Knot theory for spatial graphs

[Lecture 2]
Unknotting notions on the spatial graphs

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References for this special topics

[1] A. Kawauchi, On a complexity of a spatial graph. in: Knots and soft-matter physics, Topology of polymers and related topics in physics, mathematics and biology, Bussei Kenkyu 92-1 (2009-4), 16-19.

[2] A. Kawauchi, On transforming a spatial graph into a plane graph,in: Statistical Physics and Topology of Polymers with Ramifications to Structure and Function of DNA and Proteins, Progress of Theoretical Physics Supplement, No. 191(2011), 235-244.

2.1. A based diagram and a monotone diagram

Let Γ be a graph without degree one vertices, and $G = G(\Gamma)$ a spatial graph in R^3. Let Γ_i (i=1,2,…,r) be an ordered set of the components of Γ, and $G_i = G(\Gamma_i)$ the corresponding spatial subgraph of $G = G(\Gamma)$. Let T_i be a maximal tree of G_i.

Note: We consider a topological graph without degree 2 verticies, so that $T_i = \phi$ if G_i is a knot or link, and T_i = one vertex if G_i has just one vertex (of degree $\geqq 3$).

Let $T = T_1 \cup T_2 \cup \ldots \cup T_r$. Call it a <u>base</u> of G.

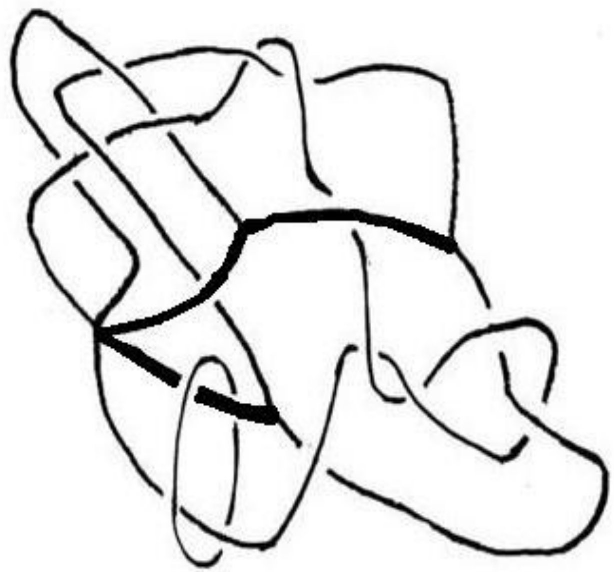
<u>Note:</u> There are only finitely many bases of G.

G is obtained from a basis T by attaching <u>edges</u> (i.e., arcs or loops) to T.

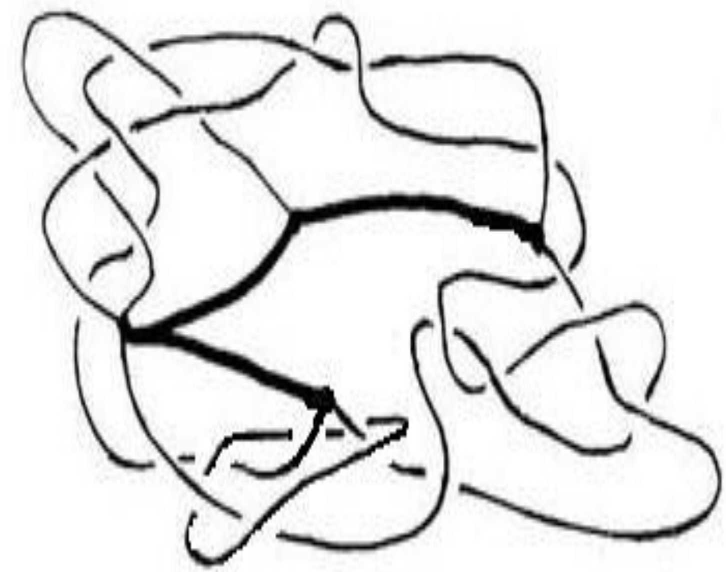
Let D be a diagram of a spatial graph G=G(Γ), and D_T the sub-diagram of D corresponding to T.

Let $c_D(D_T)$ be the number of crossing points of D whose upper or lower crossing points belong to D_T.

Definition. D is *a* based diagram (on base T), written as (D;T) if $c_D(D_T)=0$.

