UNIVERSAL $1/r^2$ POTENTIALS at short and long range avoids collapse holds bound states

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Angular Kinetic Energy $\ell^2/2mr^2$

Repulsive barrier: at small r avoids collapse, at large r holds low energy particles from penetration and/or escape

Classical Physics: Collapse for straight line $\ell = 0$

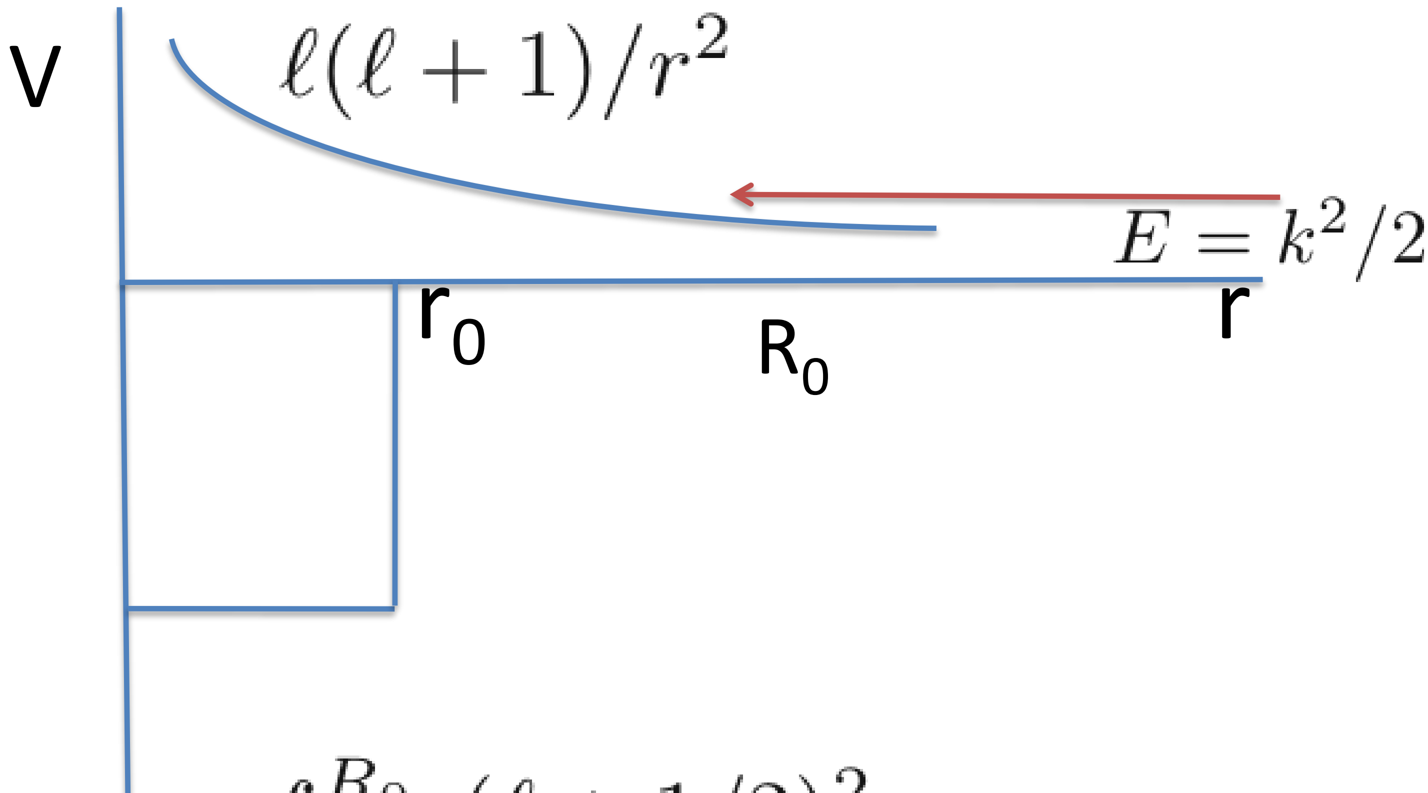
Quantum Physics, $\hbar$, always an $\hbar^2/mr^2$

