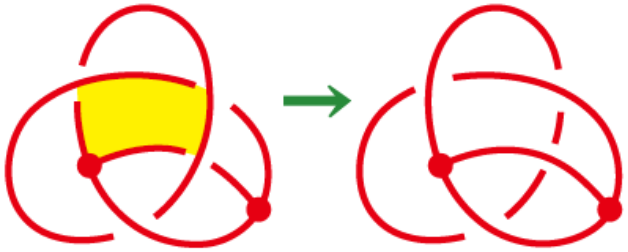
Region crossing change on knot, link and spatial-graph diagrams



Ayaka Shimizu (Gunma National College of Technology)

Advanced School and Discussion Meeting on Knot Theory and its Applications, December 13, 2013

at IISER Mohali

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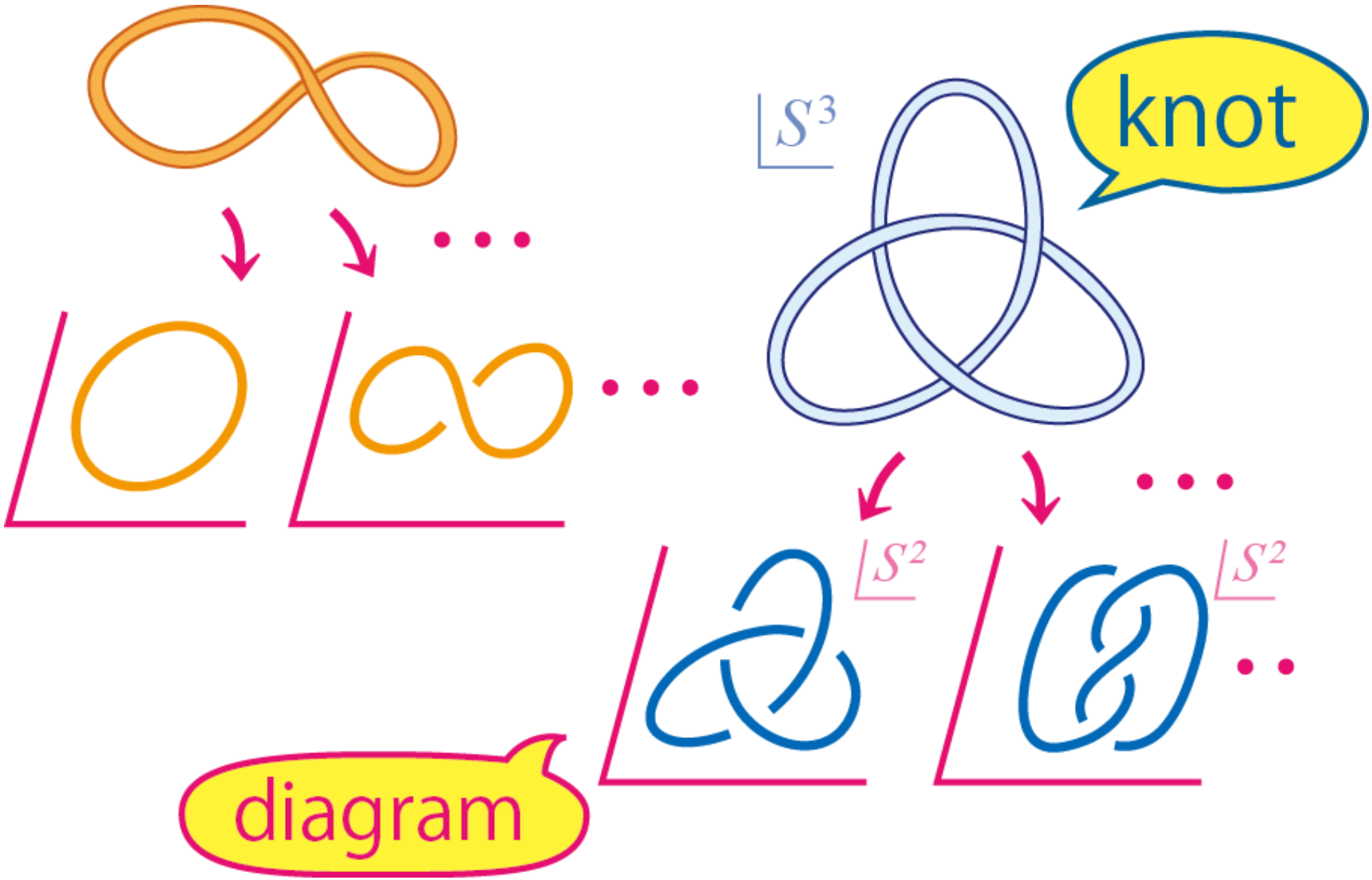
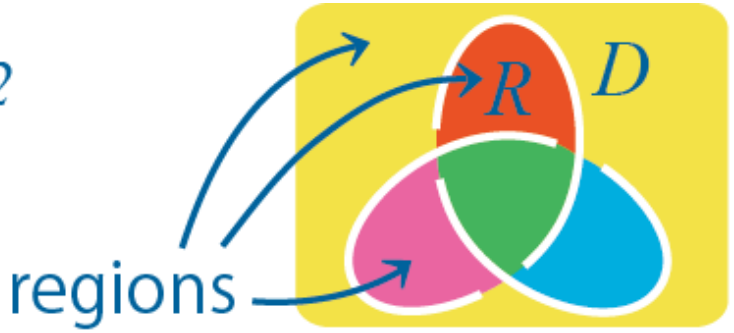

