

# New nuclear clusters

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The behavior of the mesons in the nuclear medium attracts attention rather long time. According to the fundamental theory this phenomenon sheds light on the nonperturbative sector of QCD. From the phenomenological point of view it is interesting to clarify and predict the modification of the vacuum properties of mesons, such as the mass and the width, in the nuclear medium. Below we will discuss a bit different aspect of the meson- nuclear interaction, particularly, the possibility of formation of new nuclear clusters under attraction of the meson and the nucleon.

According to the different models of the vector meson-nuclear interaction [1-4] there is attraction of a  $\phi$ - meson and a nucleon at low energies. The intensity of that attraction certainly in a large extent depends on the model which is used in the calculations. The physical nature of this attraction may have the same origin as attraction in the  $K^-N$  system, where the  $\Lambda(1405)$  and  $\Sigma(1385)$  subthreshold resonances contribute. Indeed the location of the two body  $K + \Lambda(1405)$  subthreshold state with respect to the  $\phi N$  threshold has the same order of magnitude as the location of the  $\Lambda(1405)$  with respect to the  $K^-N$  threshold.

Having this in mind we will consider the three particle systems like  $\phi NN$  and  $\phi\phi N$  focusing ourself on the existence of the bound states. The four body ( $2\phi 2N$ ) system will be considered with the folding model. In the treatment of the  $\phi NN$  and  $\phi\phi N$  systems the Faddeev equations in the differential form (i.e. in coordinate space) will be used.

First, let us shortly remind what the Faddeev equations are and why they are necessary.

It turns out that the Lippmann- Schwinger equation for the three or more particles has the non-square-integrable kernels for the pairwise potentials. Indeed, in the case of the three particles the matrix element of the pair potential  $V$  has a form

$$\langle \vec{p} \vec{q} | V | \vec{p}' \vec{q}' \rangle = \langle \vec{p} | V | \vec{p}' \rangle \delta(\vec{q} - \vec{q}')$$

here  $\vec{p}$ ,  $\vec{q}$  are the Jacobi variables for the three- body system. To avoid this problem one should rearrange the equation in order to get the fredholmian kernel. For this purpose let us introduce the components of the total wave function  $\Psi$  in the following way:

$$\Psi_{\alpha} = G_0(E) V_{\alpha} \Psi \quad (1)$$

with the condition

$$\sum_{\alpha=1}^3 \Psi_{\alpha} = \Psi \quad (2)$$

Inserting (2) into (1) and transposing the "diagonal" term in the equation from the right to the left hand side one arrives with Faddeev equations for the components in the differential form

$$(E - H_{\alpha})\Psi_{\alpha} = V_{\alpha} \sum_{\beta \neq \alpha} \Psi_{\beta} \quad (3)$$

here  $H_{\alpha} = H_0 + V_{\alpha}$  are the so called channel hamiltonians.



Inverting the operator on the left hand side of the equation (3) and combining the result with the Lippmann-Schwinger equation for the two-body  $t$ -matrix

$$t_\alpha = V_\alpha + V_\alpha G_0 t_\alpha$$

one immediately arrives with the Faddeev equation in the integral form:

$$\Psi_\alpha = G_0 t_\alpha \sum_{\beta \neq \alpha} \Psi_\beta$$

Below we proceed with the system of equations (3) since we use only local potentials given in the coordinate space.



The Jacobi coordinates are as usual:

$$\vec{r}_i - \vec{r}_j = \frac{\vec{\eta}_\alpha}{a_\alpha} \quad (4)$$

$$\frac{m_i \vec{r}_i + m_j \vec{r}_j}{m_i + m_j} - \vec{r}_k = \frac{\vec{\xi}_\alpha}{b_\alpha} \quad (5)$$

where  $\vec{r}_i$ ,  $m_i$  denote the radius-vector and the mass of particle  $i$ ,

$$a_\alpha = \sqrt{\frac{m_i m_j}{(m_i + m_j) M}}, \quad b_\alpha = \sqrt{\frac{m_k (m_i + m_j)}{M^2}}$$
$$M = m_1 + m_2 + m_3$$

and indices  $\alpha$  take on the following values:  $\alpha = 3$  for  $(ij)k = (12)3$ ,  $\alpha = 1$  for  $(ij)k = (23)1$ ,  $\alpha = 2$  for  $(ij)k = (31)2$ .

