Fermi surfaces and the AdS/CFT correspondence

Indian Institute of Science, Bangalore, Dec 8, 2010

Lecture notes
arXiv:1010.0682
arXiv: 1012.0299

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Bilayer ^{3}He: A Simple Two-Dimensional Heavy-Fermion System with Quantum Criticality

Science **317**, 1356 (2007)

Michael Neumann, Ján Nyéki, Brian Cowan, John Saunders*

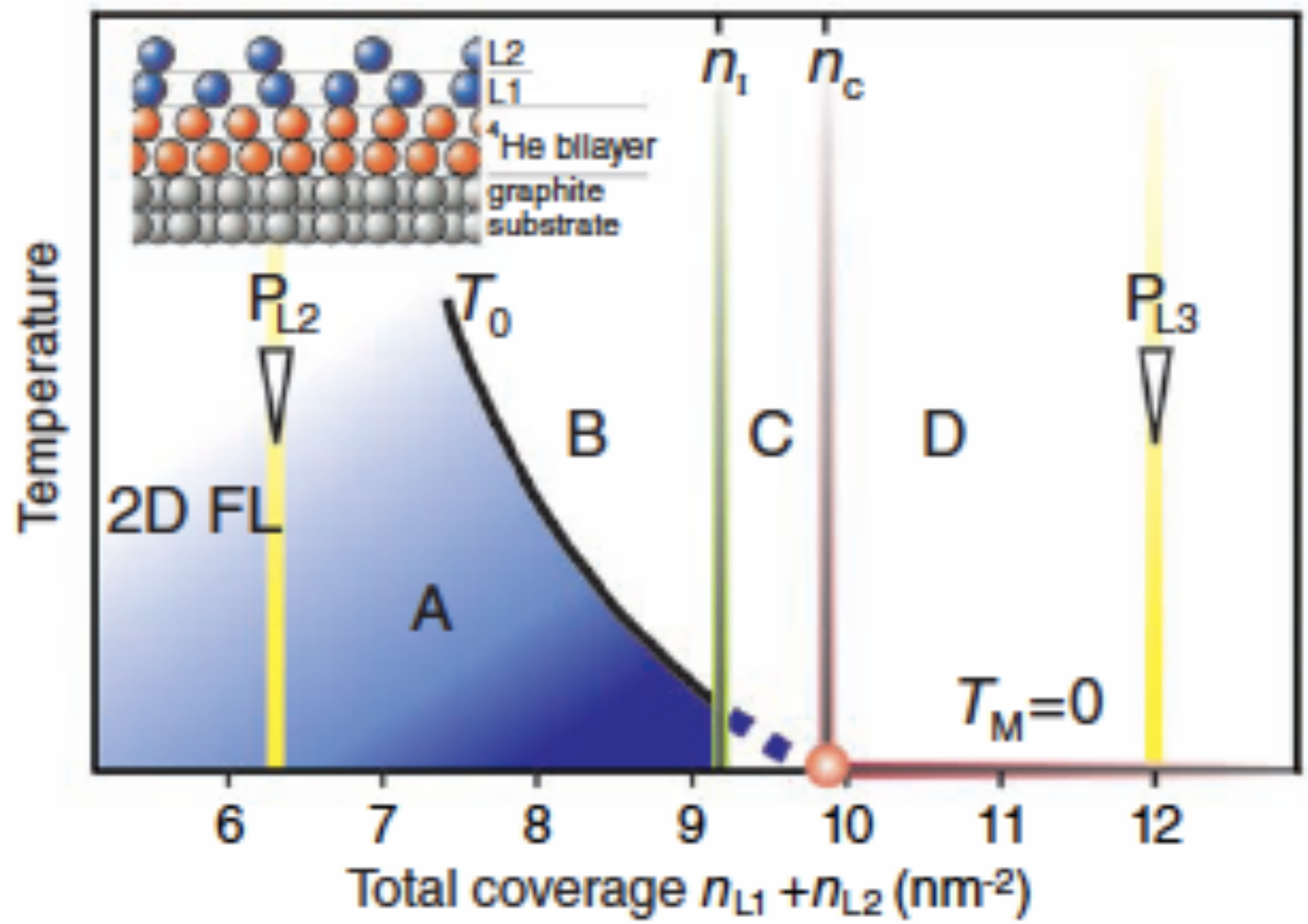
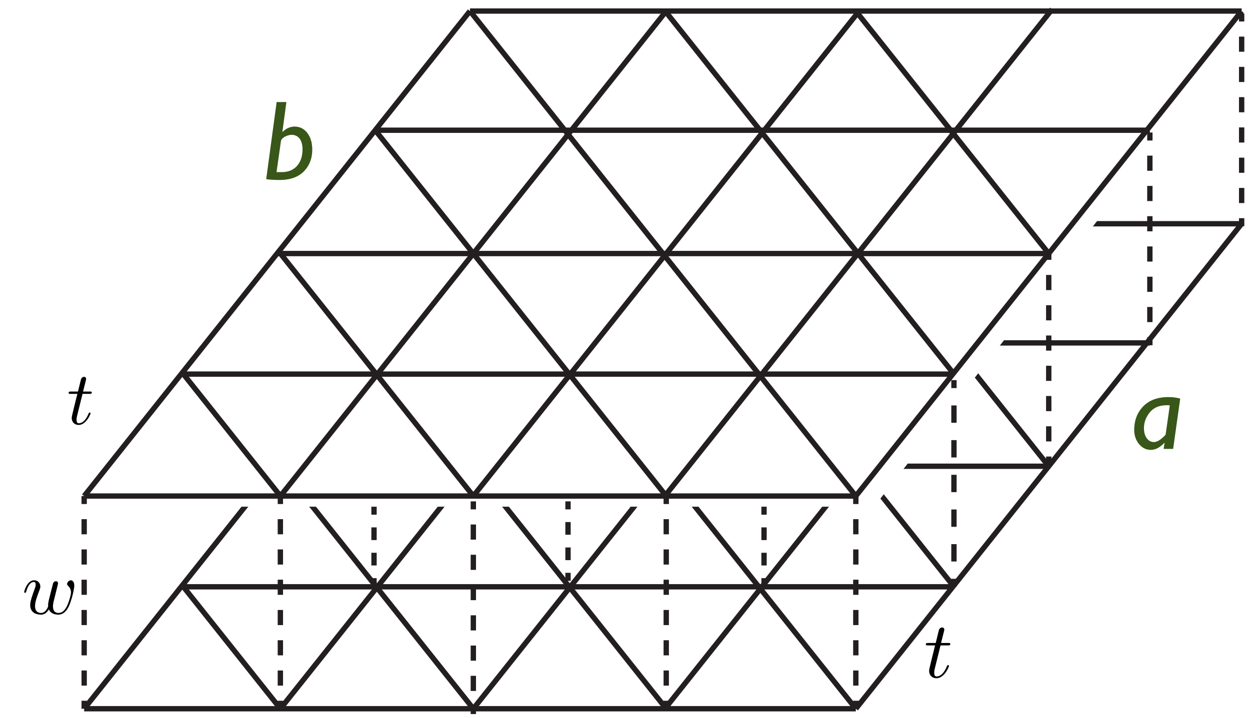


Fig. 1. Proposed phase diagram of ^{3}He film adsorbed on graphite plated by a bilayer of ^{4}He. P_{L2} indicates formation of the ^{3}He bilayer. Region A: $T < T_0$. L1 and L2 form a two-band heavy-fermion system. Region B: $T > T_0$. The two layers progressively decouple into a strongly correlated, nearly localized narrow-band Fermi system (L1) and a weakly correlated 2D Fermi fluid (L2). Region C: $n_I < n < n_c$, intervening phase. Region D: The two layers decouple into a 2D local-moment magnet (Mott insulator, L1) with frustrated intralayer spin exchange and no magnetic phase transition at finite temperature, $T_M = 0$, and a weakly correlated 2D Fermi fluid overlayer (L2). P_{L3} indicates the formation of a third ^{3}He layer. Inset is a schematic of the bilayer ^{3}He system (see text for details).

Hubbard model on a bilayer triangular lattice

$$\epsilon_a < \epsilon_b < \epsilon_a + U$$

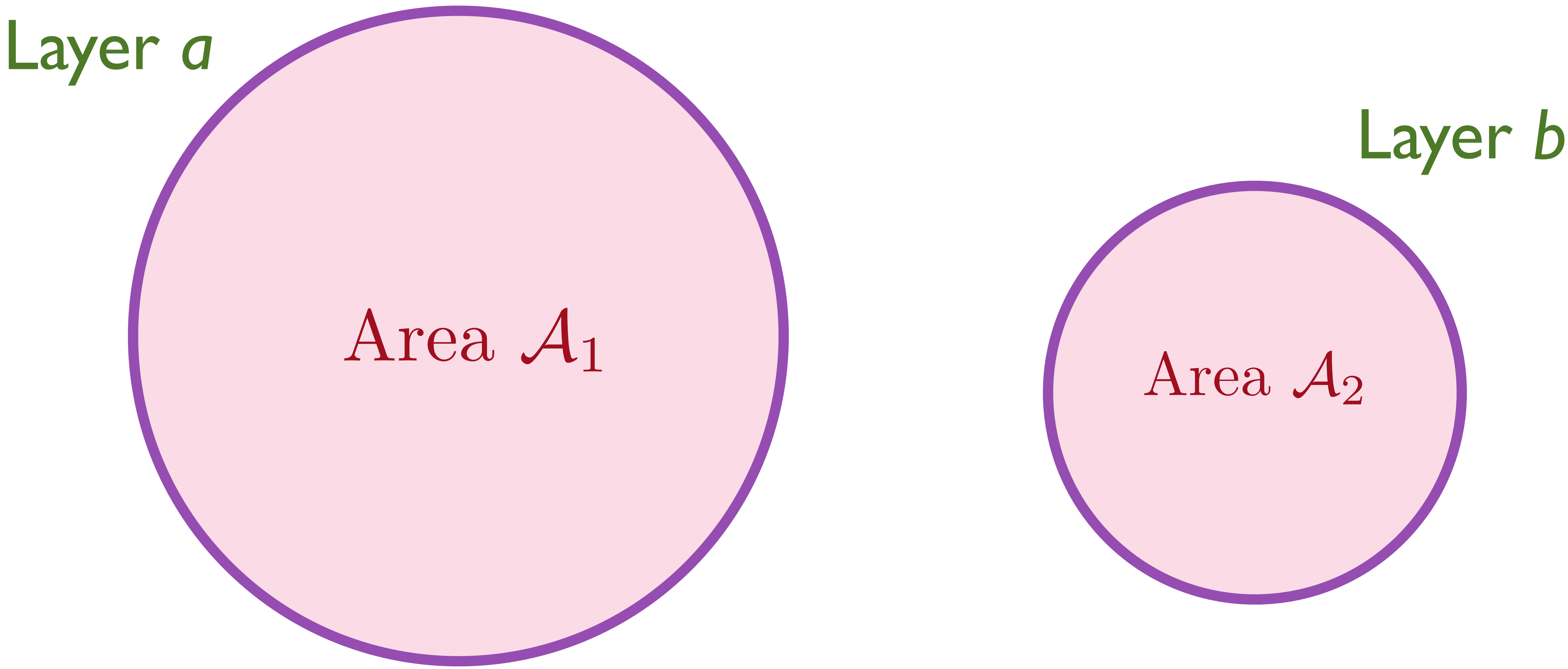


$$
\begin{aligned}
H &= H_a + H_b + H_{ab} \\
H_a &= -t \sum_{\langle ij \rangle} c^\dagger_{ai\alpha} c_{aj\alpha} + \text{H.c.} + (\epsilon_a - \mu) \sum_i (n_{ai\uparrow} + n_{ai\downarrow}) \\
&\quad + U \sum_i \left(n_{ai\uparrow} - \frac{1}{2} \right) \left(n_{ai\downarrow} - \frac{1}{2} \right) \\
H_b &= -t \sum_{\langle ij \rangle} c^\dagger_{bi\alpha} c_{bj\alpha} + \text{H.c.} + (\epsilon_b - \mu) \sum_i (n_{bi\uparrow} + n_{bi\downarrow}) \\
&\quad + U \sum_i \left(n_{bi\uparrow} - \frac{1}{2} \right) \left(n_{bi\downarrow} - \frac{1}{2} \right) \\
H_{ab} &= -w \sum c^\dagger_{ai\alpha} c_{bi\alpha} + \text{H.c.}
\end{aligned}
$$

