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Theory of cross-section and polarization effects in A(d,p)B reactions

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Outline

- A(d,p)B reactions with exotic beams

  Why are these reactions interesting?

  Rich source of a range of nuclear properties:

  Nuclear Structure and V_{np}
- Nuclear Reaction Theory

  Few-body models.

  Faddeev methods. How are these linked?

  DWBA method

  ADW method
- Polarized Deuterons

  New information: nuclear structure and V_{np}.

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Why study (d,p) reactions? Nuclear structure aspects.

d+A ⟶ p+B   B=A+n

(i) Angular distribution of p gives orbital angular momentum, $l_{n,}$, of neutron orbit in B. Polarization may give j_n.

(ii) Cross-section magnitude reveals single-particle nature of neutron state in B.

(iii) Gives insight into the way single particle strength is distributed among states of a given nucleus and over the periodic table.

(iv) Modern studies seek to answer these questions for exotic nuclei near the drip lines. This may involve experiments in inverse kinematics.

(v) It is essential that we have credible reaction theories if reliable nuclear structure information is to result.

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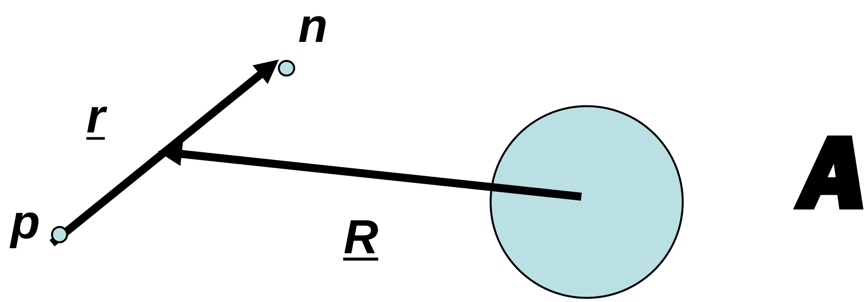
Three-body model



$$
\begin{aligned}
H &= T_R + H_{np} + V(\vec{R}, \vec{r}), \\
V(\vec{R}, \vec{r}) &\equiv V_{nA}(\vec{R} + \vec{r}/2) + V_{pA}(\vec{R} - \vec{r}/2)
\end{aligned}
$$

$$H_{np} = K_r + V_{np}$$

$$H_{np}\phi_d(\vec{r}) = -\epsilon_d \phi_d(\vec{r}), \qquad \epsilon_d = 2.225\,\mathrm{MeV}$$

$$H_{np}\phi^{(+)}_{\vec{k}}(\vec{r}) = \epsilon_k \phi^{(+)}_{\vec{k}}(\vec{r}), \quad 0 < \epsilon_k < \infty,$$

$$\phi_d(\vec{r}) \rightarrow \exp(-\gamma r)/r, \quad r \rightarrow \infty, \quad \gamma = 0.236\ \mathrm{fm}^{-1}$$

Connection between transition operators of Alt, et al., and Timofeyuk and Johnson

Alt, et al., NPB2(1967)167 :

$$U_{pd} = G_0^{-1} + V_{pA} + (V_{pA} + V_{pn})G(V_{pA} + V_{nA})$$

Equivalent integral equation:

$$U_{pd} = \bar{U}_{pd}(1 - G_0 V_{np}) + \bar{U}_{pd} G_0 V_{np} G_p U_{pd}.$$

Johnson, PRC80(2009)044616

Definition :

$$\bar{U}_{pd} = G_0^{-1} + V_{pA} + V_{pA}\bar{G}(V_{pA} + V_{nA})$$

Alternative:

$$\bar{U}_{pd} G_0 = 1 + V_{pA}\bar{G}$$

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On Shell reduction

$$(1 - G_0 V_{np}) \mid \phi_{\vec{k}_d,\, d}\rangle = 0.$$

Three-body scattering wavefunction:

$$G_p U_{pd} \left| \phi_{\vec{k}_d, d} \right\rangle = \left| \Psi^{(+)}_{\vec{k}_d, d} \right\rangle$$

(d,p) transition matrix:

$$\begin{aligned} \left\langle \phi_{\vec{k}_p, B} \middle| U_{pd} \middle| \phi_{\vec{k}_d, d} \right\rangle &= \left\langle \phi_{\vec{k}_p, B} \middle| \bar{U}_{pd} G_0 V_{np} G_p U_{pd} \middle| \phi_{\vec{k}_d, d} \right\rangle \\ &= \left\langle \bar{\Psi}^{(-)}_{\vec{k}_p, B} \middle| V_{np} \middle| \Psi^{(+)}_{\vec{k}_d, d} \right\rangle, \end{aligned}$$

$$(E - K - V_{nA} - V^{*}_{pA}) \bar{\Psi}^{(-)}_{\vec{k}_p, B} = 0.$$

Timofeyuk and Johnson, PRC59(1999)1545

Limit $m_n/m_A=0$

$$
\begin{aligned}
&\langle \phi_{\vec{k}_p,B} | \bar{U}_{pd} G_0 \\
&= \langle \phi_{\vec{k}_p,B} | \left[1 + \left(V_{pA} \frac{1}{E^+ - K_{nA} - V_{nA} - K_{pA} - V_{pA}} \right) \right] \\
&= \langle \phi_{\vec{k}_p,B} | \left[1 + \left(V_{pA} \frac{1}{E_p^+ - K_{pA} - V_{pA}} \right) \right] \\
&= \langle \chi^{(-)}_{\vec{k}_p} \phi_B |, \qquad (7)
\end{aligned}
$$

Proton distorted wave:

$$(E - K_{pA} - V^*_{pA})\chi^{(-)}_{\vec{k}_p}(\vec{r}_{pA}) = 0,$$

Neutron bound state:

$$(-\epsilon_B - K_{nA} - V_{nA})\,\phi_B(\vec{r}_{nA}) = 0$$

Many-body generalization

(i) Exact many-body:

$$T_{AB}(d,p) = \langle \Psi^{(-)}_{pB} \mid \; V_{np} \; \mid \Psi^{(+)}_{dA} \rangle$$

(ii) Direct reaction assumptions:

$$\Psi^{(+)}_{dA} \; \rightarrow \; \chi^{(+)}_{d}(n,p)\Phi_A(1...A)$$

$$\Psi^{(-)}_{pB} \; \rightarrow \; \chi^{(-)}_{p}(p)\Phi_B(1...A, N+1)$$

$$T_{AB}(d,p) \; \rightarrow \; \langle \chi^{(-)}_{p}(p)\Phi_B(1...A,N+1) \mid \; V_{np} \; \mid \chi^{(+)}_{d}(n,p)\Phi_A(1...A)\rangle$$

$$T_{AB}(d,p) \; \rightarrow \; (\sqrt{N+1})^{-1}\langle \chi^{(-)}_{p}(p)\phi_{BA}(r) \mid \; V_{np} \; \mid \chi^{(+)}_{d}(n,p)\rangle$$

Overlap Functions-the link with nuclear structure

Definition of Overlap Function for A(d,p)B:

$$\phi_{BA}(r) = \sqrt{N+1} \int d1..dA \; \phi_A^*(1..A) \; \phi_B(1..A, r_{N+1} = r)$$

Definition of Spectroscopic Factor: $S_{BA} = \int dr \mid \phi_{BA}(r) \mid^2$

Approximation: $\phi_{BA}(r) = \sqrt{S_{BA}} \phi^{HF}(r)$

Definition of ANC:

$$\phi_{BA}(r) \;\; \to \;\; C_{BA} \exp(-\lambda r)/r, \;\; r \to \infty.$$

$$\lambda \;\; = \;\; \sqrt{2mB_n/\hbar^2}.$$

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Inhomogeneous Equation for the Overlap Function.

Timofeyuk, PRL 103(2009)242501, PRC81(2010)064306

$$(S_{BA} + T_r)\langle \phi_A \mid \phi_B \rangle = \langle \phi_A \mid (V_A - V_B) \mid \phi_B \rangle$$

$$\langle \phi_A \mid \phi_B \rangle = (S_{BA} + T_{r_{nA}})^{-1} \langle \phi_A \mid (V_A - V_B) \mid \phi_B \rangle$$

(i) S_{BA} is the neutron separation energy.
(ii) $V_B - V_A$ is the interaction between the last neutron in B and the nucleons in A.

$$\phi_{BA}(\vec{r}_{nA}) \rightarrow C_{BA} \exp(-\lambda_{BA} r_{nA})/r_{nA}$$

$$\lambda_{BA} = \sqrt{\frac{2\mu_{nA} S_{nA}}{\hbar^2}}$$

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The DWBA

$$\chi_d^{(+)}(n,p) \rightarrow (\chi_d^{(+)})_{\rm elastic}(\frac{n+p}{2})\phi_d(n-p)$$

Deuteron Optical potential

Deuteron Ground State

Beyond the DWBA.
3-body models

Faddeev equations give $\chi_d^{(+)}(n,p)$

Fonseca, Deltuva, Cravo, Crespo(Lisbon), Moro(Sevilla), Nunes (MSU)

Effective 3-body Hamiltonian.

$$H_{eff} = T_R + H_{np} + \langle \phi_A \mid U \mid \phi_A \rangle$$

$$U = (v_{nA} + v_{pA}) + (v_{nA} + v_{pA}) \frac{Q_A}{e} U$$

$$v_{NA} = \sum_{i=1}^{A} v(N,i)$$

$$U = (U_{nA} + U_{pA}) + U_{nA} \frac{Q_A}{e} U_{pA} + U_{pA} \frac{Q_A}{e} U_{nA} + ..$$

$$U_{nA} = v_{nA} + v_{nA} \frac{Q_A}{e} U_{nA}, \quad U_{pA} = v_{pA} + v_{pA} \frac{Q_A}{e} U_{pA}$$

Link to n and p optical potentials

$$U_{nA} = v_{nA} + v_{nA}\frac{Q_A}{e}U_{nA}$$

where

$$e = (E^+ - T_R - H_{np} - H_A)$$

Approximation:

$$e \approx (1/2E_d^+ - T_n - H_A)$$

$$U_n \approx \nu_{nA} + \nu_{nA}\frac{Q_A}{(1/2E_d^+ - T_n - H_A)}v_{nA} \ldots$$

$$= U_{nA}^{opt}(1/2E_d^+)$$

$$V_{nA}^{opt}(r_n, 1/2E_d) = \langle \phi_A \mid U_{nA}^{opt}(1/2E_d^+) \mid \phi_A \rangle$$

Effective 3-body Hamiltonian.