radial kinetic energy

Stability of H atom $\hbar^2/(2mr^2) - e^2/r$ Bohr radius a_0

Wigner Threshold Law $k^{2\ell+1}$ tunneling at large r

Tunneling through angular potential barrier



$$\exp(-2\int_{r_0}^{R_0} [\frac{(\ell+1/2)^2}{r^2} - k^2]^{1/2} dr)$$

$k^{2\ell+1}$

elastic scattering phaseshift
inelastic cross-section

Attractive $-a/2r^2$ potential, effective complex ang momentum

$$\lambda = -\frac{1}{2} + i\alpha \qquad \alpha = \sqrt{a - \frac{1}{4}}$$

Number of bound states: Infinite number for long range

$$\hbar^2/2mr^2 \quad \text{vs} \quad -e^2/r$$

1/r^2 the dividing falloff between long/short range, infinite/finite #

for 1/r^2 itself, the strength parameter a decides

Infinite sequence described by sin ( ) = 0 => eigenvalues

$$\epsilon = -\kappa^2/2$$

Coulomb : $\pi Z/\kappa = n\pi, \epsilon_n = -Z^2/2n^2$

Dipole bound states: molecules (OH, H_2O), H atom in n > 1 :

$$\alpha \ln(2/\kappa) - arg\Gamma(1 - i\alpha) = (n + 1/2)\pi \qquad \epsilon_n = \epsilon_0 e^{-2n\pi/\alpha}$$

D-dimensional Laplacian:

$$\frac{\hbar^2}{2M}\left\{-\frac{d^2}{dR^2}+\frac{D-1}{R}\frac{d}{dR}+\frac{\Lambda^2(angles)}{R^2}\right\}$$

radial angular

$$\Lambda^2:\lambda(\lambda+D-2),\ D=2:m^2,\ D=3:\ell(\ell+1)$$

N – body problem : D = 3(N-1) three-body: 6

$R^2 = r_1^2 + r_2^2 + \ldots$ R overall size of the system, rest angles

$$R=(\sum_i m_i r_i^2/M)^{1/2}$$

$$\Psi \rightarrow R^{(D-1)/2}\Psi$$

$$-\frac{d^2}{dR^2}+\frac{(D-1)(D-3)}{4R^2}$$

radial kinematic potential

D = 2 attractive

D > 3 repulsive

V. Efimov, Phys. Lett. **33**B, 563 (1970) L.H.Thomas (1935)

Two-body with range r_0 , scattering length a
Effective attractive $1/R^2$ potential as scattering length gets large

Number of bound states as |a| gets large: $(1/\pi) \ln (|a|/r_0)$

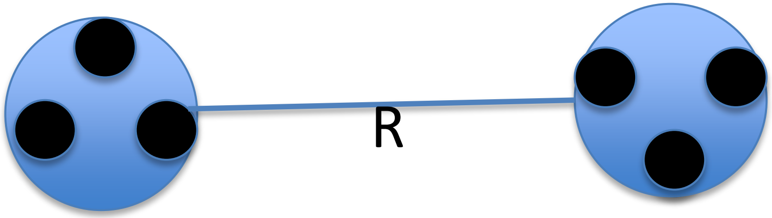
As two-body binding increases, Efimov states disappear

N – N Potential   Repulsive Barrier/Core at small R

1969 U. Fano Wannier 3-body threshold law with Coulomb

→ $H^+ + e + e$

Repulsive $1/R^2$ Potential
at small R



Not really a two-body but an effective potential of a many-body system

Number of particles not defined, a conjugate phase as in BCS

Born approximation: $\sigma_{el} \sim [E^2 \ln \vec{p}^2 - \vec{p}^2]/\vec{p}^2$

$$E^2 = \vec{p}^2 + m^2$$

Near zero-energy structures only from long range

Wigner R-matrix Reaction zone) Asymptotic Far zone

Two-body threshold laws

Exactly at threshold, longest range potential gives energy dependence

For short range interactions, angular momentum $k^{2\ell+1}$

For attractive Coulomb, that potential controls, irrespective of ℓ

Photoionization cross-sections start finite at threshold for any

Reaction zone) Coulomb zone) Far asymptotic

Z/r $r_C = 2Z/k^2$ $\varepsilon = k^2/2$

Three-body Coulomb, two-electron escape

Wannier Phys. Rev. 90, 817 (1953), Rau Phys. Rev. A 4, 207 (1971), Peterkop J. Phys. B 4, 513 (1971)

