Intriguing Growth/Breakdown Dynamics and Strong Memory Aspect in the RRTN Model of Many Complex Systems of Nature

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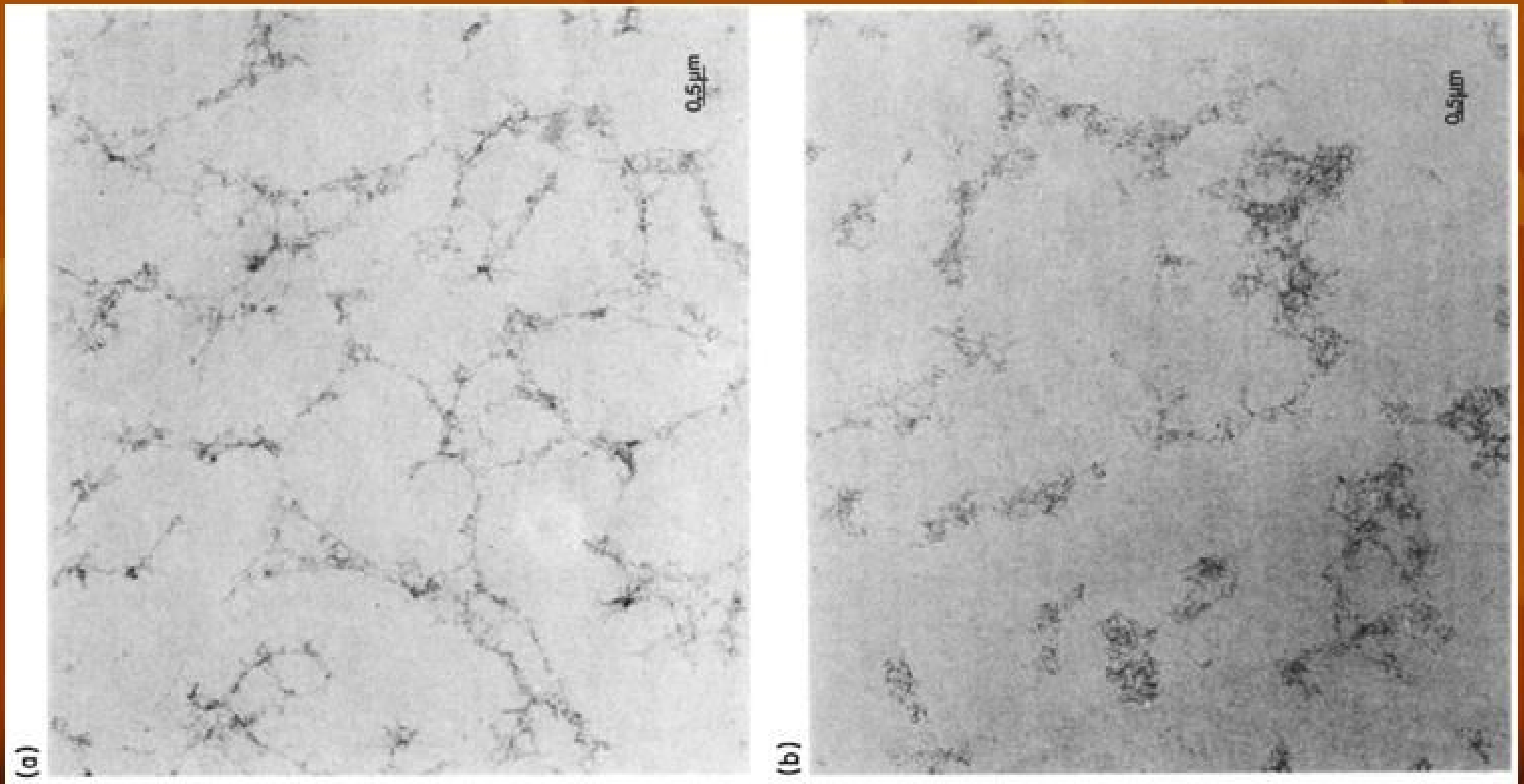
Example of PANI-CSA in PMMA

Transmission-electron micrographs of extracted PANI-CSA/PMMA polyblend films containing (a) p=0.005 and (b) p=0.0025 of PANI-CSA. It has an ultra-low pecolation threshold of p_c = 0.003 !!! [Transport in polyaniline networks near the percolation threshold; Reghu et. al.; Phys. Rev.B, v.50, p.13931 (1994)].



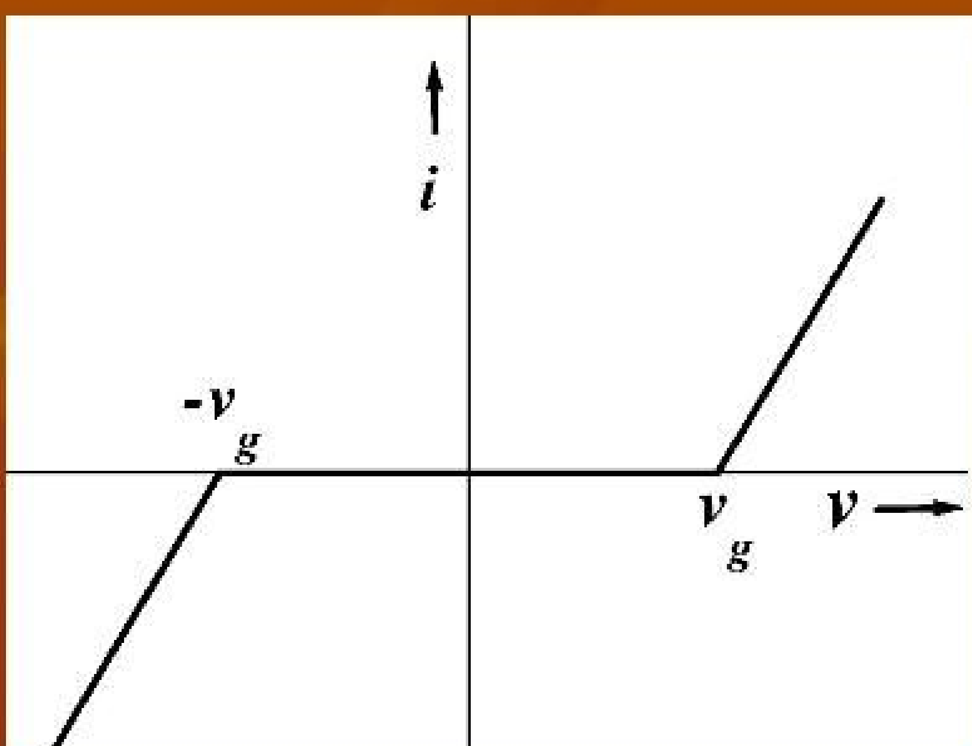
So, what is the main idea about TUNNELING?

It helps to cross a barrier (insulating wall between two charges)

Dielectric Breakdown

Spark Discharge

Example: a Zener diode gives a non-zero response only above or a threshold voltage.



In the air between two clouds, it is **lightning!**

BUT,
systems we consider need not be electrical.

Other systems:

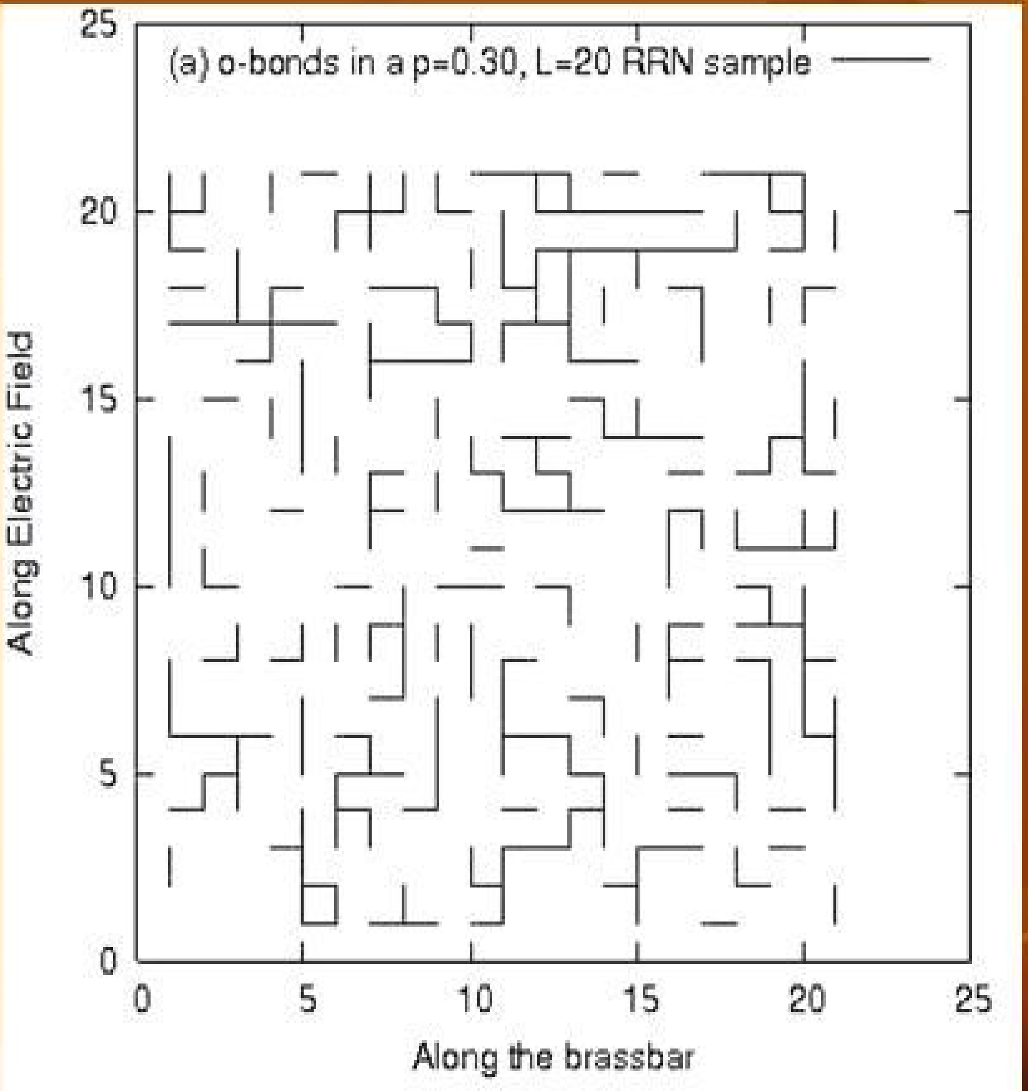
Fluid flow through narrow porous media due to capillary/ surface tension forces (important in off-shore oil recovery)
Motion of objects on surfaces or inside sticky fluids (due to *friction* or *viscosity*)
etc. ....

