Physics of Neutron Stars: from the crust to the core

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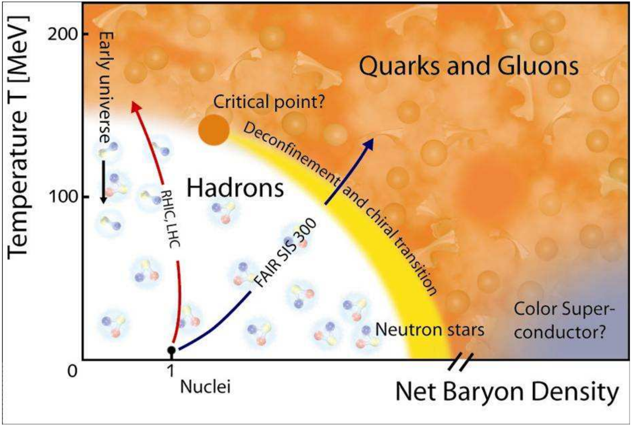
Phase Diagram



Plan of My Talk

- Introduction
- Supernova explosion and birth of a neutron star
- Outer and Inner crusts of neutron stars
- Exotic forms of Matter
- Non-relativistic and relativistic models of neutron star matter
- Symmetry Energy and Equation of State (EoS)
- Constraining dense matter from heavy ion collisions and observations of neutron stars

Introduction

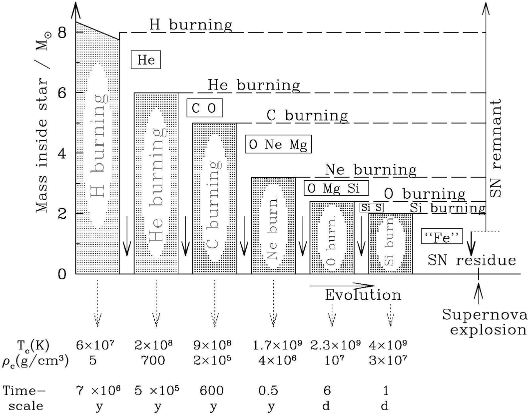
Understanding the final journey of a massive star, after its fuel has been exhausted, is a challenging problem. The outcome of it may be a supernova explosion and the residue may take the form of a neutron star.

Neutron star matter is cold and highly dense. The matter density in the core exceeds by a few times normal nuclear matter density.

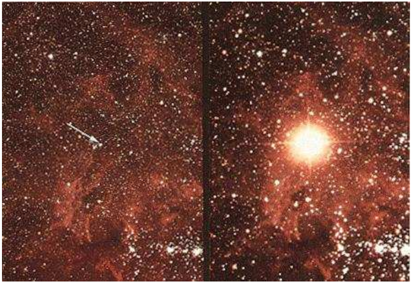
The theoretical mass-radius relationships of compact stars are directly compared with measured masses and radii from various observations.

Consequently, the composition and EoS of dense matter in a neutron star interior might be probed.

Synthesis of Heavy Elements in Stars

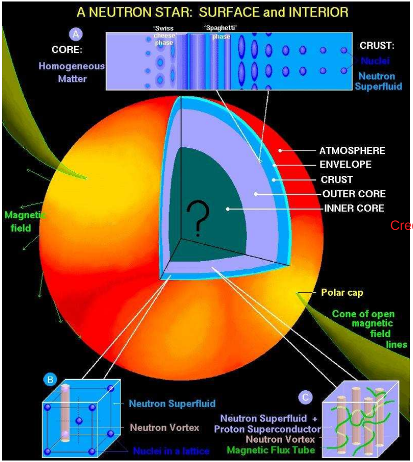


Supernova Explosion



- Mass of the core exceeds Chandrasekhar mass (1.44 $M_{\odot}$)
- Collapse is accelerated due to electron capture ($p + e^{-} \rightarrow n + \nu_e$)
- Bounce off and shock wave generation
- Trapped neutrinos in dense matter (4 x 10^{11} gms/cm^3)
- Resistance from normal nuclear matter (0.16 fm^{-3} or 2.7 x 10^{14} gms/cm^3)
- Prompt SN explosion and neutrinos diffuse out
- Revival of the shock and delayed SN explosion

Structure of a Neutron Star



Credit: D. Page

- Atmosphere (atoms)
  $n \leq 10^4 \; g/cm^3$
- Outer Crust (free e $^-$ s, lattice of nuclei)
  $10^4 - 4 \times 10^{11} \; g/cm^3$
- Inner crust (lattice of nuclei with free e$^-$s and n's)
- Outer core (atomic particle fluid)
- Inner core (exotic subatomic particles)
  $n \geq 10^{14} \; g/cm^3$

Role of Crust

- Crust thermal conductivity is important to determine the relation between observed X-ray flux and the temperature of the core.
- Electrical resistivity of the crust is important for the evolution of NS magnetic field.
- The presence of crystal lattice of atomic nuclei is mandatory for modelling of radio-pulsar glitches.
- Can make rapidly rotating pulsar a source of gravitational waves.
- Instabilities in the fusion of light elements in the outer crust of a accreting NS, are thought to be responsible for X-ray bursts.