First, the Faddeev components of the wave function are expanded into partial waves:

$$\Psi_{\alpha}(\vec{\eta}_{\alpha}, \vec{\xi}_{\alpha}) = \sum_{LMl\lambda} \frac{1}{\eta_{\alpha} \xi_{\alpha}} U_{\alpha l \lambda}^L(\eta_{\alpha}, \xi_{\alpha}) Y_{l\lambda}^{LM}(\hat{\eta}_{\alpha}, \hat{\xi}_{\alpha})$$

$$\xrightarrow{L=l=\lambda=0} \frac{1}{\eta_{\alpha} \xi_{\alpha}} U_{\alpha}(\eta_{\alpha}, \xi_{\alpha}) Y_{00}^{00}(\hat{\eta}_{\alpha}, \hat{\xi}_{\alpha}) \quad (6)$$

where  $\eta_{\alpha} = |\vec{\eta}_{\alpha}|$ ,  $\xi_{\alpha} = |\vec{\xi}_{\alpha}|$ ,  $\hat{\eta}_{\alpha} = \vec{\eta}_{\alpha}/|\vec{\eta}_{\alpha}|$ ,  $\hat{\xi}_{\alpha} = \vec{\xi}_{\alpha}/|\vec{\xi}_{\alpha}|$ ,  $Y_{l\lambda}^{LM}$  are the bispherical harmonics. Jacobi coordinates  $\vec{\eta}_{\alpha}, \vec{\xi}_{\alpha}$  were used, and only the lowest partial wave is taken into account.

Since there are two identical particles in the system (nucleon mass difference is neglected), the following two coupled-differential Faddeev equations survive:

$$\left\{ \begin{array}{l} \left[ \widehat{D} + V_1 \left( \frac{\rho \cos \varphi}{a_1} \right) - E \right] U_1(\rho, \varphi) = \\ \quad - V_1 \left( \frac{\rho \cos \varphi}{a_1} \right) \sum_{\alpha' \neq 1} \frac{1}{\sin(2\gamma_{\alpha'1})} \int_{c^-}^{c^+} U_{\alpha'}(\rho, \varphi') d\varphi' \\ \left[ \widehat{D} + V_2 \left( \frac{\rho \cos \varphi}{a_2} \right) - E \right] U_2(\rho, \varphi) = \\ \quad - V_2 \left( \frac{\rho \cos \varphi}{a_2} \right) \sum_{\alpha' \neq 2} \frac{1}{\sin(2\gamma_{\alpha'2})} \int_{c^-}^{c^+} U_{\alpha'}(\rho, \varphi') d\varphi' \end{array} \right. \quad (7)$$

$$(U_3 \equiv U_2)$$

where polar coordinates  $\rho = \sqrt{\eta_\alpha^2 + \xi_\alpha^2}$ ,  $\tan \varphi_\alpha = \xi_\alpha / \eta_\alpha$  are introduced and

where polar coordinates  $\rho = \sqrt{\eta_\alpha^2 + \xi_\alpha^2}$ ,  $\tan \varphi_\alpha = \xi_\alpha/\eta_\alpha$  are introduced and

$$V_1 = V_{NN}, \quad V_2 = V_{\phi N},$$

$$\widehat{D} = -\frac{\hbar^2}{2M} \left( \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2}{\partial \varphi^2} \right)$$

$$c_+ = \text{Min} \{ |\varphi + \gamma_{\alpha'\alpha}|, \pi - (\varphi + \gamma_{\alpha'\alpha}) \},$$

$$c_- = |\varphi - \gamma_{\alpha'\alpha}|$$

$$\gamma_{ij} = \arcsin s_{ij}, \quad s_{ij} = \sqrt{\frac{m_k M}{(m_i + m_k)(m_j + m_k)}},$$

$$(ijk = 123, 231, 312)$$

indices correspond to 1 for  $\phi$ -meson, 2 and 3 for nucleons.

## Inputs of the $\phi N$ interaction

Let us shortly describe the physical content of the quark model [5] developed for the description of the meson - baryon interaction.

The interaction in this model is given by the  $q - q$  and the  $q - \bar{q}$  potentials. The  $qq$  - potential consists of three terms:

$$V_{qq} = V^{OGE} + V^{conf} + V^{ch} \quad (8)$$

where  $V^{OGE}$  - the contribution from the one gluon exchange,  $V^{conf}$  - the confinement potential  $V^{ch}$  - the effective  $qq$  potential induced by the chiral fields.

The parameters of the above potentials are taken in order to fit the  $NN$  scattering phase shifts, the deuteron binding energy and the available  $YN$  cross sections. The  $N\phi$  and  $\Lambda K^*$  coupling are also included.

The 5-quark problem ( $N + \phi$ ) treated by the MRG method [6], where the wave function of the 5-body system has a form

$$\Psi_{N\phi} = \hat{A}\{\chi_N \chi_\phi \chi_{N\phi}\} \quad (9)$$

where  $\hat{A}$  is the antisymmetrization operator,  $\chi_N$  and  $\chi_\phi$  are the wave functions of the clusters  $N$  and  $\phi$ ,  $\chi_{N\phi}$  - the unknown function of the relative distance between two clusters. In the frame of the ansatz (9) a few MeV binding energy for the  $N\phi$  system was obtained.



