Hadron-Hadron Interactions from Lattice QCD

Takumi Doi

(Univ. of Tsukuba)

for HAL QCD Collaboration

S. Aoki, K. Sasaki (Univ. of Tsukuba)
T. Hatsuda, N. Ishii (Univ. of Tokyo)
Y. Ikeda (RIKEN)
T. Inoue (Nihon Univ.)
K. Murano (KEK)
H. Nemura (Tohoku Univ.)

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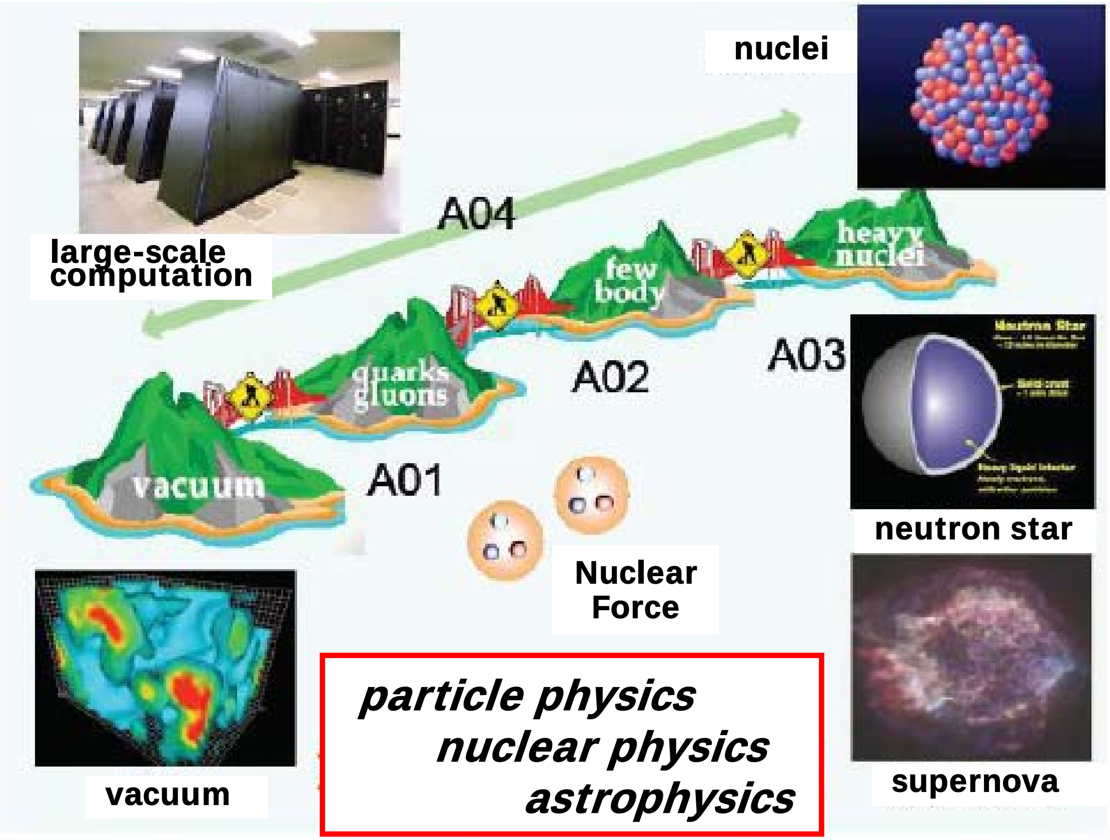
- **Motivation**
- **Formulation for NN potential in Lattice QCD**
- **Extension to YN, YY potentials**
- **Recent progress on Three Nucleon Force (TNF)**
- **Summary and Outlook**

Motivation



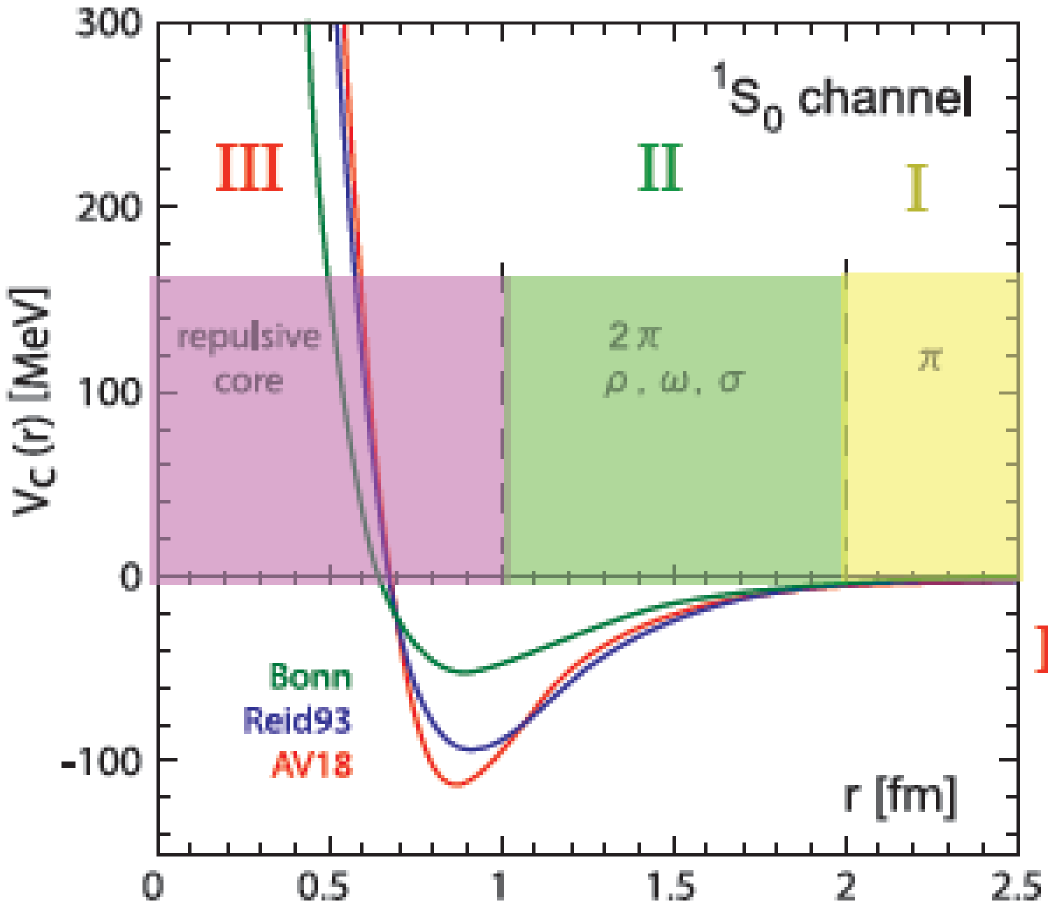
Understand the various phenomena from fundamental theory

- Nuclei
- Neutron star
- SuperNova

Nuclear Force is the key concept which **bridges** (effective) DOF in **different hierarchy**

Phenomenological NN potential

(~40 parameters to fit 5000 phase shift data)



I One-pion exchange

Yiukawa(1935)

II Multi-pions

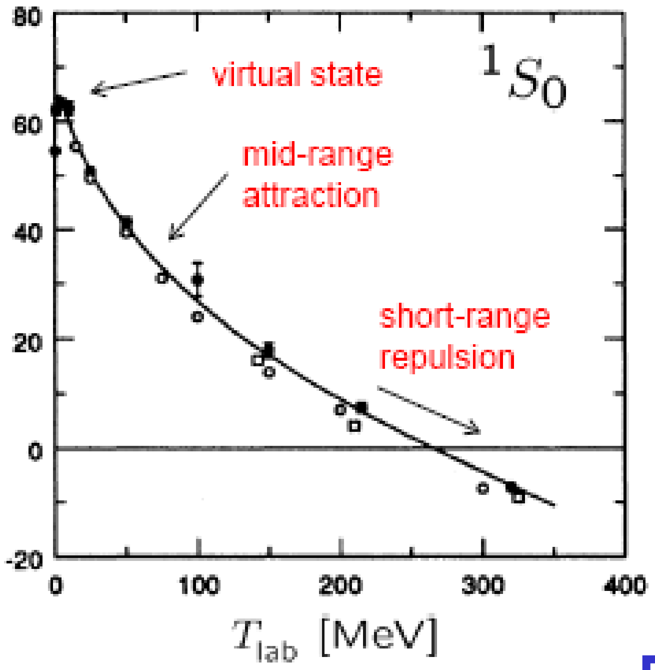
Taketani et al.(1951)

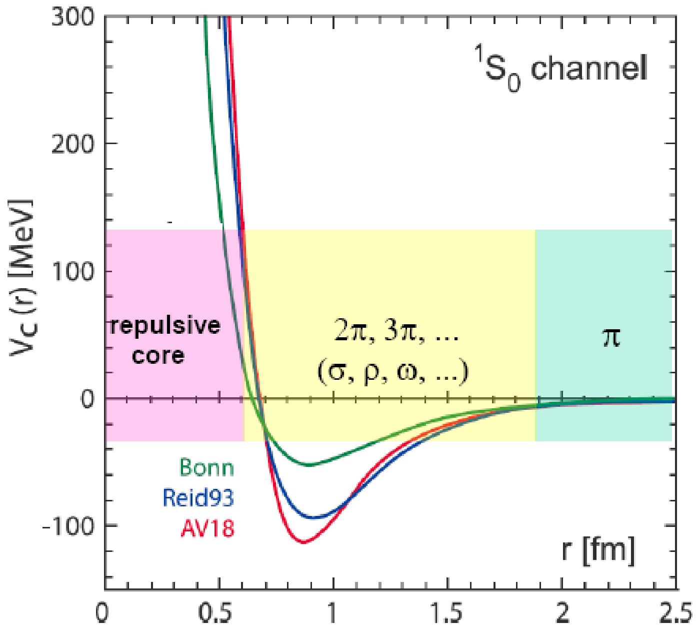
III Repulsive core

Jastrow(1951)

Nuclear Force from Experiments

- Potential is constructed so as to reproduce the NN phase shift (or, S-matrix)

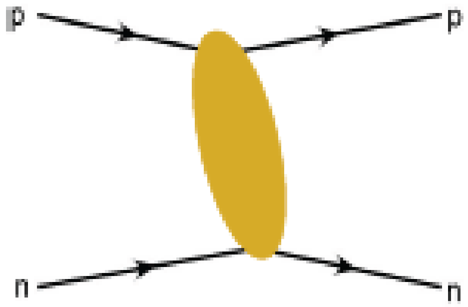


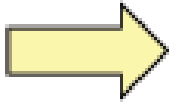


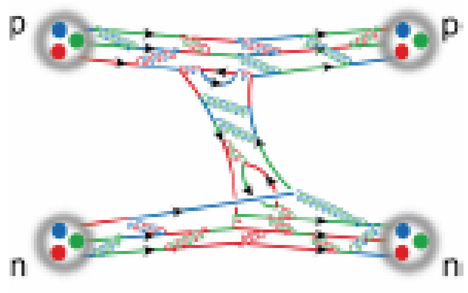
Various applications: few / many-body system of nuclei, EOS of Nuclear matter..

Nuclear Force from QCD

- First principle calculation of QCD



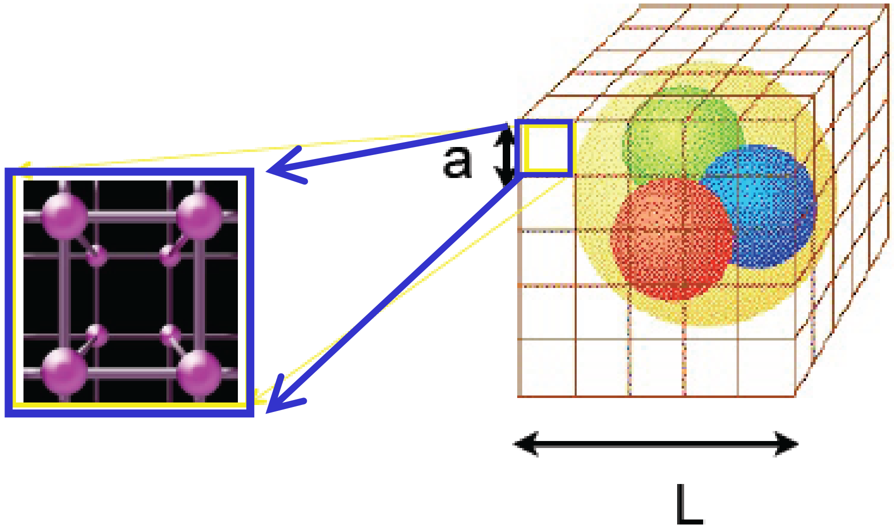




Y. Nambu, "Quarks : Frontiers in Elementary Particle Physics", World Scientific (1985)

"Even now, it is impossible to completely describe nuclear forces beginning with a fundamental equation. But since we know that nucleons themselves are not elementary, this is like asking if one can exactly deduce the characteristics of a very complex molecule starting from Schroedinger equation, a practically impossible task."

Lattice QCD as 1st principle calc



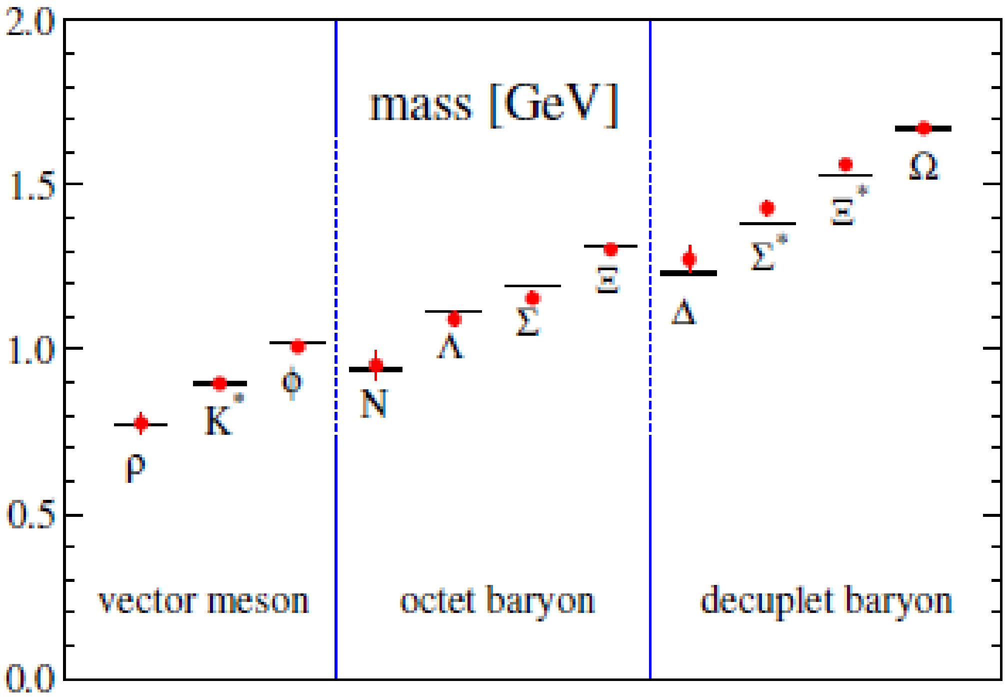
- well-defined statistical system (finite a and L)
- gauge invarinat
- fully non-perturbative

→ Monte-Calro simulations

Quenched QCD : neglects creation-anihilation of quark-anitiquak pair
Full QCD : includes creation-anihilation of quark-anitiquak pair

Status of Lattice QCD

- hadron masses (g.s.)

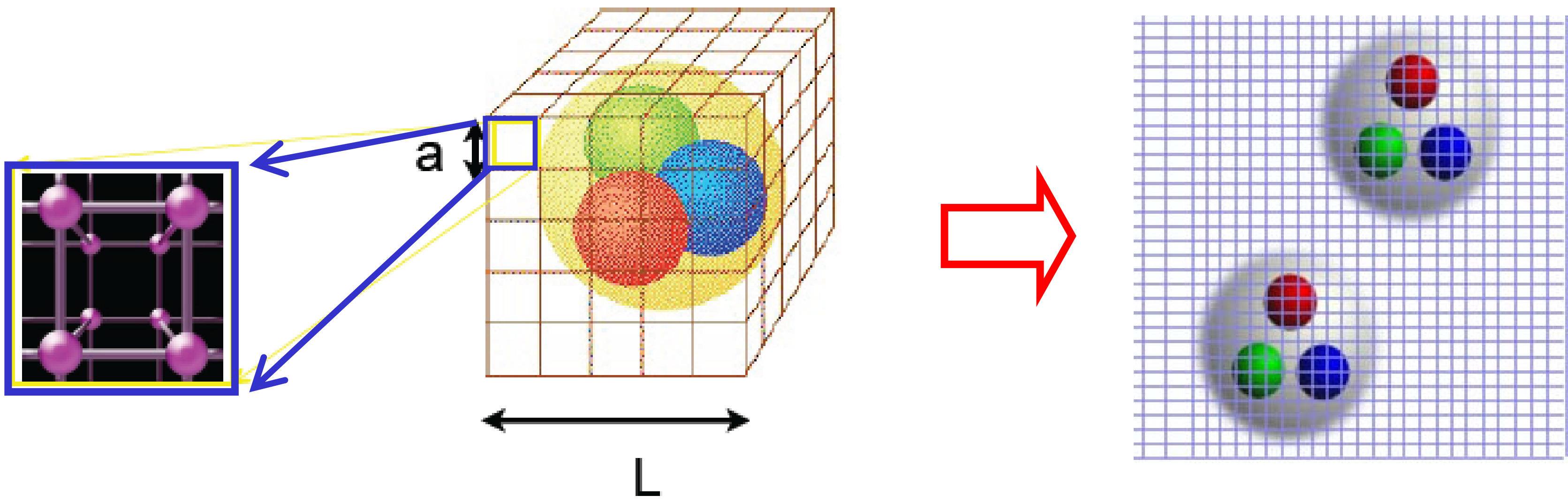


Nf=2+1 clover fermion
$V=32^3 X64$, L=2.9fm,
a=0.09fm (1/a = 2.18GeV)
m_π(min) = 156MeV

PACS-CS Collab.,
PRD79(2009)034503

3% accuracy at present

Lattice QCD as 1st principle calc



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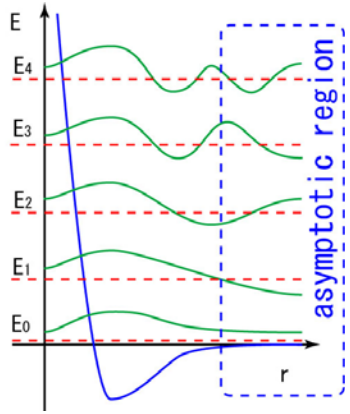
Nuclear Force from Lattice QCD [HAL QCD strategy]

- Potential is constructed so as to reproduce the NN phase shift (or, S-matrix)
- Nambu-Bethe-Salpeter(NBS) wave function

$$\psi(\vec{r}) = \langle 0 | N(\vec{x}+\vec{r},t) N(\vec{x},t) | 2N \rangle$$

  - Key concept: asymptotic region ←→ phase shift

$$(\nabla^2 + k_\delta^2)\psi(\vec{r}) = 0, \quad r > R$$



Luscher, NPB354(1991)531

C.-J.Lin et al., NPB619(2001)467
CP-PACS Coll., PRD71(2005)094504

- Define the potential at interaction region

$$(\nabla^2 + k_\delta^2)\psi(\vec{r}) = \int d\vec{r'} U(\vec{r},\vec{r'}) \psi(\vec{r'}), \quad r < R$$

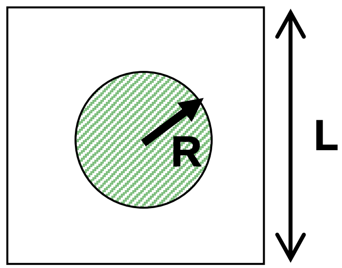
  - Non-local, but E-independent potential

- Velocity expansion Okubo-Marshak(1958)

$$U(\vec{r},\vec{r'}) = V_c(r) + S_{12}V_T(r) + \vec{L}\cdot\vec{S}V_{LS}(r) + \mathcal{O}(\nabla^2)$$

LO LO NLO NNLO

  - Truncation in expansion introduces E-dep (only practically), but we can **improve order by order**



Aoki-Hatsuda-Ishii
PTP123(2010)89

Effective Schrodinger equation with E-independent potential

$K(\vec{x};E) \equiv \left(\vec{\nabla}^2 + k^2\right)\psi(\vec{x};E)$ **[START] local but E-dep pot. ($L^3 x L^3$ dof)**

(1) We assume $\psi(x; E)$ for different E is linearly independent with each other.

