S_3 Symmetric qqq Force Dominance In High Momentum Limit of QCD For Understanding Proton Spin

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1 Introduction

This little contribution aims to provide an example of *permutation symmetry*, whose 'physics' seems to apply to the structure of the proton itself ! The the-

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oretical background is provided by a recent paper[1], hereafter designated as the qqq paper, wherein the concept of a fundamental 3-body force is introduced at the quark-gluon level, as a folding of a ggg vertex with 3 distinct $\bar{q}gq$ vertices, making up a Y-shaped diagram (see fig 1). This is of course in addition to the standard pairwise qq force that is usually considered for baryon spectroscopy, but it turns out that in the high momentum limit of QCD, where the confining *qq* force may be neglected, the direct *qqq* force *dominates* over the pairwise *qq* forces, thus offering a new basis for exploring the structure of the proton in the high momentum limit of QCD, one in which the concept of permutation (S_3) symmetry plays a central role. While referring the interested reader to [1] for mathematical details, in the context of the proton's spin anomaly [2] the essential features are the following.

The basic dynamics is provided by a

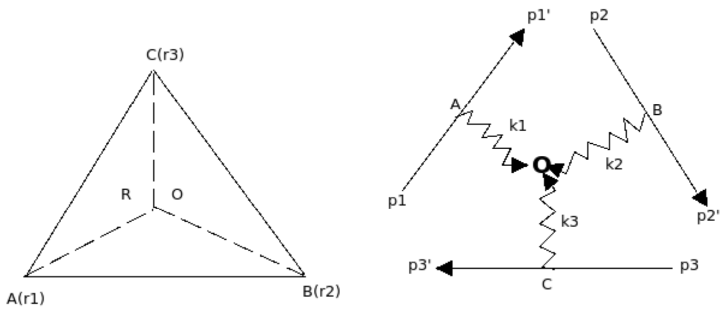

Figure 1: (a) Pictorial view of 2- & 3- body interactions; (b) 'Mercedez-Benz' diagram for qqq-force

covariant Salpeter-like equation [3] governed by what is termed in the literature as the Markov-Yukawa Transversality Principle (MYTP for short)[4,5] which specifies that all qq forces are *transverse* to the direction of the total hadron 4-momentum P_μ–a gauge principle in disguise [6]! The Salpeter equation has a remarkable property of a 3D-4D interlinkage [7] which can be formulated within Dirac's LF Dynamics [8,9] to take advantage of its bigger stability group. Using the notation, phase and normalization of [1], but skipping most of the details, the qqq force in 3D form reads

$$V_{qqq} = \frac{-g_s^4}{3} \frac{[SSS]}{k_1^2 k_2^2 k_3^2} \qquad (1.1)$$

where the dominant (spin-spin-spin) term SSS works out as

$$SSS = 2i\Sigma.(\eta - \eta') \times (\xi - \xi')$$
$$[k_1^2 + k_2^2 + k_3^2]/\sqrt{3} - 4[(\eta - \eta')$$

$$\times(\xi - \xi')]^2 \tag{1.2}$$

Σ being twice the total hadron spin operator. The procedure is now to insert this term within an interlinked 3D-4D BSE formalism wherein the complete 4D wave function Ψ satisfies a covariant Salpeter Eq a la MYTP on the light front (LF); The Ψ function whose spin dependence is fully described in terms of standard Dirac matrices, can be related through a sequence of transformations to a 3D scalar function ϕ which (on reduction of the above 4D BSE) satisfies a 3D Schroedinger-like (albeit LF covariant) equation. Next, a *reconstruction* of Ψ in terms of ϕ is achieved by Green's function techniques, so that the 4D spin structure of the former is recovered. Since the qqq paper [1] had ended at this stage, we need to start from this stage to collect (in Sect 2) some essential material outlining the solution for the ϕ function, along with the reconstructed function Ψ. In Sect 3 we offer a first application of this qqq formalism to the evaluation, using 4D loop diagrams, of some basic quantities like magnetic moments, $g_{3,8,0}$ in the notation and normalization of [2] and the (less trivial) baryon structure function.