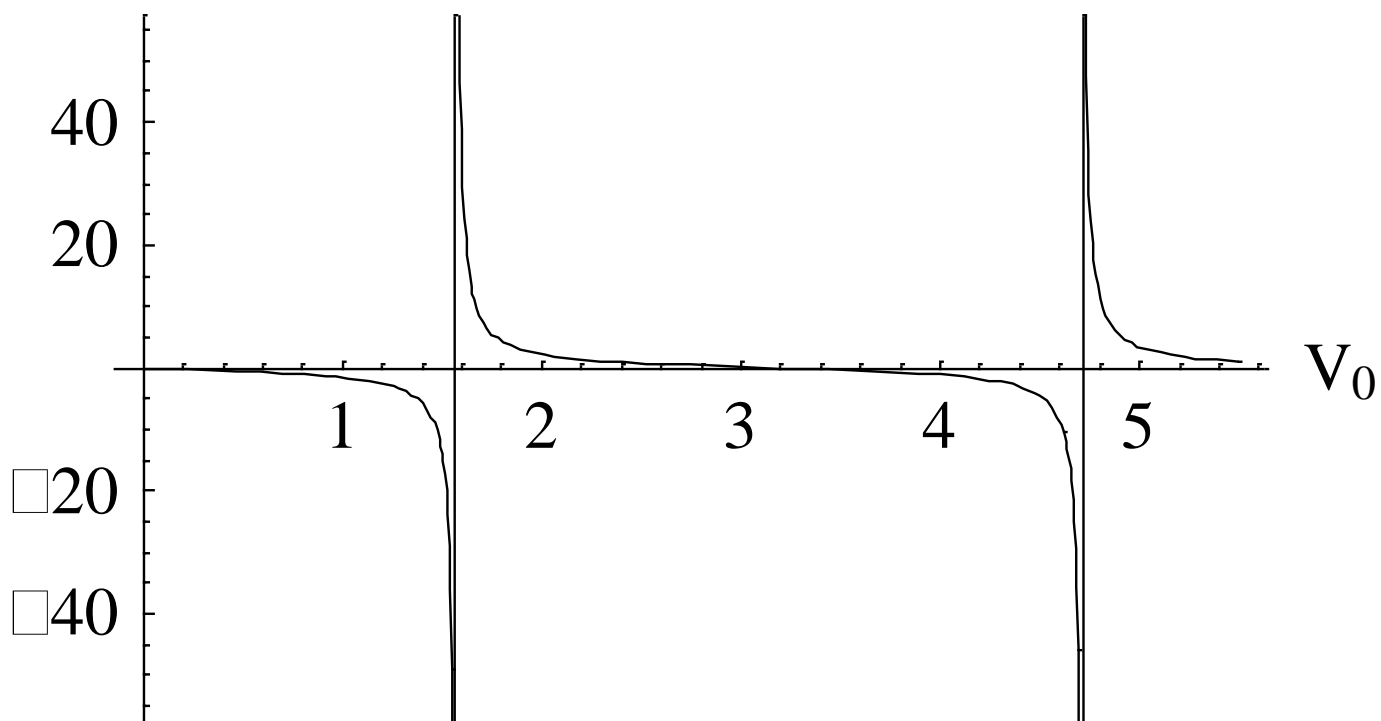
In our calculations we have used the  $\phi N$  potential of the form

$$V_{\phi N} = ae^{-\mu r}/r \quad (10)$$

with  $a = -1.25 \hbar c$ ,  $\mu = (600 \text{ MeV})/(\hbar c)$  suggested in [7], which reproduces the binding energy in the  $\phi N$  system found in the framework of the quark model.

The Binding Energies of the three-body systems, MeV				$a_{\varphi N}$ , fm	the $\varphi N$ depth	The $\varphi N$ binding energy, MeV
$\varphi n n$	$\varphi p p$	$(\varphi n \rho)$ singlet	$(\varphi n \rho)$ triplet			
21.8	20.9	22.3	37.9	2.37 [Gao]	1.25	9.46
no	no	no	no	- 0.15 [Koike]	0.23	no
<u>~570</u>				+ 0.10 [Klingl]	2.62	547
<u>~780</u>				- 0.15 [Koike]	2.89	756

a fm



It should be noticed that for this binding energy ( $-37.9$  MeV) in the  $(\phi np)_{triplet}$  system both the main  $\phi$ -meson decay channels on  $K$ -mesons are closed. Let us comment this value of the energy which is appeared rather large. From the naive reasons one would expect the binding of the order of  $2 E_{\phi N} + E_d \sim -20$  MeV in the  $\phi + d$  configuration, which is much smaller than the calculated value. However, due to the strong attraction ( $E_{\phi N} \sim -9$  MeV) in the  $\phi N$  subsystem one can expect that in the 3-particle  $\phi np$  system the configuration  $\phi + d$  is rather suppressed. Hence, it follows that in the above system there is no strong cancellation between potential and kinetic energies of nucleons, like in deuteron, and a strong attractive triplet  $N - N$  potential ( $V_{triplet} \sim 100$  MeV) shows its full value.