(2) $\psi(x; E)$ has a "left inverse" as an integration operator as

$$E \equiv 2\sqrt{m_N^2 + k^2}$$

$$\int d^3x \tilde{\psi}(\vec{x}; E')\psi(\vec{x}; E) = 2\pi\delta(E - E')$$

(3) K(x; E) can be factorized as

$$\begin{aligned} K(\vec{x}; E) &= \int \frac{dE'}{2\pi} K(\vec{x}; E') \times \int d^3y \tilde{\psi}(\vec{y}; E')\psi(\vec{y}; E) \\ &= \int d^3y \left\{ \sum_\alpha \int \frac{dE'}{2\pi} K(\vec{x}; E')\tilde{\psi}(\vec{y}; E') \right\} \psi(\vec{y}; E) \end{aligned}$$

$$\equiv m_N U(\vec{x}, \vec{y})$$

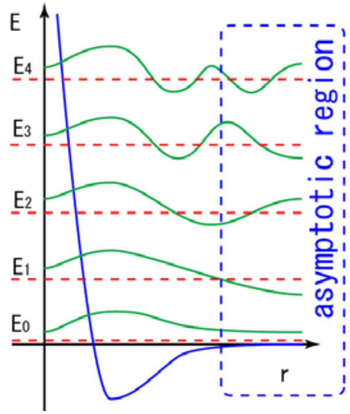
(4) We are left with an effective Schrodinger equation with an E-independent potential U.

$$\left(\vec{\nabla}^2 + k^2\right)\psi(\vec{x};E) = m_N \int d^3y\, U(\vec{x},\vec{y})\psi(\vec{y};E)$$

Intuitive understanding

[GOAL] non-local but E-indep pot. ($L^3 x L^3$ dof)

Nuclear Force from Lattice QCD [HAL QCD strategy]



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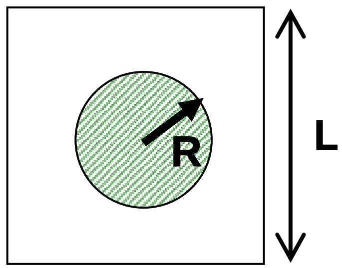

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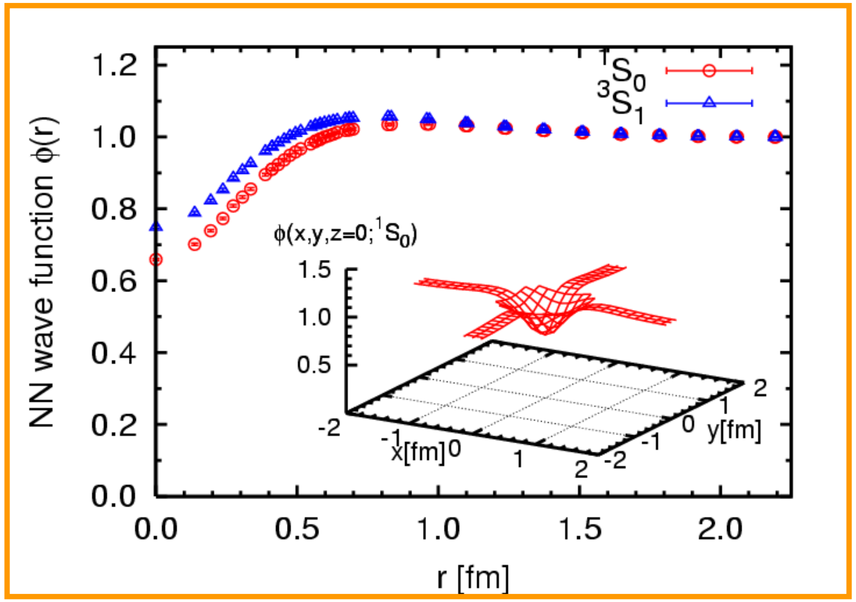
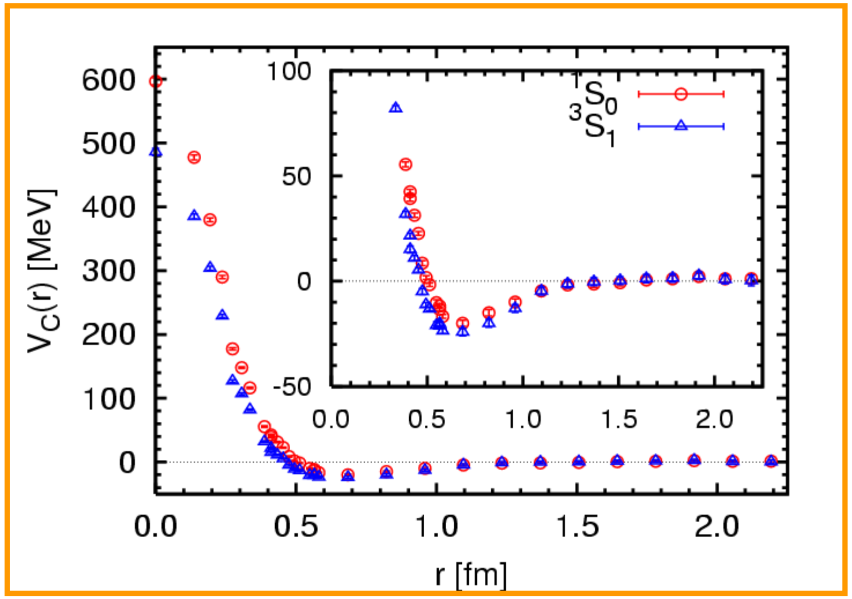
Aoki-Hatsuda-Ishii
PTP123(2010)89

Nuclear Potential (from Lat QCD)

NBS wave function



Nuclear Force



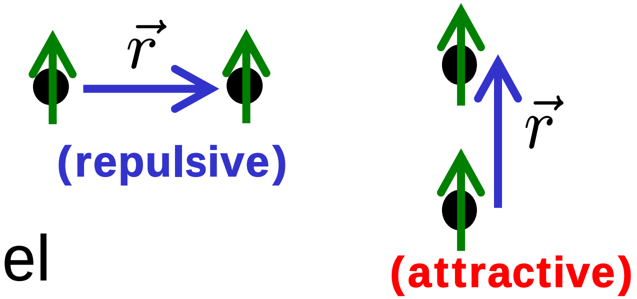
<u>**Quenched**</u> QCD

m_π = 530MeV, L=4.4fm

Ishii-Aoki-Hatsuda,
PRL99(2007)022001

Tensor Potential from Lat QCD

- Tensor operator $S_{12} = 3(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r})/r^2 - (\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
  - Essential to understand the nuclei
  - Responsible for deuteron binding
  - Hyper nuclei binding (ΛN-ΣN)



- Coupled channel study in $^3S_1$-$^3D_1$ channel

$$(H_0 + V_C + V_T S_{12})\psi \;=\; E\psi$$

$$\psi = \psi_S + \psi_D$$

$$\psi_S(\vec{r}) \;=\; P\psi(\vec{r}) = \frac{1}{24}\sum_{g\in O}\psi(g^{-1}\vec{r})$$

$$\psi_D(\vec{r}) \;=\; Q\psi(\vec{r}) = (1-P)\psi(\vec{r})$$

P	: projection to L=0
Q=(1-P)	: projection to L=2

$\Rightarrow$

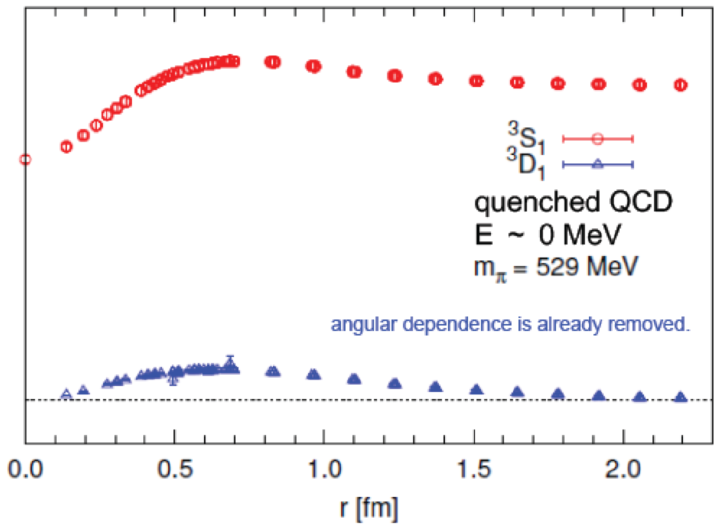
$$P(H_0 + V_C + V_T S_{12})\psi \;=\; EP\psi$$

$$Q(H_0 + V_C + V_T S_{12})\psi \;=\; EQ\psi$$

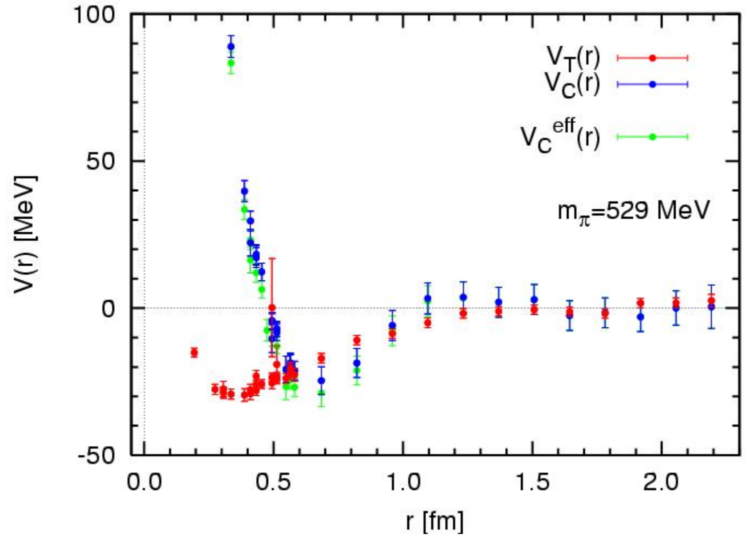
Tensor Potential from Lat QCD

- Coupled channel study in $^3S_1$-$^3D_1$ channel

Wave function



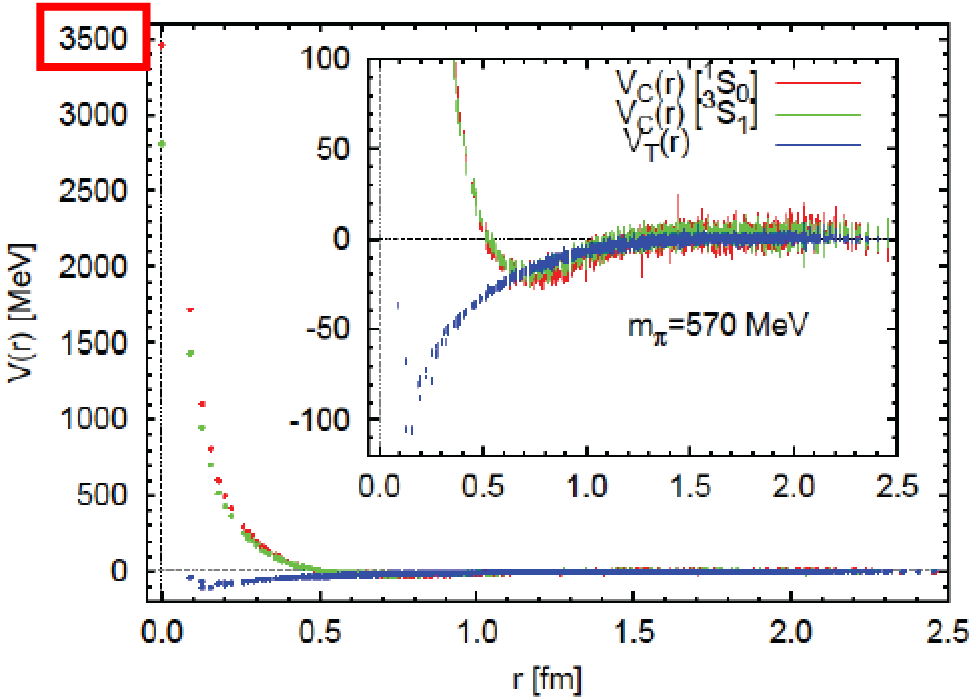
Potentials



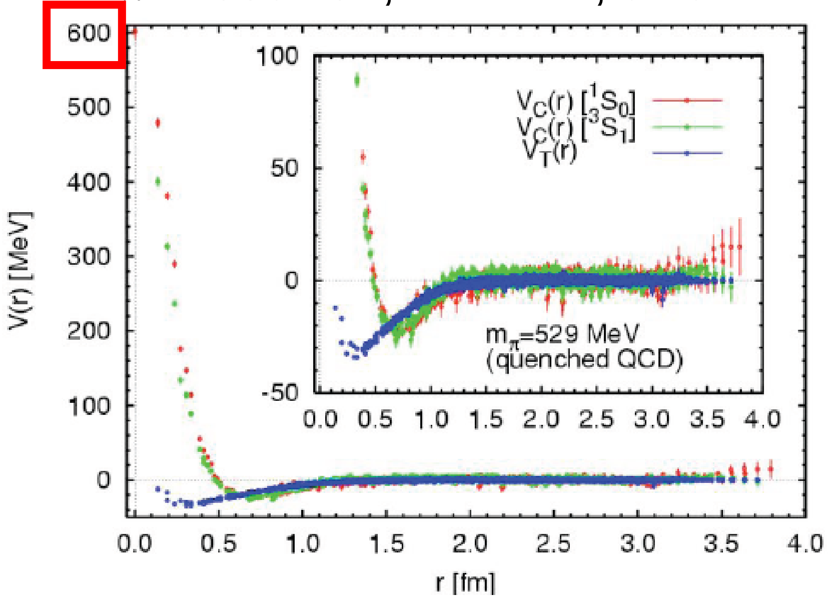
Aoki-Hatsuda-Ishii,
PTP 123 (2010) 89

Results from Full QCD Lattice

Nf=2+1 Full QCD
m_π = 570MeV, L=2.9fm, a=0.1fm



Quenched QCD
m_π = 530MeV, L=4.4fm, a=0.14fm

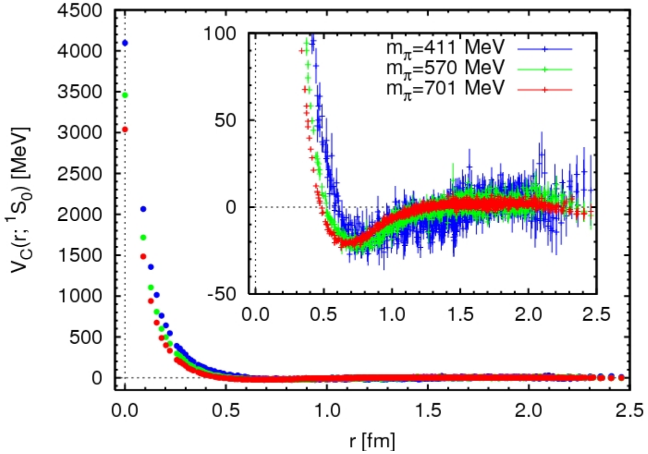


- Larger Repulsive Core
- Larger Tensor force

(N.B. 1/a is also different)