S^3
knot
S^2
S^2
diagram

Region crossing change

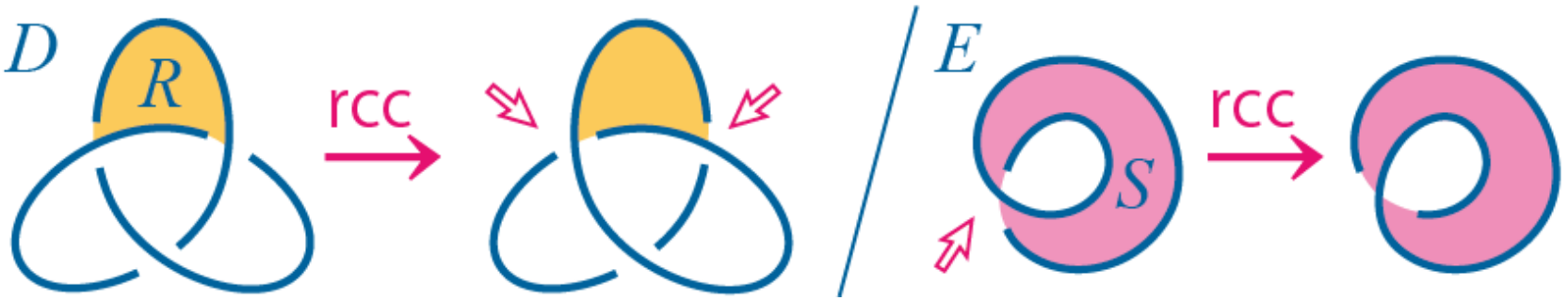
D: a link diagram on S^2

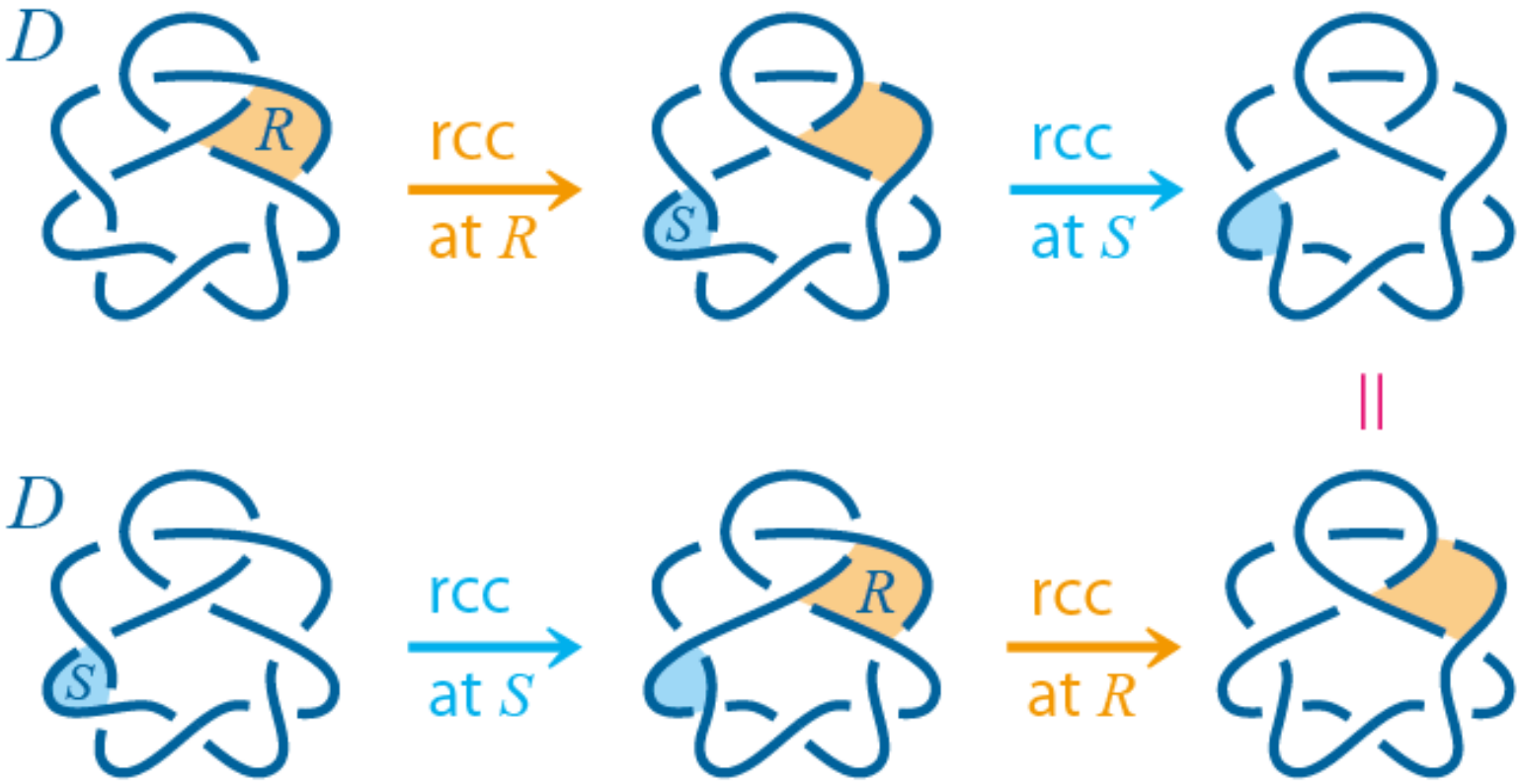
R: a region of D



A region crossing change at R changes all the crossings on ∂R.

(defined by Kishimoto)





(Region crossing change does not depend on the order.)

Theorem 1 (S)