2 Structure of Full 4D Ψ

The LF momenta in 3D form are [7]:

$$p_{iz}; p_{i0} = \frac{Mp_{i+}}{P_+}; \frac{Mp_{i-}}{2P_-}$$
$$\hat{p}_i \equiv \{p_{i\perp}, p_{iz}\} \quad (2.1)$$

$$\sqrt{2}\xi = p_3 - p_2; \quad \sqrt{6}\eta = -2p_3 + p_1 + p_2; \tag{2.2}$$

There are three main steps

Step A: Define an auxiliary 4D scalar fn Φ:

$$\Psi = \Pi_{123} S_{Fi}^{-1}(-p_i)\Phi(p_i p_2 p_3)W(P) \tag{2.3}$$

$$W(P) = [\chi'\phi' + \chi''\phi'']/\sqrt{2} \tag{2.4}$$

$$|\chi' >; |\chi'' > = [\frac{M - i\gamma.P}{2M}[i\gamma_5; i\hat{\gamma}_\mu/\sqrt{3}]C/\sqrt{2}] \otimes [[1; \gamma_5\hat{\gamma}_\mu]u(P)] \tag{2.5}$$

Step B: Set up the Master Eq for Φ with Gordon reduction.

Step C: Make a reduction of the Master Eq for 4D Φ to one for 3D ϕ; then *reconstruct* Φ in terms of ϕ,via Green's fn method adapted to LF formalism [7]. The final result for reconstructed 4D spinor Ψ in terms 3D scalar ϕ is

$$\Psi(\xi, \eta) = \Pi_{123} S_F(p_i) D_{123} W(P) \sum_{123} \frac{\phi(\hat{\xi}, \hat{\eta})}{(2\pi i)^2} \tag{2.6}$$

where

$$\frac{1}{D_{123}} = \int \frac{P_+^2 dq_{12-} dp_{3-}}{4M^2 (2i\pi)^2 \Delta_1 \Delta_2 \Delta_3} \tag{2.7}$$

and the 3D wave fn ϕ satisfies a 6D Differential equation in coordinate space with S_3 symmetric variables

$$\sqrt{2}s_3 = r_1 - r_2; \quad \sqrt{6}t_3 = -2r_3 + r_1 + r_2 \tag{2.8}$$

Pending a completely satisfactory solution of Eq.(2.7), we have attempted an approximate representation of Ψ in a two-component matrix form ψ (due to the spin-dependence (1.1) of the qqq force, ψ with its 3D spin dependence, is a more convenient alternative to ϕ, and is more readily comparable to the Orsay group's [10]), on the lines of the Orsay group [10] who had used a mixture of 56 and 70 states, each with $L = 0^+$. In the present case, due to the high degree of qqq symmetry, it seems more appropriate to take a mixture of $56; 0^+$ and $20; 1^+$, each with $J = 1/2$, but taking account of the full qqq symmetry :

$$\psi = \psi_s + i\frac{\sqrt{2}\Sigma.\eta \times \xi}{\xi^2 + \eta^2}\psi_0 \qquad (2.9)$$

where ξ, η are given by Eq.(2.2), and the norm of the second term is for later convenience. It turns out that ψ_s, ψ_0 satisfy almost identical equations represented symbolically as

$$D_{123}[\psi_s;\psi_0] = \int V_s[4(\eta \times \xi)^2 - 4X + \pm 2X](\psi_s;\psi_0); \quad (2.10)$$

where

$$X = \rho(\eta \times \xi.\hat{P}/\sqrt{3}; \quad \rho = \xi^2 + \eta^2 \quad (2.11)$$

If as a first approximation, the term $\pm 2X$ in Eq.(2.10) is dropped, there arises a remarkable simplification wherein the ratio of the ψ_s and ψ_0 components is almost independent of the dynamics, except for a constant, while the dynamics is almost entirely contained in a common function ψ satisfying an equation of the form (2.10). The effect of this simplification is best expressed by the relations

$$\psi_s = \cos\beta\psi; \quad \psi_0 = \sin\beta\psi \quad (2.12)$$

where the phase factor β, plays the role of the mixing angle ϕ of [10]).