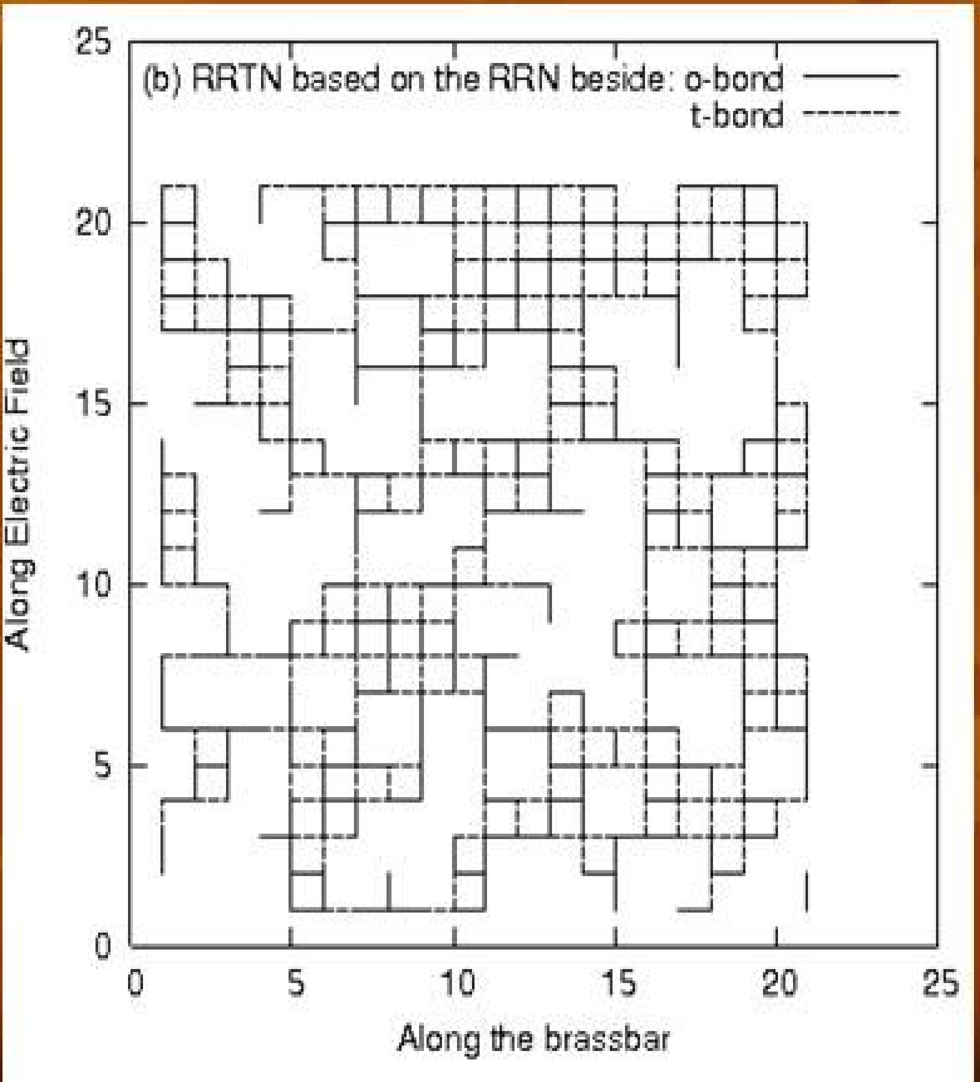
Tunneling induced ultra-low percolation threshold:

Our Random Resistor cum Tunneling-bond Network (RRTN) Model – Semi-classical tunneling introduced only between nearest-neighbour metallic (ohmic) bonds; and no further [AKS and A. Kar Gupta; Lecture Notes in Physics, v.437, p.271 (Springer, Berlin, 1994)].

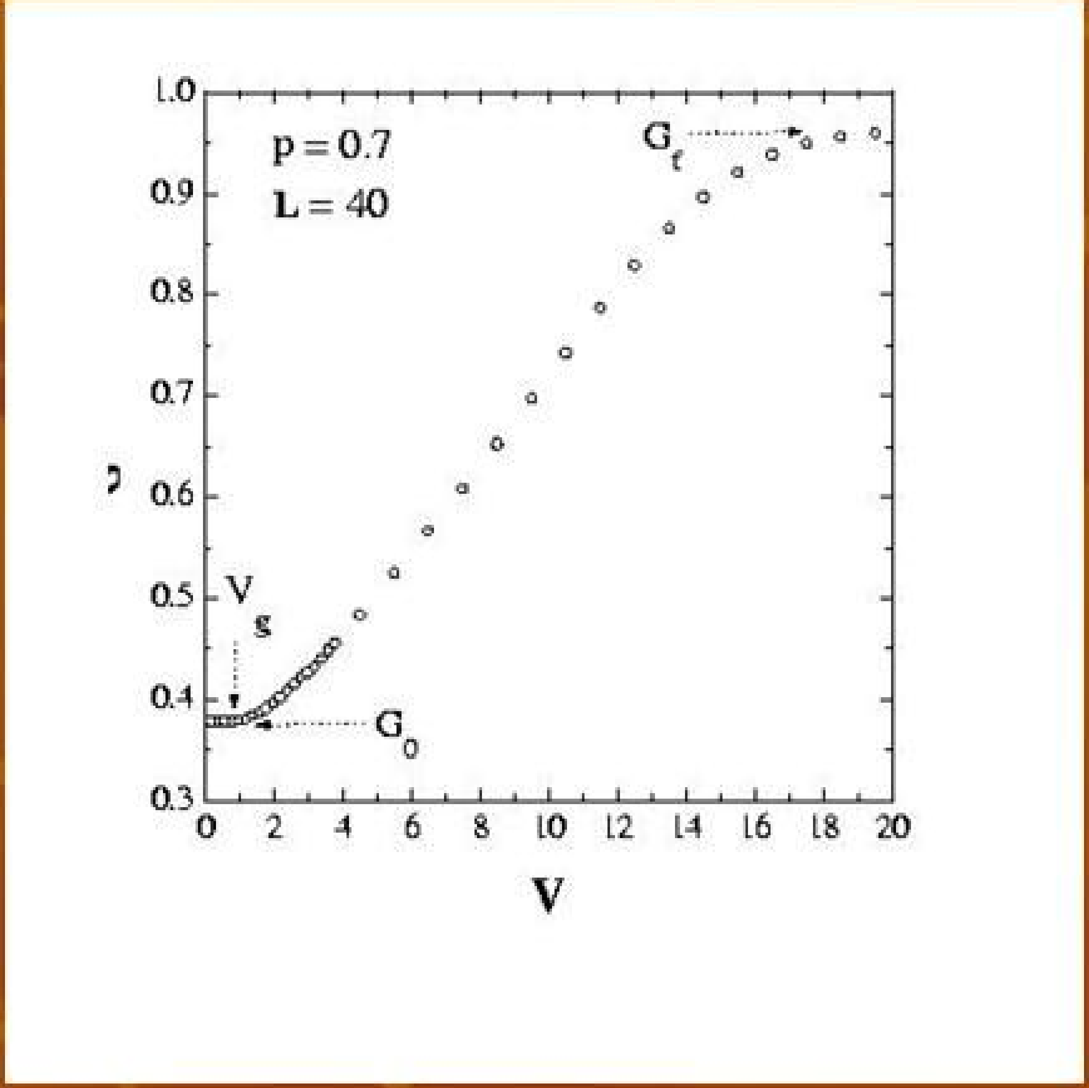
Standard method w/ finite-size scaling analysis gives a new percolation threshold for a maximal RRTN (on a 2D square lattice) to be p_{ct} = 0.181 (cf. for sq. latt. RRN p_c=0.5).

[A. Kar Gupta and AKS; Physica A, v.215, p.1 (1995)].





Recapitulation of static G-V response:



V<Vg: **Lower Linear Regime (LLR)**

V >Vg: **Non Linear Regime (NLR)**

V >> Vg: **Upper Linear Regime (ULR)**

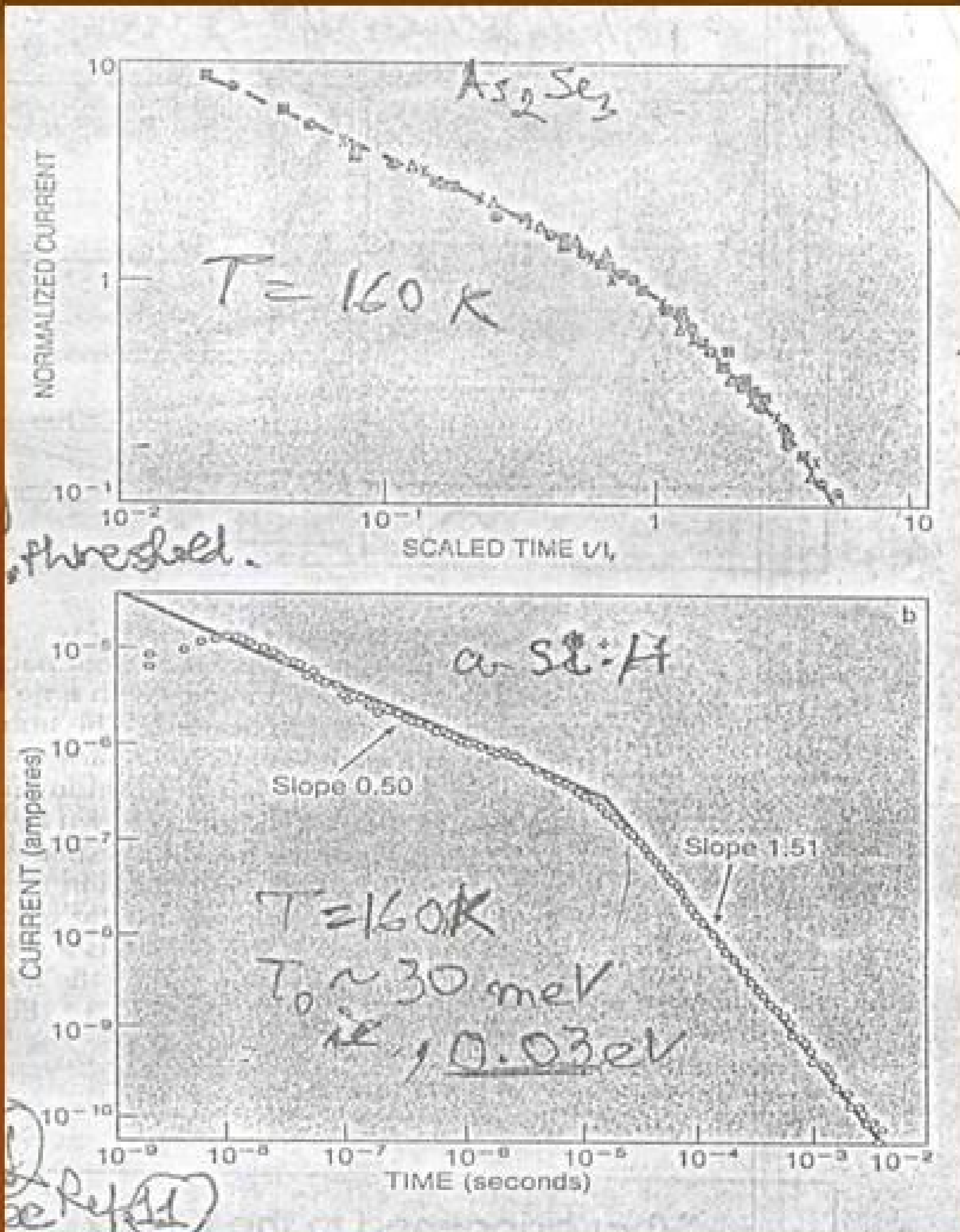
Kar Gupta and Sen, PRB 57, 3375 (1998)