Fermi liquid (FL) phase

Adiabatically connected to the state at *U=0*

Small w



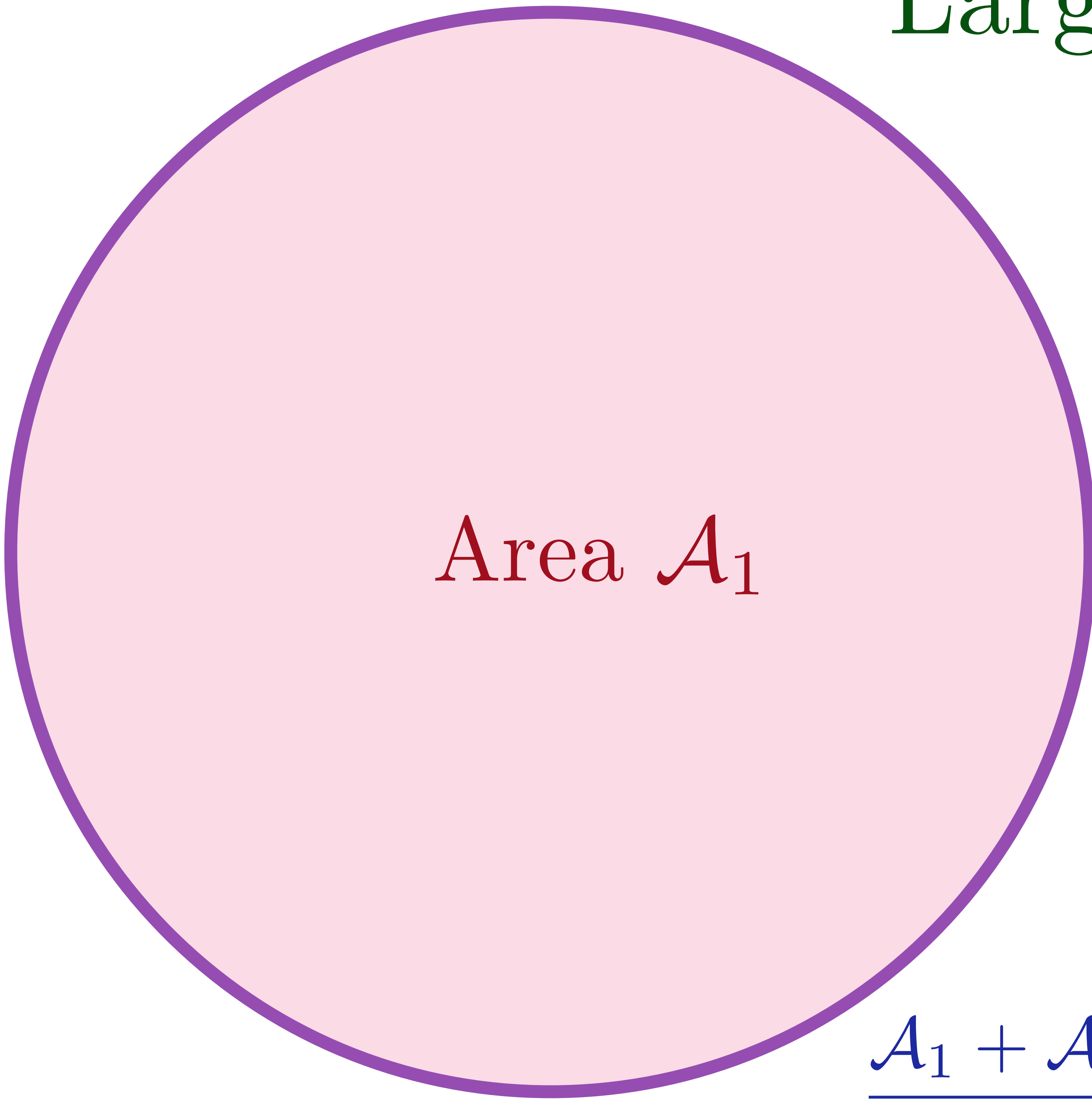
$$\frac{\mathcal{A}_1 + \mathcal{A}_2}{2\pi^2} = \mathcal{N}$$

$\mathcal{N}$ = total density of electrons

Fermi liquid (FL) phase

Adiabatically connected to the state at *U=0*

Large w



$\mathcal{A}_2 = 0$

$$\frac{\mathcal{A}_1 + \mathcal{A}_2}{2\pi^2} = \mathcal{N}$$

$\mathcal{N}$ = total density of electrons

Outline

1. The Kondo lattice model
   The fractionalized Fermi liquid (FL) phase*

2. Mean field theories of the FL* phase
   Infinite-range Kondo lattice models and AdS_2 x R^2

3. Gauge theory of the FL and FL* phases
   U(1)xU(1) theories and semi-holographic phases

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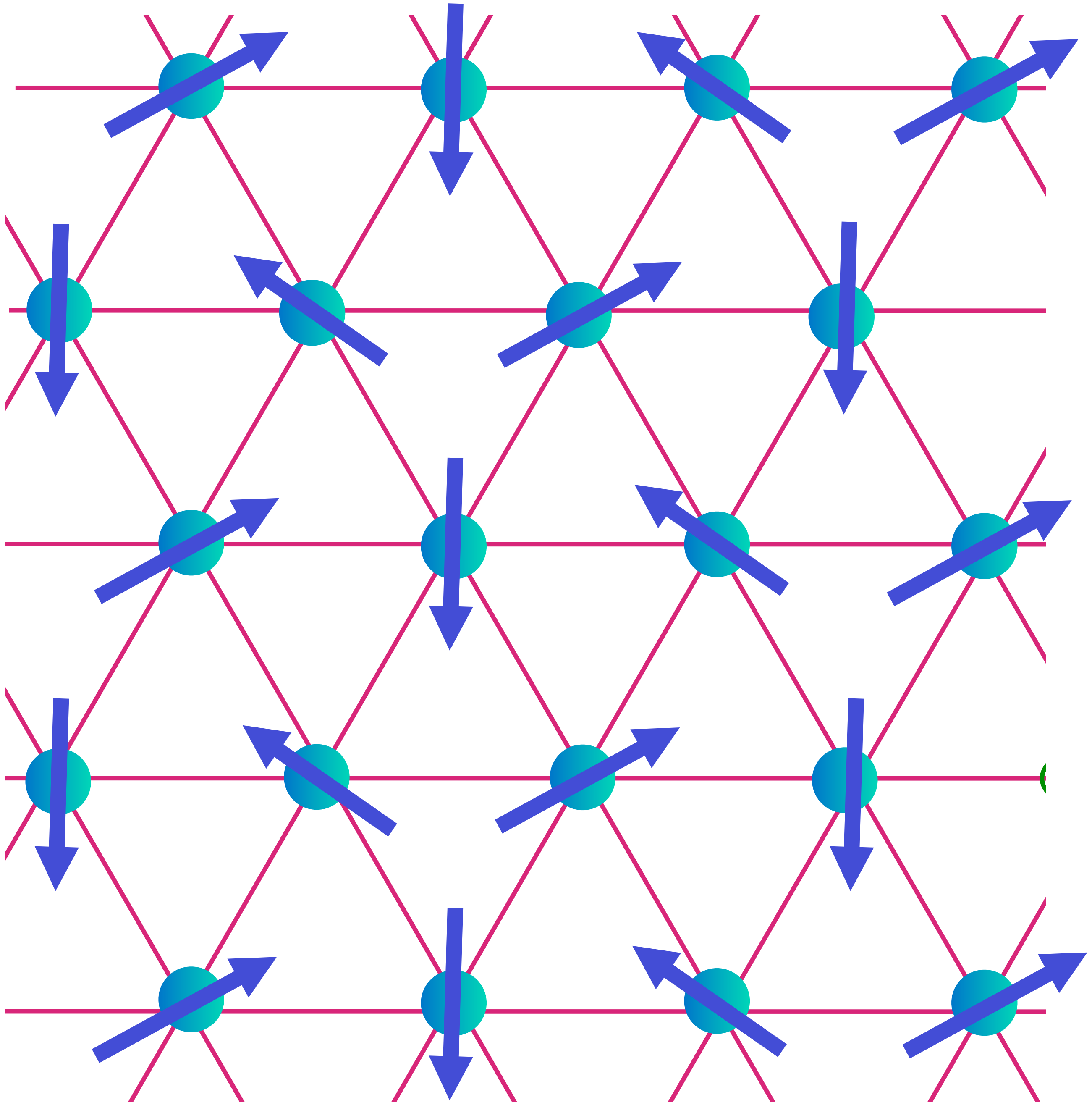
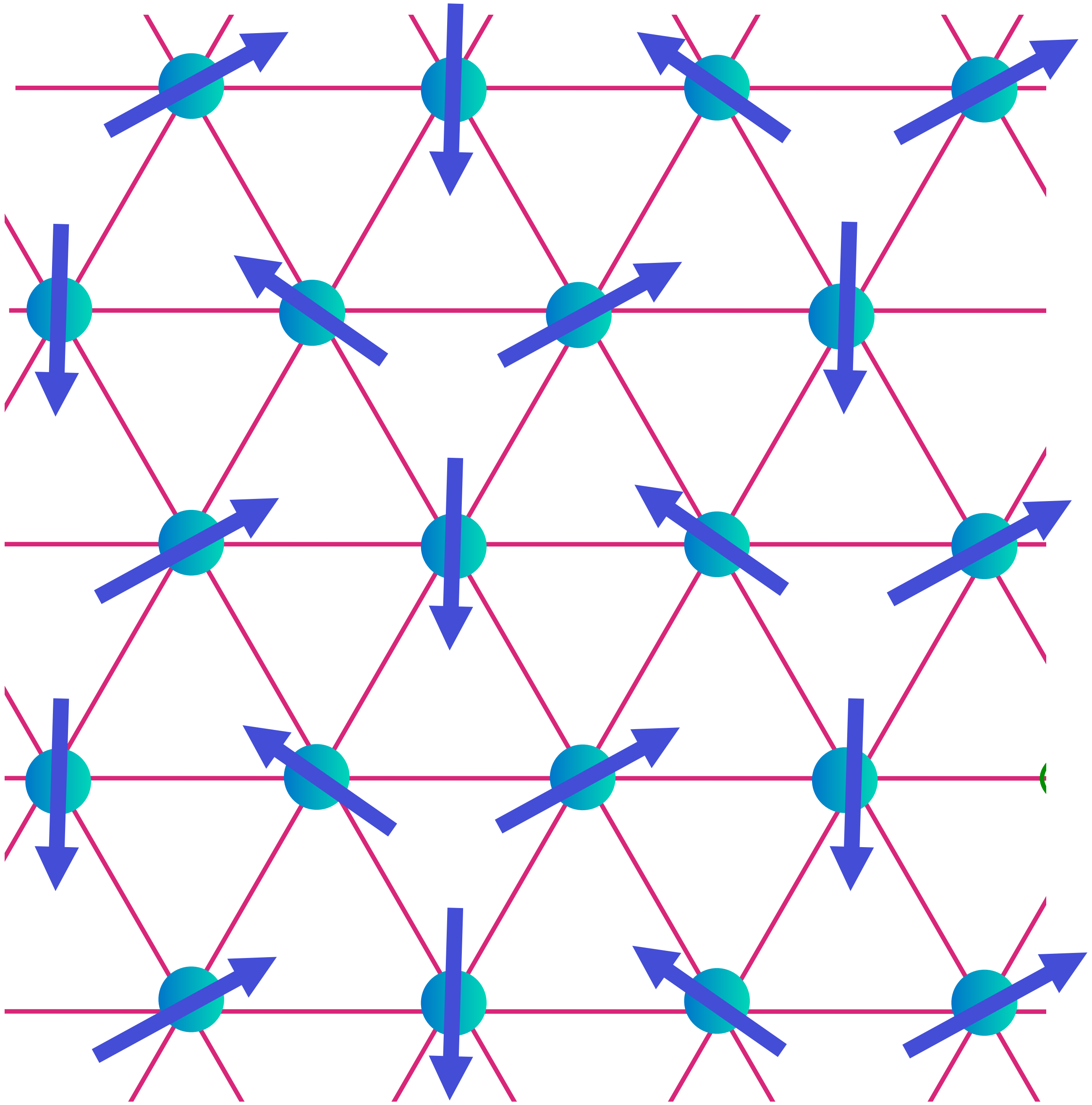
Kondo lattice model



