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NN Meeting, Mumbai, Nov. 2010

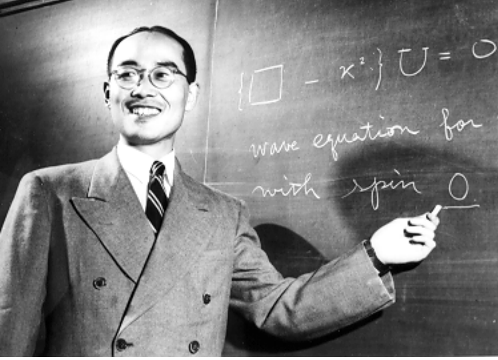
RUB

Effective Field Theory and Nuclear Forces

Lecture 1: Introduction & first look into ChPT

- The history
- How to cook up a “realistic” potential
  - General structure of the 2N force
  - Modern “high-precision“ NN potentials
  - Beyond two nucleons
- Chiral Perturbation Theory
  - Introduction
  - Chiral symmetry of QCD
  - Effective Lagrangian
  - Power counting
- 2 nucleons at very low energy (π-EFT)

Lecture 2: Chiral EFT for two nucleons



Historical overview

Yukawa's theory

Proca
Kemmer
Moller
Rosenfeld
Schwinger
Pauli …

discovery of pions

two-pion exchange, meson theory…

discovery of heavy mesons

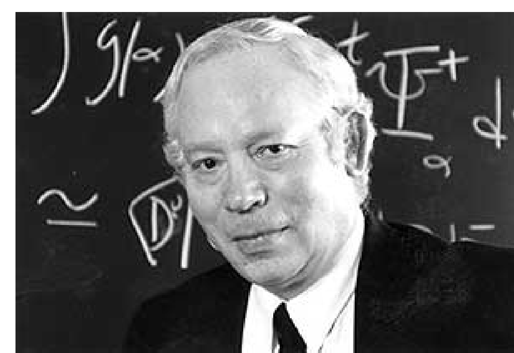
1930 | 1940 | 1950 | 1960 | 1970

BE models
inverse scattering
dispersion theory
quark cluster models
phenomenology
…

AV18
CD Bonn
Nijm I,II
Reid93
…

(Chiral) Effective Field Theory
Lattice QCD
Covariant OBE
$V_{low\text{-}K}$
…

1980 | 1990 | 2000 | 2010



2N force: operator structure

Available vectors: $\vec{r}_1,\ \vec{r}_2,\ \vec{p}_1,\ \vec{p}_2,\ \vec{\sigma}_1,\ \vec{\sigma}_2$ and isovectors: $\boldsymbol{\tau}_1\,,\ \boldsymbol{\tau}_2$

Invariance under translations and Galilei boost: $V_{2N}(\vec{r}_1, \vec{r}_2, \vec{p}_1, \vec{p}_2) = V_{2N}(\vec{r}, \vec{p})$

where $\vec{r} = \vec{r}_1 - \vec{r}_2\,,\quad \vec{p} = \frac{1}{2}(\vec{p}_1 - \vec{p}_2) = -i\vec{\nabla}_r$

Invariance under rotations, space reflection, time reversal & isospin rotations

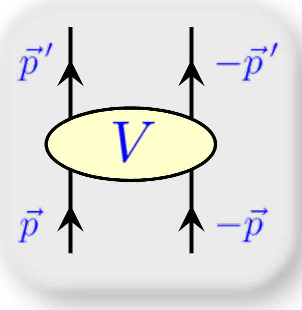
$$\Longrightarrow \quad \underbrace{\left\{1,\ \vec{\sigma}_1\cdot\vec{\sigma}_2,\ S_{12}(\vec{r}),\ S_{12}(\vec{p}),\ \vec{L}\cdot\vec{S},\ (\vec{L}\cdot\vec{S})^2\right\}}_{\textit{spin-space}} \otimes \underbrace{\left\{1,\ \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\right\}}_{\textit{isospin}}$$

where: $\vec{L} = \vec{r}\times\vec{p},\quad \vec{S} = 1/2(\vec{\sigma}_1 + \vec{\sigma}_2)\,,\quad S_{12}(\vec{x}) = 3(\vec{\sigma}_1\cdot\hat{x})(\vec{\sigma}_2\cdot\hat{x}) - \vec{\sigma}_1\cdot\vec{\sigma}_2$

Operators are to be multiplied with scalar functions of $r^2,\ p^2,\ \vec{r}\cdot\vec{p}$ or, equivalently, $r^2,\ p^2,\ L^2$

Momentum space: $\langle \vec{p}\,'|V|\vec{p}\rangle$

$$\underbrace{\left\{1,\ \vec{\sigma}_1\cdot\vec{\sigma}_2,\ S_{12}(\vec{q}),\ S_{12}(\vec{k}),\ i\vec{S}\cdot\vec{q}\times\vec{k},\ \vec{\sigma}_1\cdot\vec{q}\times\vec{k}\,\vec{\sigma}_2\cdot\vec{q}\times\vec{k}\right\}}_{\textit{spin-momentum}} \otimes \underbrace{\left\{1,\ \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\right\}}_{\textit{isospin}}$$



where $\vec{q} = \vec{p}\,' - \vec{p},\quad \vec{k} = \vec{p}\,' + \vec{p}$.

Operators are to be multiplied with scalar functions of $\vec{q}^{\,2},\ \vec{k}^{\,2},\ \vec{q}\cdot\vec{k}$.

2N force: isospin dependence

Henley, Miller '79

Class I (isospin invariant forces): $[V_I^{2N}, \boldsymbol{T}] = 0$ ⟹ $V_I^{2N} = \alpha + \beta(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)$

Class II (charge independence breaking):

$[V_{II}^{2N}, \boldsymbol{T}] \neq 0, \quad [V_{II}^{2N}, P_{cr}] = [V_{II}^{2N}, (\boldsymbol{T})^2] = 0$ ⟹ $V_{II}^{2N} = \alpha\, \tau_1^3 \tau_2^3$

$P_{cr} = \exp(i\pi T_2)$

Evidence: $1/2(\delta^{\alpha}_{nn} + \delta^{\alpha}_{pp,\, str}) \neq \delta^{\alpha}_{np}$

In particular: $a_{nn}^{1S0} \simeq -18.9$ fm, $a_{pp,\, str}^{1S0} \simeq -17.5$ fm, $a_{np}^{1S0} = -23.74(2)$ fm

Class III (charge symmetry breaking, no isospin mixing):

$[V_{III}^{2N}, \boldsymbol{T}] \neq 0, \quad [V_{III}^{2N}, P_{cr}] \neq 0, \quad [V_{III}^{2N}, (\boldsymbol{T})^2] = 0$ ⟹ $V_{III}^{2N} = \alpha\,(\tau_1^3 + \tau_2^3)$

Evidence: $\delta^{\alpha}_{nn} \neq \delta^{\alpha}_{pp,\, str}$, BE difference of mirror nuclei, …

Class IV (charge symmetry breaking and isospin mixing):

$[V_{IV}^{2N}, \boldsymbol{T}] \neq 0, \quad [V_{IV}^{2N}, P_{cr}] \neq 0, \quad [V_{IV}^{2N}, (\boldsymbol{T})^2] \neq 0$ ⟹ $V_{IV}^{2N} = \alpha\,(\tau_1^3 - \tau_2^3) + \beta[\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2]^3$

Evidence: different neutron/proton analyzing powers in np scattering, …

Long-range electromagnetic interactions

Electromagnetic interaction between point-like nucleons up to and including $\mathcal{O}(\alpha^2)$ - and $\mathcal{O}(1/m_N^2)$ -terms:

$$V_{\rm EM}(pp) = V_{\rm C}^{\rm improved} + V_{\rm VP} + V_{\rm MM}(pp)\,, \qquad V_{\rm EM}(np) = V_{\rm MM}(np)\,, \qquad V_{\rm EM}(nn) = V_{\rm MM}(nn)$$

- **Improved Coulomb potential (leading $1/m_N^2$-corrections to 1γ + 2γ-exchange)**

  Austin, de Swart '83

$$V_{\rm C}^{\rm improved} = \frac{\alpha'}{r}\left(1 - \frac{\alpha}{m_p r}\right) \quad \text{with} \quad \alpha' = \alpha \frac{m_p^2 + 2k^2}{m_p\sqrt{m_p^2 + k^2}}$$

scattering (on-shell) momentum

- **Vacuum polarization**

  Ueling '35, Durand III '57

$$V_{\rm VP} = \frac{2\alpha}{3\pi}\frac{\alpha'}{r}\int_1^\infty dx\, e^{-2m_e r x}\left(1 + \frac{1}{2x^2}\right)\frac{(x^2-1)^{1/2}}{x^2}\,,$$