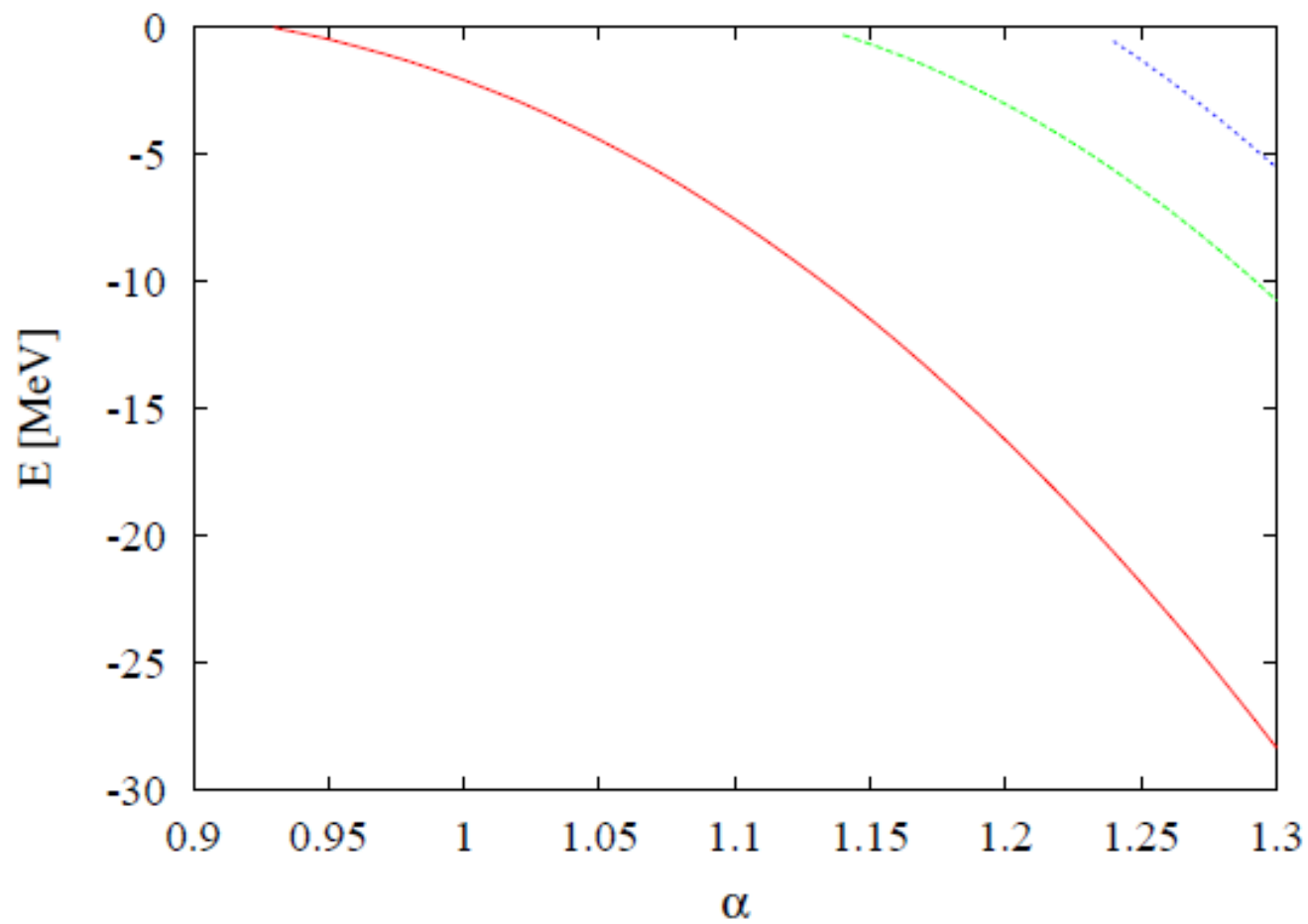


Figure 1: The dependence of the binding energy of the  $\phi nn$  system on the parameter  $\alpha$  of the  $\phi - N$  interaction.

The dependence of the  $\phi nn$  binding energy on the parameter  $\alpha$  of the  $\phi - N$  interaction is investigated. It is shown in the Figure 1 that the excited states appear in this system. As can be seen from these results, the binding in 3-particle systems like  $\phi NN$  is possible even for weaker  $\phi - N$  attraction than the one of potential (10).

Now let us consider the  $\phi\phi N$  system. To describe the interaction in the  $\phi + \phi$  subsystem we will use the central potential of the form:

$$V_{\phi\phi} = V_0 e^{-(r/r_0)^2} \quad (11)$$

with  $V_0 = -93.75$  MeV and  $r_0 = 1.2$  fm.

Above parameters are chosen to reproduce the position and the width of the  $f_2(2010)$  d-wave resonance [see PDG], decaying into the  $\phi + \phi$  channel.

