K;x,y)$$

Quantum super-A-polynomials

[Fuji-Gukov-Sulkowski –Awata '12]

$$\hat{x}\mathcal{P}_n(K;a,q,t) = q^n \mathcal{P}_n(K;a,q,t)$$

$$\hat{y}\mathcal{P}_n(K;a,q,t) = \mathcal{P}_{n+1}(K;a,q,t)$$

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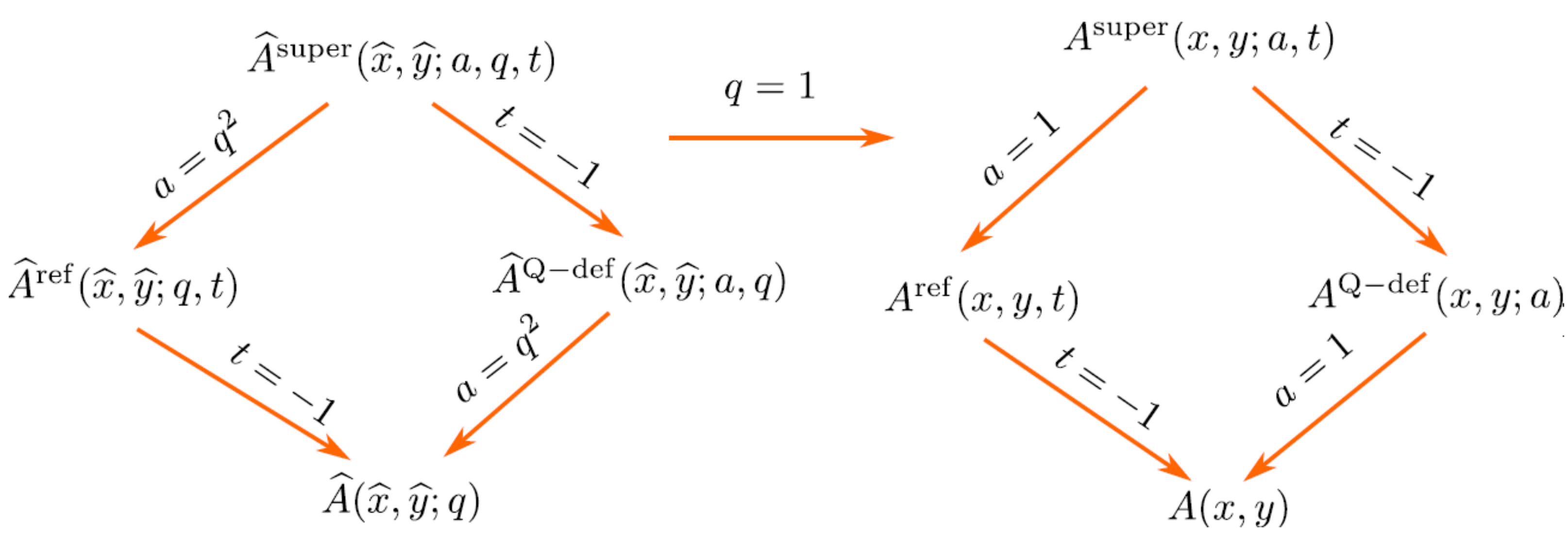