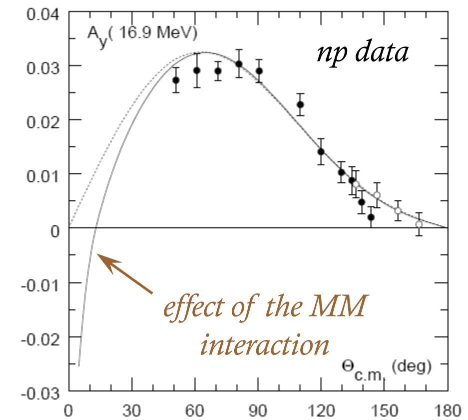
- **Magnetic moment interaction**

  Schwinger'48; Breit'55,'62; Stoks, de Swart, PRC 42 (1990) 1235

$$V_{\rm MM}(pp) = -\frac{\alpha}{4m_p^2 r^3}\left[\mu_p^2 S_{12} + (6 + 8\kappa_p)\vec{L}\cdot\vec{S}\right],$$

$$V_{\rm MM}(np) = -\frac{\alpha\kappa_n}{2m_n r^3}\left[\frac{\mu_p}{2m_p}S_{12} + \frac{1}{m}\left(\vec{L}\cdot\vec{S} + \frac{1}{2}\vec{L}\cdot(\vec{\sigma}_1 - \vec{\sigma}_2)\right)\right],$$

$$V_{\rm MM}(nn) = -\frac{\alpha\mu_n^2}{4m_n^2 r^3}S_{12}$$



NN potential: cookbook recipe

Strategy: take into account the known longest-range physics due to EM force and 1π-exchange

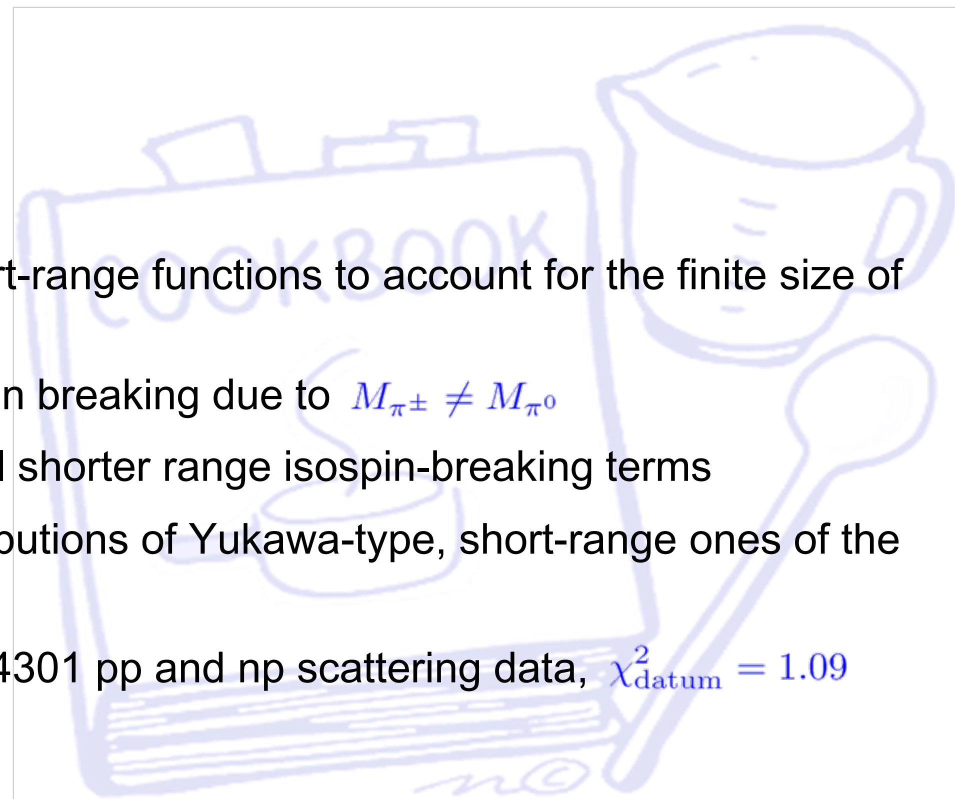
$$V_{1\pi}(\vec{q}) \propto \frac{\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^{\,2} + M_\pi^2}\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \quad \text{or in r-space:} \quad V_{1\pi}^{\text{long}}(\vec{r}) \propto \frac{e^{-M_\pi r}}{r} \left[S_{12} \left(1 + \frac{3}{M_\pi r} + \frac{3}{(M_\pi r)^2} \right) + \vec{\sigma}_1 \cdot \vec{\sigma}_2 \right] \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$$

and parametrize the medium- and short-range contributions in a most general way.

Example: AV18 potential

Wiringa, Stoks, Schiavilla '94

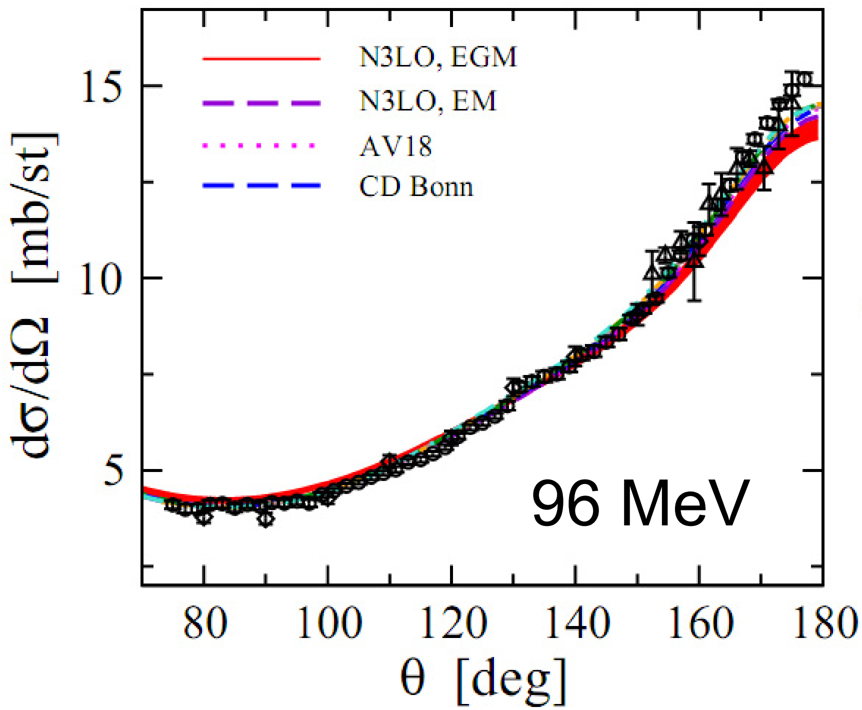
- Local r-space potential
- EM contributions multiplied by short-range functions to account for the finite size of the nucleons
- Regularized OPEP including isospin breaking due to $M_{\pi^\pm} \neq M_{\pi^0}$
- Some additional phenomenological shorter range isospin-breaking terms
- Medium-range ($r \sim (2M_\pi)^{-1}$) contributions of Yukawa-type, short-range ones of the Woods-Saxon type
- 40 adjustable parameters fitted to 4301 pp and np scattering data, $\chi^2_{\text{datum}} = 1.09$