1.Multiple scattering corrections

$$H = T_R + T_r + V_{np}(r) \quad + \quad V_{nA}^{opt}(r_n, 1/2E_d) + V_{pA}^{opt}(r_p, 1/2E_d)$$
$$+ \quad \langle \phi_A \mid U_{nA} \frac{Q_A}{e} U_{pA} \mid \phi_A \rangle + (n \leftrightarrow p) + \ldots$$

High energy: corrections suppressed by the weak correlation of n and p in the deuteron

Effective 3-body Hamiltonian.

1.Multiple scattering corrections

$$H = T_R + T_r + V_{np}(r) \quad + \quad V_{nA}^{opt}(r_n, 1/2E_d) + V_{pA}^{opt}(r_p, 1/2E_d)$$
$$\quad + \quad \langle \phi_A \mid U_{nA} \frac{Q_A}{e} U_{pA} \mid \phi_A \rangle + (n \leftrightarrow p) + \ldots$$

2. Pauli blocking. The Bethe-Goldstone equation.

$$H \quad \rightarrow \quad T_R + T_r + Q_F V_{np}(r) + V_{nA}^{opt} + V_{pA}^{opt}$$
$$Q_F \mid n, p \rangle \quad = \quad 0, \quad \text{if } \epsilon_n \text{ or } \epsilon_p < \epsilon_F .$$

Ioannides and Johnson Phys. Rev. C17 (1978)1331.
Binding energy of a d propagating in nuclear matter.

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Conclusion from many-body considerations.

The usual 3-body model Hamiltonian ignores many-body effects whose importance is not understood quantitatively, especially at low energies. The validity of the model requires that break-up into states of high relative n-p energy (>20MeV) must not play an important role in the 3-body scattering wavefunction.

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What can we learn from Faddeev solutions of 3-body models?

- Important developments in the study of **3-body models** of nuclear reactions using **the Faddeev equations**

(e.g.,Alt,et al, PRC75(2007)05403, Deltuva, et al, PRC76(2007)064602).

- Need to connect **3-body models and real laboratory experiments**.

(e.g., Catford, NPA701(2002)1), Thomas, et al, PRC76(2007)044302, Lee,al,PRC73(2006)044608.).

- We cannot estimate spectroscopic factors by simply taking ratio of experimental cross-sections and Faddeev predictions!

- We use Faddeev equations to understand the validity of approximate evaluations of the (d,p) transition matrix.

- How well do CDCC calculations represent the 3-body dynamics when used in an appropriate matrix expression for the (d,p) transition matrix? (Moro, Nunes and Johnson, 2009)

Approximate 3-body models

$$T_{AB}(d,p) \rightarrow (\sqrt{N+1})^{-1}\langle\chi_p^{(-)}(p)\phi_{BA}(r)\mid V_{np} \mid \chi_d^{(+)}(n,p)\rangle$$

$\chi_d^{(+)}(n,p)$ is needed for r_{np} inside range of V_{np} only.

$$V_{np}\,\chi_d^{(+)}(\vec{r},\vec{R}) \longrightarrow V_{np}\phi_0(\vec{r})\;\tilde{\chi}_d^{(+)}(\vec{R})$$

Zero range V_{np}:

$$(K_R + V_{pA}(R) + V_{nA}(R))\tilde{\chi}_d^{(+)}(\vec{R}) = E_d\,\tilde{\chi}_d^{(+)}(\vec{R})$$

N.B. $\tilde{\chi}_d^{(+)}(\vec{R}) \neq (\chi_d^{(+)})_{\text{elastic}}(\vec{R})$

Johnson and Soper (1970)

Separable V_{np}

$$V_{np} = -\sum_{SJM} | f_{SJM}\rangle\langle f_{SJM} |$$

Exact:

$$V_{np} | \phi_{k,\,11M}\rangle = V_{np} | \phi_{d,M}\rangle \times g(k)$$

Where:

$$g(k) = \frac{\langle f_{11M} | \phi_{k,\,11M}\rangle}{\langle f_{11M} | \phi_{d,M}\rangle}$$

The 3-body Wave function inside the range of V_{np}

Johnson and Soper,PRC1(1970)976, Johnson, 2nd RIA Workshop 2005 (AIP Conf Proc 791)

$$\chi_d^{(+)}(\vec{r},\vec{R}) = (\chi_d^{(+)}(\vec{R}))_{\mathrm{elastic}}\phi_d(\vec{r}) + \int d\vec{k}\ (\chi_{\vec{k}}(\vec{R}))_{\mathrm{b-up}}\phi_{\vec{k}}(\vec{r})$$

$$V_{np}\phi_{\vec{k}}(\vec{r}) \approx g(k)V_{np}\phi_d(\vec{r}) \qquad r < \mathrm{range}\ V_{np}.$$

$$\begin{aligned} V_{np}\chi_d^{(+)}(\vec{r},\vec{R}) &= V_{np}\phi_d(\vec{r})[(\chi_d^{(+)}(\vec{R}))_{\mathrm{elastic}} \\ &+ \int d\vec{k}\ g(k)(\chi_{\vec{k}}(\vec{R}))_{\mathrm{b-up}}] \\ &= V_{np}\phi_d(\vec{r})[\tilde{\chi}_d^{(+)}(\vec{R})] \end{aligned}$$

$$\begin{aligned} \tilde{\chi}_d^{(+)}(\vec{R}) &= (\chi_d^{(+)}(\vec{R}))_{\mathrm{elastic}} \\ &+ \int d\vec{k}\ g(k)(\chi_{\vec{k}}(\vec{R}))_{\mathrm{b-up}} \end{aligned}$$

Application to (d,p)

Johnson and Soper PRC1(1970)976); Harvey and Johnson, PRC3(1971)636; Wales and Johnson, NPA274(1976))168.

$$V_{np}\Psi^{ad}_{\vec{K}_d}(\vec{R},\vec{r}) = V_{np}\phi_0(\vec{r})\chi^{ad(+)}_{\vec{K}_d}(\vec{R},\vec{r}) = V_{np}\phi_0(\vec{r})\chi^{ad(+)}_{\vec{K}_d}(\vec{R},0)$$

$$T_{d,p} = \int d\vec{R}\chi_p^{(-)*}(\vec{R})\phi_n^*(\vec{R})[V_{np}\phi_0(\vec{r})]\chi^{ad(+)}_{\vec{K}_d}(\vec{R},0)$$

$$(T_R + V(\vec{R},0) - E_d)\chi^{ad(+)}_{\vec{K}_d}(\vec{R},0) = 0$$

V(R,0) = $V_{nA}(R) + V_{pA}(R)$

N.B. $V(R,0)$ not designed to fit elastic deuteron scattering. Designed to generate amplitude for finding n and p at R with r less than range of V_{np} and not necessarily in the form of a deuteron in its ground state.

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The CDCC Method

$$\Psi^{(+)}_{\vec{K}}(\vec{r},\vec{R}) = \sum_s \psi_s(\vec{r})\chi_s(\vec{R})$$

$$\langle \psi_s \mid H_{np} \mid \psi_{s'} \rangle = \epsilon_s \delta_{s,s'}$$

$$(E_d - \epsilon_s - T_R)\chi_s(\vec{R}) = \sum_{s'} \langle \psi_s \mid (V_{nA} + V_{pA}) \mid \psi_{s'} \rangle \chi_{s'}(\vec{R})$$

$$\Psi^{(+)}_{\vec{K}}(\vec{r},\vec{R}) \stackrel{\epsilon_s \rightarrow 0}{\rightarrow} \Psi^{\rm ad}_{\vec{K}}(\vec{r},\vec{R})$$

Austern,et al, Phys.Rep.154(1987)125.

Al-Khalili and Tostevin, in "Scattering" Academic Press 2002, p1373.

Comparison with Faddeev calculations:
Deltuva, Moro, Cravo, Nunes and Fonseca, PRC76(2007)064602
Alt, Blokhintsev, Mukhamedzhanov and Sattarov, PRC75(02007)054003.