Quantum super-A-polynomials

Knot	$\widehat{A}^{\text{super}}(K;\hat{x},\hat{y};a,q,t)$
$\mathbf{5_2}$	$\hat{y}^4 - \frac{a(1+aq^4t^3\hat{x}^2)}{q(1+aq^2t^3\hat{x})(1+aqt^3\hat{x}^2)}(1+q-q^3\hat{x}+q^3t\hat{x}-q^2t^2\hat{x}-q^3t^2\hat{x}+q^4t^2\hat{x}^2+aq^4t^2\hat{x}^2+q^5t^2\hat{x}^2+q^6t^2\hat{x}^2+aqt^3\hat{x}^2+aq^2t^3\hat{x}^2+aq^5t^3\hat{x}^2+aq^6t^3\hat{x}^2-aq^4t^3\hat{x}^3-aq^8t^3\hat{x}^3+aq^4t^4\hat{x}^3+aq^8t^4\hat{x}^3+aq^5t^5\hat{x}^3+aq^6t^5\hat{x}^3+a^2q^5t^5\hat{x}^4-aq^8t^5\hat{x}^4+a^2q^9t^5\hat{x}^4+a^2q^6t^6\hat{x}^4+a^2q^7t^6\hat{x}^4-a^2q^9t^6\hat{x}^5+a^2q^9t^7\hat{x}^5+a^3q^{10}t^8\hat{x}^6)\hat{y}^3 - \frac{a^2(-1+q^2\hat{x})(1+aq^2t^3\hat{x}^2)(1+aq^5t^3\hat{x}^2)}{q(1+aqt^3\hat{x})(1+aq^2t^3\hat{x})(1+at^3\hat{x}^2)(1+aqt^3\hat{x}^2)}(1+q^2t\hat{x}-qt^2\hat{x}-q^2t^2\hat{x}+q^3t^2\hat{x}^2+q^4t^2\hat{x}^2+at^3\hat{x}^2+aqt^3\hat{x}^2-q^3t^3\hat{x}^2+aq^3t^3\hat{x}^2-q^4t^3\hat{x}^2+aq^4t^3\hat{x}^2+q^3t^4\hat{x}^2+aq^2t^4\hat{x}^3-q^4t^4\hat{x}^3-aq^4t^4\hat{x}^3-q^5t^4\hat{x}^3-q^6t^4\hat{x}^3+aq^6t^4\hat{x}^3-aqt^5\hat{x}^3-aq^2t^5\hat{x}^3+aq^3t^5\hat{x}^3+aq^4t^5\hat{x}^3-aq^5t^5\hat{x}^3-aq^6t^5\hat{x}^3+aq^3t^5\hat{x}^4+aq^4t^5\hat{x}^4+aq^7t^5\hat{x}^4+aq^8t^5\hat{x}^4+a^2qt^6\hat{x}^4-aq^3t^6\hat{x}^4+a^2q^3t^6\hat{x}^4-aq^4t^6\hat{x}^4+2a^2q^4t^6\hat{x}^4+a^2q^5t^6\hat{x}^4-aq^7t^6\hat{x}^4+a^2q^7t^6\hat{x}^4-aq^8t^6\hat{x}^4-aq^4t^7\hat{x}^4-2aq^5t^7\hat{x}^4-aq^6t^7\hat{x}^4-a^2q^4t^7\hat{x}^5+aq^6t^7\hat{x}^5+a^2q^6t^7\hat{x}^5+aq^7t^7\hat{x}^5+aq^8t^7\hat{x}^5-a^2q^8t^7\hat{x}^5+a^2q^3t^8\hat{x}^5+a^2q^4t^8\hat{x}^5-a^2q^5t^8\hat{x}^5-a^2q^6t^8\hat{x}^5+a^2q^7t^8\hat{x}^5+a^2q^8t^8\hat{x}^5+a^2q^7t^8\hat{x}^6+a^2q^8t^8\hat{x}^6+a^3q^4t^9\hat{x}^6+a^3q^5t^9\hat{x}^6-a^2q^7t^9\hat{x}^6+a^3q^7t^9\hat{x}^6-a^2q^8t^9\hat{x}^6+a^3q^8t^9\hat{x}^6+a^2q^7t^{10}\hat{x}^6-a^3q^8t^{10}\hat{x}^7+a^3q^7t^{11}\hat{x}^7+a^3q^8t^{11}\hat{x}^7+a^4q^8t^{12}\hat{x}^8)\hat{y}^2 + \frac{a^3qt^3\hat{x}^2(-1+q\hat{x})(-1+q^2\hat{x})(1+aq^4t^3\hat{x}^2)(1+aq^5t^3\hat{x}^2)}{(1+at^3\hat{x})(1+aqt^3\hat{x})(1+aq^2t^3\hat{x})(q+at^3\hat{x}^2)(1+aqt^3\hat{x}^2)}(q+q^2t\hat{x}-q^2t^2\hat{x}+at^3\hat{x}^2-q^3t^3\hat{x}^2+aq^4t^3\hat{x}^2+aqt^4\hat{x}^2+aq^2t^4\hat{x}^2+aqt^4\hat{x}^3+aq^5t^4\hat{x}^3-aqt^5\hat{x}^3-aq^5t^5\hat{x}^3-aq^2t^6\hat{x}^3-aq^3t^6\hat{x}^3+aq^3t^6\hat{x}^4+a^2q^3t^6\hat{x}^4+aq^4t^6\hat{x}^4+aq^5t^6\hat{x}^4+a^2t^7\hat{x}^4+a^2qt^7\hat{x}^4+a^2q^4t^7\hat{x}^4+a^2q^5t^7\hat{x}^4+a^2q^4t^7\hat{x}^5-a^2q^4t^8\hat{x}^5+a^2q^3t^9\hat{x}^5+a^2q^4t^9\hat{x}^5+a^3q^3t^{10}\hat{x}^6+a^3q^4t^{10}\hat{x}^6)\hat{y} - \frac{a^5q^2t^{11}(-1+\hat{x})\hat{x}^7(-1+q\hat{x})(-1+q^2\hat{x})(1+aq^3t^3\hat{x}^2)(1+aq^4t^3\hat{x}^2)(1+aq^5t^3\hat{x}^2)}{(1+at^3\hat{x})(1+aqt^3\hat{x})(1+aq^2t^3\hat{x})(1+at^3\hat{x}^2)(q+at^3\hat{x}^2)(1+aqt^3\hat{x}^2)}$

Colored Kauffman homology

Colored Kauffman homology for trefoil and figure of eight knot colored by symmetric representation

arXiv:1310.220

[Satoshi,Rama,Z '13]

Thank you