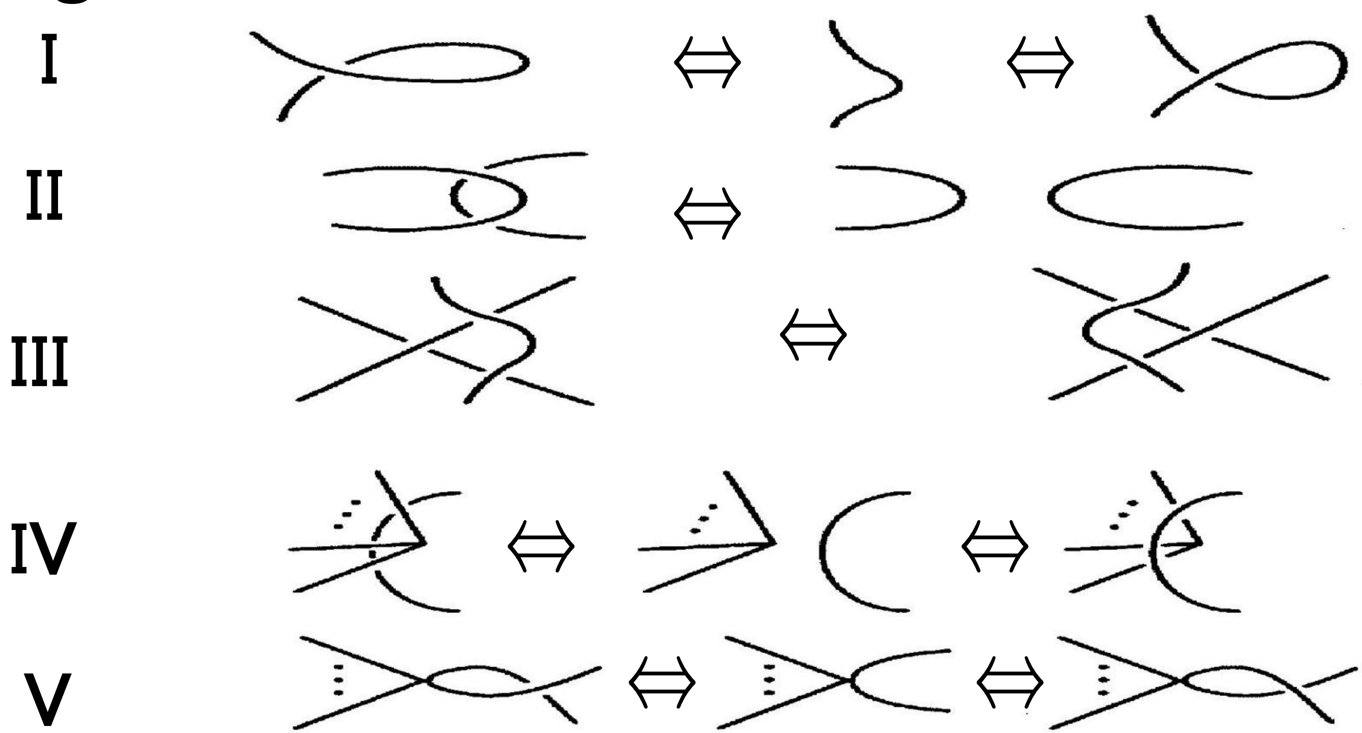


$\Rightarrow$



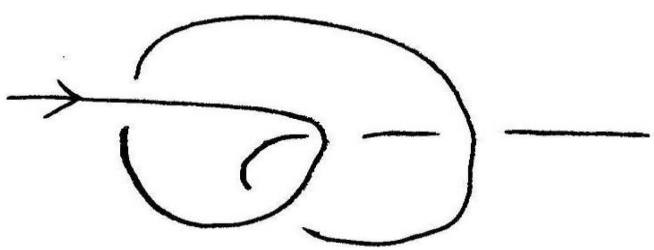
Lemma. **For ∀ base T of G, ∀ diagram D of G is deformed into a based diagram on T by generalized Reidemeister moves.**

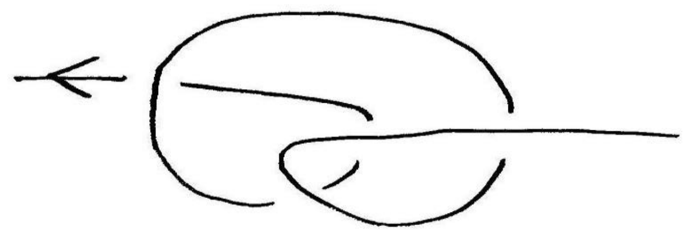
The generalirez Reidemeister moves:



Let α be an edge of G=G(Γ) attaching to a base T.

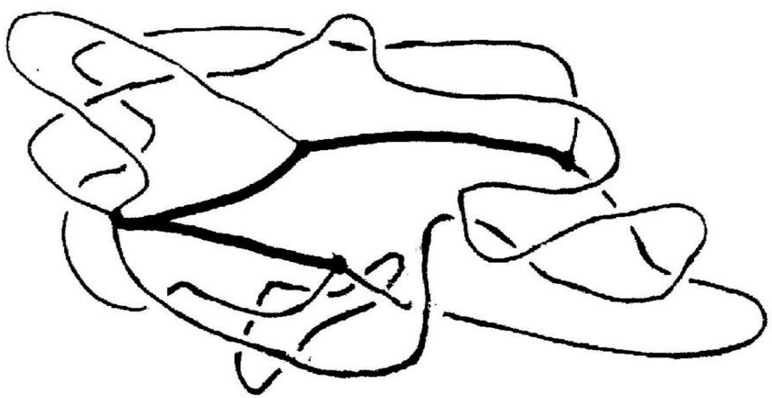
Definition. An edge diagram D_α in a diagram D of G is **monotone** if:

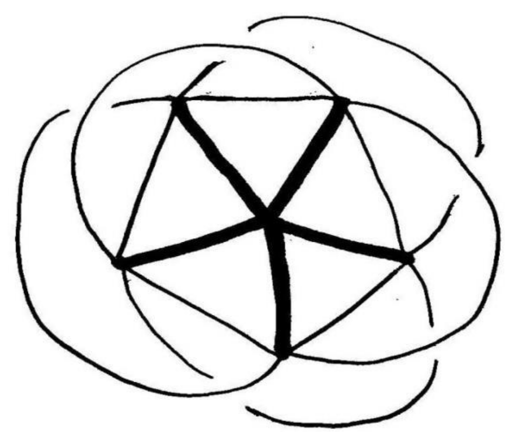




A sequence on the edges of a based graph (G ,T) is regularly ordered if an order on the edges such that any edge belonging to G_i is smaller than any edge belonging to G_j for $i<j$ is specified.

Definition. A based diagram (D;T) is monotone if there is a regularly ordered edge sequence α_i (i=1,2,⋯,m) of (G ,T) such that D_{α_i} is monotone and D_{α_i} is upper than D_{α_j} for i<j .





2.2. Complexity

Definition.

The warping degree d(D;T) of a based diagram (D;T) is the least number of crossing changes on edge diagrams attaching to T needed to obtain a monotone diagram from (D;T).

The crossing number of (D;T) is denoted by c(D;T).

If D is a knot or link diagram or an edge diagram, then the warping degree and crossing number of D are denoted by d(D) and c(D), respectively.

A similar notion for a knot or link is given in :

[Lickorish-Millett 1987] W. B. R. Lickorish and K. C. Millett, A polynomial invariant of oriented links, Topology 26(1987), 107-141.

[Fujimura 1988] S. Fujimura, On the ascending number of knots, thesis, Hiroshima University, 1988.

[Fung 1996] T. S. Fung, Immersions in knot theory, a dissertation, Columbia University, 1996.

[Kawauchi 2007] A. Kawauchi, Lectures on knot theory (in Japanese),
Kyoritu Shuppan, 2007.

[Ozawa 2010] M. Ozawa, Ascending number of knots and links.
J. Knot Theory Ramifications 19 (2010), 15-25.

[Shimizu 2010] A. Shimizu, The warping degree of a knot diagram,
J. Knot Theory Ramifications 19(2010), 849-857.