We restrict ourself in the consideration of the  $\phi\phi N$  system with the total angular momentum  $L = 2$ . In that case after the partial wave expansion of the Faddeev components we arrive with the 5 two-dimensional equations for the partial Faddeev amplitudes:



$$\left. \begin{aligned}
& \widehat{D}_1^{20} U_1^{220} = \\
& \quad V_1 \left( \frac{\eta_1}{a_1} \right) \Sigma_{\alpha' \nu \lambda'} \int \frac{\eta_1 \xi_1}{\eta_{\alpha'} \xi_{\alpha'}} U_{\alpha'}^{2l' \lambda'} (\eta_{\alpha'}, \xi_{\alpha'}) \\
& \quad Y_{l' \lambda'}^{20} (\hat{\eta}_{\alpha'}, \hat{\xi}_{\alpha'}) Y_{20}^{*20} (\hat{\eta}_1, \hat{\xi}_1) d\Omega(\hat{\eta}_1) d\Omega(\hat{\xi}_1) \\
& \\
& \widehat{D}_1^{22} U_1^{222} = \\
& \quad V_1 \left( \frac{\eta_1}{a_1} \right) \Sigma_{\alpha' \nu \lambda'} \int \frac{\eta_1 \xi_1}{\eta_{\alpha'} \xi_{\alpha'}} U_{\alpha'}^{2l' \lambda'} (\eta_{\alpha'}, \xi_{\alpha'}) \\
& \quad Y_{l' \lambda'}^{20} (\hat{\eta}_{\alpha'}, \hat{\xi}_{\alpha'}) Y_{20}^* (\hat{\eta}_1) Y_{20}^* (\hat{\xi}_1) d\Omega(\hat{\eta}_1) d\Omega(\hat{\xi}_1) \\
& \quad (-\sqrt{2/7}) \\
& \\
& \widehat{D}_1^{24} U_1^{224} = \\
& \quad V_1 \left( \frac{\eta_1}{a_1} \right) \Sigma_{\alpha' \nu \lambda'} \int \frac{\eta_1 \xi_1}{\eta_{\alpha'} \xi_{\alpha'}} U_{\alpha'}^{2l' \lambda'} (\eta_{\alpha'}, \xi_{\alpha'}) \\
& \quad Y_{l' \lambda'}^{20} (\hat{\eta}_{\alpha'}, \hat{\xi}_{\alpha'}) Y_{20}^* (\hat{\eta}_1) Y_{40}^* (\hat{\xi}_1) d\Omega(\hat{\eta}_1) d\Omega(\hat{\xi}_1) \\
& \quad (\sqrt{2/7}) \\
& \\
& \widehat{D}_2^{02} U_2^{202} = \\
& \quad V_2 \left( \frac{\eta_2}{a_2} \right) \Sigma_{\alpha' \nu \lambda'} \int \frac{\eta_2 \xi_2}{\eta_{\alpha'} \xi_{\alpha'}} U_{\alpha'}^{2l' \lambda'} (\eta_{\alpha'}, \xi_{\alpha'}) \\
& \quad Y_{l' \lambda'}^{20} (\hat{\eta}_{\alpha'}, \hat{\xi}_{\alpha'}) Y_{02}^{*20} (\hat{\eta}_2, \hat{\xi}_2) d\Omega(\hat{\eta}_2) d\Omega(\hat{\xi}_2) \\
& \\
& \widehat{D}_3^{02} U_3^{202} = \\
& \quad V_3 \left( \frac{\eta_3}{a_3} \right) \Sigma_{\alpha' \nu \lambda'} \int \frac{\eta_3 \xi_3}{\eta_{\alpha'} \xi_{\alpha'}} U_{\alpha'}^{2l' \lambda'} (\eta_{\alpha'}, \xi_{\alpha'}) \\
& \quad Y_{l' \lambda'}^{20} (\hat{\eta}_{\alpha'}, \hat{\xi}_{\alpha'}) Y_{02}^{*20} (\hat{\eta}_3, \hat{\xi}_3) d\Omega(\hat{\eta}_3) d\Omega(\hat{\xi}_3)
\end{aligned} \right\} \tag{12}$$

$$\widehat{D}_\alpha^{l\lambda} = \frac{\hbar^2}{2M} \left( \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2}{\partial \varphi_\alpha^2} - \frac{l(l+1)}{(\rho \cos \varphi_\alpha)^2} - \frac{\lambda(\lambda+1)}{(\rho \sin \varphi_\alpha)^2} \right) + E_0 \quad (13)$$

$$\rho = \sqrt{\eta_\alpha^2 + \xi_\alpha^2}, \quad \tan \varphi_\alpha = \xi_\alpha / \eta_\alpha$$

$$V_\alpha = V_{ij}$$

To simplify the solution of the above equations let us make the following approximations.

Approximation 1.

