

# Some Problems in Protein Folding

## A prejudiced View

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NSF

### Is the Protein Folding Problem Solved?

“Many have solved the problem as they have defined it” --- Eaton

# Many Facets of Folding

1. Structure Prediction
2. Protein & Enzyme Design
3. Folding Kinetics & Mechanisms
4. Crowding Effects
5. Relation to aggregation
6. Molecular Chaperones

Folding at the center stage

# Folding Kinetics

## EXPERIMENTS

- PROT ENGG (TSE)
- SAXS/NMR (DSE)
- FAST FOLDING (T JUMP;  
RAPID MIXING)
- SM FRET (FOLDING/  
UNFOLDING)
- LOT/AFM (FORCE RAMP  
FORCE QUENCH)

## Theory

- **Statistical Mechanics**  
(Energy Landscape)
- **Minimal Models**  
(Lattice/Off-Lattice)
- **MD Simulations**
- **Bioinformatics**  
(Evolutionary Imprint)

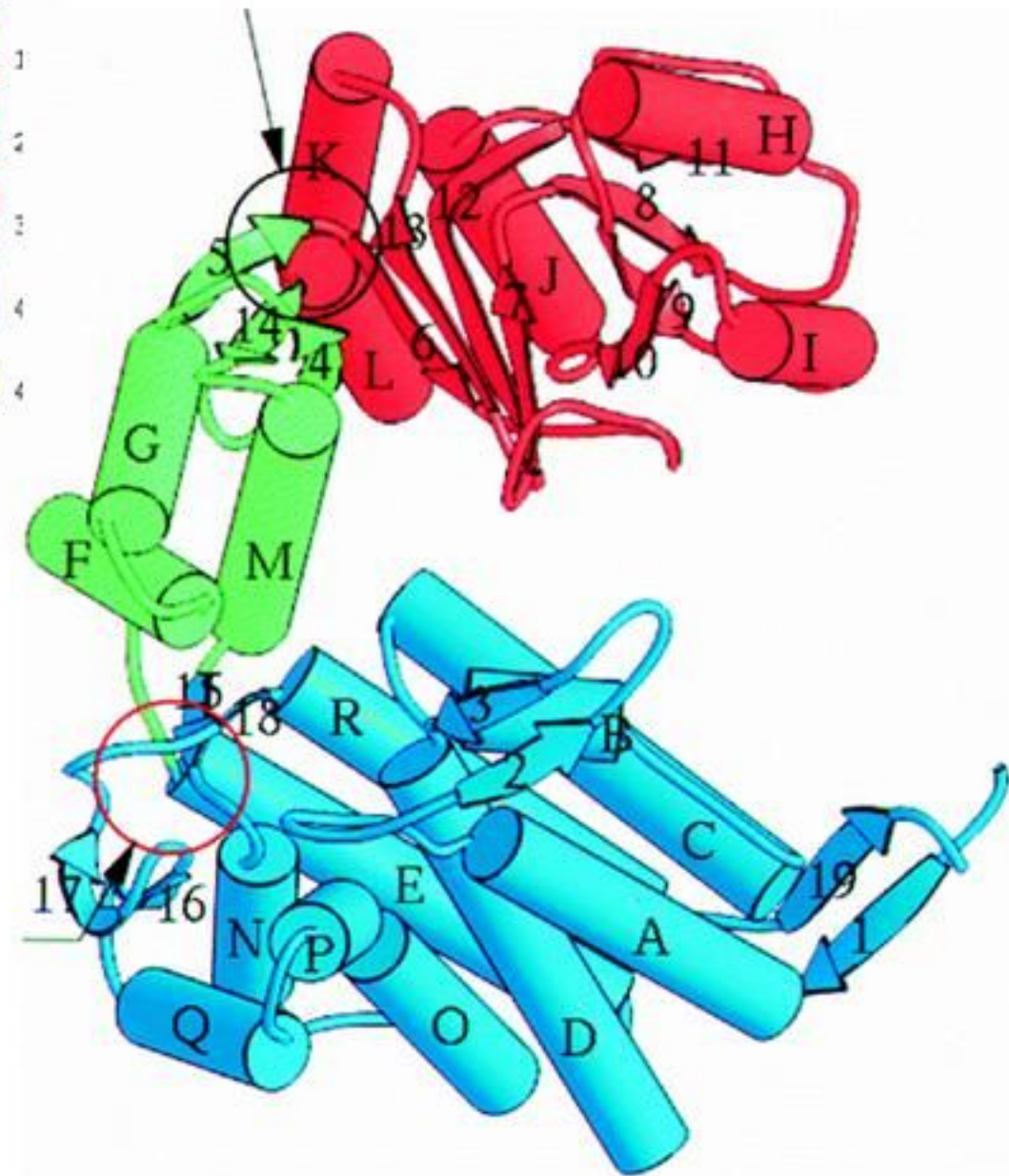
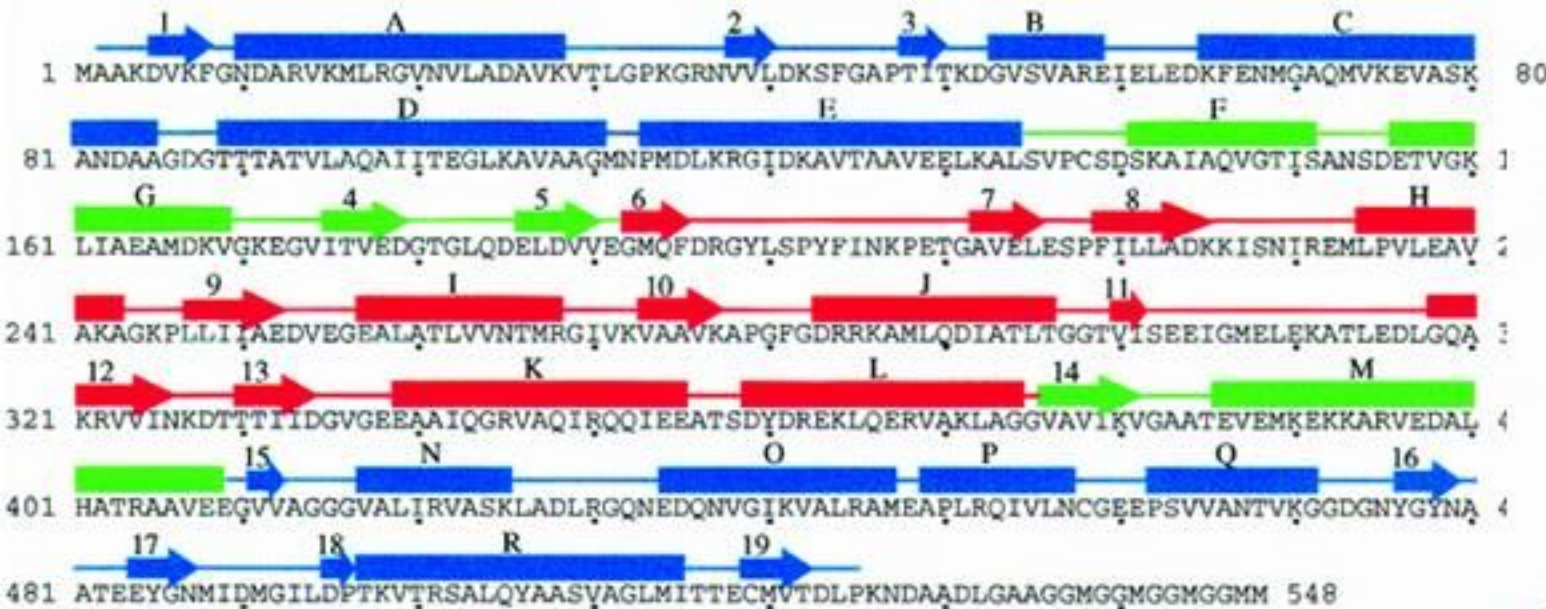
# Some concepts from physics and physical chemistry

Multiple folding scenarios, Folding funnels, nucleation-collapse, speed limit, designability and foldability  
Misfolding and kinetic traps, foldons, roughness, frustration topological and energetic, contact order  
TSE and  $\alpha$ -values, pre-factors and Kramers theory, entropy barriers, glass transition, collapse, Flory  
Laws, Master Equation, Finite size effects and consequences

Collapse transition, Folding in terms of changes in distribution functions  
SMFS as a tool for folding

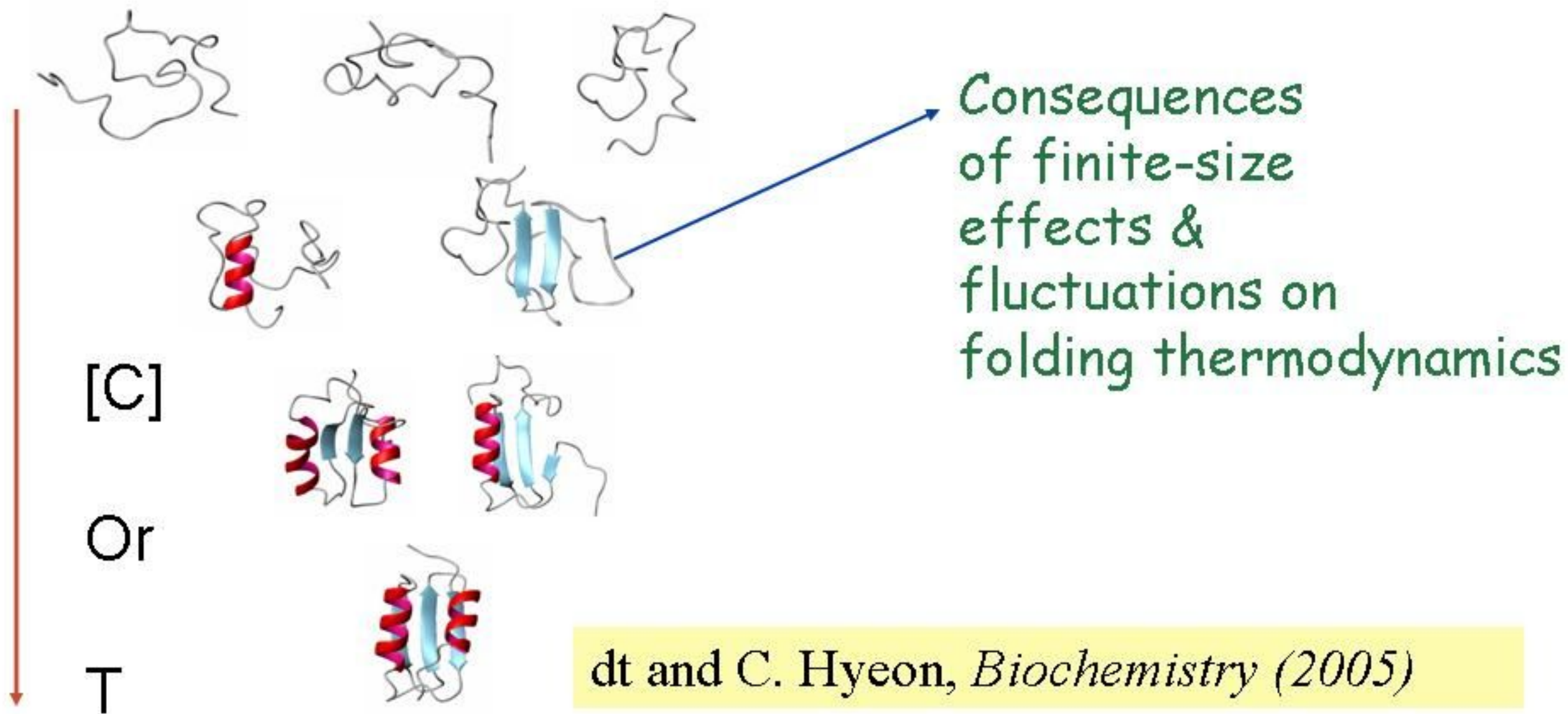
# Protein folding problems

## GroEL



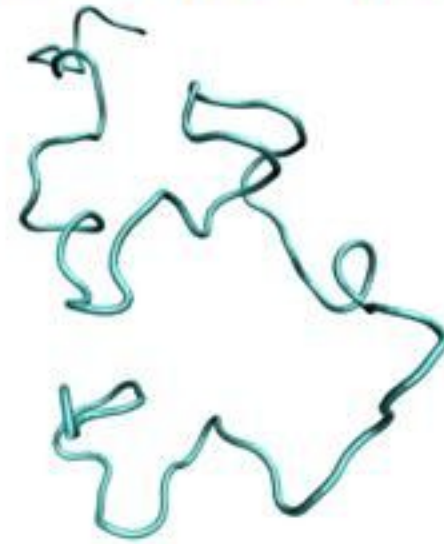
- Structure Prediction
- Assembly Mechanism
- Protein Design
- Crowding Effects

# Monomeric (spontaneous) folding Mechanisms



# Characteristic Temperatures in Proteins

HIGH T  
or [C]



Random  
Coil (Flory)



$T \approx T_{\theta}$   
Or  $[C_{\theta}]$



Compact

$T \approx T_F$   
 $[C_F]$



Native  
State

$$\sigma = (T_{\theta} - T_F)/T_{\theta}$$

# Size ( $R_g$ ) of Proteins: Flory Laws

## Unfolded state

$$R_g \approx a_D N^{\nu} \text{ “good solvent” } \nu \approx 3/(d + 2)$$

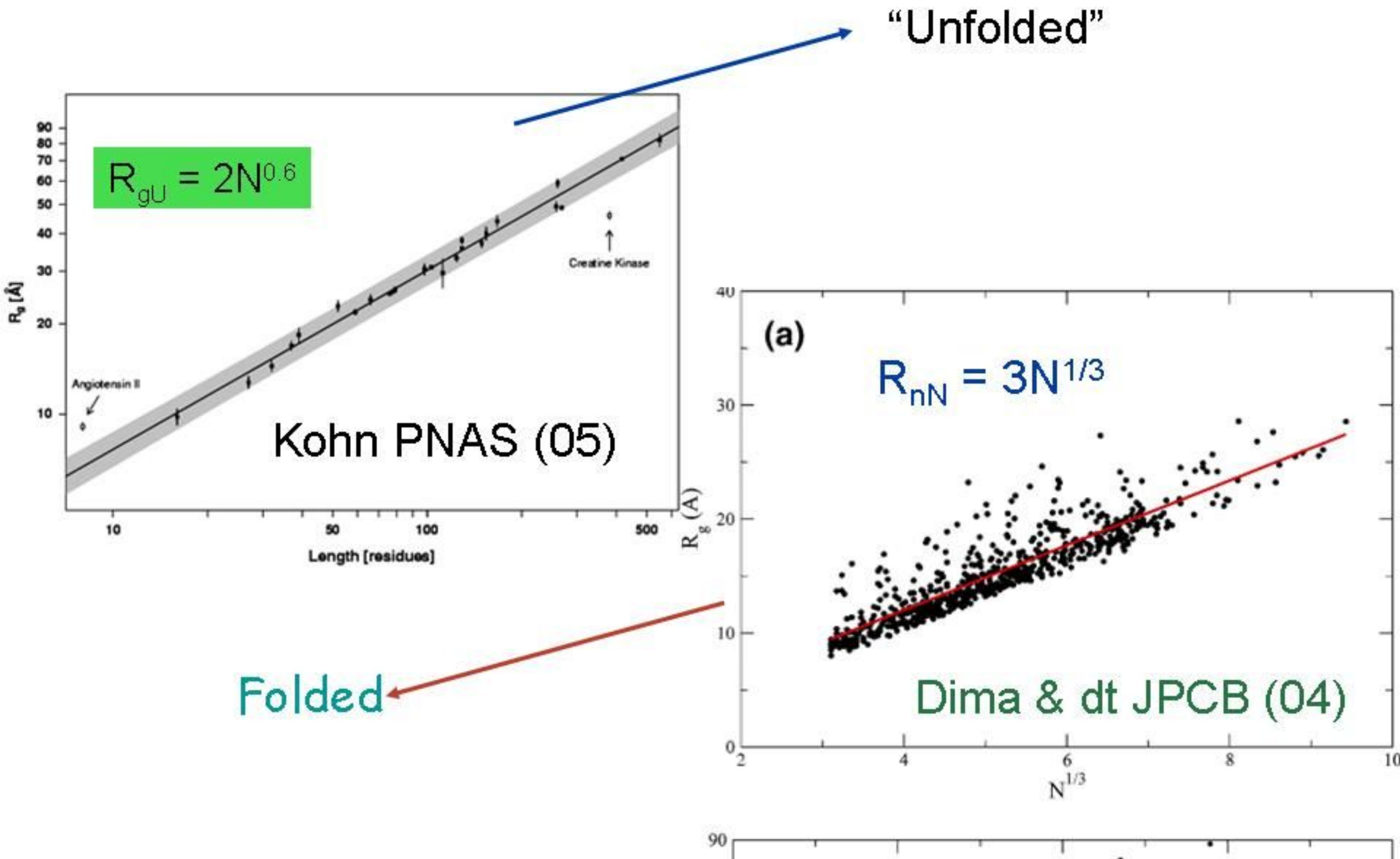
## Folded and Collapsed Globule

$$R_g \approx a_N N^{\nu} \text{ “poor solvent” } \nu = 1/d$$

How good are Flory Laws for proteins?



# Protein Collapse : $R_g$ follows Flory law



# Folding as Changes in Distribution Function

O'Brien PNAS (2008)

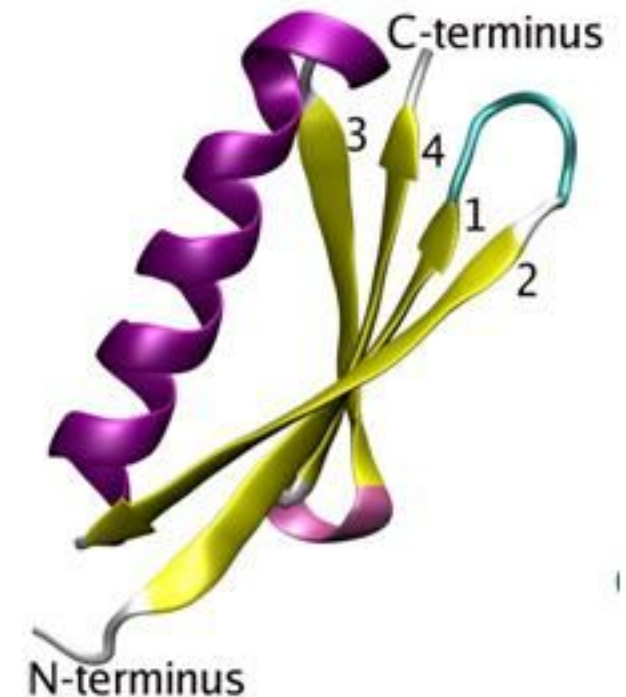
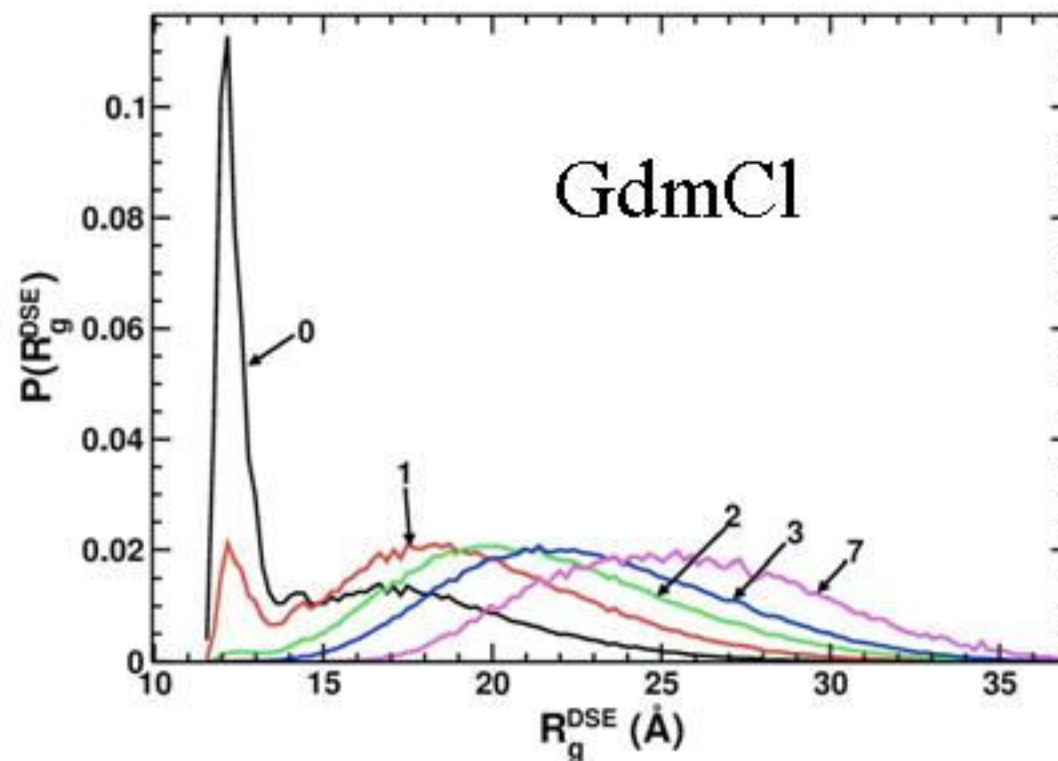


Fig. S2. Distribution of  $P(R_g^{DSE})$  for protein L at various GdmCl concentrations at 328 K. The concentration ( $[C]$ ) of GdmCl in molar units are shown in the curves.

Folding reaction as a phase transition: A rationale  $N$   
= number of amino acids

## Order Parameter Description

$\rho = N/R_g^3$  ;  $\chi$  = Overlap with NBA (0 for NBA)

Unfolded (U), Collapsed Globules (CG);  
Folded (NBA)

U:  $\rho$  (small),  $\chi$  Large (“vapor”)

CG:  $\rho \sim O(1)$ ,  $\chi$  Large (Dense no order “Liquid”)

NBA:  $\rho \sim O(1)$ ,  $\chi$  Small (Dense order “Solid”)

## Developing a “nucleation” picture

### Free Energy of Creating a Droplet

$$\Delta G(R) = -\Delta\mu R^3 + \gamma R^2$$

Driving force + Opposing

What are these forces in proteins?