Properties of the warping degree

For the warping degree $\vec{d}$ of an *oriented* edge diagram D_α,

$$\vec{d}(D_\alpha) + \vec{d}(-D_\alpha) = c(D_\alpha),$$

$$d(D_\alpha) = \min\{\vec{d}(D_\alpha), \vec{d}(-D_\alpha)\}.$$

Example. **d(**  **) =1, for**

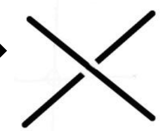
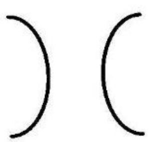
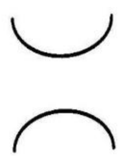
$\vec{d}$ ()=1, $\vec{d}$ ()=3.

Definition.

The complexity of a based diagram (D;T) is the pair cd(D;T)= (c(D;T), d(D;T)). The complexity of a spatial graph G is

$$\gamma(G) = \min\{cd(D;T) \mid (D;T) \in [D_G]\}$$

in the dictionary order. Let $\gamma(G)= (c_\gamma(G), d_\gamma(G))$.

Our basic viewpoint of complexity. This complexity is reducible by a crossing change  ⇔  or a splice  ⇒  or  until we obtain a graph in a plane.

2.3. The warping degree and an unknotted graph

Definition.

The warping degree of G is :

$$d(G)= \min\{d(D;T) \mid (D;T)\in[D_G]\}$$

Definition.

G is unknotted if $d(G)= 0$.

When Γ consists of loops,

G is unknotted ⇔ G is a trivial link.

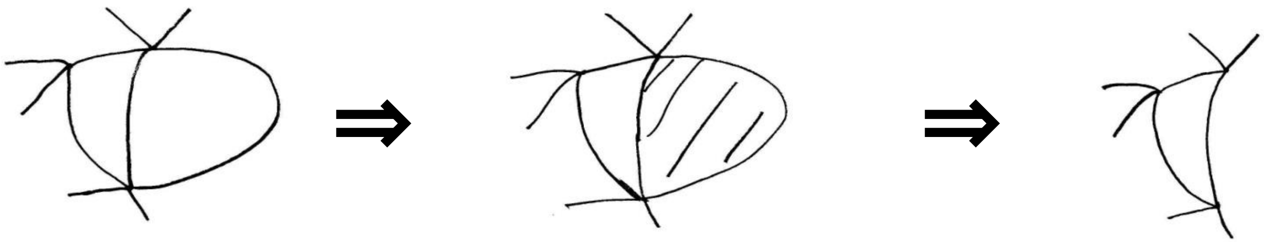
Assume Γ has a vertex of degree ≧ 3.

<u>Lemma 2.3.1</u>. For ∀ G, ∃ finitely many crossing changes on G to make G with d(G)=0.

<u>Lemma 2.3.2</u>. For ∀ given graph Γ, ∃ only finitely many G of Γ with d(G)=0 up to equivalences.

<u>Lemma 2.3.3</u>. If d(G)=0, then ∃ T such that G/T is equivalent to $S^1 \vee S^1 \vee \dots \vee S^1 \subset R^2$.

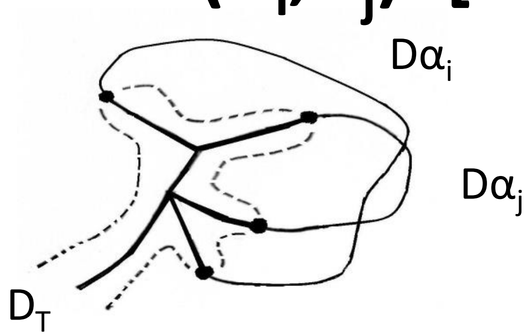
Lemma 2.3.4. A connected G with d(G)=0 is deformed into a basis T by a sequence of edge reductions:



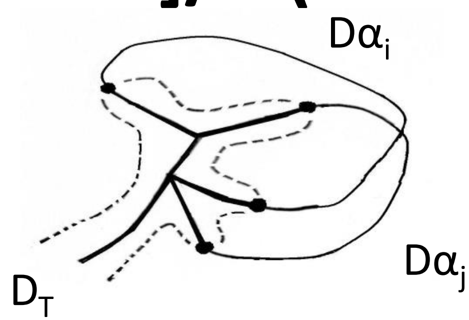
Corollary 2.3.5. For ∀ G with d(G)=0, ∃ T such that every edge (arc or loop) attaching to T is in a trivial constituent knot.

Given D_T, the <u>cross index</u> of α_i and α_j ($i \neq j$):

$$\varepsilon(\alpha_i, \alpha_j) = [1-(-1)^{\#(D\alpha i \cap D\alpha j)}]/2 \ (=0 \text{ or } 1).$$



cross index =0



cross index =1

The <u>total cross index</u> of Γ on D_T :

$$\varepsilon(\Gamma; D_T) = \sum_{i<j} \varepsilon(\alpha_i, \alpha_j).$$

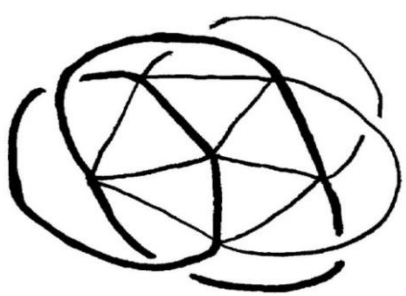
<u>Lemma 2.3.6.</u> Let $d(G)=0$. Then

$$\min\{c(D;T) \,|\, (D;T) \in [D_G],\ d(D;T)=0\} = \varepsilon(\Gamma; D_T).$$

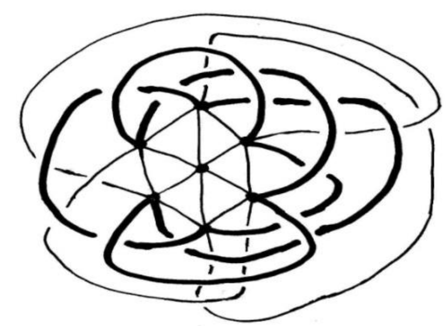
Conway-Gordon Theorem.

Every spatial 6-complete graph K_6 contains a non-trivial constituent link.

Every spatial 7-complete graph K_7 contains a non-trivial constituent knot.



An unknotted K_6



An unknotted K_7

2.4. The γ-warping degree and a γ-unknotted graph

Definition.

The γ-warping degree of G is the number $d_\gamma(G)$ for the complexity $\gamma(G)= (c_\gamma(G), d_\gamma(G))$ of G.

Definition. G is γ-unknotted if $d_\gamma(G) = 0$.

γ-unknotted ⇒ unknotted

2.5. A Γ-unknotted graph and the (γ,Γ)-warping degree

Let $\gamma(\Gamma) = \min\{\gamma(G) \mid G$ is a spatial graph of $\Gamma\}$.