Two-electron escape at threshold

Photon + H^- → $H^+ + e + e$

e + H → $H^+ + e + e$

Final state in static screening models: "full" $\epsilon^{3/2}$, "partial" ϵ

"Dynamical" screening : sequence of screening determined by energy partitioning between the electrons and determining partitioning

Wannier configuration e —— H^+ —— e

Saddle point of configuration, tight angular and radial correlations, reaching their extreme at threshold for double escape

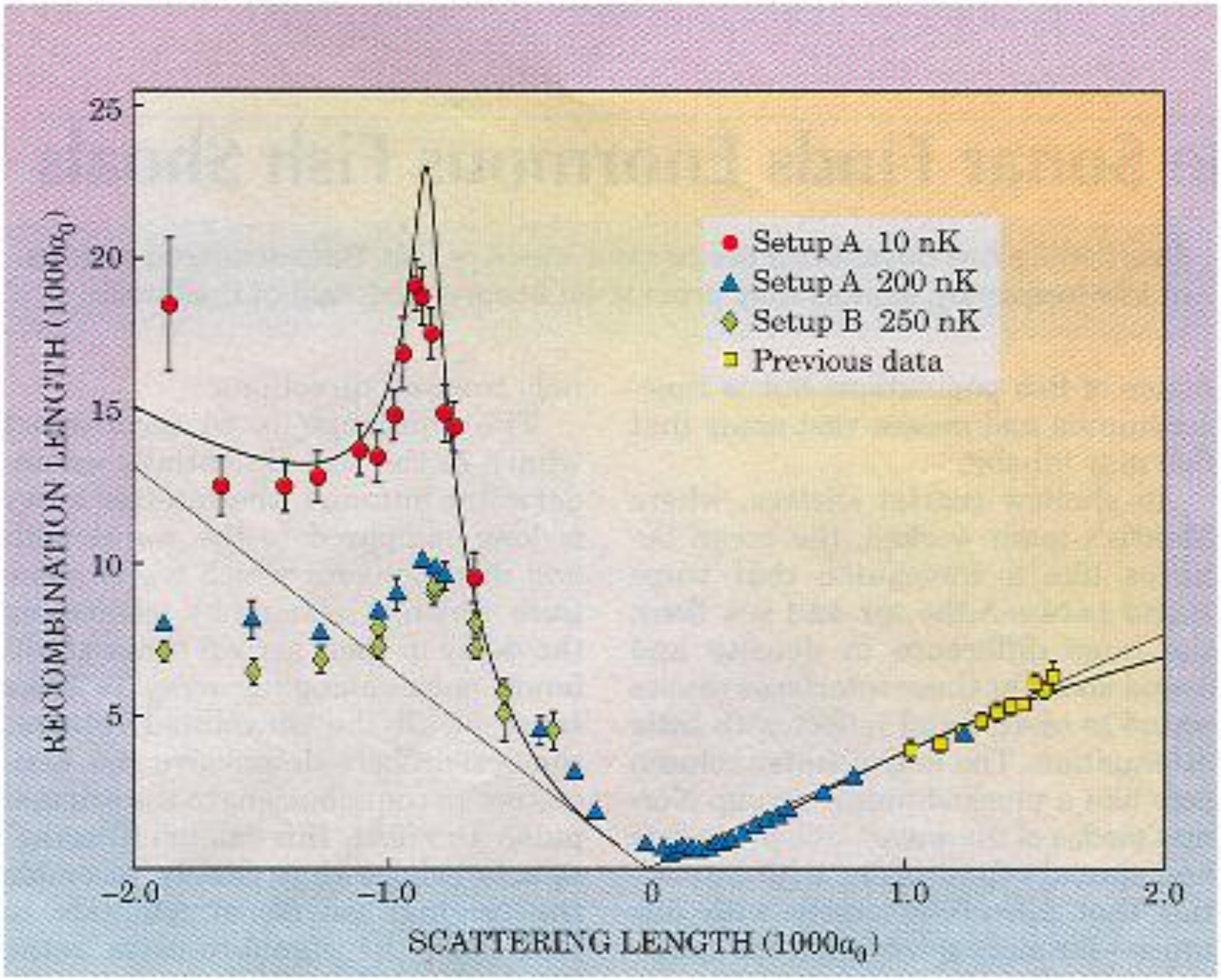
cross-section $\epsilon^{\mu(Z)}$ $\mu = 1.127 \quad Z = 1$