Outer Crust

To find an equilibrium nucleus (A,Z), we minimize the Gibbs free energy per nucleon varying A and Z at a fixed pressure P,

$$g = \frac{E_{tot} + P}{n_b} = \frac{W_N + 4/3 W_L + Z\mu_e}{A} \ .$$

The total energy density is given by

$$E_{tot} = n_N(W_N + W_L) + \varepsilon_e(n_e) \ .$$

The energy of the nucleus (including rest mass energy of nucleons) is

$$W_N = m_n(A - Z) + m_p Z - bA \ ,$$

where b is the binding energy per nucleon. The lattice energy of the cell is given by

$$W_L = -\frac{9}{10}\frac{Z^2e^2}{r_C}\left(1 - \frac{5}{9}\left(\frac{r_N}{r_C}\right)^2\right) \ .$$

The total pressure

$$P = P_e + \frac{1}{3}W_L n_N .$$

Baym, Pethick, Sutherland, ApJ170 (1971) 299

Sequence of nuclei in the outer crust

element	Z	N	ρ_{max} (g/cm^3)	n_e (cm^{-3})	μ_e (MeV)	g (MeV)
^{56}Fe	26	30	8.01×10^6	2.24×10^{30}	0.95	930.600
^{62}Ni	28	34	2.71×10^8	7.38×10^{31}	2.61	931.317
^{64}Ni	28	34	1.33×10^9	3.51×10^{32}	4.34	932.040
^{66}Ni	28	34	1.50×10^9	3.82×10^{32}	4.46	932.090
^{86}Kr	36	50	3.10×10^9	7.80×10^{32}	5.64	932.557
^{84}Se	34	50	1.06×10^{10}	2.58×10^{33}	8.39	933.617
^{82}Ge	32	50	2.79×10^{10}	6.54×10^{33}	11.43	934.749
^{80}Zn	30	50	6.11×10^{10}	1.37×10^{34}	14.63	935.901
^{78}Ni	28	50	9.25×10^{10}	1.99×10^{34}	16.56	936.565
^{126}Ru	44	82	1.29×10^{11}	2.69×10^{34}	18.30	937.119
^{124}Mo	42	82	1.86×10^{11}	3.78×10^{34}	20.50	937.825
^{122}Zr	40	82	2.64×10^{11}	5.18×10^{34}	22.76	938.530
^{120}Sr	38	82	3.77×10^{11}	7.13×10^{34}	25.33	939.305
^{118}Kr	36	82	4.34×10^{11}	7.91×10^{34}	26.22	939.566

Inner Crust

- Inner crust of a Neutron Star begins at neutron drip point defined by $\mu_n = m_n c^2$, $\mu_n =$ neutron chemical potential.
- It contains nuclear cluster immersed in electron and neutron gas under the condition of charge neutrality ($Y_p = Y_e$) and β stability ($\mu_n = \mu_p + \mu_e$).
- The nuclear clusters are assumed to be arranged in a bcc lattice which we approximate by Wigner-Seitz cells defined as spheres with radius R_c.
- Electrons are extremely relativistic and can be assumed to be uniformly distributed in the cell.

Negele and Vautherin, NPA207 (1973) 298

We minimise the free energy $\mathcal{F}$ given by

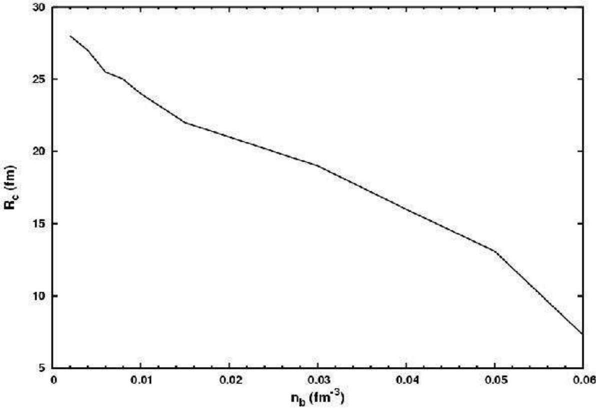
$$\mathcal{F}(\langle\rho\rangle, Y_p, T) = \int [\mathcal{H}(r) - Ts(r) + \mathcal{E}_c(r) + f_e(\rho_e)]\, d\mathbf{r}.$$

