

Nuclear Forces and Light Nuclei: (Some) Recent Developments

Outline

- 3N scattering at N²LO
- Chiral EFT for nuclear forces with explicit $\Delta(1232)$
- Pion production in NN collisions
- Probing light nuclei with photons
- Nuclear lattice simulations
- Summary and outlook



Nuclear forces in Δ-less EFT



Three-nucleon force at N²LO

First nonvanishing 3N-force contribution appears at next-to-next-to-leading order

Cannot be fixed in the NN system



D-term figures prominently in various reactions



Three nucleons up to N²LO

E.E. et al.'02; Kistryn et al.'05; Witala et al.'06; Ley et al.'06; Stephan et al.'07; ...

Differential cross section in elastic Nd scattering



Polarization observables in elastic Nd scatering



Deuteron breakup at N²LO









3N force: first corrections (N³LO)

Three-nucleon force at N³LO Ishikawa, Robilotta '07; Bernard, E.E., Krebs, Meißner '07, to appear

- parameter-free
- \bigcirc mainly finite shifts of c_i, D
- new structures (also from 1/m-terms)
- 3N scattering in progress...

$$\begin{vmatrix} -\phi - \phi \\ = & \begin{vmatrix} -1 \\ -1 \\ -1 \\ -1 \\ + \\ \end{vmatrix} + \begin{vmatrix} -1 \\ -1 \\ -1 \\ + \\ \end{matrix} + \begin{vmatrix} -1 \\ -1 \\ -1 \\ + \\ \end{matrix}$$

Partial-wave decomposition

Too many terms (> 100 !) for doing PWD "manually" let computer do the job... Golak et al.'09

$$\underbrace{\langle p'q'\alpha'|V|pq\alpha\rangle}_{matrix, \sim 10^{5} \times 10^{5}} = \int \underbrace{d\hat{p}' d\hat{q}' d\hat{p} d\hat{q}}_{5 \text{ dim. integral}} \sum_{m_{l}, \dots} \left(\text{CG coeffs.} \right) \left(Y_{l,m_{l}}(\hat{p}) Y_{l',m_{l}'}(\hat{p}') \dots \right) \underbrace{\langle m_{s_{1}}'m_{s_{2}}'m_{s_{3}}'|V|m_{s_{1}}m_{s_{2}}m_{s_{3}}'\rangle}_{depends on \ \vec{p}, \ \vec{q}, \ \vec{p}', \ \vec{q}', \ spin \ \& \ isospin \ addle \ addle$$

feasible task for modern supercomputers, work in progress...

Faddeev equations without PWD ?

talk by Charlotte Elster

3N force: first corrections (N³LO)

Elastic Nd scattering at 28 MeV

Ishikawa, Robilotta '07

0.2 $V_{2\mathrm{N}}^{\mathrm{AV18}} + V_{3\mathrm{N}}^{(3)} + V_{3\mathrm{N}}^{(4)}$ 0.0 $V_{2\mathrm{N}}^{\mathrm{AV18}}$ -0.2 60 180 120 θ (deg)

Preliminary results (incomplete) indicate that the two-pion exchange corrections at N³LO are rather small

Nuclear forces from chiral EFT with explicit $\Delta(1232)$

in collaboration with Hermann Krebs (Bochum) Ulf-G. Meißner (Bonn/Jülich)

Chiral expansion of the NN force up to NNLO (Q³): $V = V_{1\pi} + V_{2\pi} + V_{cont}$ where $V_{1\pi} = V_{1\pi}^{(0)} + V_{1\pi}^{(2)} + V_{1\pi}^{(3)} + \dots$; $V_{2\pi} = V_{2\pi}^{(2)} + V_{2\pi}^{(3)} + \dots$; $V_{cont} = V_{cont}^{(0)} + V_{cont}^{(2)} + \dots$

The 2π -exchange potential in coordinate space has the structure:

 $V(r) = \tilde{V}_C + \tilde{W}_C \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + [\tilde{V}_S + \tilde{W}_S \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2] \,\vec{\sigma}_1 \cdot \vec{\sigma}_2 + [\tilde{V}_T + \tilde{W}_T \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2] \,(3\vec{\sigma}_1 \cdot \hat{r} \,\vec{\sigma}_2 \cdot \hat{r} - \vec{\sigma}_1 \cdot \vec{\sigma}_2)$



Neutron-proton peripheral phase shifts up to N²LO (Born approximation)



⇒ big corrections at NNLO

Similar observation made by the Nijmegen Group Rentmeester et al. '99, '03

	#BC	$\chi^2_{ m min}$
Nijm78	19	1968.7
OPE	31	2026.2
OPE + TPE(l.o.)	28	1984.7
$\text{OPE} + \chi \text{TPE}$	23	1934.5



Similar convergence pattern for charge-symmetry breaking 2π -exchange



Similar convergence pattern for 3π-exchange and 2πγ-exchange *Kaiser* '01, '06

Why is the order-Q³ (i.e. subleading) 2π -exchange NN potential so strong?

- Solution Loop integrals at order Q³ yield accidentally one power of π less than expected in the denominator (chiral expansion is an expansion in $Q^2/(4\pi F_{\pi})^2$)
- Solution Unnaturally large LECs c_3 and c_4 understood in terms of resonance saturation. In particular, Δ-isobar yields an important contribution:

 $\delta c_3 = -2\delta c_4 = -\frac{4h_A^2}{9\Delta}$ (Bernard, Kaiser & Meißner '97)

including ∆ as an explicit DOF is expected to yield a more natural size of LECs, better convergence & applicability at higher energies caveats: calculations more involved; more LECs...

• standard chiral expansion: $Q \sim M_{\pi} \ll \Delta \equiv m_{\Delta} - m_N = 293 \text{ MeV}$

 $_{\odot}$ small-scale expansion: $Q \sim M_{\pi} \sim \Delta$ (Hemmert, Holstein & Kambor '98)

<u>To be studied</u>: convergence of the EFT expansion, effects beyond resonance saturation of c_i , isospin violating effects, ...

Krebs, E.E., Meißner EPJA 32 (2007) 127

Two-nucleon force in EFT with and without $\boldsymbol{\Delta}$



Notice: Δ-contributions to the OPEP and contact interactions only lead to shifts in the corresponding low-energy constants

Krebs, E.E., Meißner EPJA 32 (2007) 127

Determination of the LECs from πN threshold coefficients

πN amplitude up to NLO:

Input:

• fit 1: $h_A = \frac{3g_A}{2\sqrt{2}} \sim 1.34$ (SU(4), large N_c) • fit 2: $h_A = 1.05$ (Fettes & Meißner '01)

Determinations of the LECs

LECs	Q^2 , no Δ	Q^2 , fit 1	Q^2 , fit 2
c_1	-0.57	-0.57	-0.57
c_2	2.84	-0.25	0.83
c_3	-3.87	-0.79	-1.87
c_4	2.89	1.33	1.87
h_A	-	1.34^{\star}	1.05^{\star}
$b_3 + b_8$	-	1.40	2.95

Values of the S- and P-wave threshold param.

	Q^2 , no Δ	Q^2 fits 1, 2	EM98
a_{0+}^+	0.41	0.41	0.41 ± 0.09
b_{0+}^+	-4.46	-4.46	-4.46
a_{0+}^{-}	7.74	7.74	7.73 ± 0.06
b_{0+}^{-}	3.34	3.34	1.56
a_{1-}^{-}	-0.05	-1.32	-1.19 ± 0.08
a_{1-}^+	-2.81	-5.30	-5.46 ± 0.10
a_{1+}^{-}	-6.22	-8.45	-8.22 ± 0.07
a_{1+}^+	9.68	12.92	13.13 ± 0.13

all values in units $10^{-2}M_{\pi}^{-n}$

We found:

- improved description of P-wave threshold parameters when Δ is included;
- resulting c_i's depend strongly on h_A while the thresh. param. do not
-) strongly reduced values for c_i 's;

Chiral 2π -exchange up to NNLO with and without explicit Δ



 \Rightarrow much better convergence when Δ is included explicitly!