Perform canonical transformation of Hubbard model on layer a to a Heisenberg antiferromagnet

$$\sum_{i<j} J_H(i,j) \vec{S}_i \cdot \vec{S}_j$$

Kondo lattice model

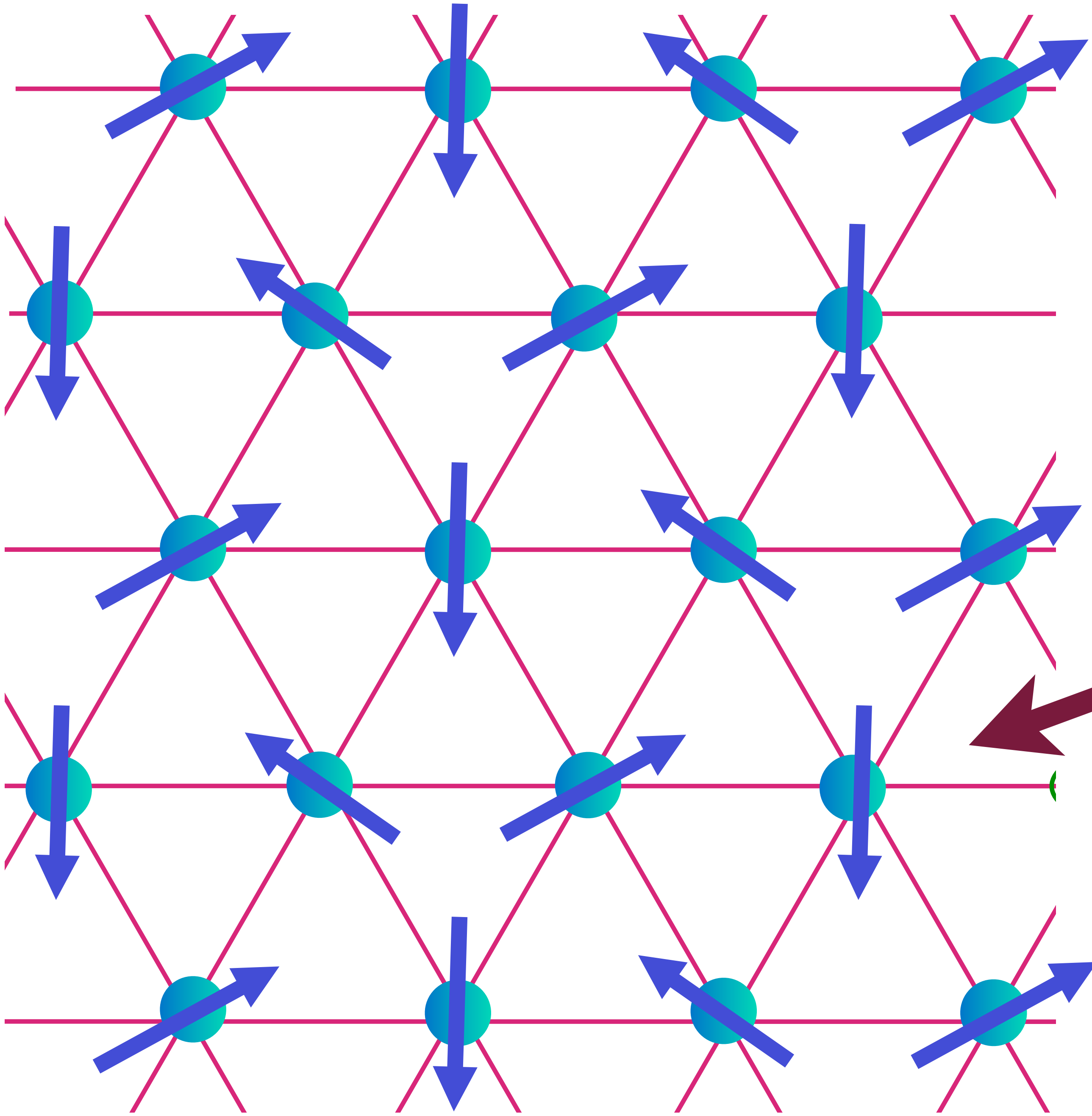


$$\sum_{i<j} J_H(i,j)\vec{S}_i \cdot \vec{S}_j$$

Conduction electrons from layer b

$$\sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} c_{\mathbf{k}\alpha}$$

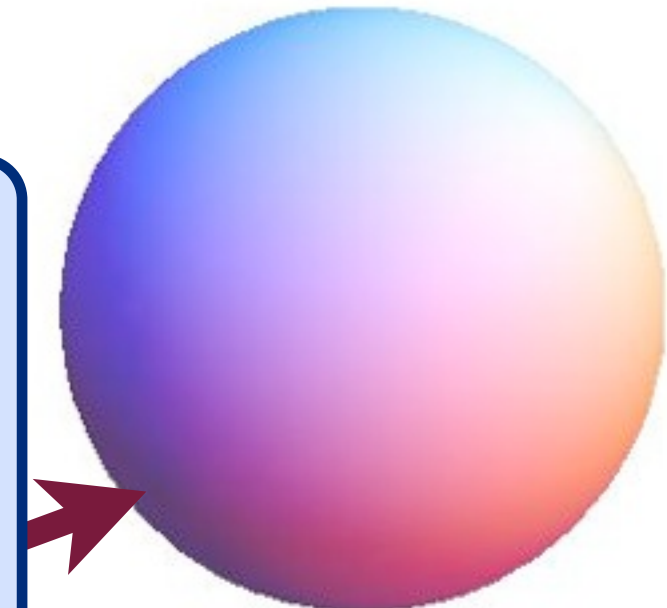
Kondo lattice model



Kondo exchange

$$J_K \sum_i \vec{S}_i \cdot c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}$$

$$J_K \sim \frac{w^2}{U}$$



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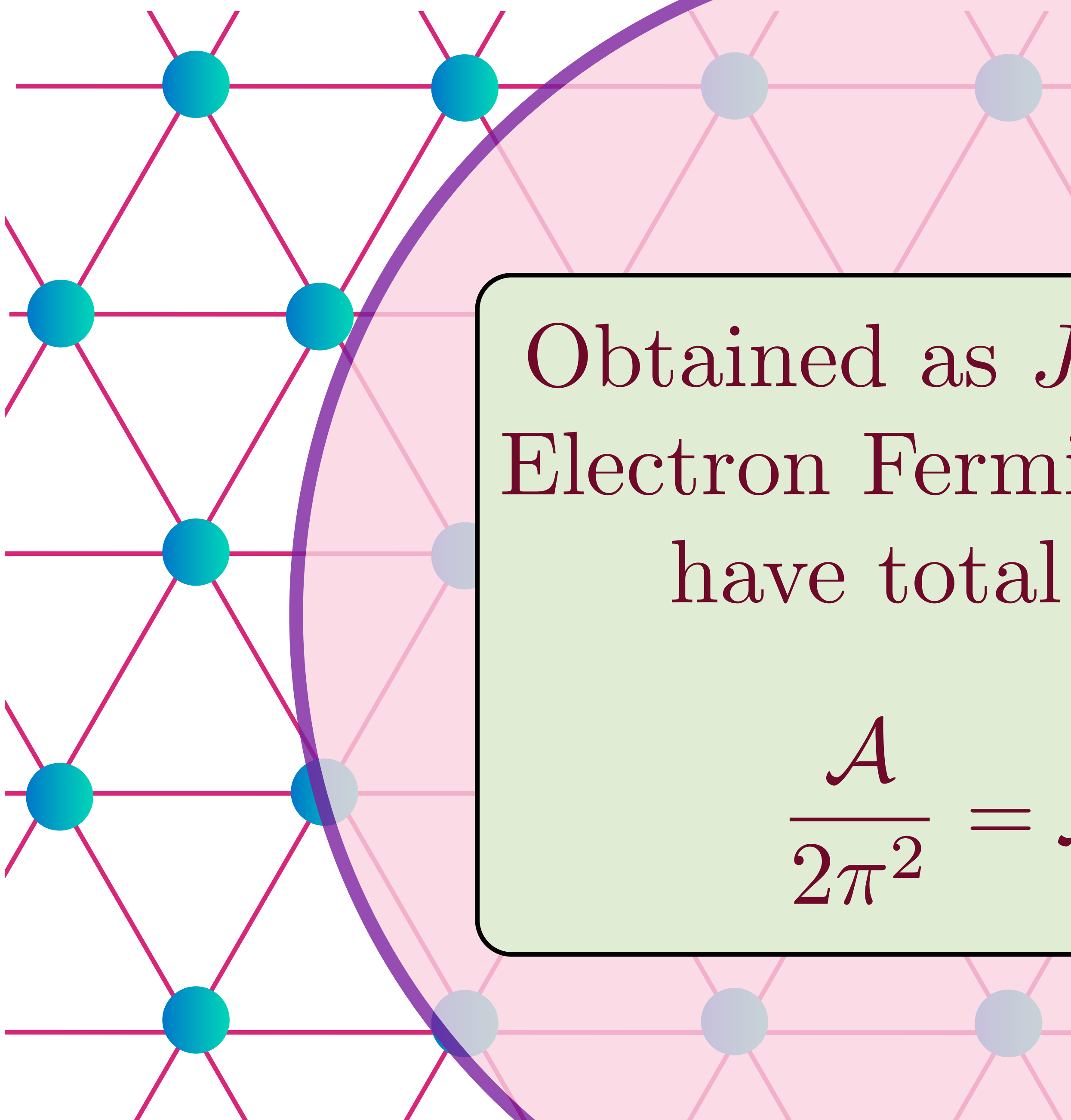


Obtained as $J_K \to \infty$.
Electron Fermi surfaces have total area

$$\frac{\mathcal{A}}{2\pi^2} = \mathcal{N}$$

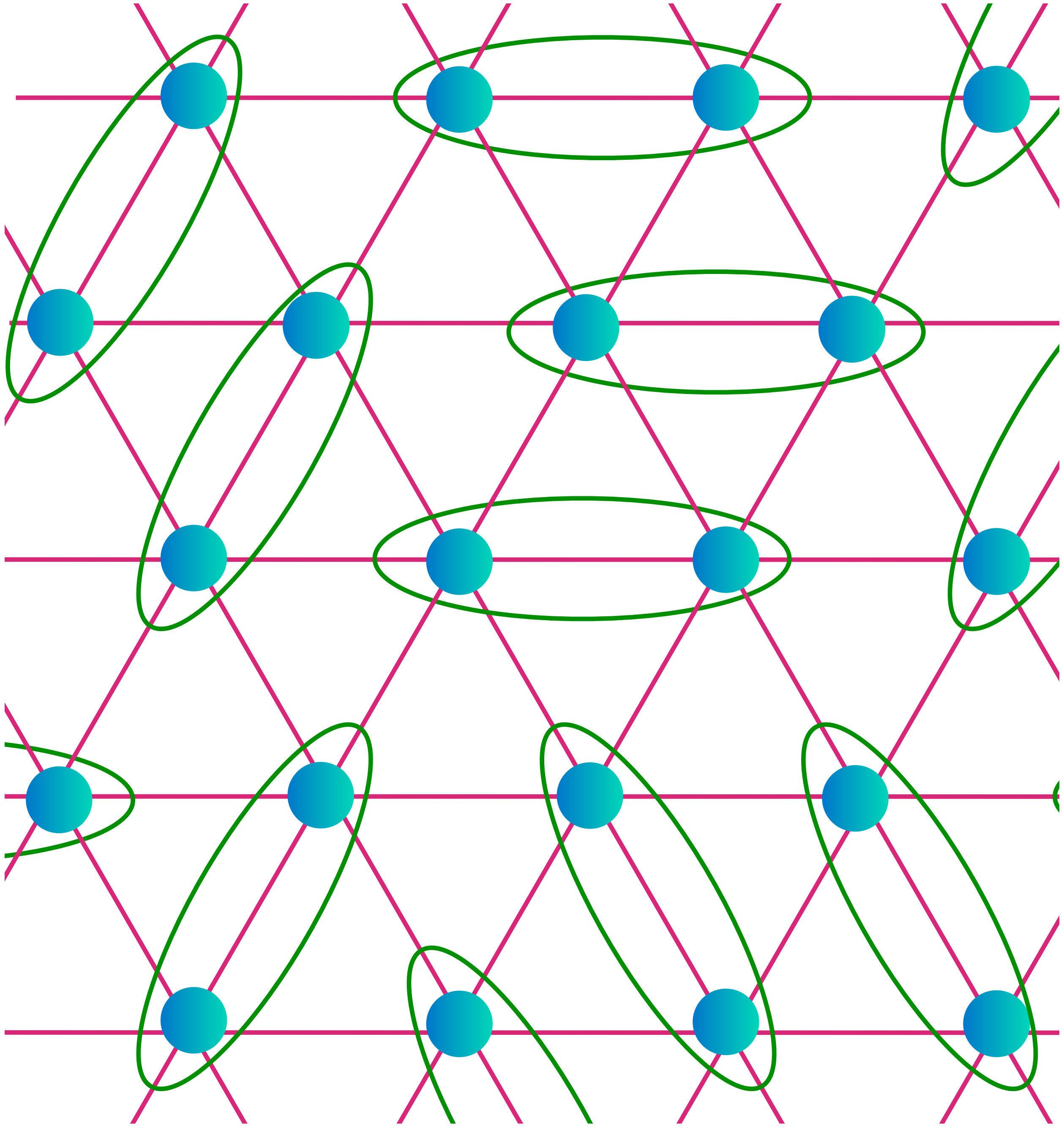
Conduction electrons from layer b

$$\sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^{\dagger} c_{\mathbf{k}\alpha}$$