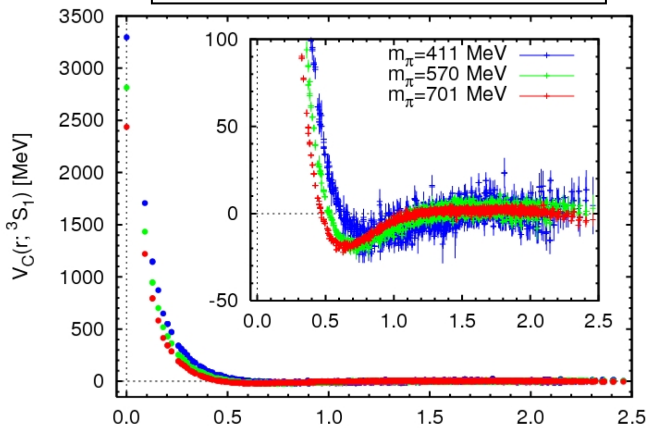
Quark mass dependence

Central in $^1S_0$

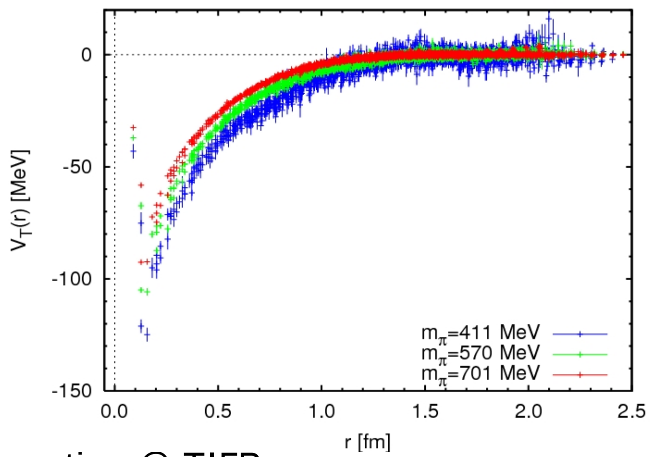


$^3S_1$-$^3D_1$ channel

Central



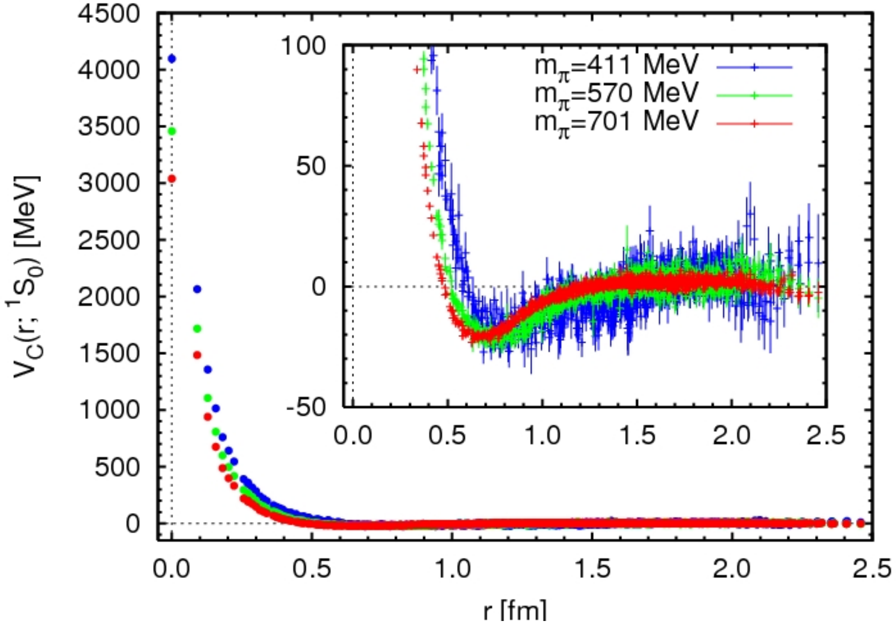
Tensor

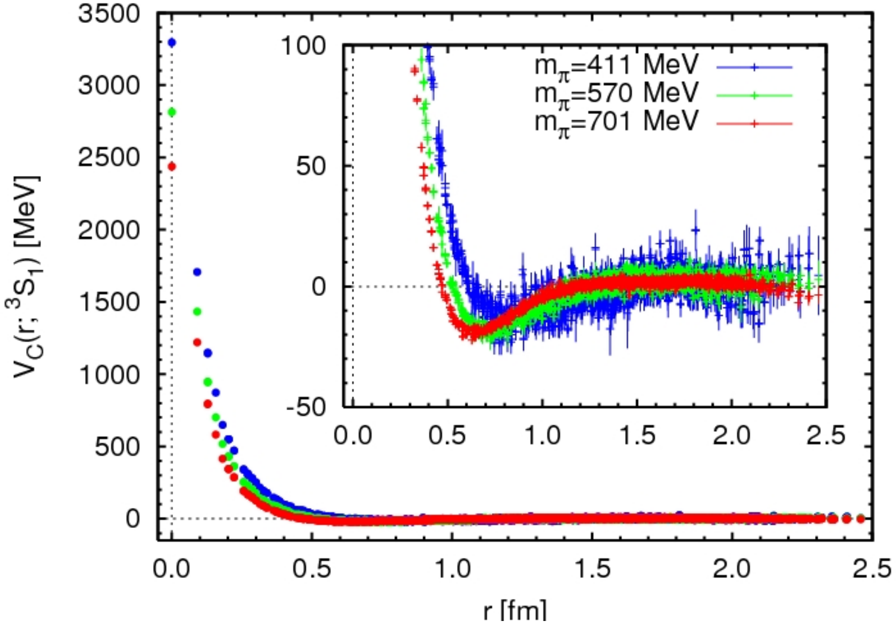


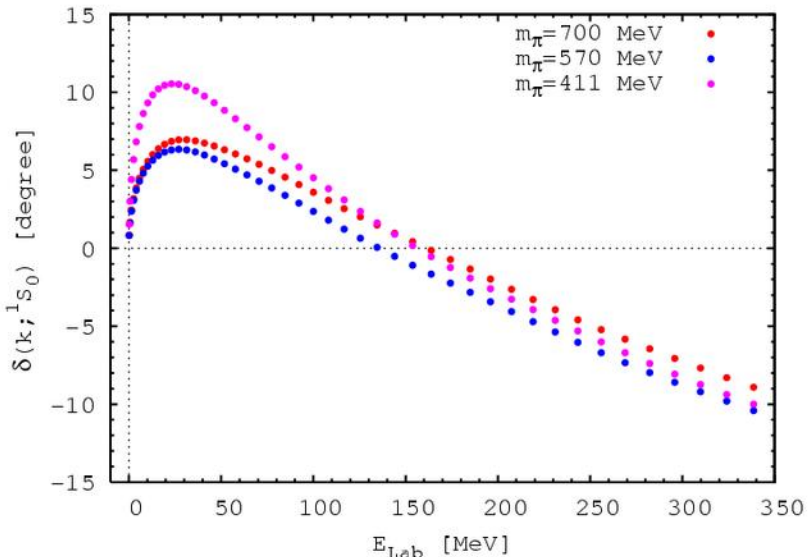
Lighter mass corresponds to…

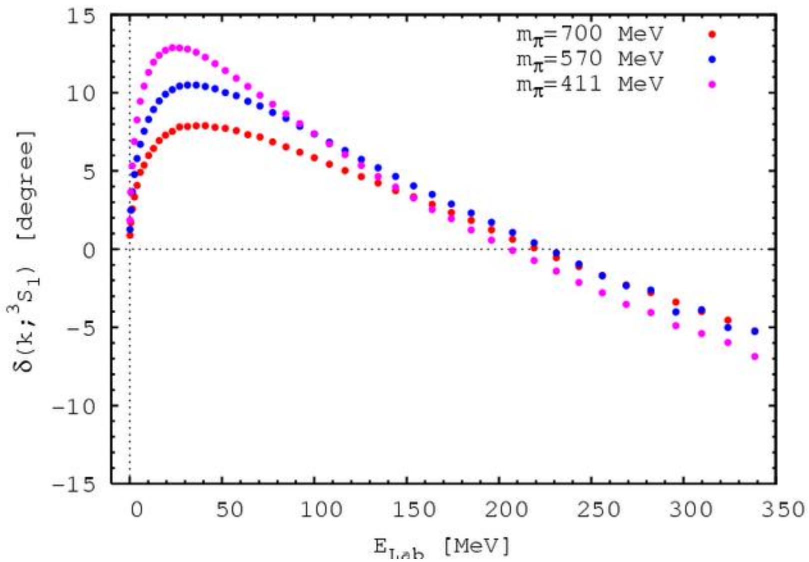
- **Longer interaction range**
- **Larger Repulsive Core**
- **Stronger Tensor Force**
- **(stronger attraction in Center Force)**

Phase shift from potential

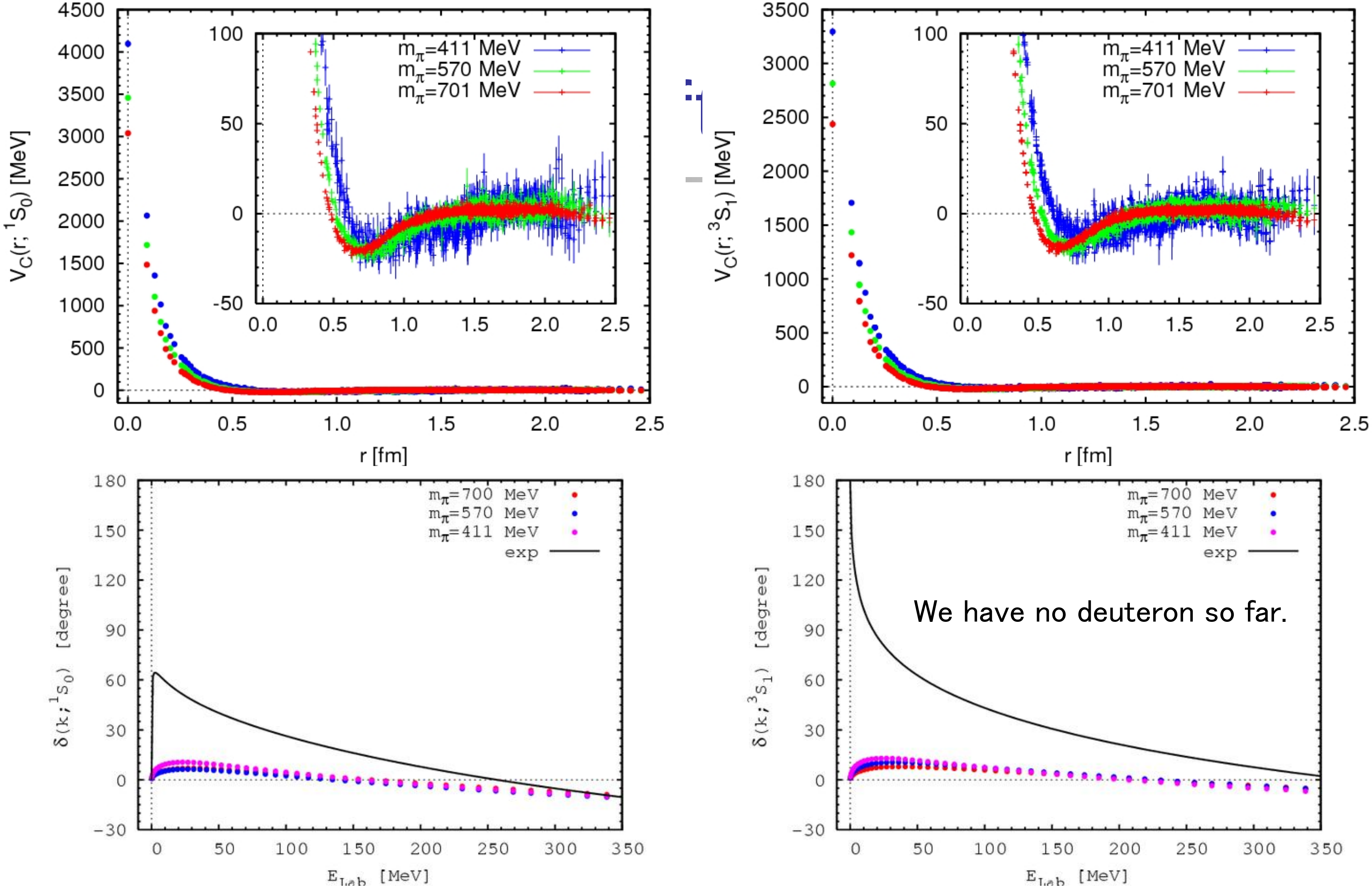








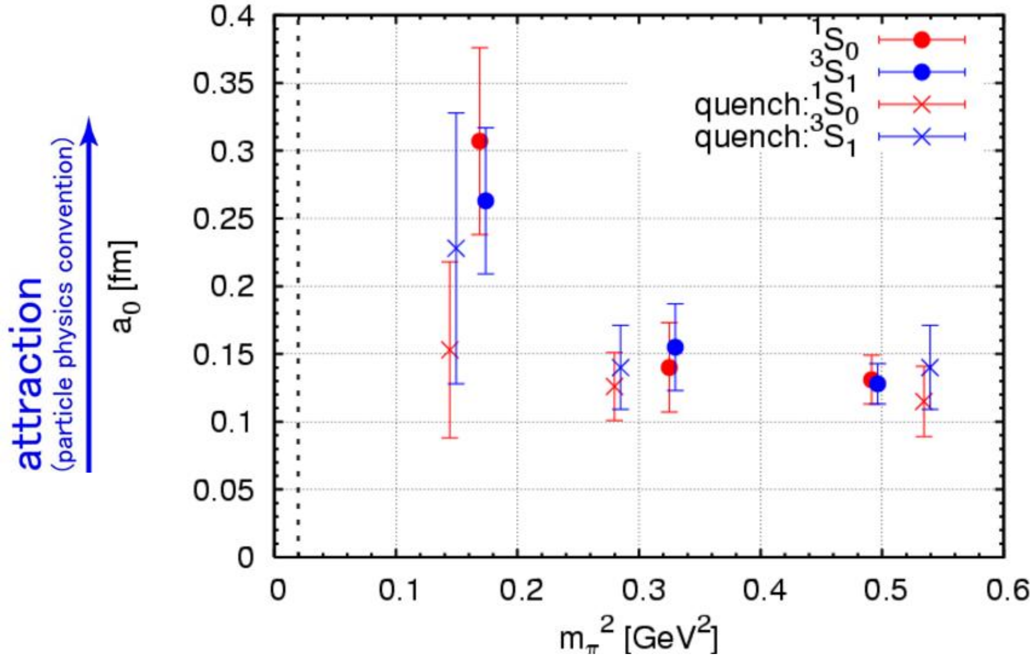
Phase shift from potential



Scattering Length

Scatt. Length

(w.f. ➔ k^2 ➔ Luscher's formula)



Attractive Scatt. Length

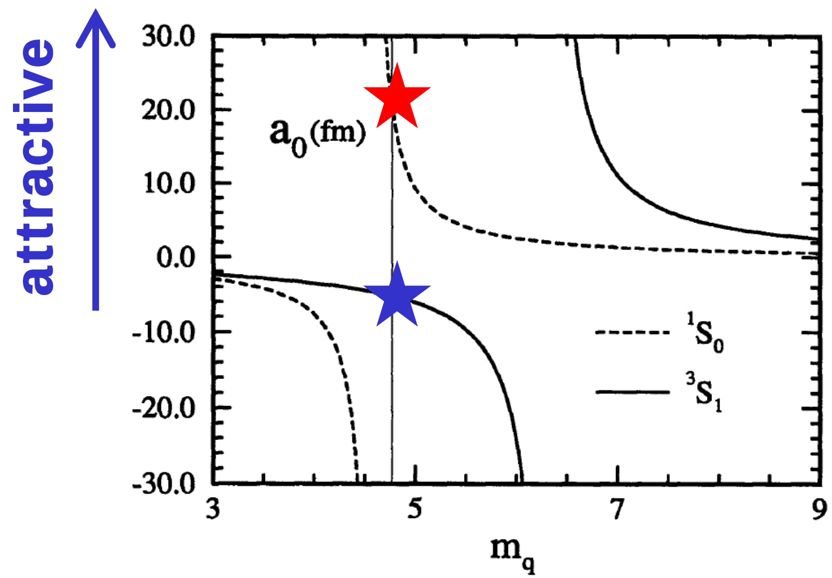
Further quantitative refinement in progress:
precise determination of E and
long-range w.f. behavior is essential

Yet, much smaller compared to the experimental values

$$a_0(^1S_0) \sim 20\text{fm}$$
$$a_0(^3S_1) \sim -5\text{fm}$$

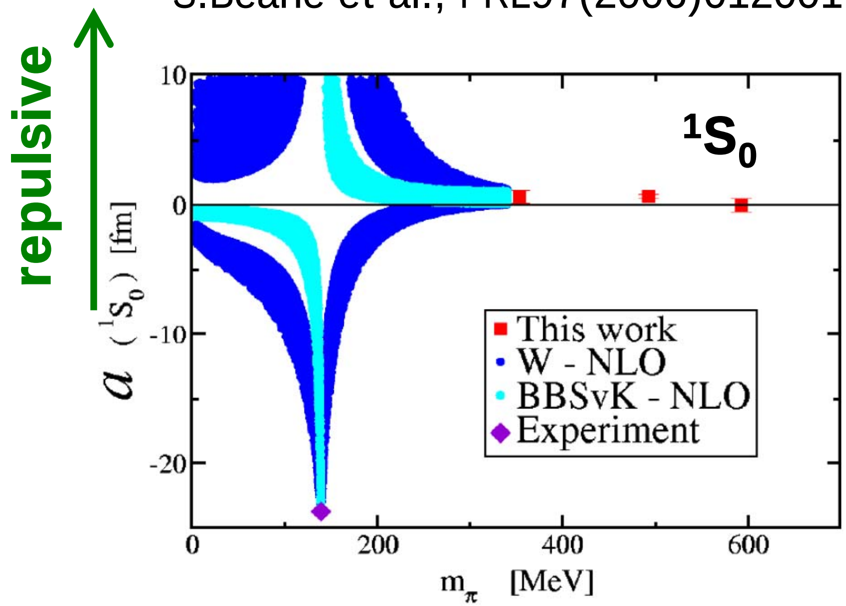
Scattering Length

Y.Kuramashi, PTPS122(1996)153



OBEP + lattice hadron masses

S.Beane et al., PRL97(2006)012001

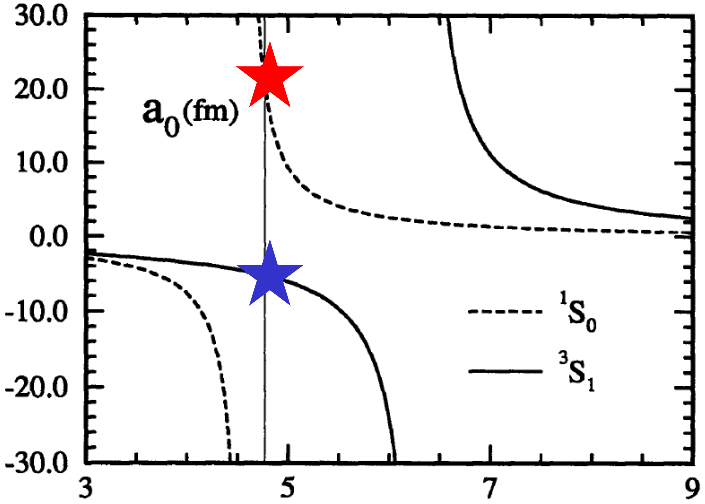


EFT

Scattering Length

Y.Kuramashi, PTPS122(1996)153

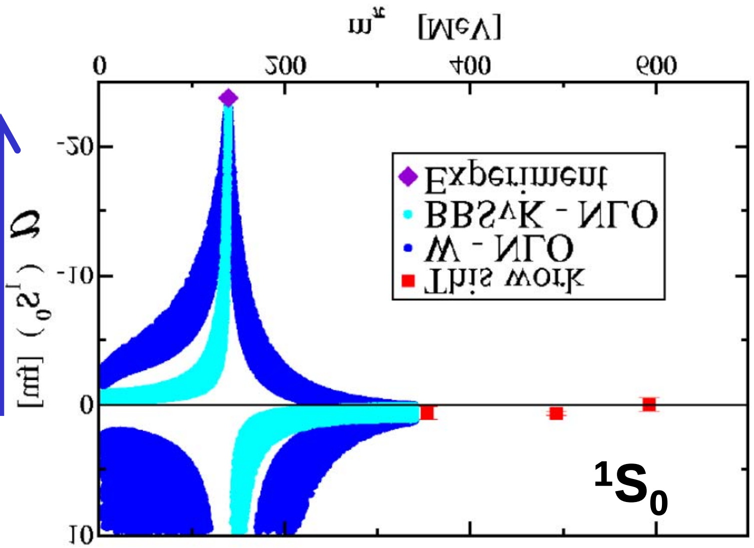
attractive



OBEP + lattice hadron masses

"Unitary region"