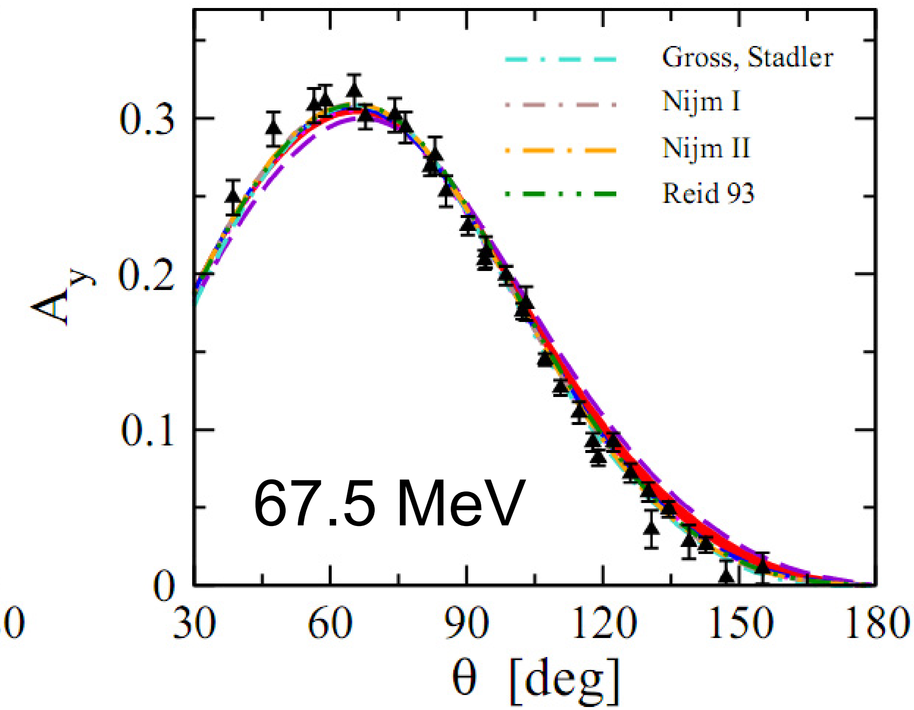


Phenomenological NN potentials

Other phenomenological potentials

- OBE motivated nonlocal (Nijm I, 41 parameters; CD Bonn, 43 parameters) and local (Nijm II, 47 parameters; Reid93, 50 parameters) potentials, covariant OBE *(Gross, Stadler)*.
- BE models (Nijm93, Bonn): less parameters but higher χ^2_{datum}





Three-nucleon force

JUNE 15, 1939 PHYSICAL REVIEW VOLUME 55

Many-Body Interactions in Atomic and Nuclear Systems

H. Primakoff, *Polytechnic Institute of Brooklyn, Brooklyn, New York*

AND

T. Holstein,* *New York University, University Heights, New York, New York*

(Received March 28, 1938)

„…replacement of field interactions by two-body action-at-a-distance potentials is a poor approximation in nuclear physics.“

„Evidence“ for missing 3NF: ^{3}H based on 2NF is typically underbound by about 1 MeV.

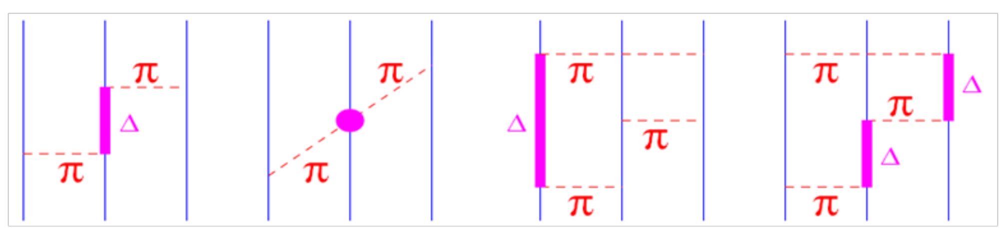
Phenomenological parametrization of the 3NF seems not feasible:

- too many possible structures (> 100)
- too scarce data base available
- too involved calculations

⇒ need guidance from a theory

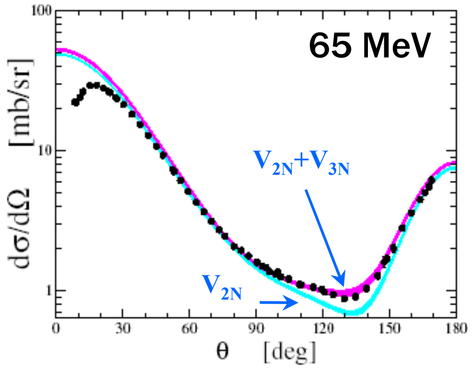
Three-nucleon force models

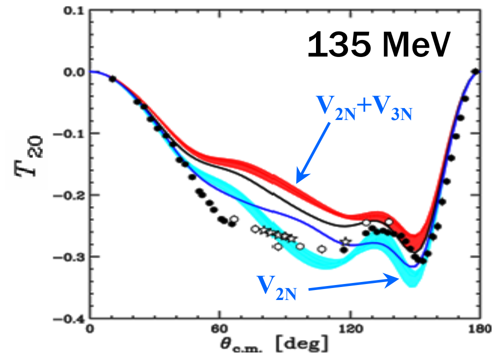
Fujita-Miyazawa, Brazil,
Tucson-Melbourne,
Urbana IX,
Illinois, …



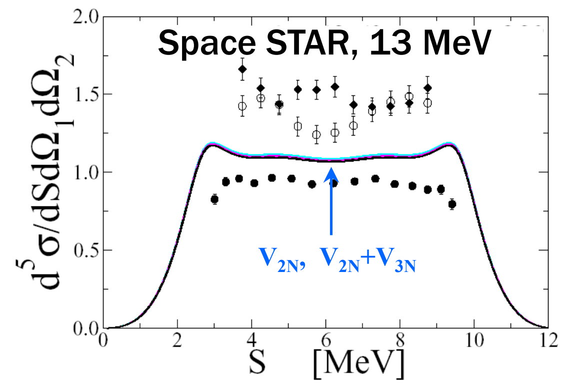
Successes and failures

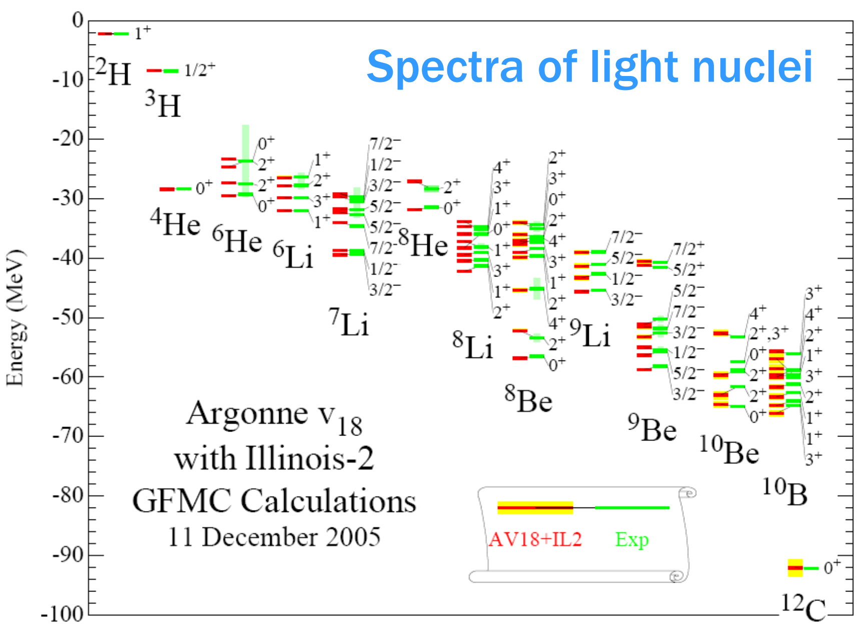
Elastic scattering observables





Deuteron breakup





Inclusion of the 3NF sometimes leads to improvements, sometimes — not. Situation, in part, chaotic.

Need a theoretical approach which would:

- be based on QCD,
- yield consistent many-body forces,
- be systematically improvable,
- allow for error estimation

⇨ chiral effective field theory

Further reading

Some modern high-precision nucleon-nucleon potentials

- *Stoks, Klomp, Terheggen, de Swart, Phys. Rev. C49 (1994) 2950 [Nijmegen 93, Nijm I,II, Reid 93]*
- *Wiringa, Stoks, Schiavilla, Phys. Rec. C51 (95) 38 [Argonne V18]*
- *Machleidt, Phys. Rev. C63 (01) 024001 [CD Bonn 2000]*
- *Machleidt, Slaus, J. Phys. G27 (01) R69 [review article]*
- *Gross, Stadler, Phys. Rev. C78 (2008) 014005 [covariant OBE]*

Three-nucleon force models

- *Fujita, Miyazawa, Prog. Theor. Phys. 17 (57) 360 [Fujita-Miyazawa 3NF model]*
- *Coon, Han, Few-Body Syst. 30 (01) 131 [Tucson-Melbourne 3NF model]*
- *Coelho, Das, Robilotta, Phys. Rev. C28 (83) 1812 [Brazilian 3NF model]*
- *Pudliner, Pandharipande, Carlson, Pieper, Wiringa, Phys. Rev. C56 (97) 1720 [Urbana IX 3NF model]*
- *Pieper, Wiringa, Ann. Rev. Nucl. Part. Sci. 51 (01) 53 [Illinois 3NF model]*