Krebs, E.E., Meißner EPJA 32 (2007) 127

 $^{3}F_{3}$ partial waves up to NNLO with and without Δ



E.E., Krebs, Meißner NPA 806 (2008) 65



- Δ contributions at N³LO are large!
- Long-range part is parameter free
- Much richer spin/isospin structure compared to the Illinois model
- Complete analysis still to be done Krebs, E.E., in progress



Electromagnetic currents

in collaboration with Stefan Kölling (Jülich/Bonn) Hermann Krebs (Bochum) Dagmara Rozpedzik, Jacek Golak (Cracow) Ulf-G. Meißner (Bonn/Jülich)

Probing few nucleons with photons



• Leading 1-loop expressions for exchange currents in the threshold kinematics known since long time (Park, Min, Rho) Application to $np \rightarrow d\gamma$ at threshold:

 $\sigma_{1N} = 306.6 \text{ mb} \longrightarrow \sigma_{1N+2N} = 334 \pm 3 \text{ mb}$ to be compared with $\sigma_{exp} = 334.2 \pm 0.5 \text{ mb}$

 Leading 2π-exchange charge and current densities worked out (parameter-free)

Pastore, Schiavilla, ... ; Kölling, Krebs, E.E., Meißner

 1π-exchange and short-range contributions to one loop in progress (Kölling et al., in preparation)

Leading 2π -exchange contributions to 2N current



Deuteron photodisintegration Rozpedzik et al. '10

Cross section and photon analyzing power at E_v =30 MeV



large sensitivity to MEC; short-range & 1π -exchange terms still to be taken into account

Pion production in NN collisions

in collaboration with Vadim Baru, Arseny Filin, Christoph Hanhart, Johan Haidenbauer (Jülich) Vadim Lensky (Manchester) Alexander Kudryavtsev (Moscow) Ulf-G. Meißner (Bonn/Jülich)

Pion production in NN collisions

Considerably more challenging due to the appearance of a new "soft" scale $|\vec{p}| \gtrsim \sqrt{M_{\pi}m_N} \sim 350 \; {
m MeV}$

slower convergence of the chiral expansion (expansion parameter $\sqrt{M_{\pi}m_N}/\Lambda_{\chi}$ vs M_{π}/Λ_{χ} in the few-N sector)

Current state-of-the-art

- Solution With Hybrid approach (EFT description of the 2N system for $|\vec{p}| \sim \sqrt{M_{\pi}m_N}$ not yet available)
- $= \Delta(1232)$ isobar plays an important role = must be included as an explicit DOF



results for $pp \rightarrow d\pi^+$



Proper separation of irred. contributions crucial! Lensky et al. '01

Near threshold: $\sigma = \alpha \eta + \mathcal{O}(\eta^3)$ with $\eta \equiv k_{\pi}/M_{\pi}$



p-wave π -production and the D-term

Hanhart, van Kolck, Miller '00; Baru, EE, Haidenbauer, Hanhart, Kudryavtsev, Lensky, Meißner '09

- Loops start to contribute at N³LO
- Up to N²LO, *D* is the only unknown LEC
- Simultaneous description of $pn \rightarrow pp\pi^{-}$, $pp \rightarrow pn\pi^{+}$ and $pp \rightarrow d\pi^{+}$



 $pn \rightarrow pp\pi^{-}, pp \rightarrow pn\pi^{+}$ and $pp \rightarrow d\pi^{+} \implies$ nontrivial consistency check of chiral EFT

In the future: implications for the 3NF and for weak reactions with light nuclei

$$|\vec{p}'| \sim M_{\pi} \implies i S_{1} \text{ for } pn \to pn\pi^{+}, pp \to d\pi^{+}; {}^{3}S_{1} \text{ for } pn \to pp\pi^{-}$$
$$|\vec{p}| \sim \sqrt{M_{\pi}m_{N}} \implies i S_{1} \text{ for } pn \to pn\pi^{+}, pp \to d\pi^{+}; {}^{1}S_{0} \text{ for } pn \to pp\pi^{-}$$