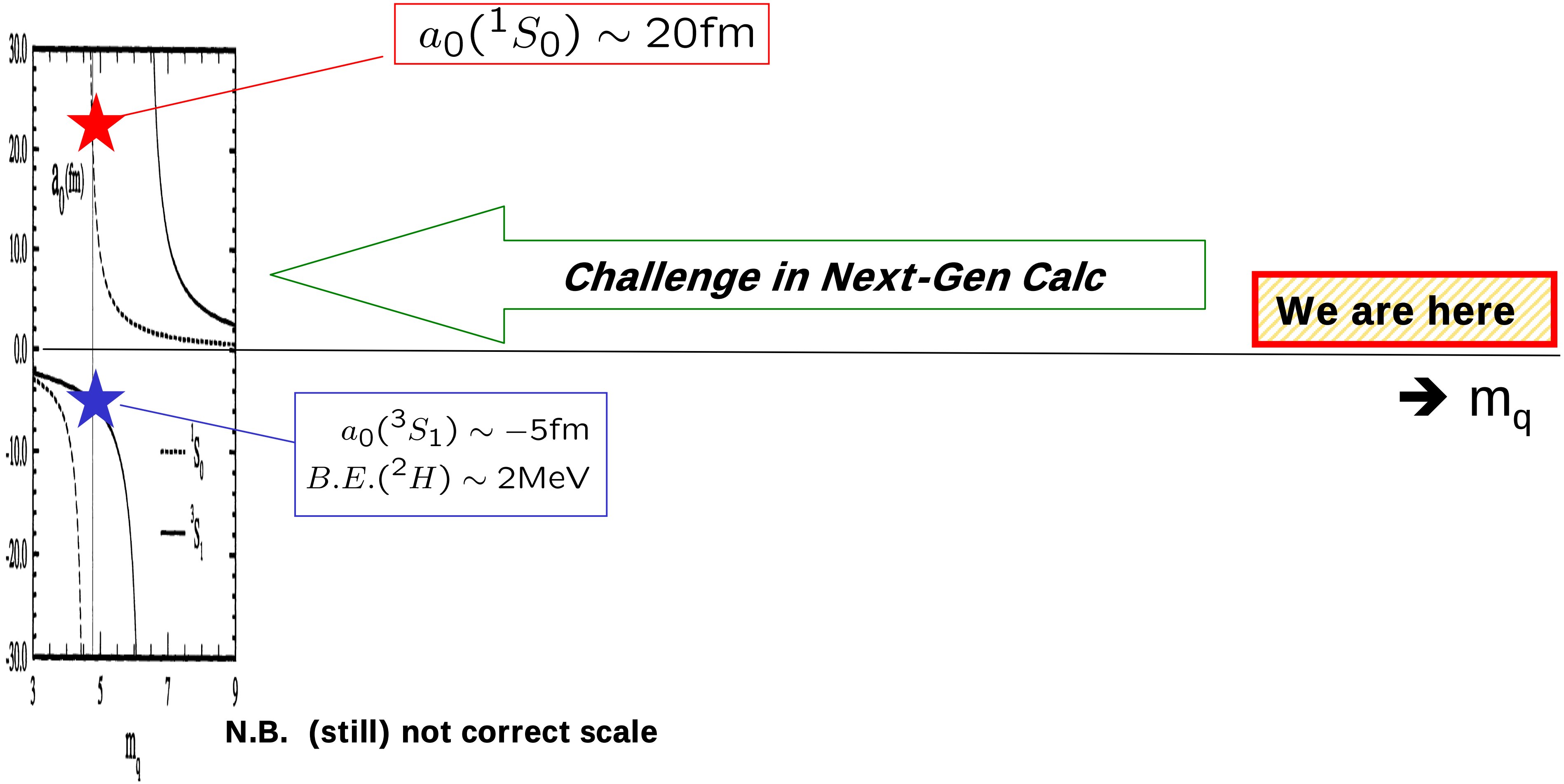
attractive



S.Beane et al., PRL97(2006)012001

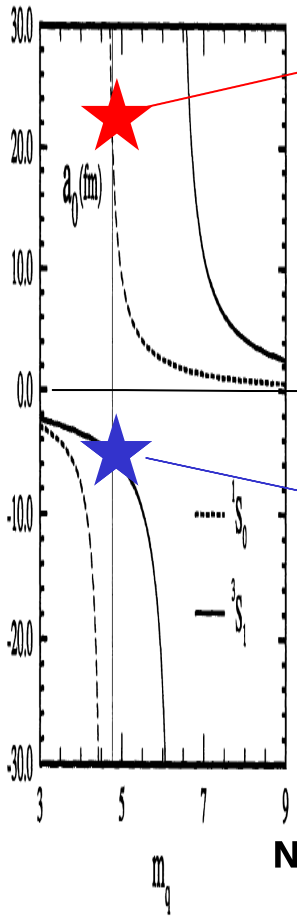
It is crucial to go to physical quark mass region

Challenge in Next-Gen Simulation



N.B. (still) not correct scale

Challenge in Next-Gen Simulation



$a_0(^1S_0) \sim 20\text{fm}$

$a_0(^3S_1) \sim -5\text{fm}$
$B.E.(^2H) \sim 2\text{MeV}$

Temporal info (Luscher's formula)

$$\delta E = -\frac{2\pi a_0}{\mu L^3}\left(1 + c_1\frac{a_0}{L} + c_2\frac{a_0^2}{L^2}\right) + \mathcal{O}(L^{-6})$$

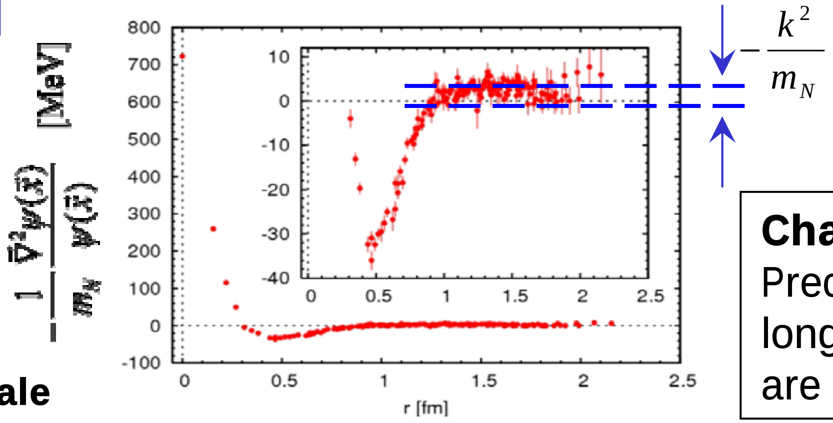
Spacial info (potential/phase shift)

$$V(r \to \infty) \sim e^{-m_\pi r}/r$$ *Localized !*

Challenge in Next-Gen Calc

We are here

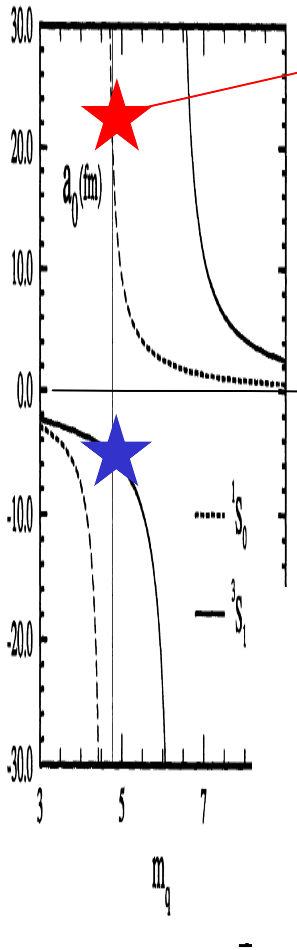
$\rightarrow m_q$



N.B. (still) not correct scale

Challenge:
Precise results at long range part are necessary

Challenge in Next-Gen Simulation



$a_0(^1S_0) \sim 20\text{fm}$

- **np : -23.7 fm**
- **pp : -17.9 fm (w/o Coulomb)**
- **nn : -16.5 fm**

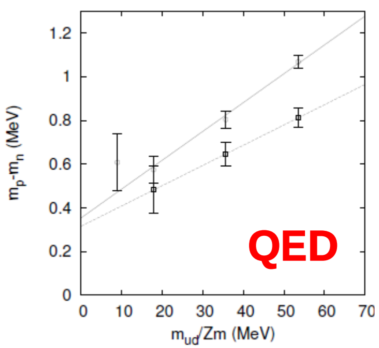
SU(2) breaking

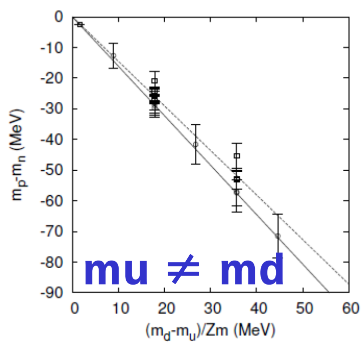
Challenge in Next-Gen Calc

We are here

RBC(em)
Nf= 2+1 DWF w/ quenched QED

T.Blum, R.Zhou, T.D., M.Hayakawa, T.Izubuchi, S.Uno, N.Yamada, arXiv:1006.1311





	$m_p - m_n$(MeV)
QED	$+0.383(68)$
$m_u \neq m_d$	$-2.51(14)$
total	$-2.13(16)(70)$

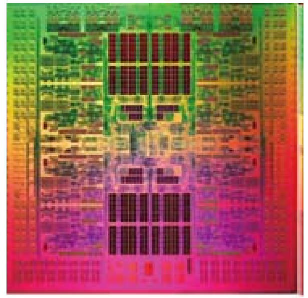
$m(\pi^+)$-m(“π^0”)[QED]=4.50(23)MeV
m_u=2.24(10)(34), m_d=4.65(15)(32)MeV

NEXT

Nf=1+1+1 w/ full QED, RBC(em), PACS-CS, …

Japan's next gen computer

- K computer at Kobe, Japan
  - 10PFlops (2012)



K computer

K (Kei) = 10^{16} = 10 Peta

Frequently Asked Questions

[Q1] Is potential observable ? Just give me phase shifts !

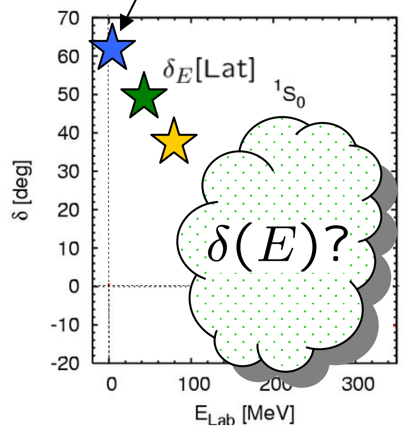
- Potential U(x,y) is NOT observable, and is NOT unique. However, combination of (Φ(x),U(x,y)) gives observable, which is unique.
  - Same situation for QM(Φ,U), QFT(Φ(asym),vertices), EFT(eff. dof, LECs) … Yet, we use “wave function Φ(x)” in QM, etc.
- We study potential (in addition to phase shift), because:
  - Convenient framework/concept to understand the physics
  - Potential is essential to study many-body systems
    - c.f. QM: Matrix mechanics vs. Wave mechanics

$$Lat \rightarrow \delta_E \rightarrow U(x,y) \rightarrow \text{many-body}$$

$$Lat \rightarrow \rightarrow \rightarrow U(x,y) \rightarrow \text{many-body}$$

  - It is very difficult to calculate phase shift at high energy
    - Lattice ➔ only ground state + a few excited energy states
  - Potential (hopefully) contains “useful” off-shell information
    - Sys. error by velocity expansion can be checked order by orde

δ[by Lat] $= \delta$[by $U(x)$]



Frequently Asked Questions

[Q1] Is potential observalbe ? Just give me phase shifts !

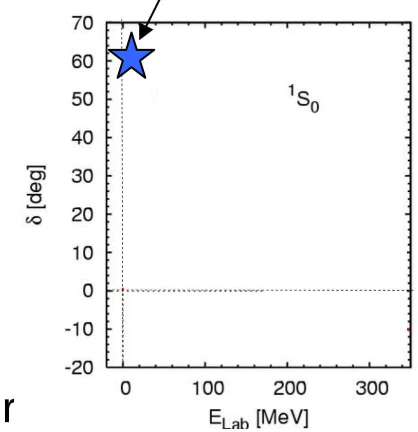
- Potential U(x,y) is NOT observable, and is NOT unique. However, combination of (Φ(x),U(x,y)) gives observable, which is unique.
  - Same situation for QM(Φ,U), QFT(Φ(asym),vertices), EFT(eff. dof, LECs) … Yet, we use "wave function Φ(x)" in QM, etc.
- We study potential (in addition to phase shift), because:
  - Convenient framework/concept to understand the physics
  - Potential is essential to study many-body systems
    - c.f. QM: Matrix mechanics vs. Wave mechanics

$$Lat \rightarrow \boxed{\delta_E \rightarrow U(x,y)} \rightarrow \text{many-body}$$

$$\boxed{Lat \rightarrow \rightarrow \rightarrow U(x,y)} \rightarrow \text{many-body}$$

  - It is very difficult to calculate phase shift at high energy
    - Lattice ➔ only ground state + a few excited energy states
  - Potential (hopefully) contains "useful" off-shell information
    - Sys. error by velocity expansion can be checked order by order

$$\delta[\text{by Lat}] = \delta[\text{by } U(x)]$$



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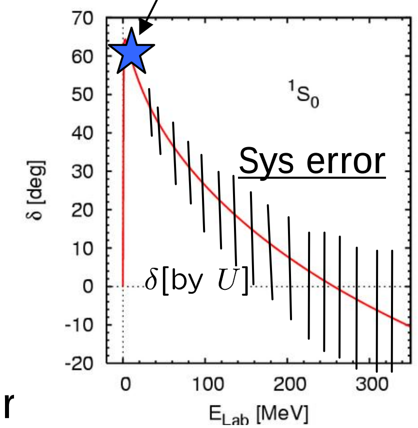
δ[by Lat] $= \delta$[by $U(x)$]



Frequently Asked Questions

[Q2] Isn't Potential dependent on the sink operator ?

- Yes, the potential is dependent on the choice of the sink operator, since Potential U(x,y) is NOT observable. (➔ go back to the 1st Q&A)
  - One can choose any sink opeartor, and the physical observables (at least phase shift) calculated from that potential remain same
  - We choose local operator as convenient choice for the reduction formula
  - Good operator ←➔ small non-locality in potential
    - We check the velocity expansion convergence *a posteriori*

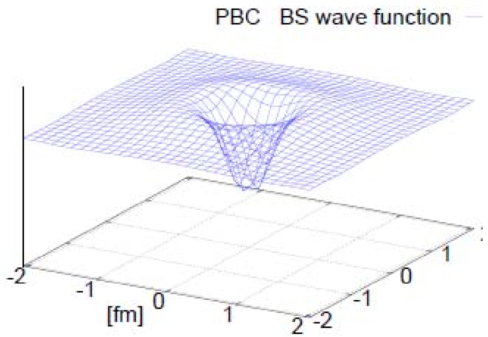
[Q3] How good is velocity expansion of potential ?

- We explicitly checked the validity of expansion using two methods:
  - By Energy dependence of LO potential $V_C(r)$
  - By L^2 dependence of $V_C(r)$

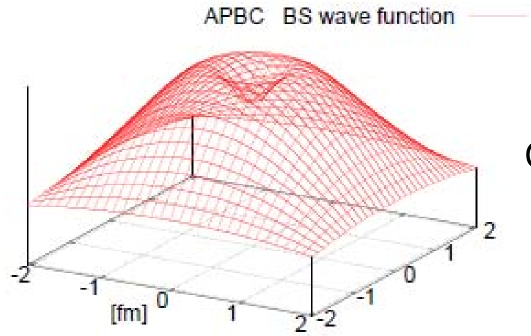
K.Murano (HAL Collab.)
@ Lattice2009, Lattice2010

"Energy dependence" of LO $V_c(r)$ in velocity expansion

E ～ 0 MeV

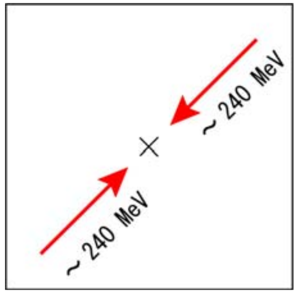


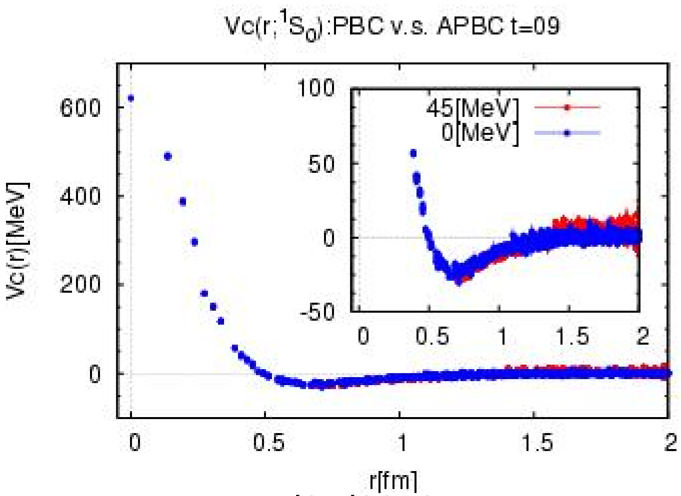
E ～ 45 MeV



Anti-periodic BC to achieve E ≠ 0

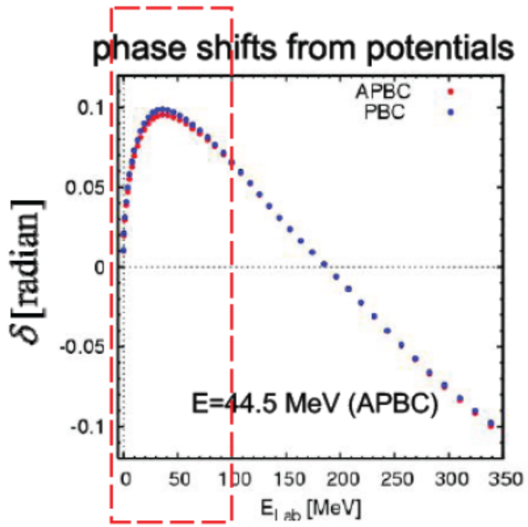
Quenched QCD
m_π =0.53GeV
a=0.137fm





In our choice of wave function,
E-dependence of the local potential
turns out to be very small at low E.

➔ *Velocity expansion is good !*

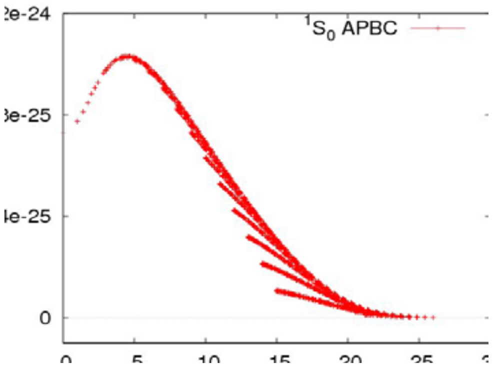


O.K. !

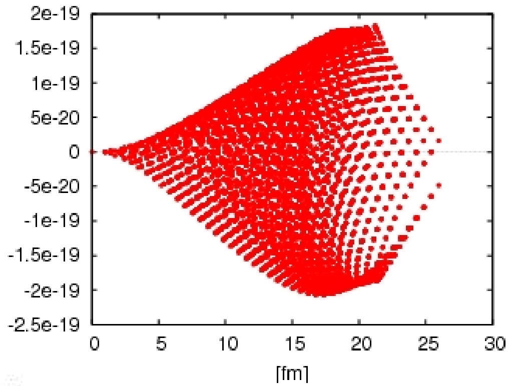
K.Murano et al., (HAL QCD Collab.)
PoS Lattice 2009 (2009) 126

L^2 dependence of $V_c(r)$ in S=0

A_1^+ NBS wave ($^1S_0$)



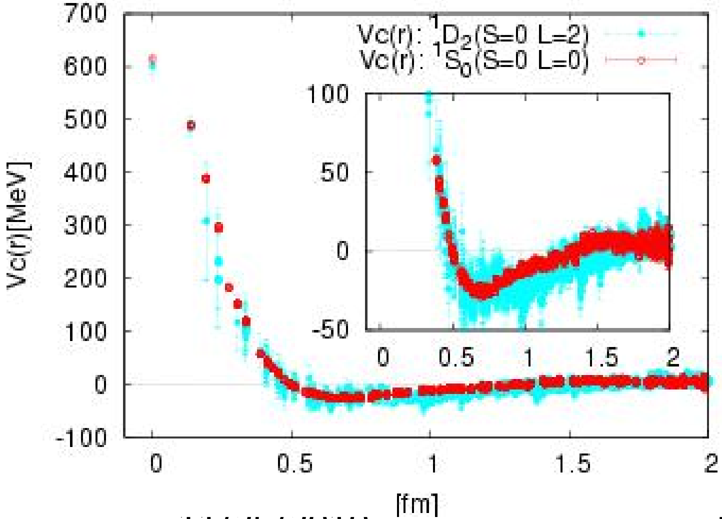
T_2^+ NBS wave ($^1D_2$)



Anti-periodic BC

$$\sim \sum Y_{2m}(\vec{r})$$

Quenched QCD
$m\pi$ =0.53GeV
a=0.137fm
E ~= 45MeV



In our choice of wave function,
L^2-dependence of the local potential
turns out to be small at low E.