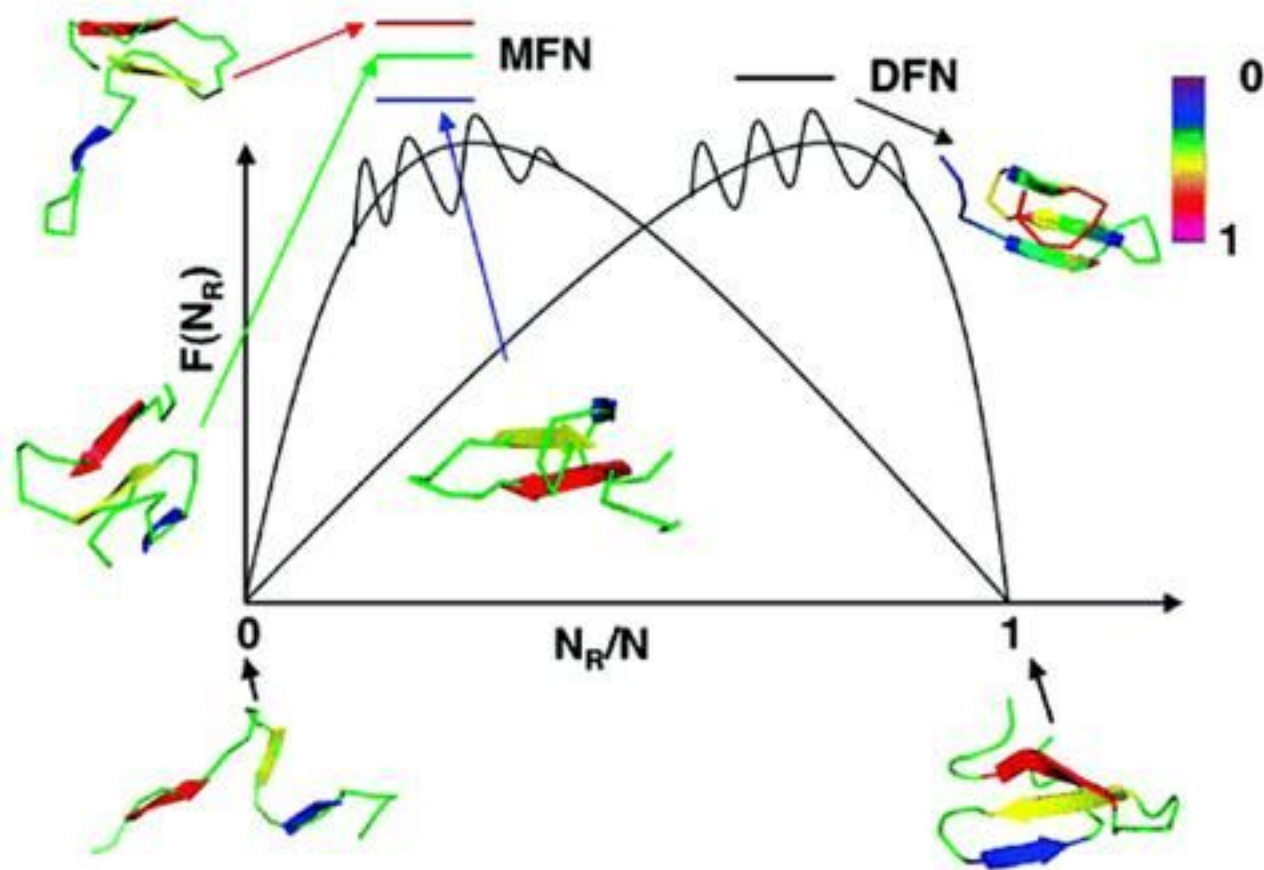
Driving force: Hydrophobic Collapse

Burying H bonds

Opposing: “Droplet with nonconstant  $\gamma$ ”

Entropy loss due to looping

# Tentative Models + Slight refinement



Cost of creating a region with  $N_R$  ordered residues out of  $N$ ?

**Rugged Landscape with Many possibilities**

## Some phenomenological Models

$$\Delta G_{\text{BW}}(N_{\text{R}}) \approx -\Delta f(T)N_{\text{R}} + \gamma a^2 N_{\text{R}}^{2/3}$$

$$N_{\text{R}}^* \approx (8\pi\gamma a^2/3 \Delta f(T))^3$$

$N_{\text{R}}^*$  too large for typical  $\gamma$  and  $\Delta f(T)$  values

$$\Delta G_{\text{GT}}(N_{\text{R}}) \approx \varepsilon_{\text{h}}(\varepsilon_{\text{h}} - 1)N_{\text{R}}^2 + \gamma a^2 N_{\text{R}}^{2/3}$$

$$N_{\text{R}}^* \approx (8\pi\gamma a^2/\varepsilon_{\text{h}})^{3/4}$$

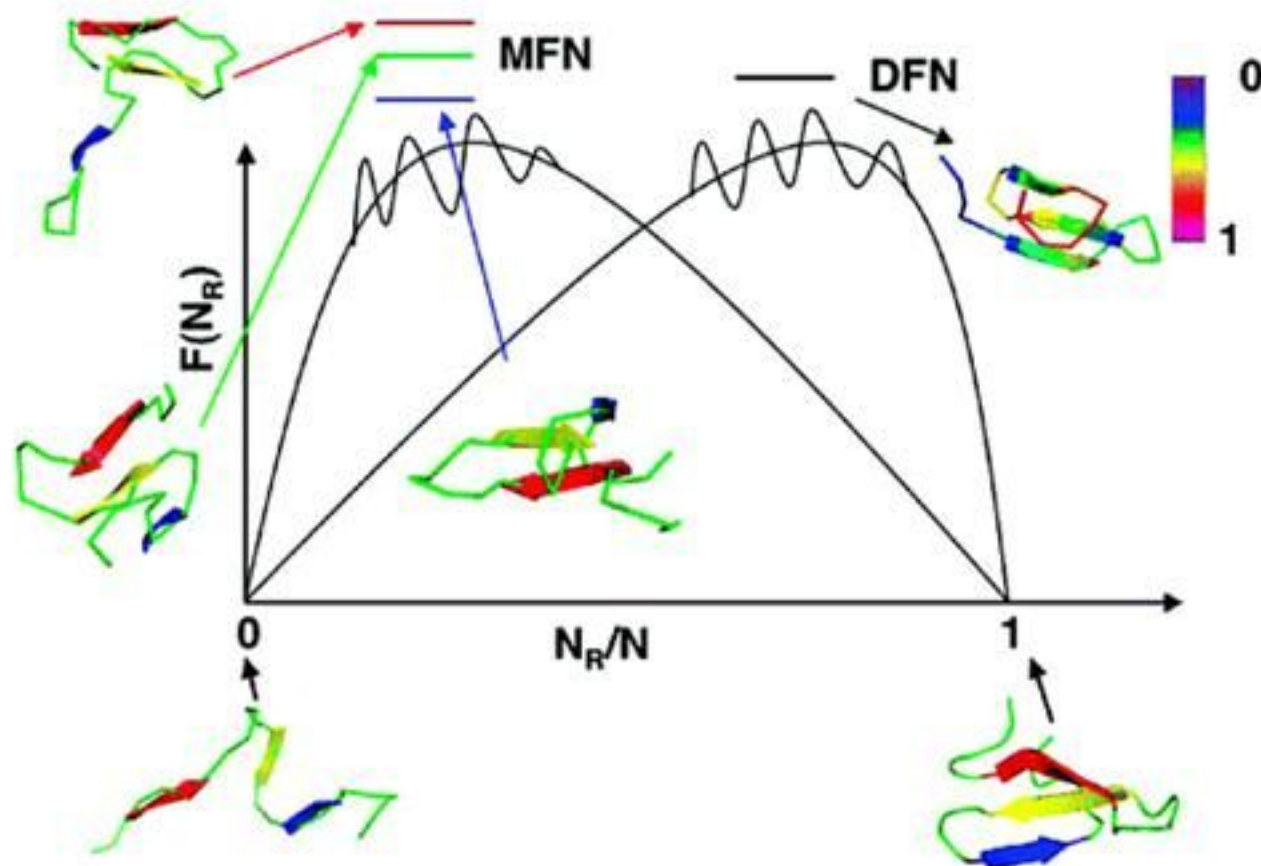
$$N_{\text{R}}^* \approx 15 \text{ or so...}$$

Using experimental parameters

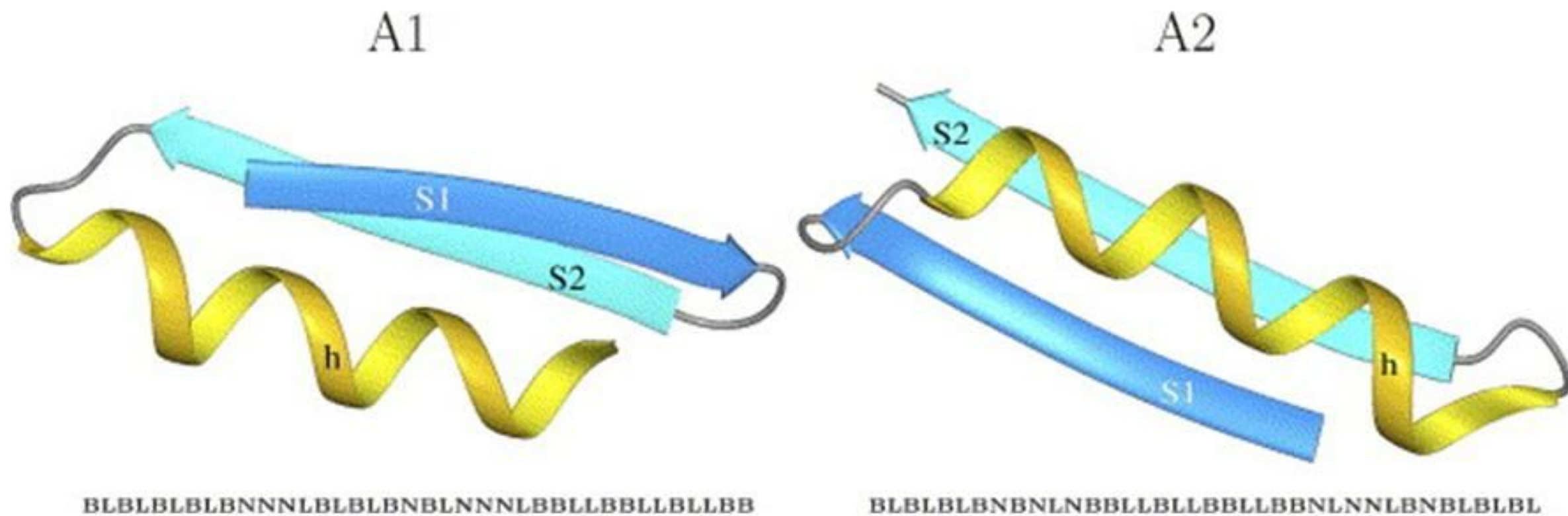
$$N_{\text{R}}^* \approx 27 \text{ or so..}$$

# Folding trajectories to MFN to transition state ensemble (TSE)

Structures near Barrier top or TSE  
Simulations

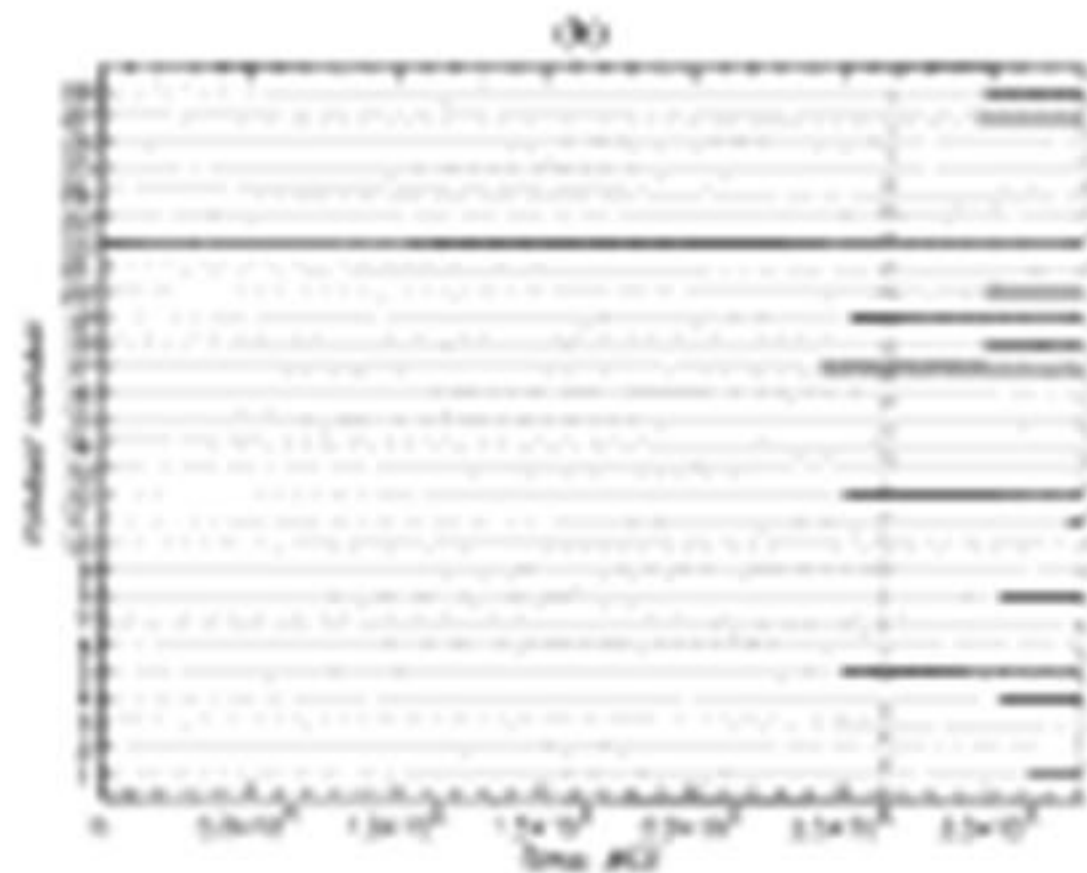
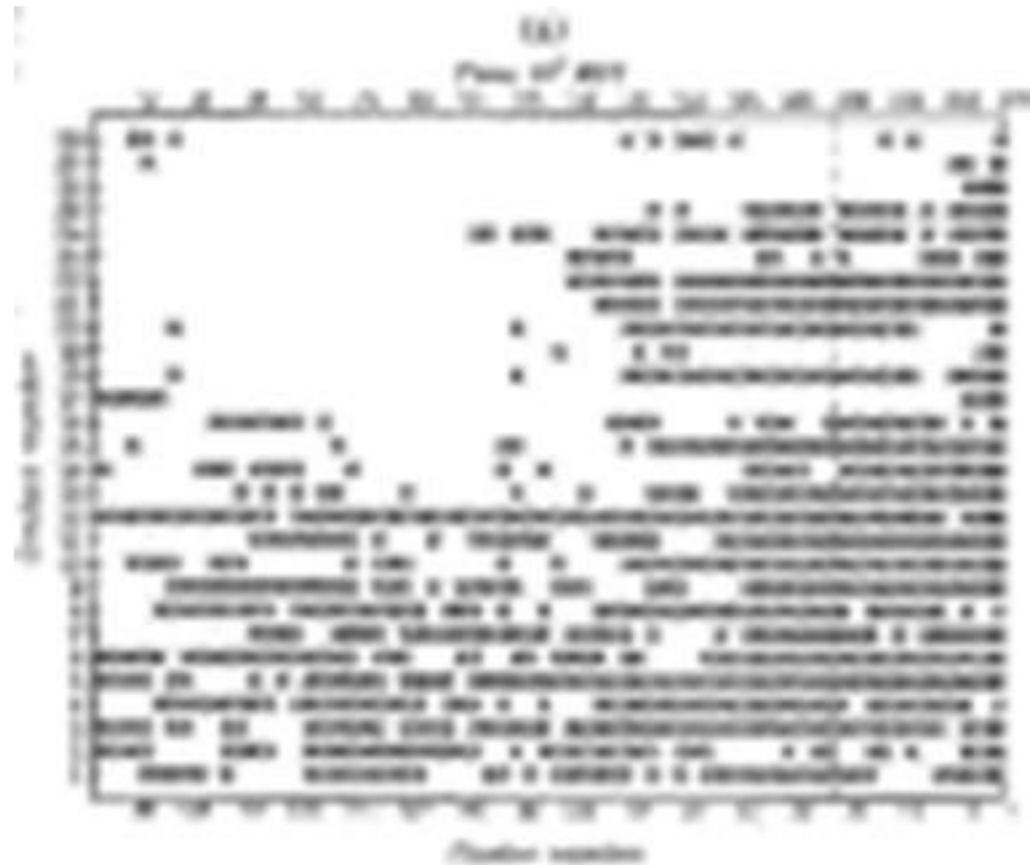


# Folding trajectories Simple Topology Brownian Dynamics Simulations

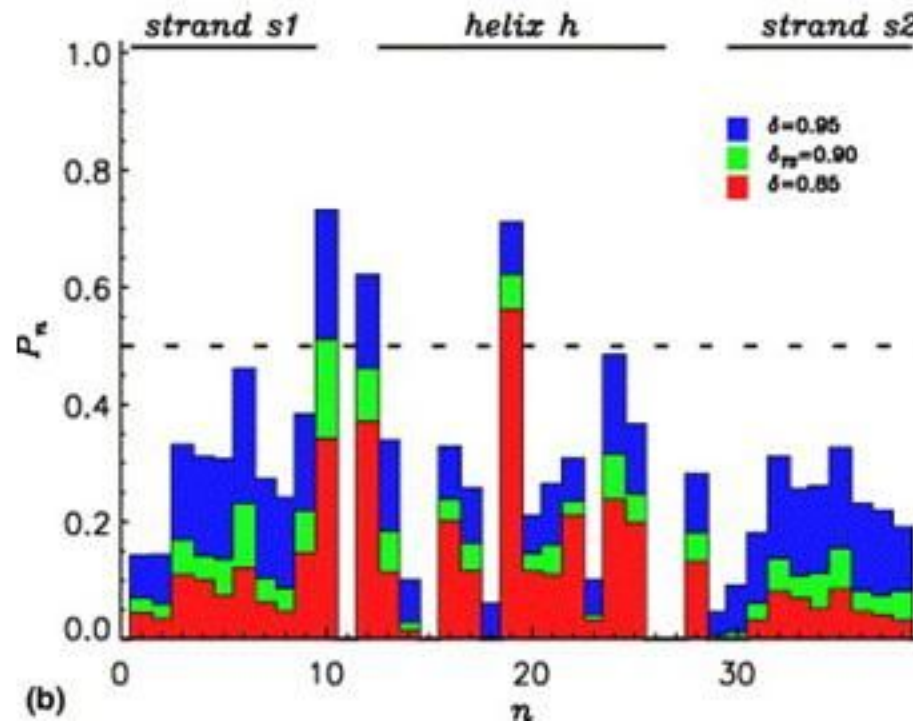
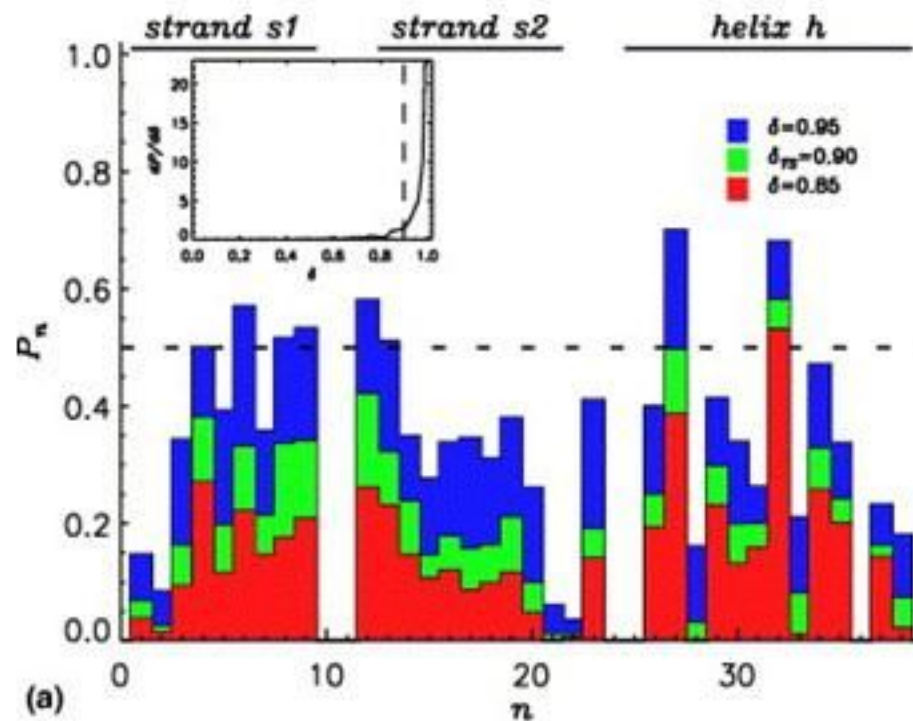




# Heterogeneity in Folding Trajectories



# Multiple Folding Nuclei (MFN) Model



No unique nucleus

Different molecule  
Follows distinct  
Growth pattern

Structural plasticity

## Refinement (Hiding Ignorance)

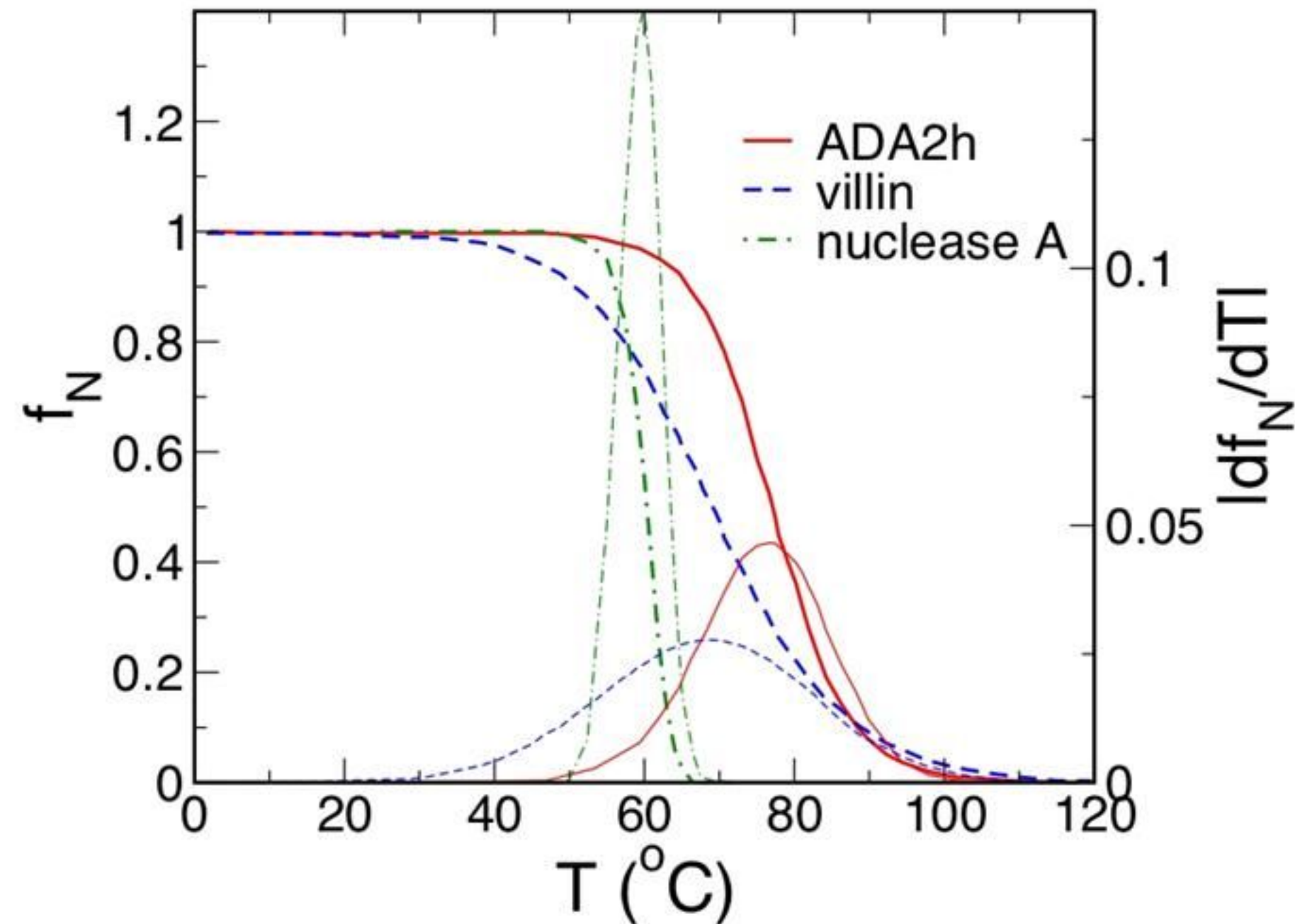
$$\Delta G(N_R) \approx -\alpha_1 N_R^\delta + \gamma N_R^\theta + S \text{ (loop)}$$