Debye relaxation:

- Temporal rate of change in response is proportional to the response itself,

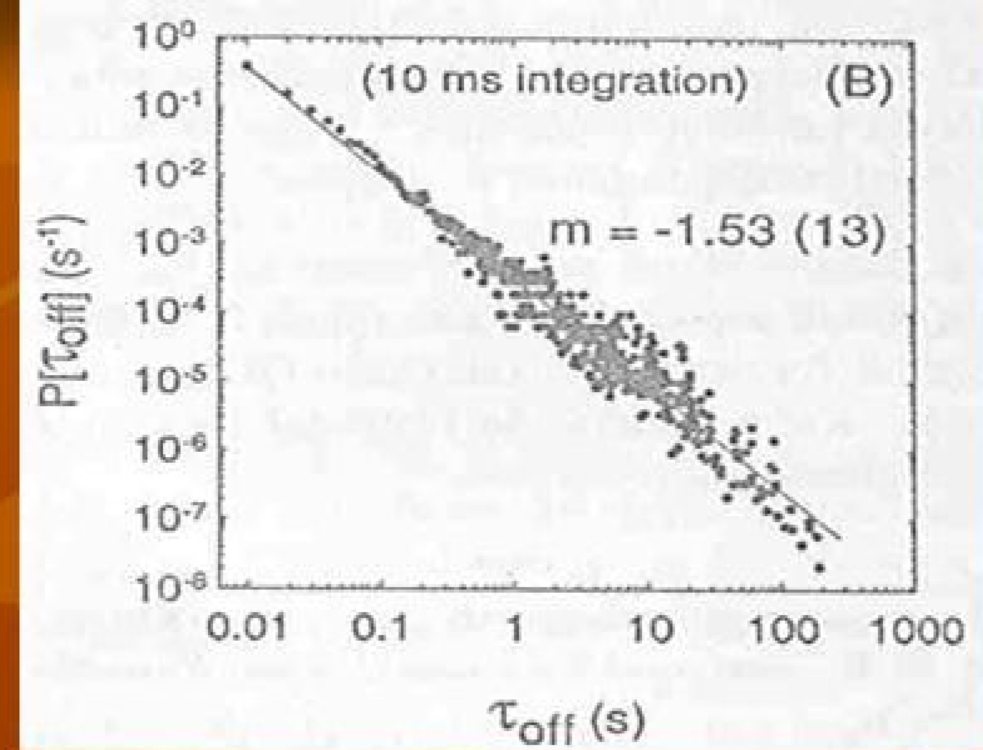
$$\frac{dI}{dt} = -\frac{1}{\tau} I \implies I = I_0 \exp\left(-\frac{t}{\tau}\right)$$

- Also called Boltzmann's relaxation time approximation
- Simplest dynamics; possesses a single time-scale τ (Boltzmann-Gibbs-Shannon Statistics)
- Obviously LLR of the RRTN (indeed a RRN) follows this dynamics

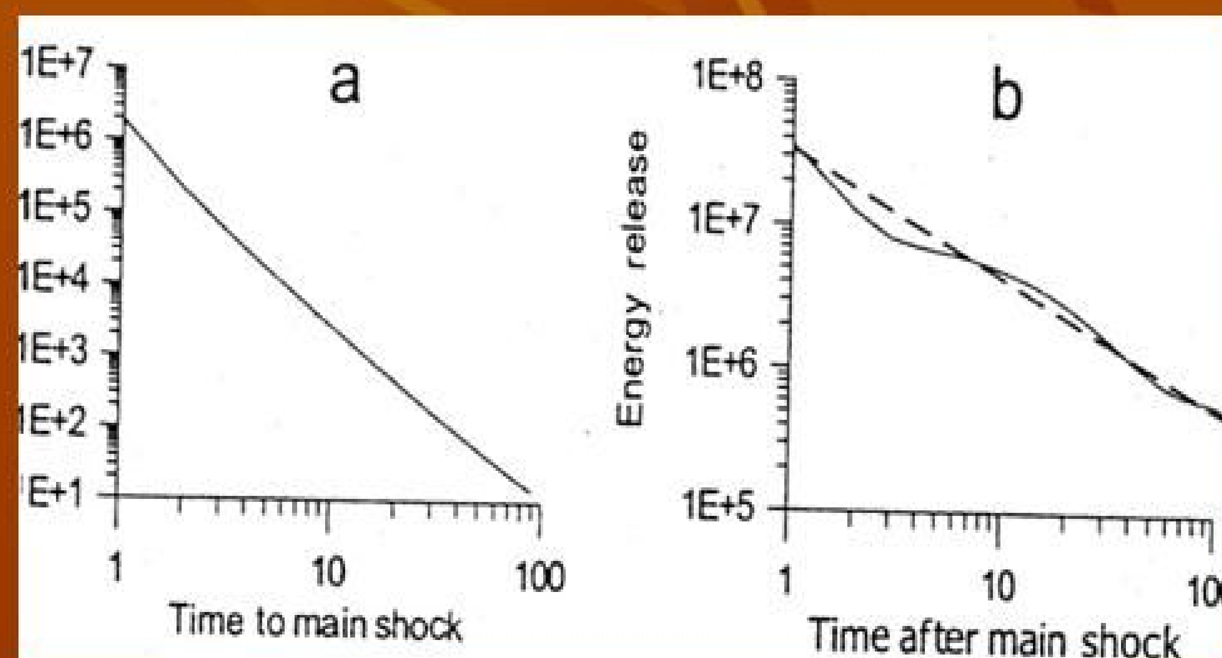


Photocurrent in Amorphous Si:H and As_2Se_3

Re-association of ligands of Fe in folded heme-Proteins; Parak et al, Physica A**201**, 332 (1993)

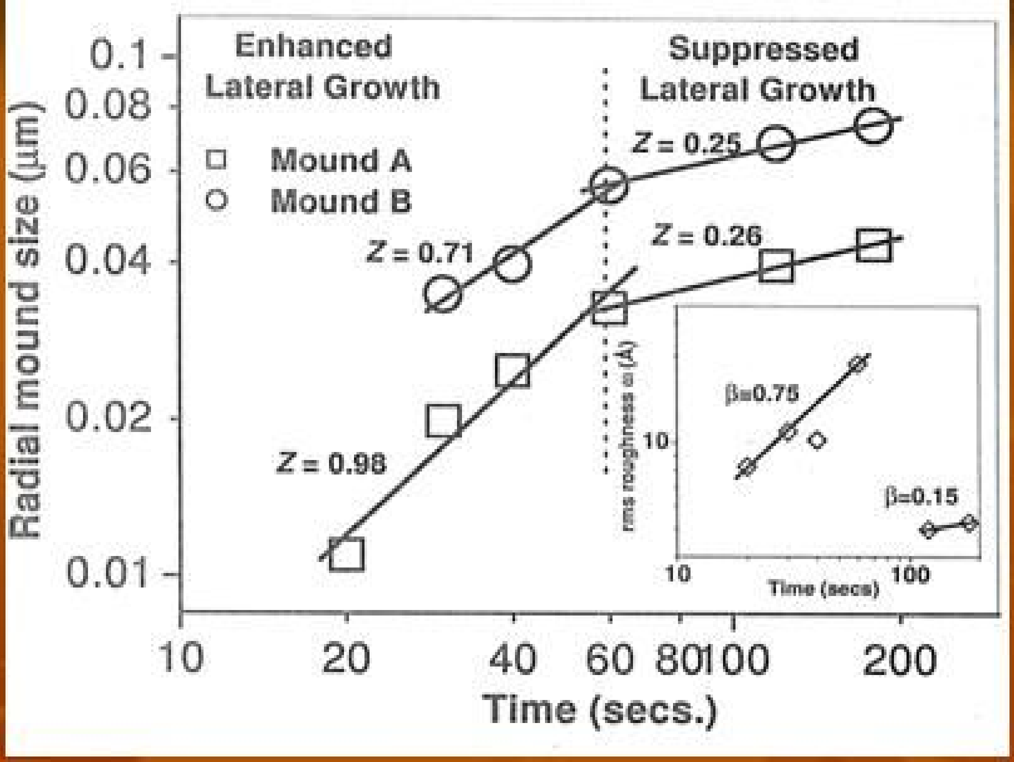


Blinking kinetics in CdSe Quantum Dots

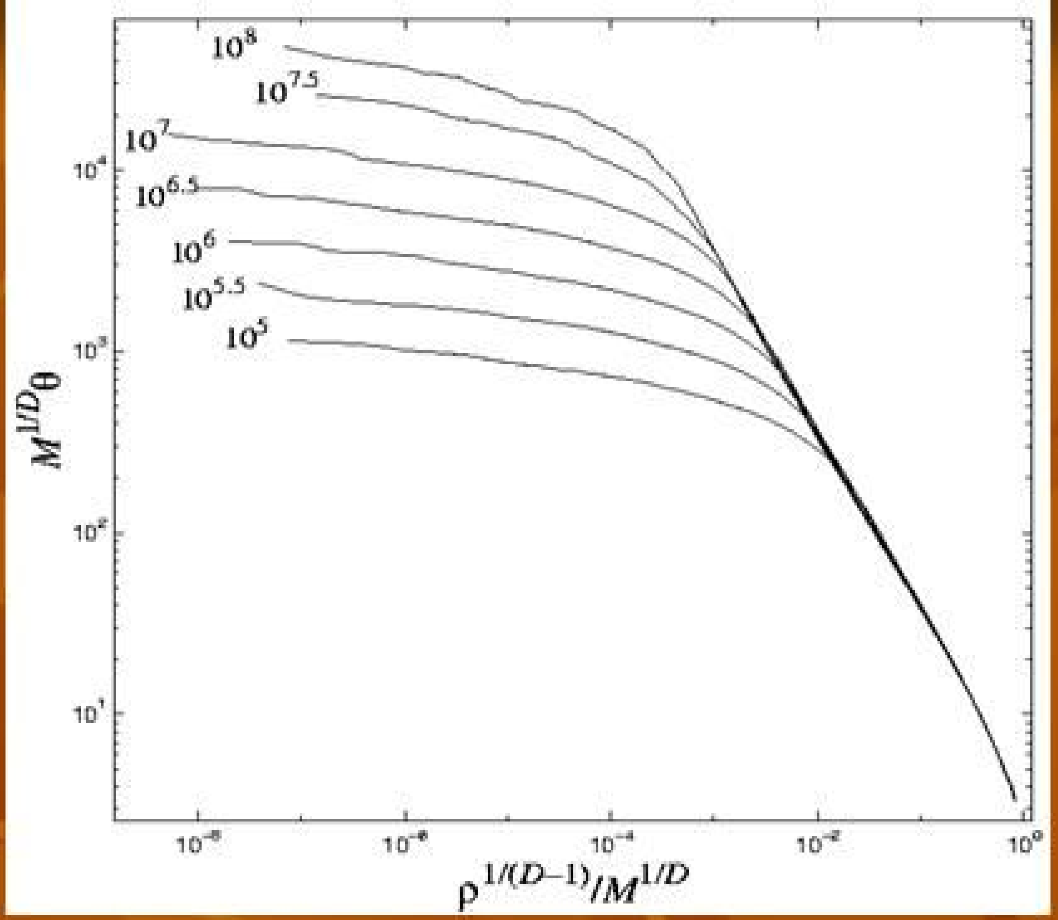


Earthquake (a) fore- and (b) after-shocks;