• Reaction $pp \rightarrow d\pi^+$

Near threshold: Heimberg A_{y} $\frac{d\sigma}{d\Omega} \simeq A_0 + A_2 P_2(\cos\theta_\pi)$ -0.1 n = 0.21 $4_2/A_0$ Natural units for *D*: -0.2 $D = rac{d \leftarrow dimensionless \ coefficient \sim 1}{F_\pi^2 \ m_N}$ -0.30.1 0.4 45 135 0 90 180 η θ_{π}

p-wave π -production and the D-term

• Reaction $pn \rightarrow pp\pi^{-}$

- The final *pp* relative momentum is restricted to
 be: |*p*[']| < 38 MeV *pp* p-waves suppressed
- Data only available at $\eta = 0.66 \implies$ expect only qualitative description...



New data at lower energies will be taken at COSY.

• Reaction $pp \rightarrow pn\pi^+$

- The relevant amplitude $({}^{1}S_{0} \rightarrow {}^{3}S_{1}p)$ is suppressed compared to the dominant ${}^{1}D_{2} \rightarrow {}^{3}S_{1}p$ amplitude
 - → minor sensitivity to the D-term...

Overall best results are achieved for d ~ 3



Isospin breaking & few-N systems



Charge-symmetry-breaking nuclear forces and BE differences in ³He – ³H

Coulomb	Breit	K.E.	Two-Body	Three-body	Theory	Experiment
648	28	14	65(22)	5	760(22)	764

Friar et al. PRC 71 (2005) 024003

 \bigcirc dd → απ⁰ measured at IUCF: σ = 12.7 ± 2.2 / 15.1 ± 3.1 pb @ 228.5 / 231.8 MeV Stephenson et al. '03

Theoretical analysis challenging; first estimations yield the right order of magnitude. *Gardestig et al.* '04; Nogga et al.'06

 \bigcirc CSB forward-backward asymetry in $np \rightarrow d\pi^0$ @ 279.5 MeV at TRIUMF

$$A_{\rm fb} = \frac{\int [d\sigma/d\Omega(\theta) - d\sigma/d\Omega(\pi - \theta)] d[\cos\theta]}{\int [d\sigma/d\Omega(\theta) + d\sigma/d\Omega(\pi - \theta)] d[\cos\theta]} = \left[17.2 \pm 8(\text{stat}) \pm 5.5(\text{sys})\right] \times 10^{-4} \quad \text{(Opper et al. '03)}$$

$np \rightarrow d\pi^0$ & the np mass difference

Niskanen '99; van Kolck et al. '00; Bolton, Miller '09; Filin, Baru, E.E., Haidenbauer, Hanhart, Kudryavtsev, Meißner '09

The goal: use A_{fb} measured at TRIUMF to extract the strong/em contributions to the neutron-to-proton mass shift.

$$\begin{cases} \delta m_N^{\text{str}} \equiv (m_n - m_p)^{\text{str}} = 2.05 \pm 0.3 \text{ MeV} \\ \delta m_N^{\text{em}} \equiv (m_n - m_p)^{\text{em}} = -0.76 \pm 0.3 \text{ MeV} \\ \text{Gasser, Leutwyler '82} \end{cases}$$

(based on the Cottingham sum rule)

$$\frac{d\sigma}{d\Omega} = A_0 + \underbrace{A_1 P_1(\cos \theta_{\pi})}_{gives \ rise \ to \ A_{fb} \ nonzero \ only \ for \ pn \to d\pi^0}_{due \ to \ interference \ of \ IB \ and \ IC \ amplitudes} + A_2 P_2(\cos \theta_{\pi}) + \dots \implies A_{fb} \simeq \frac{A_1}{2A_0}$$

A₀ can be determined from the pionic deuterium lifetime measurement @ PSI:

 $\sigma(np \to d\pi^0) = \frac{1}{2}\sigma(nn \to d\pi^-) = \frac{1}{2} \times 252^{+5}_{-11} \eta \ [\mu b] \implies A_0 = 10.0^{+0.2}_{-0.4} \eta \ [\mu b]$