(consistent within stat. error)

➔ *Velocity expansion is good !*

K.Murano (HAL QCD Collab.) @ Lattice 2010

Towards the prediction from Lattice QCD

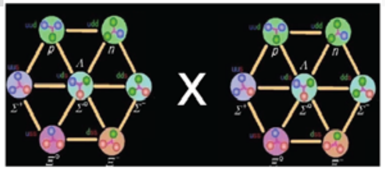
- “Realistic” NN potentials have achieved quite a good precision
  - ~40 parameters for ~5000 (high prec) phase shifts, χ^2/dof ~ 1

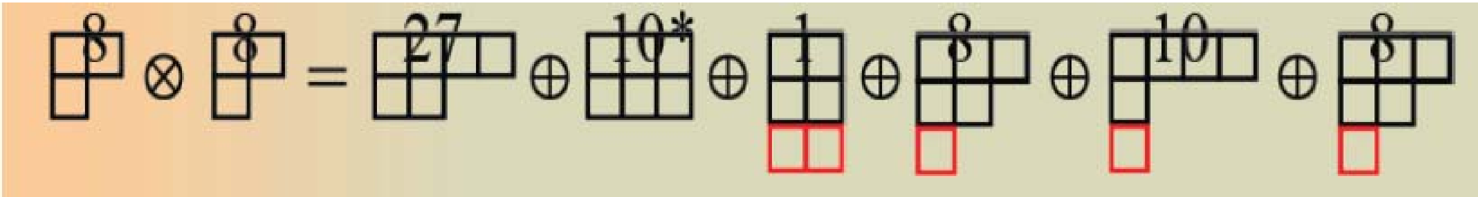
- Hyperon-Nucleon(YN), Hyperon-Hyperon(YY) potentials
  - Large uncertainties in YN, YY potentials, and theoretical predictions are highly awaited
    - Huge impact on EoS in high density, Neutron Star Core / Supernova
  - “Generalization” of the nuclear force
    - ➔ what is universal, what is not universal in hadron-hadron interactions ? (e.g. origin of repulsive core)

- Three Baryon Potentials
  - The Lattice study of Three Nucleon Force (TNF)

Hyperon potentials (YN, YY)

- Generalized BB force

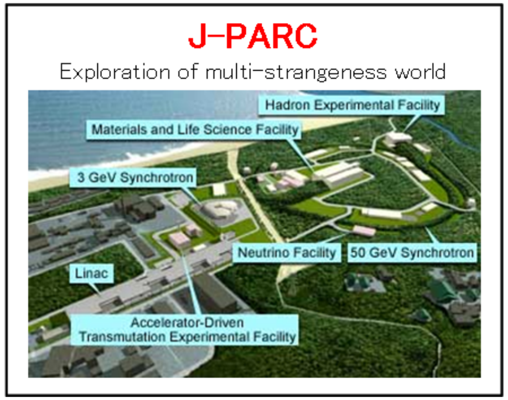




8 X 8 = 27 + 8s + 1 + 10* + 10 + 8a

symmetric — anti-symmetric

NN channel

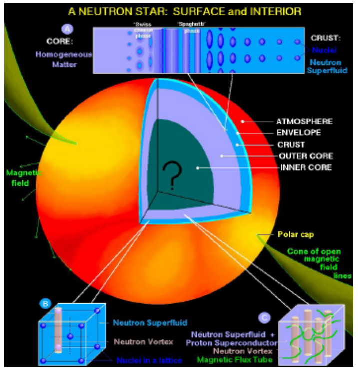


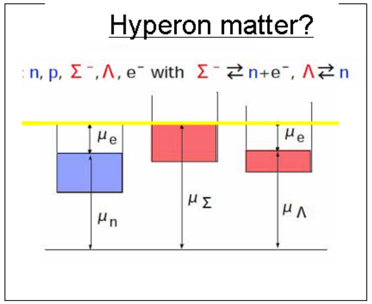
- No phase shift available for YN and YY scatterings
- Plenty of hyper-nucleus data will be soon available at J-PARC

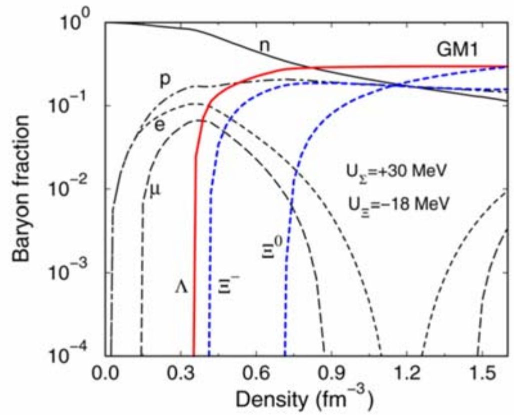
"Strange World" is opening !

Hyperon potentials (YN, YY)

- Equation of State at high density



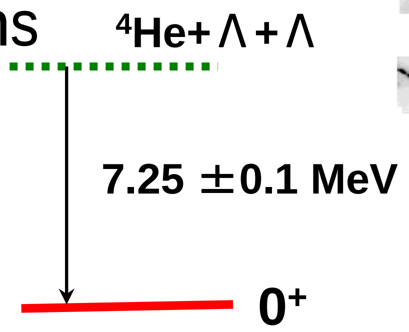


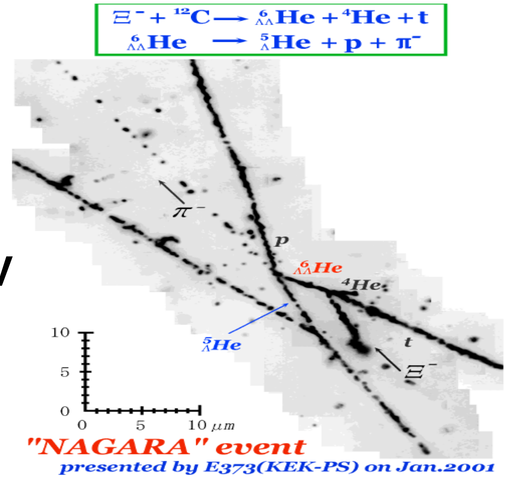


J.Schaffner−Bielich, NPA804(□f08)309.

- Hyper nuclei
  - ←→ Hyperon interactions

$^{6}_{\Lambda\Lambda}$He





- ΛN attraction
- ΛΛ weak attraction
- No deeply bound H-dibaryon

Baryon-baryon interaction

- SU(3) × SU(2)spin ⇒ SU(6) classification

MO, K. Shimizu, K. Yazaki, PLB130 (1983), NPA464 (1987)

S=0

1	×	0	[33]	ΛΛ, NΞ, ΣΣ → H dibaryon	
8_s	×	0	[51]	Pauli forbidden	ΣN (I=1/2, S=0)
27	×	0	[33], [51]	NN $^1S_0$	

S=1

8_a	×	1	[33], [51]		
10	×	1	[33], [51]	Nearly forbidden	ΣN (I=3/2, S=1)
10*	×	1	[33], [51]	NN $^3S_1$	

- The SU(6) symmetry predicts a strong spin-isospin dependence of the ΣN interaction.
- It also predicts state dependences of the spin-orbit interaction.

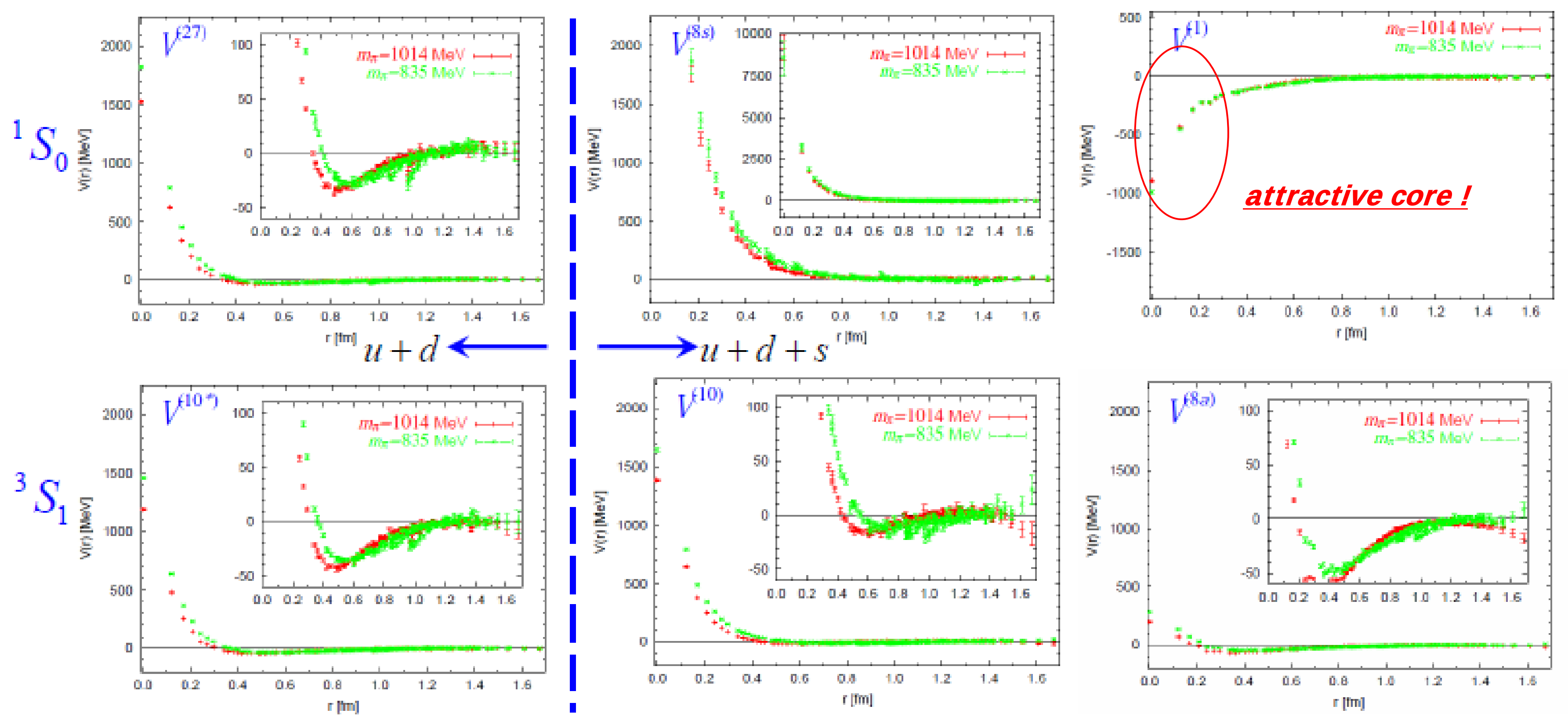
Oka-Shimizu-Yazaki, NPA464(1987)700

M. Oka, J-PARC Hadron Salon talk (06/17/2010)

SU(3) study

BB potentials

a=0.12fm, L=2fm, m(PS)= 0.84, 1.01GeV



27,10*:
Same as NN

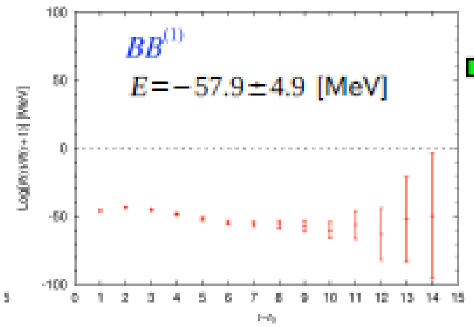
8s,10:
strong repulsive core

8a: weak repulsive core
1s: deep attractive pocket

Pauli principle at work !

T.Inoue et al., (HAL QCD Collab.), PTP124(2010)591, arXiv:1007.3559

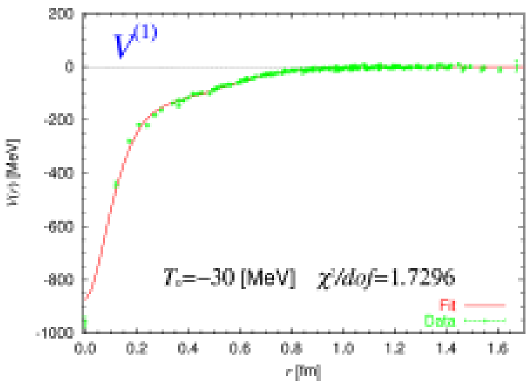
H-dibaryon in the SU(3) limit world ?



Bound state in 1(singlet) channel ? H-dibaryon ?

However, it is difficult to determine E precisely, due to contaminations from excited states.

Singlet potential with a certain value of E



$$V(r) = a_1 e^{-a_2 r^2} + a_3 \left(1 - e^{-a_4 r^2}\right)^2 \left(\frac{e^{-a_5 r}}{r}\right)^2$$

Schroedinger eq. predicts a bound state at E < -30 MeV

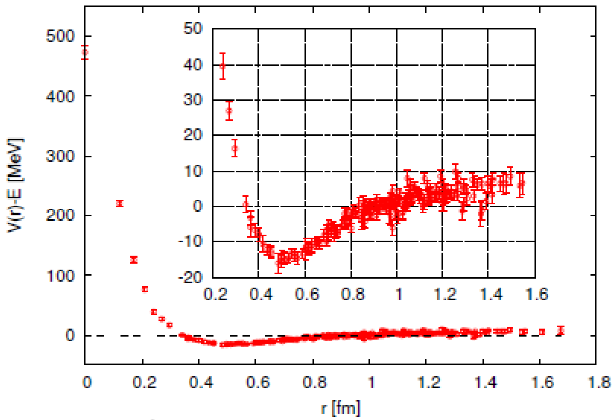
E [MeV]	E_0 [MeV]	$\sqrt{\langle r^2 \rangle}$ [fm]
E = −30	−0.018	24.7
E = −35	−0.72	4.1
E = −40	−2.49	2.3

finite size effect is very large on this volume.
(consistent with previous results.)
simulations on larger volume is in progress.

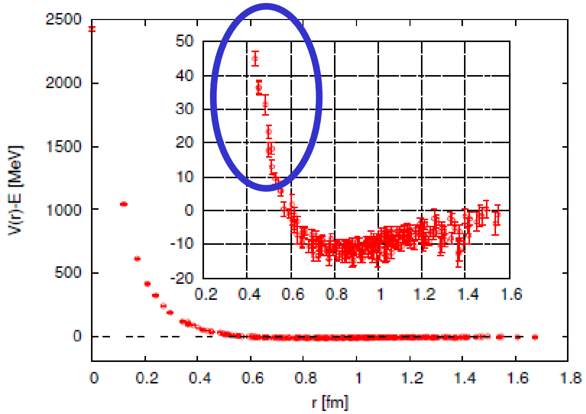
31

Meson-Baryon system

- KN interaction (S-wave) in I=0, 1



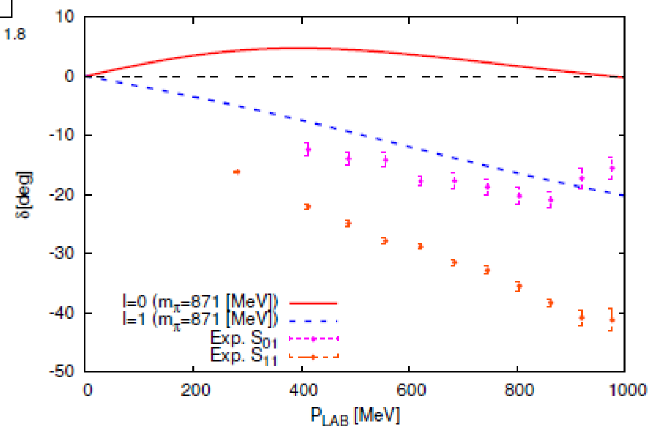
$$I = 0(u\bar{s} + udd, d\bar{s} + uud)$$



$$I = 1(u\bar{s} + uud)$$

Nf=2+1 clover (CP-PACS/JLQCD)
a=0.12fm, L=2fm, m_π=0.87GeV,
m_K=0.91GeV, m_N=1.8GeV

Phase shift



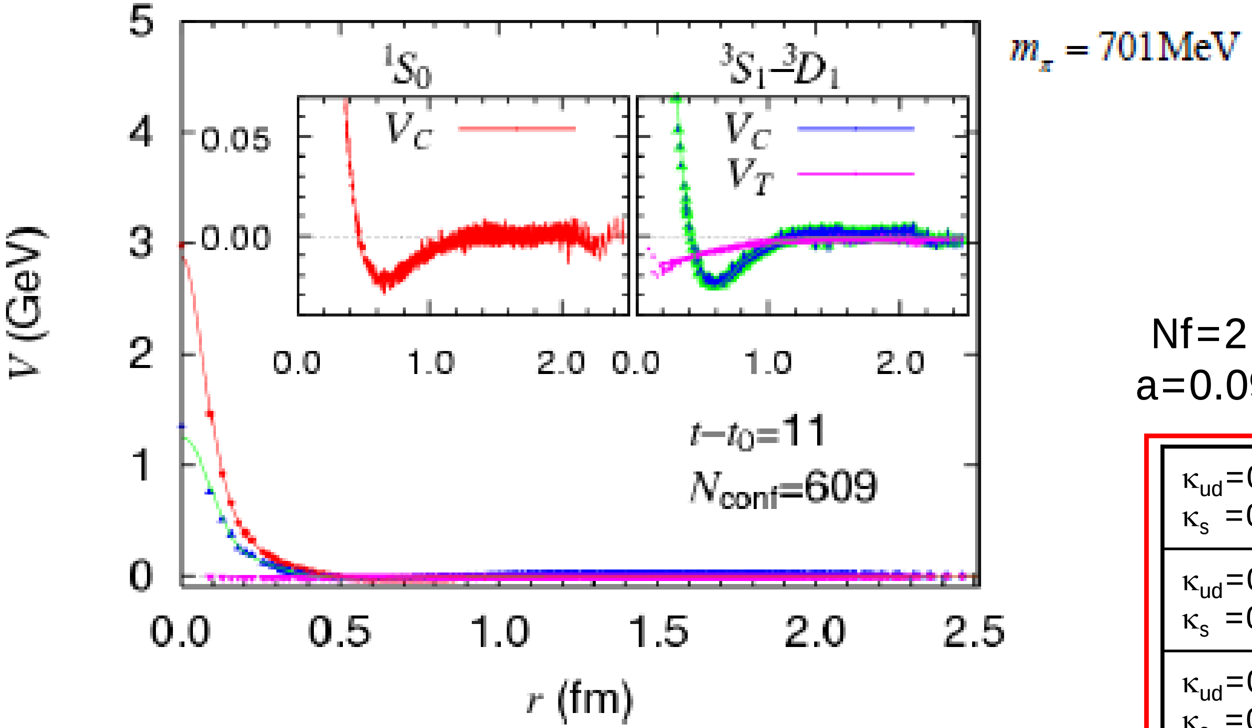
Pauli principle works for meson-baryon system as well !