$$\sum J_H(i,j) \vec{S}_i \cdot \vec{S}_j$$

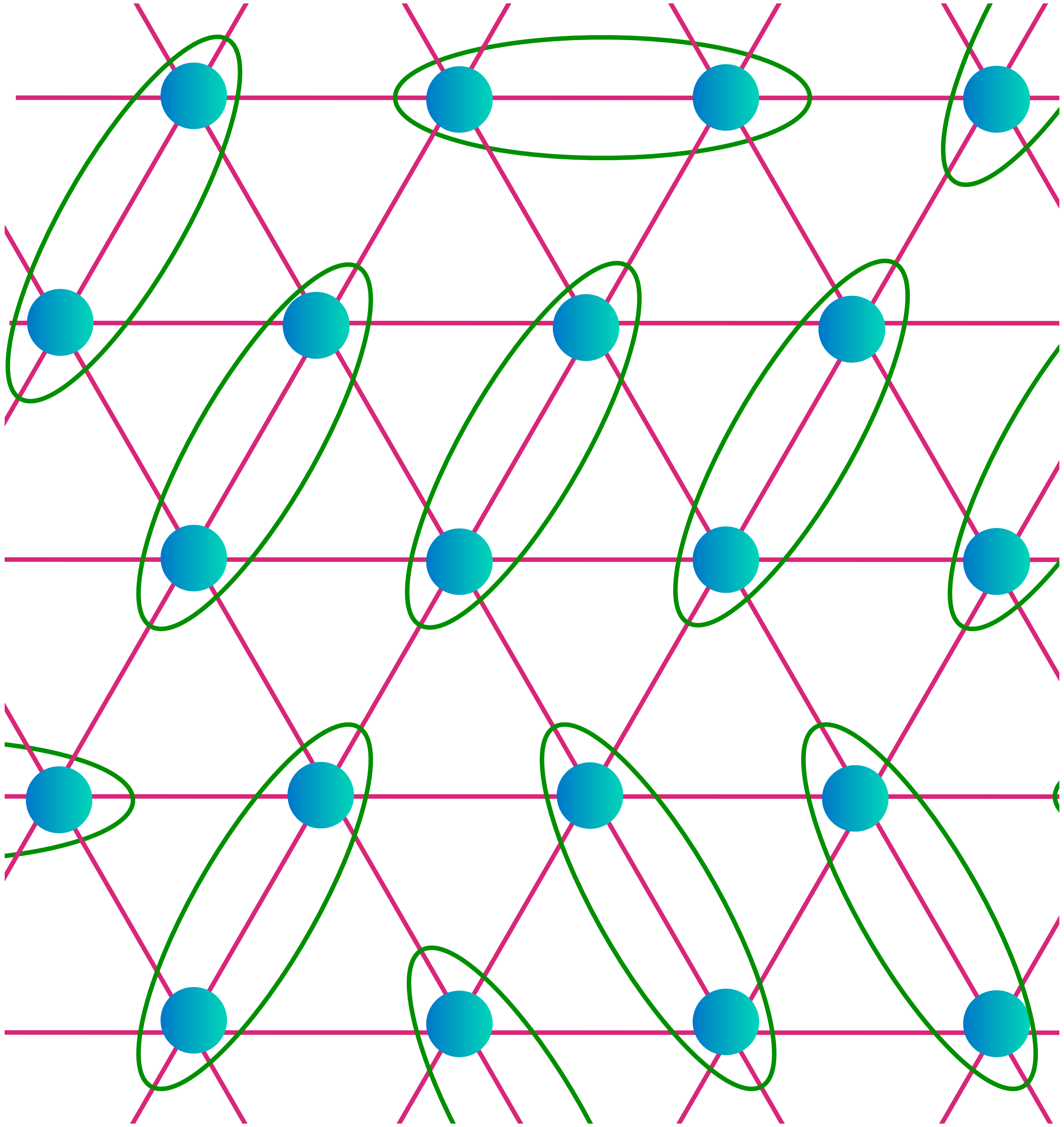
Large Fermi surface Fermi liquid (FL)

Kondo lattice model



$$\sum_{i<j} J_H(i,j)\vec{S}_i \cdot \vec{S}_j$$

Kondo lattice model



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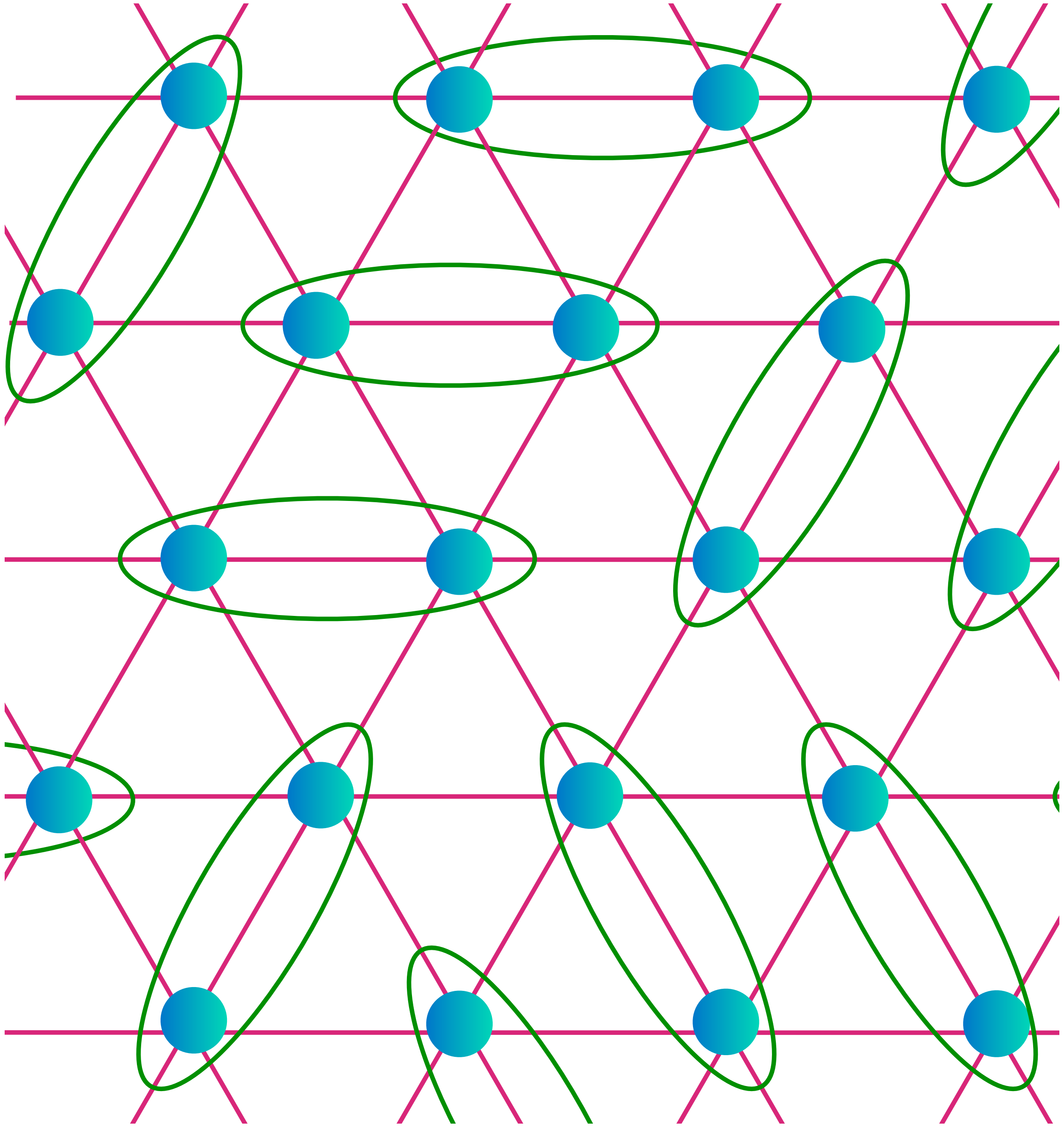
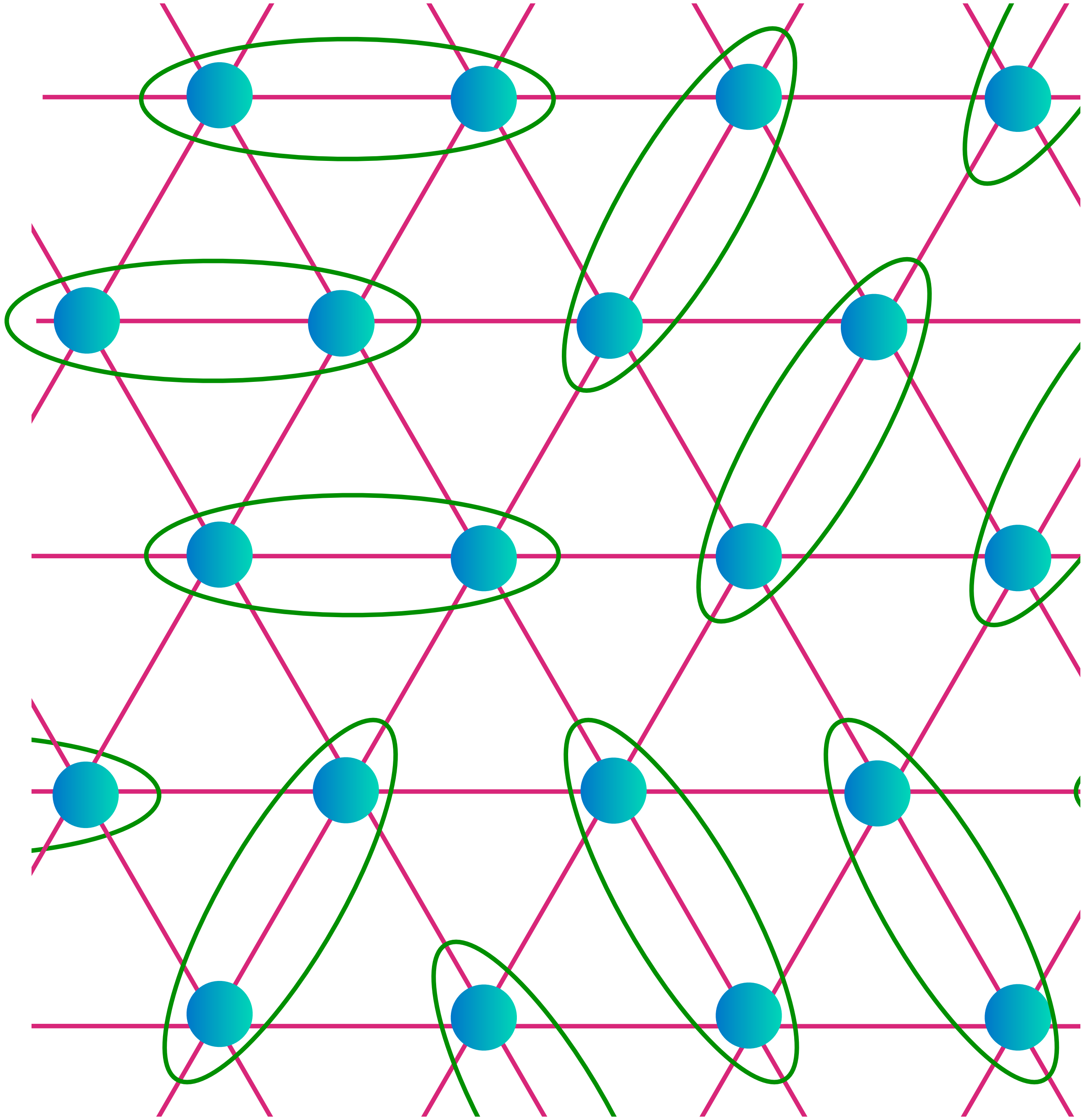
Kondo lattice model