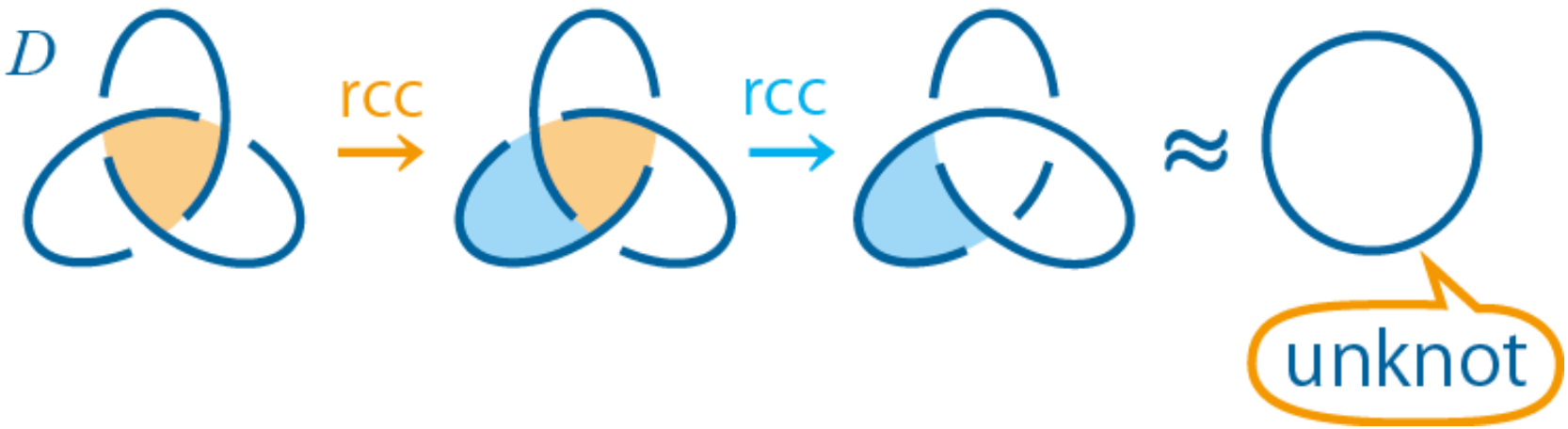
We can make any crossing change on a knot diagram by region crossing changes.

Reference

A. Shimizu, Region crossing change is an unknotting operation, to appear in JSMJ.

Corollary 2.

A region crossing change on a knot diagram is an unknotting operation.

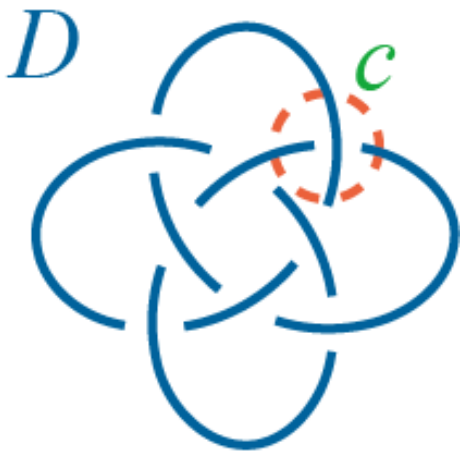


A knot diagram D is reduced if D has no reducible crossings.

Proof of Theorem 1.

(1) for reduced diagrams

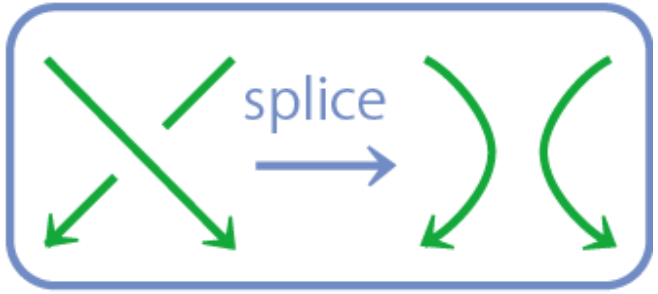
D: a reduced knot diagram
c : a crossing point of D

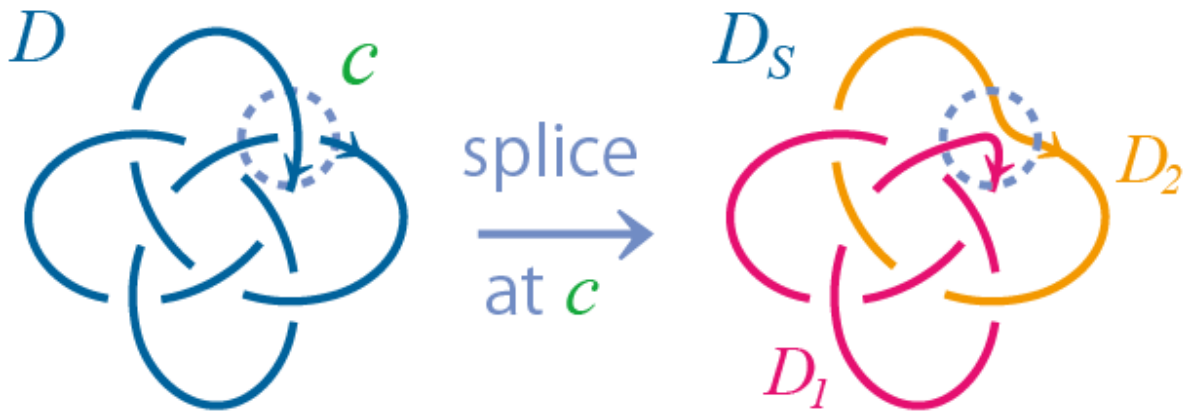


To obtain the regions which change only c by the region crossing changes...

Step 1.

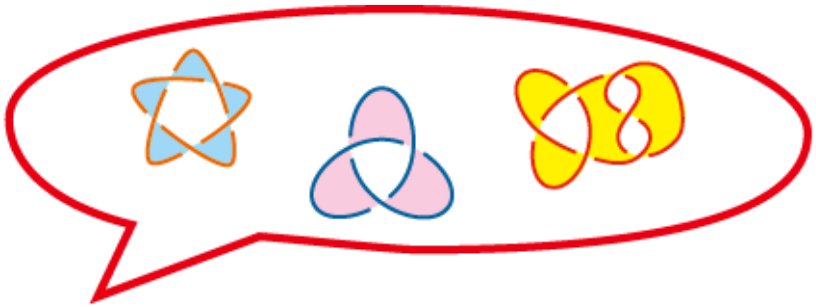
Splice D at c.



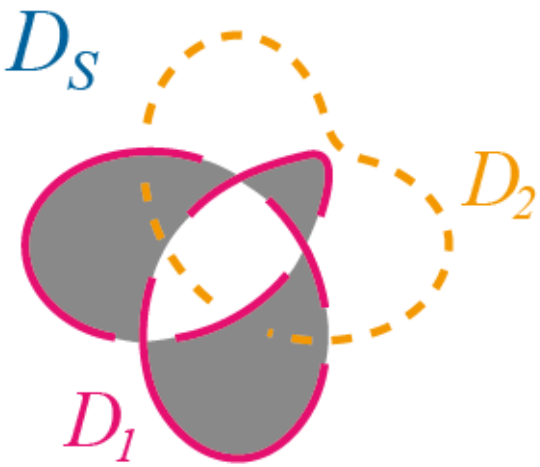


Then we obtain a two-component link diagram $D_S = D_1 \cup D_2$.