Y.Ikeda et al., (HAL QCD Collab.)
arXiv:1002.2309

YN, YY potentials beyond SU(3) limit

NΛ potential (2+1 flavor QCD)

[Nemura@lattice2009] (38)



Nf=2+1 clover, L=2.9fm, a=0.091fm (1/a=2.18GeV)

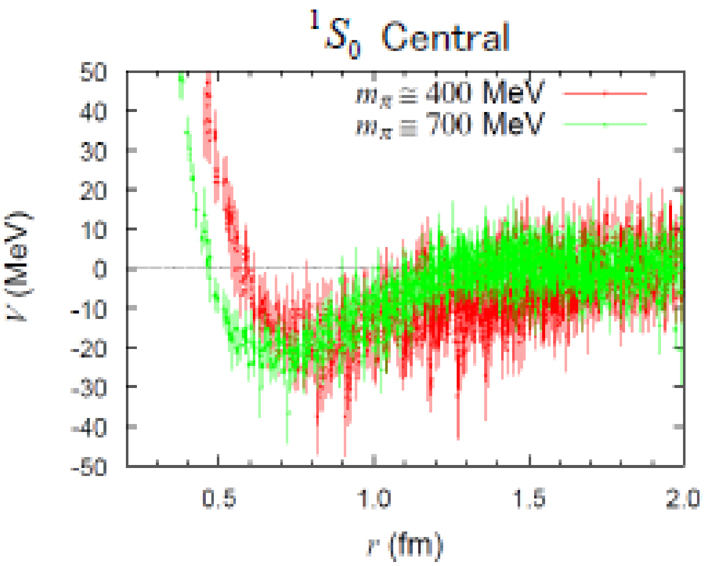
κ_{ud}=0.13700 κ_s =0.13640	m_π=701MeV m_K=789MeV
κ_{ud}=0.13727 κ_s =0.13640	m_π=570MeV m_K=713MeV
κ_{ud}=0.13754 κ_s =0.13640	m_π=411MeV m_K=635MeV
κ_{ud}=0.13754 κ_s =0.13660	m_π=384MeV m_K=582MeV
κ_{ud}=0.13770 κ_s =0.13640	m_π=296MeV m_K=594MeV
κ_{ud}=0.13781 κ_s =0.13640	m_π=156MeV m_K=554MeV

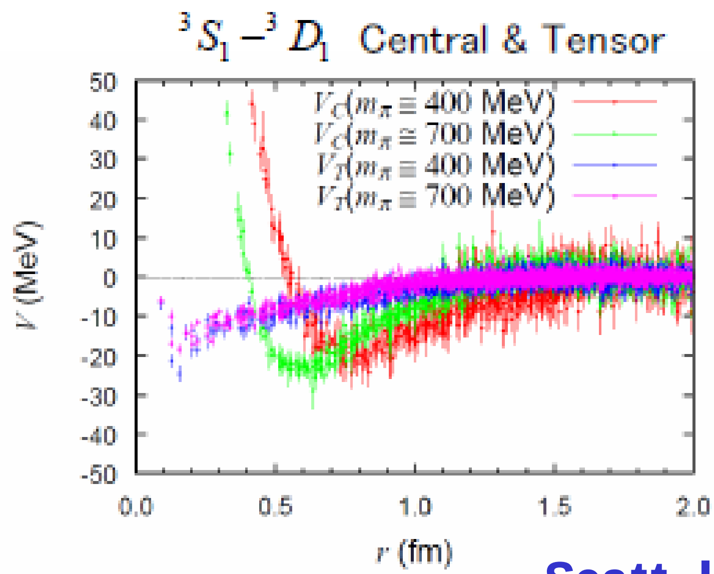
- Repulsive core is surrounded by attractive well.
- Large spin dependence of repulsive core
- Weak tensor force
- Net interaction is attractive.

PACS-CS Collab. PRD79(2009)034503

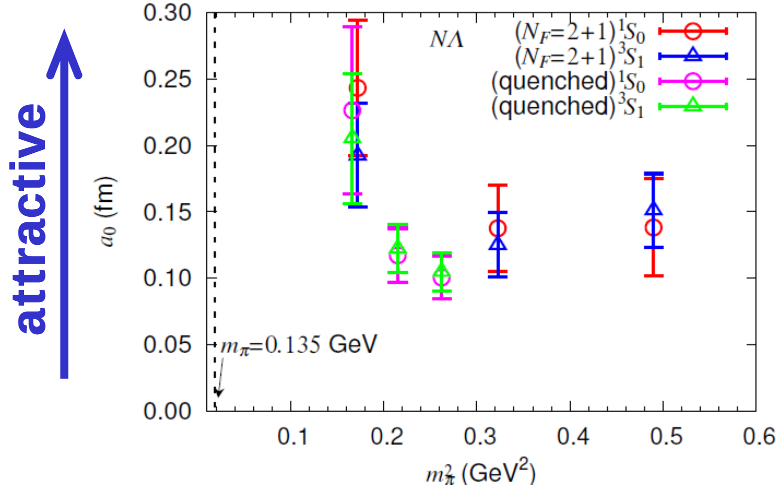
Quark mass dependence of NΛ potential

(39)





Scatt. length



With decreasing u and d quark masses,

- Repulsive core is enhanced.
- Attractive well moves to outer region.
- Small quark mass dependence of tensor potential

Nf=2+1 clover, L=2.9fm,
a=0.091fm (1/a=2.18GeV)
m(pi)=410-700MeV

H.Nemura (HAL QCD Collab.)
PoS Lattice2009 (2009)152

c.f. NPLQCD: repulsive scatt. length ?
PRD81(2010)054505

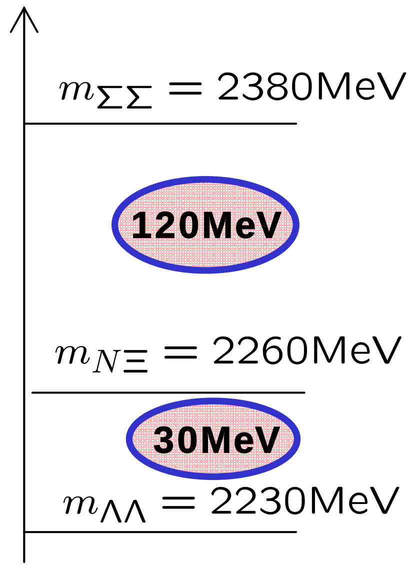
Coupled channel study

- BB system (S=-2, I=0)
  - small energy difference ➔ coupled channel

$$\begin{cases} \psi_\alpha^{\Sigma\Sigma} = \langle 0|\Sigma(\vec{r})\Sigma(\vec{0})|E_\alpha\rangle \\ \psi_\alpha^{N\Xi} = \langle 0|N(\vec{r})\Xi(\vec{0})|E_\alpha\rangle \\ \psi_\alpha^{\Lambda\Lambda} = \langle 0|\Lambda(\vec{r})\Lambda(\vec{0})|E_\alpha\rangle \end{cases}$$

$|E_1\rangle, |E_2\rangle, |E_3\rangle$ $(\alpha = 1, 2, 3)$

(variational method)



Coupled channel eq.

$$\begin{aligned} (\nabla^2 + k_\alpha^2)\psi_\alpha^{\Lambda\Lambda}(\vec{x}) &= U_{\Lambda\Lambda,\Lambda\Lambda}(\vec{x})\psi_\alpha^{\Lambda\Lambda}(\vec{x}) + U_{\Lambda\Lambda,N\Xi}(\vec{x})\psi_\alpha^{N\Xi}(\vec{x}) + U_{\Lambda\Lambda,\Sigma\Sigma}(\vec{x})\psi_\alpha^{\Sigma\Sigma}(\vec{x}) \\ (\nabla^2 + p_\alpha^2)\psi_\alpha^{N\Xi}(\vec{x}) &= U_{N\Xi,\Lambda\Lambda}(\vec{x})\psi_\alpha^{\Lambda\Lambda}(\vec{x}) + U_{N\Xi,N\Xi}(\vec{x})\psi_\alpha^{N\Xi}(\vec{x}) + U_{N\Xi,\Sigma\Sigma}(\vec{x})\psi_\alpha^{\Sigma\Sigma}(\vec{x}) \\ (\nabla^2 + q_\alpha^2)\psi_\alpha^{\Sigma\Sigma}(\vec{x}) &= U_{\Sigma\Sigma,\Lambda\Lambda}(\vec{x})\psi_\alpha^{\Lambda\Lambda}(\vec{x}) + U_{\Sigma\Sigma,N\Xi}(\vec{x})\psi_\alpha^{N\Xi}(\vec{x}) + U_{\Sigma\Sigma,\Sigma\Sigma}(\vec{x})\psi_\alpha^{\Sigma\Sigma}(\vec{x}) \end{aligned}$$

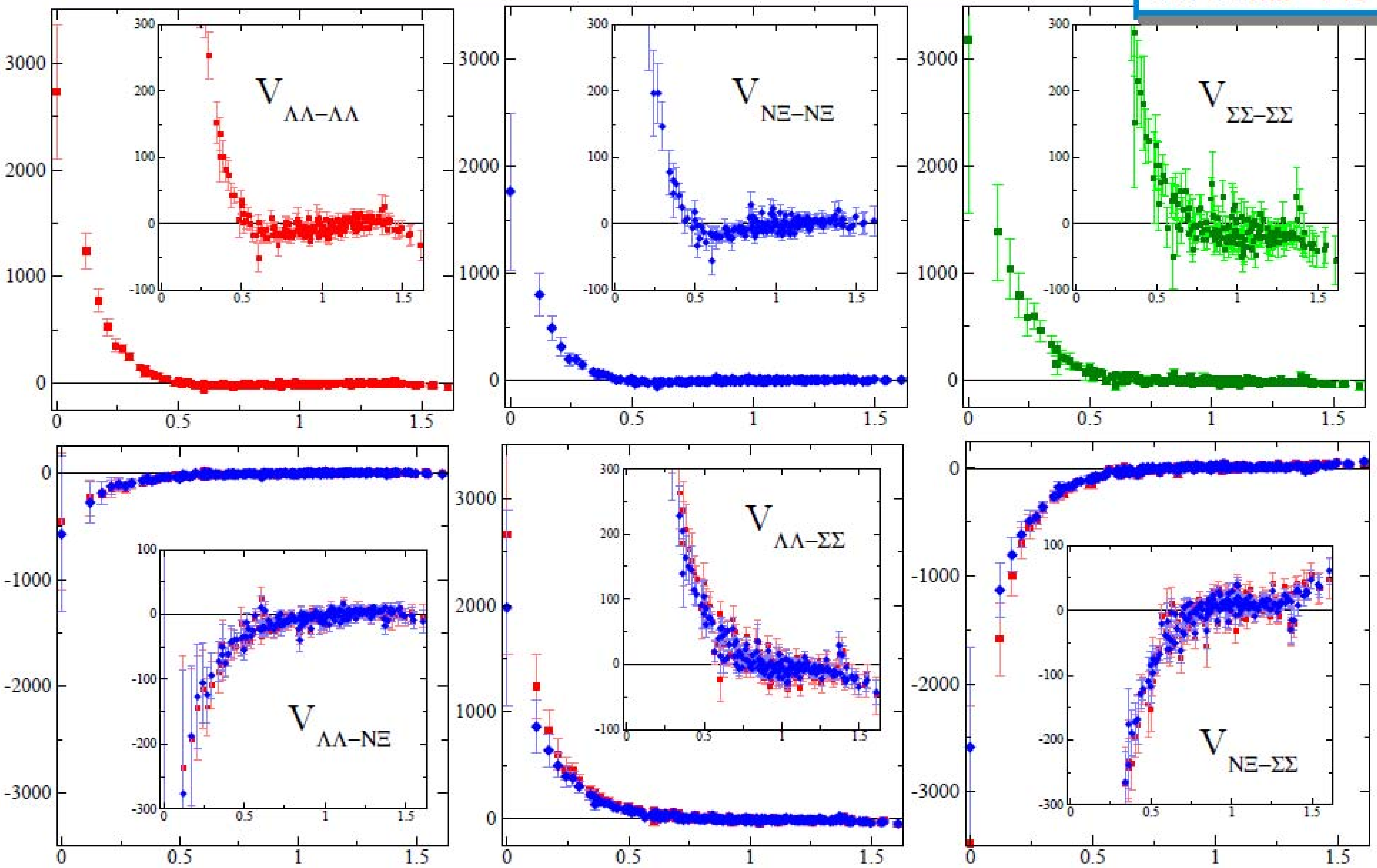
$(\alpha = 1, 2, 3)$

$$\begin{aligned} E_\alpha &= 2\sqrt{m_\Lambda^2 + k_\alpha^2} \\ &= \sqrt{m_N^2 + p_\alpha^2} + \sqrt{m_\Xi^2 + p_\alpha^2} \\ &= 2\sqrt{m_\Sigma^2 + q_\alpha^2} \end{aligned}$$

(asymptotic region)

Potential matrix

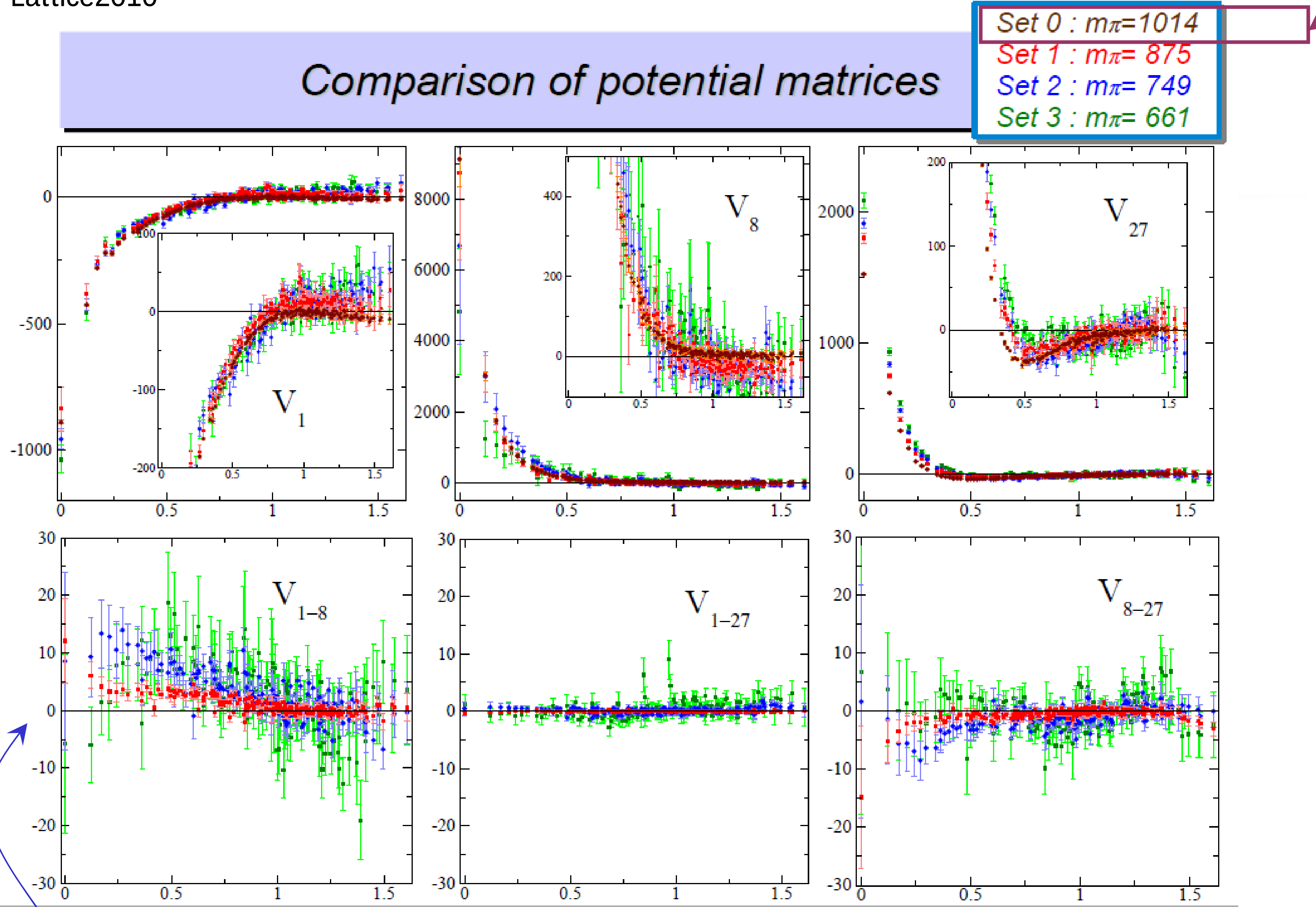
Set 2 : m_π= 749



K.Sasaki (HAL QCD Collab.)
@ Lattice2010

Comparison of potential matrices

Set 0: SU(3) limit



Irreducible flavor representations begin to mix with each other with decreasing u,d quark mass.

Three Nucleon Force (TNF)

Importance of Three Nucleon Force (TNF)

- Precise few-body calc:
  - e.g. benchmark calc of ^{4}He by 7 methods (NN only)

Method	$\langle T \rangle$	$\langle V \rangle$	E_b	$\sqrt{\langle r^2 \rangle}$
FY	102.39(5)	−128.33(10)	−25.94(5)	1.485(3)
CRCGV	102.30	−128.20	−25.90	1.482
SVM	102.35	−128.27	−25.92	1.486
HH	102.44	−128.34	−25.90(1)	1.483
GFMC	102.3(1.0)	−128.25(1.0)	−25.93(2)	1.490(5)
NCSM	103.35	−129.45	−25.80(20)	1.485
EIHH	100.8(9)	−126.7(9)	−25.944(10)	1.486

➔ **0.5% prec. for B.E.**

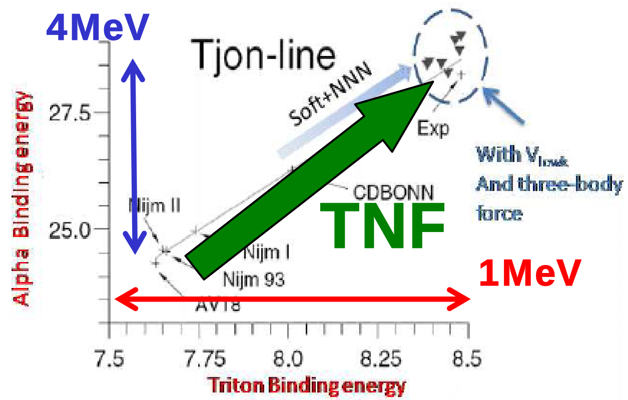
H.Kamada et al., PRC64(2001)044001

- NN force cannot reproduce B.E.