$$\sum_{i<j} J_H(i,j) \vec{S}_i \cdot \vec{S}_j$$

Spin liquid
of electrons
on layer *a*

Kondo lattice model



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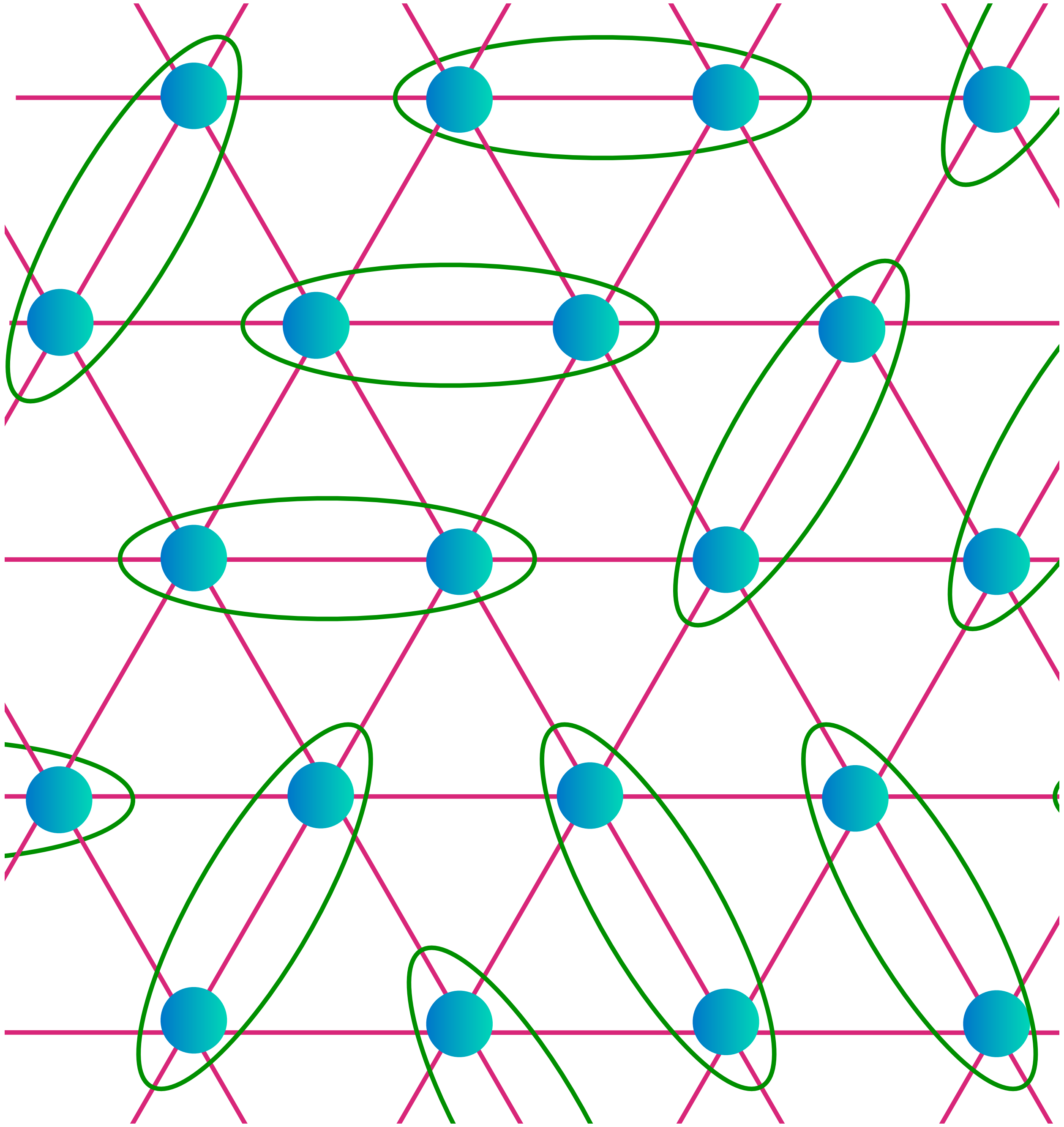

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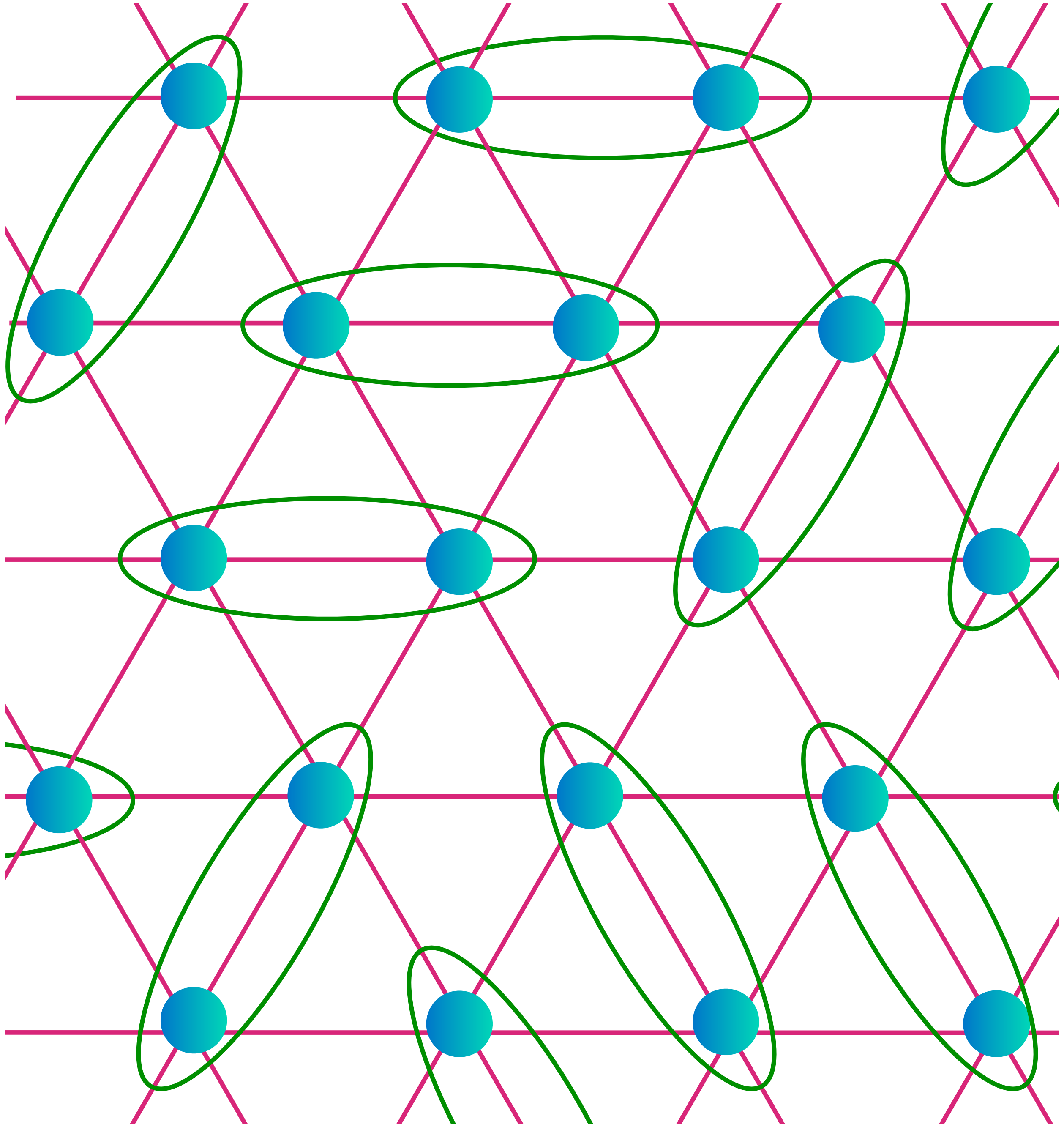

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Conduction electrons from layer b

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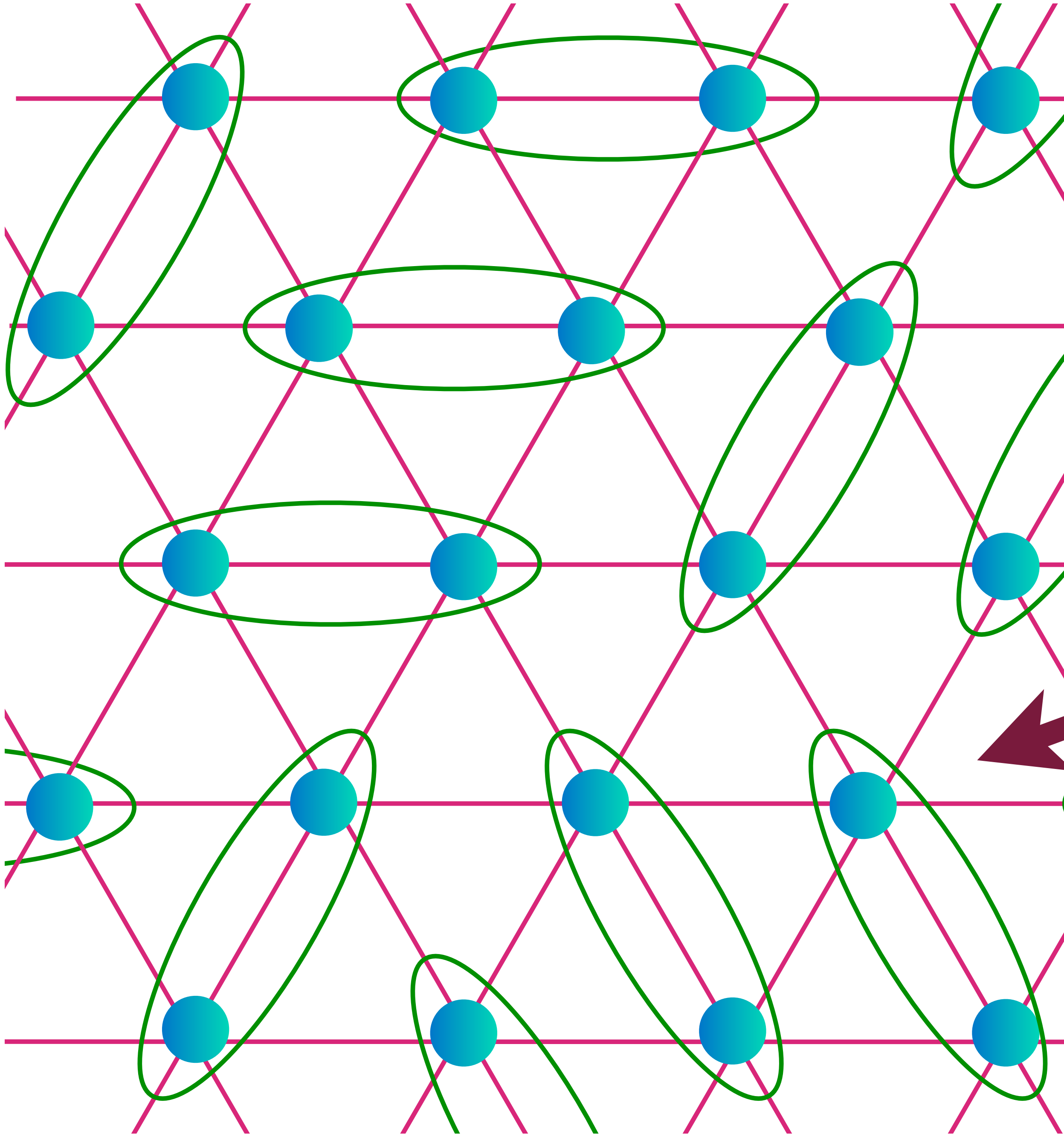
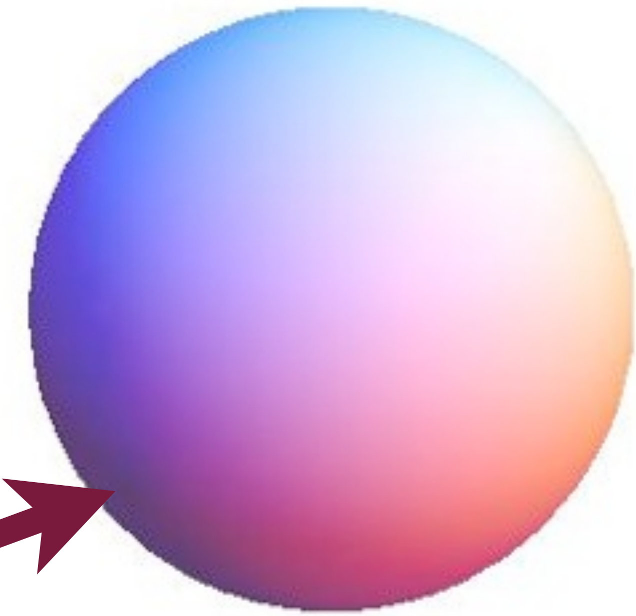
Kondo lattice model