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The Sturmian Method of Johnson and Tandy

Johnson and Tandy NuclPhys A235(1974)56

$$\Psi^{(+)}_{\vec{K}}(\vec{r},\vec{R}) = \sum_{0}^{\infty} \bar{\phi}_i(\vec{r})\bar{\chi}_i(\vec{R})$$

$$(T_r + \alpha_i V_{np})\bar{\phi}_i = -\epsilon_0 \bar{\phi}_i \qquad \langle \bar{\phi}_i \mid V_{np} \mid \bar{\phi}_j \rangle = -\delta_{i,j}$$

$$V_{np}\Psi^{(+)}_{\vec{K}}(\vec{r},\vec{R}) \approx V_{np}\bar{\phi}_0(\vec{r})\bar{\chi}_0(\vec{R})$$

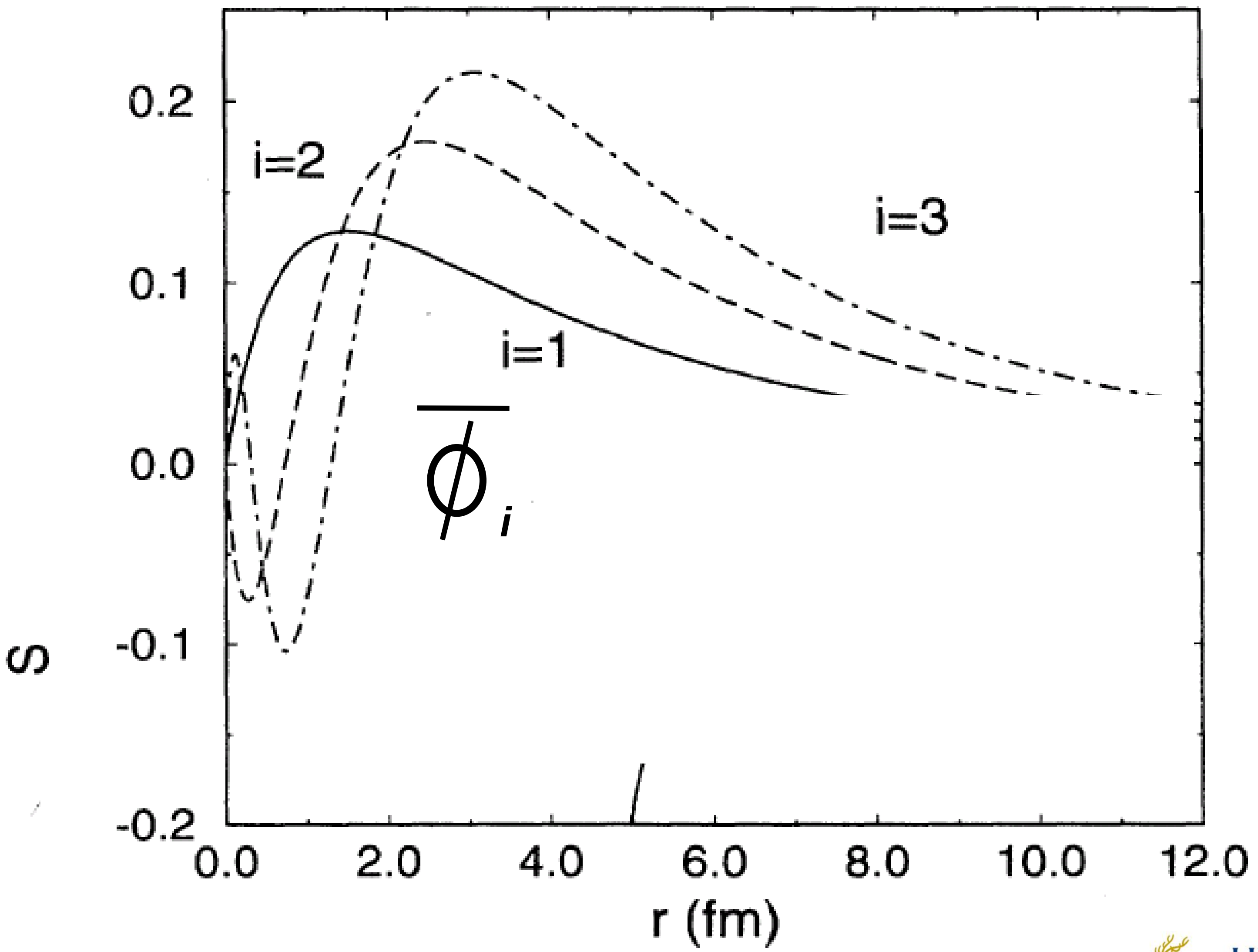
$$\bar{V}(R) = \frac{\langle \phi_0 \mid V_{np}(V_{nA} + V_{pA}) \mid \phi_0 \rangle}{\langle \phi_0 \mid V_{np} \mid \phi_0 \rangle}$$

Finite range version of Johnson-Soper:

$$(T_R + \bar{V}(\vec{R}) - E_d)\bar{\chi}_0(\vec{R}) = 0$$

Zero range V_{np}: $$\bar{V}(R) \rightarrow (V_{nA}(\vec{R}) + V_{pA}(\vec{R}))$$

Sturmian States for V_{np} (Hulthen)



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Laid, Tostevin and Johnson, PRC48(1993)1307

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Adiabatic Distorted Wave model

$$T_{AB}(d,p) \rightarrow (\sqrt{N+1})^{-1} \langle \chi_p^{(-)}(p)\phi_{BA}(r) \mid V_{np} \mid \chi_d^{(+)}(n,p)\rangle$$

$$V_{np}\,\chi_d^{(+)}(\vec{r},\vec{R}) \longrightarrow V_{np}\phi_0(\vec{r})\ \tilde{\chi}_d^{(+)}(\vec{R})$$

$$\bar{V}(R) = \frac{\langle \phi_0 \mid V_{np}(V_{nA}+V_{pA}) \mid \phi_0\rangle}{\langle \phi_0 \mid V_{np} \mid \phi_0\rangle}$$

Some applications of the ADW model to (d,p) and (p,d) reactions.

1 Cadmus and Haeberli, Nucl Phys A327(1979)419; A349(1980)103. E_d=12.9Mev, Sn target. DWBA fails. ADW works.

2. Liu, et al, Phys.Rev.C69(2004)064313 E_d=12-60MeV, ^{12}C(d,p)^{13}C and ^{13}C(p,d)^{12}C. Consistent spectroscopic factors with ADW.

5. Lee,et al, Phys.Rev.C73(2006)064320. Links with Hartree-Fock densities

3.Jones, et al., Nature, 465(2010)454. Magic nature of ^{132}Sn with ^{132}Sn(d,p)^{133}Sn. DWBA used!

4. Catford, et al, Phys.Rev.Lett. 104(2010)192501 Study of nuclear shell gaps through ^{24}Ne(d,p)^{25}Ne.

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Application of ADW method

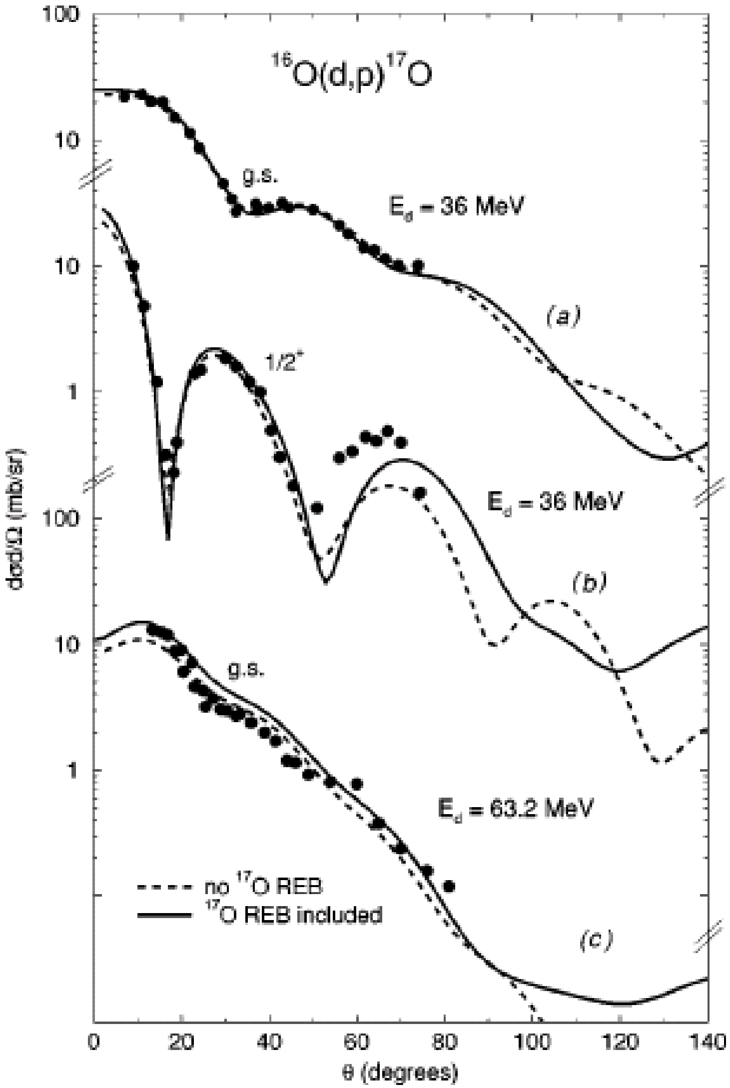

FIG. 2. (a) $^{16}O(d,p)^{17}O$(g.s.) at E_d=36 MeV, (b) $^{16}O(d,p)^{17}O(1/2^+)$ at E_d=36 MeV, and (c) $^{16}O(d,p)^{17}O$(g.s.) at E_d=63.2 MeV calculated with deuteron breakup taken into account. Solid lines denote REB and dashed, no-REB calculations.

Timofeyuk and Johnson
Phys.Rev.C 59(1999)1545

Data:Cooper, Hornyak and Roos,
Nucl.Phys.A218(1974)249

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Dependence of (d,p) crosssections on V_{np}

(i) Explicit in (d,p) matrix element

$$\langle \vec{k} \mid V_{np} \mid \phi_d \rangle = D_0 (1 - \frac{k^2}{\beta^2} \ldots).$$

$$D_0 = \int d\vec{r} V_{np} \phi_d,$$

$$\frac{1}{\beta^2} = \frac{1}{6} \frac{\int d\vec{r} r^2 V_{np} \phi_d}{\int d\vec{r} V_{np} \phi_d}.$$

(ii) In the distorted wave

Nguyen, Nunes and Johnson, PRCC82(2010)014611

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Deuteron Analyzing Powers

Cross-section for polarized d beam :

$$d\sigma/d\Omega = (d\sigma/d\Omega)_0[1 + \frac{3}{2}\vec{A}.\vec{P}_d + \text{2nd rank tensor terms}]$$

$$\vec{A} = \vec{n}\begin{cases} \frac{\ell}{3(\ell+1)}\frac{\langle\vec{\ell}.\vec{n}\rangle}{\ell} & \text{if } j = \ell + 1/2 \\ -\frac{1}{3}\frac{\langle\vec{\ell}.\vec{n}\rangle}{\ell} & \text{if } j = \ell - 1/2 \end{cases}$$

$$\langle\vec{\ell}.\vec{n}\rangle = \frac{\sum_m m \mid B_{\ell m} \mid^2}{\sum_m \mid B_{\ell m} \mid^2} \qquad \vec{n} = \vec{k}_d \times \vec{k}_p / \mid \vec{k}_d \times \vec{k}_p \mid$$

$$B_{\ell m} = \langle\bar{\Psi}^{(-)}_{\vec{k}_p}(p) u_{\ell j} Y_{\ell m}(n) \mid V_{np} \mid \phi_d \chi^{(+)}_{\vec{k}_d, d}\rangle$$