Definition.

A Γ-unknotted graph G is a spatial graph of Γ with $\gamma(G) = \gamma(\Gamma)$.

Note.

(1) Let $\gamma(\Gamma) = (c_\gamma(\Gamma), d_\gamma(\Gamma))$. Then $d_\gamma(\Gamma)=0$.

Γ-unknotted ⇒ γ-unknotted ⇒ unknotted.

(2) $c_\gamma(\Gamma)=0$ if and only if Γ is a plane graph.

(3) A spatial plane graph G is Γ-unknotted

⇔ G is equivalent to a graph in a plane.

Definition.

O = {unknotted graphs of Γ}.

O_γ^G = {γ-unknotted graphs on $(D;T) \in [D_G]$ with $cd(D;T)=\gamma(G)$}.

$O_\gamma = \cup\{O_\gamma^G \mid G$ is a spatial graph of $\Gamma\}$
= {γ-unknotted graphs of Γ}.

O_Γ = {Γ-unknotted graphs}.

Then $O \supset O_\gamma \supset O_\Gamma$.

Note: $O_\gamma^G \subset O_\Gamma$ or $O_\gamma^G \cap O_\Gamma = \phi$ for every G.

Definition.

The (γ,Γ)-warping degree $d_\gamma^\Gamma(G)$ of G is:

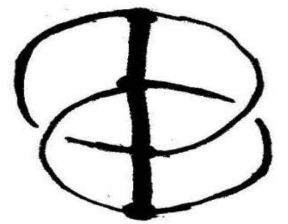
$$d_\gamma^\Gamma(G) = d_\gamma(G) + \rho(O_\gamma^G, O_\Gamma).$$

(ρ denotes the Gordian distance.)

By definition, $d(G) \leqq d_\gamma(G) \leqq d_\gamma^\Gamma(G)$.

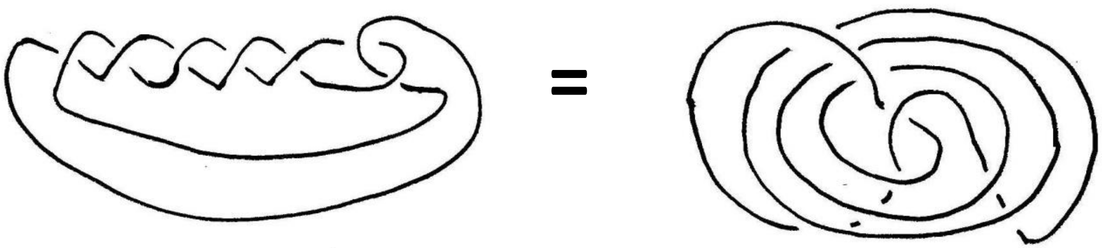
$d_\gamma^\Gamma(G) = 0$ **if and only if G is Γ-unknotted.**

2.6. Examples

Example 1.6. 1. Let $G =$  .

G has $c_{\gamma}(G)=2$, for G has a Hopf link as a constituent link.
$d(G)=d_{\gamma}(G)= 0$.
Because G is a planar graph, if G is Γ-unknotted, then $c_{\gamma}(G)=0$, a contradiction.
Hence $d^{\Gamma}_{\gamma}(G) =1$.

<u>Lemma 2.6.2.</u> (1) (**[Fung 1996] , [Ozawa 2010]**)
If K is a knot with d(K)=1, then K is a non-trivial twist knot.



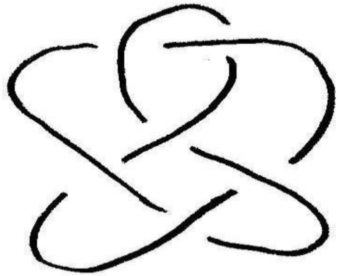
(2) If G is a θ-curve with d(G)=1, then the 3 constituent knots of G consist of two trivial knots and one non-trivial twist knot .

<u>Example 2.6.3.</u> (([Fung 1996] , [Ozawa 2010], [Shimizu 2010])

For K=  **5_2 , we have**

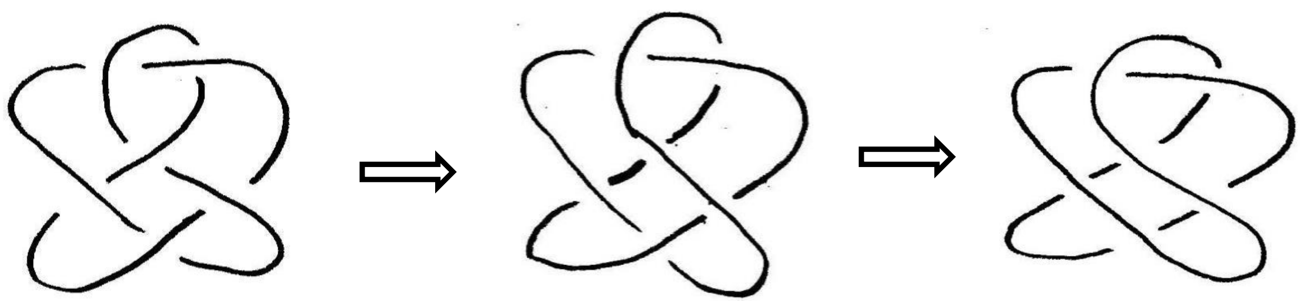
$$c_{\gamma}(K)=5,\ \ d(K)=1< d_{\gamma}(K)= d^{\Gamma}_{\gamma}\ (K)=2.$$

Example 2.6.4.

For K =  6_2,

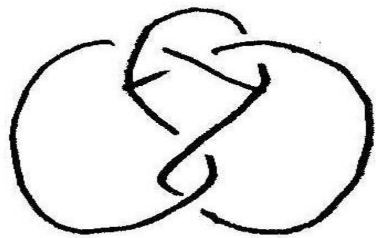
$c_\gamma(K)=6$, $d(K)= d_\gamma(K)= d^\Gamma_\gamma(K)=2$.

In fact, $d^\Gamma_\gamma(K) \leqq 2$:

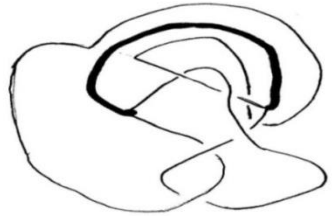


By Lemma, $d(K) \geqq 2$ (, for K is not any twist knot).