For the rest of the paper the 3D LF function ψ of Eq.(2.12) will play a passive role (of normalization), so it will be dropped from further consideration.

3 Basic Matrix Elements

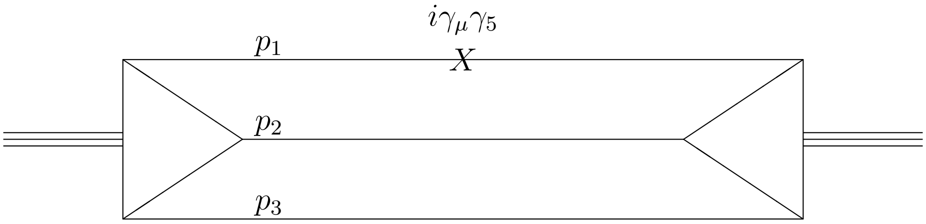

Figure 2: Schematic baryon spin diagram, with internal quark momenta p_1, p_2, p_3 ; basic spin operator $i\gamma_\mu\gamma_5$ is inserted in line p_1.

The baryonic matrix elements of a given operator A, are symbolically expressed by quantities like $< B_{1,2} \mid A \mid B_{1,2} >$, where $B_{1,2}$ are the two wave functions (56 and 20) of Eq.(2.9), whose full degrees of freedom (including spin and isospin) are

$$
\begin{aligned}
|B_1> &= \psi \cos\beta(\phi'\chi' + \phi''\chi'')/\sqrt{2} \\
|B_2> &= \psi \sin\beta f_{20}(\phi'\chi'' - \phi''\chi')/\sqrt{2}; \\
f_{20} &= i\frac{\sqrt{2}\Sigma.\eta \times \xi}{\rho} \qquad (3.1)
\end{aligned}
$$

the last line representing the angular effect of the 20 state. For a given form of A, after evaluating the isospin part of the matrix element, the spin part involves overlaps like $A' =< \chi'|A|\chi' >$ and $A'' =< \chi''|A|\chi'' >$ for the 'direct' [56 $\rightarrow$ 56 and 20 $\rightarrow$ 20] transitions, while the mixed transitions involve the overlap $A_m =< \chi'|A|\chi'' >$, etc. Using the Dirac forms of the spin functions as given by (2.5), such overlaps correspond to loop integrals like in fig.2. A typical matrix element with $A = 1$(corresponding to a normalization integral) is of the form

$$N' = N'_{1;23} + 5 cyclicperms$$
$$N'' = N''_{1;23} + 5 cyclicperms (3.2)$$

$$N'_{1;23} = \bar{u}(P) S_F(p_1)\{1\} S_F(p_1)\gamma_5 C$$
$$S_F(p_2) C^{-1}\gamma_5 P_E$$

$$S_F(p_3)u(P)/2 \qquad (3.3)$$

Similarly for $N''_{1;23}$. Same pattern for other physical quantities : electric charge ; spin $i\gamma_\mu\gamma_5$

$$1 \Rightarrow ei\gamma_\mu[1/6+\tau_3/2] \qquad (3.4)$$

3.1 Spin Matrix Elements in Lowest Order

The spin matrix elements g_i in the notation of [2] are obtained with $A = i\gamma_\mu\gamma_5\lambda_i$ within this formalism, where the $\lambda' s$ are the Gellmann matrixes as given in Lichtenberg [11]. The 56-20 mixing affects g_3 but not g_8 and g_0. The results in lowest order are

$$g_3 = (5/3)\cos^2\beta - \sin^2\beta; \quad g_8 = g_0 = 1/\sqrt{3} \tag{3.5}$$

Note that only g_3 depends on β, and agrees with the observed value 1.248 for $\beta \approx 24^{\text{deg}}$. The other two quantities (at ≈ 0.58) agree with [2]. Of the latter, only g_0 is affected by the 2-gluon anomaly, but not g_8.