There are four equations containing operators  $\hat{D}_1^{2\lambda}$  with  $\lambda > 0$ . The corresponding equations include terms with more centrifugal repulsion and due to this one can neglect the corresponding components of the wave function. Thus, three equations are left two of which are identical. As a result, we arrive at the following two two-dimensional integrodifferential equations:

$$\left\{ \begin{array}{l}
\left( \widehat{D} + \frac{\hbar^2}{2M} \frac{6}{(\rho \cos \varphi)^2} + V_1 \left( \frac{\rho \cos \varphi}{a_1} \right) - E \right) \\
U_1(\rho, \varphi) = -V_1 \left( \frac{\rho \cos \varphi}{a_1} \right) \Sigma_{\alpha' \neq 1} \\
\frac{2}{\sin(2\gamma_{\alpha'1})} \int_{c^-}^{c^+} U_{\alpha'}(\rho, \varphi') h_{\alpha'1}(\varphi, \varphi') d\varphi' \\
\\
\left( \widehat{D} + \frac{\hbar^2}{2M} \frac{6}{(\rho \sin \varphi)^2} + V_2 \left( \frac{\rho \cos \varphi}{a_2} \right) - E \right) \\
U_2(\rho, \varphi) = -V_2 \left( \frac{\rho \cos \varphi}{a_2} \right) \Sigma_{\alpha' \neq 2} \\
\frac{2}{\sin(2\gamma_{\alpha'2})} \int_{c^-}^{c^+} U_{\alpha'}(\rho, \varphi') h_{\alpha'2}(\varphi, \varphi') d\varphi' \\
\\
(U_3 \equiv U_2)
\end{array} \right. \quad (14)$$

where  $U_1 = U_1^{220}$ ,  $U_2 = U_2^{202}$ ,  $V_1 = V_{\phi\phi}$ ,  $V_2 = V_{\phi N}$ ,

$$\widehat{D} = -\frac{\hbar^2}{2M} \left( \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2}{\partial \varphi^2} \right)$$

$$c_+ = \text{Min} \{ |\varphi + \gamma_{\alpha'\alpha}|, \pi - (\varphi + \gamma_{\alpha'\alpha}) \}$$

$$c_- = |\varphi - \gamma_{\alpha'\alpha}|$$

$$\gamma_{ij} = \arcsin s_{ij},$$

$$s_{ij} = \sqrt{\frac{m_k M}{(m_i + m_k)(m_j + m_k)}},$$

$$(ijk = 123, 231, 312)$$

indices correspond to 1 for the nucleon, 2 and 3 for  $\phi$ -mesons. From the expression which is given in [8] one gets

$$h_{\alpha'1} = 1/8 + (3/8) \cos 2\theta_{\vec{\xi}_{\alpha'}} \\ h_{\alpha'2} = 1/8 + (3/8) \cos 2(\theta_{\vec{\xi}_2} - \theta_{\vec{\eta}_{\alpha'}})$$

where

$$\cos \theta_{\eta_{\alpha'}} = (-\cos \gamma_{\alpha'\alpha} \cos \varphi + \epsilon_{\alpha'\alpha} \sin \gamma_{\alpha'\alpha} \cos \theta_{\xi_{\alpha}} \sin \varphi) / \cos \varphi'$$

$$\cos \theta_{\xi_{\alpha'}} = -(\epsilon_{\alpha'\alpha} \sin \gamma_{\alpha'\alpha} \cos \varphi + \cos \gamma_{\alpha'\alpha} \cos \theta_{\xi_{\alpha}} \sin \varphi) / \sin \varphi'$$

$$\cos \theta_{\xi_{\alpha}} = \frac{-\cos \varphi'^2 + \cos \gamma_{\alpha'\alpha}^2 \cos \varphi^2 + \sin \gamma_{\alpha'\alpha}^2 \sin \varphi^2}{\epsilon_{\alpha'\alpha} \cos \gamma_{\alpha'\alpha} \sin \gamma_{\alpha'\alpha} \sin 2\varphi}$$

$$\epsilon_{\alpha'\alpha} = \begin{cases} 1 & \alpha'\alpha = (13), (32), (21) \\ -1 & \alpha'\alpha = (31), (23), (12) \end{cases}$$

Approximation 2.

One may notice that  $m_N \approx m_\phi$ . Therefore, it seems reasonable to make another simplification and put  $m_i = m = m_\phi$ .

Approximation 3. Now let us reduce the system of the two 2-dimensional equations (14) to the system of one-dimensional equations in the variable  $\rho$ -hyperradius of the system considered. This is possible due to the following observation. The potential  $V_{\phi n}$  is a shortrange, strongly attractive and acts in the s-state. The potential  $V_{\phi\phi}$  contains a centrifugal barrier, so one can expect for the  $\phi\phi N$  - system the equilibrium configuration when



nucleon is in the centrum and  $\phi$ -mesons are on the opposite sides. This configuration corresponds to the values of variable  $\varphi$  different for different Jacobi sets. The equilibrium values are  $\varphi_{eq} = 0$  for  $U_1(\rho, \varphi)$  and  $\varphi_{eq} = \pi/3$  for  $U_2(\rho, \varphi)$ . Expanding  $U_1(\rho, \varphi)$  and  $U_2(\rho, \varphi)$  around equilibrium values

$$U_1(\rho, \varphi) = \sum_{n=0}^{\infty} c_1^n(\rho) \varphi^n$$

$$U_2(\rho, \varphi) = \sum_{n=0}^{\infty} c_2^n(\rho) (\varphi - \pi/3)^n$$

and putting the expansion into the equation (14) one immediately arrives at the system of 2 one-dimensional equations on the variable  $\rho$