Example 2.6.5. (Kinoshita's θ-curve)

For G =  , we have

$c_\gamma(G) = 7$ and $d(G)= d_\gamma(K)= d^{\Gamma}_{\gamma}(G)= 2$.

 $\Longrightarrow$ 

a based diagram of G **a monotone diagram**

$O^G_\gamma = O_\Gamma$ implies $\rho(O^G_\gamma, O_\Gamma)=0$. Hence $d_\gamma(G)= d^\Gamma_\gamma(G)$.
Since G is non-trivial and the 3 constituent knots are trivial, we have $d(G) \geqq 2$ by Lemma.
Hence, if $c_\gamma(G)=7$, then $d(G)= d_\gamma(G)= d^\Gamma_\gamma(G)=2$.

By the diagram, $c_\gamma(G) \leqq 7$. We show $c_\gamma(G) \geqq 7$. By the classification of algebraic tangles with crossing numbers $\leqq 6$ in:

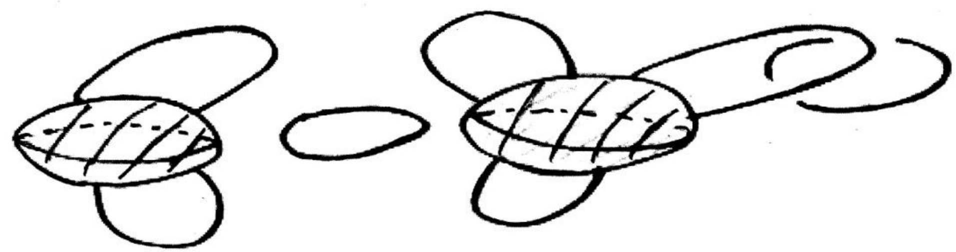
H. Moriuchi, Enumeration of algebraic tangles with applications to theta-curves and handcuff graphs, Kyungpook Math. J. 48(2008), 337-357

the Kinoshita's θ-curve G cannot have any based diagram with crossing number $\leqq 6$. Hence $c_\gamma(G)=7$.

2.7. A β-unknotted graph

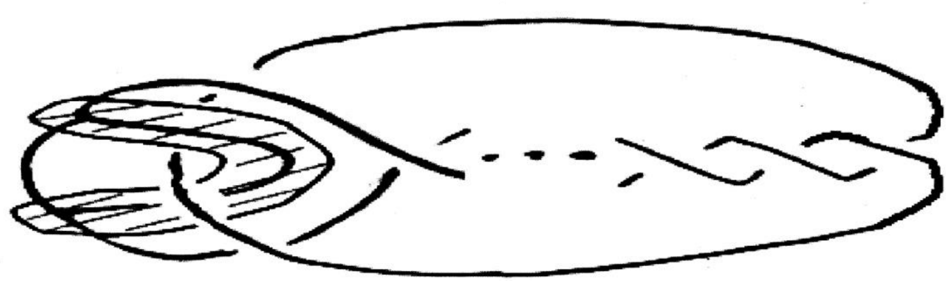
For a base $T = T_1 \cup T_2 \cup \ldots \cup T_r$ of G, let B be the disjoint union of mutually disjoint 3-ball neighborhoods B_i of T_i in S^3 ($i=1,2,\ldots,r$).
Let $B^c = cl(S^3 - B)$ be the complement domain of B with $L = B^c \cap G = a_1 \cup a_2 \cup \ldots \cup a_n$ an n-string tangle in B^c, called the complementary tangle of T.

Definition. G is **β-unknotted** if ∃ a base T of G whose complementary tangle (B^c,L) is trivial.



A trivial complementary tangle

Example 1.7.1. For a θ-curve Γ, ∃ ∞-many β-unknotted graphs G of Γ up to equivalences.



Example 1.7.2. **Triviality of the complementary tangle (B^c,L) depends on a choice of a base.**

Example 1.7.3. **If G is β-unknotted, then G is a free graph (i.e., $\pi_1(R^3-G)$ is a free group), but the converse is not true.**

A free β-knotted graph

By definitions and examples explained above, we have:

Theorem.

Γ-unknotted⇒γ-unknotted⇒unknotted

⇒ β-unknotted ⇒ free.

These concepts are mutually distinct.

Note: Given a Γ, ∃ only finitely many Γ-unknotted, γ-unknotted, or unknotted graphs of Γ.

2.8. The unknotting number

Let O = {unknotted graphs of Γ}.

Definition.

The unknotting number u(G) of a spatial graph G of Γ is the distance from G to O by crossing changes on edges attaching to a base:

$$u(G) = \rho(G,O).$$

2.9. A β-unknotting number

Let O_β = {β-unknotted graphs of Γ}.

Definition.

The β-unknotting number $u_\beta(G)$ of a spatial graph G of Γ is the distance from G to O_β by crossing changes on edges attaching to a base:

$$u_\beta(G) = \rho(G, O_\beta).$$

2.10. A γ-unknotting number

Given G, let

$$\{D_{G,\gamma}\} = \{(D;T) \in [D_G] \mid c(D;T) = c_\gamma(G)\}$$

(the set of minimal crossing based diagrams).

Definition.

The γ-unknotting number $u_\gamma(G)$ of a spatial graph G of Γ is the distance from $\{D_{G,\gamma}\}$ to O by crossing changes on edges attaching to a base:

$$u_\gamma(G) = \rho(\{D_{G,\gamma}\}, O).$$

Note. G is γ-unknotted ⇔ $u_\gamma(G) = 0$.

2.11. Γ-unknotting number

Let O_Γ={Γ-unknotted graphs}.

Definition.

The Γ-unknotting number $u^\Gamma(G)$ of G is the distance from G to O_Γ by crossing changes on edges attaching to a base:

$$u^\Gamma(G) = \rho(G, O_\Gamma)$$

Definition.