Same correlations in high doubly-excited states,

Pauli principle, S and L, irrelevant except $^1P^e$ and $^3S^e$ more suppressed

C.H.Greene and ARPR, Phys. Rev. Lett. 48, 533 (1982)

Efimov states of Cesium trimers



Kraemer et al, Nature **440**, 315 (2006), Esry-Greene, p.289

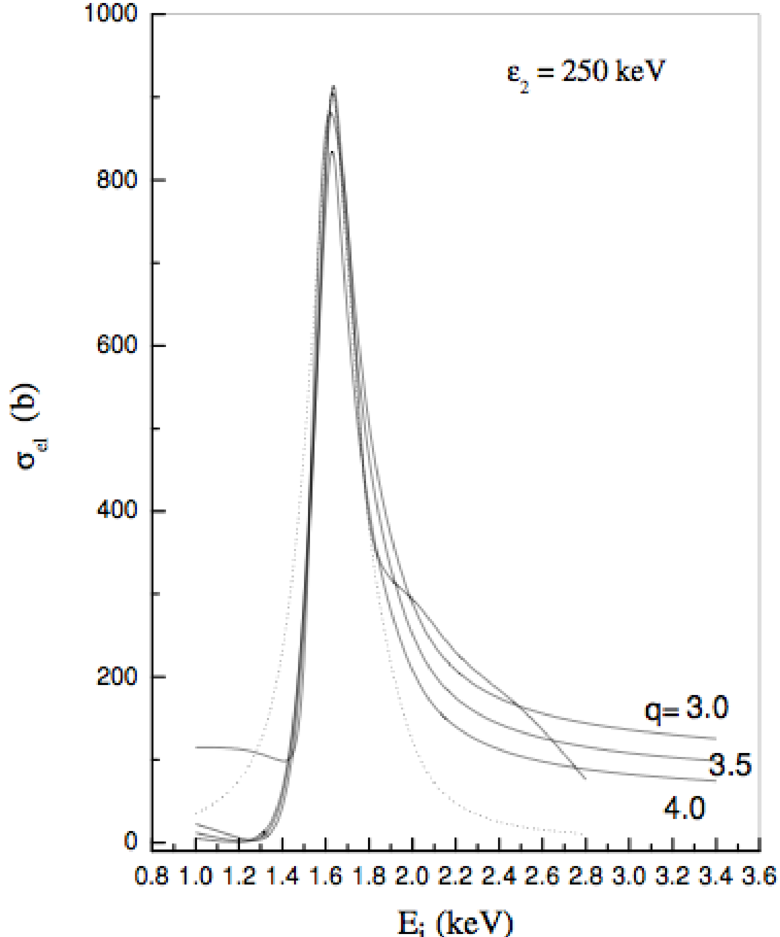
C. H. Greene, Phys. Today **63** (3), 40 (2010)