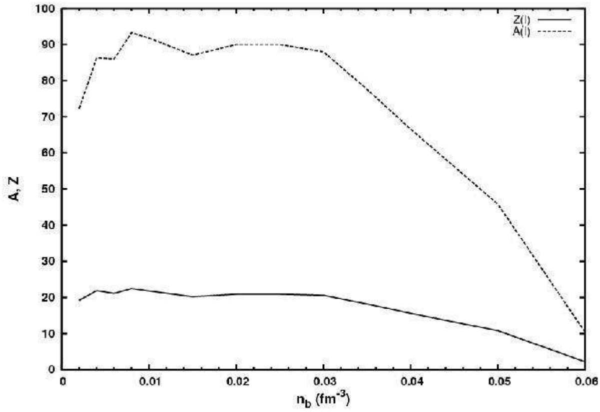
The Skyrme energy density functional

$$\begin{aligned}
\mathcal{H}(r) \quad = \quad & \frac{\hbar^2}{2m_n^*}\tau_n + \frac{\hbar^2}{2m_p^*}\tau_p + \frac{1}{2}t_0\left[\left(1+\frac{x_0}{2}\right)\rho^2 - \left(x_0+\frac{1}{2}\right)\left(\rho_n^2+\rho_p^2\right)\right] \\
& -\frac{1}{16}\left[t_2\left(1+\frac{x_2}{2}\right) - 3t_1\left(1+\frac{x_1}{2}\right)\right](\nabla\rho)^2 \\
& -\frac{1}{16}\left[3t_1\left(x_1+\frac{1}{2}\right) + t_2\left(x_2+\frac{1}{2}\right)\right]\left[(\nabla\rho_n)^2 + (\nabla\rho_p)^2\right] \\
& +\frac{1}{12}t_3\rho^{\alpha}\left[\left(1+\frac{x_3}{2}\right)\rho^2 - \left(x_3+\frac{1}{2}\right)\left(\rho_n^2+\rho_p^2\right)\right]
\end{aligned}$$

where $\rho = \rho_n + \rho_p$ and m_q^* are the effective mass of nucleons given as

$$\begin{aligned}
\frac{m}{m_q^*(r)} \quad = \quad & 1 + \frac{m}{2\hbar^2}\left\{\left[t_1\left(1+\frac{x_1}{2}\right) + t_2\left(1+\frac{x_2}{2}\right)\right]\rho\right. \\
& \left. + \left[t_2\left(x_2+\frac{1}{2}\right) - t_1\left(x_1+\frac{1}{2}\right)\right]\rho_q\right\}.
\end{aligned}$$


30
25
20
15
10
5
R_c (fm)
0
0.01
0.02
0.03
0.04
0.05
0.06
n_b (fm^{-3})



Z(l)
A(l)
A, Z
n_b (fm^{-3})
0
10
20
30
40
50
60
70
80
90
100
0.01
0.02
0.03
0.04
0.05
0.06

Various exotic components of matter such as hyperons, Bose-Einstein Condensates (pion or kaon) & quarks, may appear in the neutron star core. Hyperons

- Hyperons produced at the cost of the nucleons.

$$n + p \longrightarrow p + \Lambda + K^0, \quad n + n \longrightarrow n + \Sigma^- + K^+$$

- Chemical equilibrium in compact star interior through weak processes,
- $p + e^- \longrightarrow \Lambda + \nu_e, \quad n + e^- \longrightarrow \Xi^- + \nu_e$

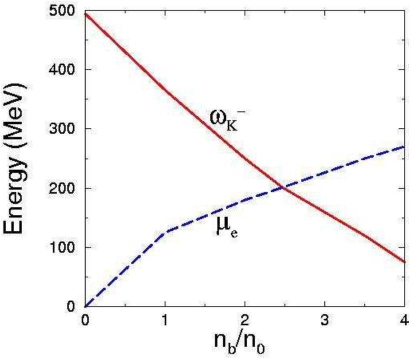
$$\mu_i = b_i \mu_n - q_i \mu_e$$

- Threshold Condition for Hyperons

$$\mu_n - q_i \mu_e \geq m_B^* + g_{\omega B}\omega_0 + g_{\rho B}\rho_{03}\tau_3$$

Bose-Einstein condensates

The processes responsible for *p*-wave pion condensate/*s*-wave kaon condensate in compact stars,



- $n \longrightarrow p + \pi^-$ $\qquad n \longrightarrow p + K^-$
- $e^- \longrightarrow \pi^- + \nu_e$ $\qquad e^- \longrightarrow K^- + \nu_e$

  • Threshold conditions for Bose-Einstein condensation of mesons

  For K^-

- $\omega_{K^-} = \mu_e$.
- For π^- $\qquad \omega_{\pi^-} = \mu_e$.

S. Banik, D.B., Phys.Rev.C63 (2001) 035802

S. Banik, D.B., Phys.Rev.C64 (2001) 055805

S. Banik, D.B., Phys.Rev.C66 (2002) 065801

Quark Matter

Witten Conjecture: u, d, s quark matter is the ground state of matter (energy/baryon $<$ 939 MeV at finite density).