Case (b) is called Omori law

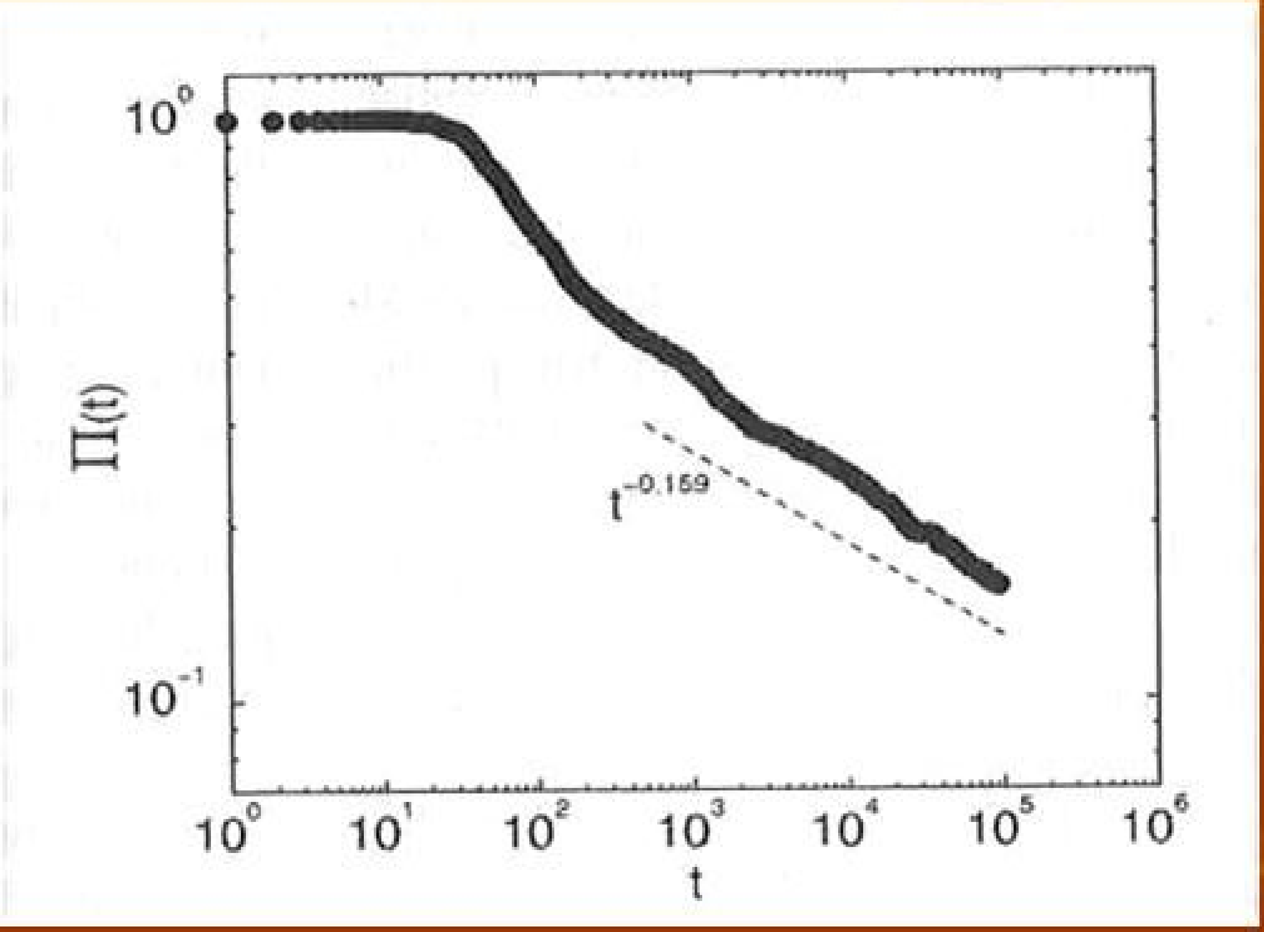


Sputtering of Ag particles on Si(001) surface

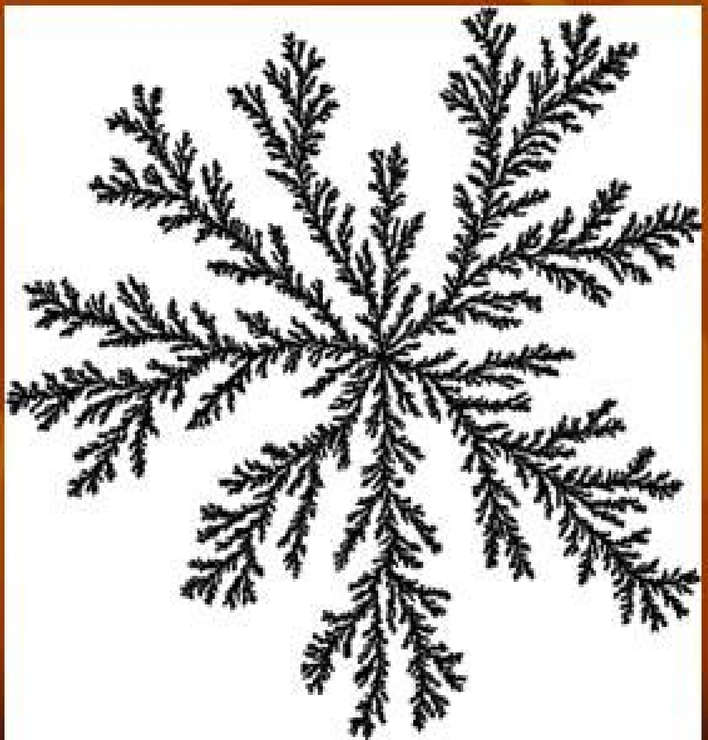


Comp-expt On radial DLA growth ($N=10^5$ - 10^8)

x-axis is time-like; a DLA is shown below.



Ca^{+2} channel dynamics in Living Cells



Relaxation in the RRTN model; some typical parameters:

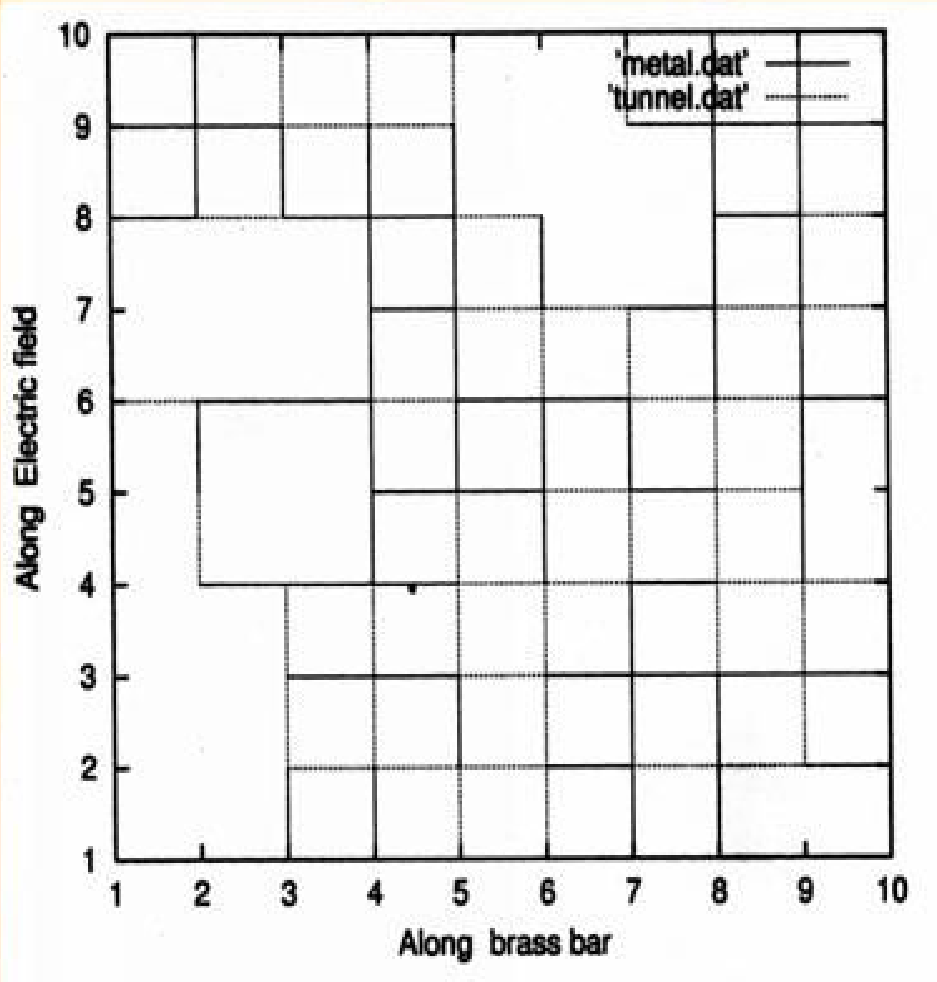
$g_o = 1.0$, $g_t = 0.01$, $c = 10^{-5}$, displacement current : $i_{dis} = c\, dv/dt$ for t-bonds with $v < v_g$

- Use a graded random initial voltage configuration v_{ij} at each node

- Update the microscopic voltages at each node using the Continuity Eqn. $\Sigma_{<ij>} I_{ij} = 0$ locally; i.e., a *lattice Kirchhoff's dynamics*:

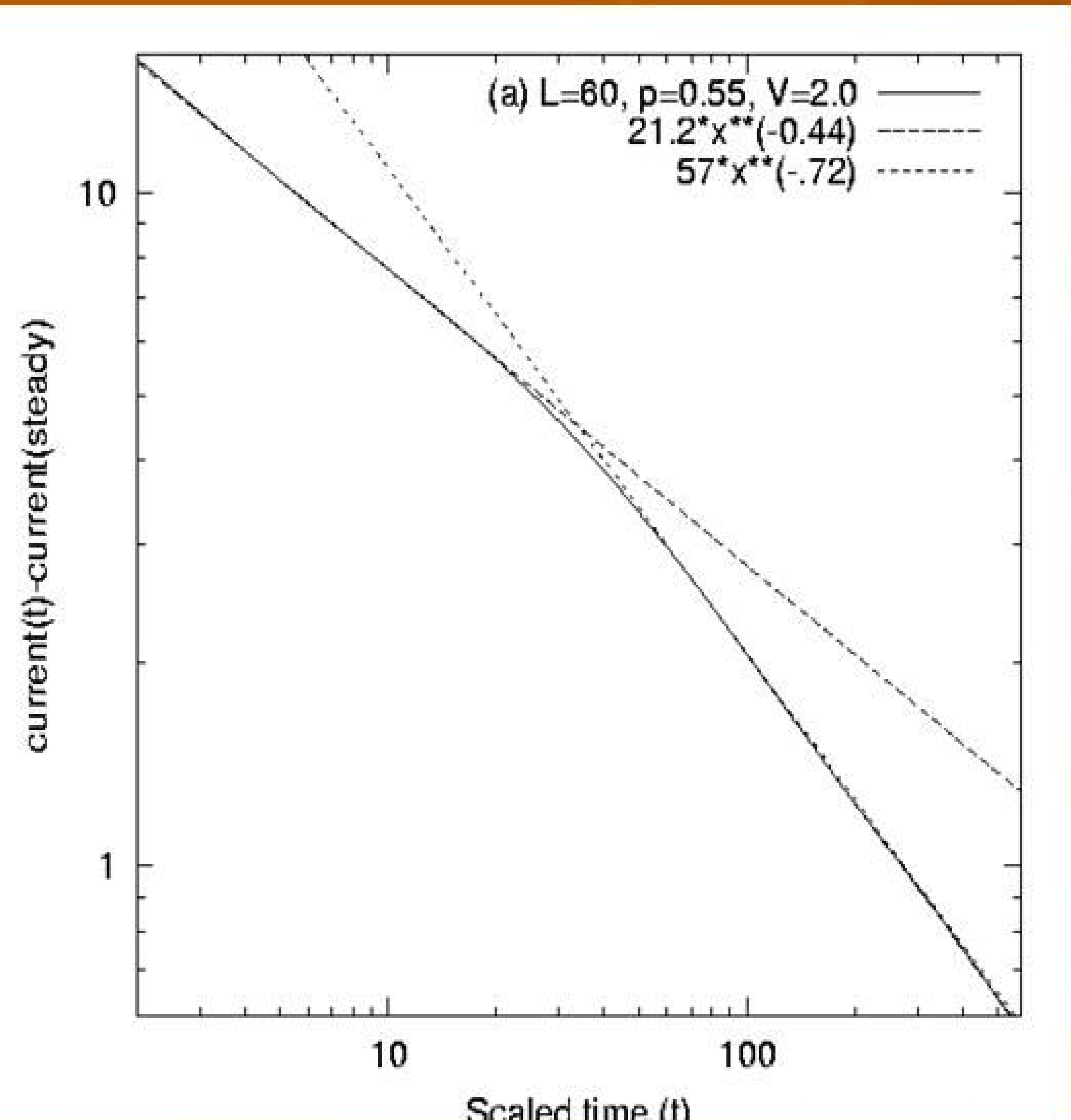
$$v_{ij}(t+1) \Leftarrow v_{ij}(t) + \Sigma_{<ij>} I_{ij} / \Sigma_{<ij>} g_{ij}$$

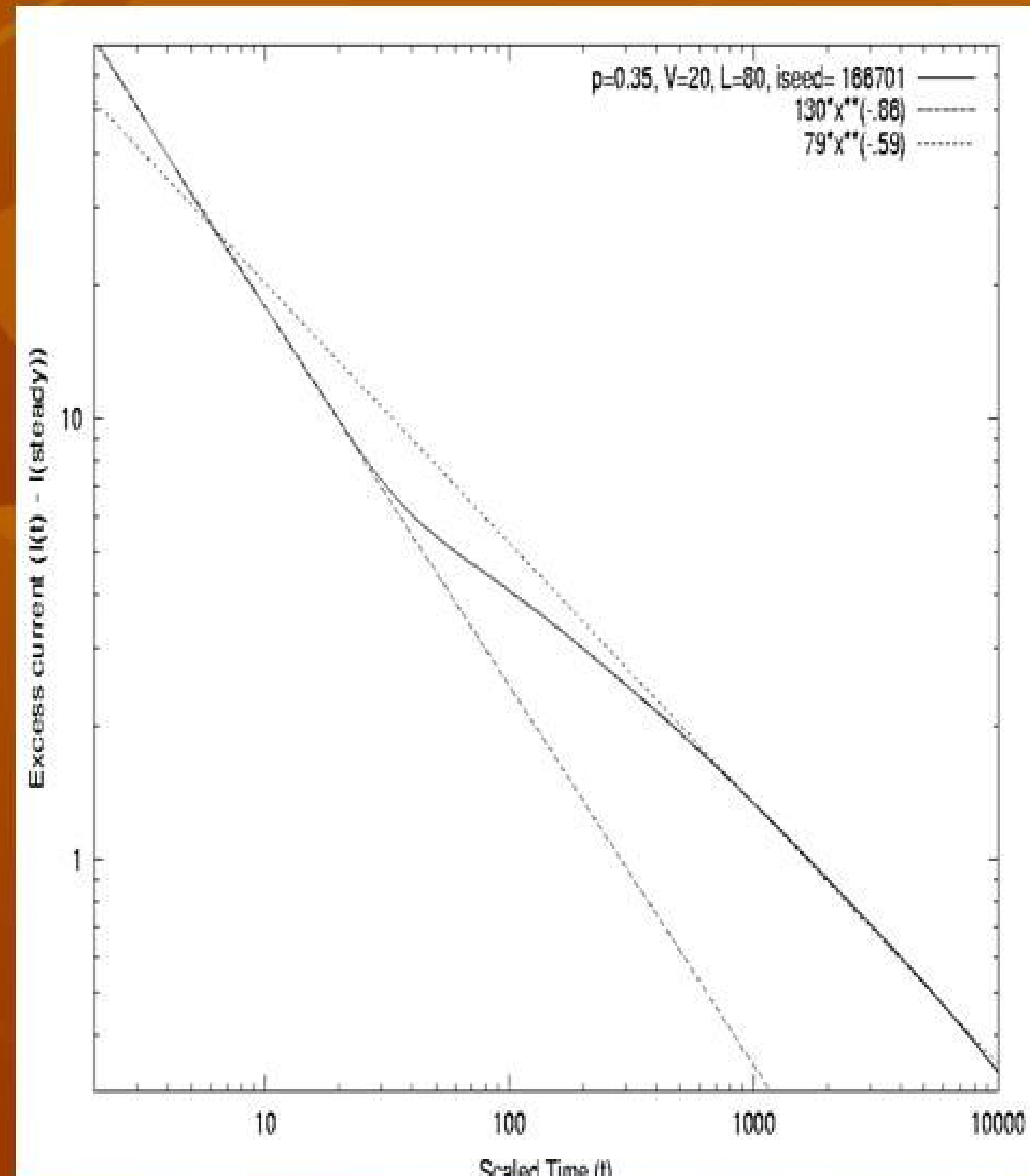
- Check the global continuity to obtain the final steady state, i.e., stop iteration when $| I(1^{st}\text{ layer}) - I(N^{th}\text{ layer}) | \leq \varepsilon$, a pre-assigned small +ve number (for controlling precision)



Bhattacharya and Sen, Europhys. Lett. **71**, 797 (2005)

Two early-stage power-laws in RRTN current dynamics





$\alpha_1 < \alpha_2$ (appear in most expts.)

$\alpha_1 > \alpha_2$ (somewhat rare in expts.)

S. Bhattacharya and A.K. Sen, Europhys. Lett. **71**, 797 (2005)

Two initial Power-law Relaxations:

Appear, in general, due to a sequence of

- **Re-distribution of local clusters/fields in time**

crossing over to a

- **re-distribution of global clusters/fields in time**

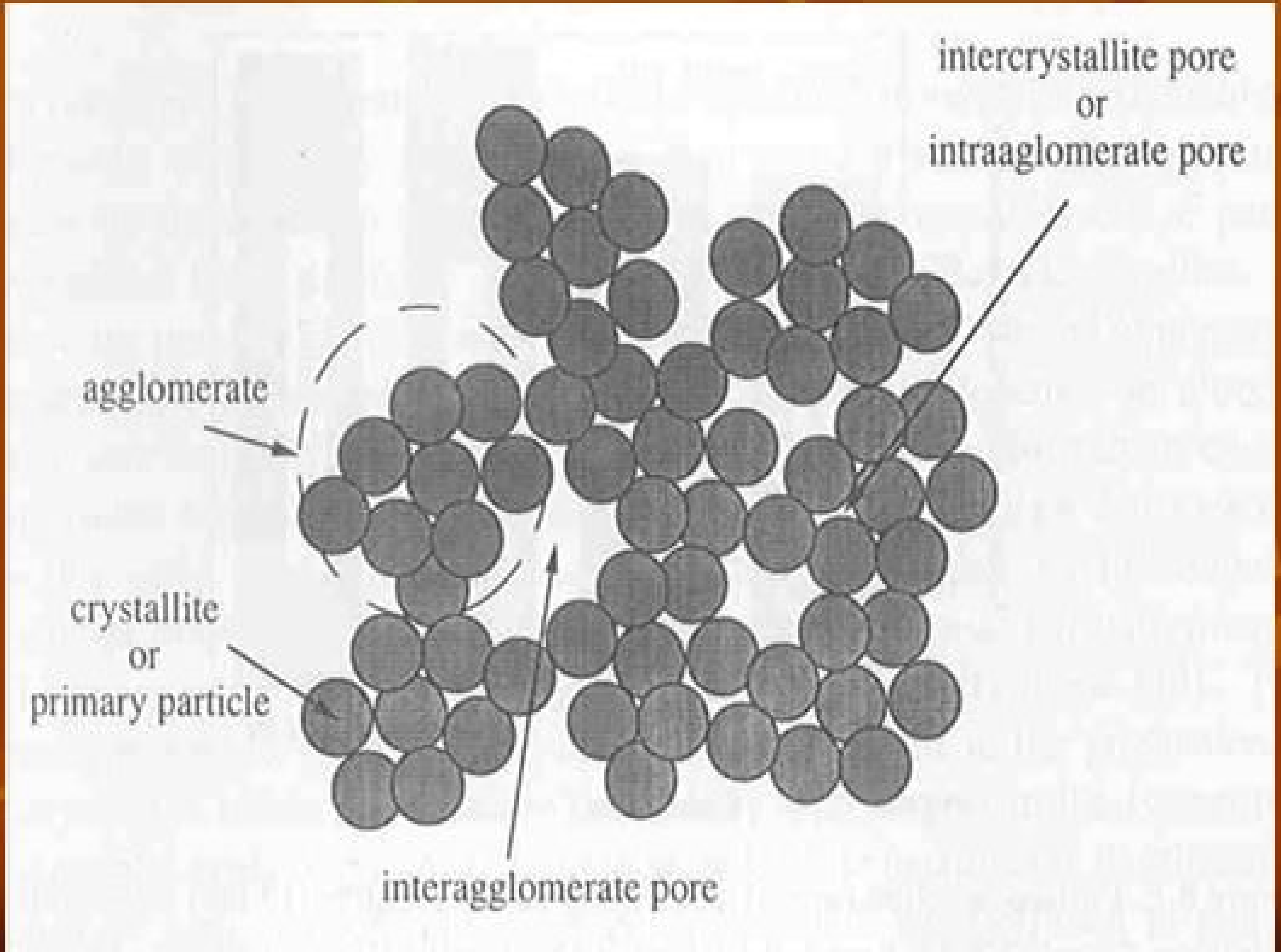
- Eventually,

$\phi(t)$ for $t \rightarrow \infty$,

crosses over to an

exponential dynamics,

towards a steady-state



Power-law Relaxation/s:

- Belongs to a non-Debye class, as an outcome of an inherent property of the system having multiple/infinite time-constants τ
- An example: a relaxation function $\phi(t)$ with multiple τ's, weighted by a self-similar probability density function (p.d.f.), $w(\kappa t) = t^{-\alpha} w(\kappa)$, where $\kappa = 1/\tau$

$$\varphi(t) = \int \exp(-t/\tau)\, w(\tau)\, d\tau$$

- With a proper choice of the p.d.f., one finds two power-law relaxations; one at a short-time scale and another at an asymptotically long-time scale

Ref.: Weron and Jurlewicz, J. Phys A, **26**, 395 (1993)

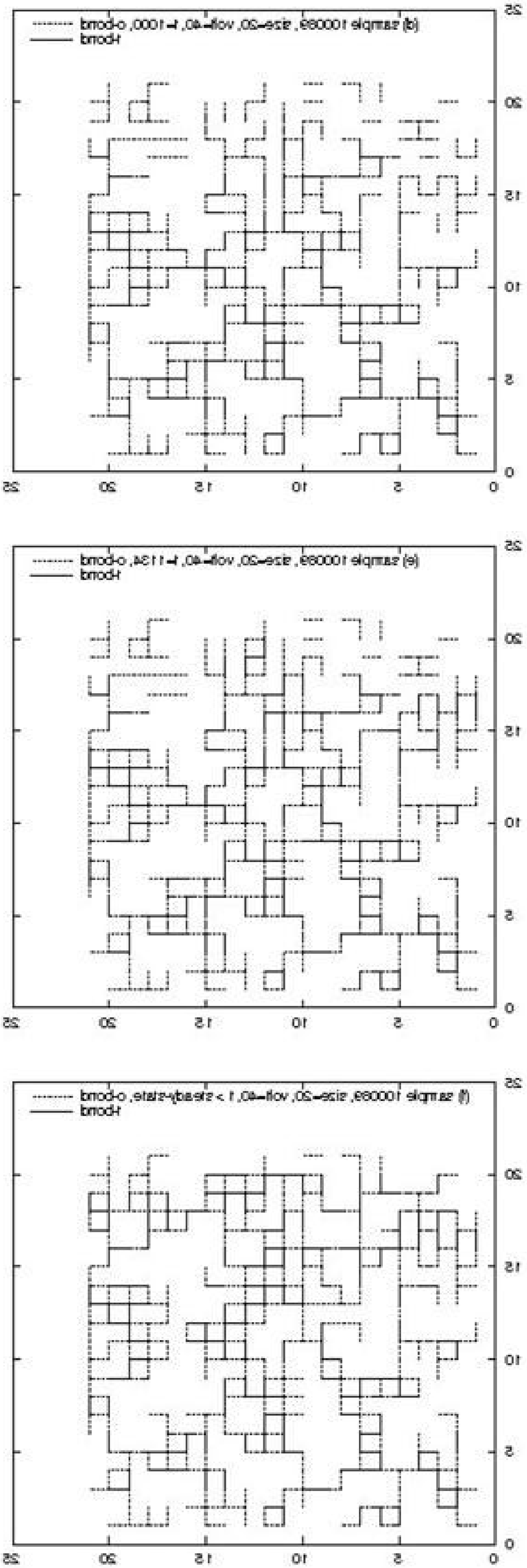
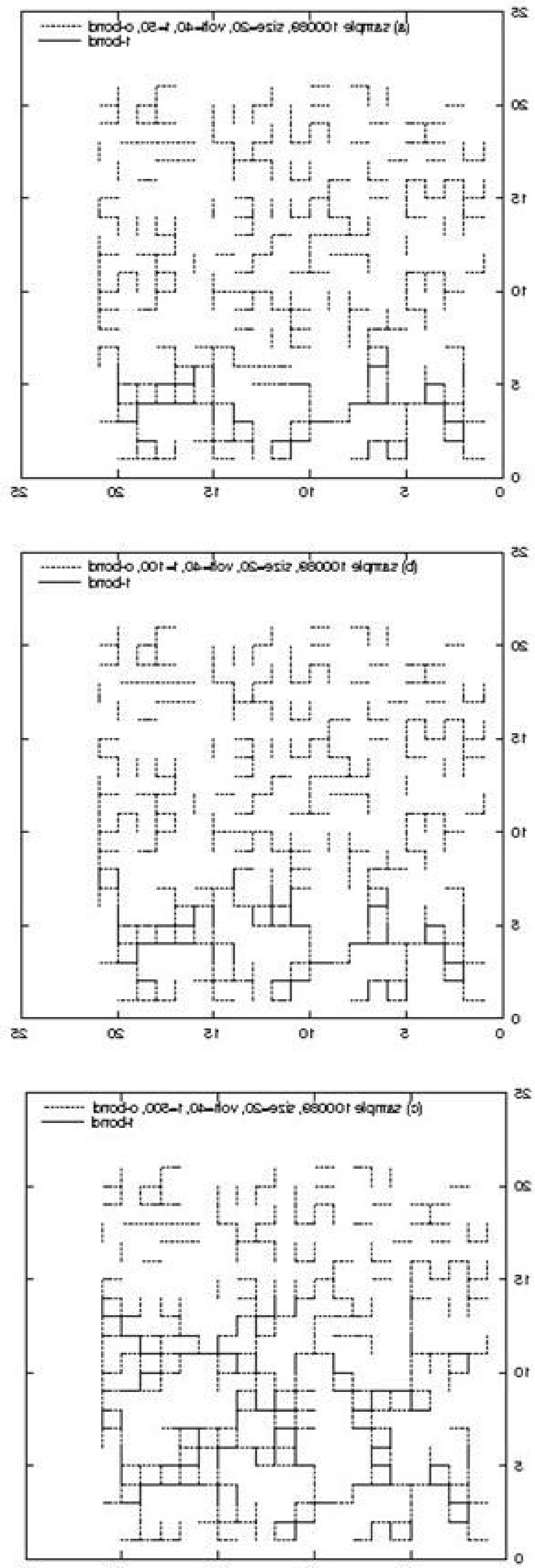
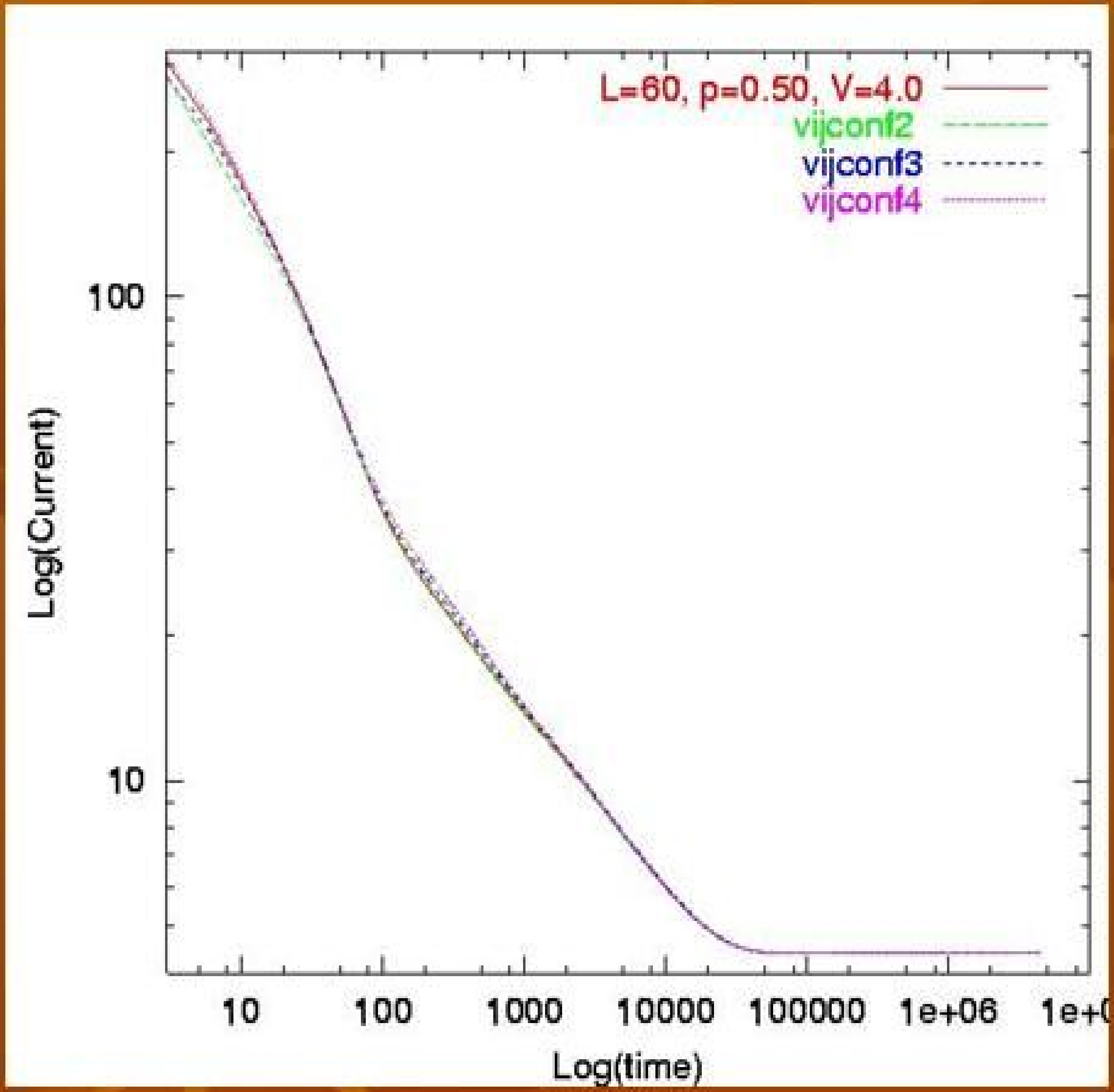
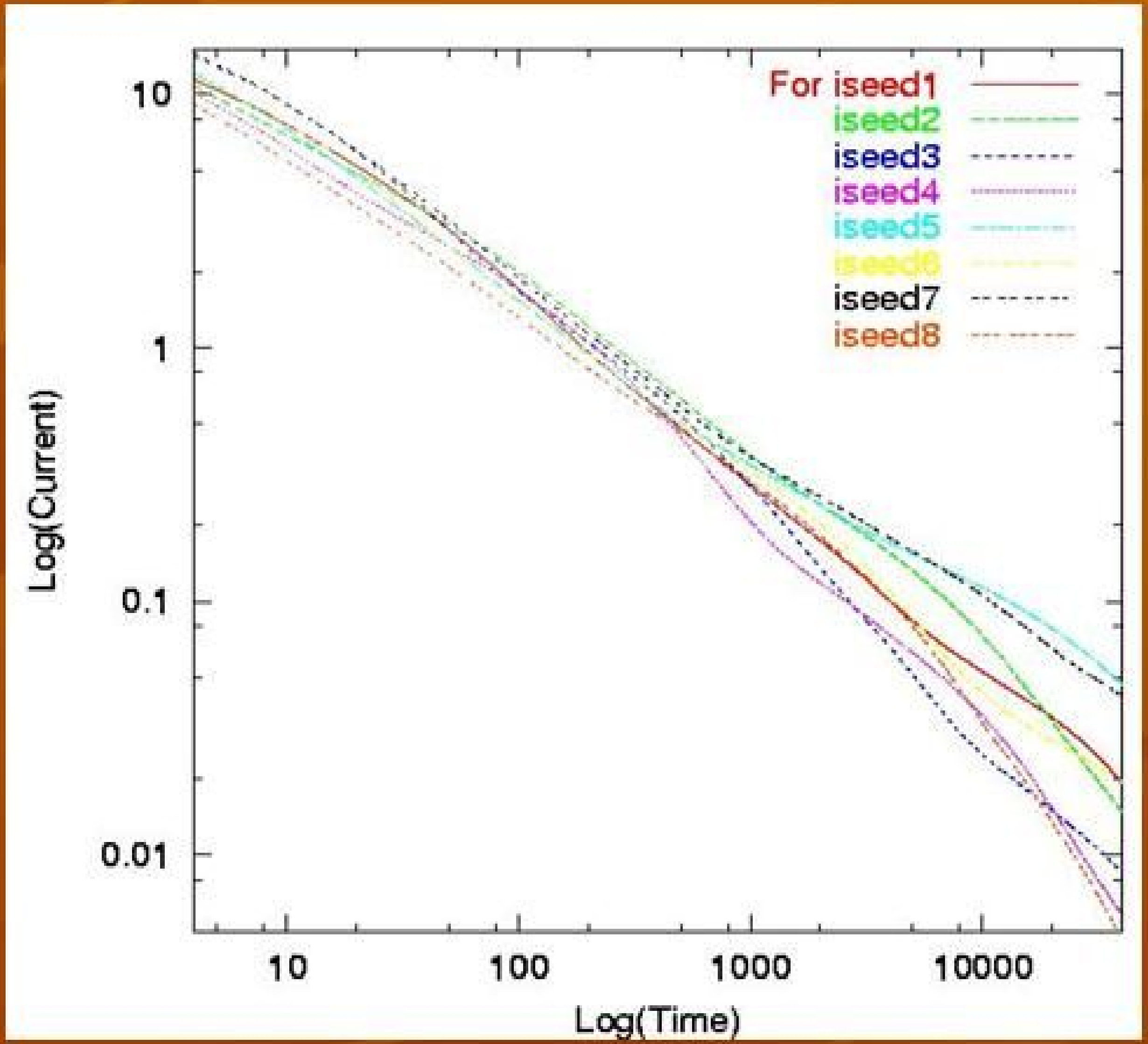