$\delta \approx \theta$  small barrier (downhill folding)

Surface tension cannot be a constant  
Multiple Folding Nuclei (Structural  
Plasticity)

# Finite Size Effects on Folding

## Order parameters matter



# Scaling of $\Omega_C$ with N (number of aa)

Two points:

1)  $T_F = \max$  in  $\Delta\chi$  (suceptibility)

$\Delta\chi = T(d\langle\chi\rangle/dh; h = \text{ordering field (analogy to mag system)}$   
 $\Delta\chi$  is dimensionless  $\Rightarrow h \sim T$  (in proteins or [C])

2) Efficient folding  $T_F \approx T_\theta$  (collapse Temp; Camacho & dt  
PNAS (1993))  $\Rightarrow \Omega_C$  controlled by protein DSE at  $T \approx T_F \approx T_\theta$

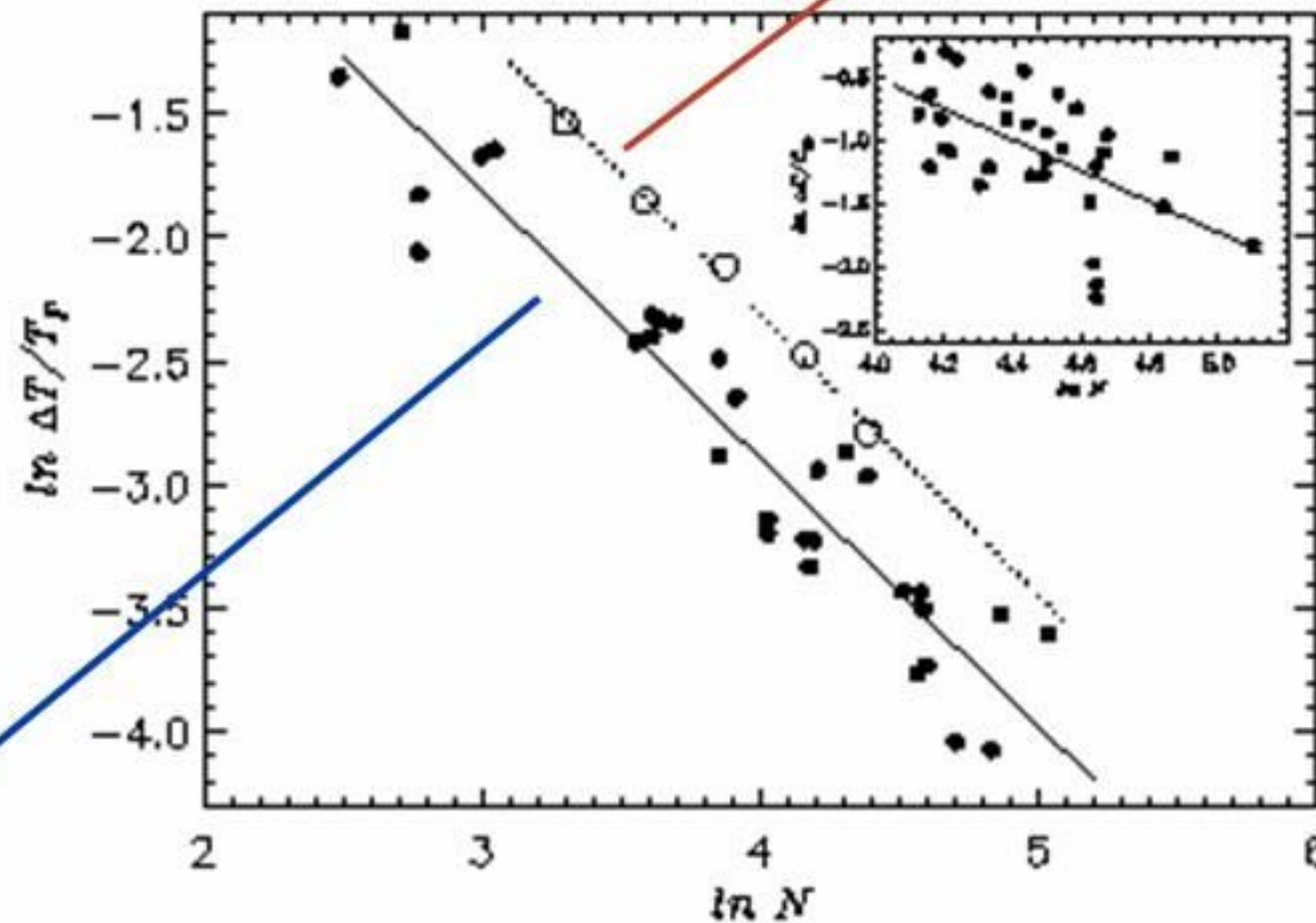
$R_g \sim (\Delta T/T_F)^{-\nu} \sim N^\nu$  (DSE a SAW & manget analogy)

$\Delta T/T_F \sim 1/N$  (Result I)

# Finite-size effects on $T_F$

Lattice models  
Side Chains

$$\Delta T/T_F \sim 1/N$$



Experiments

Li, Klimov & DT Phys. Rev. Lett. (04)

# Scaling of $\Omega_c$ with N

## Magnet-Polymer analogy

$$\Omega_c = (T_F/\Delta T) [T_F(d\langle\chi\rangle/dT)]$$

“disp in  $T_F$ ” X “susceptibility”

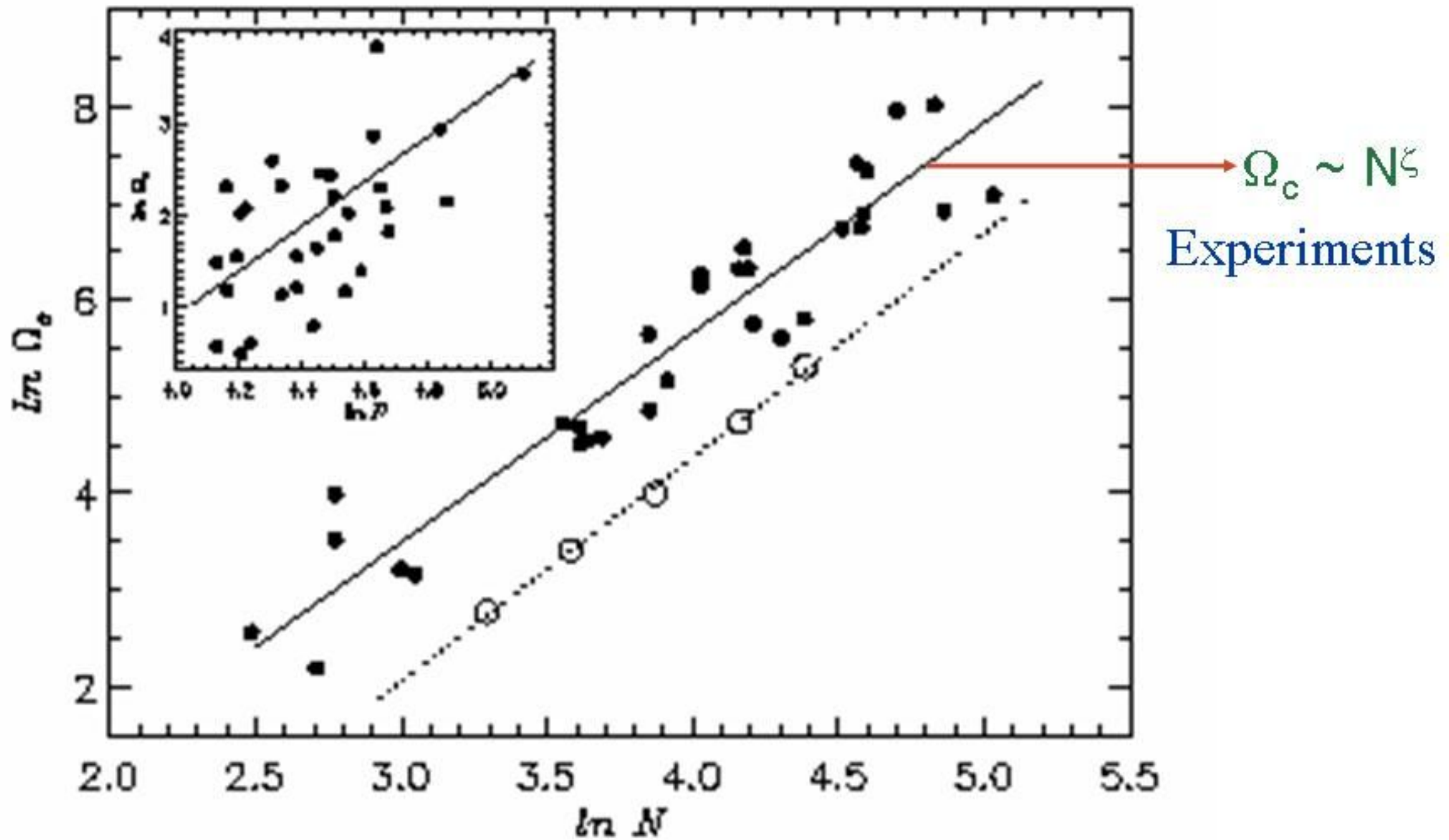
$$T \approx T_F \approx T_\theta$$

$$\Delta\chi \approx N^\gamma$$

$$\Omega_c \approx N^\zeta ; \zeta = 1 + \gamma \text{ (Universal); } \gamma \approx 1.2$$

### Result II

# Universality in Cooperativity





# Folding as Changes in Distribution Function (of what?)

O'Brien PNAS (2008)

Finite-size Fluctuations give rise to dispersions in  $T_m$

Different probes (order parameter) can give different information...

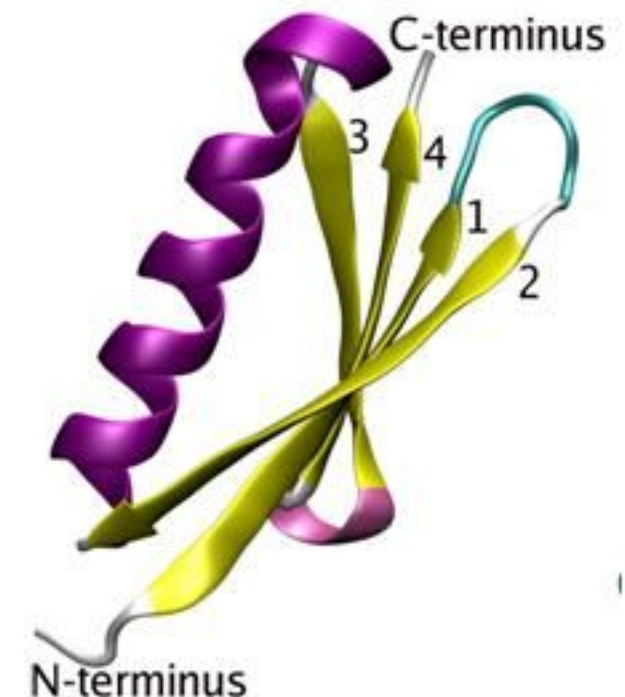
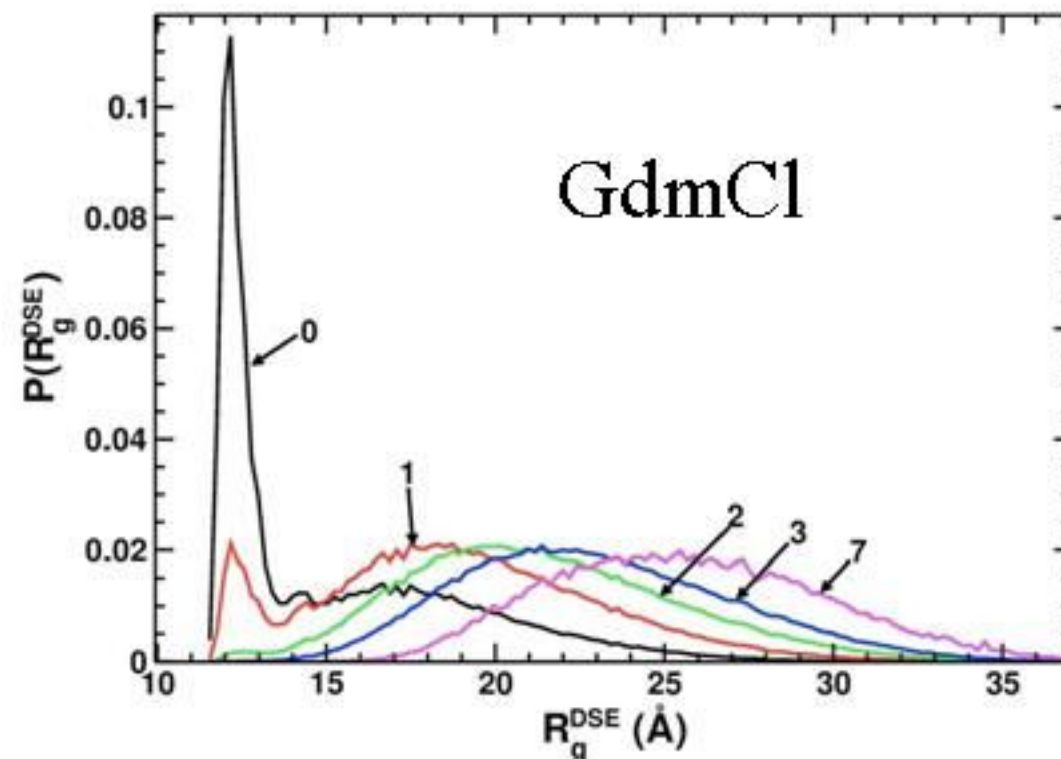


Fig. S2. Distribution of  $P(R_g^{DSE})$  for protein L at various GdmCl concentrations at 328 K. The concentration ( $[C]$ ) of GdmCl in molar units are shown in the curves.

# Order parameter description

## Macroscopic System

Ferromagnetism  $M$

Nematic Phases  $S = P_2(\cos\theta)$

Smectic Phases  $S, \text{tilt angle}$

Spin Glasses:  $M; q_{EA}$

Paramagnet  $M = 0; q_{EA} = 0$

Spin Glass  $M = 0; q_{EA} \neq 0$

Ferromagnet  $M \neq 0; q_{EA} \neq 0$

Physics dictates OP

Proteins a lot of choices  
OP is in the eye of the beholder

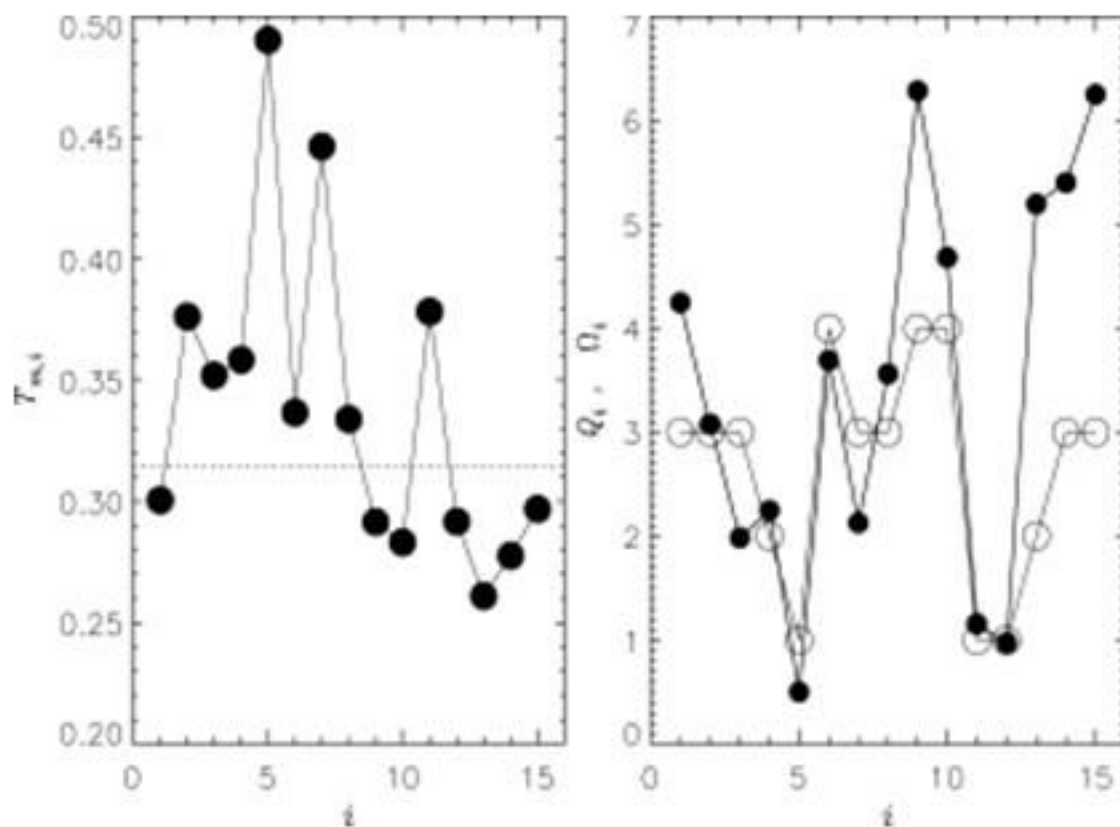
$\rho = N/R_g^3$ ;  $\chi$  (overlap)  
“unfolded” (Small, big)

Compact non-native  
(O(1), big)

Native  
(O(1), small)

Other Choices  
Helix/sheet content;  
Distribution of contacts  
.....

## Residue-dependent melting $T_m$ -Holtzer Effect

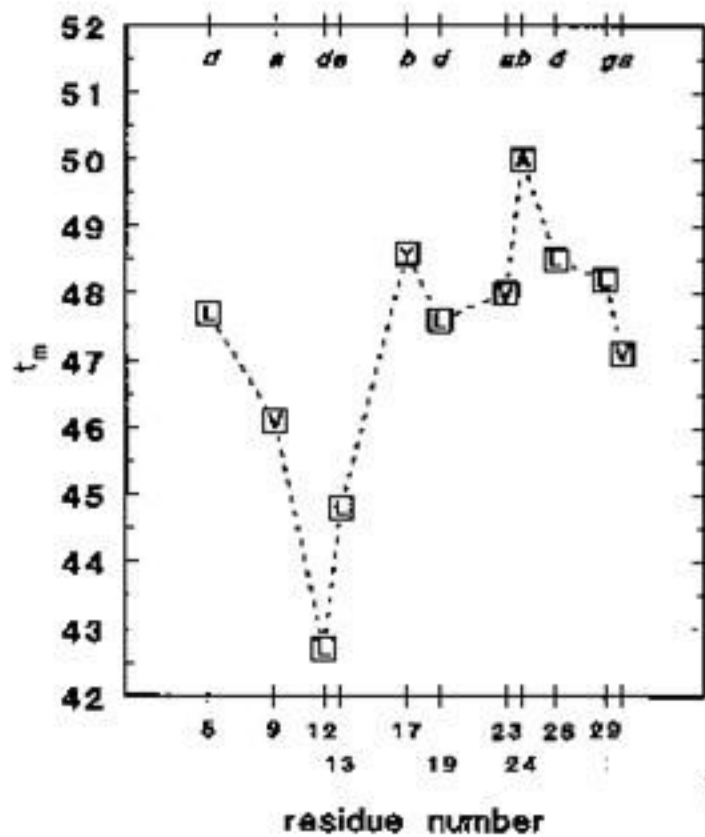


$$f_m(T_{mi}) = 0.5$$

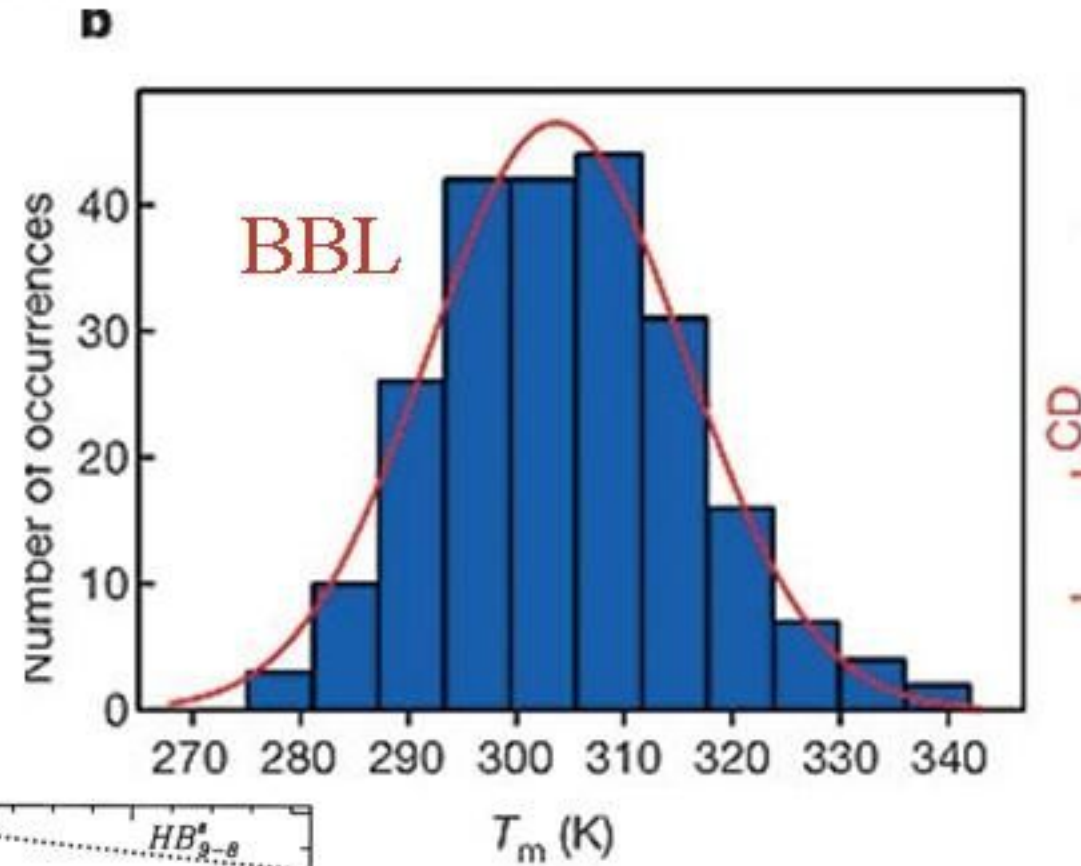
Lattice Models  
Side Chains

Klimov & dt J. Comp. Chemistry (2002)