$$\left\{ \begin{array}{l} \hbar^2/(2M) ((c_1)'' + (c_1)'/\rho - 6c_1/\rho^2) + (E - \\ \quad V_1(\rho\sqrt{6}))c_1 = -8/\sqrt{3} V_1(\rho\sqrt{6}) c_2 \\ \hbar^2/(2M) ((c_2)'' + (c_2)'/\rho - 8c_2/\rho^2) + (E + \\ \quad (1 + 4/\sqrt{3}\nu_1) V_2(\rho\sqrt{3/2})) c_2 = -4/\sqrt{3} \\ \quad \nu_2 V_2(\rho\sqrt{3/2}) c_1 \end{array} \right. \quad (15)$$

where  $\nu_1 = 0.110$ ,  $\nu_2 = 0.0555$ , which was solved for the eigenvalue problem.

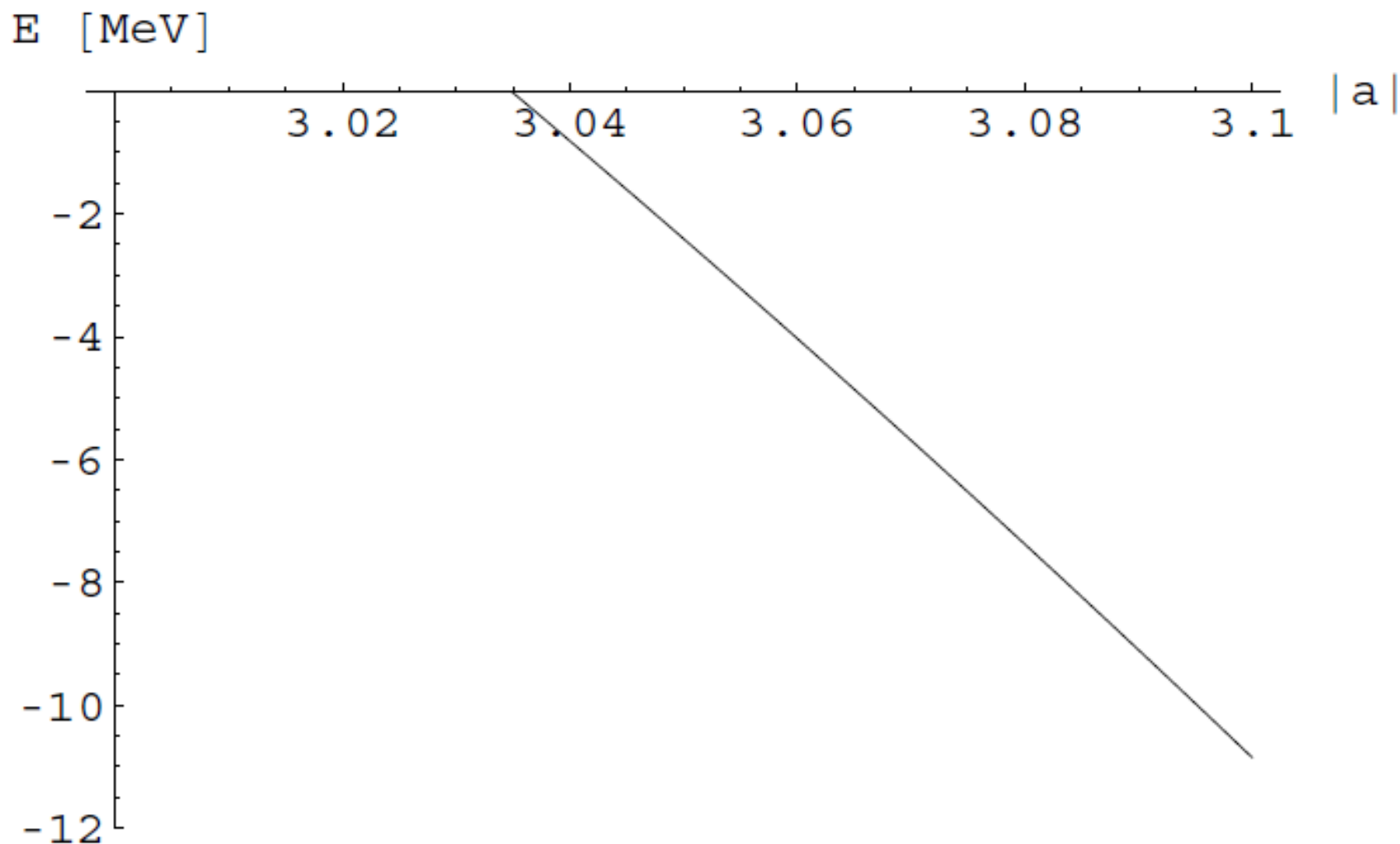


Figure 2: The dependence of the binding energy of the  $\phi\phi n$  system on the parameter  $|a|$  of the  $\phi - N$  interaction ( $\hbar = c = 1$ ).