Ref: E. Witten, Phys. Rev. D30 (1984) 272

Quarks are in chemical equilibrium:

$$d \longrightarrow u + e^- + \bar{\nu}_e, \quad s \longrightarrow u + e^- + \bar{\nu}_e;$$

$$\mu_d = \mu_u + \mu_e, \quad \mu_s = \mu_d$$

MIT Bag model: $P \longrightarrow P - B, \quad \& \quad \epsilon \longrightarrow \epsilon + B$

Recently it has been predicted that quark matter might be a color superconductor. Quarks near their Fermi surfaces form Cooper pairs due to the attractive quark-quark interaction in color antisymmetric channel.

Many-body theories of dense matter in Neutron Stars

Neutron star matter is a many-body system

- Two classes of models: non-relativistic and relativistic models
  i) Microscopic models :
- Brueckner Hartree-Fock and Dirac-Brueckner-Hartree-Fock theories (R. Brockmann and R. Machleidt, PRC42 (1990) 1965)
- Variational many-body approach (A. Akmal, V. Pandharipande and D.G. Ravenhall, PRC58 (1998) 1804)
  ii) Effective Field theory approach:
- Density functional theory (R.J. Furnstahl, Lect. Notes Phys. 641 (2004) 1)
- Chiral perturbation theory (K. Hebeler, PRL105 (2010) 161102)
  iii) Phenomenological theories:
- Effective two-body interactions (Skyrme or modified Seyler-Blanchard interactions)
- Relativistic Mean Field (RMF) models (J. D. Walecka, Adv. Nucl. Phys. 16 (1986) 1)

Momentum and Density Dependent two-body interaction

The Seyler-Blanchard effective baryon-baryon interaction is given by

$$V_{eff}(r,\rho,p) = -C_{B_1B_2}[1 - \frac{p^2}{b^2} - d^2(\rho_1+\rho_2)^n]\frac{e^{-r/a}}{r/a},$$

D.B. and S.K. Samaddar, NPA484 (1988) 315 S. Banik and D.B., J. Phys. G 26 (2000) 1495

The energy per nucleon in asymmetric matter may be written as

$$E(\rho,\beta) = E(\rho,\beta=0) + \beta^2 E_{sym}(\rho),$$

where $\beta = \frac{(\rho_n - \rho_p)}{\rho}$ is the asymmetry parameter.
The symmetry energy is an essential ingredient in understanding dense matter. The expression of nuclear symmetry energy follows from

$$\mu_n - \mu_p = 4\beta E_{sym}(\rho).$$

- The EoS becomes superluminal at higher densities

D.B. et al., NPA511 (1990) 1

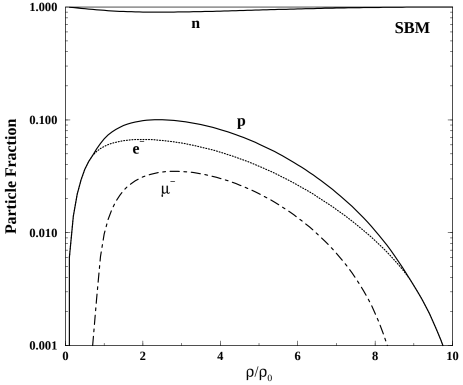
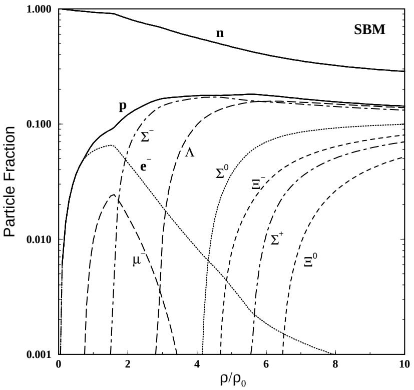


Fig.1



1.000
0.100
0.010
0.001
Particle Fraction
n
SBM
p
Σ⁻
Λ
e⁻
Σ⁰
Ξ⁻
Σ⁺
μ⁻
Ξ⁰
0
2
4
6
8
10
ρ/ρ0