Nuclear Structure: J-dependence of Vector Analyzing Power

$$\frac{\vec{A}.\vec{n}(\theta)_{j=\ell+\frac{1}{2}}}{\vec{A}.\vec{n}(\theta)_{j=\ell-\frac{1}{2}}} = -\frac{\ell}{\ell+1}.$$

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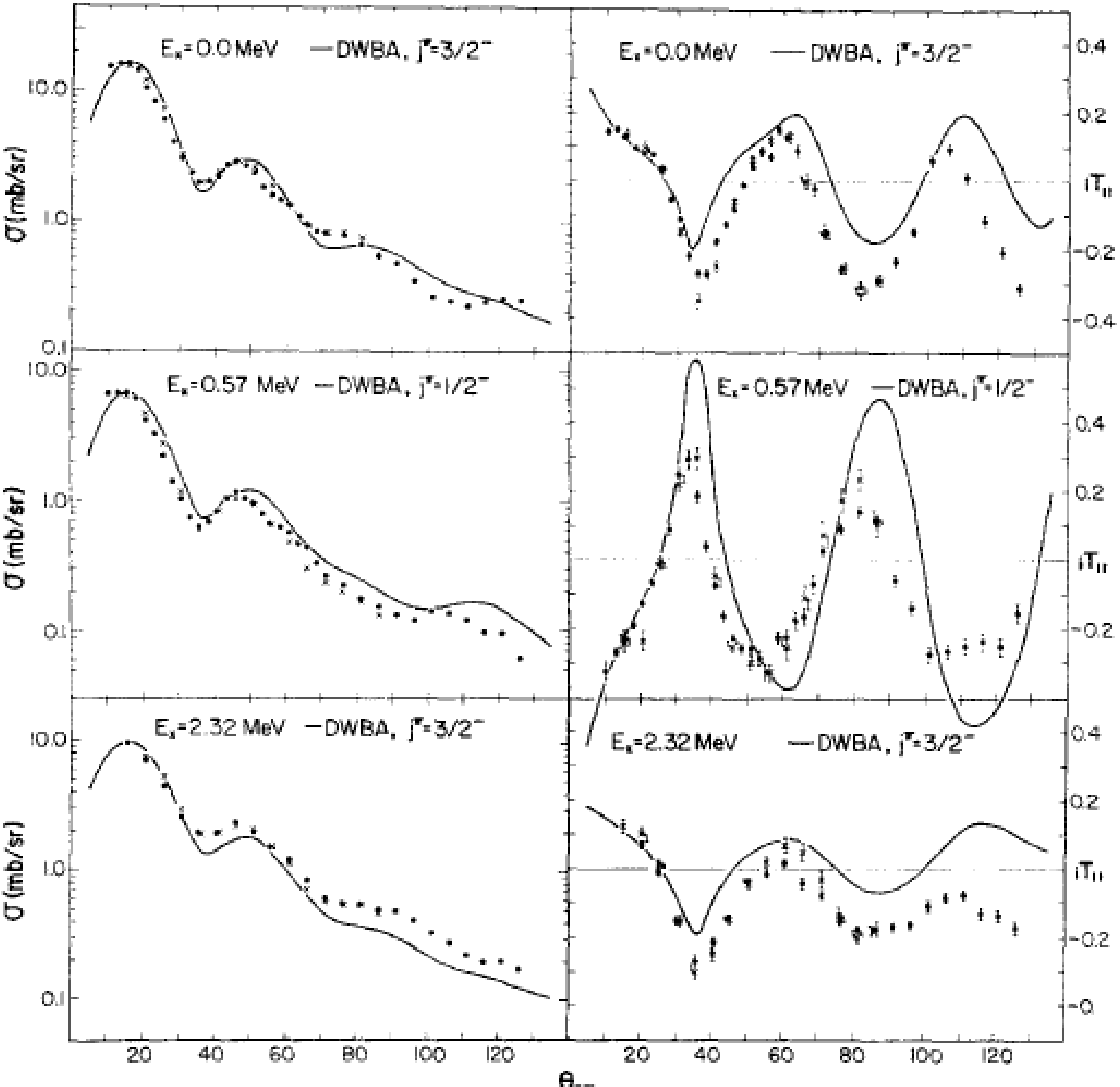
$^{52}Cr(d,p)^{53}Cr$ $E_d = 10$ MeV



Kocher and Haeberli Nucl.Phys.A196 (1972)225

$IT_{11} = 3^{1/2}/2 \; A_y$

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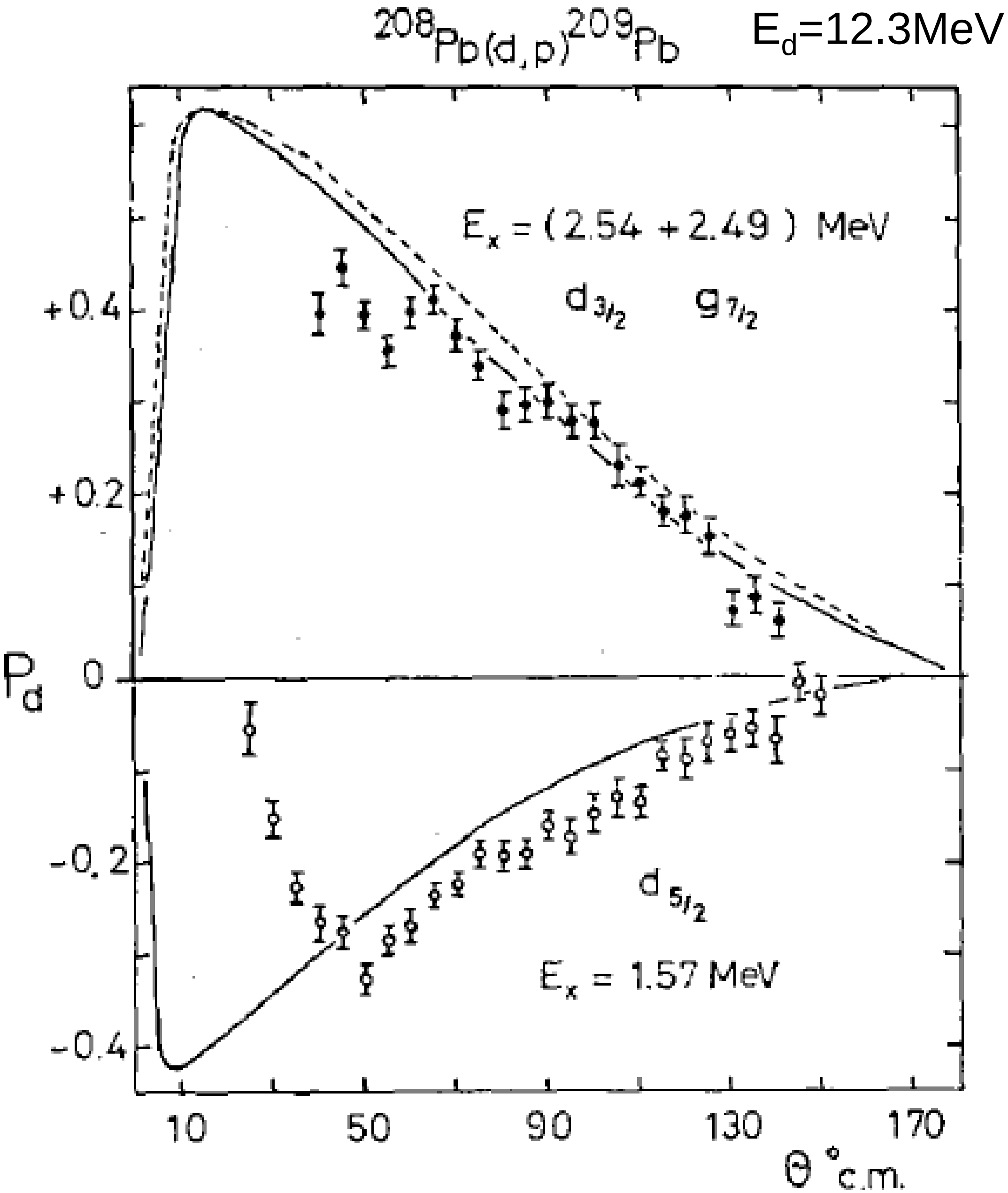
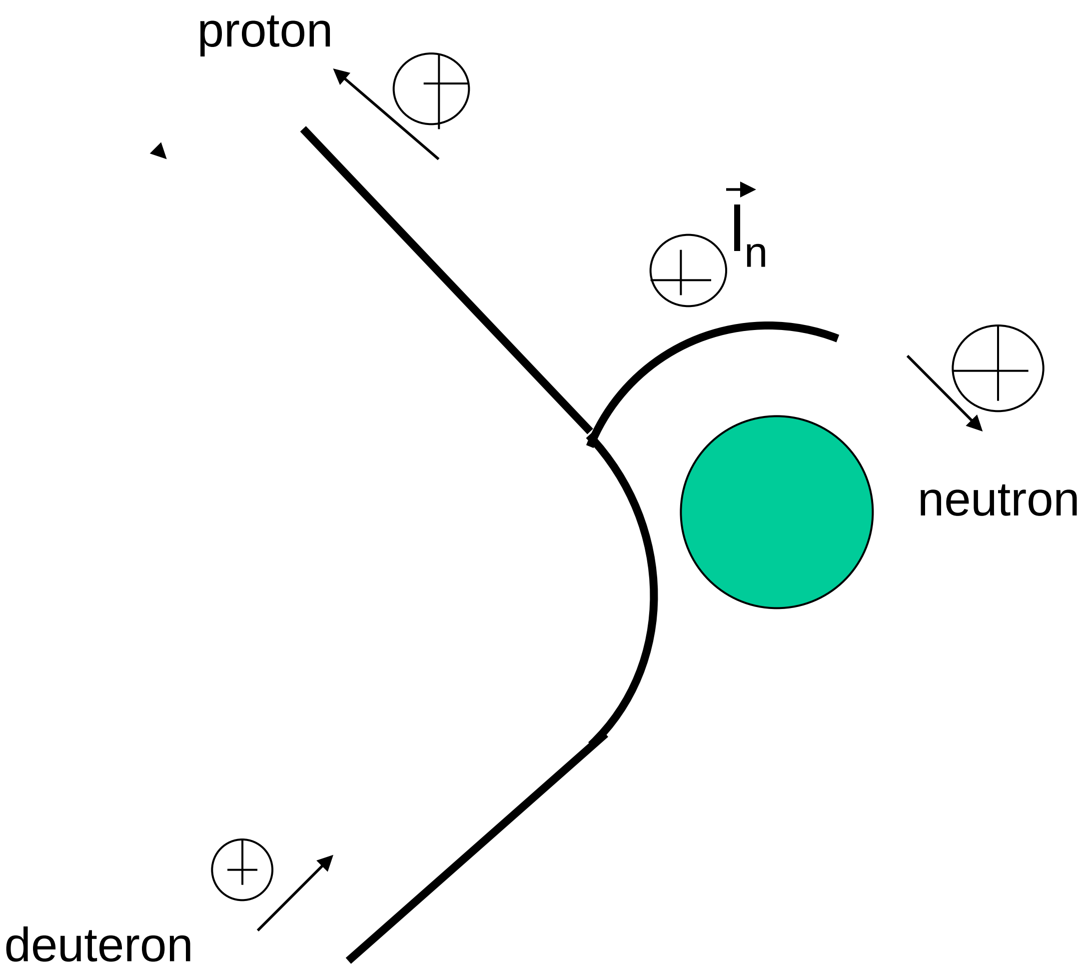
Sub-Coulomb Analyzing Power (Pd=A)