# Is the melting temperature Unique? Finite-size effects!

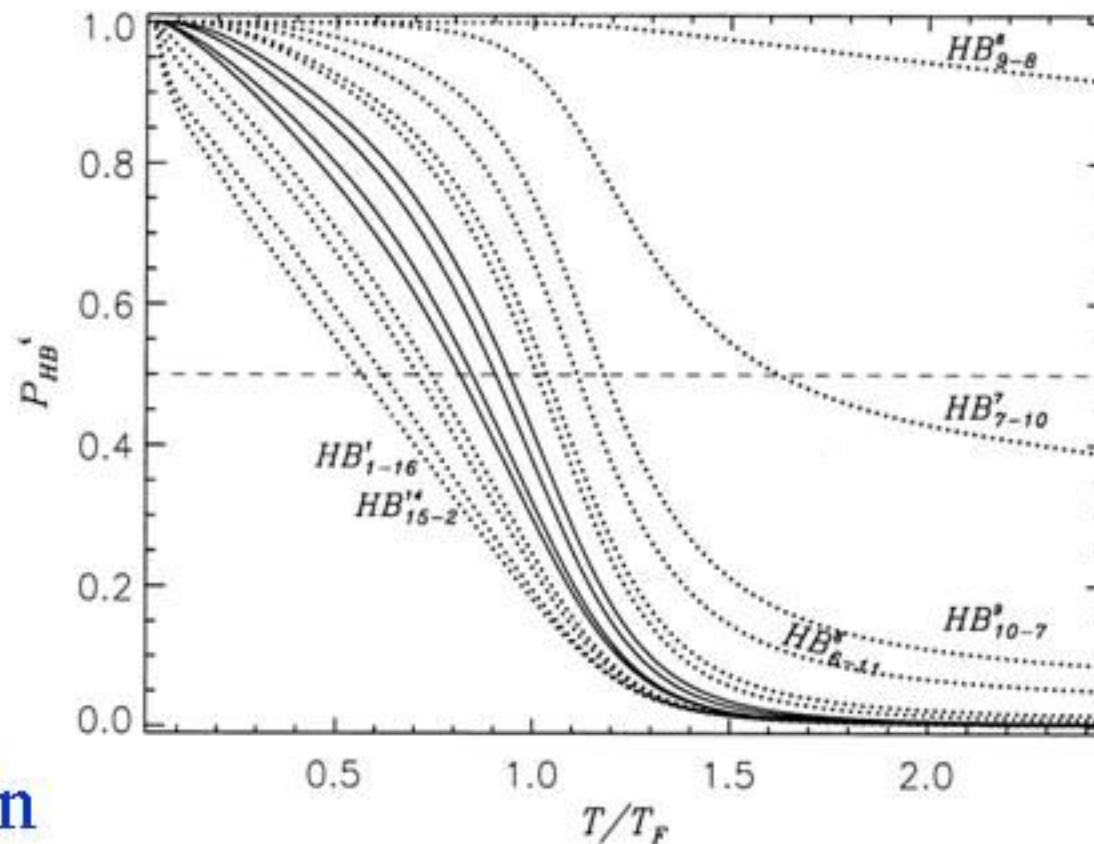


$\Delta T$  large



Holtzer  
Leucine  
Zipper  
Biophys J  
1997

Udgaonkar  
Barstar Monnelin



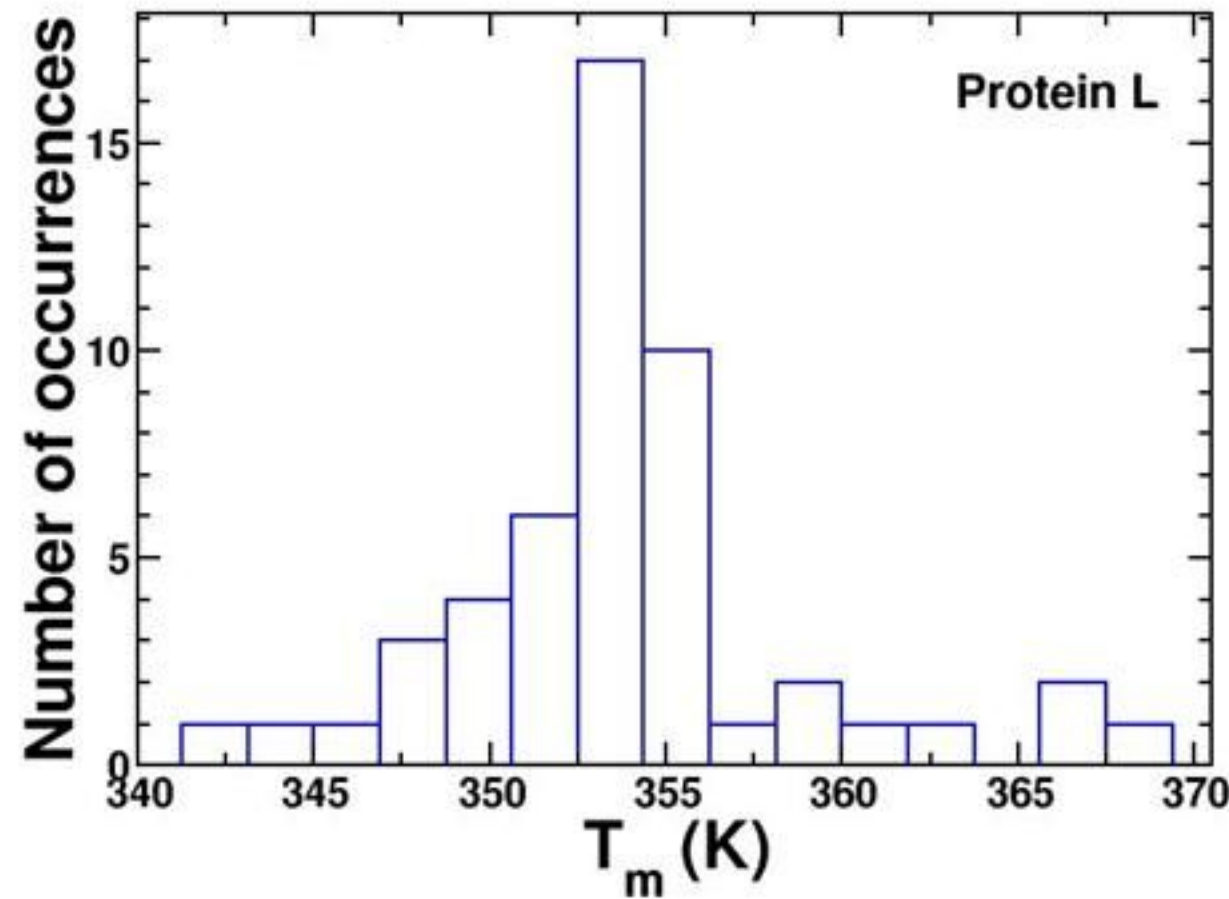
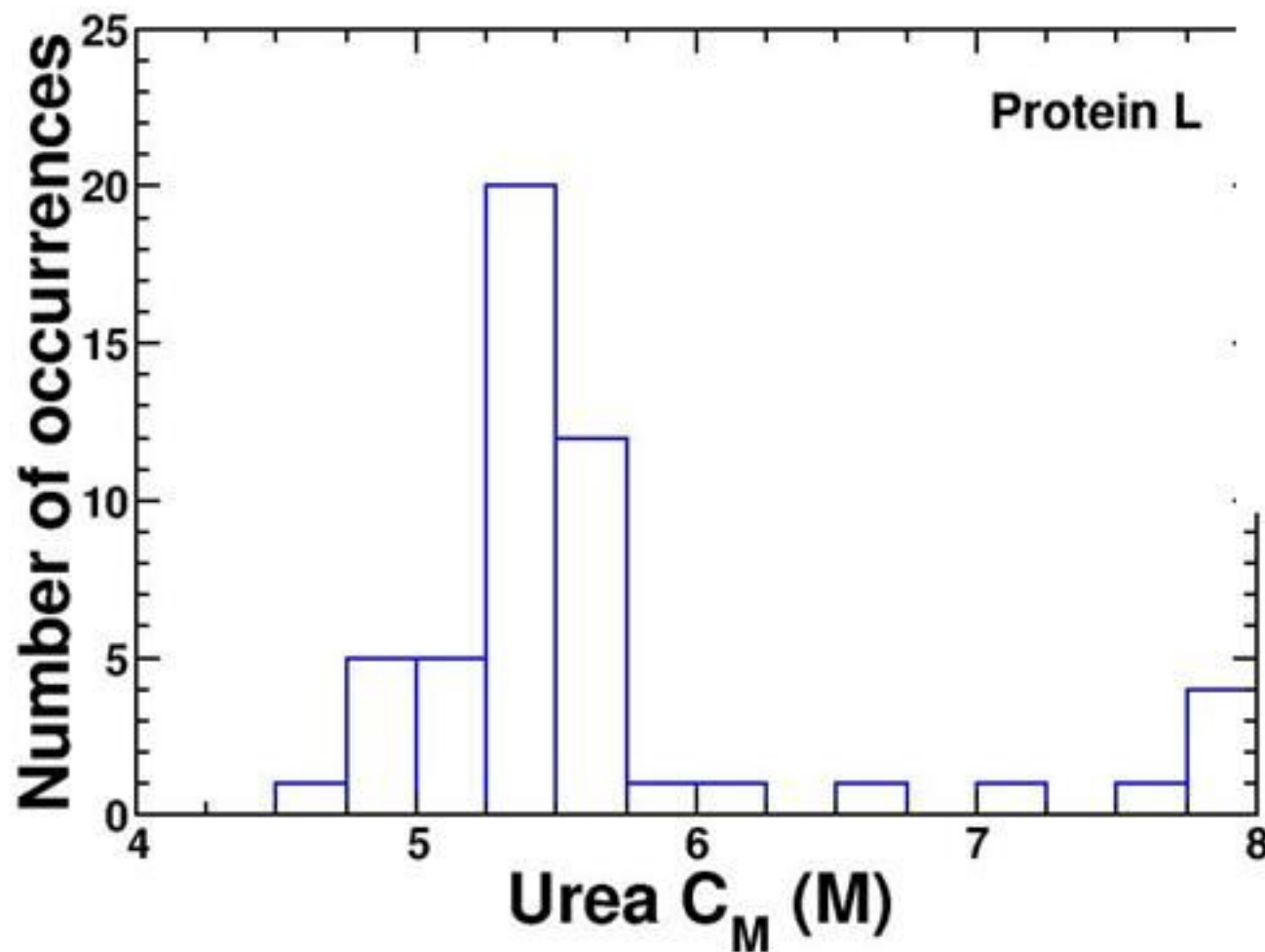
Munoz  
Nature  
2006

$\beta$ -hairpin  
PNAS 2000  
Klimov & dt

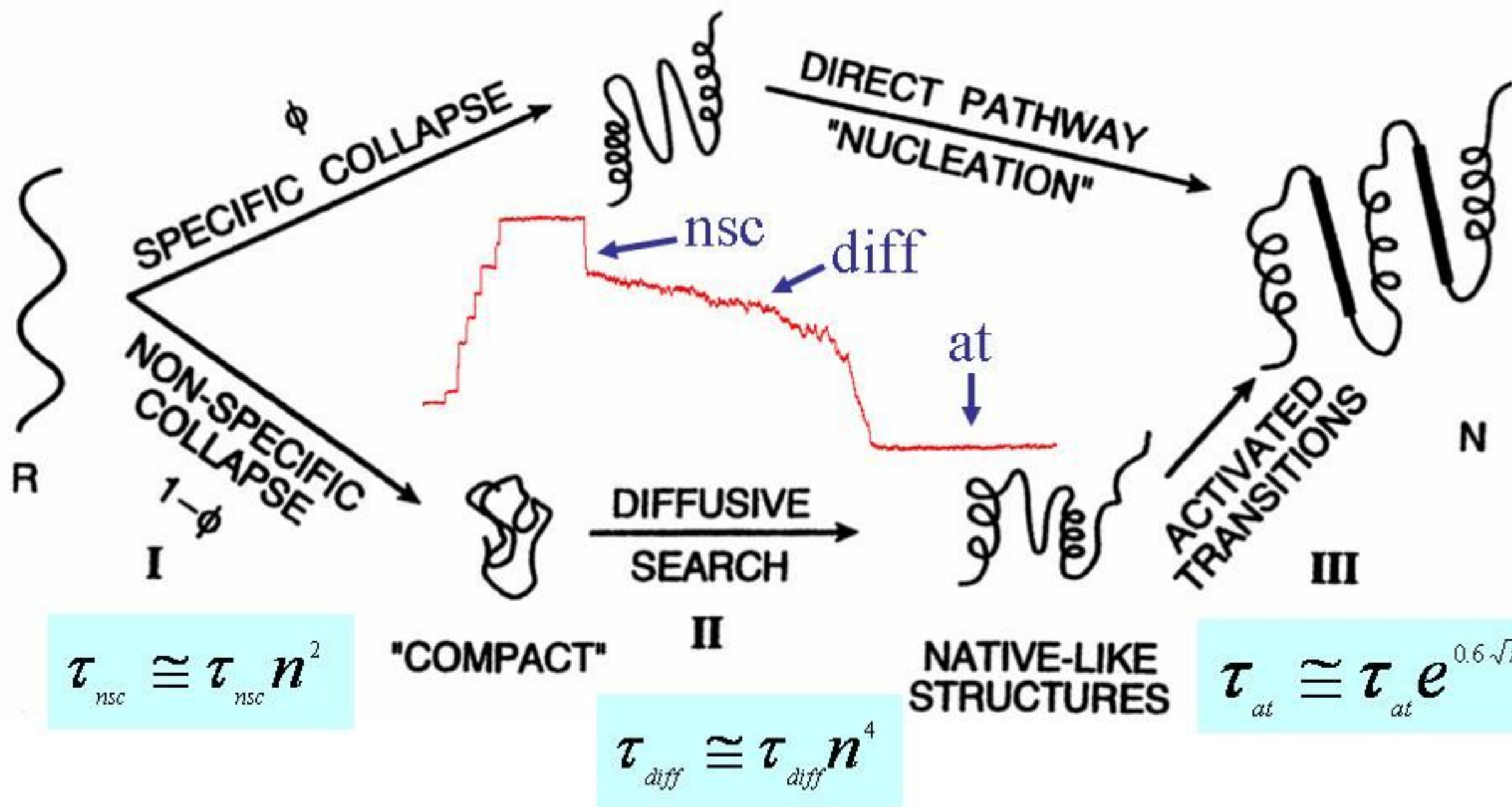
# Residue dependent ordering Protein L

O'Brien, Brooks & dt Biochemistry (2009)

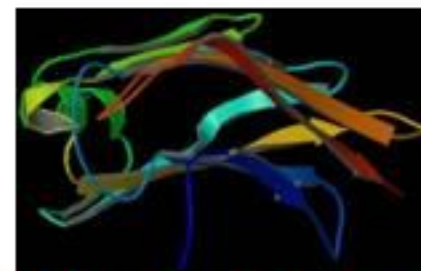
Spread decreases as  
N decreases....finite-size  
effects



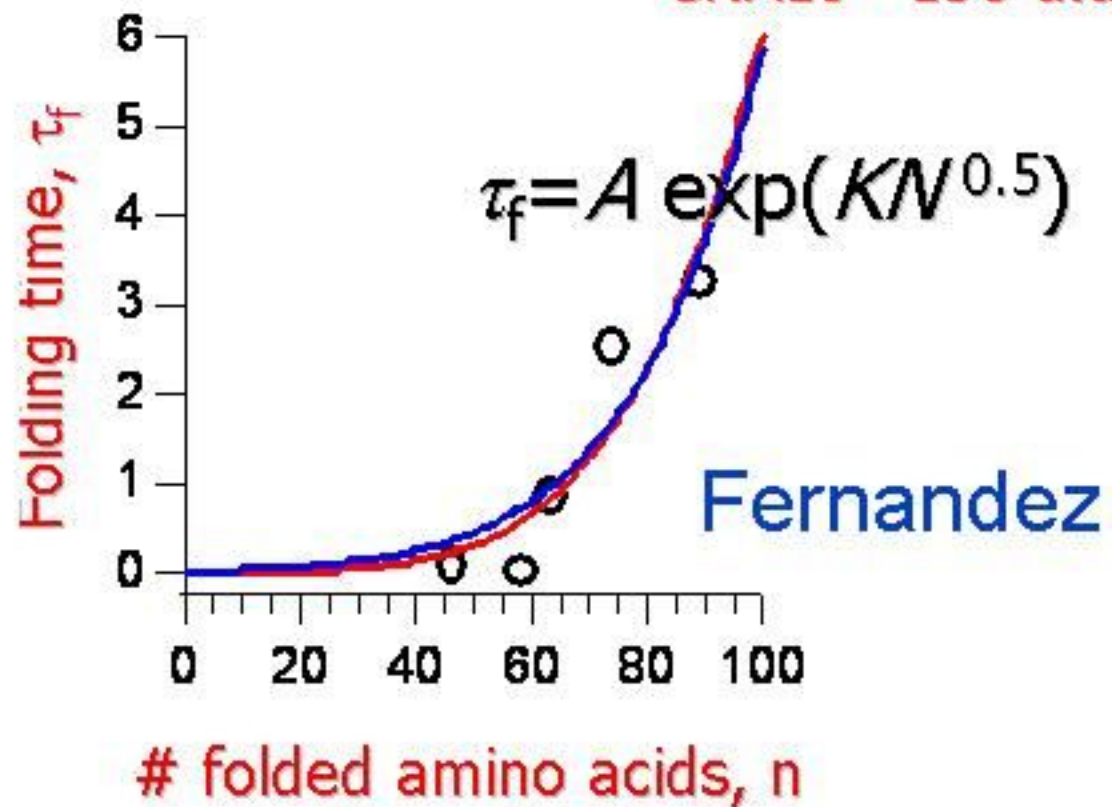
# Folding Pathways and times



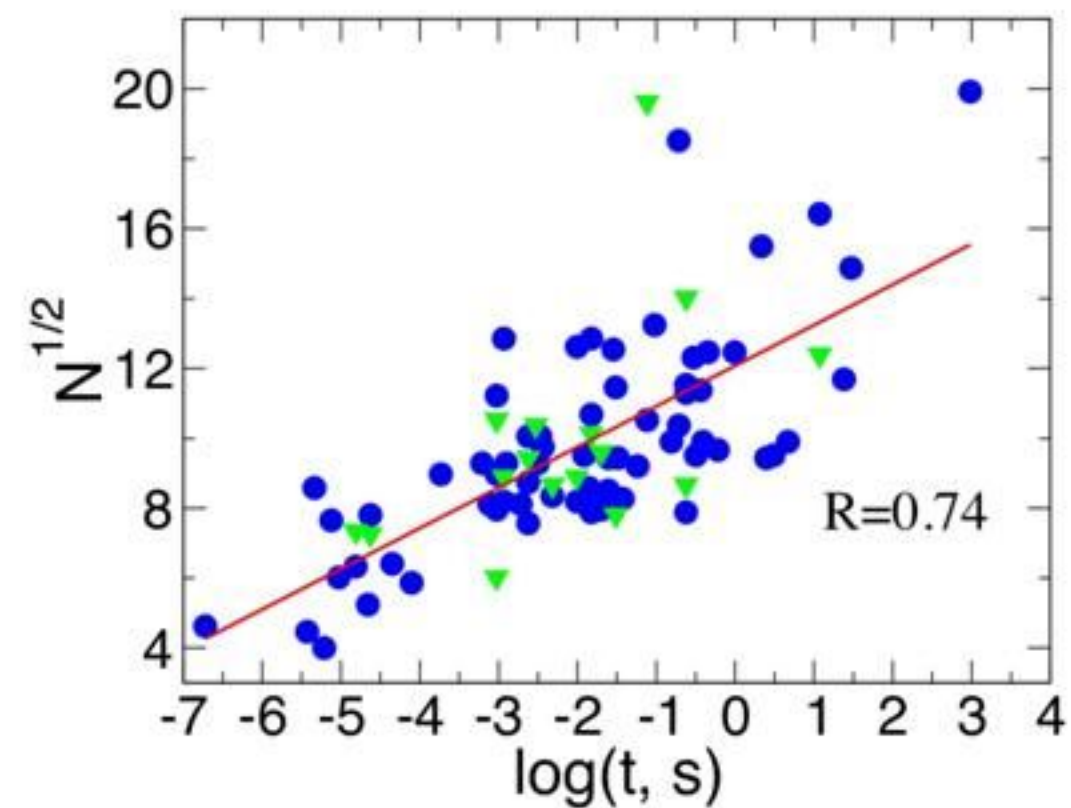
# Folding Time vs N



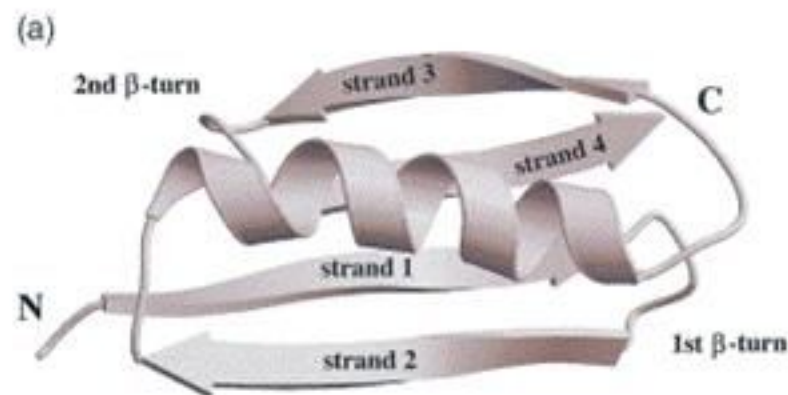
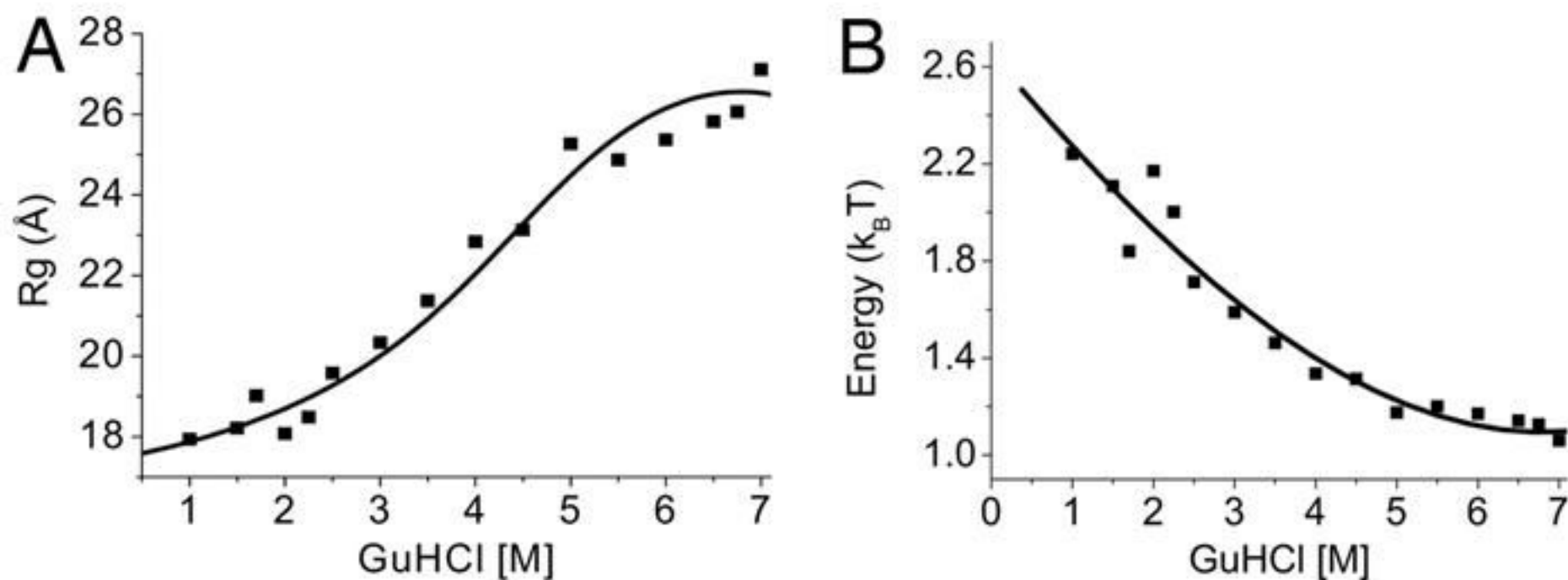
CNA19 -150 a.a.