$$\sum_{i<j} J_H(i,j)\vec{S}_i \cdot \vec{S}_j$$

Kondo exchange perturbative



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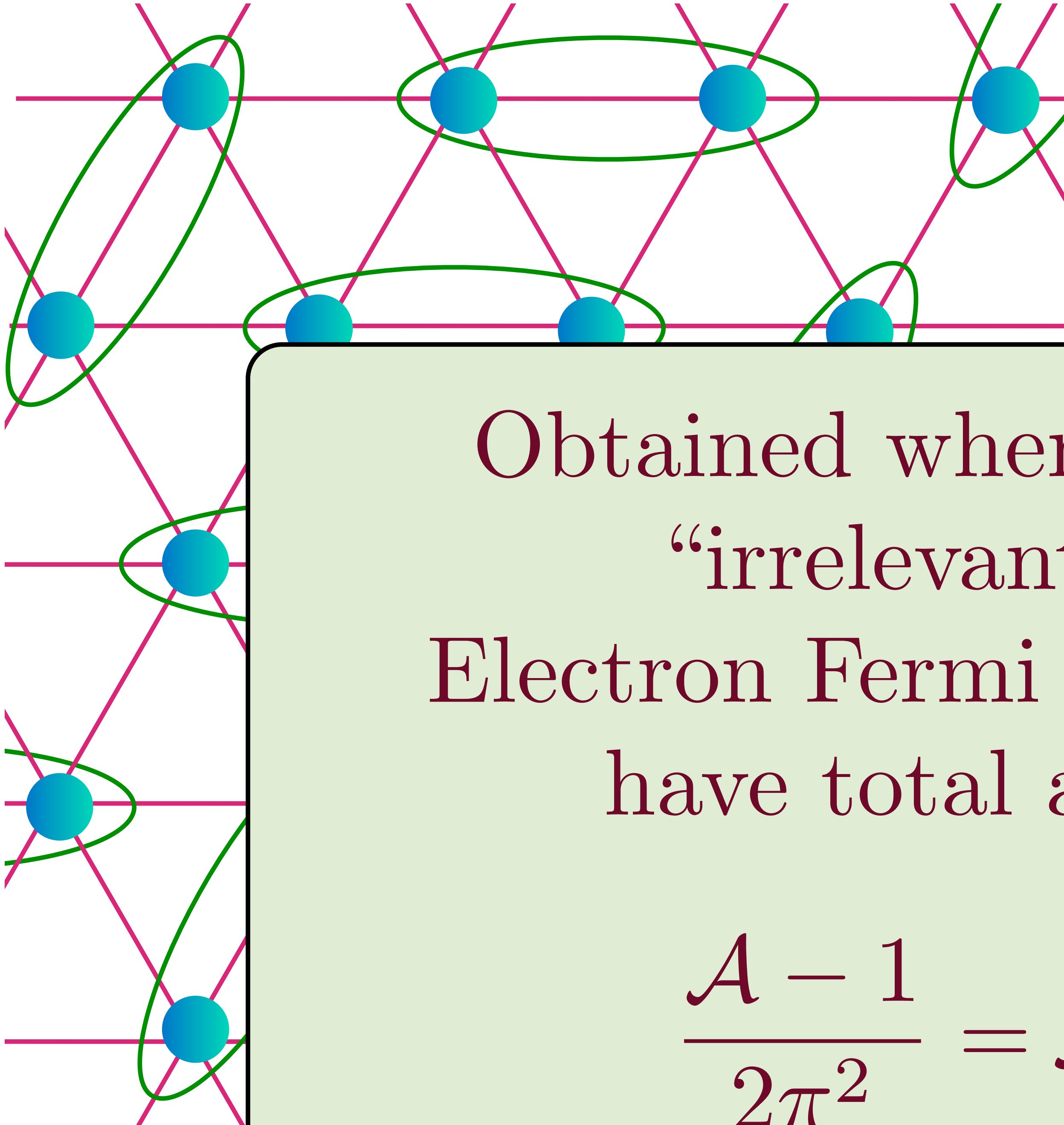

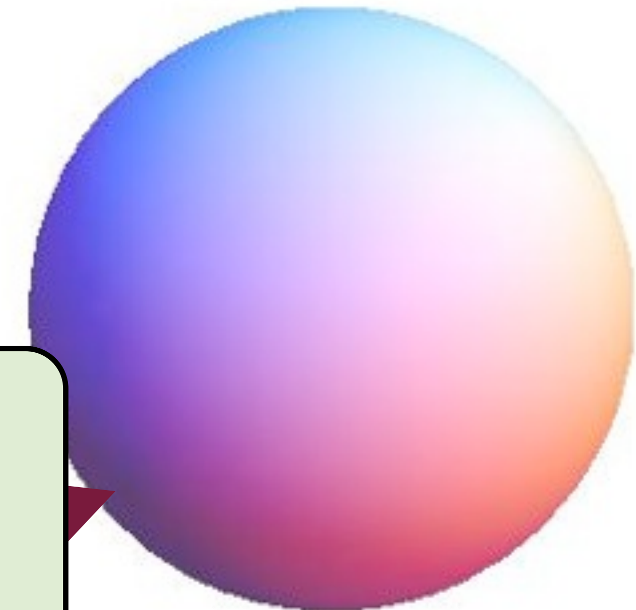

Obtained when J_K is "irrelevant".
Electron Fermi surfaces have total area

$$\frac{\mathcal{A}-1}{2\pi^2} = \mathcal{N}$$

T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003).
T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004).

Conduction electrons from layer b

$$\sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\alpha} c_{\mathbf{k}\alpha}$$

$$\sum_{i<j}$$

Fractionalized Fermi liquid (FL*)

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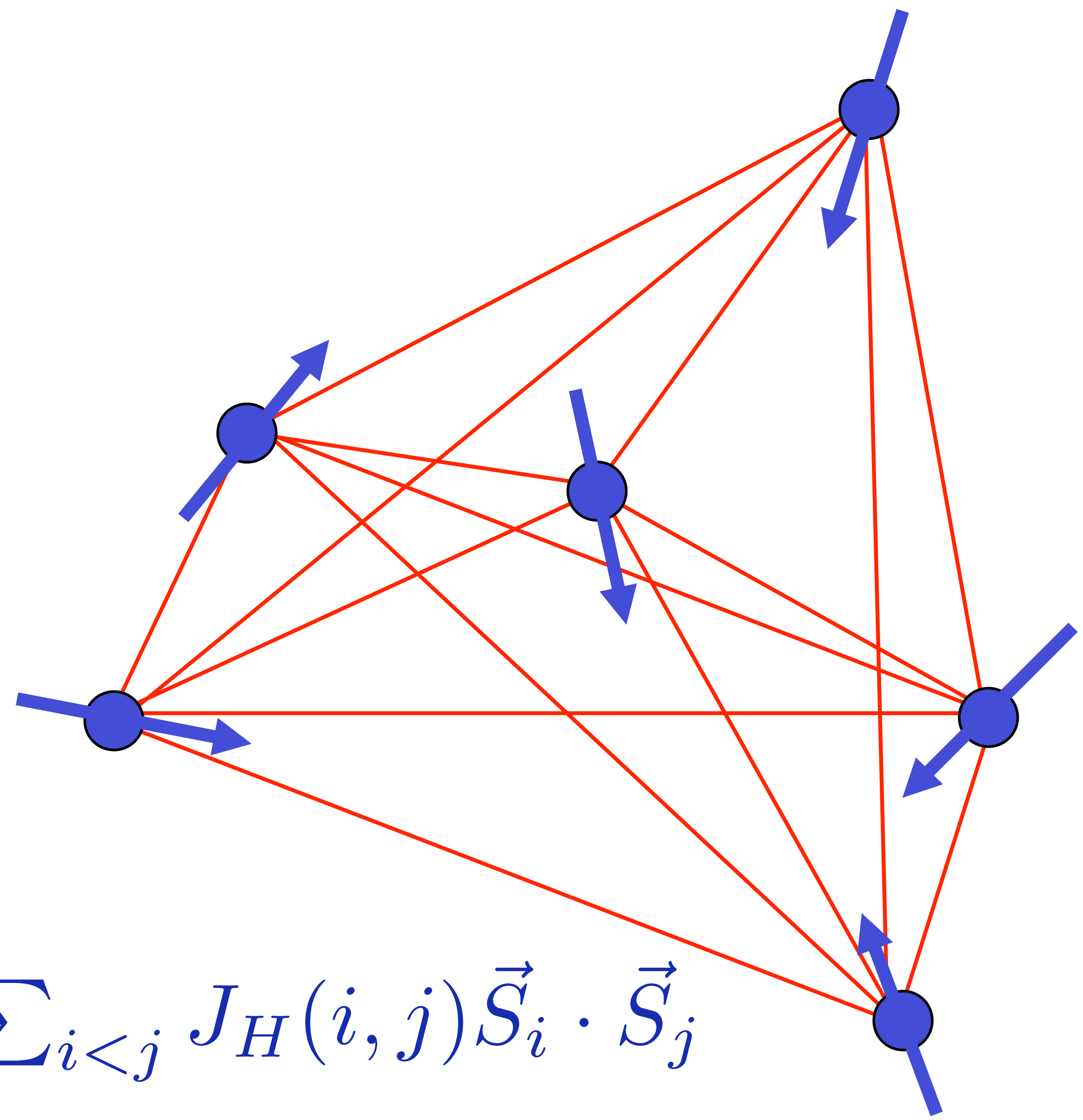
A mean-field theory of a spin liquid



$$\sum_{i<j} J_H(i,j)\vec{S}_i \cdot \vec{S}_j$$

$J_H(i,j)$ Gaussian random variables.
A quantum Sherrington-Kirkpatrick
model of SU(N) spins.

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993).
A. Georges, O. Parcollet, and S. Sachdev, Phys. Rev. B **63**, 134406 (2001).

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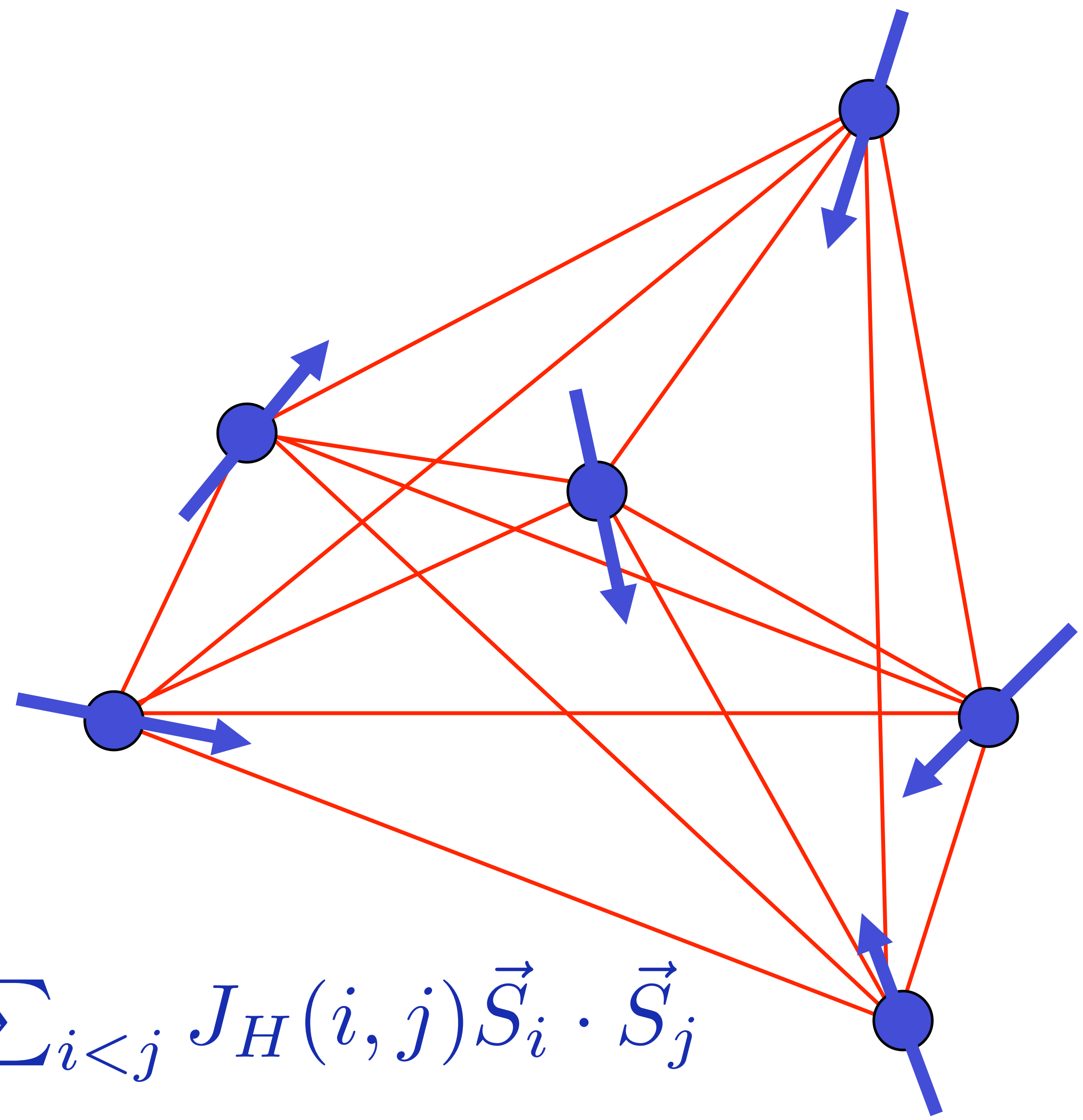