Debenham, Griffith,
Irshad and Roman,
Nucl.Phys.A151(1970)81

Sub-Coulomb (d,p) with $j_n = l_n + 1/2$



$$\vec{P}_{\mathbf{inc}} \cdot \vec{n} = -1$$

Tensor analyzing powers

$$T_{2\,q}(\theta),\ q=-2,-1,0,1,2.\quad T_{2\,-q}=(-1)^q\,T^*_{2\,q},$$

Completely aligned along beam direction:

$P_{M=+1} = P_{M=-1}$, $P_{M=0} = 0$.

$$d\sigma/d\Omega=(d\sigma/d\Omega)_0[1+\frac{1}{\sqrt{2}}T_{20}(\theta)]$$

Novel Information about V_{np}.

Plane wave limit (Johnson, Nucl.Phys.A90(1967)289)

$$T_{20} \;=\; -2\frac{\Delta + \sqrt{1/8}\Delta^2}{1+\Delta^2}$$

$$\Delta(K) \;=\; \frac{u_2(K)}{u_0(K)}, \quad \vec{K} = \frac{1}{2}\vec{k}_d - \vec{k}_p.$$

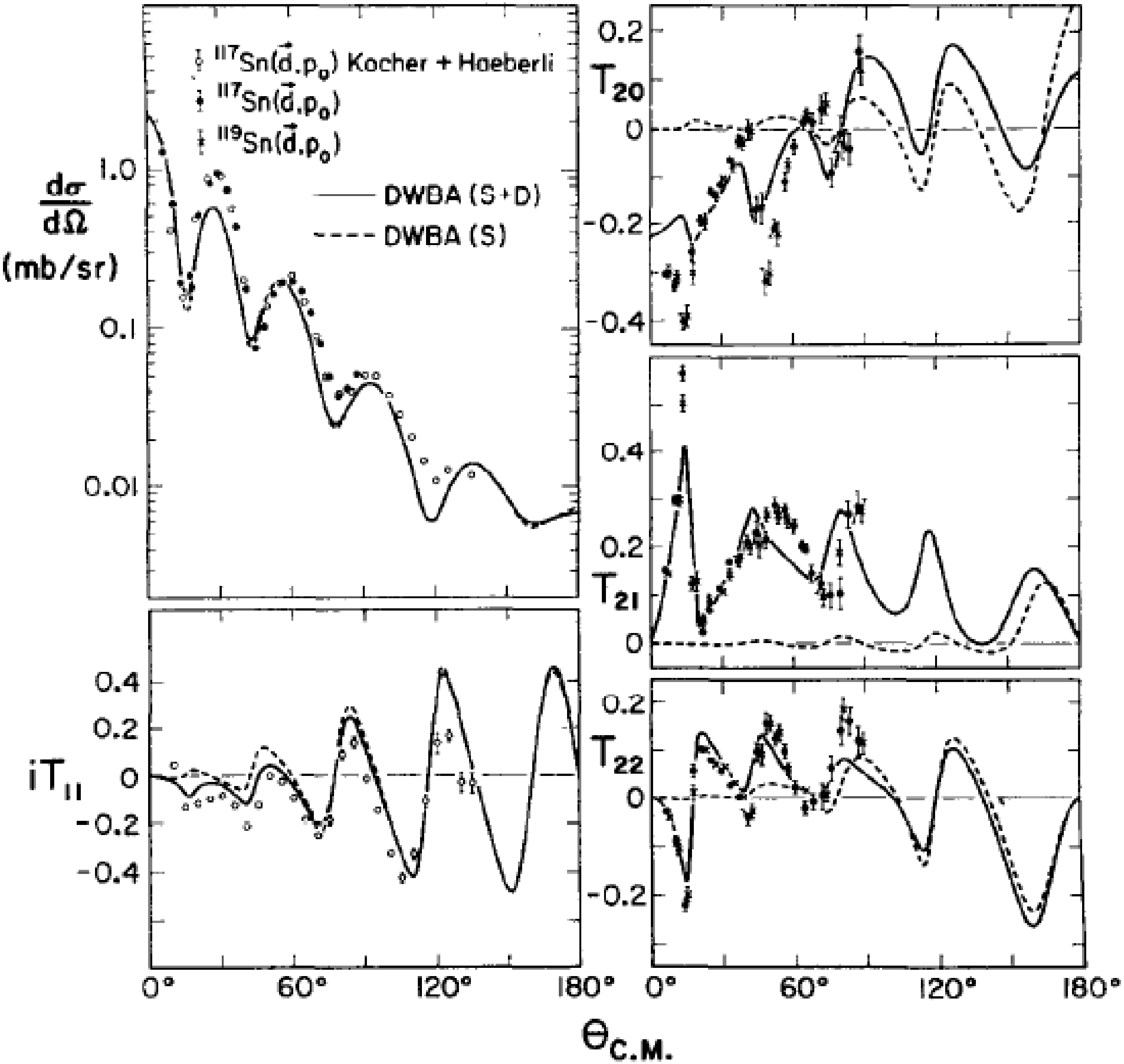
$$\Delta(K) = D_2\, K^2 + \ldots\text{for small } K.$$