Step 2.

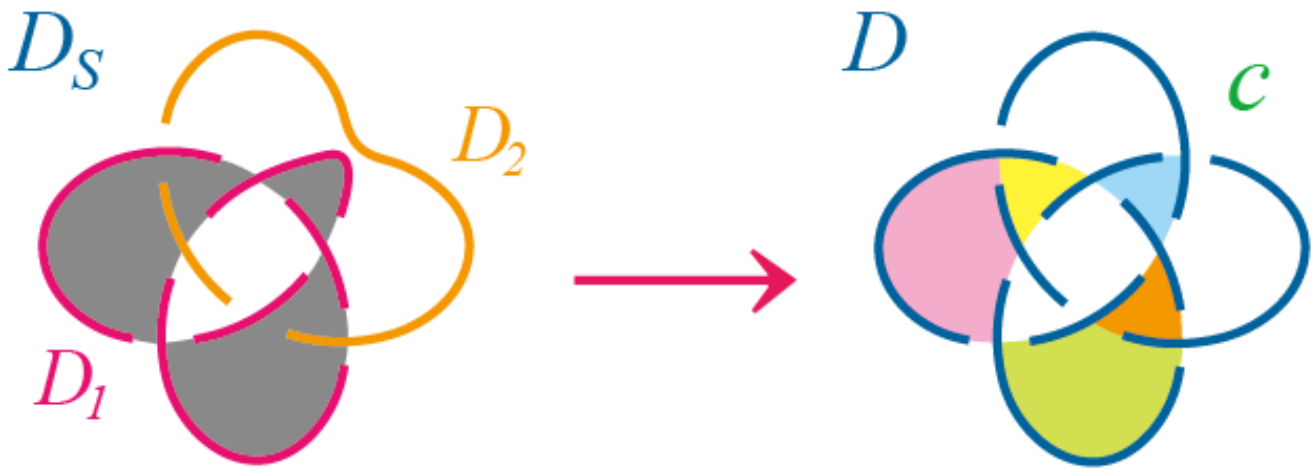


Apply a checkerboard coloring to only D_1.



Step 3.

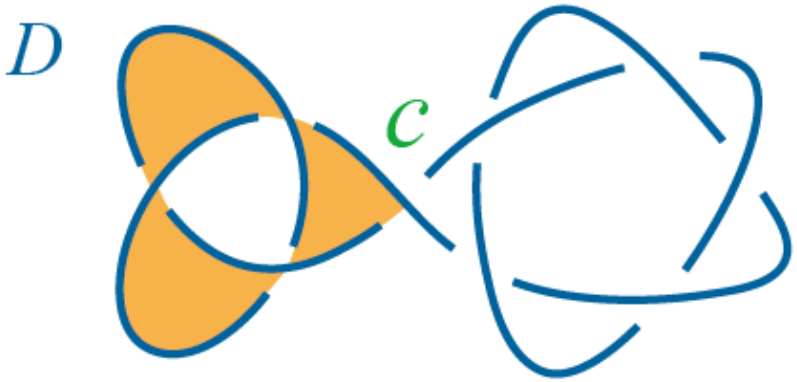
Take the regions of D corresponding to the shaded regions of D_S .



Thus, we obtain the regions.

(2) for non-reduced diagrams

★ If D has just one reducible crossing, and c is the reducible crossing,

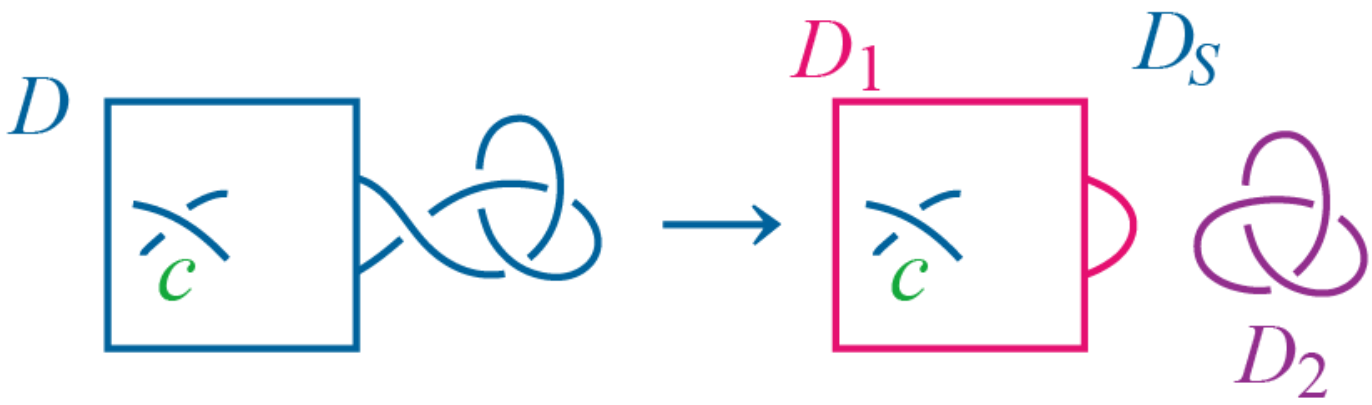


we can change c by region crossing changes.

★ For other cases, we prove by an induction on the number k of reducible crossings.

- When $k=0$, D is reduced.
- We assume that the theorem holds for D with k reducible crossings.
- We consider D with $k+1$ reducible crossings.