The (γ,Γ)-unknotting number $u^{\Gamma}_{\gamma}(G)$ of G is the distance from $\{D_{G,\gamma}\}$ to O_{Γ} by crossing changes on edges attaching to a base:

$$u^{\Gamma}_{\gamma}(G) = \rho(\{D_{G,\gamma}\}, O_{\Gamma}).$$

2.12. Dsitinctness of the unknotting numbers

Theorem 2.5.1. The unknotting numbers

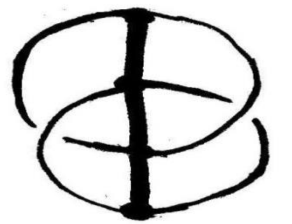
$$u_\beta(G),\ u(G),\ u^\Gamma(G),\ u_\gamma(G),\ u_\gamma^\Gamma(G)$$

of ∀ spatial graph G of ∀ graph Γ are mutually distinct topological invariants and satisfy the following inequalities :

$$u_\beta(G) \leqq u(G) \leqq \{u_\gamma(G), u^\Gamma(G)\} \leqq u_\gamma^\Gamma(G).$$

Proof. The inequalities are direct from definitions. We show that these invariants are distinct.

(1)

G= 

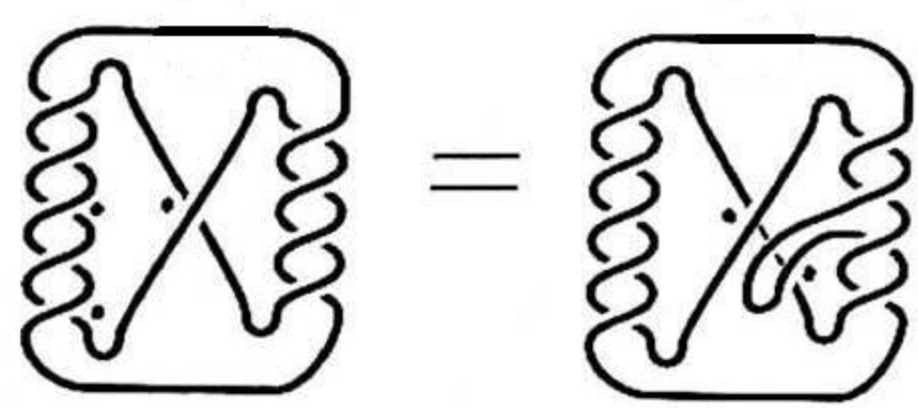
G has $c_\gamma(G)=2$ and hence $u_\beta(G)=u(G)=u_\gamma(G)=0$.
On the other hand, we have

$$u^\Gamma(G)=u^\Gamma_\gamma(G)=1,$$

for G is a spatial graph of a plane graph with a Hopf link as a constituent link and hence not Γ-unknotted.

(2)

Let G=

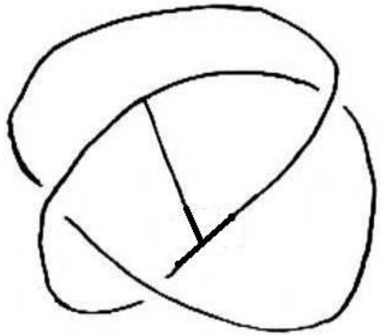


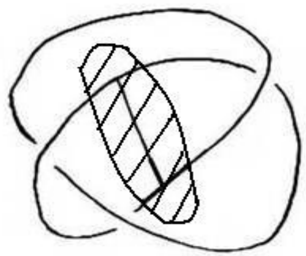
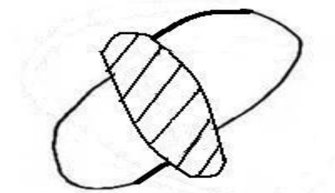
G=10_8 has $u(10_8)=2$ and $u_\gamma(10_8)=3$ by [Nakanishi 1983] and [Bleiler 1984]. Hence

$$u_\beta(G)= u(G) =u^\Gamma(G)=2< u_\gamma(G)=u^\Gamma_\gamma(G)=3.$$

[Nakanishi 1983] Y. Nakanishi, Unknotting numbers and knot diagrams with the minimum crossings, Math. Sem. Notes Kobe Univ. 11 (1983), no. 2, 257-258.
[Bleiler 1984] S. A. Bleiler, A note on unknotting number, Math. Proc. Cambridge Philos. Soc. 96 (1984), 469-471.

(3)

$G =$  Then $u_\beta(G) = 0$.

In fact:  $=$ 

Since G is a Θ-curve,

$u(G)=0 \Leftrightarrow$ G is isotopic to a plane graph.

G has a trefoil constituent knot.

Hence $u(G) \geqq 1$.

Thus, we have $u(G) = u^\Gamma(G) = u_\gamma(G) = u^\Gamma_\gamma(G) = 1$.//

2.13. The values of the unknotting numbers

Theorem 2.13.1. For ∀ given graph Γ and ∀ integer n ≧ 1, ∃ ∞-many spatial graphs G of Γ such that

$$u_{\beta}(G)= u(G)= u_{\gamma}(G)= u^{\Gamma}(G)= u^{\Gamma}_{\gamma}(G)=n.$$

Infinite cyclic covering homology of a spatial graph

For a spatial graph G of Γ in $S^3=R^3\cup\{\infty\}$with a base T and oriented edges α_i(i=1,2,…,s) attaching to T.
Let $E(G)=cl(S^3-N(G))$ for a regular neighborhood N(G) of G in S^3.
Let χ: $H_1(E(G))\to Z$ be the epimorphism sending the meridians of α_i (i=1,2,…,m) to $1\in Z$.
Let $E(G)_\infty\to E(G)$ be the ∞-cyclic cover of E(G) associated with χ.

Let $\Lambda=Z[t,t^{-1}]$.

The homology $H_1(E(G)_\infty)$ is a finitely generated Λ-module which we denote by $M(G,T)_\infty$.

We take an exact sequence (over Λ)

$$\Lambda^a \to \Lambda^b \to M(G,T)_\infty \to 0,$$

where we take $a \geqq b$. A matrix $A(G,T)_\infty$ over Λ representing the homomorphism $\Lambda^a \to \Lambda^b$ is called a presentation matrix of the module $M(G,T)_\infty$.

For an integer d$\geqq$0, the <u>dth ideal</u> $\varepsilon_d(G,T)_\infty$ of M$(G,T)_\infty$ is the ideal generated by all the (b-d)-minors of A$(G,T)_\infty$.
The ideals $\varepsilon_d(G,T)_\infty$ (d=0,1,2,3,…) are invariants of the Λ-module M$(G,T)_\infty$.
Let (Δ_d) be the smallest principal ideal containing $\varepsilon_d(G,T)_\infty$. Then the Laurent polynomial $\Delta_d \in \Lambda$ is called the <u>dth Alexamder polynomial</u> of M$(G,T)_\infty$.
If G is a knot (with T=ϕ), then $\Delta_0 \in \Lambda$ is called the <u>Alexander polynomial</u> of the knot G.