$$D_2 = \sqrt{2}F \mid Q_0 \mid$$

F=1 to 1 percent

$$D_2 = \frac{\eta}{M\epsilon_d/\hbar^2}$$

$$u_2(r)/u_0(r) \stackrel{r\to\infty}{\longrightarrow} \eta$$



Knutson, Thomson, Meyer, Nucl.Phys. A241(1975)36

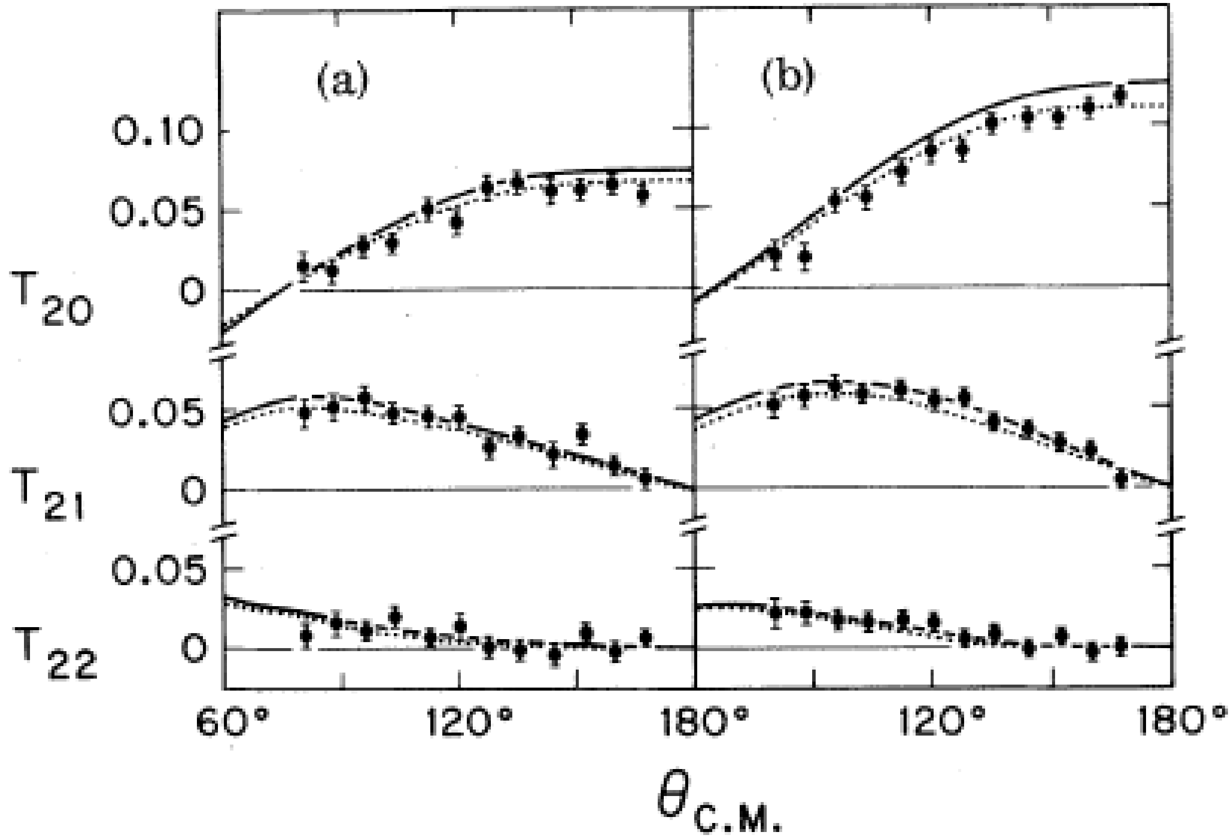


FIG. 2. Angular distributions of the tensor analyzing powers T_{20}, T_{21}, and T_{22} for the reaction $^{208}Pb(d,p)^{209}Pb$ at 9 MeV leading to states with (a) $J^\pi = \frac{1}{2}^+$, $E_x = 2.03$ MeV and (b) $J^\pi = \frac{5}{2}^+$, $E_x = 1.57$ MeV. The displayed errors are statistical only. The curves show the result of DWBA calculations which include the effects of the deuteron D state. The solid curves were obtained with $D_2 = 0.484$ fm^2. The dotted curves show the best fit to the measurements, which was obtained with $D_2 = 0.432$ fm^2.

Knutson, Haeberli
Phys.Rev.Lett.
35(1975)560

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Overview of ADW and adiabatic methods

1.ADW method gives a simple prescription for taking into account coherent , `entangled', effects of d break-up on (d,p) reactions.

2.Johnson and Soper sought to justify their assumptions (`adiabatic') by applying the adiabatic idea to 20 MeV elastic d scattering, with some success.

3. They were able to explain some outstanding discrepancies between old DWBA calculations and experiment. New theory could use existing codes and was simple to implement. Need for deuteron optical potentials disappeared.

4.Johnson and Tandy introduced a new approach which made clear that the adiabatic assumption was not a necessary condition for the validity of the J-S method for transfer. Implemented by Laid, Tostevin and Johnson.

5.For application to low energy the big question is `What is the correct 3-body Hamiltonian?'

6.Validity of adiabatic approximation for elastic scattering investigated by comparison with CDCC calculations. Generalised to 3-body projectiles.

Concluding Remarks

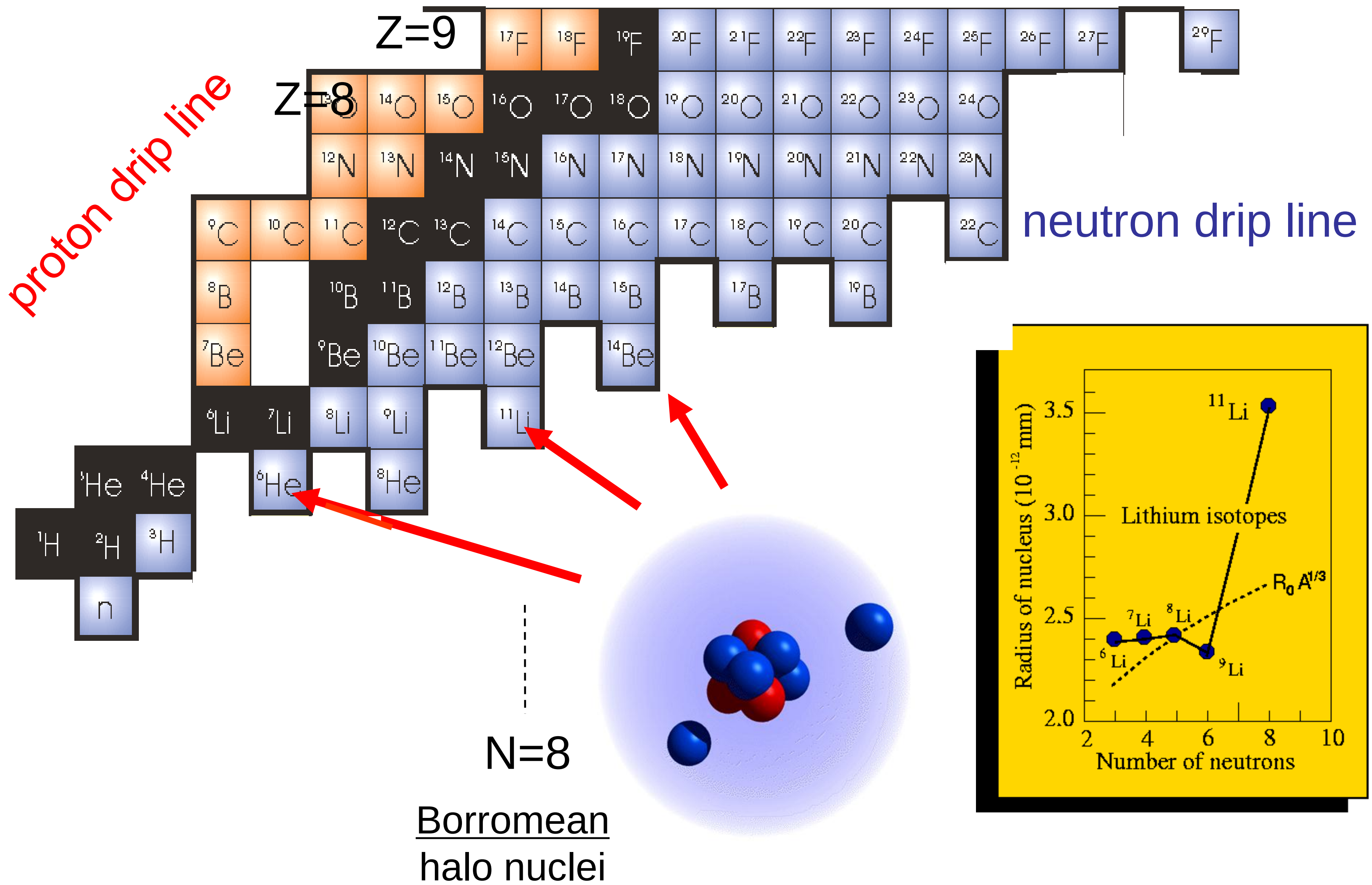
1. 3-body aspects of (d,p).
   DWBA does not work.
   3-body wavefunction needed within V_{np} only.
   Adiabatic approximation is sufficient but not necessary condition for the J-T distorting potential to be valid for transfer.
   Evidence for validity at low energy(4 MeV/A).
   Need to extend work of Laid et al PRC48(1993)1307 on Johnson-Tandy expansion to low energy.
   Validity of adiabatic approximation for elastic d scattering is a separate question. (See Summers, et al, PRC66(2002)014614).

2. Overlap functions.
What properties of this quantity are measured in any one (d,p) experiment?
See Nunes,et al, PRCC72(2005)017602, C75(2005)024601.

3. What is the effective 3-body Hamiltonian at low energy?
Multiple scattering effects?
Effective V_{np} ? Pauli blocking?

4. To deduce reliable nuclear structure information we need a co-ordinated programme of transfer and <u>relevant</u> nucleon elastic and inelastic scattering measurements, including polarization variables.

Part of the N-Z plane



Application of ADW method to $^{10}Be(d,p)^{11}Be(g.s.)$

Data: Auton,Nucl.Phys. A157(1970)305

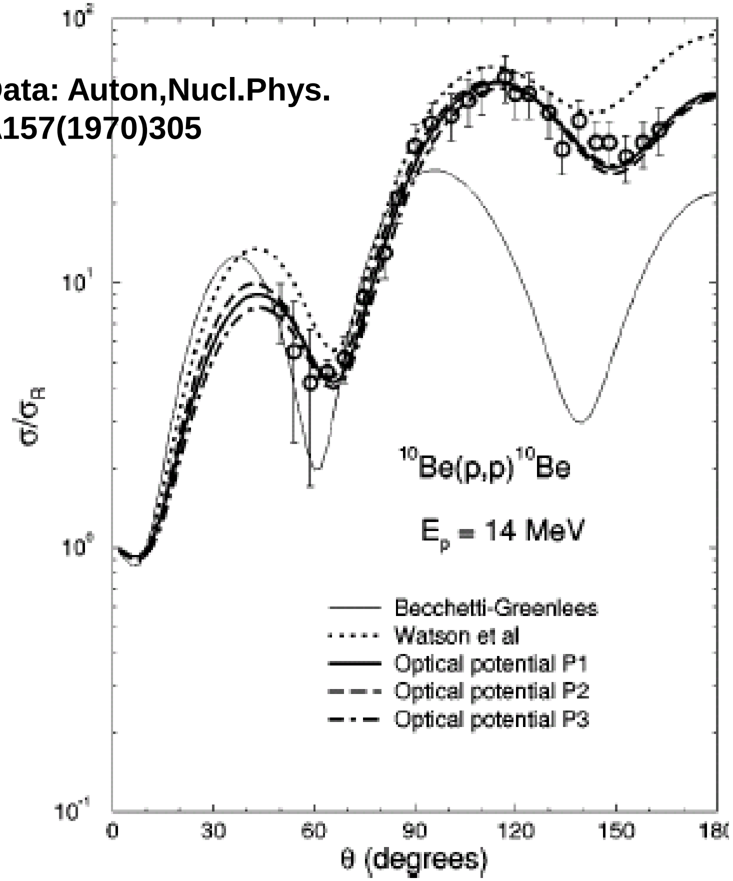


FIG. 3. Elastic scattering of the $p+{}^{10}$Be at $E_p=14$ MeV ca culated with different proton optical potentials.