Review articles on 3N scattering & 3N force effects

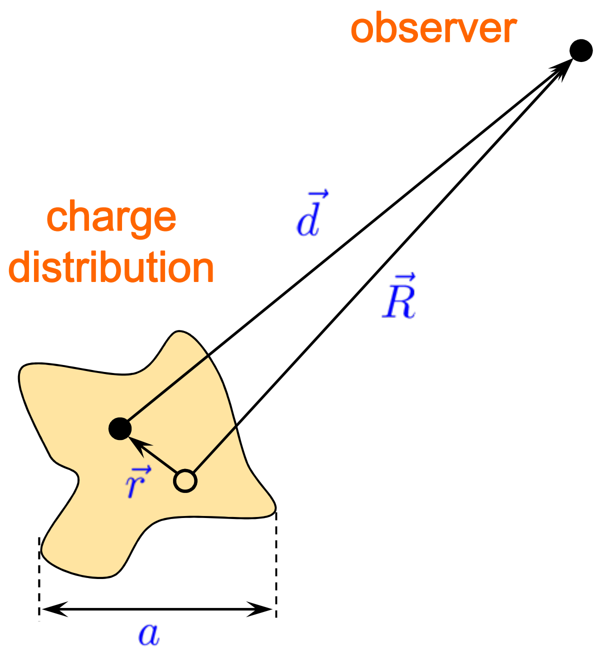
- *Glöckle, Witala, Huber, Kamada, Golak, Phys. Rept. 274 (96) 107*
- *Kalantar-Nayestanaki, E.E., Nucl. Phys. News 17 (07) 22*

Effective field theories

Effective (field) theories: approximate theories to describe phenomena which occur at a chosen length (or energy) range.

Example: multipole expansion for electric potentials

Electric potential from a localized charge distribution: $V(\vec{R}) \propto \int d^3r \, \frac{\rho(\vec{r})}{|\vec{R}-\vec{r}|}$



Multipole expansion ($a \ll R$):

$$V(\vec{R}) = \frac{q}{R} + \frac{1}{R^3}\sum_i R_i P_i + \frac{1}{6R^5}\sum_{ij}(3R_iR_j - \delta_{ij}R^2)Q_{ij} + \ldots$$

with the "low-energy constants" which depend on $\rho(\vec{r})$:

$$q = \int d^3r \, \rho(\vec{r}), \qquad P_i = \int d^3r \, \rho(\vec{r}) \, r_i, \qquad Q_{ij} = \int d^3r \, \rho(\vec{r})(3r_ir_j - \delta_{ij}r^2) \, .$$

Weinberg's Folk Theorem

"if one writes down the most general possible Lagrangian, including all terms consistent with the assumed symmetry principles, and then calculates S-matrix elements with this Lagrangian to any order in perturbation theory, the result will simply be the most general possible S-matrix consistent with analyticity, perturbative unitarity, cluster decomposition and the assumed symmetry principles"

S.Weinberg, Physica A96 (79) 327

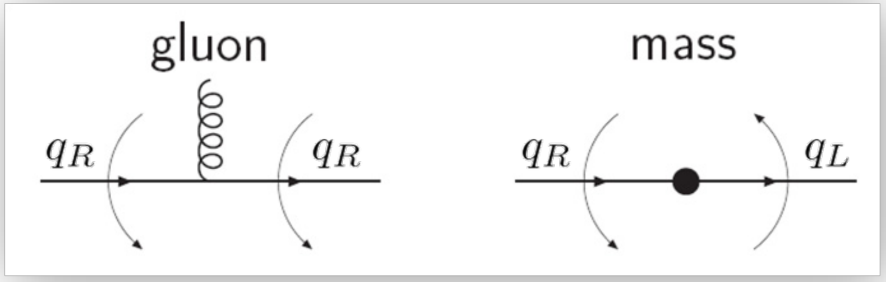
Chiral symmetry of QCD Lagrangian

$$\mathcal{L}_{\rm QCD} = -\tfrac{1}{4}G_{\mu\nu}G^{\mu\nu} + \bar{q}(i\not{D} - \mathcal{M})q \;=\; -\tfrac{1}{4}G_{\mu\nu}G^{\mu\nu} + \underbrace{\bar{q}_L i\not{D} q_L + \bar{q}_R i\not{D} q_R}_{SU(N_f)_L \times SU(N_f)_R\ \textit{invariant}} - \underbrace{\bar{q}_L \mathcal{M} q_R - \bar{q}_R \mathcal{M} q_L}_{\textit{breaks chiral symmetry}}$$

Left- & right-handed quark fields:

$$q_{L,R} = \tfrac{1}{2}(1 \pm \gamma_5)q.$$

$q_{L,\,R}$ do not "talk" to each other in the massless limit ⟹ $\mathcal{L}_{\rm QCD}$ is invariant against independent flavor rotations (chiral symmetry):



$$q_L \longrightarrow q'_L = Lq_L = \exp\left(-i\boldsymbol{\theta}_L \cdot \boldsymbol{\tau}/2\right)q_L, \qquad q_R \longrightarrow q'_R = Rq_R = \exp\left(-i\boldsymbol{\theta}_R \cdot \boldsymbol{\tau}/2\right)q_R\,,$$

$m_u \simeq 1.5 \ldots 3.3$ MeV, $m_d \simeq 3.5 \ldots 6.0$ MeV *($\overline{MS}$, μ=2 GeV)* ⟹ accurate symmetry for 2 flavors

Conserved currents *(Noether Theorem)*

$L^i_\mu = \bar{q}_L \gamma_\mu \frac{\tau^i}{2} q_L\,, \quad R^i_\mu = \bar{q}_R \gamma_\mu \frac{\tau^i}{2} q_R\,,$ or equivalently $V^i_\mu = L^i_\mu + R^i_\mu\,, \quad A^i_\mu = R^i_\mu - L^i_\mu$

Lie algebra generated by conserved charges

$$\left[Q^i_V,\, Q^j_V\right] = i\epsilon^{ijk}Q^k_V\,, \quad \left[Q^i_A,\, Q^j_A\right] = i\epsilon^{ijk}Q^k_V\,, \quad \left[Q^i_V,\, Q^j_A\right] = i\epsilon^{ijk}Q^k_A$$

Spontaneous symmetry breaking

There is a strong evidence that chiral symmetry of QCD is spontaneously broken down to the isospin group:

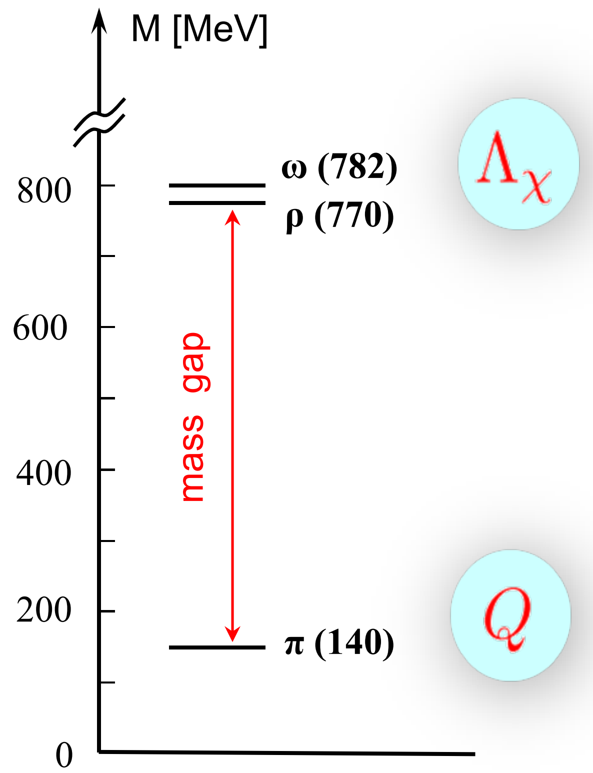
- Only isospin but not chiral multiplets are observed in the particle spectrum (axial charges would lead to parity doublets)
- Triplet of unnaturally light pseudoscalar mesons (pions) — natural candidates for Goldstone bosons
- Scalar quark condensate:

$$\langle 0|\bar{q}q|0\rangle\Big|_{\overline{MS},\,2\,GeV} = -(273 \pm 12\ \mathrm{MeV})^3$$

(Lattice QCDSF/UKQCD, *Schierholz et al. '07*)