Fig.2

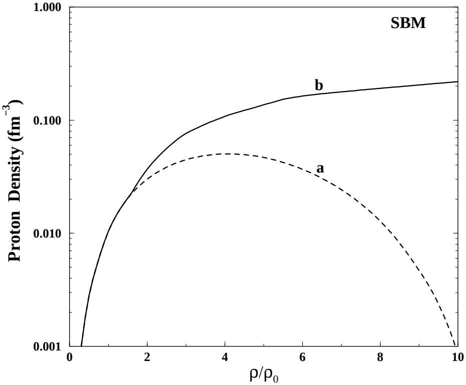

SBM
b
a
Proton Density (fm^{-3})
1.000
0.100
0.010
0.001
0
2
4
6
8
10
ρ/ρ_0

Fig.3

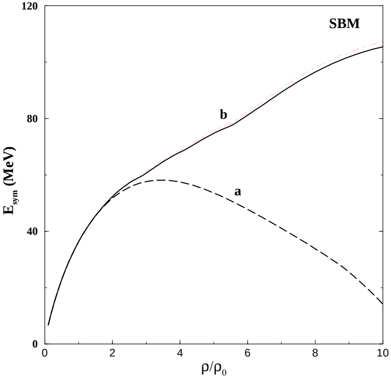
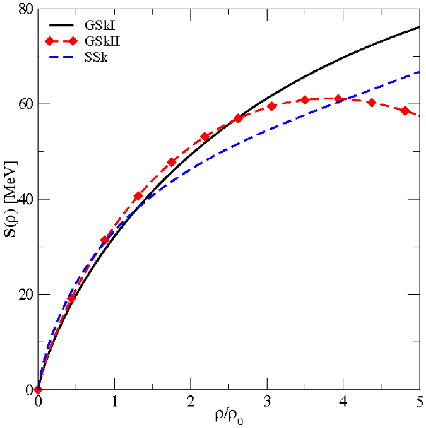


Fig.4



B. Agrawal et al., PRC73 (2006) 034319

RMF Model

$$
\begin{aligned}
\mathcal{L}_B &= \sum_B \bar{\Psi}_B \left(i\gamma_\mu \partial^\mu - m_B^* - g_{\omega B}\gamma_\mu \omega^\mu - g_{\rho B}\gamma_\mu \mathbf{t}_B \cdot \boldsymbol{\rho}^\mu \right) \Psi_B \\
&+ \frac{1}{2}\left(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2 \right) - U(\sigma) \\
&- \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4}\boldsymbol{\rho}_{\mu\nu} \cdot \boldsymbol{\rho}^{\mu\nu} + \frac{1}{2}m_\rho^2 \boldsymbol{\rho}_\mu \cdot \boldsymbol{\rho}^\mu + \mathcal{L}_{YY} \; .
\end{aligned}
$$

where, $U(\sigma) = \frac{1}{3} b m_N \left(g_{\sigma N}\sigma \right)^3 + \frac{1}{4} c \left(g_{\sigma N}\sigma \right)^4$.

- All the baryons of SU(3) octet are considered.
- Baryons interact through σ, ω meson & ρ meson exchange.
- Hyperons interact via $\sigma*$ & ϕ mesons

Hyperon-Hyperon interaction:

$$
\begin{aligned}
\mathcal{L}_{YY} &= \sum_B \bar{\Psi}_B \left(g_{\sigma^* B}\sigma^* - g_{\phi B}\gamma_\mu \phi^\mu \right) \Psi_B + \frac{1}{2}\left(\partial_\mu \sigma^* \partial^\mu \sigma^* - m_{\sigma^*}^2 \sigma^{*2} \right) \\
&- \frac{1}{4}\phi_{\mu\nu}\phi^{\mu\nu} + \frac{1}{2}m_\phi^2 \phi_\mu \phi^\mu
\end{aligned}
$$

In the mean field approximation, the meson field equations in the presence of antikaons

$$m_\sigma^2 \sigma = -\frac{\partial U}{\partial \sigma} + \sum_B g_{\sigma B} n_B^s$$

$$m_{\sigma^*}^2 \sigma^* = \sum_B g_{\sigma^* B} n_B^s$$

$$m_\omega^2 \omega_0 = \sum_B g_{\omega B} n_B$$

$$m_\phi^2 \phi_0 = \sum_B g_{\phi B} n_B$$

$$m_\rho^2 \rho_{03} = \sum_B g_{\rho B} I_{3B} n_B$$

The scalar density and baryon number density

$$n_B^S = \frac{2J_B + 1}{2\pi^2} \int_0^{k_{F_B}} \frac{m_B^*}{(k^2 + m_B^{*2})^{1/2}} k^2 \, dk \ ,$$

$$n_B = (2J_B + 1) \frac{k_{F_B}^3}{6\pi^2} \ .$$

with $m_B^* = m_B - g_{\sigma B}\sigma - g_{\sigma^* B}\sigma^*$

The total energy density, $\varepsilon = \varepsilon_B + \varepsilon_l$

$$\begin{aligned}\varepsilon &= \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4 + \frac{1}{2}m_{\sigma^*}^2\sigma^{*2} \\ &+ \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\phi^2\phi_0^2 + \frac{1}{2}m_\rho^2\rho_{03}^2 \\ &+ \sum_B \frac{2J_B+1}{2\pi^2}\int_0^{k_{F_B}} (k^2 + m_B^{*2})^{1/2} k^2\, dk \\ &+ \sum_l \frac{1}{\pi^2}\int_0^{K_{F_l}} (k^2 + m_l^2)^{1/2} k^2\, dk\end{aligned}$$