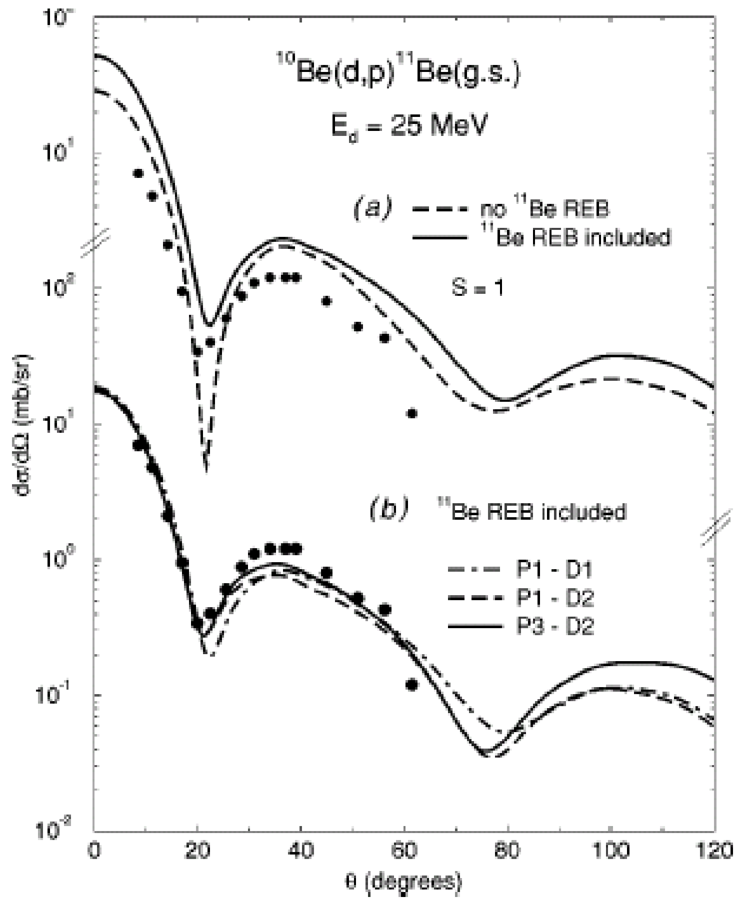


FIG. 4. Cross sections of the ^{10}Be$(d,p)^{11}$Be reaction at $E_d=25$ MeV calculated with adiabatic deuteron wave function. (a) Calculations have been done with proton optical potential P1 and deuteron adiabatic potential D1; spectroscopic factor $S=1$ was used. (b) Different sets of proton and deuteron optical potentials were used and theoretical curves are normalized to the experimental data.

Data: Zwieglinki,et al.Nucl.Phys.A3 15(1979)124

Timofeyuk and Johnson Phys.Rev. 59(1999)1545

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Proof of alternative formula for U_{pd}

$$\bar{U}_{pd}G_0 = 1 + V_{pA}G_0 + V_{pA}\bar{G}(V_{pA} + V_{nA})G_0$$
$$= 1 + V_{pA}G_0 + V_{pA}(\bar{G} - G_0)$$
$$= 1 + V_{pA}\bar{G},$$

Important identity:

$$\bar{G} - G_0 = \bar{G}\left(G_0^{-1} - \bar{G}^{-1}\right)G_0$$
$$= \bar{G}[(z - K) - (z - K - V_{nA} - V_{pA})]G_0$$
$$= \bar{G}(V_{nA} + V_{pA})G_0.$$

Integral equation for U_{pd} (2)

$$G = G_p U_{pd} G_d$$

$$G_p U_{pd} = G G_d^{-1} = G\left(G_d^{-1} - G^{-1}\right) + 1$$
$$= G(V_{nA} + V_{pA}) + 1,$$

$$U_{pd} = \bar{U}_{pd}[1 + G_0 V_{np}(G_p U_{pd} - 1)]$$
$$= \bar{U}_{pd} G_0 V_{np} G_p U_{pd} + \bar{U}_{pd}(1 - G_0 V_{np}).$$

$$(1 - G_0 V_{np})|\phi_{\vec{k}_d,\, d}\rangle = G_0\left(G_0^{-1} - V_{np}\right)\left|\phi_{\vec{k}_d,d}\right\rangle$$
$$= G_0\left(G_0^{-1} - V_{np}\right)\left|\phi_{\vec{k}_d,d}\right\rangle$$
$$= G_0(E + \imath\varepsilon - K - V_{np})\left|\phi_{\vec{k}_d,d}\right\rangle$$
$$= \imath\varepsilon G_0\left|\phi_{\vec{k}_d,d}\right\rangle, = 0$$

General: $$U_{\beta\alpha}|\phi_\alpha\rangle = \bar{U}_{\beta\alpha} G_0 V_\alpha G_\beta U_{\beta\alpha}|\phi_\alpha\rangle \quad \alpha \neq \beta$$

$$G_p U_{pd} \left| \phi_{\vec{k}_d,d} \right\rangle = \left| \Psi^{(+)}_{\vec{k}_d,d} \right\rangle$$

$$G = G_p U_{pd} G_d$$

$$\iota \varepsilon G_d \left| \phi_{\vec{k}_d,d} \right\rangle = \left| \phi_{\vec{k}_d,d} \right\rangle$$

$$\left| \Psi^{(+)}_{\vec{k}_d,d} \right\rangle = \iota \varepsilon G \left| \phi_{\vec{k}_d,d} \right\rangle$$

$$\langle \phi_{\vec{k}_p,B} | U_{pd} | \phi_{\vec{k}_d,d} \rangle = \langle \phi_{\vec{k}_p,B} | \bar{U}_{pd} G_0 V_{np} G_p U_{pd} | \phi_{\vec{k}_d,d} \rangle$$
$$= \langle \bar{\Psi}^{(-)}_{\vec{k}_p,B} | V_{np} | \Psi^{(+)}_{\vec{k}_d,d} \rangle,$$

$$(E - K - V_{nA} - V^*_{pA}) \bar{\Psi}^{(-)}_{\vec{k}_p,B} = 0.$$

Timofeyuk and Johnson(1999)

Connection between transition operators of Alt, et al., (1967) and Timofeyuk and Johnson (PRC80,044616(2009))

Alt, et al. :

$$U_{pd} = G_0^{-1} + V_{pA} + (V_{pA} + V_{pn})G(V_{pA} + V_{nA})$$

Equivalent on-shell:Johnson, PRC80(2009)044616

$$U_{pd} = \bar{U}_{pd} G_0 V_{np} G_p U_{pd}$$

Definition:

$$\bar{U}_{pd} = G_0^{-1} + V_{pA} + V_{pA}\bar{G}(V_{pA} + V_{nA})$$

Alternative:

$$\bar{U}_{pd} G_0 = 1 + V_{pA}\bar{G}$$

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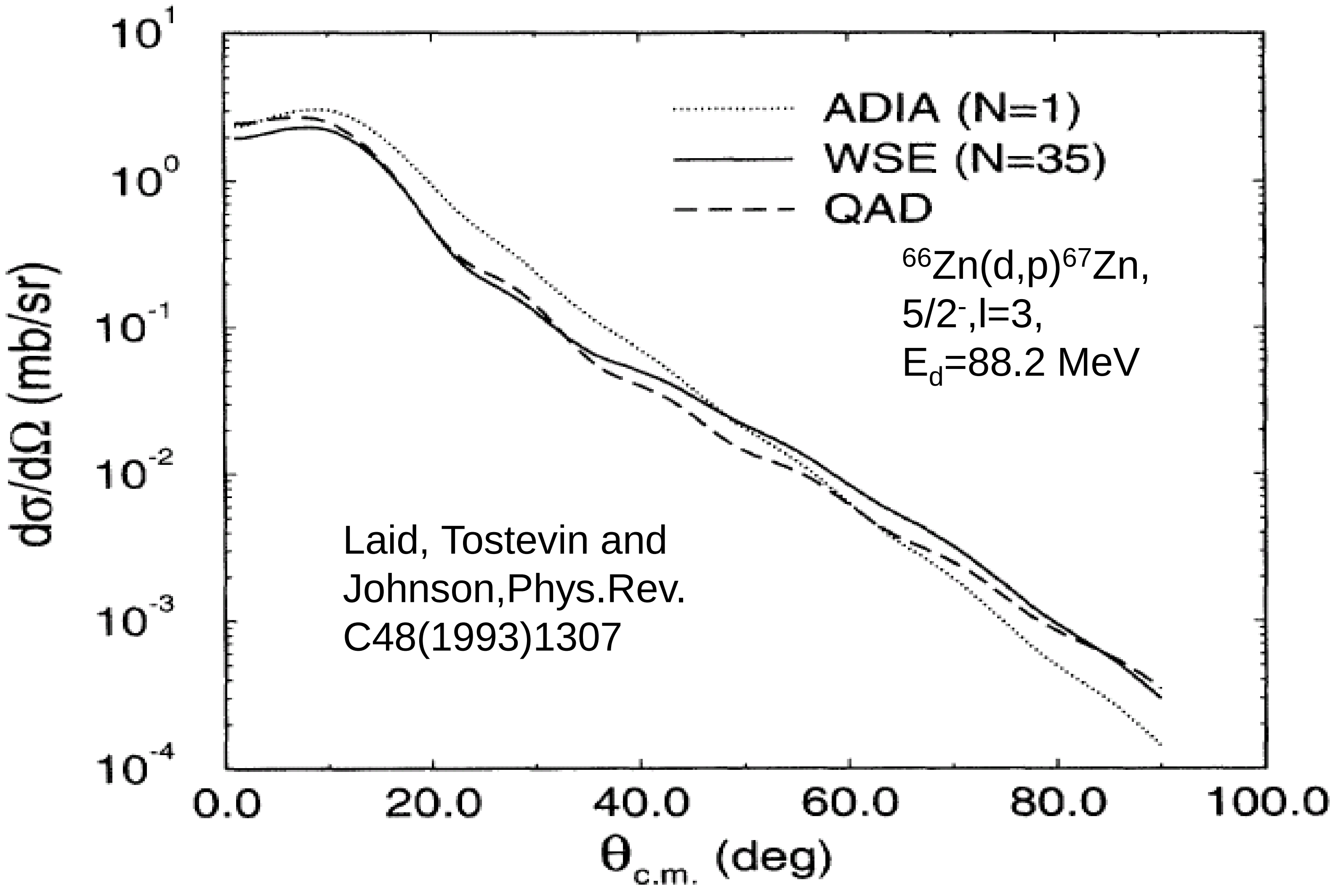
Corrections to the Adiabatic approximation at E_d=88.2 MeV