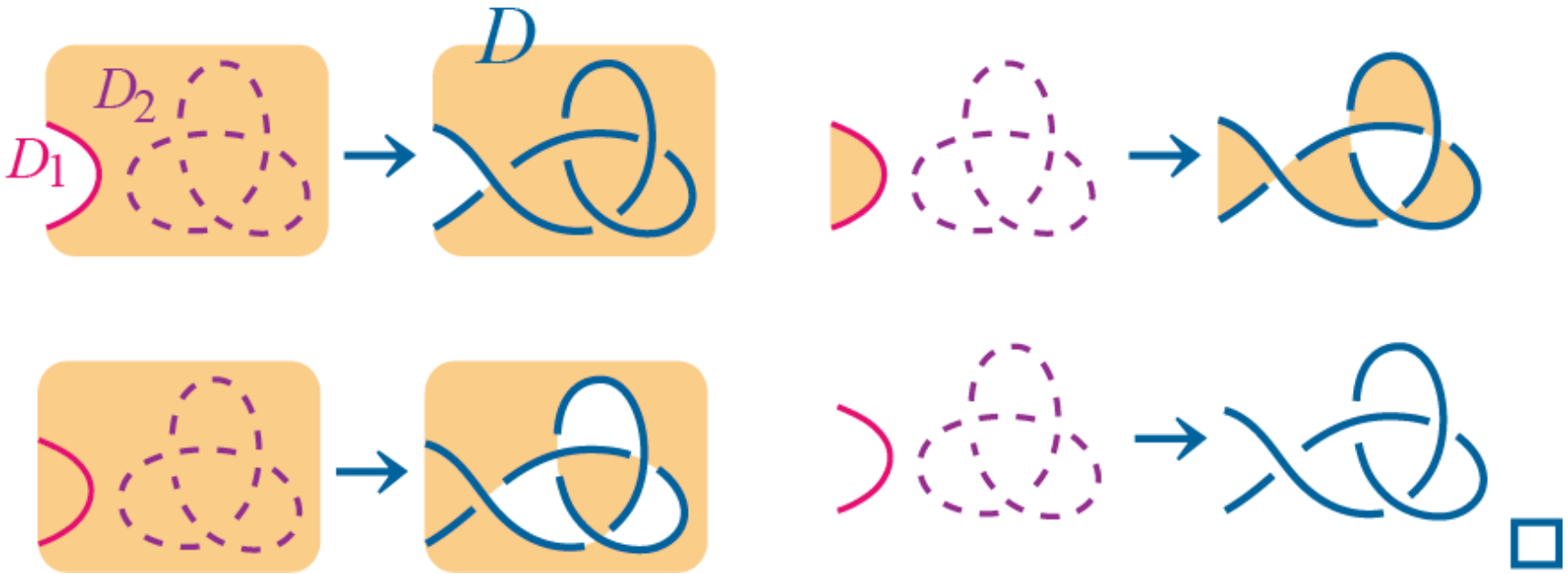
Splice D at a reducible crossing to obtain $D_S = D_1 \cup D_2$ such that D_1 has k reducible crossings and c.



By the assumption,

D_1 has the regions which change only c.

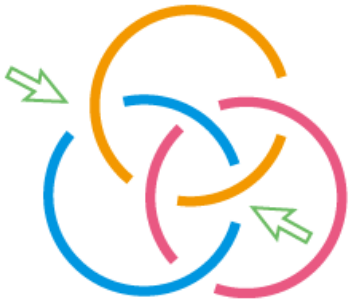
From the regions of D', we can obtain the regions of D which change only c :



For links...

Proposition 3. (Cheng & Gao)

For a link diagram, we can change any self-crossing of any knot component, and any pair of crossings between any two components by region crossing changes.



Reference

Z. Cheng and H. Gao, On region crossing change and incidence matrix, Science China Mathematics 55 (2012), 1487--1495.

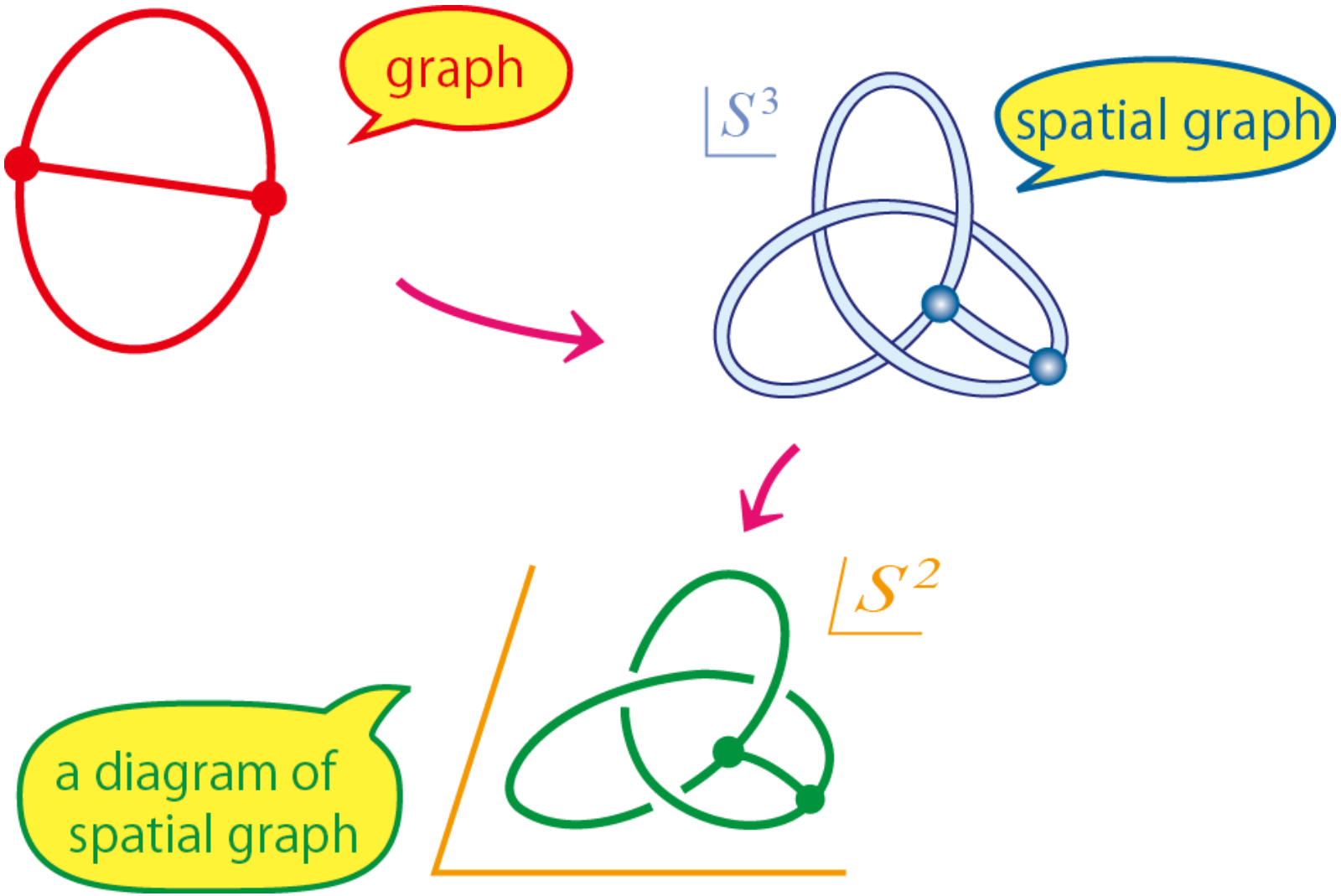
Theorem 4. (Cheng)