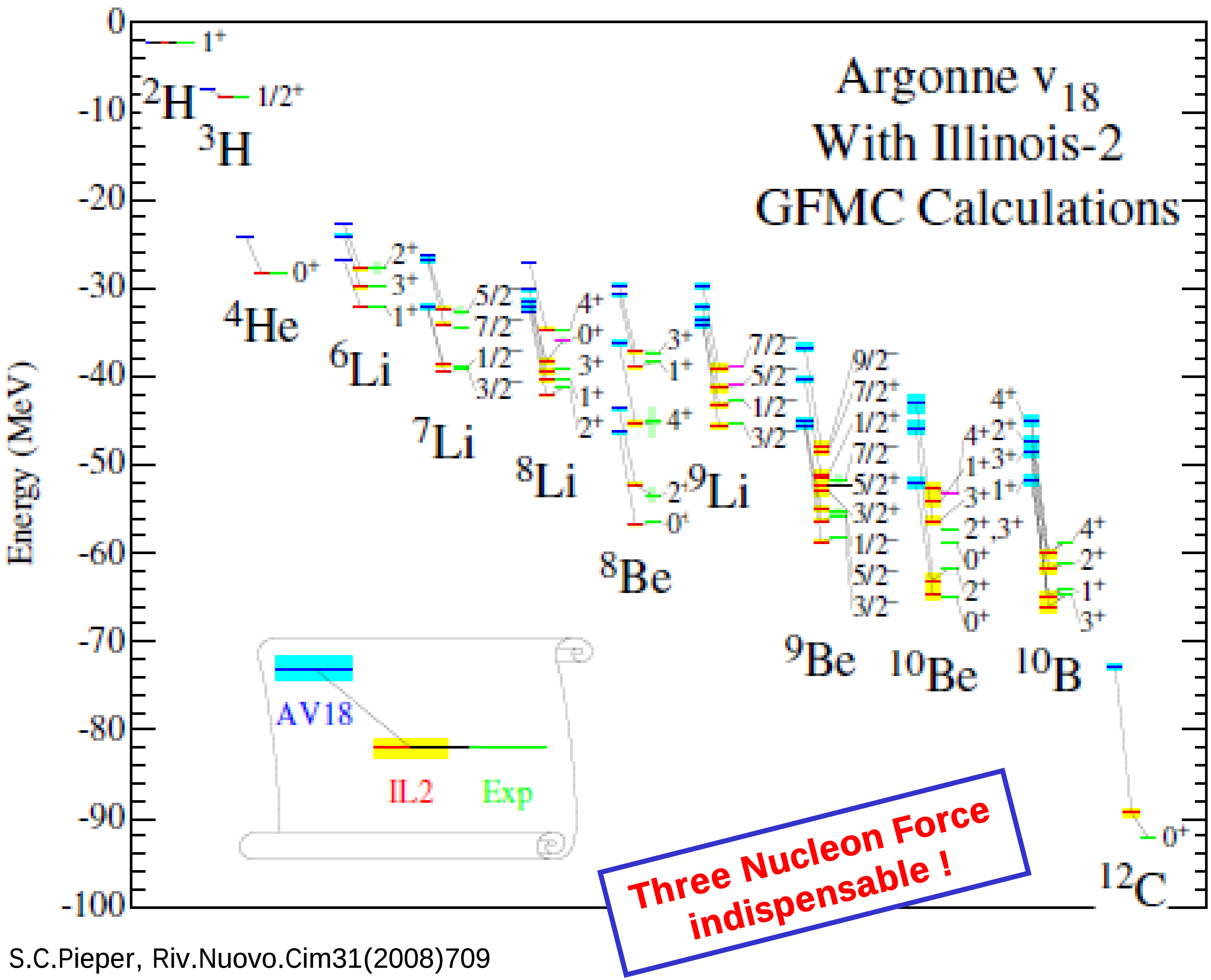
δB.E.= 0.5-1MeV for ^{3}H
δB.E.= 2-4 MeV for ^{4}He

missing

➡ Attractive TNF necessary



Nogga et al., PRL85(2000)944

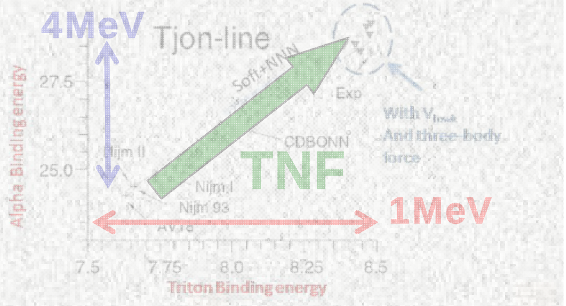


S.C.Pieper, Riv.Nuovo.Cim31(2008)709
arXiv:0711.1500

Importance of Three Nucleon Force (TNF)

- Precise few-body calc:
  NN force cannot reproduce B.E.
  - δB.E.= 0.5-1MeV for ^{3}H
  - δB.E.= 2-4 MeV for ^{4}He

Attractive TNF necessary



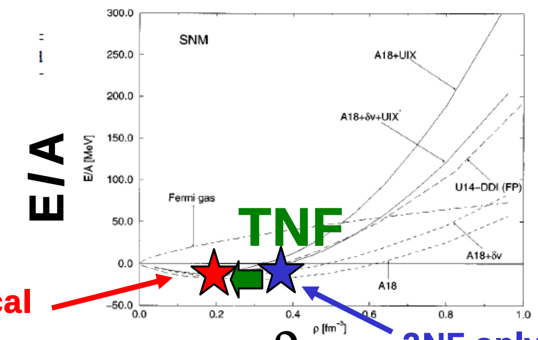
Nogga et al., PRL85(2000)944

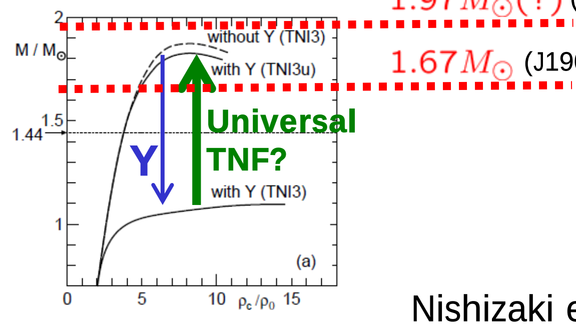
- Saturation density/energy of nuclear matter also requires TNF
  - EOS of neutron star
    - Flavor universal TNF (repulsive) ?

Repulsive TNF also necessary

A.Akmal et al., PRC58(1998)1804

Takatsuka et al., PTPS174(2008)80





$1.97 M_\odot (?)$ (J1614-2230)

$1.67 M_\odot$ (J1903+0327)

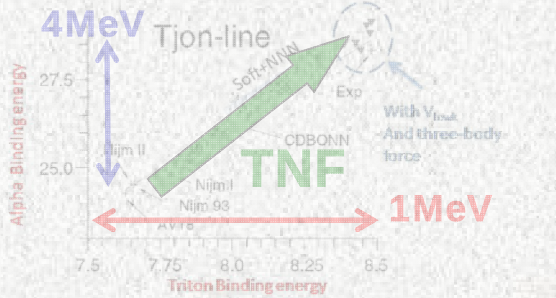
Demorest et al. (2010)
Freire (2009)

Nishizaki et al., PTP108(2002)703

Importance of Three Nucleon Force (TNF)

- Precise few-body calc:
  NN force cannot reproduce B.E.
  - δB.E.= 0.5-1MeV for ^{3}H
  - δB.E.= 2-4 MeV for ^{4}He

Attractive TNF necessary



Nogga et al., PRL85(2000)944

- Saturation density/energy of nuclear matter also requires TNF
  - EOS of neutron star
    - Flavor universal TNF (repulsive) ?

Repulsive TNF also necessary

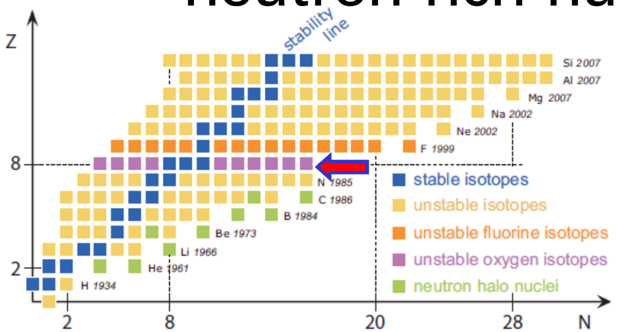
A.Akmal et al., PRC58(1998)1804

Takatsuka et al., PTPS174(2008)80

- The effect on the nuclear chart T.Otsuka et al., PRL105(2010)032501
  - anomaly in drip line and magic numbers by TNF
- Ay puzzle in N-d, N-A scatt., etc. (TNF may worsen the situation)

Importance of Three Nucleon Force (TNF)

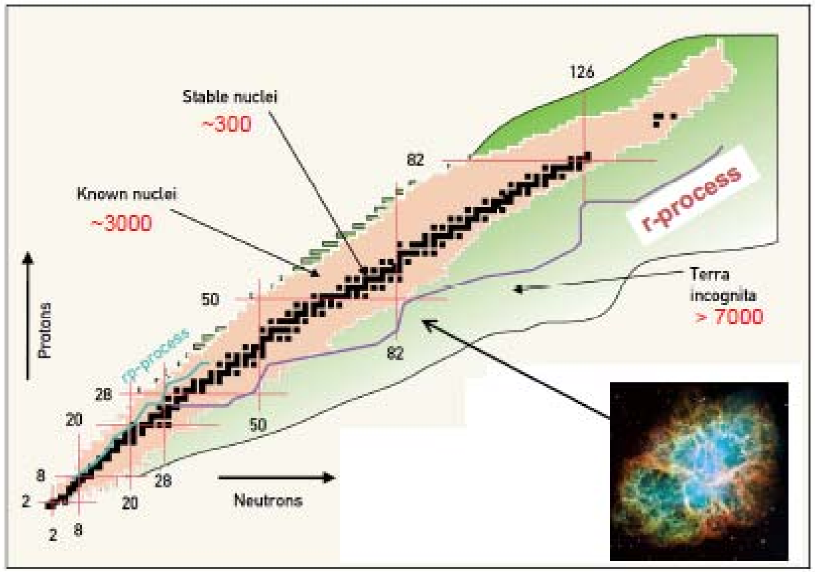
- The effect on the nuclear chart
  - anomaly in drip line and nontrivial magic number in neutron rich nuclei by TNF



drip line: $^{28}O \rightarrow {}^{24}O$

nontrivial magic number
N=28 for $_{20}Ca$

T.Otsuka et al., PRL105(2010)032501
J.D.Holt et al., arXiv:1009.5984



Nucleosynthesis by Supernova

Importance of Three Nucleon Force (TNF)

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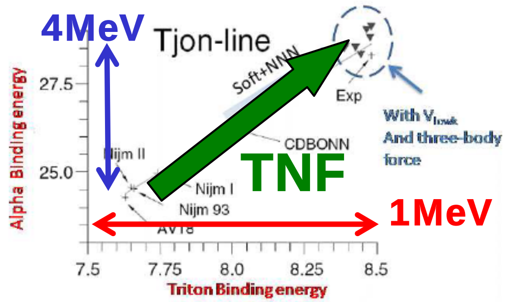

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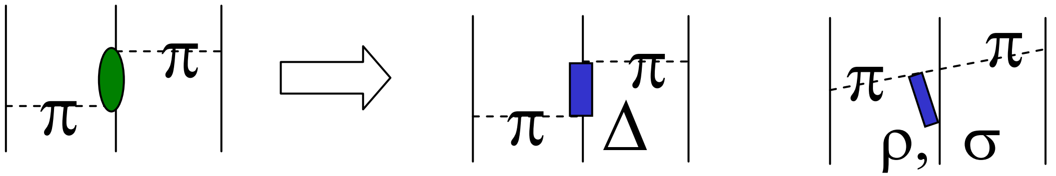
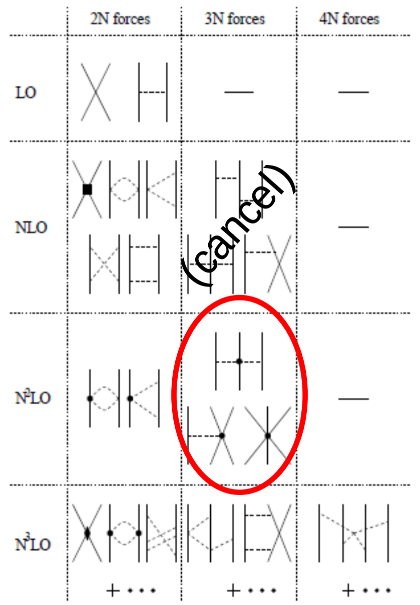
Three Nucleon Force (TNF)

- It is natural to expect the existence of TNF
- It is very nontrivial to determine TNF from QCD
- 2πE-TNF Fujita-Miyazawa, PTP17(1957)360
  - Off-energy-shell πN scatt

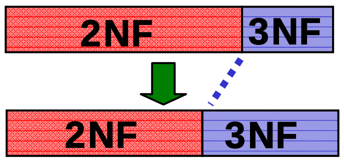


- EFT expansion ➔ TNF appears at NNLO order
- Phenomenological short-range repulsion is necessary
- 2πE-TNF too attractive, often suppressed (artificially) by form factor
- NB: the combination of (2NF,3NF) ➔ observables



U.v.Kolck, PRC49(1994)2932

Epelbaum, Prog.Part.Nucl.Phys.57(06)654



How can we tackle TNF in Lattice QCD ?

c.f. pioneering lat calc of B.E. ^{3}He(=^{3}H), ^{4}He
T.Yamazaki et al., arXiv:0912.1383

- **In the case of 2N system…**
  - Calc 4pt func ➔NBS amp.

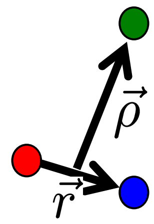
$$\psi(\vec{r}) = \langle 0|N(\vec{x}+\vec{r};t)N(\vec{x};t)|2N\rangle$$

$$|2N\rangle = \bar{N}_{src}(t=0)\bar{N}_{src}(t=0)|0\rangle$$

➔ $$(E - H_0)\psi(\vec{r}) = [V_c(r) + S_{12}V_T(r) + \cdots]\psi(r)$$

- **Extention to 3N system**
  - Calc 6pt func ➔NBS amp. of NNN

$$\psi(\vec{r},\vec{\rho}) = \langle 0|N(\vec{x}+\vec{r})N(\vec{x})N(\vec{x}+\vec{r}/2+\vec{\rho})|3N\rangle$$



  - Obtain TNF through

$$(E-H_0^r-H_0^\rho)\psi(\vec{r},\vec{\rho}) = \left[\sum_{i<j} V_{ij}(\vec{r}_{ij}) + V_{TNF}(\vec{r},\vec{\rho})\right]\psi(\vec{r},\vec{\rho})$$

by 2N calc

  - Difficulty(1): volume factor
    - 2N: naïve O(L^6) calc ➔ O(L^3 log L^3)
    - 3N: naïve O(L^9) calc ➔ O(L^6 log L^6)

O(10^4-10^5) factor

  - Difficulty(2): naïve calc of quark dof grows in factorial (~N_u! N_d!)
    - 2N: O(L^3) X N_{wick} X color/spinor loops
    - 3N: O(L^6) X N_{wick} X color/spinor loops

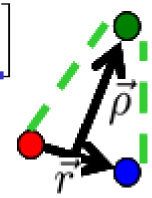
O(L^3) X O(4000) = O(10^7-10^8) factor

TNF is exceptionally challenging problem !

How can we tackle TNF in Lattice QCD ? (cont'd)

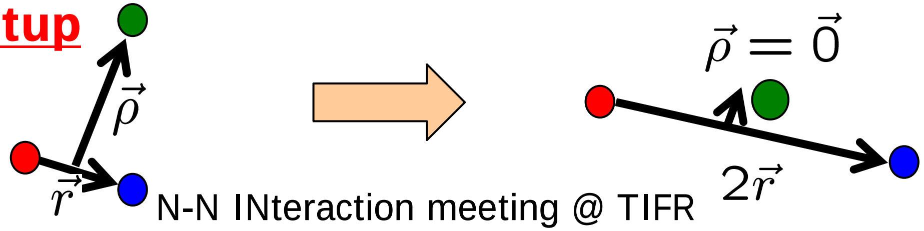
- We studied the effective 2N potential in 3N system (^{3}H)

$$(E - H_0^r)\phi(\vec{r}) = \left[V_{12}(\vec{r}) + \underline{\delta V_{eff}(\vec{r})}\right]\phi(\vec{r})$$
$$= \left[V_{12}(\vec{r})\phi(\vec{r}) + \underline{\int d\vec{\rho}\,(V_{13}(\vec{r},\vec{\rho}) + V_{23}(\vec{r},\vec{\rho}) + V_{TNF}(\vec{r},\vec{\rho}))\psi(\vec{r},\vec{\rho})}\right]$$



  - Relatively small calc cost (yet, still much expensive than 2N)
  - Good precision achieved thanks to the sum over spectator particle
  - Indirect access to TNF (due to off-diag 2N), and the effect of TNF is "smeared" by spacial average with triton wave function

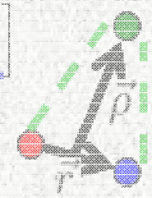
- Calculation for **fixed 3D-configuration** of 3N system
  - **<u>Direct access to TNF is possible !</u>**
  - ➔ We can explore the various features of TNF (spin/isospin/spacial, etc.)
  - Much more expensive calc cost (**O(10-100)** factor) <u>and</u> yet worse S/N
  - We study **<u>linear setup</u>**



How can we tackle TNF in Lattice QCD ? (cont'd)

- We studied the effective 2N potential in 3N system (^{3}H)

$$
\begin{aligned}
(E - H_0^{\vec r})\phi(\vec r) &= \left[V_{12}(\vec r) + \delta V_{eff}(\vec r)\right]\phi(\vec r) \\
&= \left[V_{12}(\vec r)\phi(\vec r) + \int d\vec\rho\,(V_{13}(\vec r,\vec\rho) + V_{23}(\vec r,\vec\rho) + V_{TNF}(\vec r,\vec\rho))\psi(\vec r,\vec\rho)\right]
\end{aligned}
$$



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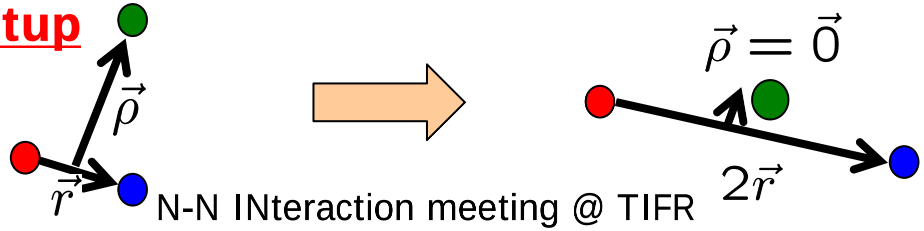
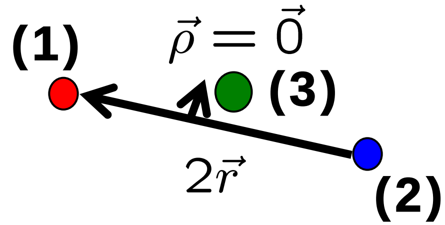


Features of Linear setup for ^{3}H



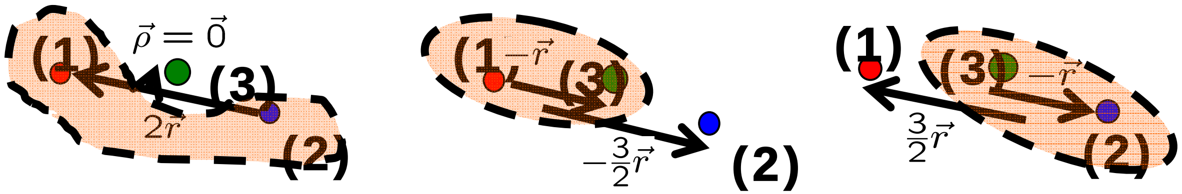
- Simplified coupled channel analysis possible
  - The vector to 3rd particle $\vec{\rho} = \vec{0}$
  - ➔ $L^{(1,2)\text{-pair}} = L^{\text{total}} = 0$ or 2 only
  - ➔ Possible **bases are only three**, which can be labeled by **1S0, 3S1, 3D1 for (1,2)-pair**
- However, in order to determine TNF in **3x3 coupled channel**, we need information of parity-odd potential
  - Although (1,2)-pair is L=even, (3,1),(2,3)-pair have L=odd components
  - Partial wave expansion with different Jacobi setup is impossible, since we do not have full wave function (only linear setup)
- Parity-odd potential from lattice QCD (still) in progress
  - ➔3X3 channel, but unknown $V_C^{I,S=0,0}$,$V_C^{I,S=1,1}$,$V_T^{I,S=1,1}$,TNF(s)