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Described by the quantum mechanics of a spin fluctuating in a self-consistent time-dependent magnetic field: a realization the finite entropy density $\text{AdS}_2 \times R^d$ state

S. Sachdev, *Physical Review Letters* **105**, 151602 (2010)

AdS$_2$ realization in the quantum SK model

Focus on a single $\vec{S}$ spin, and represent its imaginary time fluctuations by a unit vector $\vec{S} = \vec{n}(\tau)/2$ which is controlled by the partition function

$$\mathcal{Z} = \int \mathcal{D}\vec{n}(\tau)\, \delta(\vec{n}^2(\tau) - 1) \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \frac{i}{2}\int_0^1 du \int_0^{1/T} d\tau\, \vec{n} \cdot \left(\frac{\partial \vec{n}}{\partial u} \times \frac{\partial \vec{n}}{\partial \tau} \right) - \int_0^{1/T} d\tau\, \vec{h}(\tau) \cdot \vec{n}(\tau)$$

The first term is a Wess-Zumino term, with the "extra dimension" u defined so that $\vec{n}(\tau, u = 1) \equiv \vec{n}(\tau)$ and $\vec{n}(\tau, u = 0) = (0, 0, 1)$.

The field $\vec{h}(\tau)$ represents the "environment", which which we take to be a Gaussian random variable with the correlation

$$\left\langle \vec{h}(\tau) \cdot \vec{h}(0) \right\rangle = A \left| \frac{\pi T}{\sin(\pi T \tau)} \right|^{\gamma}$$

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AdS$_2$ realization in the quantum SK model

Solution of $\mathcal{Z}$ for such an $\vec{h}(\tau)$ yields

$$\langle \vec{n}(\tau) \cdot \vec{n}(0) \rangle = B \left| \frac{\pi T}{\sin(\pi T \tau)} \right|^h$$

with the exponent $h = 2 - \gamma$. The self-consistency condition for the infinite-range model requires that the two-point correlation of $\vec{h}$ is proporionaly to that of $\vec{n}$. This leads to $h = \gamma$, which implies $h = \gamma = 1$.

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993).
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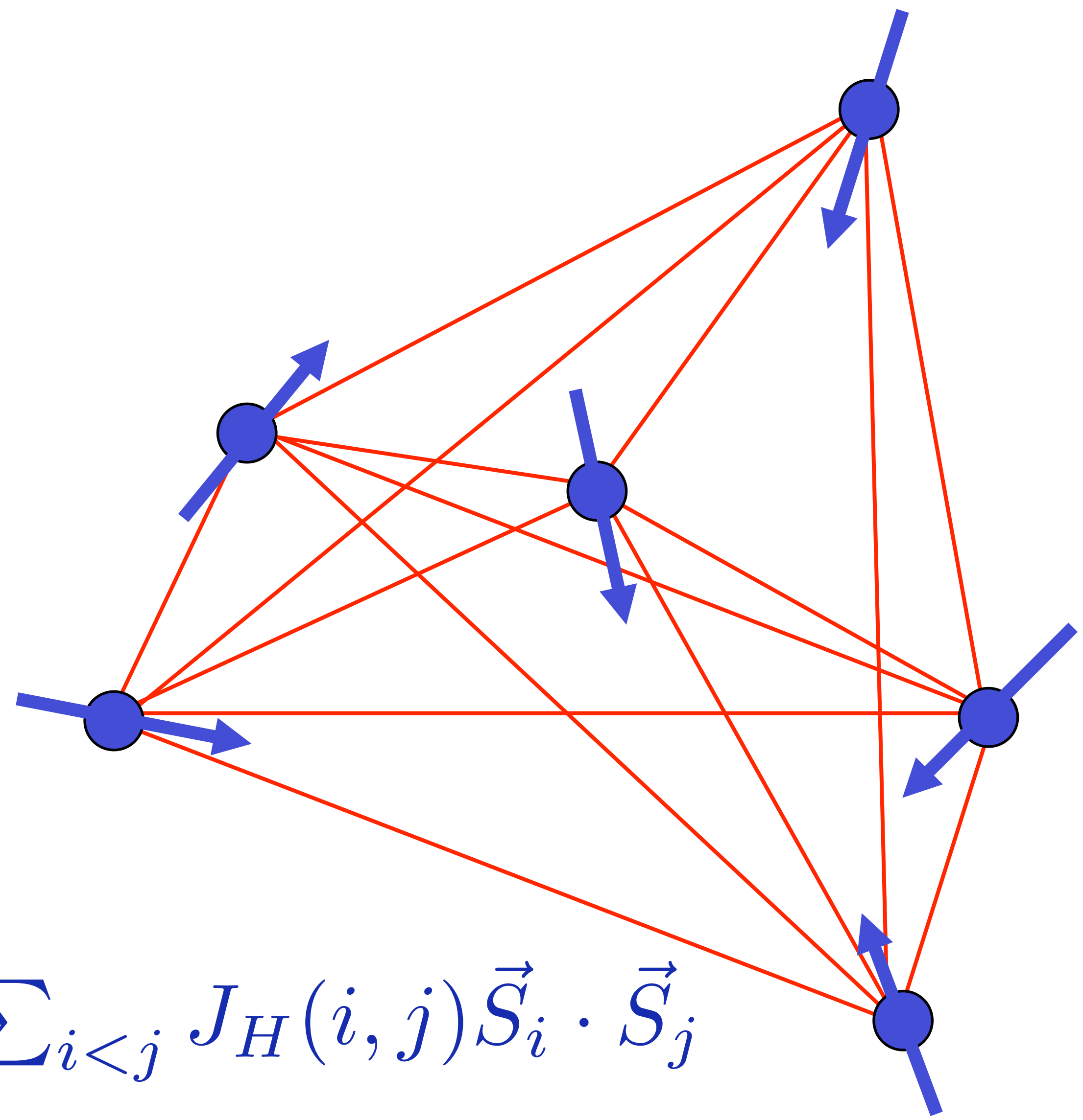

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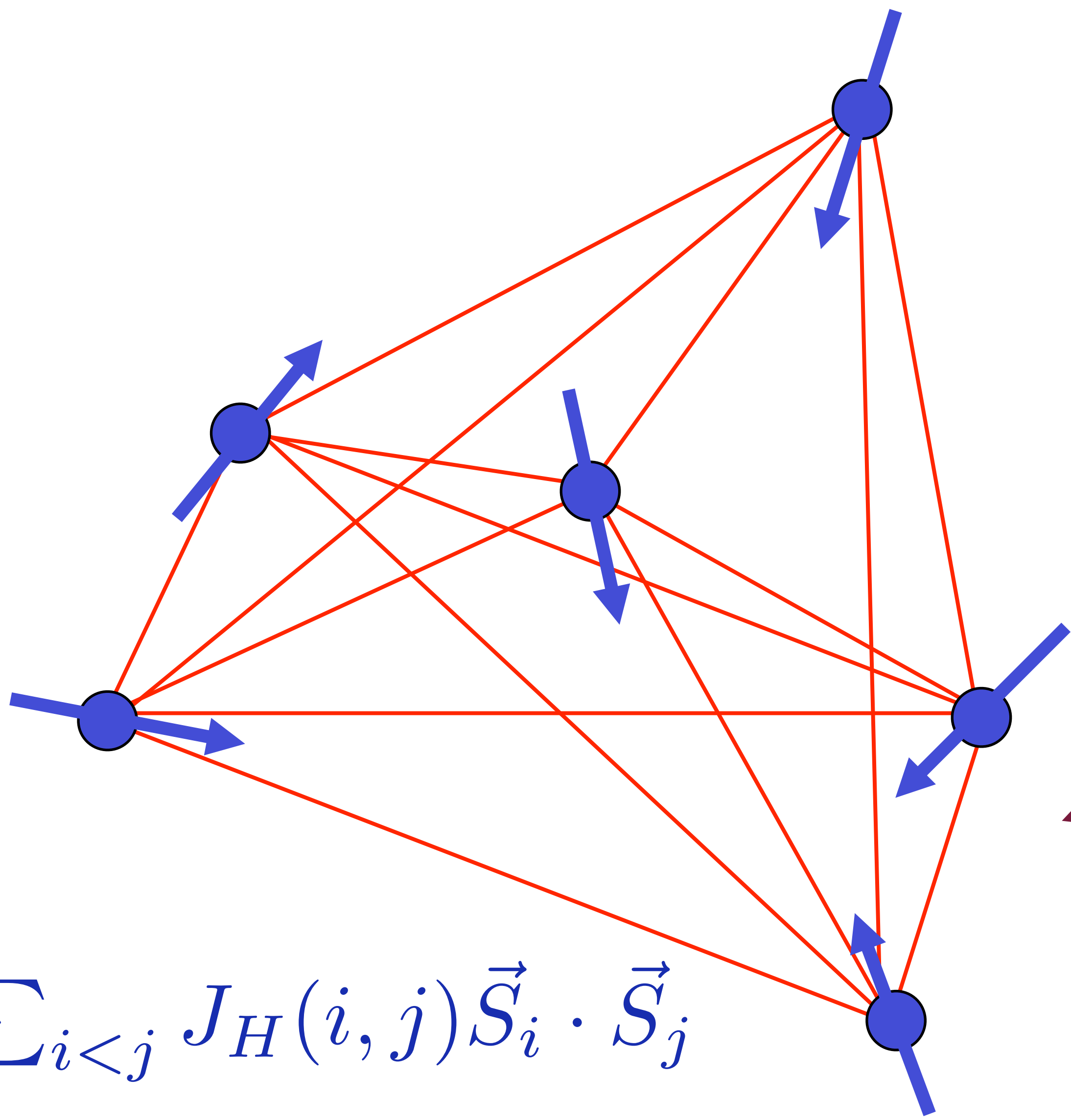
A mean-field theory of FL*



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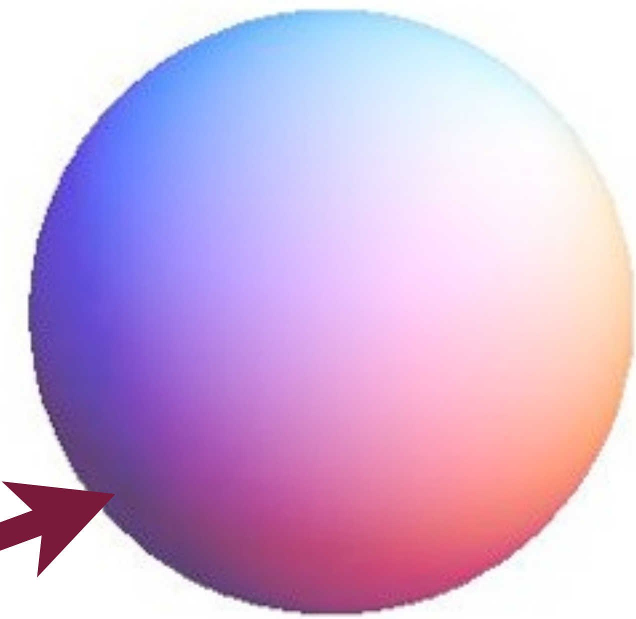
Kondo exchange perturbative



Conduction electrons from layer b

$$\sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\alpha} c_{\mathbf{k}\alpha}$$

S. Burdin, D. R. Grempel, and A. Georges, Phys. Rev. B **66**, 045111 (2002)

Effective low energy theory for conduction electrons

The operators acting on the low energy subspace are layer a electrons c_i and layer b spins $\vec{S}_i$.
For the c_i we have the effective theory

$$\mathcal{S}_c = \int \frac{d^d k}{(2\pi)^d} \int d\tau \left[c_{\mathbf{k}\alpha}^\dagger \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha} + J_K \sum_i \vec{S}_i \cdot c_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta} \right]$$

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Here the $F_{i\alpha}$ are strongly renormalized operators on layer b, which project onto the low energy theory as

$$F_{i\alpha} = \left(\vec{\sigma}_{\alpha\beta} \cdot \vec{S}_i \right) c_{i\beta}$$

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From this we obtain the conduction electron self energy

$$\Sigma_c(\tau) \sim \left[\frac{\pi T}{\sin(\pi T \tau)} \right]^{h+1}$$

This is the marginal Fermi liquid form for $h = 1$.