The pressure

$$\begin{aligned}P &= -\frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 \\ &-\frac{1}{2}m_{\sigma^*}^2\sigma^{*2} + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\phi^2\phi_0^2 + \frac{1}{2}m_\rho^2\rho_{03}^2 \\ &+ \frac{1}{3}\sum_B \frac{2J_B+1}{2\pi^2}\int_0^{k_{F_B}} \frac{k^4\, dk}{(k^2 + m_B^{*2})^{1/2}} \\ &+ \frac{1}{3}\sum_l \frac{1}{\pi^2}\int_0^{K_{F_l}} \frac{k^4\, dk}{(k^2 + m_l^2)^{1/2}} .\end{aligned}$$

Parameters of the model

- Nucleon-meson coupling constants

ρ_0	E/B	a_{sym}	m_n^*/m_n	K
$0.153 fm^{-3}$	$-16.3 MeV$	$32.5 MeV$	0.78	$240 MeV$

$g_{\sigma N}$	$g_{\omega N}$	$g_{\rho N}$	g_2	g_3	$g_{\sigma^* N}$	$g_{\phi N}$
8.7822	8.7122	8.5416	27.8812	-14.3970	0	0

Glendenning and Moszkowski, PRL67 (1991) 2414

- Hyperon-meson coupling constants

$$\frac{1}{2}g_{\omega\Lambda} = \frac{1}{2}g_{\omega\Sigma} = g_{\omega\Xi} = \frac{1}{3}g_{\omega N}, \quad \frac{1}{2}g_{\rho\Sigma} = g_{\rho\Xi} = g_{\rho N};$$

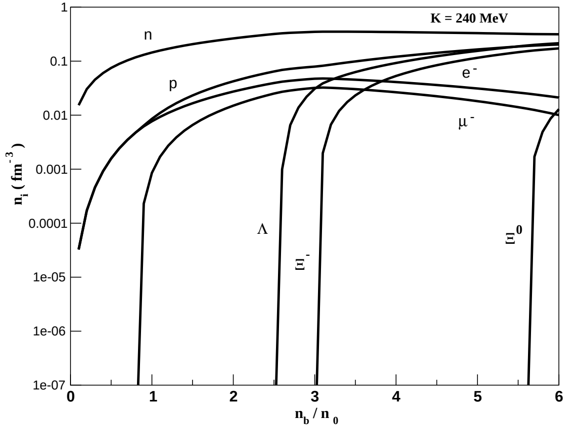
$$g_{\rho\Lambda} = 0, \quad 2g_{\phi\Lambda} = 2g_{\phi\Sigma} = g_{\phi\Xi} = -\frac{2\sqrt{2}}{3}g_{\omega N}.$$

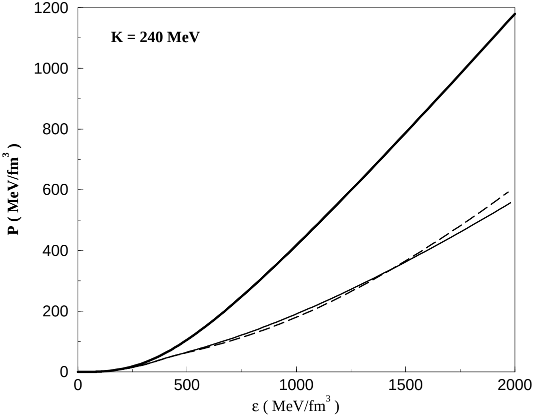
From Hypernuclei data

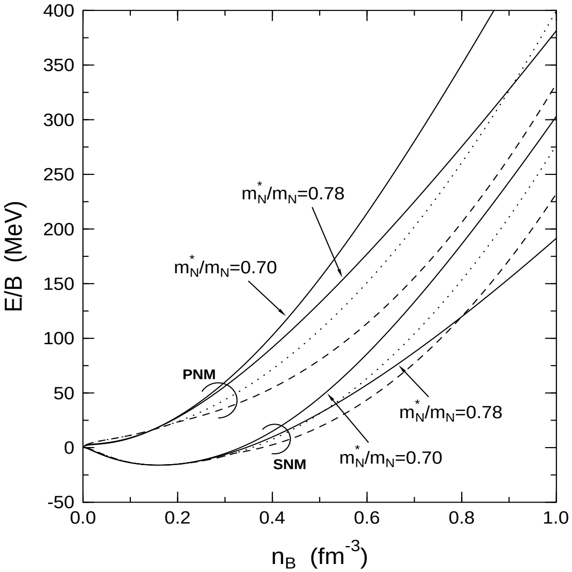
$$U_{\Lambda}^{N} = -30MeV, \quad U_{\Sigma}^{N} = +30MeV \;\; \& \;\; U_{\Xi}^{N} = -18MeV$$

$$U_{\Xi}^{(\Xi)}(n_0) = U_{\Lambda}^{(\Xi)}(n_0) = 2U_{\Xi}^{(\Lambda)}(n_0) = 2U_{\Lambda}^{(\Lambda)}(n_0) = -40MeV.$$