A region crossing change on a link diagram is an unknotting operation if and only if the diagram represents a proper link.

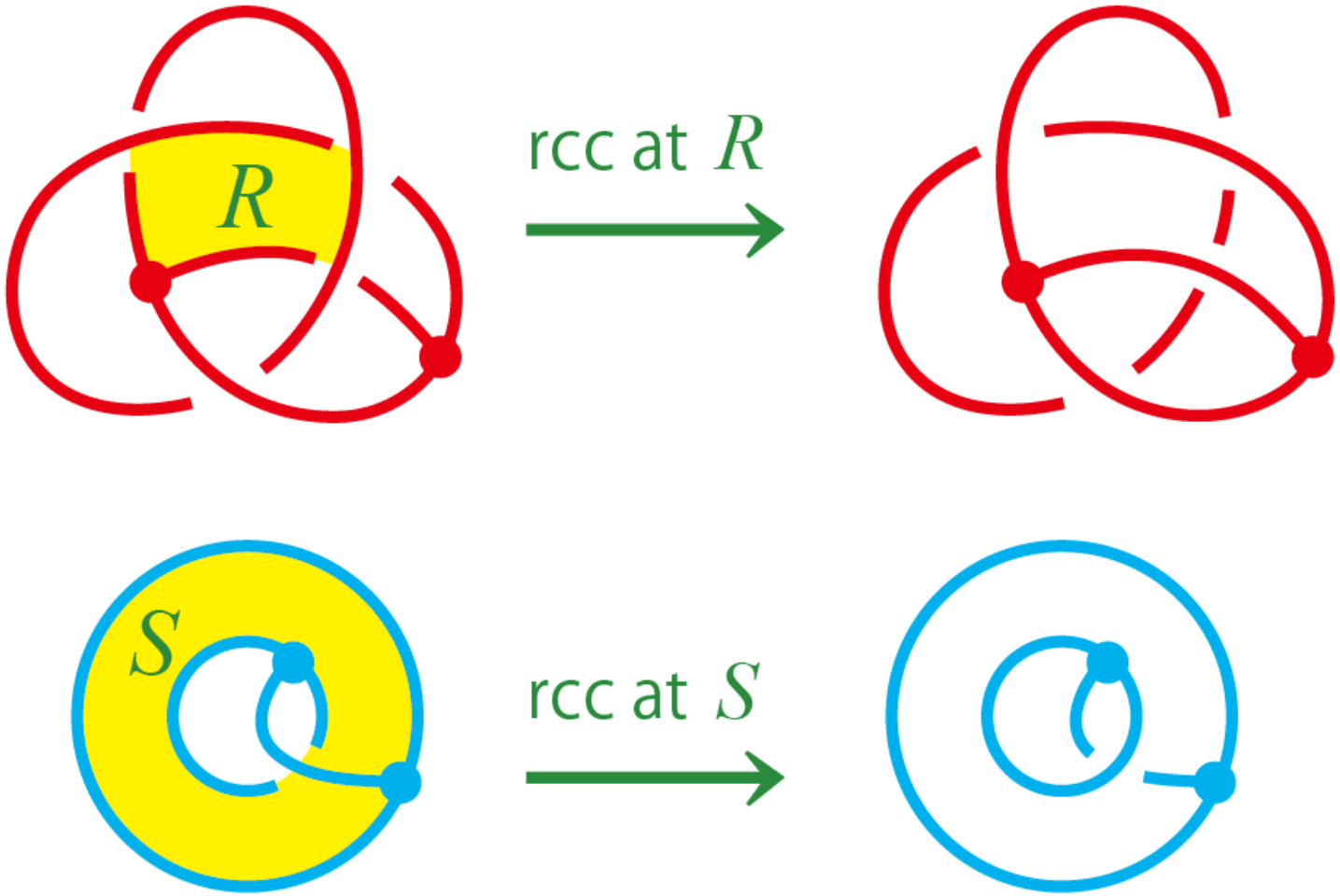
Reference

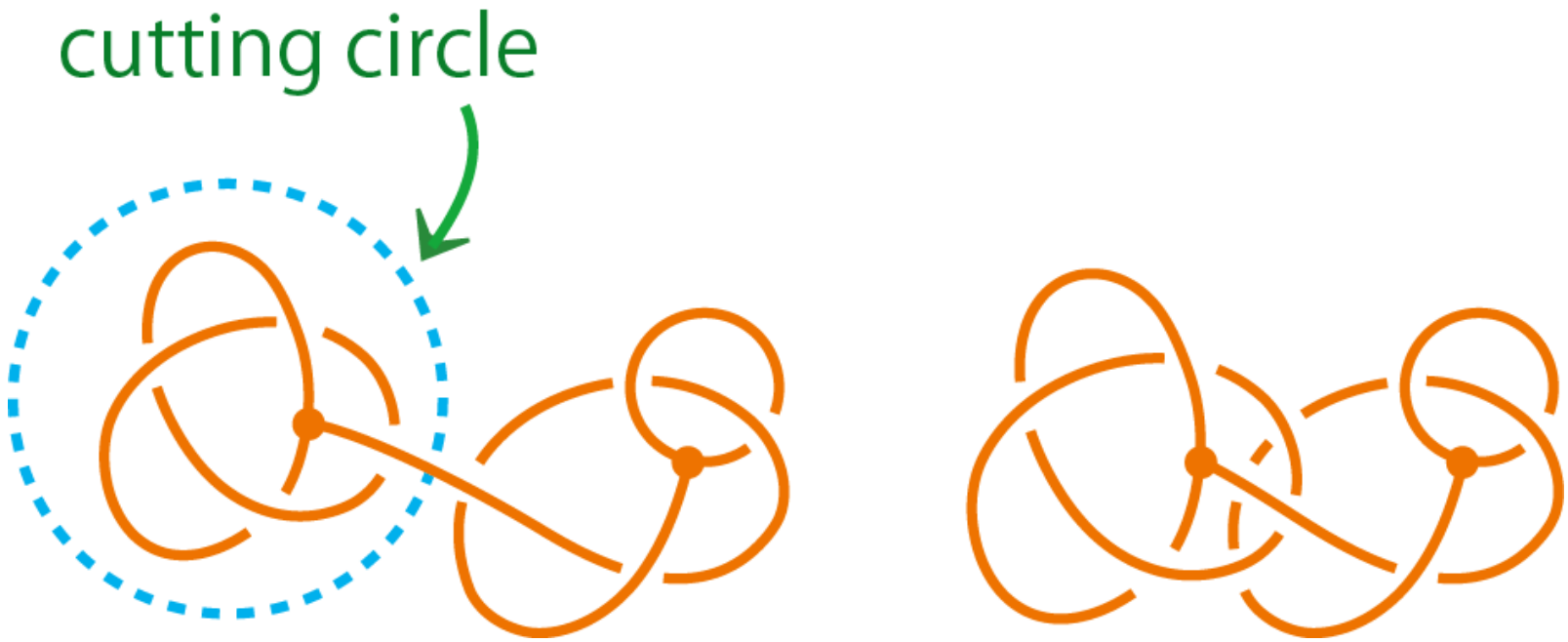
Z. Cheng, When is region crossing change an unknotting operation?, arXiv: 1201.1735.