Connection to semi-holographic metals

- The quantum SK model has $z = \infty$ conformal spin correlations and a finite ground state entropy density: similar to $\mathrm{AdS}_2 \times \mathrm{R}^d$.

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where $F_{i\alpha}$ are operators probing the $z = \infty$ correlations of $\mathrm{AdS}_2 \times \mathrm{R}^d$ (T. Faulkner and J. Polchinski, arXiv:1001.5049.)

Connection to semi-holographic metals

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$$F_{i\alpha} \sim \frac{1}{U} \left(\vec{\sigma}_{\alpha\beta} \cdot \vec{S}_{fi} \right) c_{i\beta}$$

- This leads to a 'probe fermion' self energy which is identical to the $\mathrm{AdS}_2 \times R^d$ theory of the holographic metal (T. Faulkner, H. Liu, J. McGreevy and D. Vegh, arXiv:0907.2694.)

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Gauge theory of FL and FL* phases

We want to keep better track of the charge on layer a. For this we introduce a 'fictitious' quantum rotor on each a lattice site. Each rotor has a periodic angular co-ordinate ϑ_i with period 2π; hence the states of the rotors are $e^{in_{ri}\vartheta_i}$ where n_{ri} is a rotor angular momentum, which takes all positive and negative integer values. We will use the state with all $n_{ri} = 0$ to represent the states with one electron each a lattice site. Then we write

$$c_{a\alpha} = e^{-i\vartheta} f_\alpha \qquad (1)$$

where we have dropped the implicit site index, and f_α are neutral fermions ('spinons') which keep track of the orientation of the electron.

We can now identify the 4 states on each *a* lattice site with states of the rotor and spinons:

$$\begin{aligned} |0\rangle \quad &\Leftrightarrow \quad e^{-i\vartheta}|0\rangle \\ c^\dagger_{a\alpha}|0\rangle \quad &\Leftrightarrow \quad f^\dagger_\alpha|0\rangle \\ c^\dagger_{a\uparrow}c^\dagger_{a\downarrow}|0\rangle \quad &\Leftrightarrow \quad e^{i\vartheta}f^\dagger_\uparrow f^\dagger_\downarrow|0\rangle \end{aligned} \tag{2}$$

Note that these allowed states obey the constraint

$$f^\dagger_\alpha f_\alpha - n_r = 1. \tag{3}$$

Associated with this constraint is the U(1) gauge invariance

$$f_\alpha \to f_\alpha e^{i\zeta} \quad , \quad \vartheta \to \vartheta + \zeta. \tag{4}$$

We can now write down an effective continuum U(1) gauge theory which captures the low energy physics of the Hubbard model. The degrees of freedom are the b layer electrons c_α, the a layer spinons f_α, and the bosonic rotors

$$b \sim e^{-i\vartheta}. \tag{5}$$

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}_f + \mathcal{L}_b + \mathcal{L}_c \\
\mathcal{L}_f &= f_\alpha^\dagger \left[\frac{\partial}{\partial \tau} + \epsilon_f - iA_\tau - \frac{1}{2m_f} (\boldsymbol{\nabla} - i\mathbf{A})^2 \right] f_\alpha \\
\mathcal{L}_b &= \left[(\partial_\mu - (\epsilon_r - \mu)\delta_{\mu\tau} - iA_\mu + iA_{\mathrm{ext},\mu})\, b^\dagger \right] \\
&\quad \times \left[(\partial_\mu + (\epsilon_r - \mu)\delta_{\mu\tau} + iA_\mu - iA_{\mathrm{ext},\mu})\, b \right] + s|b|^2 + u|b|^4 \\
\mathcal{L}_c &= c_\alpha^\dagger \left[\frac{\partial}{\partial \tau} - \mu - iA_{\mathrm{ext},\tau} - \frac{1}{2m_c} (\boldsymbol{\nabla} - i\mathbf{A}_{\mathrm{ext}})^2 \right] c_\alpha \\
&\quad - w \left(c_\alpha^\dagger b f_\alpha + b^\dagger f_\alpha^\dagger c_\alpha \right)
\end{aligned} \tag{6}$$

Here $A_\mu = (A_\tau, \mathbf{A})$ is an emergent U(1) gauge field; we have also introduced a non-fluctuating electromagnetic gauge field $A_{\mathrm{ext},\mu}$ as a source term which couples to the current of the globally conserved electromagnetic charge.

The continuum theory in Eq. (6) has a $\mathrm{U}(1)\times\mathrm{U}(1)_{\rm ext}$ symmetry associated with the transformations

$$\begin{array}{ccccc} f_\alpha \to f_\alpha e^{i\zeta} & , & b \to b^{-i\zeta} & , & c_{b\alpha} \to c_{b\alpha} \\ f_\alpha \to f_\alpha & , & b \to b^{i\tilde{\zeta}} & , & c_{b\alpha} \to c_{b\alpha} e^{i\tilde{\zeta}} \end{array} \tag{7}$$

There are Fermi surface area constraints associated with these two U(1) symmetries:

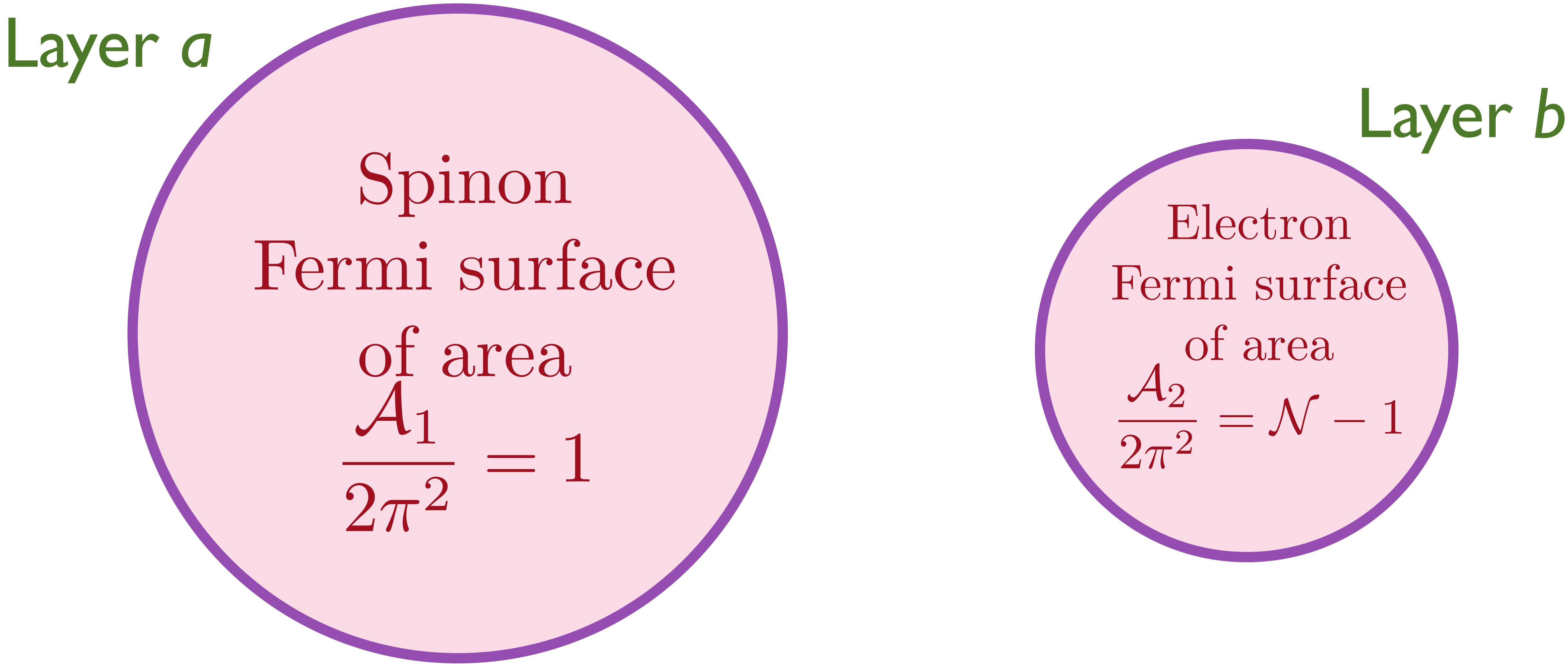
$$\sum_\alpha \left\langle f_\alpha^\dagger f_\alpha \right\rangle - \left\langle \frac{\partial \mathcal{L}_b}{\partial \mu} \right\rangle = \frac{\mathcal{A}_1}{2\pi^2} = \mathcal{N}_a. \tag{8}$$

$$\sum_\alpha \left\langle c_{b\alpha}^\dagger c_{b\alpha} \right\rangle + \left\langle \frac{\partial \mathcal{L}_b}{\partial \mu} \right\rangle = \frac{\mathcal{A}_2}{2\pi^2} = \mathcal{N} - \mathcal{N}_a. \tag{9}$$

$\mathcal{N}_a$ is the density of electrons on layer a in the projected Hilbert space: our present lattice derivation was for $\mathcal{N}_a = 1$, but the continuum theory in Eq. (6) is sensible for any value of $\mathcal{N}_a$. The operator $\partial\mathcal{L}_b/\partial\mu$ is the rotor angular momentum.

These two area constraints apply only if the $\mathrm{U}(1)\times\mathrm{U}(1)_{\mathrm{ext}}$ symmetry is not spontaneously broken. This is the case only in the FL* phase:

$$\boxed{\langle b \rangle = 0 \text{ in the FL* phase.}} \qquad (10)$$



In the FL phase the U(1)$\times$U(1)$_{\rm ext}$ symmetry is broken down to a diagonal U(1) because

$$\boxed{\langle b \rangle \neq 0 \text{ in the FL phase.}} \tag{11}$$

Only the *sum* of the constraints in Eqs. (8) and (9) applies, and this leads to expected area constraint.

Electron
Fermi surface(s)
of total area

$$\frac{\mathcal{A}_1}{2\pi^2} = \mathcal{N}$$

Connections to semi-holographic RG

FL phase

In the FL phase, we condense the b boson, and focus on the fluctuations of its phase $b = e^{-i\vartheta}$. Then the effective theory of the FL phase of Eq. (6) is

$$\mathcal{L}_{FL} = K \left(\partial_\mu \vartheta - A_\mu + A_{\mathrm{ext},\mu}\right)^2 + \Pi_f(A_\mu) + \mathcal{L}_c \tag{12}$$

where Π_f is the effective action obtain after integrating out the f spinons

The structure of Eq. (12) is nearly identical to the semi-holographic of metals by Nickel and Son. They argued that there is generically an emergent gauge field A_μ which links the UV fields (the 'electrons', c_α) to the IR fields near the horizon (the 'spinons', f_α). In addition, they had a Goldstone boson which broke the U(1)$\times$U(1)$_{\mathrm{ext}}$ symmetry to U(1) (the rotor b)

Connections to semi-holographic RG

FL* phase

Now the b field is not condensed, and so we can integrate it out, and obtain an effective theory for the electrons and the spinons

$$\mathcal{L}_{FL*} = \mathcal{L}_f + J_K \left(c^\dagger_\alpha \sigma^a_{\alpha\beta} c_\beta \right) \left(f^\dagger_\gamma \sigma^a_{\gamma\delta} f_\delta \right) + \mathcal{L}_c \qquad (13)$$

We can now rewrite this as

$$\mathcal{L}_{FL*} = \mathcal{L}_f - \frac{J_K}{2} \left[F^\dagger_\alpha c_\alpha + c^\dagger_\alpha F_\alpha \right] + \mathcal{L}_c \qquad (14)$$

where F_α is a IR fermion invariant under the emergent U(1)

$$F_\alpha \equiv - \left(\sigma^a_{\alpha\beta} f^\dagger_\gamma \sigma^a_{\gamma\delta} f_\delta \right) c_\delta \qquad (15)$$

This is precisely the semi-holographic thery of Faulkner and Polchinski.