4 Two-gluon Anomaly Operator

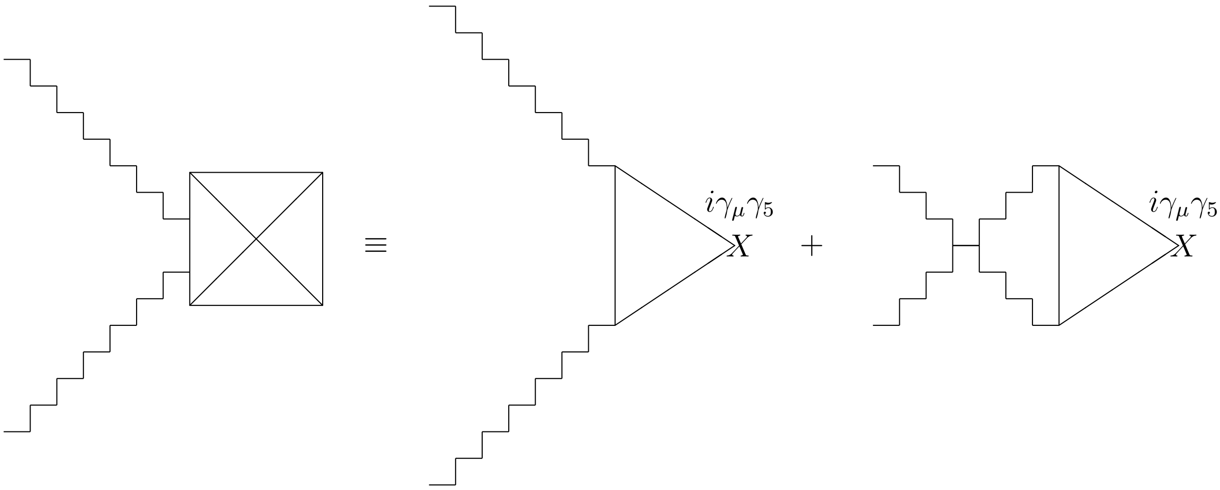

Figure 3: Two gluon operator (crossed box) representing a sum of two distinct diagrams for axial vector coupling

The 2-gluon anomaly operator $\Delta_{\mu\nu\lambda}$ 'crossed box' as a sum of two triangle diagrams, is

$$\Delta_{\mu\nu\lambda}(k) = \frac{ig_s^2}{(2\pi)^4} Tr[\int d^4 q i\gamma_\nu S_F(q+k_1)$$

$$i\gamma_\mu\gamma_5 S_F(q+k_2) i\gamma_\lambda S_F(q)]$$
$$+conj \tag{4.1}$$

Gauge Invariance duly incorporated. Result

$$\Delta_{\mu\nu\lambda} \approx \alpha_s \epsilon_{\mu\nu\lambda\sigma} k_\sigma; \tag{4.2}$$

4.1 Spin corrections from 2-gluon Anomaly

Now insert $\Delta_{\mu\nu\lambda}$ in the quark lines Fig. 3 – forward scattering amplitude for the baryon.

Two ways for insertion : 1) self-energy-like in line p_1 (Fig.3a); Designate these as Σ' , Σ'' 2) exchange -like connecting p_1 and p_2 (Fig.3b). Designate these as V', V'' respectively. And add their allowed perms as before.

$$\begin{aligned}\Sigma' = \ & \frac{2g_s^2}{3(2\pi)^4} \int d^4k \bar{u}(P) P_s S_F(p_1) \\ & \Delta_{\mu\nu\lambda} i\gamma_\nu S_F(p_1 - k) i\gamma_\lambda D^2(k) S_F(p_1) P_E \\ & \frac{1}{2}\gamma_5 C S_F(-p_2) C^{-1} \gamma_5 P_E S_F(p_3) \\ & P_s u(P) + conj \end{aligned} \tag{4.3}$$