§2. *Region crossing change on spatial-graph diagrams*


graph
S^3
spatial graph
S^2
a diagram of
spatial graph

RCC on spatial-graph diagrams

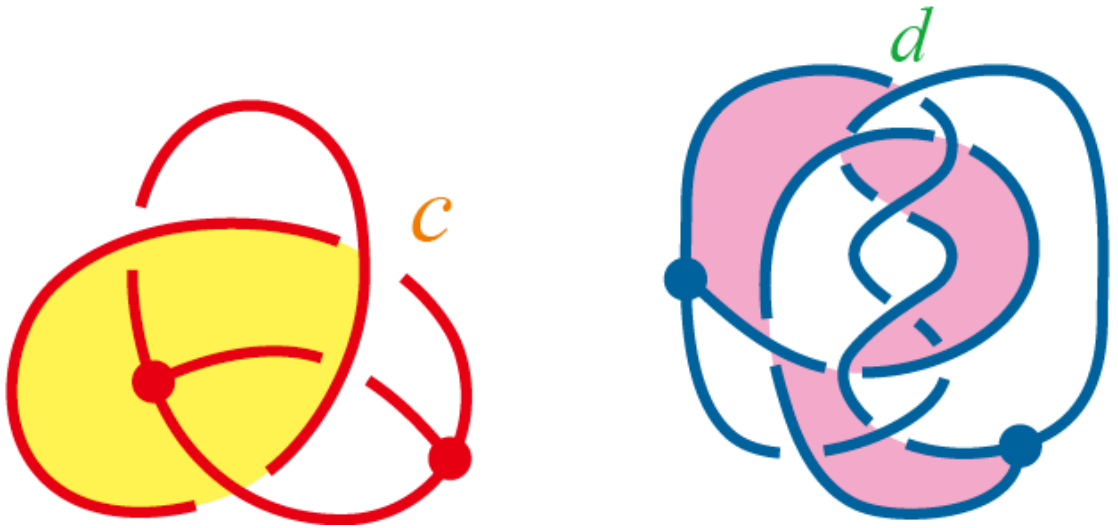



cutting circle

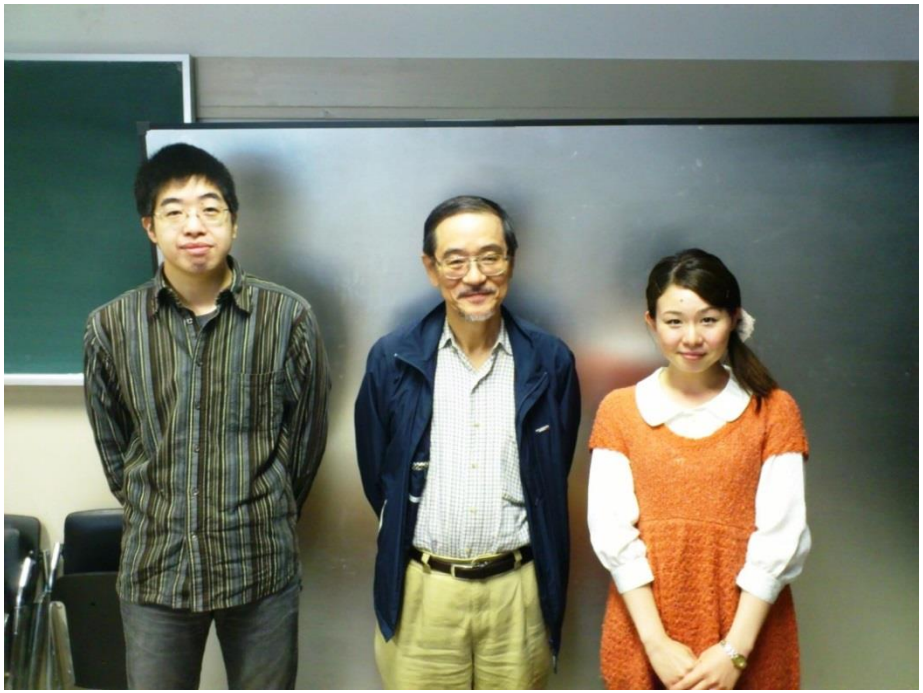
Theorem 5.