Munoz



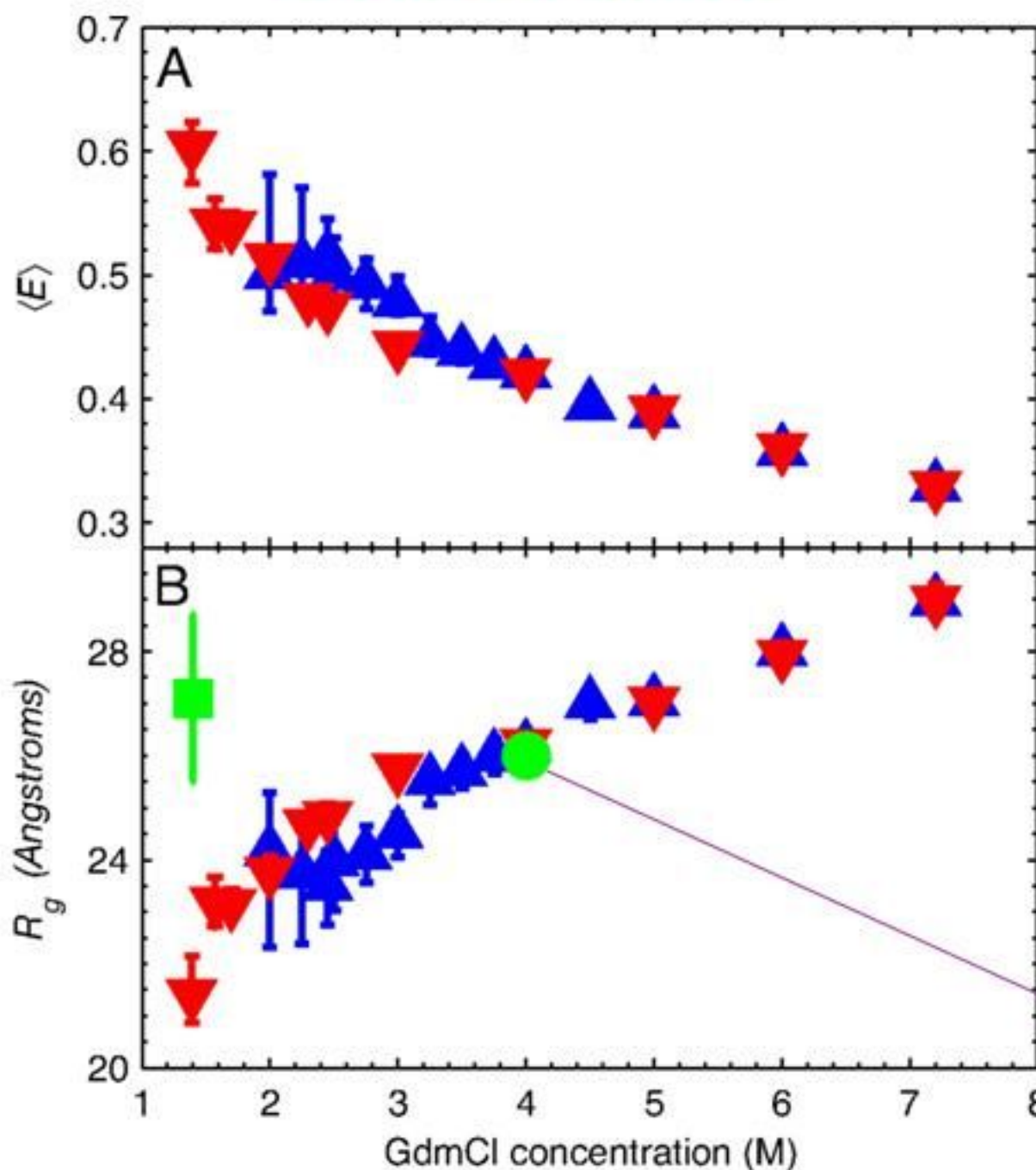
# The CG transition in denatured protein L smFRET measurements: Collapse



Sherman, Eilon and Haran, Gilad (2006) Proc. Natl. Acad. Sci. USA 103, 11539-11543



# Dependence of FRET efficiency and $R_g$ on denaturant concentration



Blue: Prot L

RED: CspTm

SAXS  
(1999)

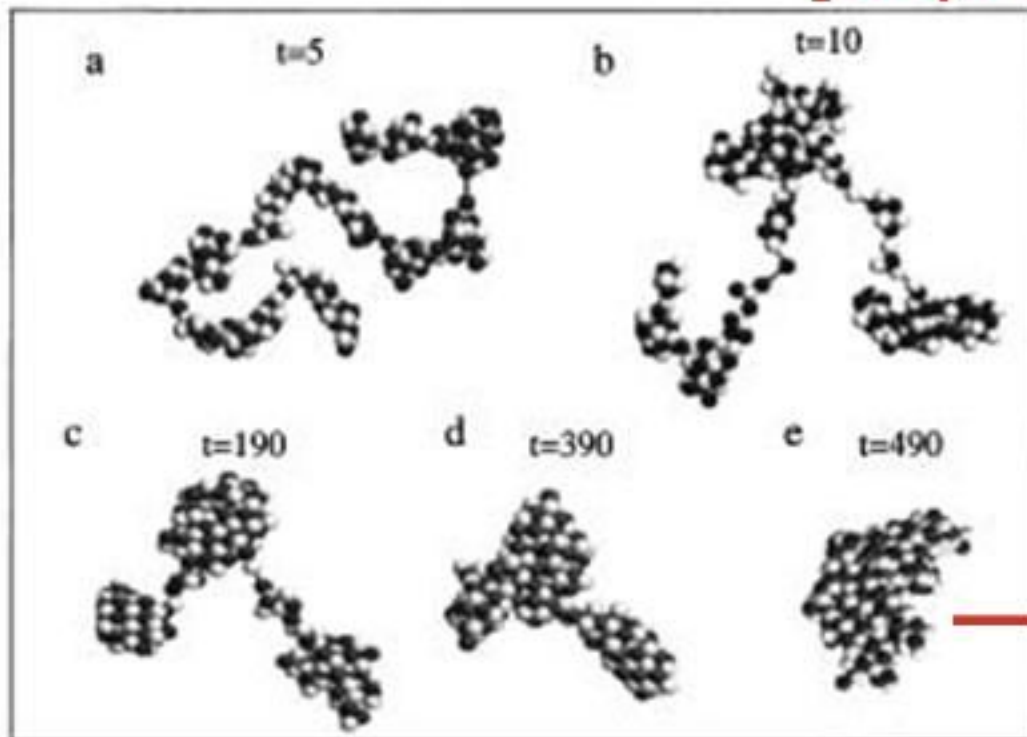
Similar N  
(65 and 66)

Merchant, Kusai A. et al. (2007) Proc. Natl. Acad. Sci. USA 104, 1528-1533

# Collapse Kinetics of Polyampholytes and polyelectrolytes

Lee and DT  
JCP (2000)

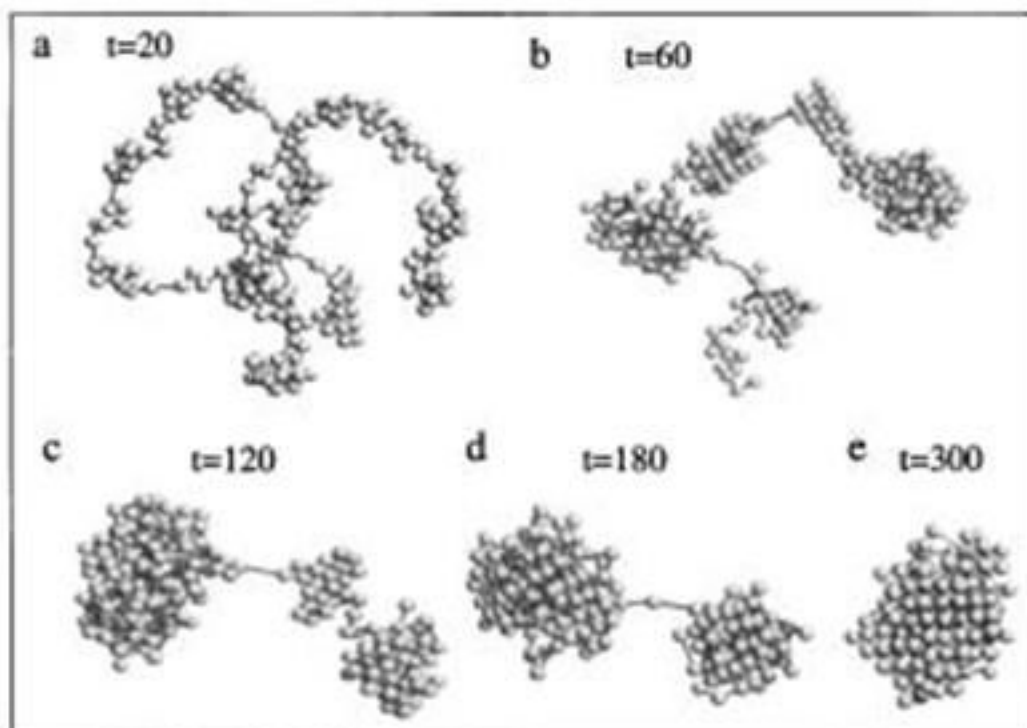
PA



$Q < Q_c = \pm\sqrt{N} e PA$   
collapses (Kardar,  
Rubenstein, Obukhov..)

Wigner crystal

PE



Two-stage transition:

1) Pearl-necklace structures form.

2) Pearls merge to form Globules.

## Second Stage: Globules grow by Lifshitz-Sluzov mechanism

*Lee and dt (2000)*

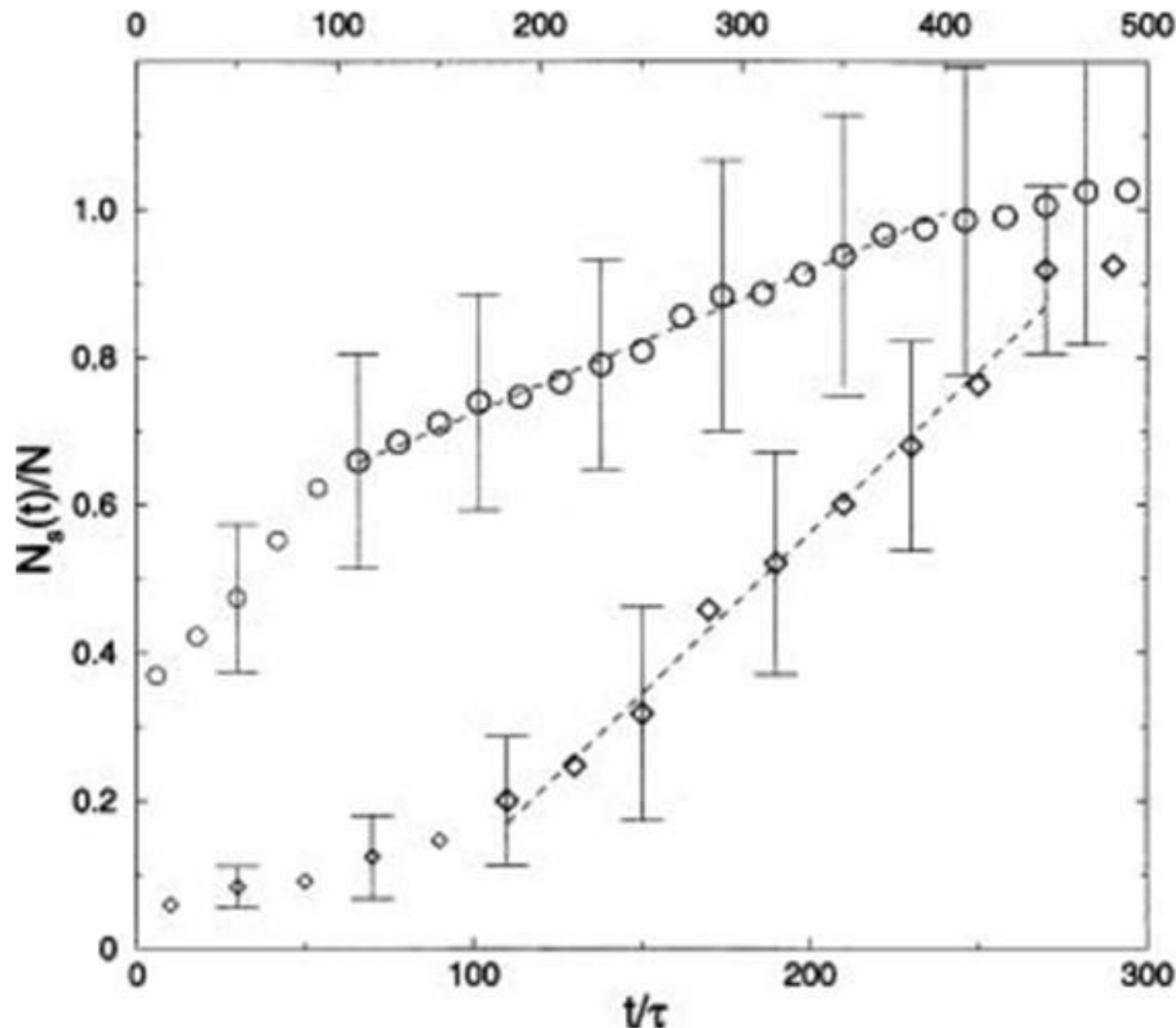
$$\tau_P \approx N^{4/5}$$

*(STAGE 1)*

$$\tau_C \approx N^\alpha$$

*(stage 2)*

*de Gennes, Orland,  
NGS, others for  
homopolymers*



# “Two-State” Folders: Stages in Collapse

Camacho and dt, PNAS (1993)

Random Coil

$\tau_C$

“Specific Collapse”