Schaffner and Mishustin, PRC53 (1996) 1416


K = 240 MeV
n
p
e -
μ -
Λ
Ξ -
Ξ 0
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
n_i (fm^-3)
0
1
2
3
4
5
6
n_b / n_0


K = 240 MeV
P (MeV/fm3)
ε (MeV/fm3)
0
200
400
600
800
1000
1200
0
500
1000
1500
2000

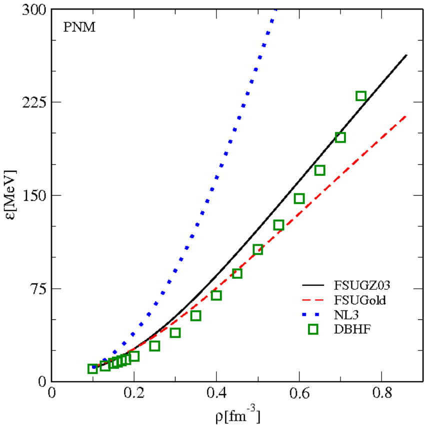


400
350
300
250
200
150
100
50
0
-50
0.0
0.2
0.4
0.6
0.8
1.0
E/B (MeV)
n_B (fm^-3)
m*_N/m_N=0.78
m*_N/m_N=0.70
PNM
SNM
m*_N/m_N=0.78
m*_N/m_N=0.70

Effective Field Theory

$$
\begin{aligned}
\mathcal{E}(r) = & \sum_\mu \phi_\mu^\dagger \left\{ -i\alpha\cdot\nabla + \beta\left[M - \Phi(r)\right] + W(r) + \frac{1}{2}\tau_3 R(r) + \frac{1+\tau_3}{2}A(r) \right\} \phi_\mu \\
& + \frac{1}{2}\frac{m_\sigma^2}{g_\sigma^2}\Phi^2(r) + \frac{\overline{\kappa}}{6}\Phi^3(r) + \frac{\overline{\lambda}}{24}\Phi^4(r) - \frac{\zeta}{24}W^4(r) - \frac{\xi}{24}R^4(r) \\
& + \frac{1}{2g_\sigma^2}\left[\nabla\Phi(r)\right]^2 - \frac{1}{2g_\omega^2}\left[\nabla W(r)\right]^2 - \frac{1}{2}\frac{m_\omega^2}{g_\omega^2}W^2(r) - \overline{\alpha}_1\Phi(r)W^2(r) \\
& - \frac{1}{2}\overline{\alpha}_1'\Phi^2(r)W^2(r) - \frac{1}{2}\frac{m_\rho^2}{g_\rho^2}R^2(r) - \frac{1}{2g_\rho^2}\left[\nabla R(r)\right]^2 + \overline{\alpha}_2\Phi(r)R^2(r) \\
& - \frac{1}{2}\overline{\alpha}_2'\Phi^2(r)R^2(r) - \frac{1}{2}\overline{\alpha}_3'W^2(r)R^2(r) - \frac{1}{2e^2}\left[\nabla A(r)\right]^2
\end{aligned}
$$

Serot and Walecka, IJMPE6 (1997) 515



B. Agrawal et al., PRC74 (2006) 034323

First order kaon Condensation

- (Anti)kaon-baryon interaction is treated in the same footing as the baryon-baryon interaction. The Lagrangian density for (anti)kaons in the minimal coupling scheme is

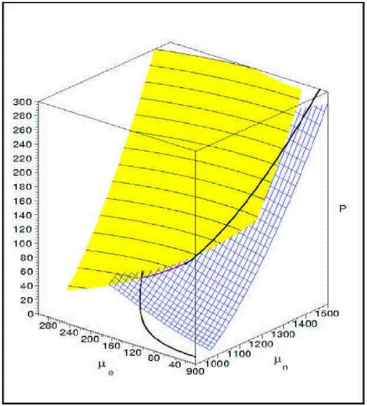
$$\mathcal{L}_K = D^*_\mu \bar{K} D^\mu K - m_K^{*2} \bar{K} K \ ,$$

where $D_\mu = \partial_\mu + i g_{\omega K} \omega_\mu + i g_{\rho K} \mathrm{t}_K \cdot \boldsymbol{\rho}_\mu$ and the effective mass of (anti)kaons is $m_K^* = m_K - g_{\sigma K} \sigma$.
- The equation of motion for (anti)kaons is