Let D be a diagram of a connected spatial graph which has no cutting circles.
We can make any crossing change on D by region crossing changes.

Example



§3. *Region Select*



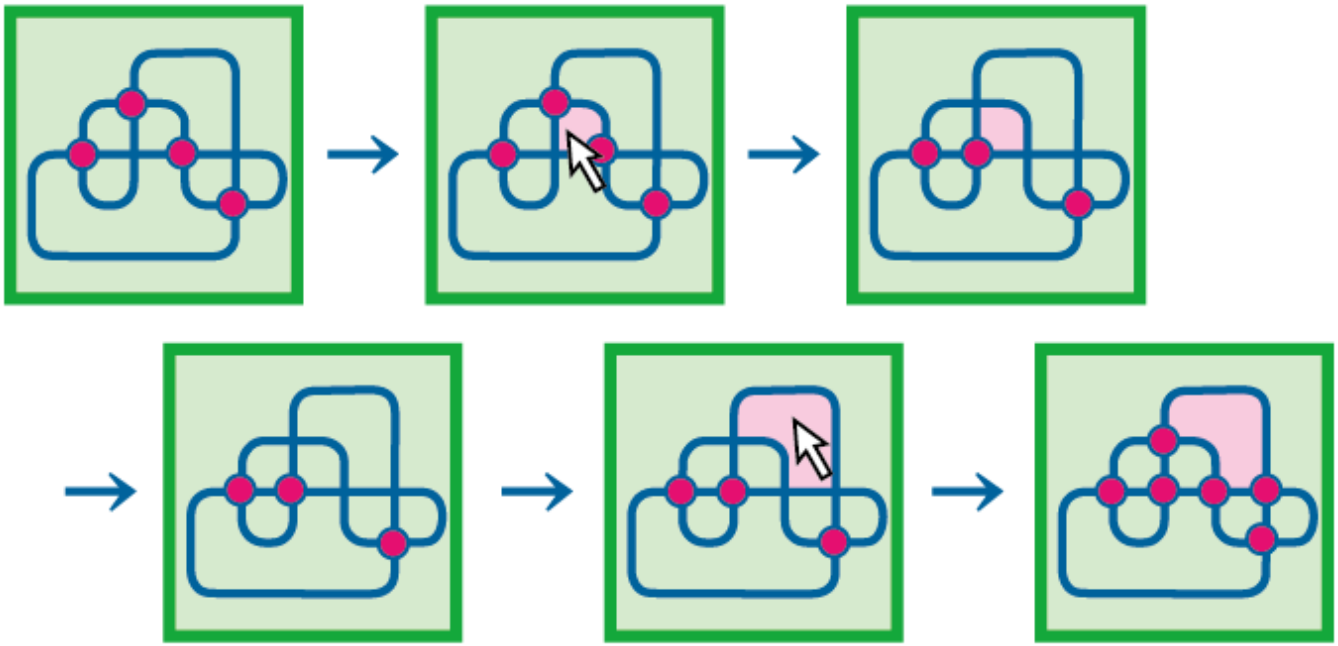
Again,

Theorem 1.

We can make any crossing change on a knot diagram by region crossing changes.

Region Select

A Game Using Knot Theory



by Akio Kawauchi, Ayaka Shimizu and Kengo Kishimoto

Japanese Patent Application (2011)

Region Select has been released as an application for Android smart phone!

Region Select for children!!

17 科学 13版 2013年(平成25年)5月27日(月曜日)

科学 MONDAY

「結び目理論」を応用した幼児用ゲーム

危険だと思う津波の高さのアンケート調査結果 (%)

ゲームで養う数学力

結び目理論で「iPad」に花が咲く

大阪市大名誉教授らが幼児用ソフト

高度な数学理論を利用した幼児用ゲームを、河内明夫・大阪市立大名誉教授らのグループが開発した。タブレット型情報端末「iPad（アイパッド）」上で、簡単なタッチ操作だけで楽しめる。数学の知識は不要で、直感力や数手先を読む思考力が試される。幼稚園児らに遊んでもらって教育効果を調べており、河内さんは「将来はSUDOKU（数独）のように世界中で親しまれてほしい」と話している。（新井清美）

iPadで結び目理論を利用したゲームをする園児たち（大阪市天王寺区で）＝近藤誠撮影

大阪市天王寺区の「すくすく保育園」の5歳児クラ

ス。園児がiPadのゲーム画面を開くと、アサガオの鉢植えのイラストが現れた。ぐるりと伸びた茎がわる点につぼみや花があり、茎で囲まれた領域をタッチすると、その領域を取り囲むつぼみは花に、花はつぼみに変わる。タッチする領域をどの順番で選ぶかがカギで、すべての花

と を知らない市民が6割以上

数字はわかりにくいだけでなく、油断のならない相手でもある。

中谷内一也・同志社大教授と大木聖子・慶応大准教授が、東日本大震災の前後に西日本の住民に対して行ったアンケート調査は、巨大津波が人々の意識にもたらした皮肉な効果を浮かび上がらせた。

問いは「どのくらいの高さの津波を危険だと思うか」。「1㍍以下」という回答は、震災前の71%から震災後は46%に、「3㍍以下」は90%から73%に減り、「5㍍以上」が4%から20%に増えた。津波の恐ろしさを実感したはずなの

知っておきたい正しい指標

に、人々は津波の高さに関するリスク評価を下げ、より無防備になっていた。

人間は提示された数字を、いかり（アンカー）のように指標として判断する傾向があるとされる。「アンカリング」と呼ばれる現象だ。東日本大震災の報道で巨大な津波の高さの数値を繰り返し耳にしたため、リスクととらえる数字が上がったとみられる。

大木准教授は「津波は1㍍程度


Thank you!