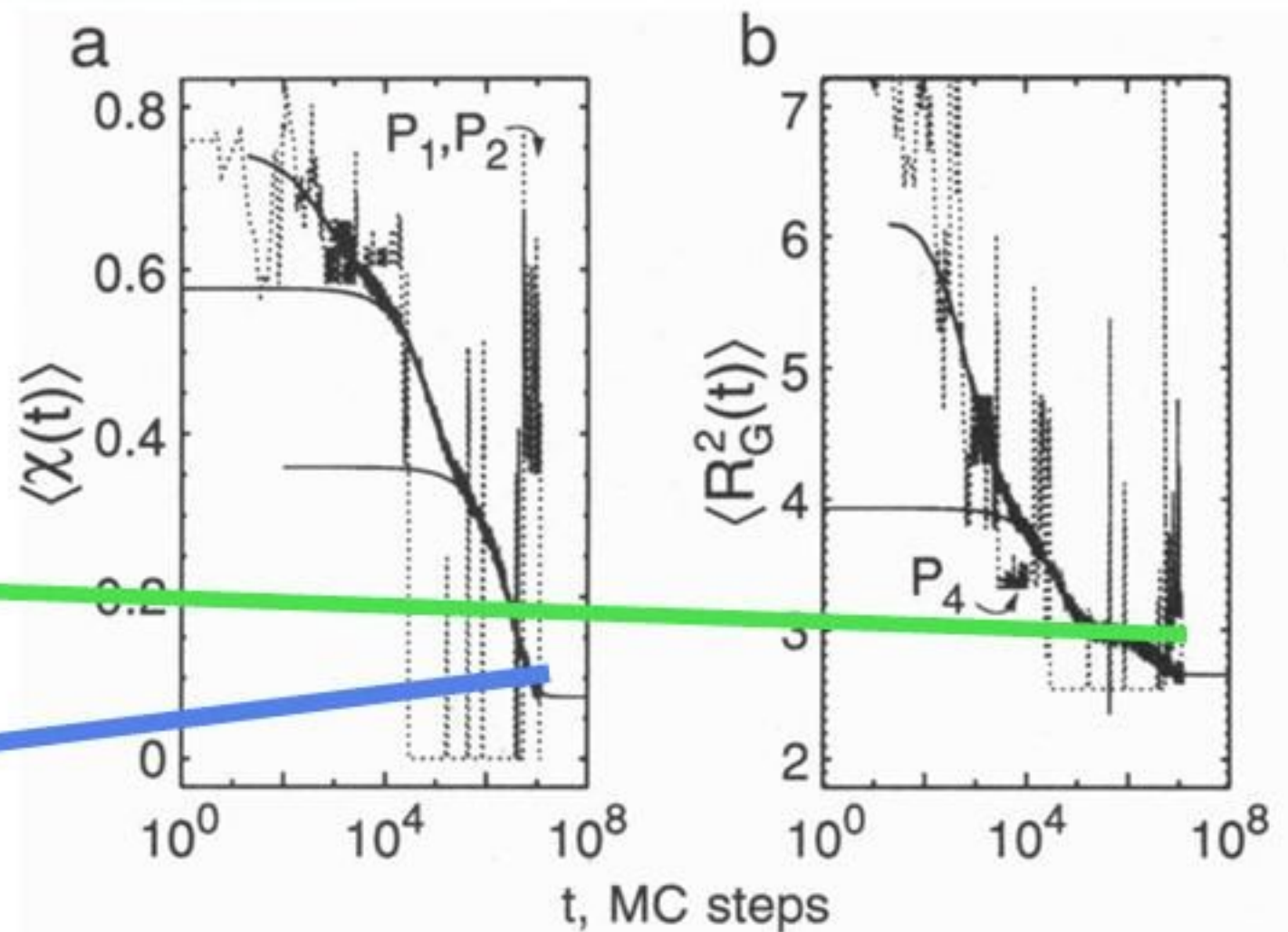
$\tau_F$

Native State

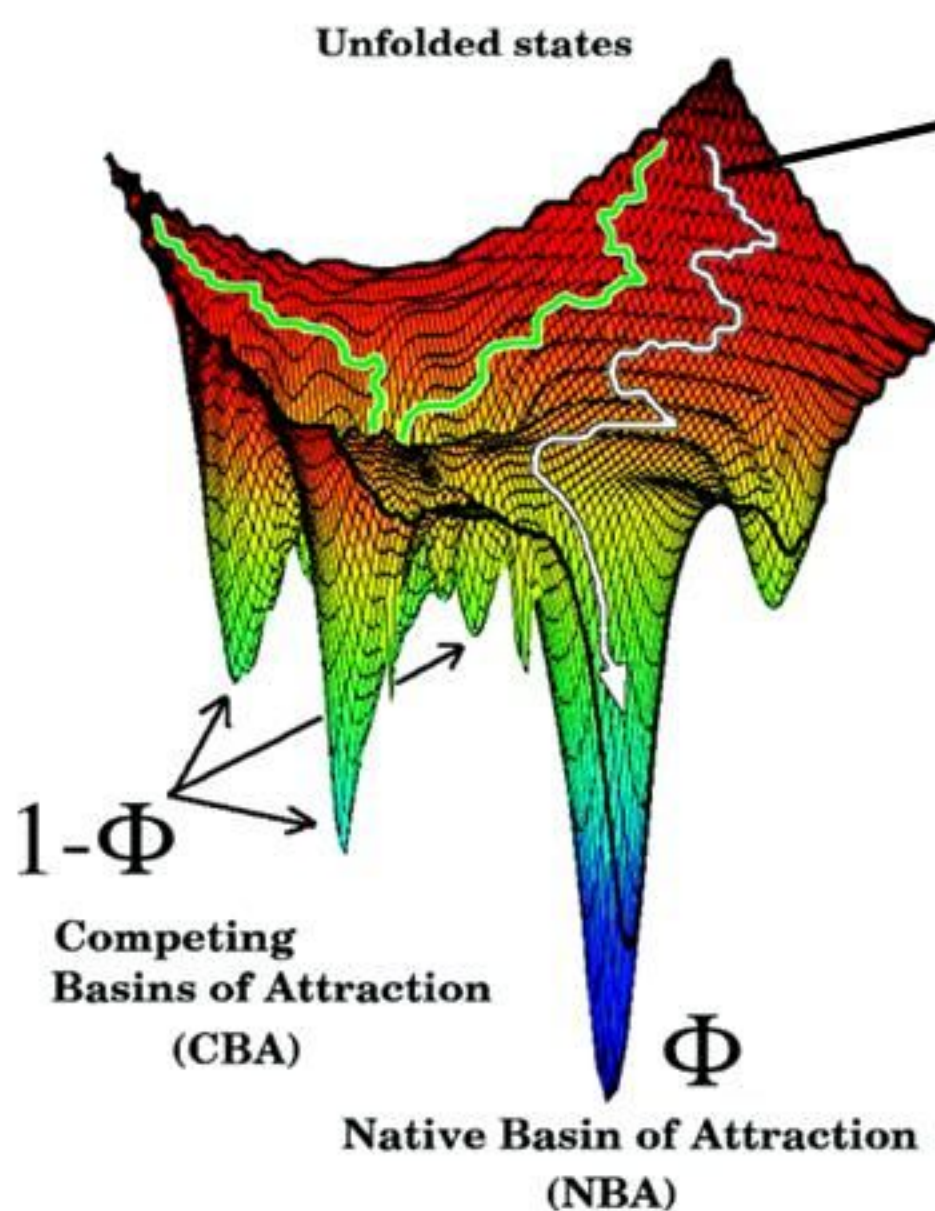
$$\tau_F / \tau_C \approx (3 - 5)$$

$\tau_C$

$\tau_F$



# Folding in rugged Landscape: KPM different collapse scenarios



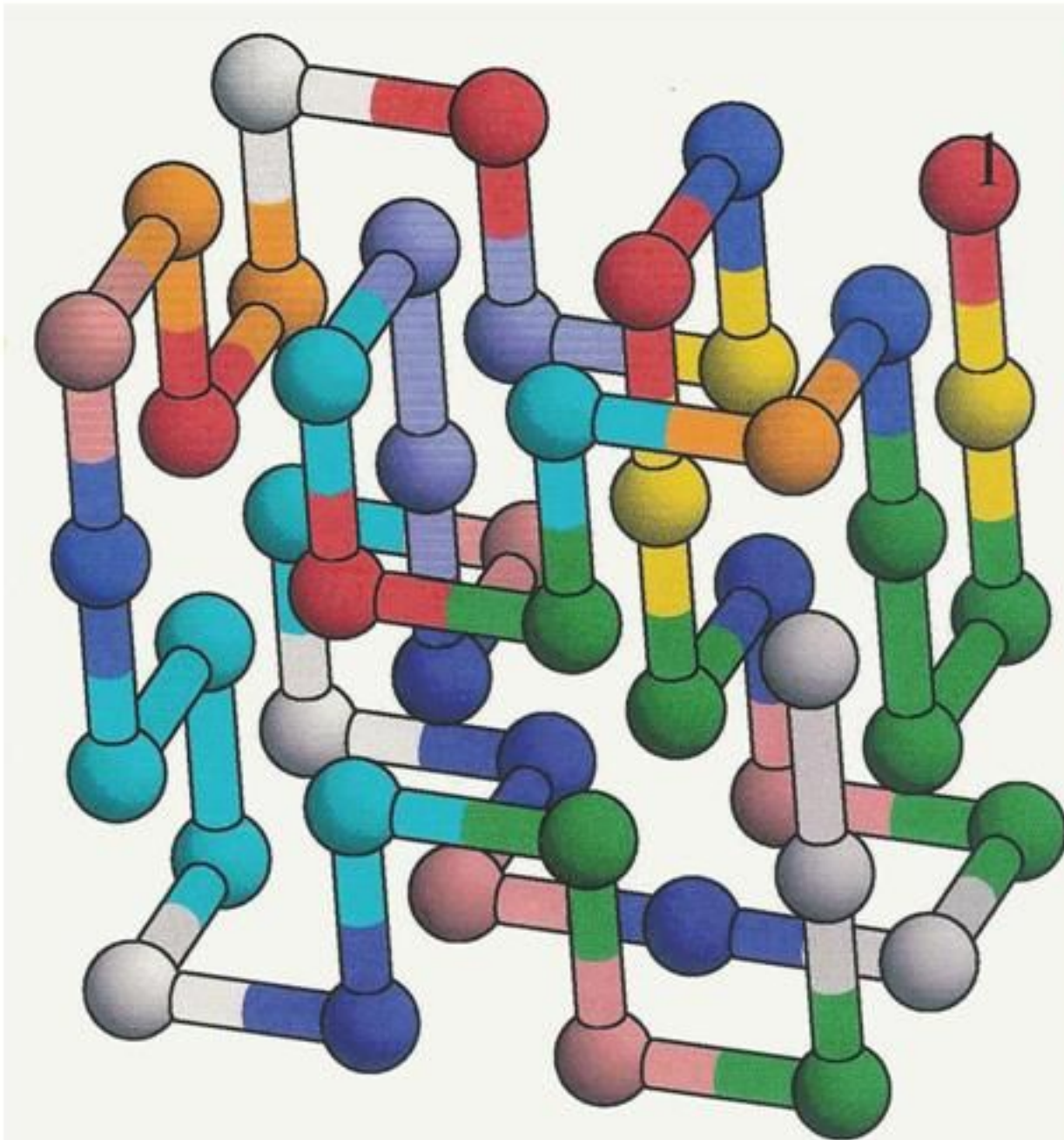
	$\Phi$	$\tau_{fast}$	Ref.
Lysozyme	0.25	50ms	PNAS (95) Kiefhaber
Rubisco	0.05	-	PNAS (96) MT/GL/DT
Pre r-RNA	0.10	1s	Pan/DT/ Woodson JMB (97)

# Lattice model with small and moderate values of $\sigma$

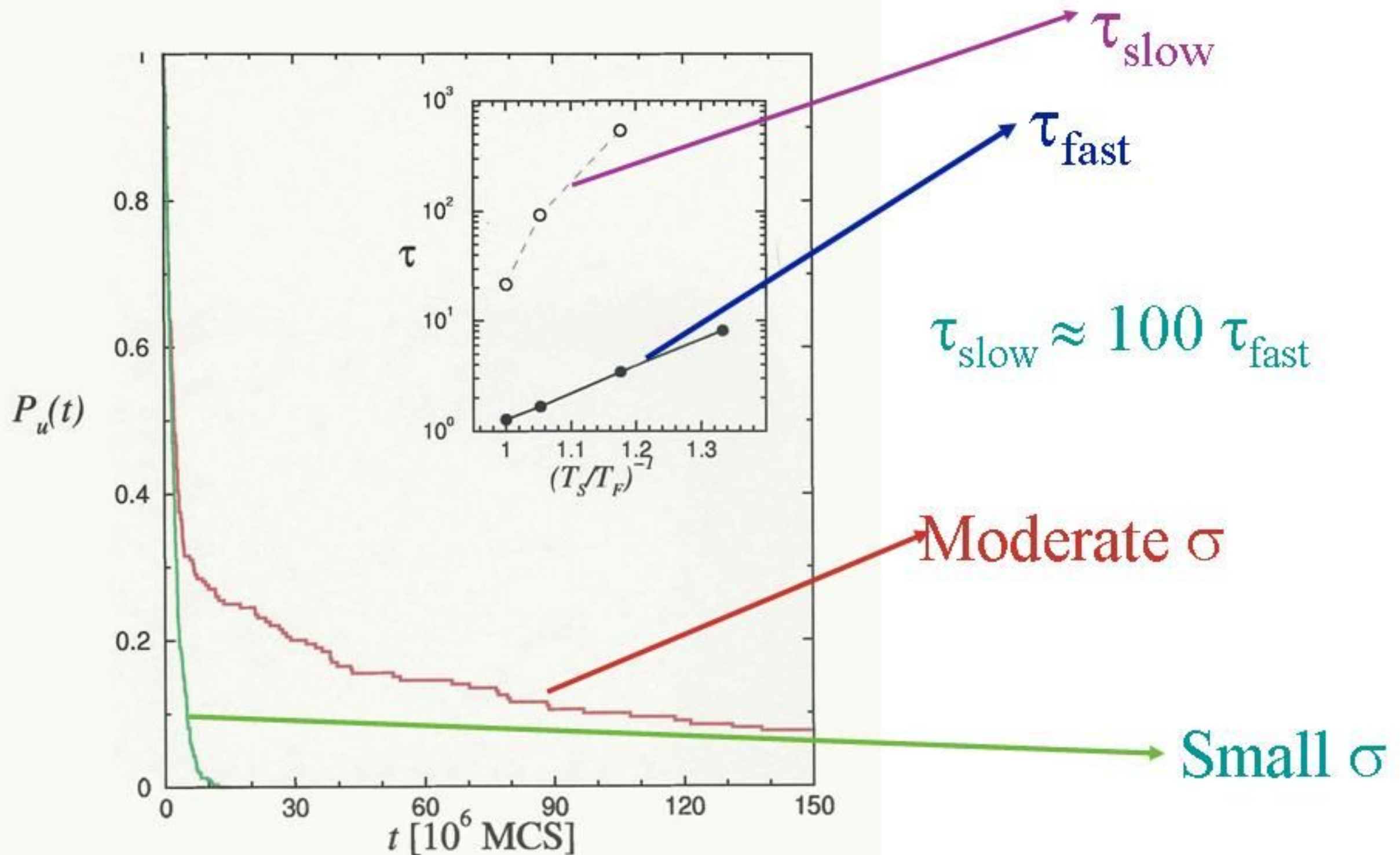
Two sequences  
Small  $\sigma$  (specific  
Collapse)

Moderate  $\sigma$  (KPM)

Fast and slow tracks



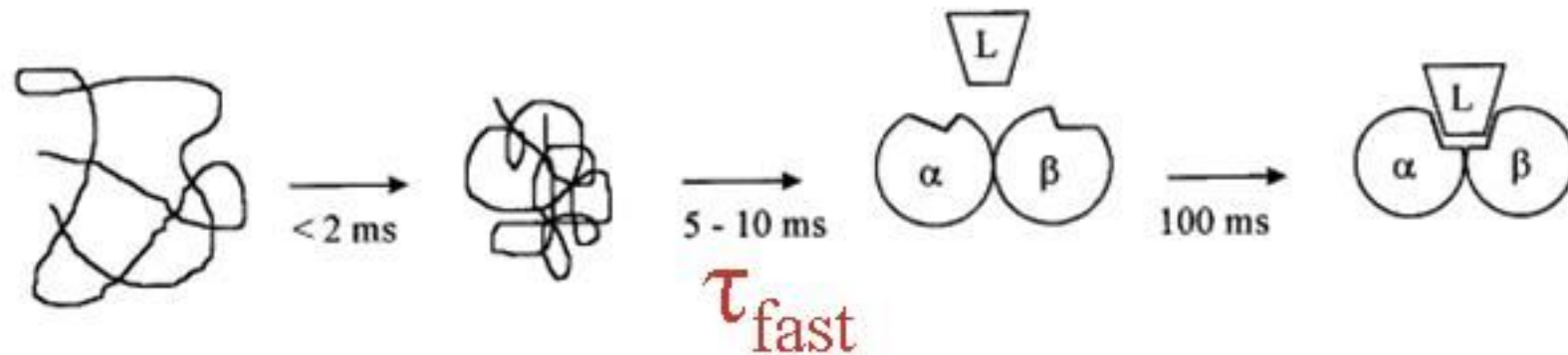
# Sequences with moderate $\sigma$ value fold by KPM



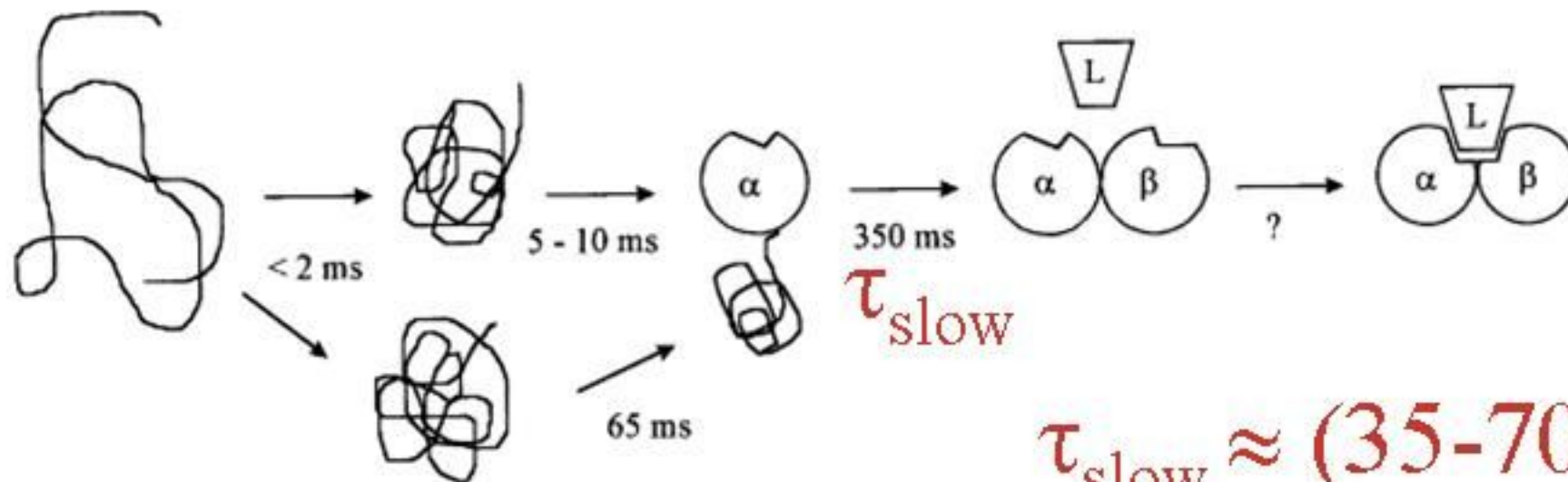
# KPM for Lysozyme

*Matagne, Radford, & Dobson JMB (1997)*

Fast Track



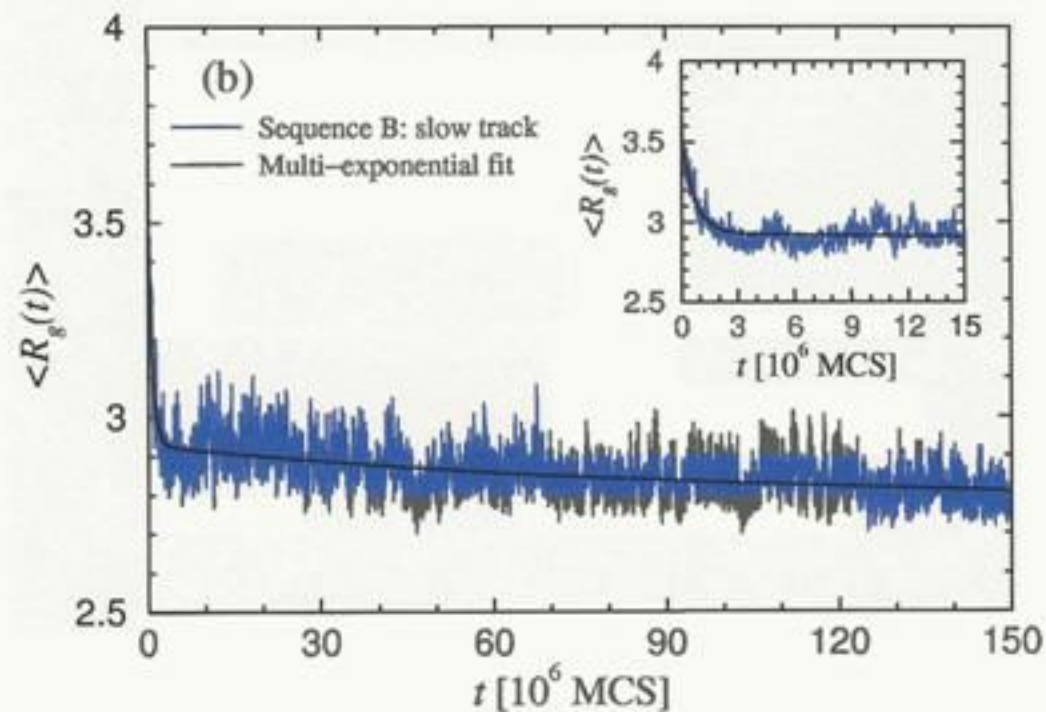
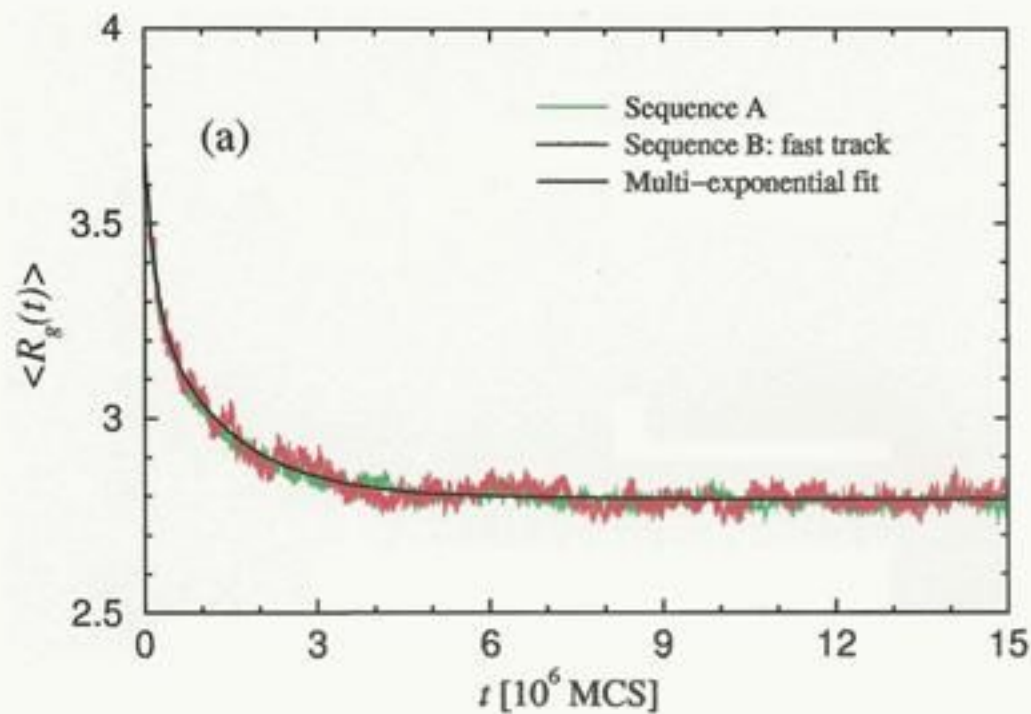
Slow Track



$\tau_{\text{slow}} \approx (35-70) \tau_{\text{fast}}$   
agrees with LM  
Simulations



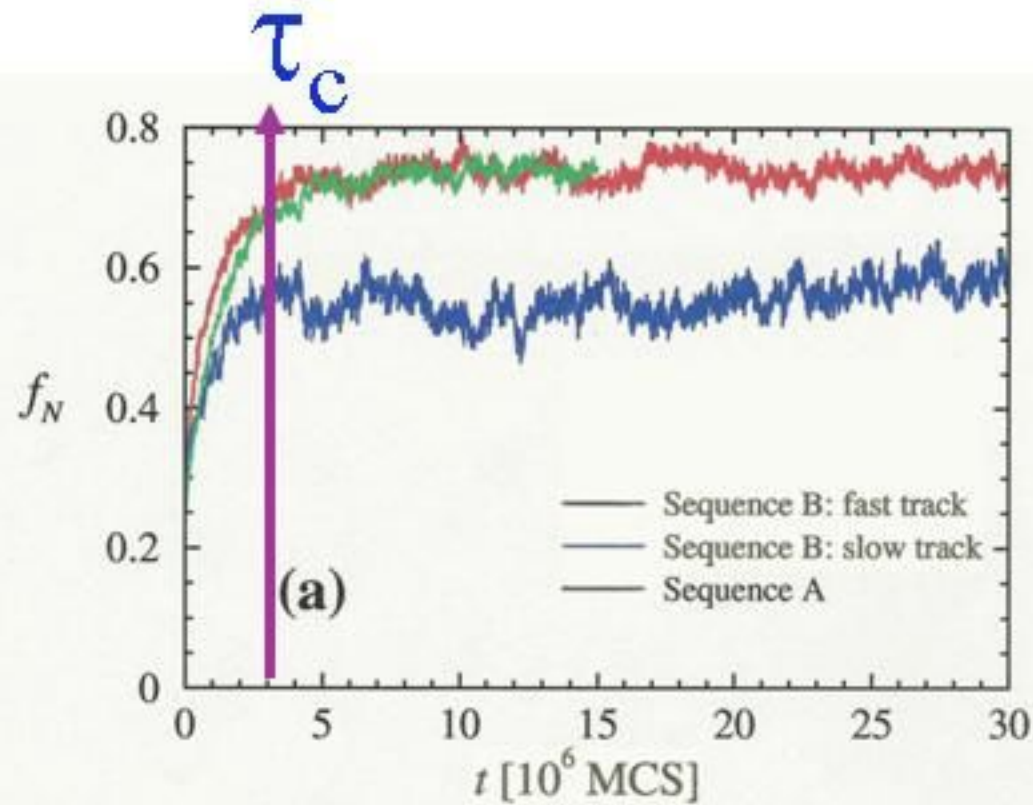
# Collapse kinetics in fast and slow track molecules are different



- Fast track collapse similar to small  $\sigma$  sequence
- Both exhibit 3 stage compaction

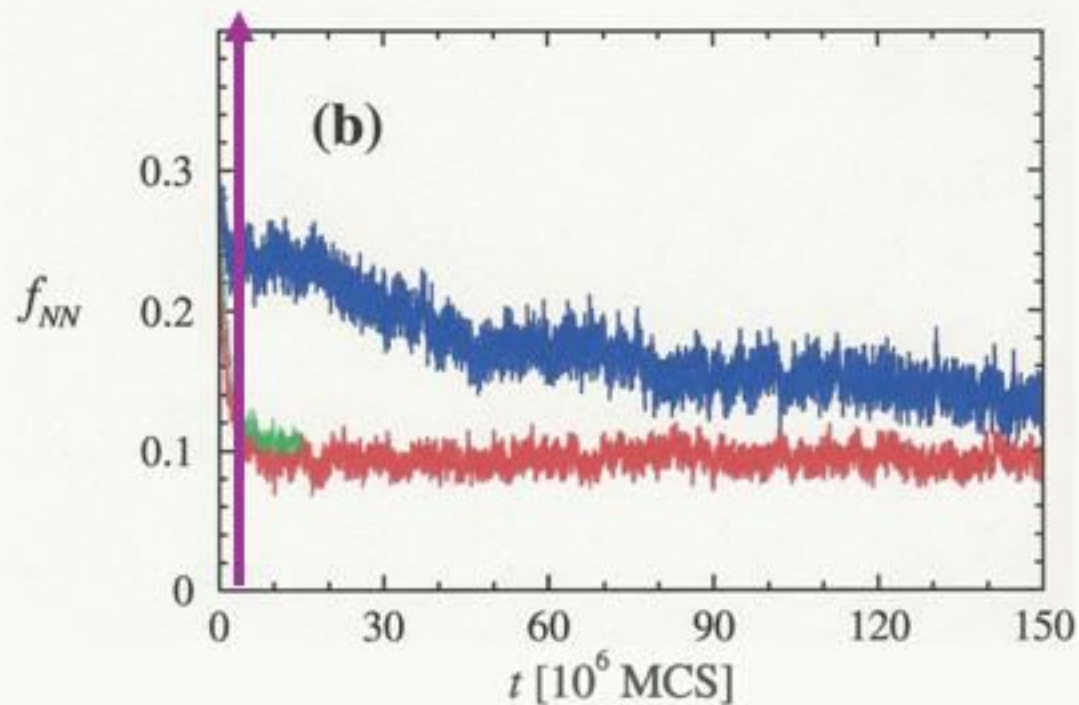
- o Collapse kinetics greatly impeded
- o Early events are similar but produce “incorrect” structures
- o Partitioning occurs early