I.Mazumdar, ARPR, V.S.Bhasin, Phys.Rev.Lett. **97**,062503 ('06)

neutron-rich, halo nuclei. $^{20}C : n + n + {}^{18}C$

two-body energy ^{19}C tuned

elastic n + ^{19}C cross-section for energy E_i



E_r =1.63 keV

Γ = 0.25 keV

q = 4

Doubly-excited states in atoms, two-neutron states in nuclei

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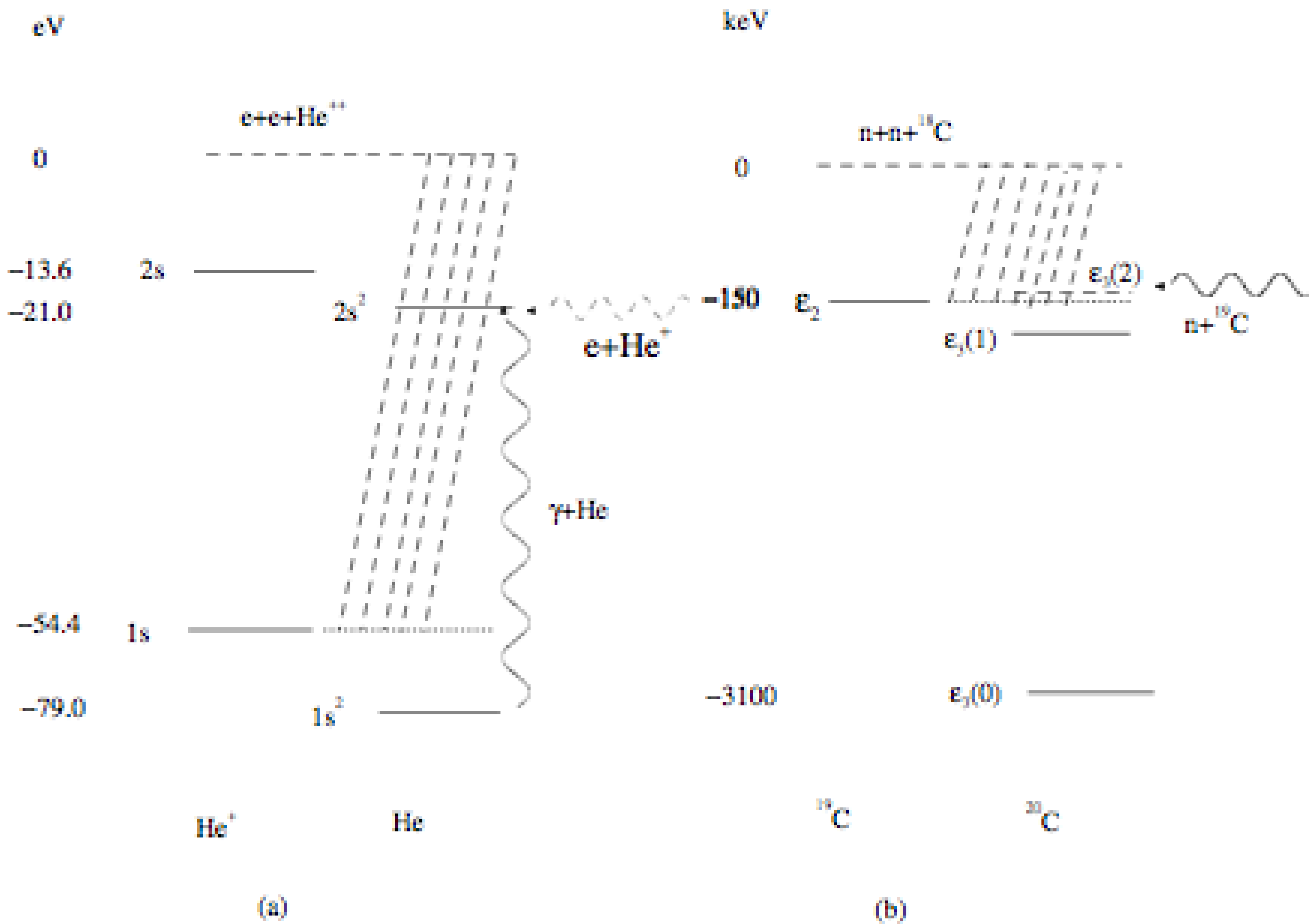

Effect of finite angular momentum and symmetry

For a non-zero λ, $\lambda(\lambda + D - 2)/R^2$ will swamp $-a/R^2$,

So Efimov states only at λ or L = 0.

Fermions, Pauli antisymmetrization relevant only if
3 identical particles forbidden in L = 0. $^4S^e$
Macek and Sternberg, Phys. Rev. Lett. 97, 023201 (2006).