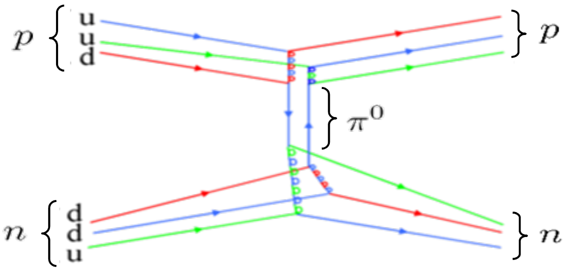
- Further theoretical arguments

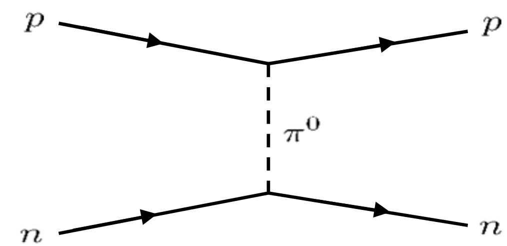
Vafa & Witten '84; 't Hooft '80; Coleman & Witten '80



Chiral Perturbation Theory

Weinberg, Gasser, Leutwyler, Bernard, Kaiser, Meißner, ...





$$\int [Dq][D\bar{q}][DG]\; e^{\int i d^4x\, \mathcal{L}_{\rm QCD}[q,\bar{q},G;\; J]} \quad \Longrightarrow \quad \int [D\Phi]\; e^{\int i d^4x\, \mathcal{L}_{\rm eff}[\Phi;\; J]}$$

Cannot derive $\mathcal{L}_{eff}$ ⟹ write down most general expression

- including all possible χ-invariant terms,
- including all terms that break χ-symmetry in the same way as $\bar{q}mq$ in $\mathcal{L}_{\rm QCD}$

The simplest case: pure Goldstone Boson sector in the chiral limit.

- How to write down most general χ-invariant $\mathcal{L}_{eff}$?
- How do π's transform under G?
- Isospin subgroup $H \in G$ realized linearly (π's build an isospin triplet).
- Chiral group necessarily realized nonlinearly:

  $SU(2)_L \times SU(2)_R \sim SO(4)$ ⟹ need ≥ 4 dimensions for a nontrivial linear realization

Chiral rotations & pion fields

Infinitesimal $\mathrm{SO}(4)$ rotation of the 4-vector $(\pi_1, \pi_2, \pi_3, \sigma)$:

$$\begin{pmatrix} \vec{\pi} \\ \sigma \end{pmatrix} \to \begin{pmatrix} \vec{\pi}' \\ \sigma' \end{pmatrix} = \left[1 + \vec{\theta}^V \cdot \vec{V} + \vec{\theta}^A \cdot \vec{A}\right] \begin{pmatrix} \vec{\pi} \\ \sigma \end{pmatrix}$$

where: $\vec{\theta}^V \cdot \vec{V} = \begin{pmatrix} 0 & -\theta_3^V & \theta_2^V & 0 \\ \theta_3^V & 0 & -\theta_1^V & 0 \\ -\theta_2^V & \theta_1^V & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ and $\vec{\theta}^A \cdot \vec{A} = \begin{pmatrix} 0 & 0 & 0 & \theta_1^A \\ 0 & 0 & 0 & \theta_2^A \\ 0 & 0 & 0 & \theta_3^A \\ -\theta_1^A & -\theta_2^A & -\theta_3^A & 0 \end{pmatrix}$

One reads off: $\vec{\pi}' = \vec{\pi} + \vec{\theta}^V \times \vec{\pi} + \vec{\theta}^A \sigma$ and $\sigma' = \sigma - \vec{\theta}^A \cdot \vec{\pi}$

Switch to the nonlinear realization of $\mathrm{SO}(4)$:
only 3 out of 4 components of the vector $(\vec{\pi}, \sigma)$ are independent, i.e. $\vec{\pi}^2 + \sigma^2 = F^2$

$$\sigma = \sqrt{F^2 - \vec{\pi}^2} \quad \Rightarrow \quad \begin{cases} \vec{\pi} \xrightarrow{\vec{\theta}^V} \vec{\pi}' = \vec{\pi} + \vec{\theta}^V \times \vec{\pi} & \longleftarrow \textit{linear under } \vec{\theta}^V \\ \vec{\pi} \xrightarrow{\vec{\theta}^A} \vec{\pi}' = \vec{\pi} + \vec{\theta}^A \sqrt{F^2 - \vec{\pi}^2} & \longleftarrow \textit{nonlinear under } \vec{\theta}^A \end{cases}$$

It is more convenient to use a 2 x 2 matrix notation:

$$U = \frac{1}{F}\left(\sigma I + i\vec{\pi} \cdot \vec{\tau}\right) \xrightarrow[\textit{realization}]{\textit{nonlinear}} U = \frac{1}{F}\left(I\sqrt{1 - \vec{\pi}^2} + i\vec{\pi} \cdot \vec{\tau}\right)$$

Chiral rotations: $U \longrightarrow U' = L U R^\dagger$ with $L = e^{-i/2(\vec{\theta}^V - \vec{\theta}^A) \cdot \vec{\tau}}$ and $R = e^{-i/2(\vec{\theta}^V + \vec{\theta}^A) \cdot \vec{\tau}}$

Effective Lagrangian

The above realization of G is not unique. How does this non-uniqueness affect S-matrix?

- All realizations of G are equivalent to each other by means of nonlinear field redefinitions $\vec{\pi} \to \vec{\pi}' = \vec{\pi}\, F[\vec{\pi}\,],\; F[0] = 1$ *(Coleman, Callan, Wess & Zumino '69)*
- Field redefinitions do not affect S-matrix *(Haag '58)*

Derivative expansion for the effective Lagrangian $\mathcal{L}_{\rm eff} = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \ldots$

- 0 derivatives: $UU^\dagger = U^\dagger U = 1$ — plays no role
- 2 derivatives: $\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) \xrightarrow{g\in G} \mathrm{Tr}(L\partial_\mu U R^\dagger R\, \partial^\mu U^\dagger L^\dagger) = \mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger)$

  $$\Longrightarrow \quad \mathcal{L}_\pi^{(2)} = \frac{F^2}{4}\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger)$$

- 4 derivatives: $[\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger)]^2\,,\;\; \mathrm{Tr}(\partial_\mu U \partial_\nu U^\dagger)\mathrm{Tr}(\partial^\mu U \partial^\nu U^\dagger)\,,\;\; \mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger \partial_\nu U \partial^\nu U^\dagger)$ ← *derivatives act only on the next U*

  (terms with $\partial_\mu\partial_\nu U,\; \partial_\mu\partial_\nu\partial_\rho U,\; \partial_\mu\partial_\nu\partial_\rho\partial_\sigma U$ can be eliminated via EOM/partial integration)

…

What is the meaning of F ?

Axial current from $\mathcal{L}_\pi^{(2)}$: $\quad J^i_{A\mu} = i\mathrm{Tr}[\tau^i(U^\dagger\partial_\mu U - U\partial_\mu U^\dagger)] = -F\partial_\mu \pi^i + \ldots$ ← *more pion fields*

$$\langle 0| J^i_{A\mu} |\pi^j(\vec{p})\rangle \equiv i p_\mu F_\pi \delta^{ij} \quad \Longrightarrow \quad F = F_\pi = 92.4\ \mathrm{MeV}$$

Effective Lagrangian

How to account for explicit χ-symmetry breaking due to nonvanishing quark masses?

external hermitian field

Trick (method of external sources): $\delta\mathcal{L}_{\rm QCD} = -\bar q \mathcal{M} q\Big|_{\mathcal{M}=m}$

$-\bar q \mathcal{M} q = -\bar q_L \mathcal{M} q_R - \bar q_R \mathcal{M} q_L$ is χ -invariant if: $\mathcal{M} \xrightarrow{G} \mathcal{M}' = g_R \mathcal{M} g_L^{-1} = g_L \mathcal{M} g_R^{-1}$

⟹ write down all possible χ–invariant terms with $\mathcal{M}$ and then set $\mathcal{M} = m$

The leading (i.e. no ∂_μ and $\propto \mathcal{M}$) SB term in $\mathcal{L}_{eff}$:

$$\mathcal{L}_{\rm SB} = \frac{BF^2}{2}{\rm Tr}[(U+U^\dagger)\mathcal{M}]\Big|_{\mathcal{M}=m} = 2BF^2 m_q - B\, m_q\, \vec\pi^{\,2} + \mathcal{O}(\vec\pi^{\,4}) \;\Longrightarrow\; M_\pi^2 = 2m_q B + \mathcal{O}(m_q^2)$$

The LEC B is related to the scalar quark condensate via $\langle 0|\bar u u|0\rangle = \langle 0|\bar d d|0\rangle = -BF^2 + \mathcal{O}(\mathcal{M})$

Notice: the generalized scenario (*Stern et al. '91*) in which $2m_q B \ll M_\pi^2$ is ruled out by recent data on $\pi\pi$ scatt. length.