# Fast track molecules undergo specific collapse



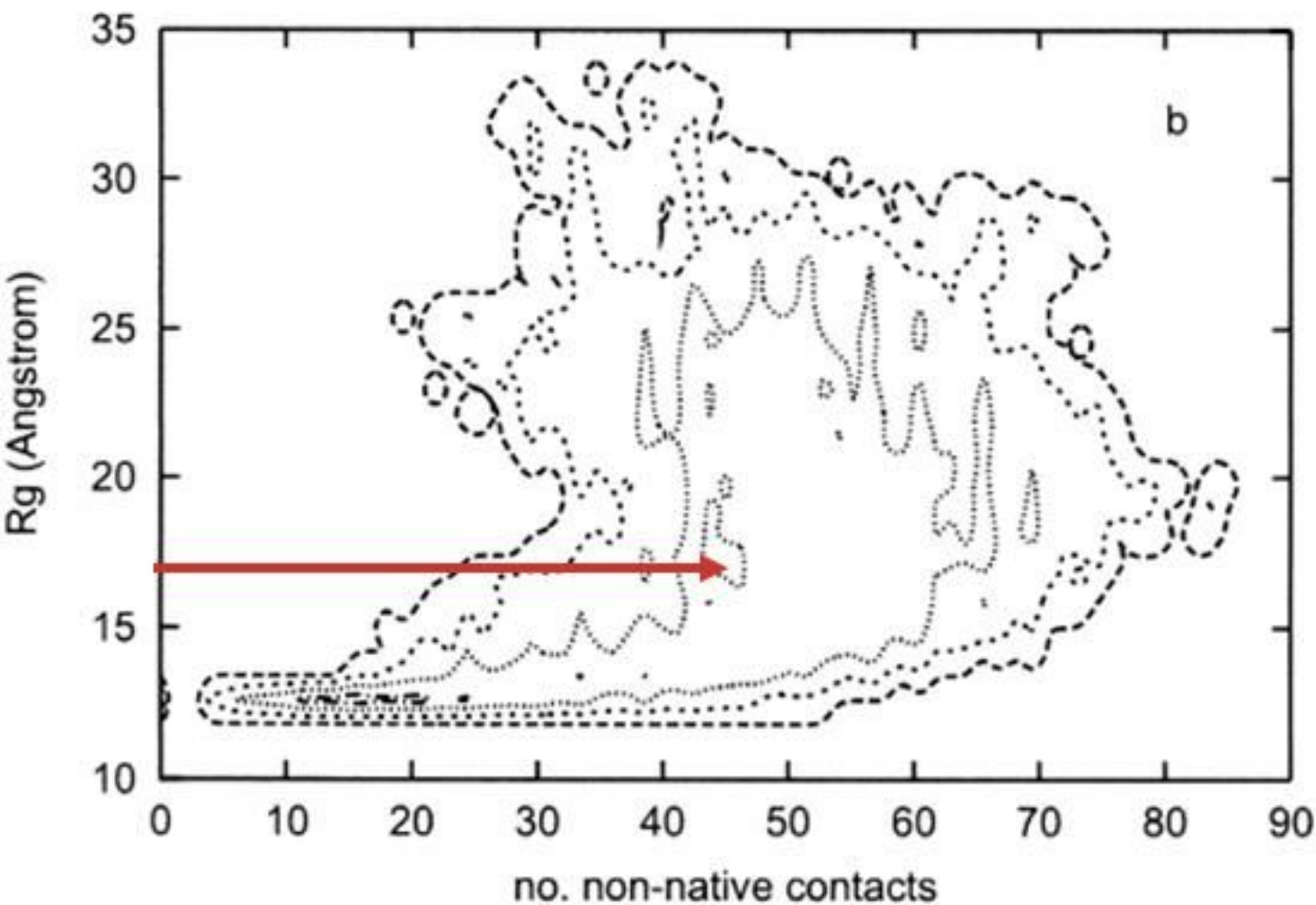
Collapse in small  $\sigma$  sequence and fast track molecules have Substantial fraction native contacts on  $\tau_c$ .

*Slow track molecules: Non-specific collapse with non-native contacts. Annealing takes much longer time.*



# Non-Native contacts in early stages of Cyt C folding

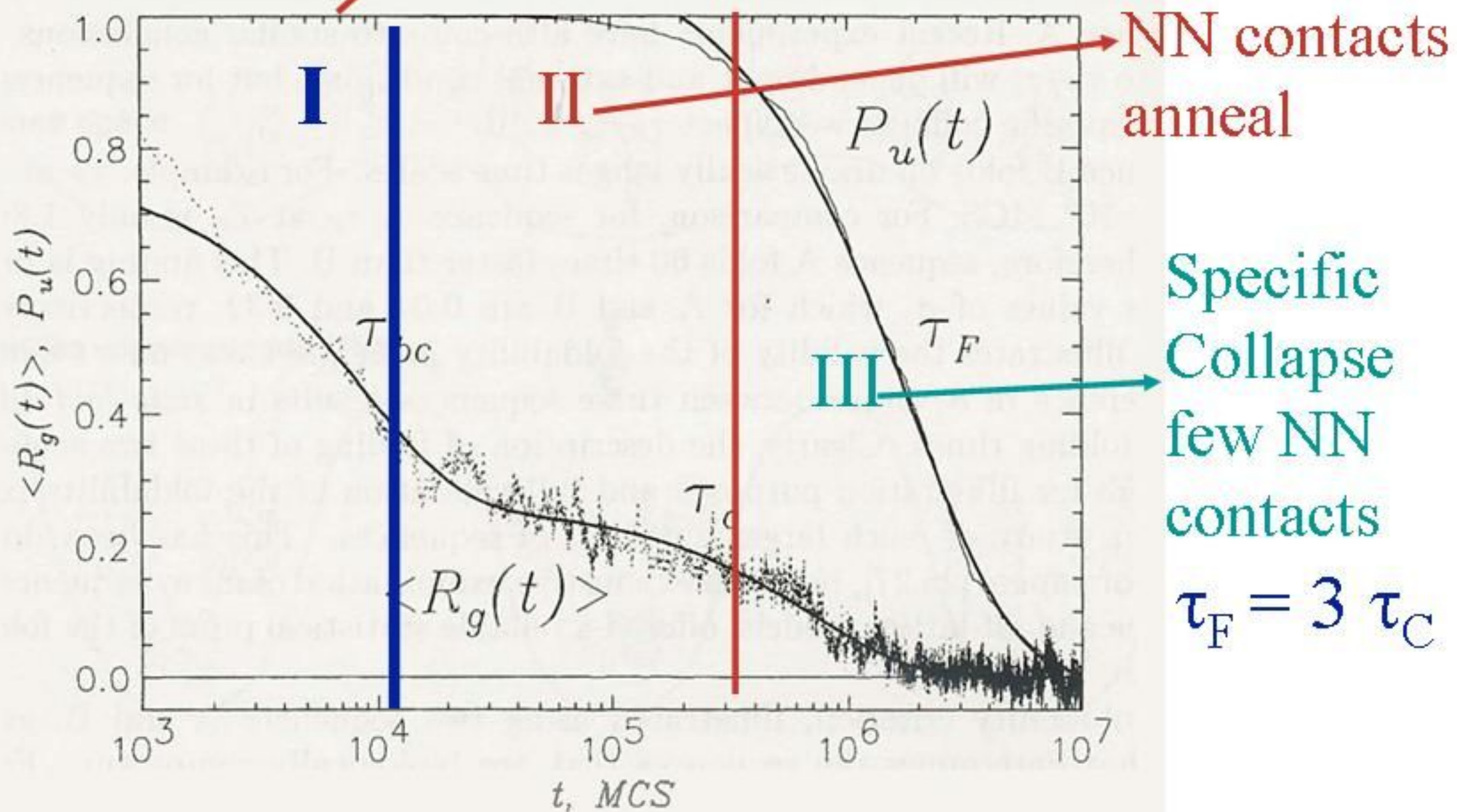
Serdal, Cardenas, dt, Elber  
All atom simulations



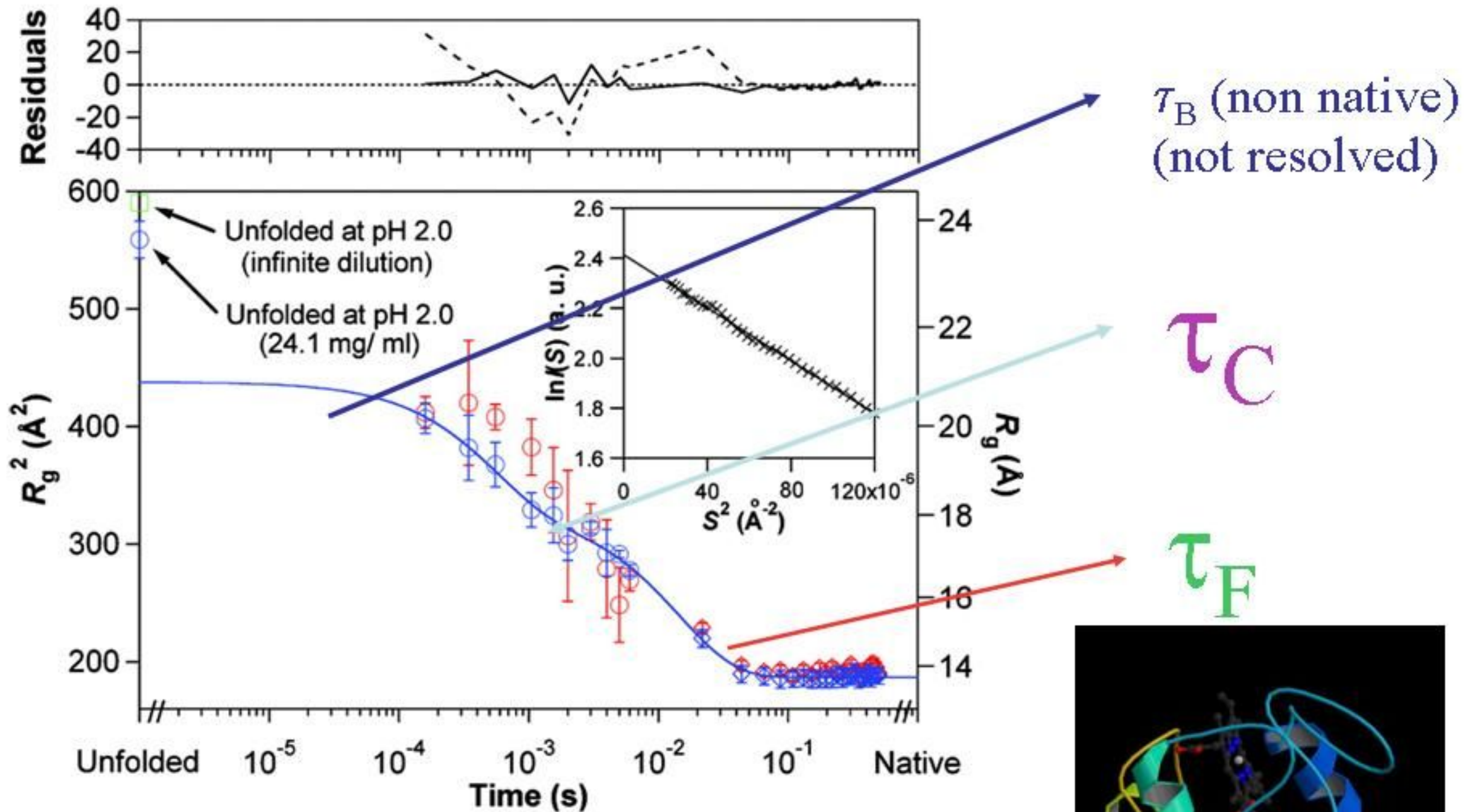
“Molten Globule”  
(40% helix content;  
< 10 native contacts)

# Relating Collapse and folding times (Lattice Models with side chains)

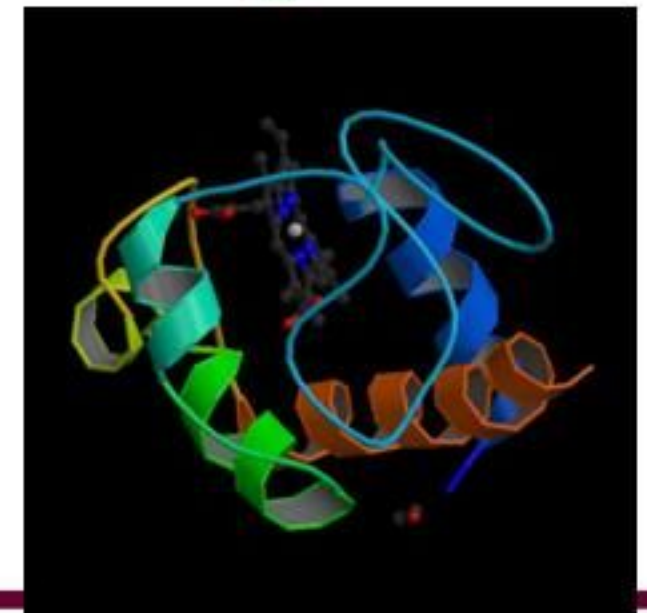
Non-native contacts form



# Collapse kinetics of Cyt c (three stages)



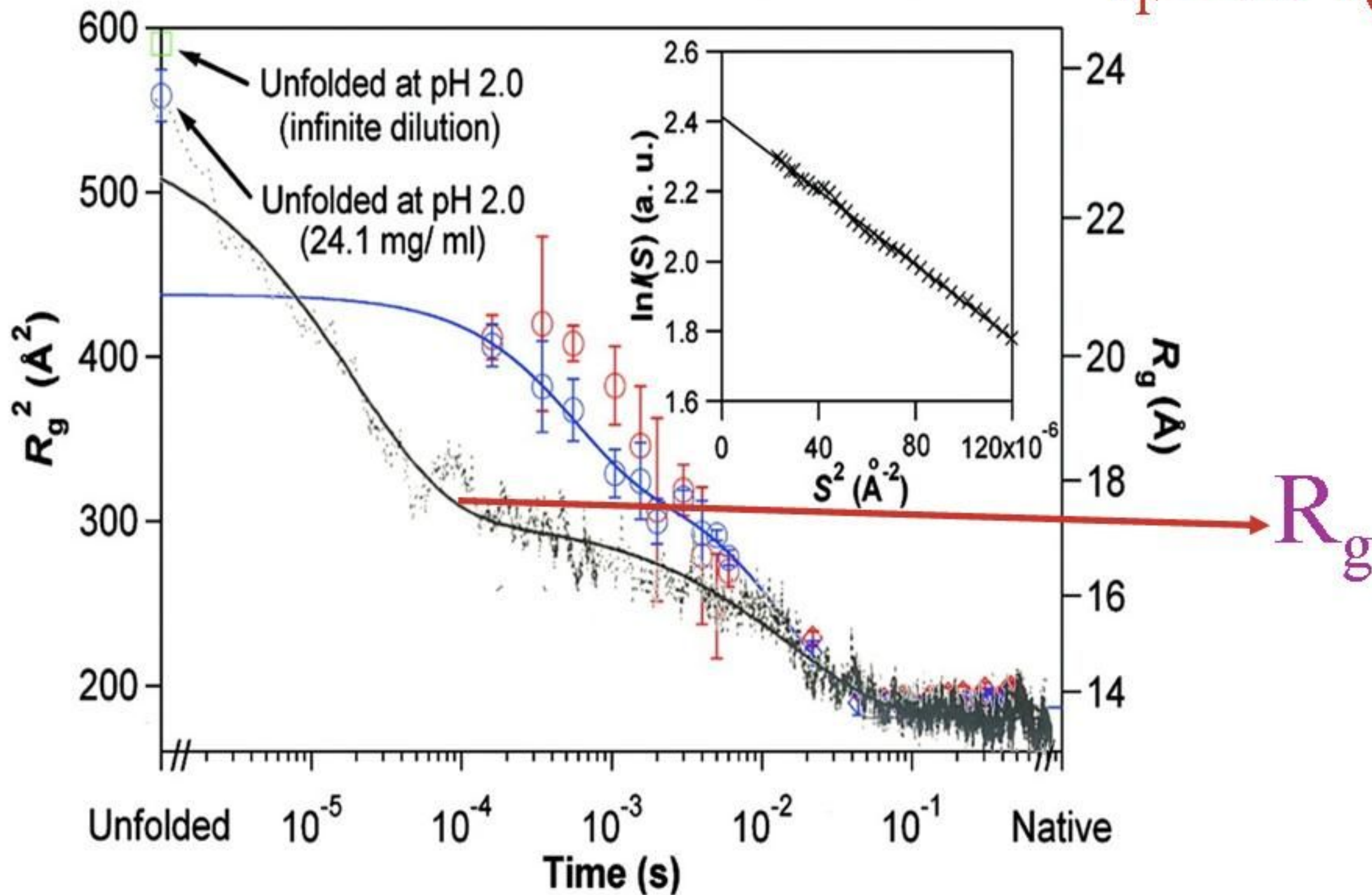
Akiyama, Shuji et al. (2002) Proc. Natl. Acad. Sci. USA 99, 1329-1334



# Comparing Cyt C data and Lattice Model with side chains

Akivama PNAS (2002)

$$\tau_F \approx 10 \tau_C$$

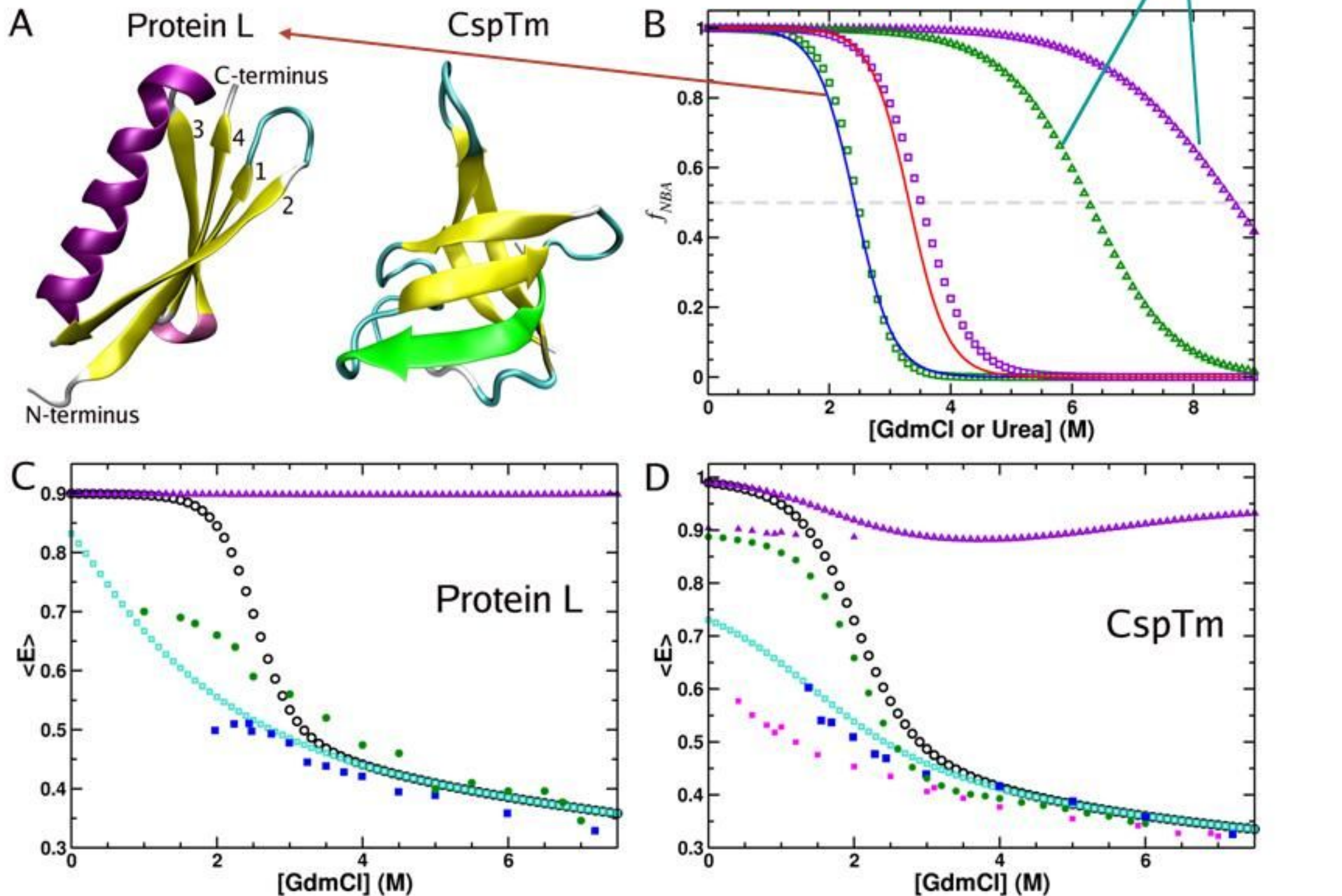


# Some Specific Unresolved Questions

- Is 8M urea a good solvent in Flory sense?  
Vitalis, dt & Pappu: No... Means partially compact
- T vs [C] collapse... Schuler type experiments
- DSE collapse SM FRET or using  $f$ . What about SAXS?
- For simulations: Models for direct comparison  
With experiments. MTM (O'Brien) is a great start

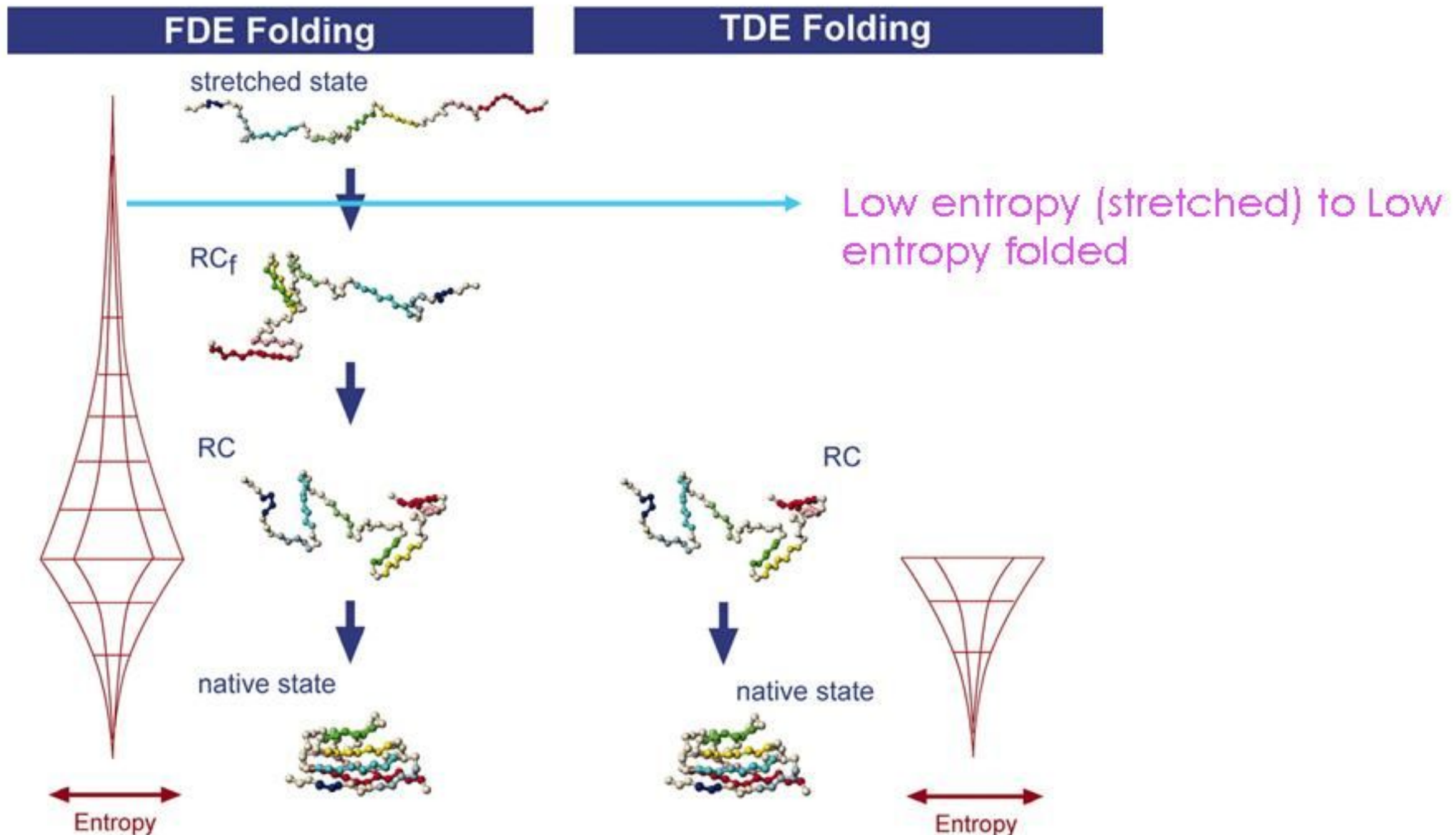
# MTM simulations & single molecule exps