Assume that G^* is obtained from G by k crossing changes on α_i $(i=1,2,\ldots,m)$. Then χ induces the epimorphism $\chi^*:H_1(E(G^*))\to Z$.

Let $m(G,T)_\infty$ and $m(G^*,T)_\infty$ be the numbers of minimal Λ-generators of the Λ-modules $M(G,T)_\infty$ and $M(G^*,T)_\infty$, respectively.

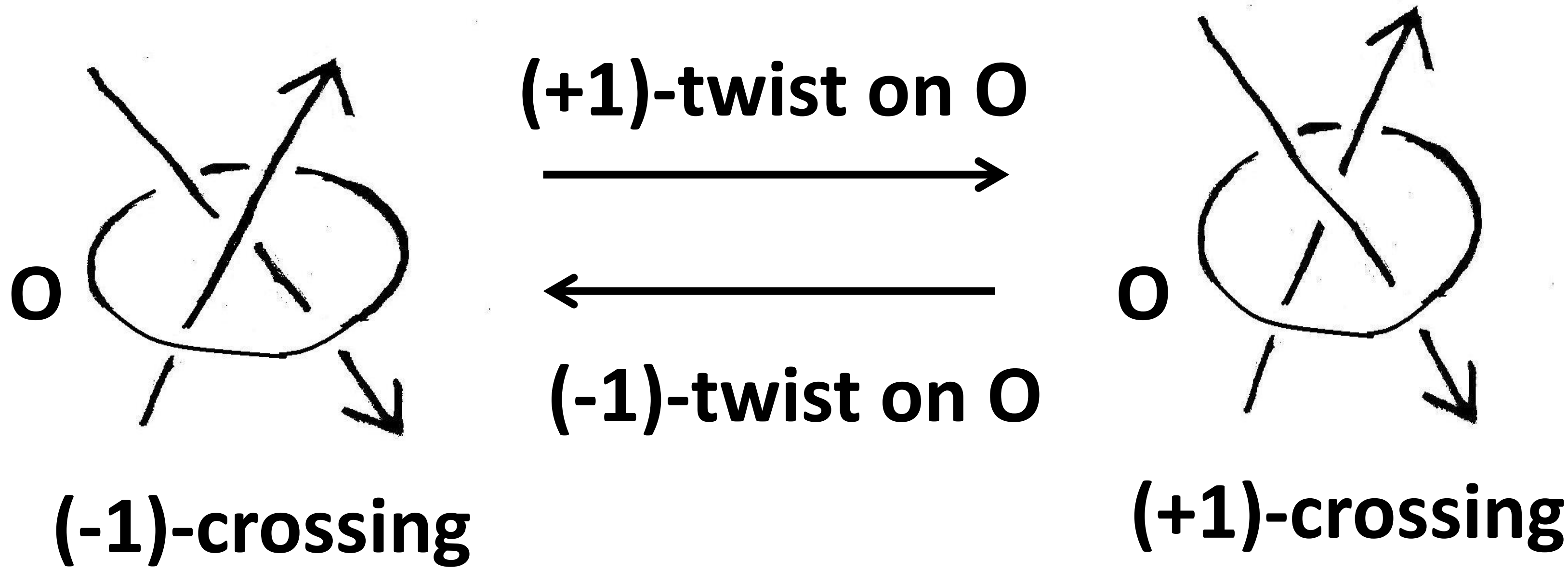
We use the following lemma:

Lemma A.

A. Kawauchi, Distance between links by zero-linking twists, Kobe J. Math.13(1996), 183-190.

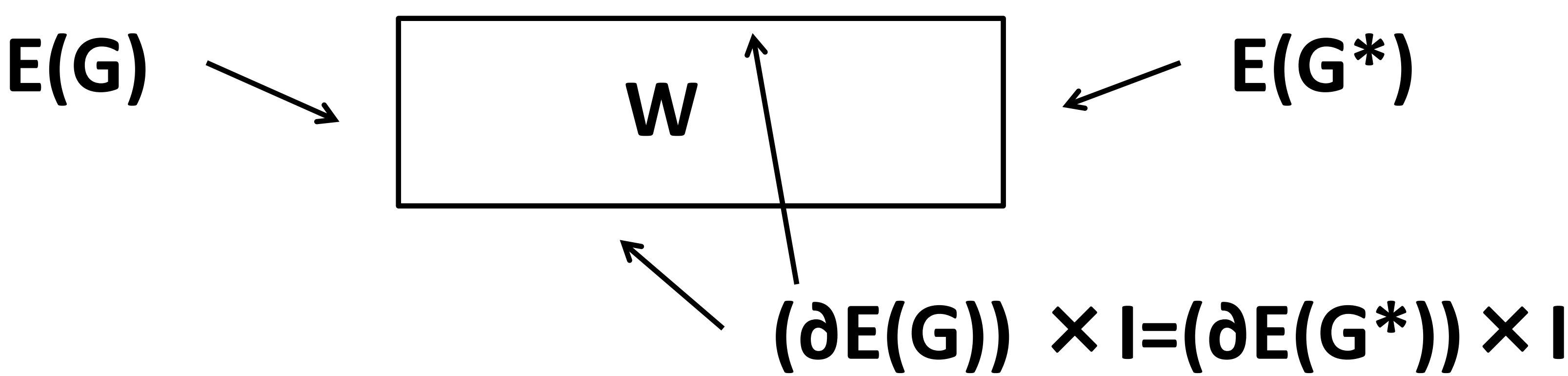
$$|m(G,T)_\infty - m(G^*,T)_\infty| \leqq k.$$

Proof.



G* is obtained from G by k crossing changes on the edges α_i (i=1,2,…,m). G is also obtained from G* by k crossing changes on the corresponding edges α_i^* (i=1,2,…,m).

Let $W = E(G) \times I \cup_{i=1}^{k} D^2 \times D^2_i$ be a surgery trace from $E(G)$ to $E(G^*)$ by 2-handles $D^2 \times D^2_i$ $(i=1,2,\ldots,k)$, which is also a surgery trace from $E(G^*)$ to $E(G)$ by the “dual” 2-handles $D^2 \times D^2_i$ $(i=1,2,\ldots,k)$.



By construction, χ and χ^* extend to an epimorphism $\chi^+: H_1(W) \to Z$.
Let $(W_\infty; E(G)_\infty, E(G^*)_\infty)$ be the ∞-cyclic cover of $(W; E(G), E(G^*))$ associated with χ^+.