Solution using "symmetric" wave function

- Rotate the basis $|\psi_{^1S_0}\rangle, |\psi_{^3S_1}\rangle, |\psi_{^3D_1}\rangle \longrightarrow |\psi_S\rangle, |\psi_M\rangle, |\psi_{^3D_1}\rangle$

$$|\psi_S\rangle = 1/\sqrt{2}\left(-|\psi_{^1S_0}\rangle + |\psi_{^3S_1}\rangle\right) \qquad |\psi_M\rangle = 1/\sqrt{2}\left(+|\psi_{^1S_0}\rangle + |\psi_{^3S_1}\rangle\right)$$

- We can construct the wave function in which <u>any 2N pair</u> is spin/isospin anti-symmetric



$$\begin{aligned}
|\psi_S\rangle \;=\; 1/\sqrt{6}\Big[& -(p_\uparrow n_\uparrow - n_\uparrow p_\uparrow)n_\downarrow & (\leftarrow I=0, S=1)\\
& -(n_\uparrow n_\downarrow - n_\downarrow n_\uparrow)p_\uparrow & (\leftarrow I=1, S=0)\\
& +1/2(p_\uparrow n_\downarrow + n_\uparrow p_\downarrow - p_\downarrow n_\uparrow - n_\downarrow p_\uparrow)n_\uparrow & (\leftarrow I=1, S=0)\\
& +1/2(p_\uparrow n_\downarrow - n_\uparrow p_\downarrow + p_\downarrow n_\uparrow - n_\downarrow p_\uparrow)n_\uparrow \Big] & (\leftarrow I=0, S=1)
\end{aligned}$$

- ➔ **<u>L=even for any 2N pair</u>** automatically guaranteed

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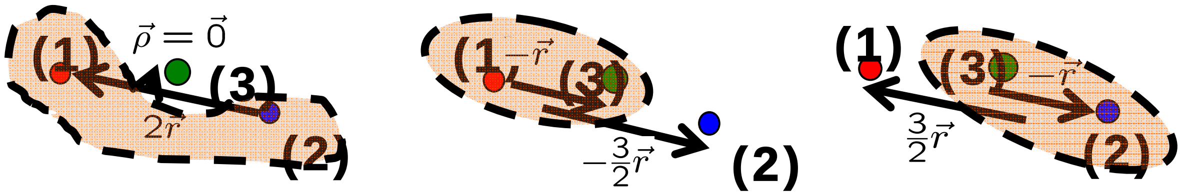
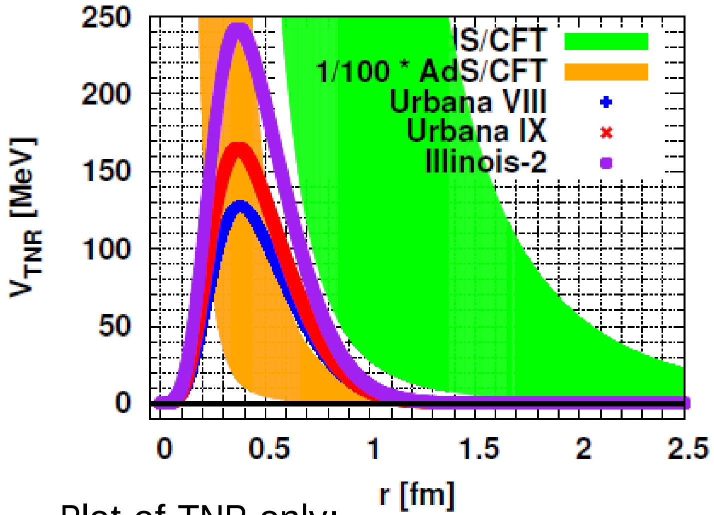


  - ➔ **<u>L=even for any 2N pair</u>** automatically guaranteed
- 3x3 coupled channel is reduced to
  - **<u>one channel with only TNF unknown</u>** (L^2-dep ignored)
  - two channels with $V_C^{I,S=0,0}$, $V_C^{I,S=1,1}$, $V_T^{I,S=1,1}$,(TNF) unknown
- ➔ Even without parity-odd V, we can determine one TNF
  - This methodology works for any fixed 3D-conf other than linear

Repulsive TNF (TNR)

- We determine TNF assuming scalar/isoscalar
  - Phenomenologically introduced to reproduce saturation density/energy of nuclear matter, etc.



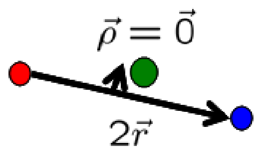
Plot of TNR only:
there is cancellation from TNA

$$V_{TNF} = V_{2\pi E} + (V_{3\pi R}) + V_{TNR}$$

Urbana/Illinois

$$V_{TNR} = U_0 \sum_{cyc} T^2(r_{12}) T^2(r_{13})$$

$$T(r) = \left(1 + \frac{3}{\mu r} + \frac{3}{\mu^2 r^2}\right) \frac{e^{-\mu r}}{\mu r} T_{cut}(r)$$



AdS/CFT: $V_{TNF} = +\text{const.} \cdot \frac{1}{r^4}$

K.Hashimoto, N.Iizuka
arXiv:1005.4412

Lattice calculation setup

- Nf=2 dynamical clover fermion + RG improved gauge configs (CP-PACS)
  - 598 configs X 16 measurements
  - beta=1.95, (a^{-1}=1.27GeV, a=0.156fm)
  - 16^3 X 32 lattice, L=2.5fm
  - Kappa(ud)=0.13750
    - $M(\pi)$ = 1.13GeV
    - M(N) = 2.15GeV
    - $M(\Delta)$ = 2.31GeV
  - Techniques
    - **Automatic Wick contraction code** to handle 4 up- and 5 down-quarks
    - Non-rela limit op is used to create 3N state at source

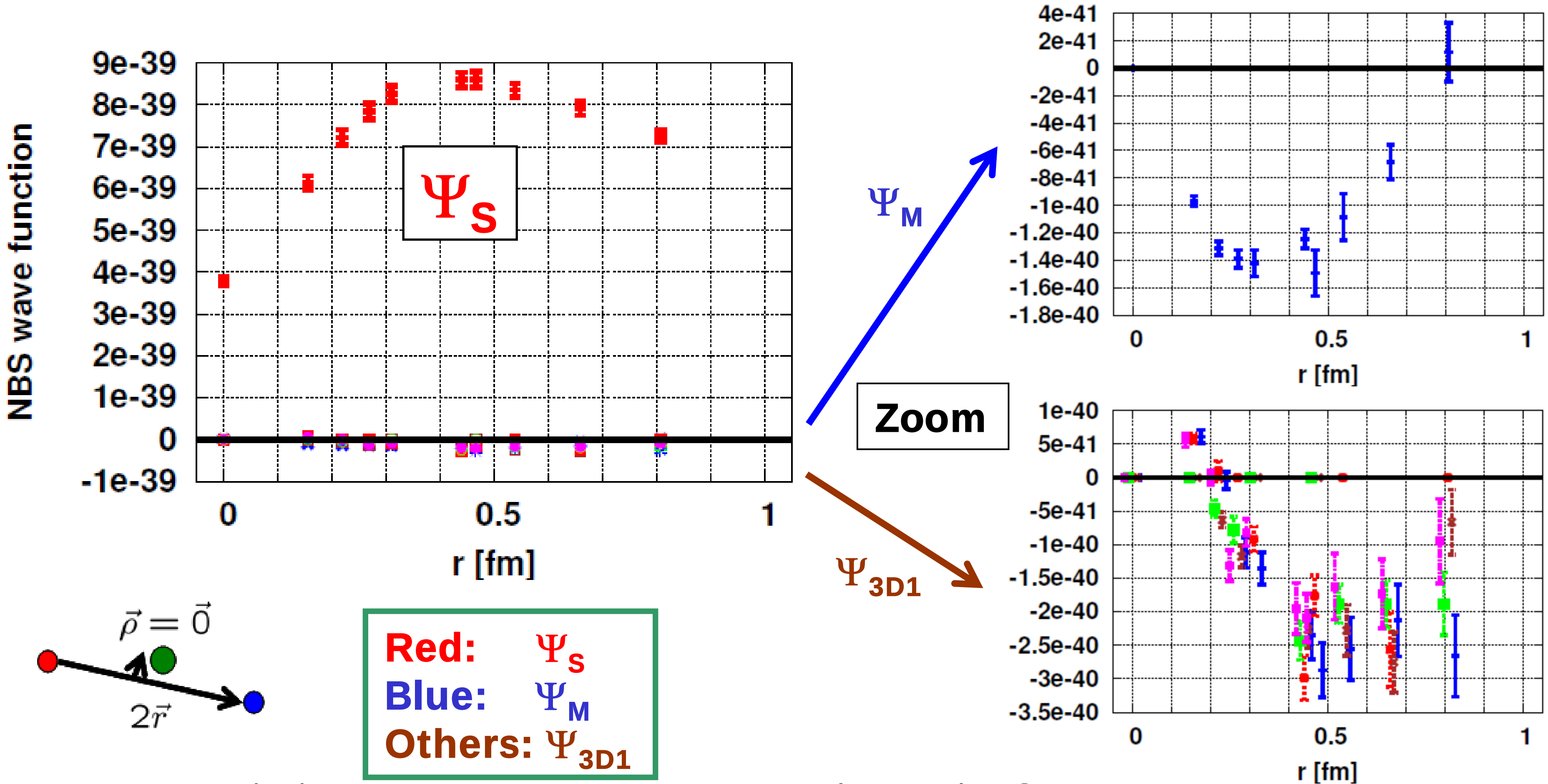
CP-PACS Coll. S. Aoki et al.,
Phys. Rev. D65 (2002) 054505
[E: D67 (2003) 059901]

BGL@KEK

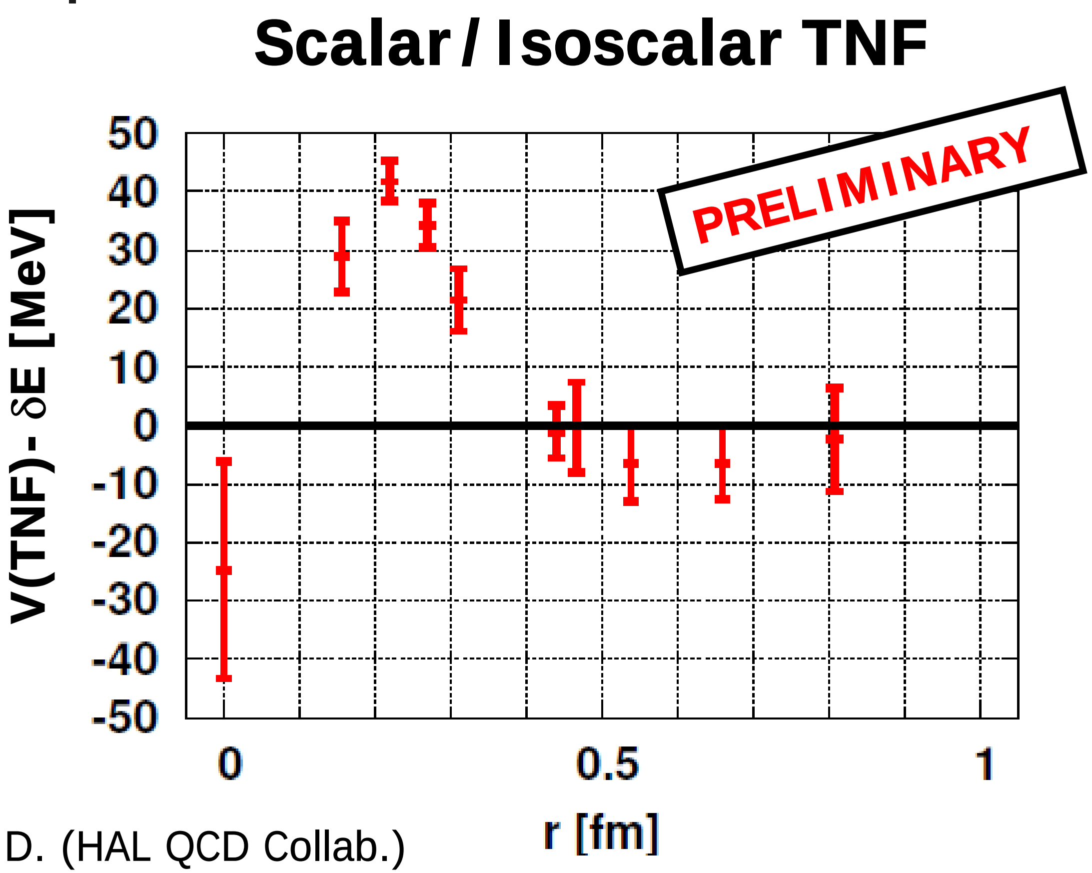
$$N^{src} = \epsilon_{abc}(u_a^T C\gamma_5 \frac{1+\gamma_4}{2} d_b)\frac{1+\gamma_4}{2} u_c$$

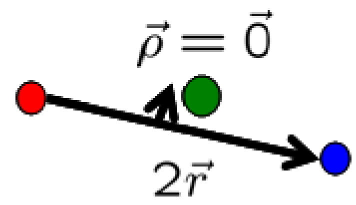
→ Factor of 2^3=8 faster

Results for wave functions



Genuine Three Nucleon Force





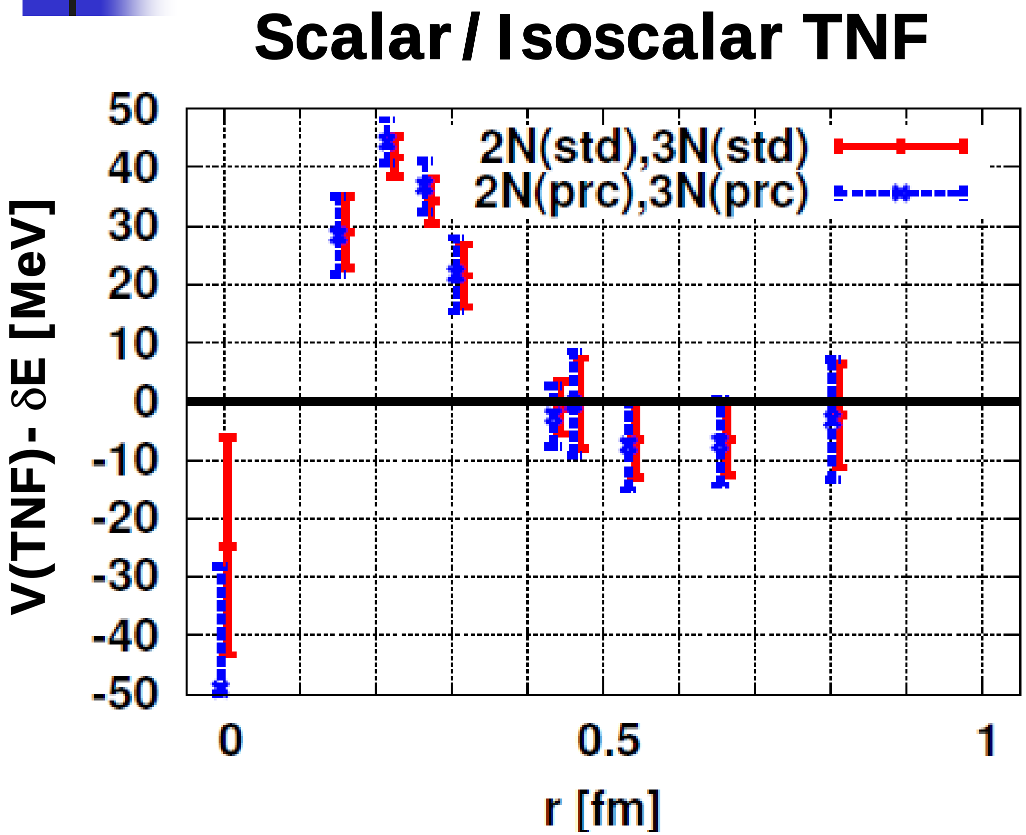
Indication of short-range repulsive TNF

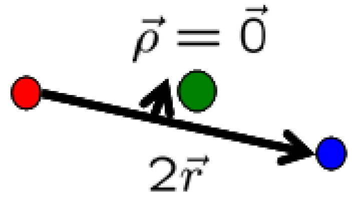
Note:
GS saturation may not be sufficient
Constant energy shift not included
➔ *O(10)MeV shift possible*

T.D. (HAL QCD Collab.)
arXiv:1011.0657

M(π)=1.13GeV

Genuine Three Nucleon Force





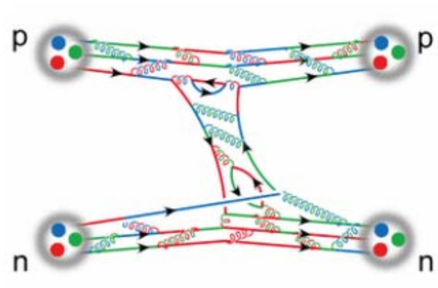
Comparison with improved Laplacian op.

Discretization error in Laplacian op. is small

$$\nabla^2_{std} f(\vec{x}) = \frac{1}{a^2} \sum_i [f(\vec{x}+a_i) + f(\vec{x}-a_i) - 2f(\vec{x})] = \nabla^2 f(\vec{x}) + \mathcal{O}(a^2)$$

$$\nabla^2_{prc} f(\vec{x}) = \frac{1}{12a^2} \sum_i [-(f(\vec{x}+2a_i) + f(\vec{x}-2a_i)) + 16(f(\vec{x}+a_i) + f(\vec{x}-a_i)) - 30f(\vec{x})] = \nabla^2 f(\vec{x}) + \mathcal{O}(a^4)$$

Summary/Outlook



- **Potentials from Lattice QCD using NBS wave function**
  - Central and tensor potentials in parity-even channel
  - Qualitative features of NN potentials are reproduced, Velocity expansion checked
  - Significant step toward **Nuclear Physics from QCD**
- Lattice QCD can give **useful predictions** on unknown potentials
  - **YN, YY**: Strangeness physics, hyperon matter in neutron star, SU(3) & beyond SU(3)
  - Meson-Baryon: N-K, N-ccbar (Kawanai-Sasaki), Q-Qbar: (Iida-Ikeda)
- **The First** calculation on **Three Nucleon Force (TNF)** from Lattice QCD
  - 2N subtraction is possible using only parity-even potentials
  - Calculation of linear setup of 3N (^{3}H) system
    - **Indication of Repulsive TNF at short distance, further studies ongoing**
- Various complementary approaches useful
  - Operator-Product-Expansion (Aoki-Balog-Weisz)
  - Lattice nuclei (Yamazaki-Kuramashi-Ukawa)
  - Strong-coupling limit (de Forcrand-Fromm)
- **Outlook**
  - Realistic potentials (and phase shifts) with **physically light masses w/ large volume**
  - Parity-odd potentials, Higher derivative terms (LS-force and more) ➔ More TNFs
  - Understand the insight of nuclei: lattice nuclei vs. lattice potentials + ab initio calc.
  - Resonances from potentials ? (ρ, Δ, H-dibaryon, exotics…)
  - TNF: other 3D-conf (triangle etc.) for spacial info, I=3/2, $SU(2)_f$ ➔ $SU(3)_f$: Astro physics

A Space Odyssey
From QCD to Nuclear / Astro Physics
has just begun !