Fig. 5. Evolution of current in a 20×20 RRTN square lattice at times t = (a) 50, (b) 100, (c) 500, (d) 1000, (e) 1134, and (f) in the steady state, in units of one lattice scan.

Two early-stage power-law dynamics for a p=0.50 RRTN sample





A typical two power-law dynamics ($t^{-\alpha_1}$ and $t^{-\alpha_2}$) and final exponential dynamics to an unique steady current (strong memory). The α_1 and α_2 are app. robust; for some separate classes of $v_{ij}(t=0)$; thus on the edge of chaos

The steady current is subtracted out to treat all cases under the same footing; also the final exponential dynamics is not shown further. For different samples (iseeds), α_1 and α_2 vary widely; non-self averaging

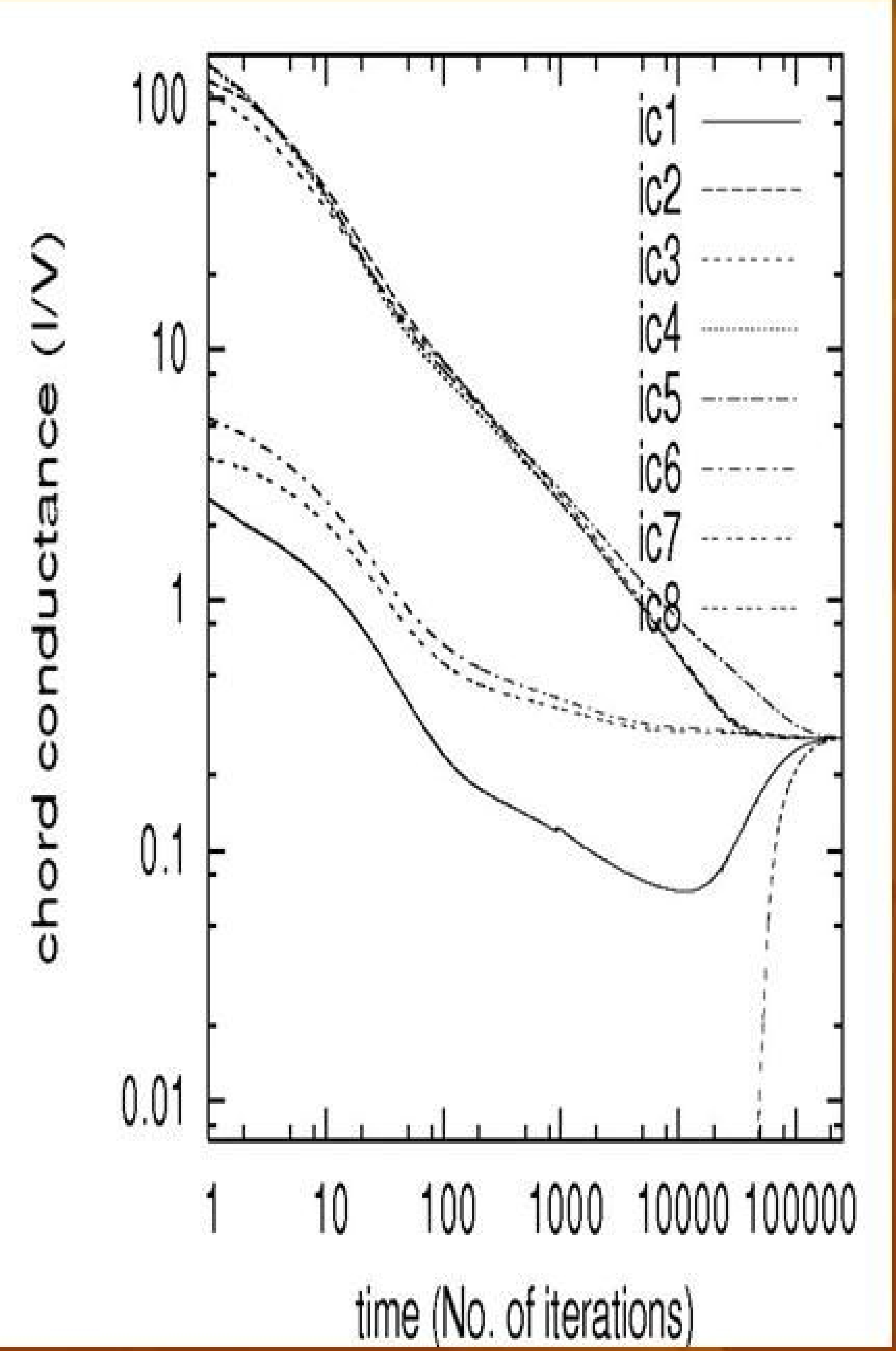
Thus, the out-of-equilibrium dynamics of various systems of nature as well as the same in the RRTN model, suggests that:

- ✓The 1st. order D.E. for the relaxation is strongly non-Debye type, and
- ✓In particular, it should have the empirical form,

$$dI/dt = -\lambda_q I^q - \lambda_r I^r - (1/\tau)\, I; \quad \text{with } q, r > 1$$

[following Tsallis, Bemski and Mendes; Phys Lett A**257**, 93 (1999), and adding a q=1, τ >> 1 (Boltzmann-Gibbs-Shannon) term explicitly].

- ✓As expected it gives rise to two early power-law relaxations with the following exponents $\alpha_1 = 1/(q-1)$ and $\alpha_2 = 1/(r-1)$ for $\tau >> 1$
- ✓Eventually for t >> τ, RRTN's dc-response is in the Upper Linear Regime (ULR), and there is expnl. relxn. (consistent with B-G-S)
- ✓Very strong memory of the steady state (ULR regime); due to perfect correlation in the placements of the t-bonds !!!