Let $m(W_\infty)$ be the minimal number of Λ-generators of the Λ-module $H_1(W_\infty)$.

Then we have

$$m(W_\infty) \leqq m(G,T)_\infty ,$$

$$m(W_\infty) \leqq m(G^*,T)_\infty .$$

Because, the natural homomorphisms

$$\pi_1(E(G)) \to \pi_1(W) \text{ and } \pi_1(E(G^*)) \to \pi_1(W)$$

are onto, so that the natural homomorphisms

$$H_1(E(G)_\infty) \to H_1(W_\infty) \text{ and } H_1(E(G^*)_\infty) \to H_1(W_\infty)$$

are onto.

By the exact sequence of the pair $(W_\infty, E(G)_\infty)$

$$H_2(W_\infty, E(G)_\infty) \to H_1(E(G)_\infty) \to H_1(W_\infty) \to 0$$

and $H_2(W_\infty, E(G)_\infty) = \Lambda^k$, we obtain

$$m(G,T)_\infty \leqq k + m(W_\infty) \leqq k + m(G^*,T)_\infty .$$

Similarly,

$$m(G^*,T)_\infty \leqq k + m(W_\infty) \leqq k + m(G,T)_\infty .$$

Thus, we have

$$|m(G,T)_\infty - m(G^*,T)_\infty| \leqq k. \quad //$$

Proof of Theorem 2.13.1.

Let G_0 be a Γ-unknotted graph.

Let K be a trefoil knot, and K(n) the n-fold connected sum of K. Then

$$u(K(n))=u_\gamma(K(n))=n \quad \text{for } \forall n \geqq 1.$$

Let $G = G_0 \# K(n)$ be the connected sum of K(n) and an edge attaching to a base T_0 of G_0.

Then $u_\gamma^{\Gamma}(G) \leqq n$ since $c_\gamma(G) = c_\gamma(G_0) + c_\gamma(K(n))$.

We show $u_\beta(G) \geqq n$.

Assume that $u_\beta(G)=k$. Then a β-unknotted graph G^* is obtained from G by k crossing changes on edges $\alpha_i(i=1,2,\ldots,m)$ attaching to a base T in G.

We choose orientations on α_i $(i=1,2,\ldots,m)$ as it is stated in the following two cases.

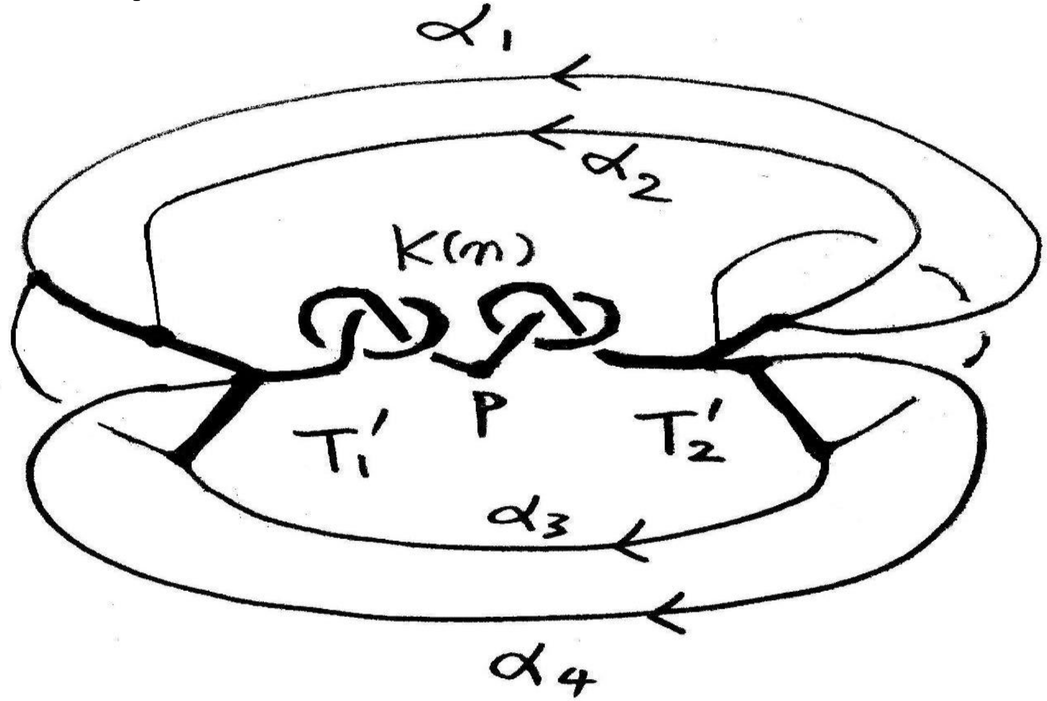
Case (I): K(n) is in an edge α_i.

Case (II): K(n) is in a component T' of the base T.

In Case (I), take any orientations on α_i $(i=1,2,\ldots,m)$.

In Case (II), let T'_1 and T'_2 be the components of $T'-\{p\}$ for a point $p \in K(n)$, and α_i (i=1,2,…,u) the edges joining T'_1 and T'_2.

We take orientations of the edges α_i (i=1,2,…,u) going from T'_2 to T'_1 and any orientations of the other edges α_i (i=u+1,u+2,…,m).



Let χ: $H_1(E(G)) \to Z$ be the epimorphism sending the oriented meridians of α_i (i=1,2,…,m) to $1 \in Z$.

Then we have

in Case(I), $M(G,T)_\infty = \Lambda^{m-1} \oplus [\Lambda/(\Delta_K(t))]^n$, and

in Case(II), $M(G,T)_\infty = \Lambda^{m-1} \oplus [\Lambda/(\Delta_K(t^u))]^n$.

In either case, we have $m(G,T)_\infty = m+n-1$.

On the other hand, $\pi_1(E(G^*))$ is a free group of rank m and hence $M(G^*,T)_\infty = \Lambda^{m-1}$.

Thus, $m(G^*,T)_\infty = m-1$.

By Lemma A, $|(m(G,T)_\infty - m(G^*,T)_\infty| = n \leqq k$.

Hence $u_\beta(G) \geqq n$ and

$u_\beta(G) = u(G) = u_\gamma(G) = u^\Gamma(G) = u^\Gamma_\gamma(G) = n$. //