Effective Lagrangian

Gasser, Leutwyler, Nucl. Phys. B250 (1985) 465

$$\mathcal{L}_\pi^{(2)} = \frac{F^2}{4}\Big[\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) + \mathrm{Tr}(U\chi + U^\dagger\chi)\Big],$$

low-energy constants

$$\begin{aligned}\mathcal{L}_\pi^{(4)} &= L_1[\mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U)]^2 + L_2\mathrm{Tr}(\partial_\mu U^\dagger \partial_\nu U)\mathrm{Tr}(\partial^\mu U^\dagger \partial^\nu U) + L_3\mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U \partial_\nu U^\dagger \partial^\nu U)\\ &+ L_4\mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U)\mathrm{Tr}(U\chi + U^\dagger\chi) + L_5\mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U(U\chi + U^\dagger\chi)) + L_6[\mathrm{Tr}(U\chi + U^\dagger\chi)]^2\\ &+ L_7[\mathrm{Tr}(U\chi - U^\dagger\chi)]^2 + L_8\mathrm{Tr}(\chi U\chi U + \chi U^\dagger \chi U^\dagger)\end{aligned}$$

where $\chi = 2B\mathcal{M}$.

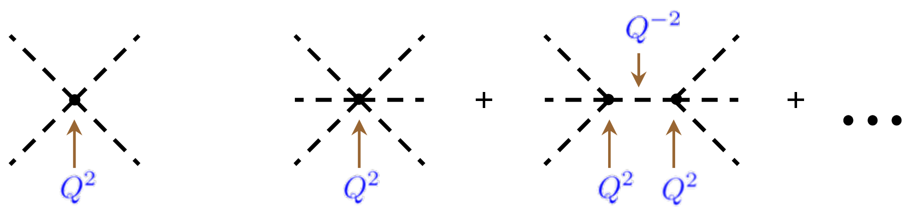
- Only those terms are shown which do not involve external sources (there are 3 more terms which describe the interaction of GBs with external fields).
- The Lagrangian is shown for the SU(3) x SU(3) case. Some terms are redundant in the case of SU(2) x SU(2) chiral symmetry.
- How to calculate observables ??

From effective Lagrangian to S-matrix

Lowest-order effective Lagrangian: $\mathcal{L}_\pi^{(2)} = \frac{F^2}{4}\left[\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) + \mathrm{Tr}(U\chi + U^\dagger \chi)\right]$

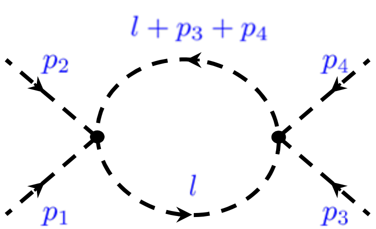
$$U = \frac{1}{F}[\sqrt{F^2 - \pi^2} + i\boldsymbol{\tau}\cdot\boldsymbol{\pi}] = 1 + \frac{i\boldsymbol{\tau}\cdot\boldsymbol{\pi}}{F} - \frac{\pi^2}{2F^2} + \ldots \quad \Longrightarrow \quad \mathcal{L}_\pi^{(2)} = \frac{\partial_\mu \boldsymbol{\pi}\cdot\partial^\mu \boldsymbol{\pi}}{2} - \frac{M^2\pi^2}{2} + \frac{(\partial_\mu \boldsymbol{\pi}\cdot\boldsymbol{\pi})^2}{2F^2} - \frac{M^2\pi^4}{8F^2} + \ldots$$

Tree-level multi-pion connected diagrams from $\mathcal{L}_\pi^{(2)}$ at low energy ($p_i \sim |\vec{p}_i| \sim M_\pi \sim Q$):



- *all diagrams scale as* Q^2
- *insertions from* $\mathcal{L}_\pi^{(4)}$, $\mathcal{L}_\pi^{(6)}$, … *suppressed by powers of* Q^2

What about quantum corrections (loop diagrams)?



$$\int \frac{d^4 l}{(2\pi)^4} \frac{p_1\cdot p_2 \; p_3 \cdot p_4}{[l^2 - M_\pi^2 + i\epsilon]\,[(l + p_3 + p_4)^2 - M_\pi^2 + i\epsilon]} \sim \mathcal{O}(E^4) \quad \Longrightarrow \quad \text{suppressed…}$$

UV divergences removed e.g. using DR, $\int d^4 l \to \mu^{d-4} \int d^d l$, and redefining LECs from $\mathcal{L}_\pi^{(4)}$

General observation: n-loop diagrams are suppressed by the factor Q^{2n} compared to the tree ones Q^2.

Power counting

Weinberg '79

amplitude → *phase space factors*

Consider S-matrix element: $S = \delta^4(p_1 + p_2 + \ldots + p_N)\, M\, \Pi$

The amplitude can be rewritten as: $M \equiv M(E, \mu, g^r) = E^D f\left(\frac{E}{\mu}, g^r\right)$

combination of LECs

Dimensional analysis:

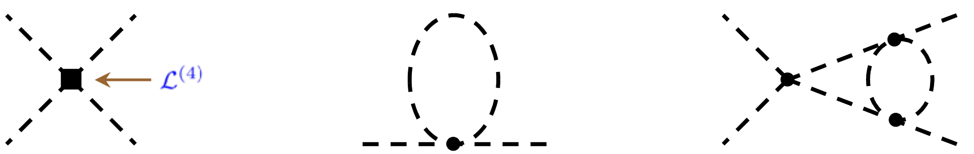
- pion propagators: $1/(p^2 - M_\pi^2) \sim 1/Q^2$
- momentum integrations: $d^4 l \sim Q^4$
- delta functions: $\delta^4(p - p') \sim 1/Q^4$
- derivatives: $\partial_\mu \sim Q$

⟹

# of loops *# of vertices with derivatives*

$$D = 2 + 2L + \sum_d N_d (d - 2)$$

Examples:



$D = 2 + 0 + 2 = 4$ $D = 2 + 2 + 0 = 4$ $D = 2 + 4 + 0 = 6$

Amplitude is obtained via expansion in E/Λ_χ. What is the value of Λ_χ?

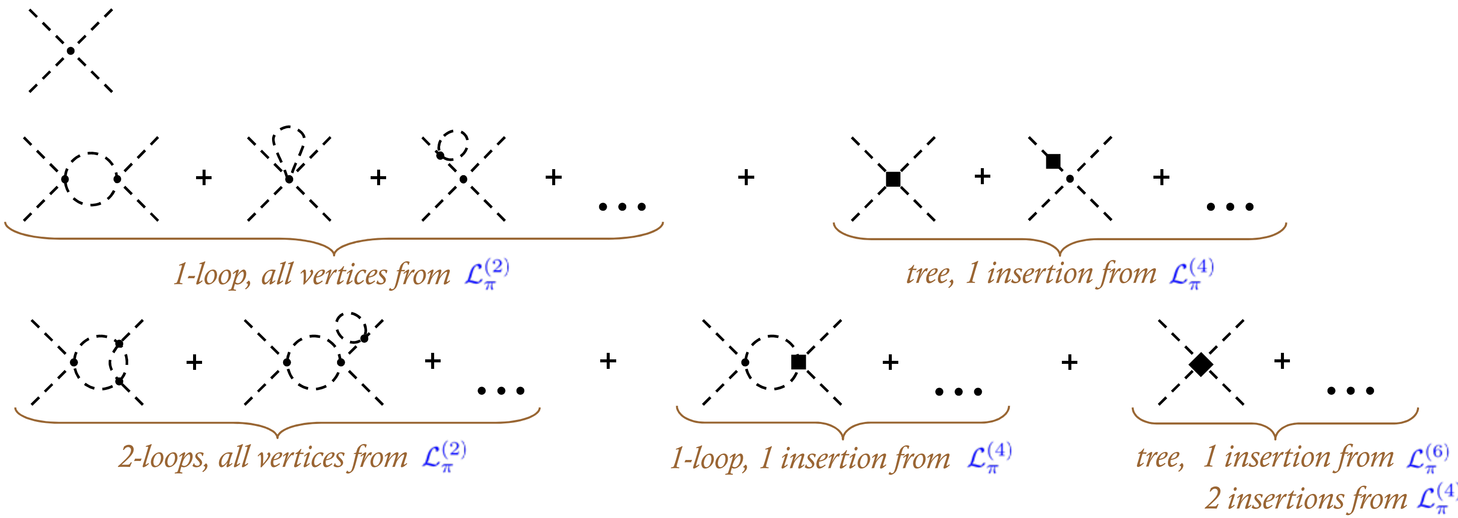
- Chiral expansion breaks down for $E \sim M_\rho$ ⟹ $\Lambda_\chi \sim M_\rho = 770$ MeV
- Consistency arguments imply: $\Lambda_\chi \leq 4\pi F_\pi = 1.2$ GeV *(Manohar & Georgi '84)*