O'Brien et. al. PNAS (2008)





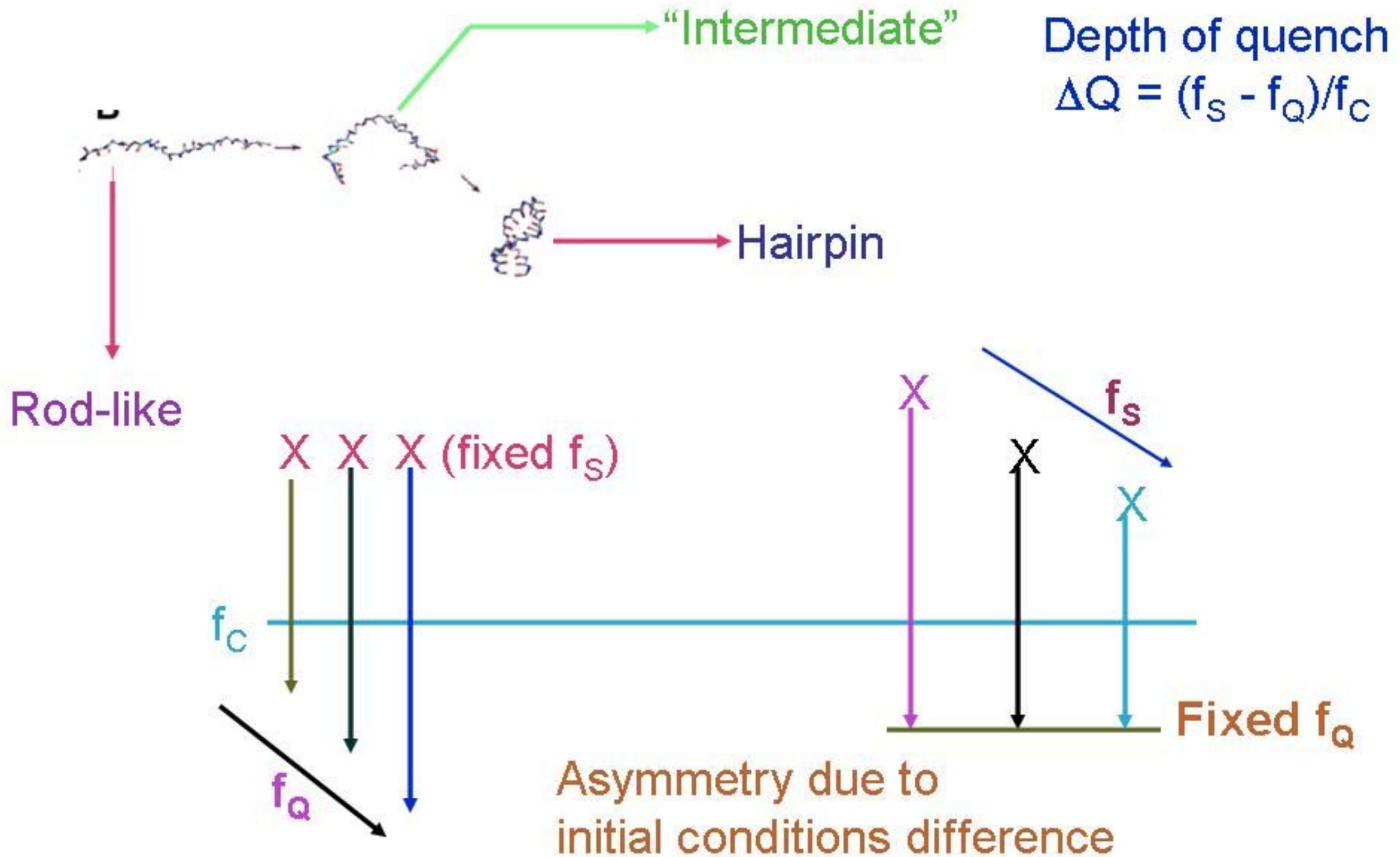
# Differences in FDE and TDE refolding



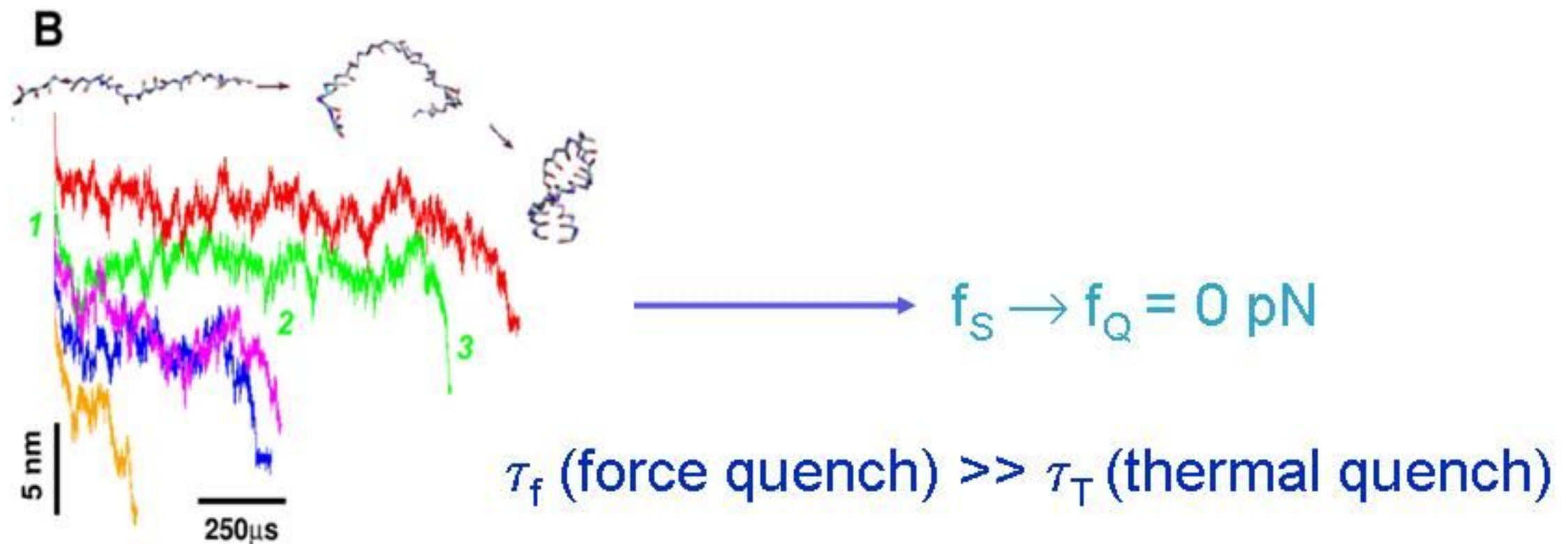
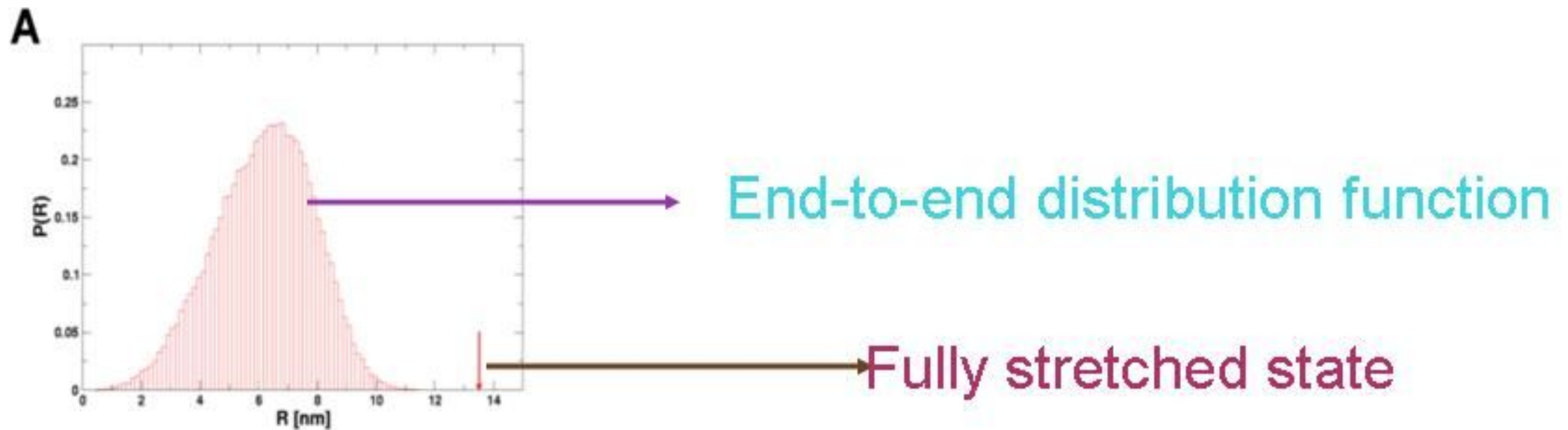
Fernandez J TIBS (2000)

# Refolding upon force quench

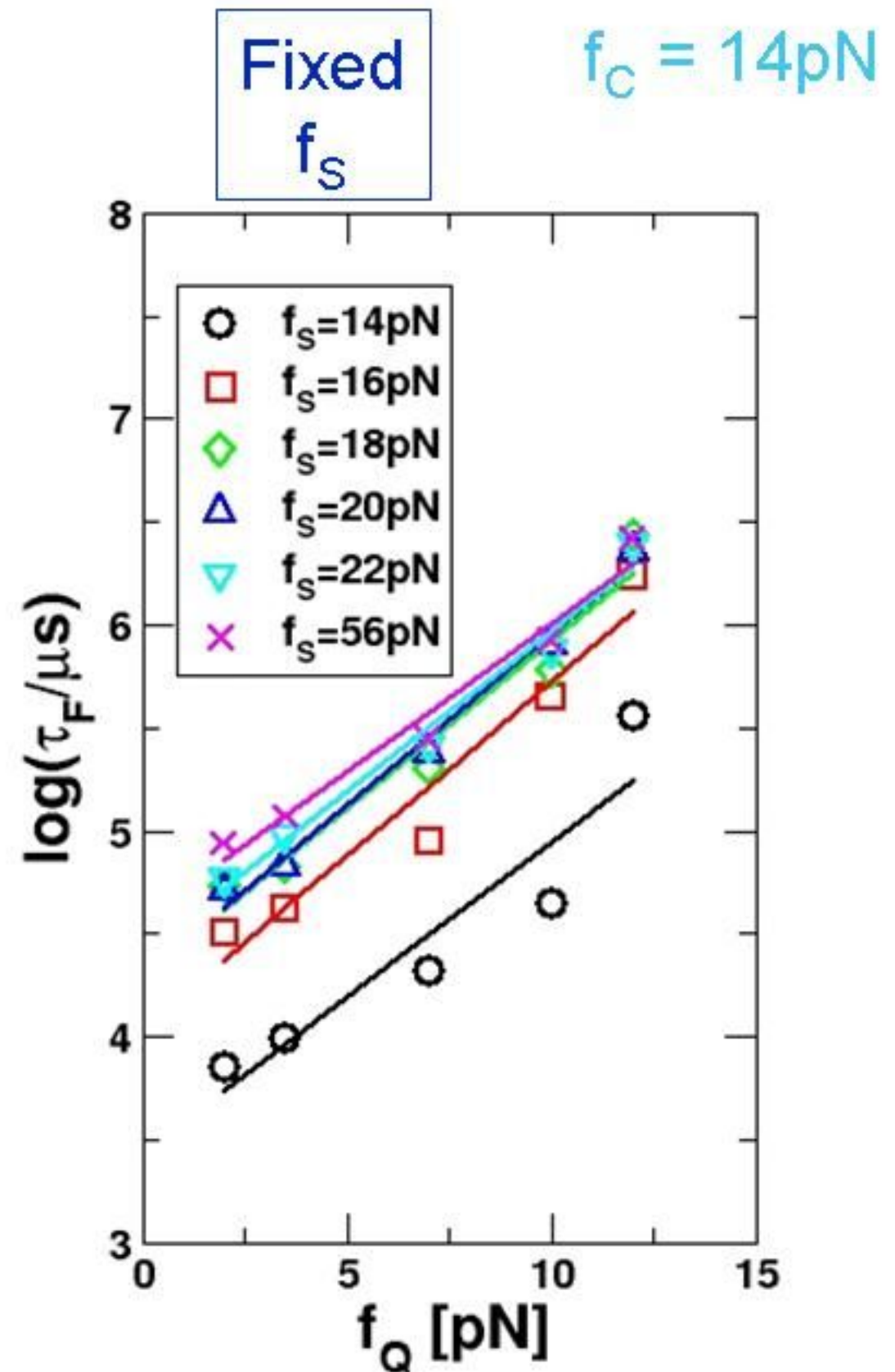
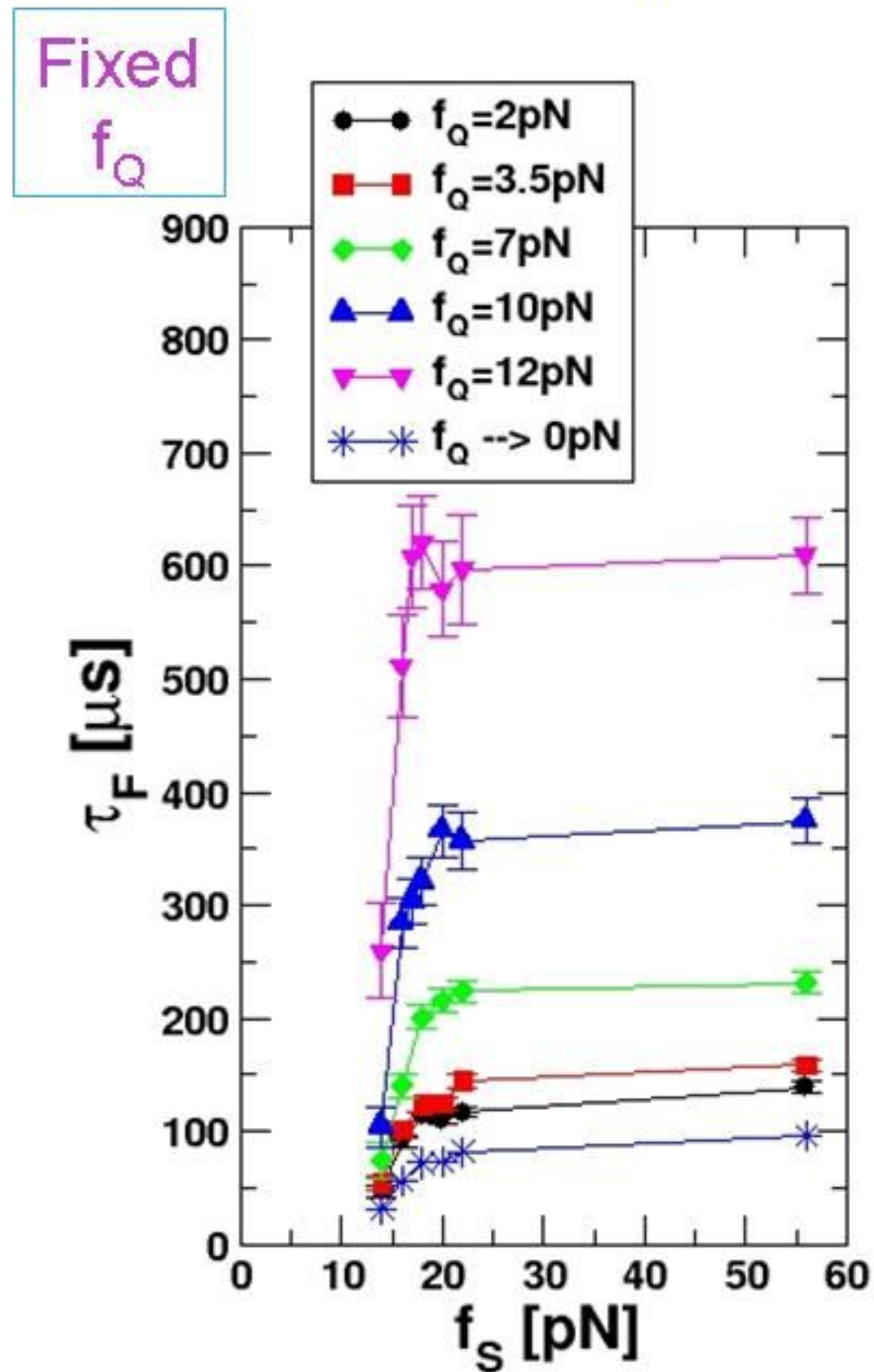
Hyeon, Morrison, Pincus & PNAS (2009)



# Refolding upon force quench

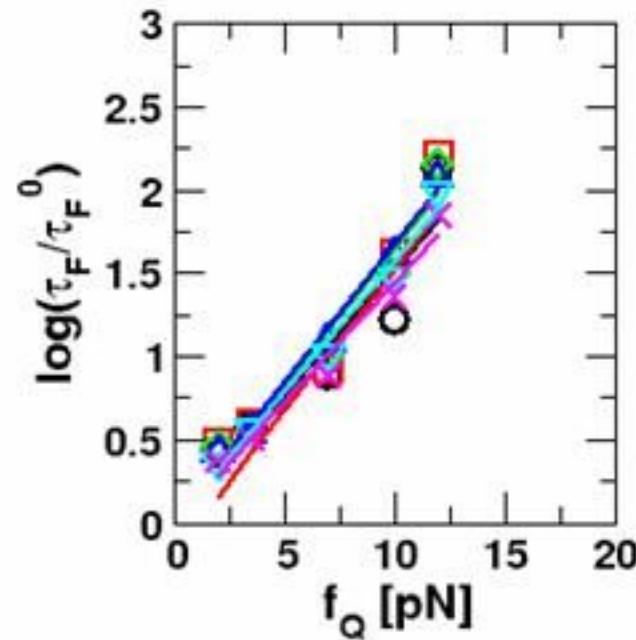
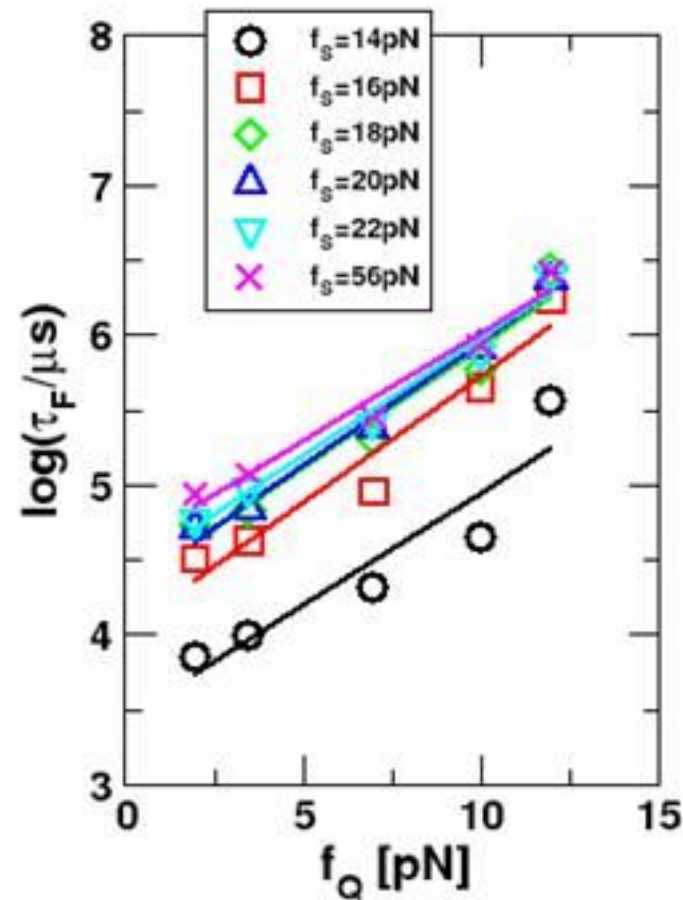


# Force quench kinetics of P5GA

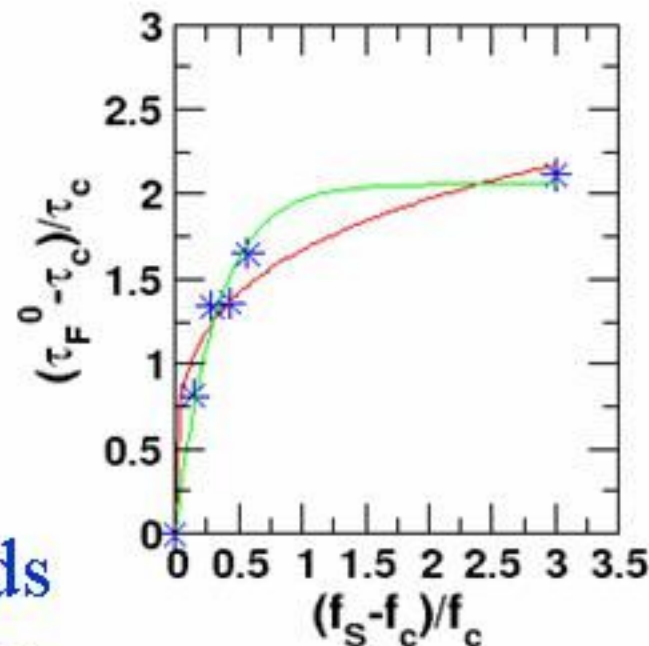


# Force quench refolding: Role of $f_C$

$$\tau_F = \tau_F^0(f_S, f_C) \exp[(f_Q \Delta x) / k_B T]$$



Complete Theory  
PNAS (2009)



$\tau_C$  hopping time at  
 $f_C$

FQ Protocol

Multiple quench methods

Force Corr Spectroscopy

Barsegov & dt PRL 2006