• A₁ at LO in chiral EFT:
$$A_1 = \frac{1}{128\pi^2} \frac{\eta M_{\pi}}{p(M_{\pi} + m_d)^2} \Re \left[(M_{^1S_0 \to ^3S_1, p} + \frac{2}{3} M_{^1D_2 \to ^3S_1, p}) M_{^1P_1 \to ^3S_1, s}^* \right]$$

IC amplitudes calculated at NLO Baru et al.'09

Our result:
$$A_{\rm fb}^{\rm LO} = (11.5 \pm 3.5) \times 10^{-4} \, \delta m_N^{\rm str} / {\rm MeV}$$

 $\implies \delta m_N^{\rm str} = 1.5 \pm 0.8 \, ({\rm exp.}) \pm 0.5 \, ({\rm th.}) \, {\rm MeV}$

Lattice: $\delta m_N^{\text{str}} = 2.26 \pm 0.57 \pm 0.42 \pm 0.10 \text{ MeV}$ Beane et al.'07



Nuclear lattice simulations

in collaboration with Dean Lee (North Carolina) Kermann Krebs (Bochum) Ulf-G. Meißner (Bonn/Jülich)

Nuclear Lattice Simulations

Lee, E.E., Krebs, Meißner



Pions and nucleons as point-like particles on the lattice (typical lattice size ~ 20 fm)

Use Monte Carlo to evaluate path integral

 $Z_A(t) = \langle \Psi^0_A | \exp(-tH) | \Psi^0_A
angle \qquad E^0_A = \lim_{t
ightarrow \infty} \left[- rac{d}{dt} \ln Z_A(t)
ight]$

systematic ab initio approach to few- & many-nucleon systems

Two-particle scattering: spherical wall method

Borasoy, E.E., Krebs, Lee, Meißner, EPJA 34 (2007) 185

Place a wall at sufficiently large R. Phase shifts & mixing angles can be extracted by measuring energy shifts from free-particle values.



Phase shifts for a toy model potential

Inclusion of the 3N force

E.E., Krebs, Lee, Meißner '09

The unknown LECs D and E fixed from the ³H binding energy and *nd* doublet S-wave.

Neutron-deuteron spin-3/2 channel

⁴He BE vs. Euclidean time

about 5% overbinding

³H-³He binding energy difference (NNLO)

E.E., Krebs, Lee, Meißner '10

Infinite-volume extrapolations via: *Lüscher* '86

$$E(L) = E(\infty) - \frac{C}{L}e^{-L/L_0} + \mathcal{O}\left(e^{-\sqrt{2}L/L_0}\right)$$

Lattice simulations of light nuclei

Simulations for ⁶Li, L=9.9 fm

LO	-32.6(9) MeV
NLO	-34.6(9) MeV
NLO + IB + EM	-32.4(9) MeV
NNLO + IB + EM	-34.5(9) MeV
$NNLO + IB + EM + 4N_{contact}$	-32.9(9) MeV
Physical (infinite volume)	-32.0 MeV

Simulations for ¹²C, L=13.8 fm

LO	-109(2) MeV
NLO	-115(2) MeV
NLO + IB + EM	-108(2) MeV
NNLO + IB + EM	-106(2) MeV
$NNLO + IB + EM + 4N_{contact}$	-99(2) MeV
Physical (infinite volume)	-92.2 MeV

Summary

Nd scattering at N²LO

Promising results at N²LO; N³LO in progress...

\odot Chiral EFT for nuclear forces with explicit $\Delta(1232)$

- Improved convergence of the chiral expansion for nuclear forces verfied
- Expect large contributions for 3NF at N3LO (work in progress)

Pion production in NN collisions

- Important consistency check from studying different channels
- $\delta m_N^{\text{str}} = 1.5 \pm 0.8 \,(\text{exp.}) \pm 0.5 \,(\text{th.}) \text{ MeV}$ extracted from A_{fb} in $np \rightarrow d\pi^0$ consistent with the value obtained using the Cottingham sum rule and Lattice QCD

Probing light nuclei with photons

NN exchange current worked out to leading one loop, applications in progress...

Nuclear lattice simulations

- formulated continuum EFT on space-time lattice
- promising results for NN scattering and light nuclei up to N²LO