Pion scattering lengths in ChPT

E^2 :

E^4 :

E^6 :



Predictive power?

$$\mathcal{L}_{\rm eff} = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \mathcal{L}_\pi^{(6)} + \ldots$$

# of LECs increasing…

S-wave $\pi\pi$ scattering length

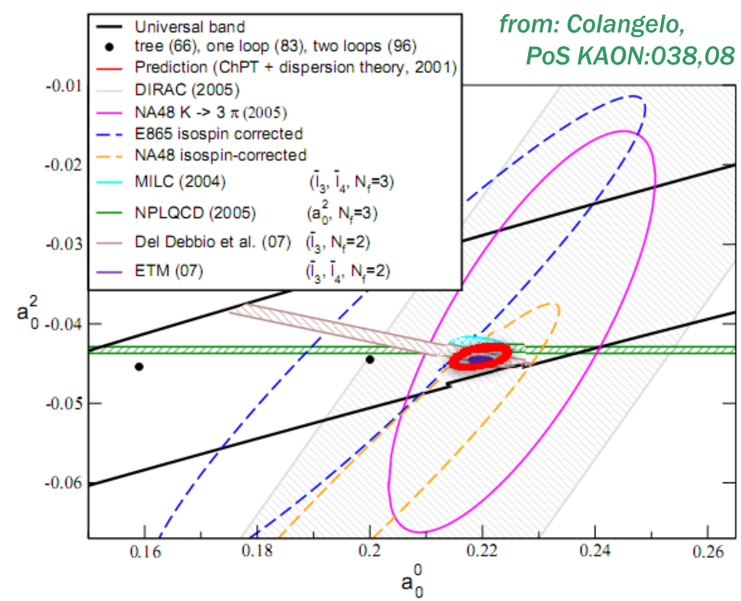
LO: $a_0^0 = 0.16$ *(Weinberg '66)*

NLO: $a_0^0 = 0.20$ *(GL '83)*

NNLO: $a_0^0 = 0.217$ *(Bijnens et al. '95)*

NNLO + disp. relations: *(Colangelo et al.)*

$$a_0^0 = 0.217 \pm 0.008\,(\text{exp}) \pm 0.006\,(\text{th})$$



from: Colangelo, PoS KAON:038,08

ChPT in the single-baryon sector

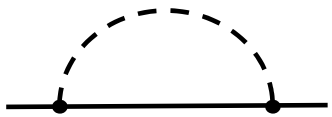
General formalim to construct the most general χ–invariant $\mathcal{L}_{\mathrm{eff}}$ with matter fields (like e.g. nucleons) is given in *Coleman, Callan, Wess, Zumino, PR 177 (1969) 2239; 2247.*

Lowest-order: $$\mathcal{L}_{\pi N}^{(1)} = \bar{N} \left(i\gamma^\mu D_\mu - m + \frac{g_A}{2} \gamma^\mu \gamma_5 u_\mu \right) N$$

known functions of the pion fields

Problem (?): new hard mass scale m ⟹ power counting ??

For example: 1-loop correction to m *(Gasser, Sainio & Svarc '88)*



⟹ $$\delta m \overset{M \to 0}{\longrightarrow} -\frac{3 g_A^2 m^3}{(4\pi F)^2} \left[\log \frac{m}{\mu} + \mu^{d-4} \left(\frac{1}{d-4} + const \right) \right]$$

Making power counting manifest: HBChPT *(Jenkins & Manohar '91; Bernard et al. '92)*

$$\mathcal{L}_{\pi N}^{(1)} = N'^\dagger \left(iD_0 + \frac{g_A}{2} \vec{\sigma} \cdot \vec{u} \right) N' + \mathcal{O}(1/m)$$

m disappeared from $\mathcal{L}_{\pi N}^{(1)}$ ⟹ power counting manifest! For example: $(\delta m)^{\mathrm{HB}} = -\frac{3 g_A^2 M_\pi^3}{32 \pi F^2}$

Notice: Lorentz invariant formulations preferable in certain cases!

Ellis & Tang '98; Becher & Leutwyler '99; Gegelia et al. '03; ...

Further reading

Construction of the effective chiral Lagrangian

- *Weinberg, Phys. Rev. 166 (1968) 1568*
- *Coleman, Wess, Zumino, Phys. Rev. 177 (1969) 2239*
- *Callan, Coleman, Wess, Zumino, Phys. Rev. 177 (1969) 2247*

Chiral perturbation theory: milestones

- *Weinberg, Physica A96 (1979) 327*
- *Gasser, Leutwyler, Ann. Phys. 158 (1984) 142; Nucl. Phys. B250 (1985) 465*

Chiral perturbation theory: (some) review articles

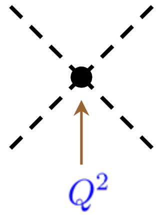
- *Bernard, Kaiser, Meißner, Int. J. Mod. Phys. E4 (1995) 193*
- *Pich, Rep. Prog. Phys. 58 (1995) 563*
- *Bernard, Prog. Part. Nucl. Phys. 60 (2007) 82*
- *Scherer, Prog. Part. Nucl. Phys. 64 (2010) 1*

Chiral perturbation theory: (some) lecture notes

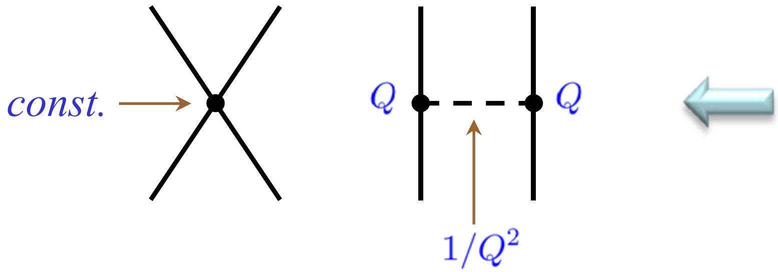
- *Scherer, Adv. Nucl. Phys. 27 (2003) 277*
- *Gasser, Lect. Notes Phys. 629 (2004) 1*

Two nucleons

In the $\pi\pi$, πN sectors, S-matrix can be evaluated in perturbation theory (Goldstone bosons do not interact at $E = 0,\ m_q = 0$)



But this is not the case for 2 and more nucleons:



do **not** vanish for $E = 0,\ m_q = 0$

The presence of shallow bound states (^{2}H, ^{3}H, ^{3}He, ^{4}He, …) indicates breakdown of perturbation theory even at very low energy!

Is EFT still useful for strongly interacting nucleons?

Effective Range Expansion

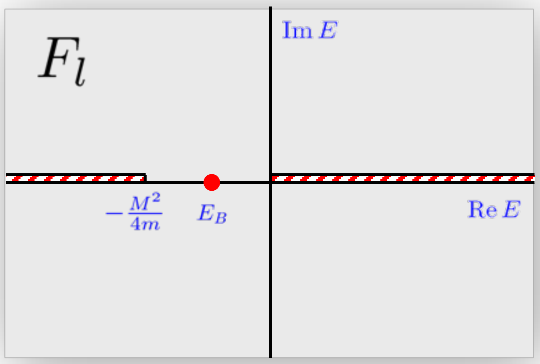
Blatt, Jackson '49; Bethe '49

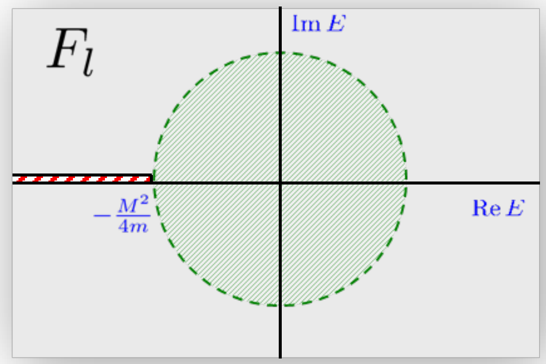
Nonrelativistic nucleon-nucleon scattering (uncoupled case):

effective-range function

$S_l(k) = e^{2i\delta_l(k)} = 1 + i\frac{mk}{2\pi}T_l(k)$ where $T_l(k) = \frac{4\pi}{m}\frac{k^{2l}}{F_l(k) - ik^{2l+1}}$ and $F_l(k) \equiv k^{2l+1}\cot\delta_l(k)$

If $V(r)$ satisfies certain conditions, F_l is a meromorphic function of k^2 near the origin