The dependence of the energy of the three-body system  $\phi\phi N$  on the depth of the  $\phi N$  potential  $|a|$  is given on Figure 1. One can see that the binding in the system appears only at  $a = -3.035 \hbar c$ . It is quite large with respect to the input value.

Let us discuss now the 4-body system  $\phi nnn$ . To make a preliminary estimate of its binding energy, we have used the folding model with the  $(\phi nn)$ -subsystem as a cluster.

By averaging the interactions of the third neutron with the particles of the  $(\phi nn)$  system over the cluster wavefunction it is easy to obtain an effective potential which has the form shown in Figure 3. Here  $r$  means the distance between the third neutron and the center of mass of the  $(\phi nn)$  cluster.

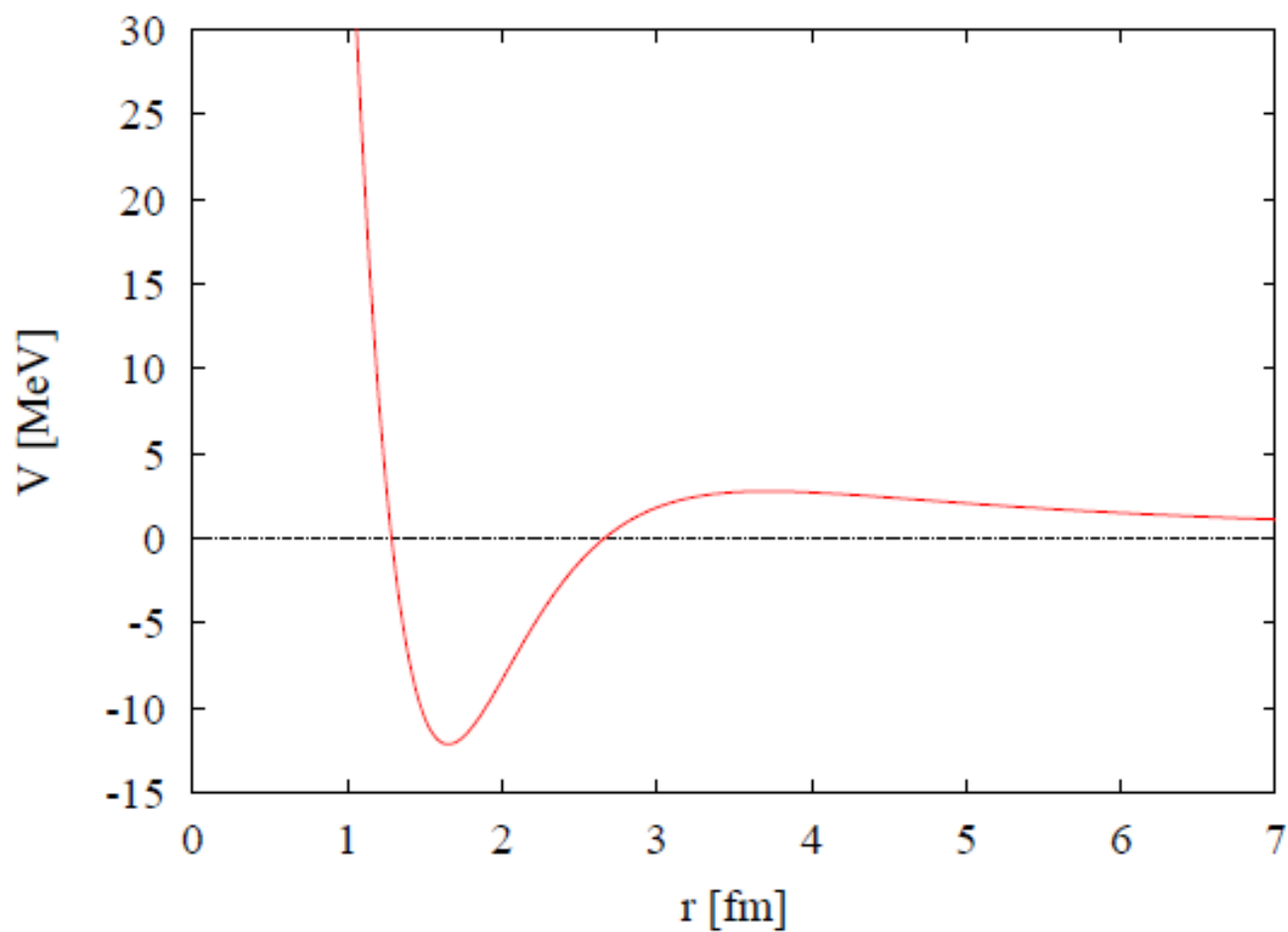


Figure 3: The folding potential with the p-wave centrifugal barrier for the four-body system  $(\phi nn) + n$ .

The solution of the Schroedinger equation with this potential shows that there are no bound states. However, the exact treatment of an analogous 4- body system ( $\eta_c + 3N$ ), performed by means of the AGS- equations [9], shows that the folding model may strongly underestimate the exact calculation.

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