A similar form exists for Σ''. For the exchange type insertions, V':

$$V' = \frac{2g_s^2}{3(2\pi)^4} \int d^4k \bar{u}(P) P_s S_F(p_1) \frac{P_E \gamma_5}{2} \Delta_{\mu\nu\lambda} S_F(-p_2 + k) \gamma_\nu D(k) S_F(-p_2) \gamma_5 P_E S_F(p_3) \gamma_\lambda D(k) S_F(p_3 + k) P_s u(P) + conj \quad (4.4)$$

Similar result for V''. These quantities, when integrated over $\int d\tau$, and divided by the normalizer N, qualify directly as 2-gluon anomaly corrections (same relative normalization) to the spin matrix element $g_A^{(0)}$. The process involves:

(a) integration over d^4k;

(b) integration over $d\tau$, as in above.

The (logarithmic) divergence of the k-integration is removed by dimensional regularization of t′Hooft and Veltmann. The result for the fractional correction to $g_A^{(0)}$ may be expressed in the form

$$\delta g_A = \theta [\frac{\alpha_s}{\pi}]^2 g_A^{(0)}. \qquad (4.5)$$

And a very crude estimate yields

$$\theta \sim -2.0$$

which with $\alpha \approx 0.39$, amounts to a small correction, albeit of the right sign.

5 Summary and Conclusion

To conclude, we have presented a first application of a new form of dynamics within the framework of QCD in the high momentum limit, viz., the role of a direct qqq force which has been shown to produce an additional singularity in the structure of the qqq wave fn ϕ. This application indicates the possibility of addressing the details of the proton structure function, including spin anomaly, through the inclusion of internal dynamics.

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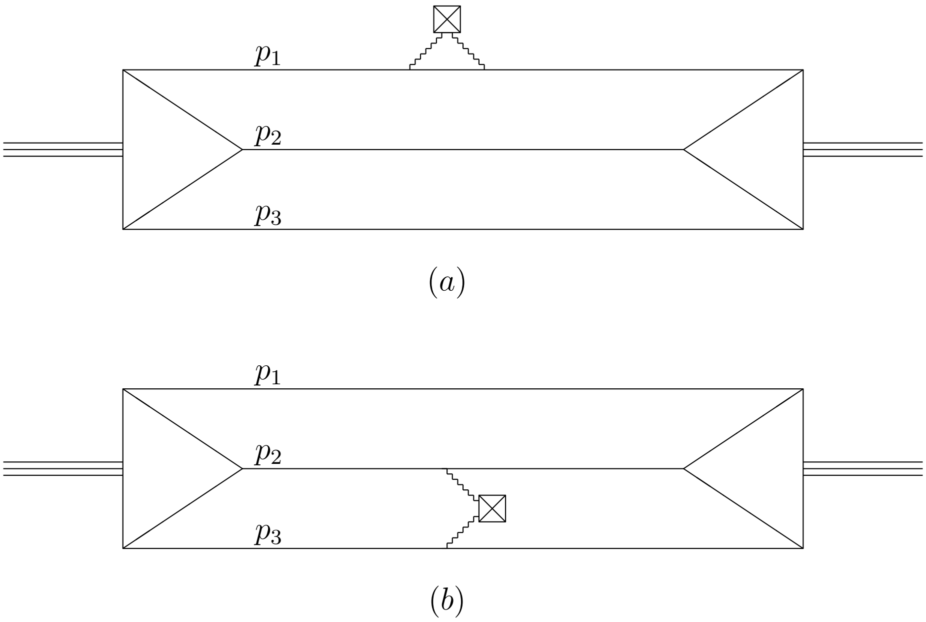

Figure 4: Two-gluon operator, fig (2), inserted in the internal quark lines of the baryon: (a) 'self-energy' like insertion in line p_1; (b) 'exchange-like' insertion connecting lines p_2 and p_3