Efimov states only for 3 bodies: Amado and Greenwood, Phys. Rev. D 7, 2517 (1973).
3 particles define a plane and radial kinematic potential attractive only in 2-dimensions.

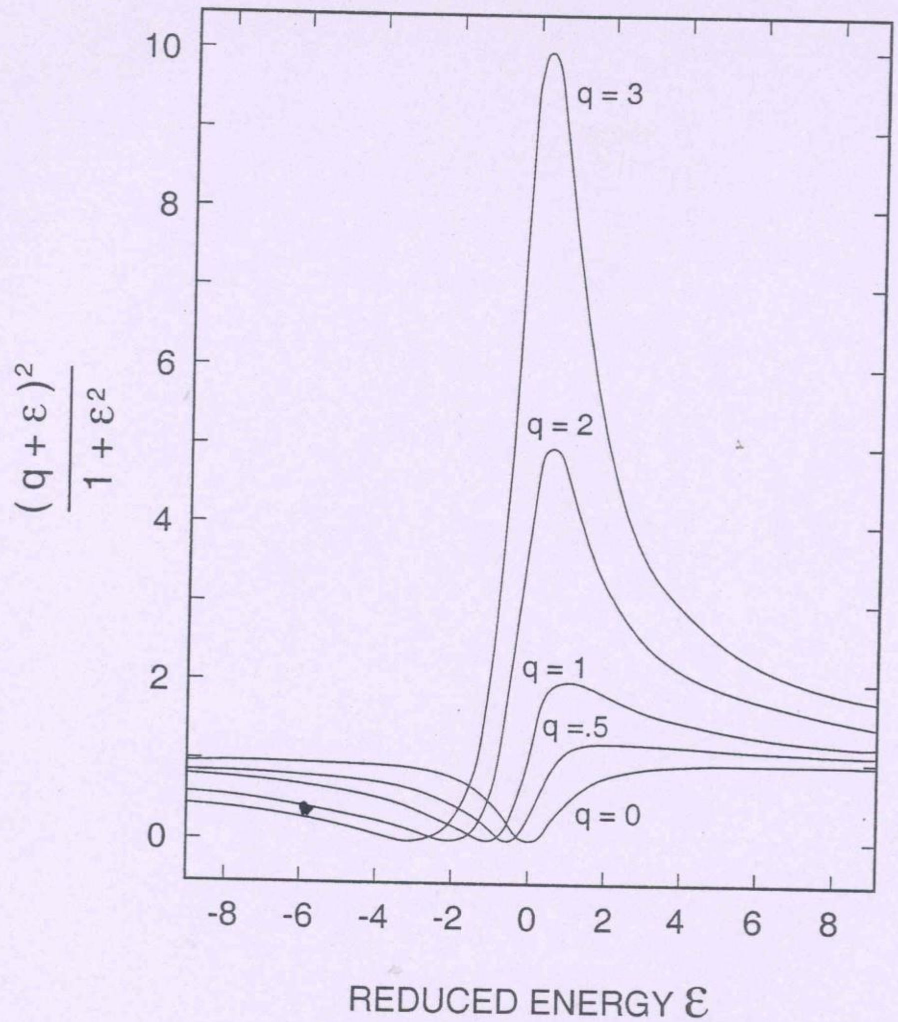

Figure 5.13. Beutler-Fano profiles with different values of the Fano q-parameter. From U. Fano, Phys. Rev. **124**, 1866 (1961).

Resonances in physics: symmetric Lorentzian shapes

Breit-Wigner in nuclear and particle physics

$$\sigma = \frac{A}{(E - E_r)^2 + (\Gamma/2)^2} = \frac{\sigma_0}{1 + \epsilon^2}$$

$$\epsilon \equiv (E - E_r)/(\Gamma/2)$$

Fano (1961) resonance as interference between two pathways

$$\sigma = \sigma_0 \frac{(q + \epsilon)^2}{(1 + \epsilon^2)}$$

Fano profile, asymmetric, zero at $\epsilon = -q$

Reduces to Lorentzian for $q \rightarrow \infty$