⟹ effective range expansion (ERE):

$$F_l(k^2) = -\frac{1}{a} + \frac{1}{2}rk^2 + v_2k^4 + v_3k^6 + v_4k^8 + \dots$$

The range of convergence of the ERE depends on the range M^{-1} of $V(r)$ defined as

$$M = \min(\mu) \quad \text{such that} \quad \int_{R>0}^{\infty} |V(r)|\, e^{\mu r} dr = \infty$$

Pionless EFT: natural scattering length

Effective Lagrangian: for $Q \ll M_\pi$ only point-like interactions

$$\mathcal{L}_{\rm eff} = N^\dagger \left(i\partial_0 + \frac{\vec\nabla^2}{2m} \right) N - \frac{1}{2} C_1^0 (N^\dagger N)^2 - \frac{1}{2} C_2^0 (N^\dagger \vec\sigma N)^2 - \frac{1}{4} C_1^2 (N^\dagger \vec\nabla^2 N)(N^\dagger N) + \text{h.c.} + \ldots$$

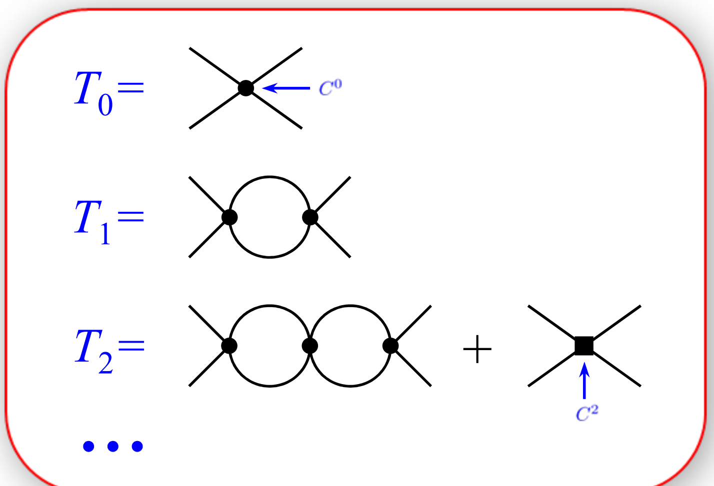
Scattering amplitude (S-waves):

$$S = e^{2i\delta} = 1 - i \left(\frac{km}{2\pi} \right) T \,, \qquad T = -\frac{4\pi}{m} \frac{1}{k\cot\delta - ik} = -\frac{4\pi}{m} \frac{1}{\left(-\frac{1}{a} + \frac{1}{2} r_0 k^2 + v_2 k^4 + v_3 k^6 + \ldots\right) - ik}$$

- **Natural case**

$$|a| \sim M_\pi^{-1} \,, \ |r| \sim M_\pi^{-1} \,, \ \ldots \quad \Longrightarrow \quad T = T_0 + T_1 + T_2 + \ldots = \frac{4\pi a}{m} \left[1 - iak + \left(\frac{a r_0}{2} - a^2 \right) k^2 + \ldots \right]$$

$\sim Q^0$ $\sim Q^1$ $\sim Q^2$



The EFT expansion can be arranged to match the above expansion for T.

Using e.g. dimensional or subtractive ragularization yields:

- perturbative expansion for T;
- scaling of the LECs: $C^i \sim Q^0$

In reality: $a_{^1S_0} = -23.741 \text{ fm} = -16.6\, M_\pi^{-1}$ $\quad a_{^3S_1} = -5.42 \text{ fm} = 3.8\, M_\pi^{-1}$

Pionless EFT: large scattering length

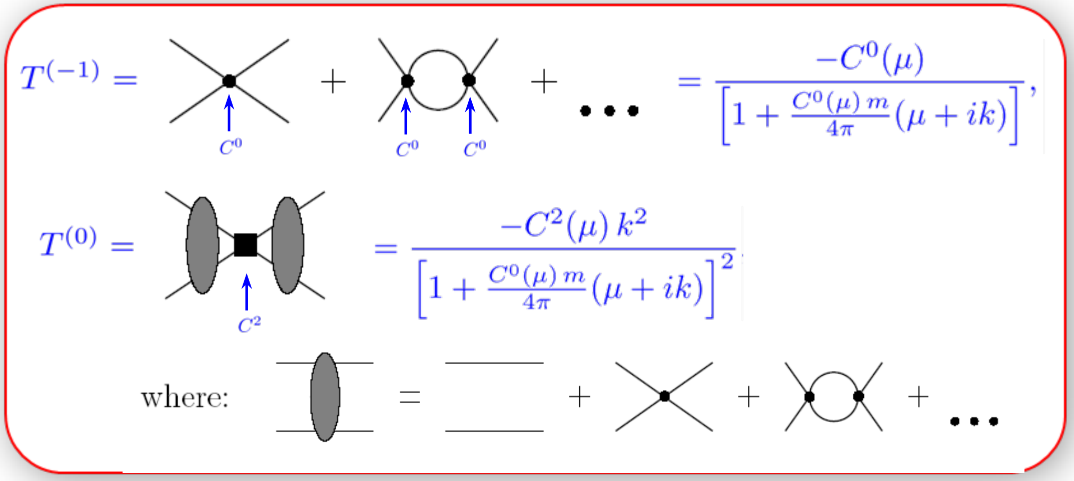
- **Unnatural case:** $|a| \gg M_\pi^{-1}$ *(Kaplan, Savage & Wise '97)*

  Keep ak fixed, i.e. count $a \sim Q^{-1}$:

$$T = -\frac{4\pi}{m}\frac{1}{\left(-\frac{1}{a}+\frac{1}{2}r_0k^2+v_2k^4+v_3k^6+\ldots\right)-ik} = \frac{4\pi}{m}\frac{1}{(1+iak)}\left[\underbrace{a}_{\sim Q^{-1}}+\underbrace{\frac{ar_0}{2(a^{-1}+ik)}}_{\sim Q^{0}}k^2+\underbrace{\ldots}_{\sim Q^{1}}\right].$$

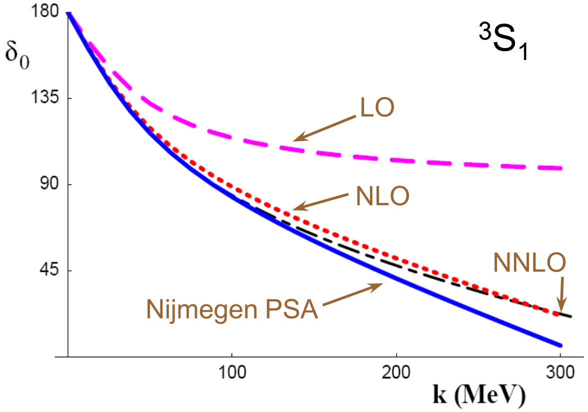
  Notice: perturbation theory breaks down
  (T has a pole at $|k| \sim |a|^{-1} \ll M_\pi$)

KSW expansion (DR + Power Divergence Subtraction $\Rightarrow$ $C^0 \sim 1/Q, \quad C^2 \sim 1/Q^2, \quad \ldots$ **)**



$$T^{(-1)} = \ldots = \frac{-C^0(\mu)}{\left[1+\frac{C^0(\mu)\,m}{4\pi}(\mu+ik)\right]},$$

$$T^{(0)} = \ldots = \frac{-C^2(\mu)\,k^2}{\left[1+\frac{C^0(\mu)\,m}{4\pi}(\mu+ik)\right]^2}$$



from: *Chen, Rupak & Savage NPA653 (1999)*

- Astrophysical applications *(Butler, Chen, Kong, Ravndal, Rupak, Savage, ...)*
- Universal properties of few-body systems for $a \to \infty$, *(Braaten & Hammer, Phys. Rept. 428 (06) 259)*
- Halo-nuclei *(Bedaque, Bertulani, Hammer, Higa, van Kolck, ...)* & many other topics…

Further reading

Pionless effective field theory: Formulation

- *Kaplan, Savage, Wise, Phys. Lett. B424 (98) 390; Nucl. Phys. B534 (98) 329*
- *Bedaque, Hammer, van Kolck, Phys. Rev. Lett. 92 (99) 463; Nucl. Phys. A646 (99) 444*

Pionless effective field theory: (some) review articles

- *Beane et al., arXiv:nucl-th/0008064, in Boris Ioffe Festschrift, ed. By M. Shifman, World Scientific*
- *Bedaque, van Kolck, Ann. Rev. Nucl. Part. Sci. 52 (02) 339*
- *Braaten, Hammer, Phys. Rept. 428 (06) 259*