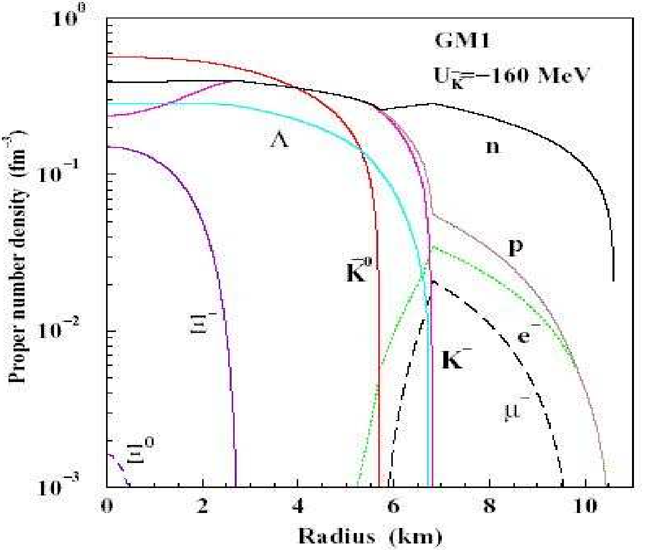
$$(D_\mu D^\mu + m_K^*) K = 0$$

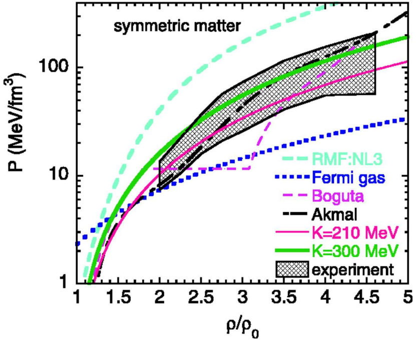
It can be solved within mean field approximation to get the in-medium energy of kaon:

$$\begin{aligned} \omega_K &= \sqrt{(p^2 + m_K^{*2})} - g_{\omega K} \omega_0 - \frac{1}{2} g_{\rho K} \rho_{03} \\ &= m_K^* - g_{\omega K} \omega_0 - \frac{1}{2} g_{\rho K} \rho_{03}, \text{ for } p = 0. \end{aligned}$$

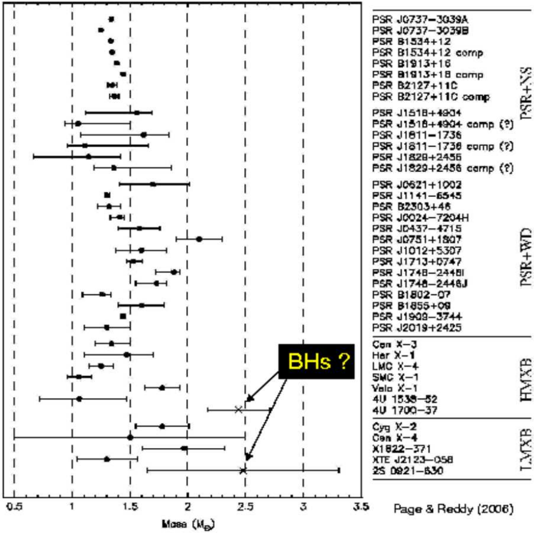


Credit: S. Banik

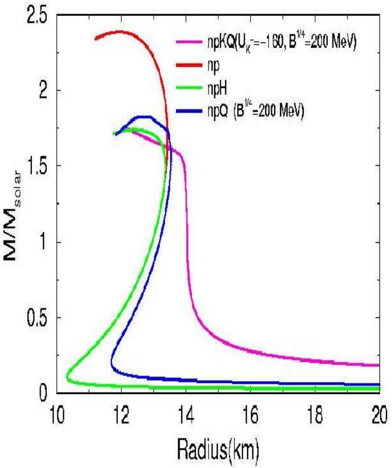

GM1
$U_{\bar{K}} = -160$ MeV
Proper number density (fm^{-3})
Radius (km)
n
Λ
p
$\bar{K}^0$
Ξ^-
e^-
K^-
μ^-
Ξ^0



- First attempt to extract EoS of dense matter in Heavy Ion Collisions (Danielewicz,Lacey and Lynch, Science 298 (2002))
- High density behaviour of the EoS is sensitive to symmetry energy
- Knowledge of the density dependence of symmetry energy is essential
- Ratio of baryons and mesons might be good probes of symmetry energy (B.A. Li, Phys. Rep. 464 (2008))



Page & Reddy (2006)



- An observed NS mass with 2 $M_\odot$ will certainly rule out all exotic forms of dense matter (P. Demorest et al, Nature 467 (2010))

Summary

- A strong interplay between the physics of dense matter in heavy ion collisions and the physics of dense matter in neutron stars
- Density dependence of the symmetry energy holds the key to the behaviour of matter at high baryon density
- Future experiments at FAIR, GSI might be shed light on hot dense matter properties