Integral equation for U_{pd} (1)

$$\begin{aligned} U_{pd} &= G_0^{-1} + V_{pA} + V_{pA}G(V_{pA} + V_{nA}) \\ &\quad + V_{pn}G(V_{pA} + V_{nA}) \\ &= G_0^{-1} + V_{pA} + V_{pA}(\bar{G} + \bar{G}V_{np}G)(V_{pA} + V_{nA}) \\ &\quad + V_{pn}G(V_{pA} + V_{nA}) \\ &= \bar{U}_{pd} + V_{pA}\bar{G}V_{np}G(V_{pA} + V_{nA}) \\ &\quad + V_{pn}G(V_{pA} + V_{nA}) \\ &= \bar{U}_{pd} + (V_{pA}\bar{G} + 1)V_{np}G(V_{pA} + V_{nA}), \end{aligned}$$

Use identity previously proved:

$$U_{pd} = \bar{U}_{pd}[1 + G_0 V_{np}G(V_{pA} + V_{nA})]$$

Concluding Remark

1. "Adiabatic" approximation. Must distinguish between

(a) Stripping and pick-up.

Johnson and Soper, Johnson and Tandy;

Laid, Tostevin and Johnson, PRC(1993)1307.

(b) Elastic Scattering.

Summers, Tostevin and Johnson, PRC66(2002)014614.

Adiabatic Distorted Waves:

2. Implementation needs optical potentials for nucleons, not deuterons.

3. CDCC is very complicated to implement. It needs excited continuum state wavefunctions.

4-body CDCC: Matsimoto, et al, PRC70,(2004)061601,
Rodriguez-Gallardo, et al, PRC77064609(2008)

4-body adiabatic: Christley, et al, Nucl.Phys.A624(1997)275.

Deuteron break-up effects

1. In 1970 Johnson-Soper and Harvey-Johnson gave a simple prescription for taking into account coherent , `entangled', effects of d break-up on (d,p) reactions. Need for Deuteron optical potentials disappears. Only need NUCLEON optical potentials at several energies. No change in the way nuclear structure parameters (overlap functions, spectroscopic factors, ANC's) appear in the theory.
2.J-S sought to justify their assumptions (`adiabatic') by applying the same ideas to 20 MeV elastic d scattering, with some success. (See Chau Huu-Tau, Nucl.Phys.A773(2006)56;A776(2006)80 for CDCC developments.)
3. H-J were able to explain some outstanding discrepancies between old DWBA calculations and experiment. New theory could use existing codes and was simple to implement..
4.Johnson and Tandy introduced a new approach which made clear that the adiabatic assumption was not a necessary condition for the validity of the J-S method for <u>transfer.</u> Implemented by Laid, Tostevin and Johnson.
5.For application to modern low energy experiments (GANIL, MSU) the big question is `What is the correct 3-body Hamiltonian?'
6.Validity of adiabatic approximation for elastic scattering investigated by comparison with CDCC calculations. Generalised to 3-body projectiles.

Calculating Transfer Amplitudes: B(p,d)A

The 3-body Schroedinger equation:

$$(E - K - V_{nA} - V_{pA} - U_i) \mid \Psi\rangle = (V_{np} - U_i) \mid \Psi\rangle$$

Exact solution of inhomogeneous equation:

$$T_{\rm prior} = \langle \Psi_{dA}^{(-)} | V_{pn} + V_{pA} - U_i | \Phi_i^{(+)} \rangle$$

$$\langle \Psi_{dA}^{(-)} | (E + i\epsilon - H) = \imath\epsilon \langle \phi_d , \mathbf{K}_d \mid$$

$$\begin{aligned}[E + i\epsilon - K \quad &- \quad V_{nA} - U_i] \mid \Phi_i^{(+)} \rangle \\ &= \quad \imath\epsilon \mid \phi_B \mathbf{K}_p \rangle\end{aligned}$$

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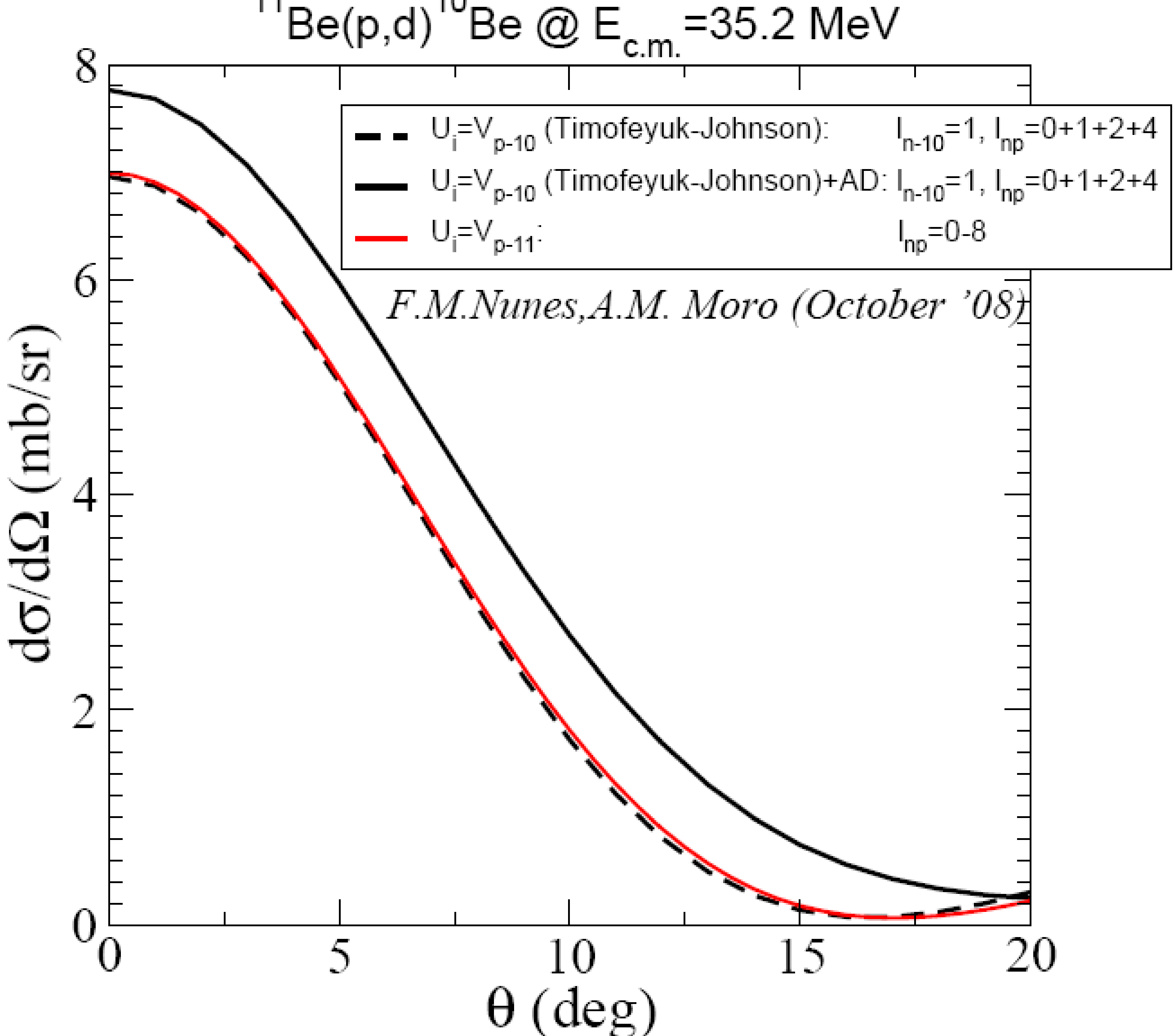

11Be(p,d)10Be @ Ec.m.=35.2 MeV
Ui=Vp-10 (Timofeyuk-Johnson): ln-10=1, lnp=0+1+2+4
Ui=Vp-10 (Timofeyuk-Johnson)+AD: ln-10=1, lnp=0+1+2+4
Ui=Vp-11: lnp=0-8
F.M.Nunes,A.M. Moro (October '08)
dσ/dΩ (mb/sr)
θ (deg)

DWBA

(i) Ignore all components of the 3-body wave function except the elastic A(d, d)A components.
(ii) Find deuteron and proton optical potentials which correctly describe elastic data.
(iii) Deuteron excitation (break-up) and excitation of A taken into account only in so far as these channels influence elastic d channel.

DWBA:

$$\Psi^{el}_{\vec{K}_d}(\vec{R},\vec{r})=\phi_0(\vec{r})\chi^{el(+)}_{\vec{K}_d}(\vec{R}),$$

$$(T_R+V^{opt}(\vec{R})-E_d)\chi^{el(+)}_{\vec{K}_d}(\vec{R})=0,$$

$$\chi^{el(+)}_{\vec{K}_d}(\vec{R})\overset{R\rightarrow\infty}{\rightarrow}\exp(\imath\vec{K}_d.\vec{R})+f^{el}(\hat{R})\frac{\exp(\imath K_d R)}{R}.$$

D
D
D
D
W

D
W
B
A

$$\begin{aligned}T_{dp} &= \langle\chi_p^{(-)}\phi_n \mid V_{np} \mid \Psi^{el}_{\vec{K}_d}\rangle,\\ &= \langle\chi_p^{(-)}\phi_n \mid V_{np} \mid \phi_0\ \chi^{el(+)}_{\vec{K}_d}\rangle,\end{aligned}$$

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