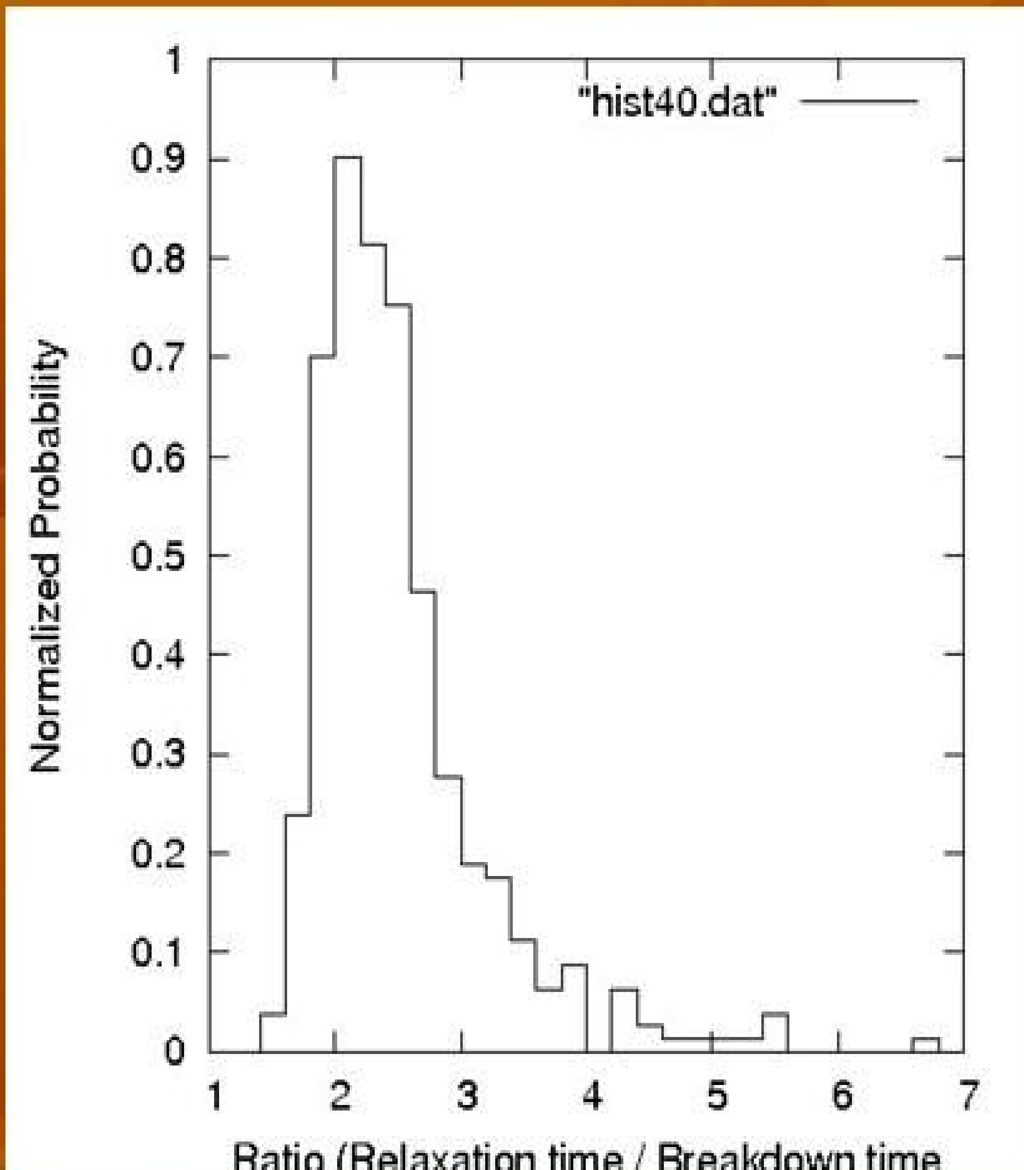
The initial configuration (both random and non-random voltage inputs) chosen and the ensuing (initial) current through the RRTN itself evolving may be different by orders of magnitude.

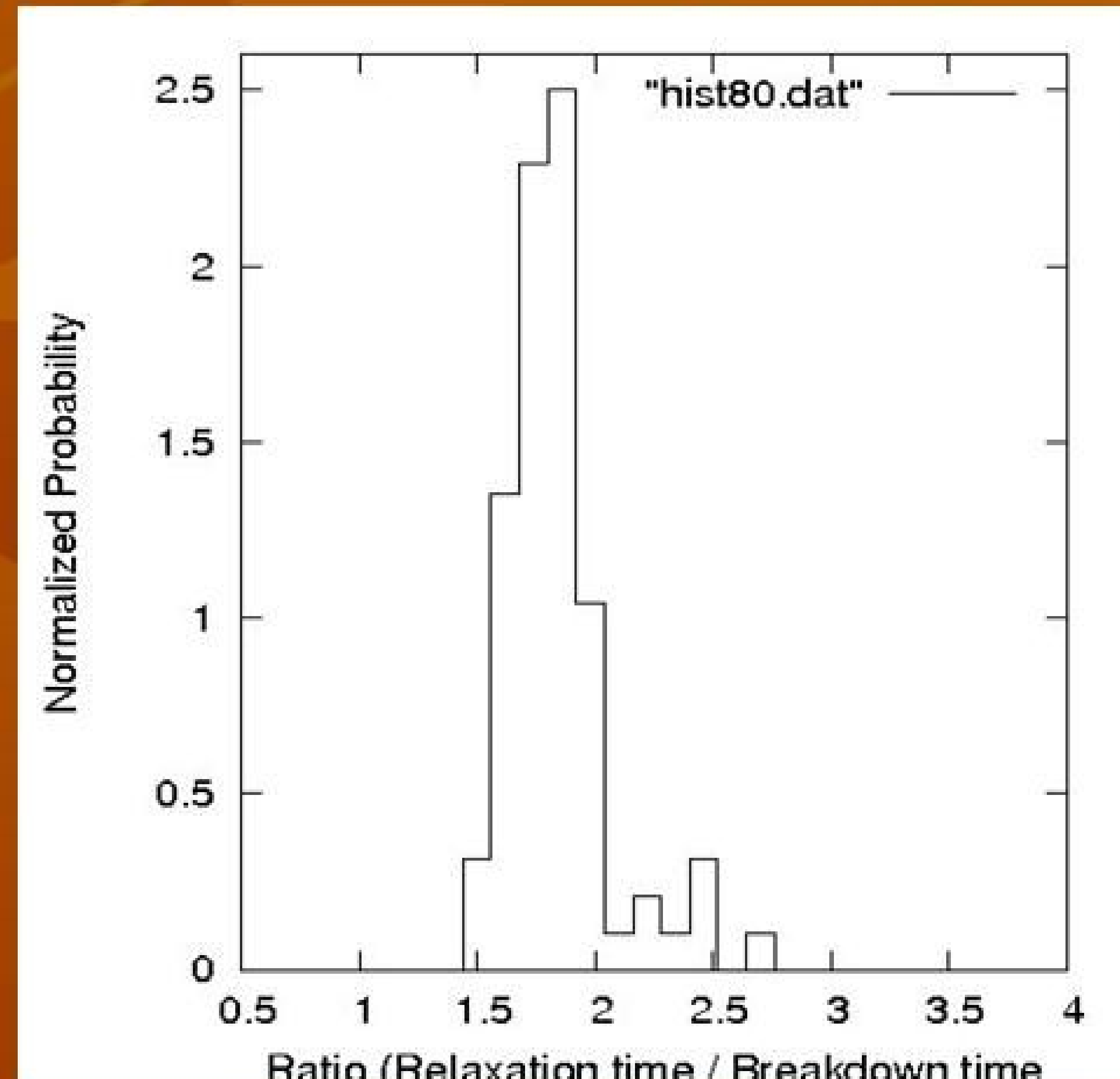
Very many different pathways (Kirchoff's dynamics with local continuity), created as a function of time (in one lattice-scan, discrete-time units).

But, the final configuration/current is UNIQUE. Very strong MEMORY!! The evolution for 'ic1' is particularly surprising! Useful for cognitive processes, learning, fault-tolerant coding, etc.

We first look at two possible time-scales in this problem. The curve 'ic8' looks for it, with the (Initial voltage at layer #1)=V; and zero elsewhere.

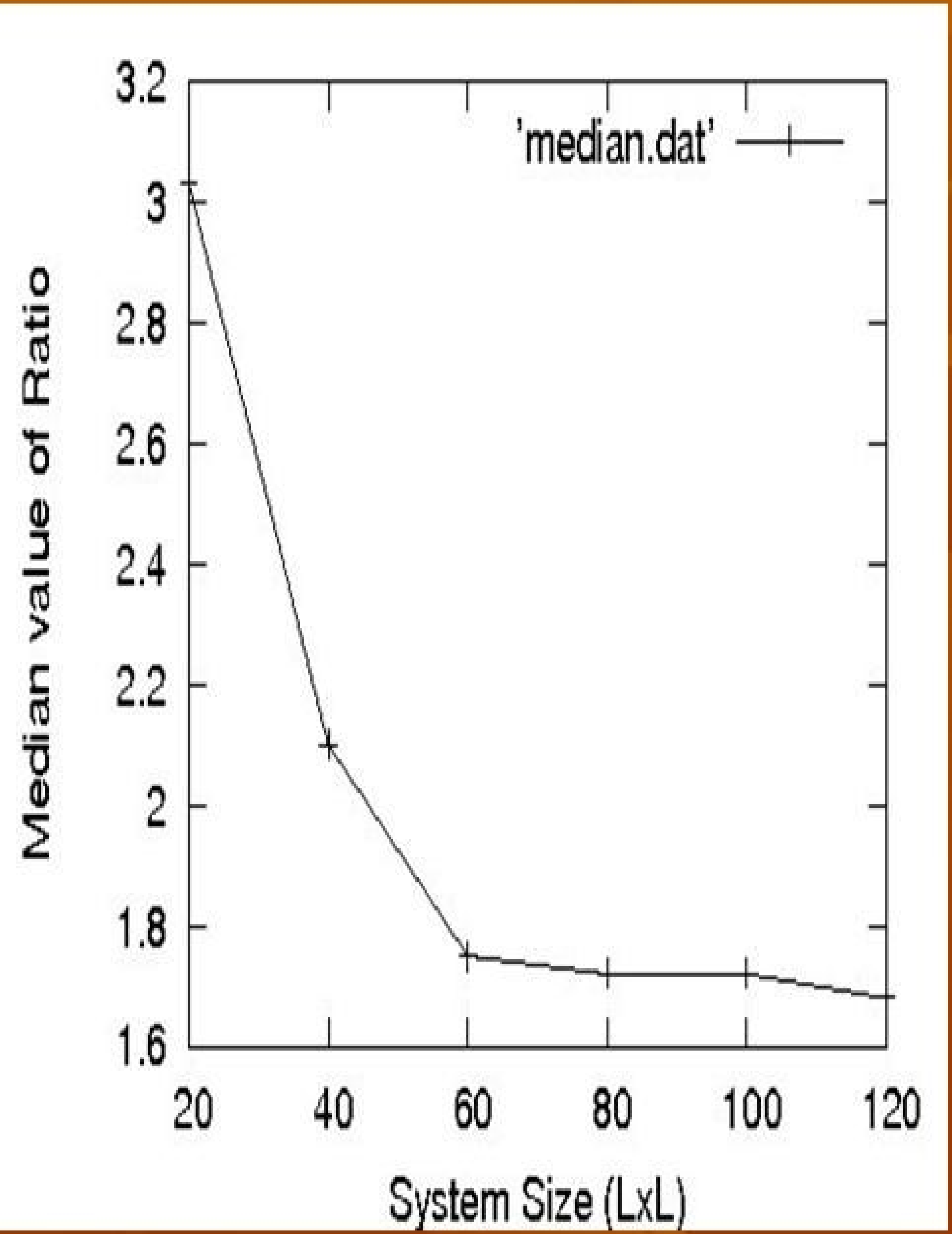
Ratio of (Relaxation time/Breakdown time) of two samples p=0.5





Self-averaging was found to fail in these systems. So, we look at the median of the histograms, and their spreads. Clearly, the spread is reducing fast. So, we study how the peak of the ratio changes with L.

The steady current is subtracted out to treat all cases under the same footing; also the final exponential dynamics is not shown further. For different samples (seeds), α_1 and α_2 vary widely: non-self averaging



The median value also approaches a constant value, very fast (within a system size of about L=200 ! Thus,

This has important implications. First,

(Breakdown time/relaxation time)
--> a constant value (V and p-dep.),
i.e., only one time-scale exists (cf. One-parameter scaling theory in Anderson localizn.) asymptotically.

Second, and more importantly, the result states that if we know the relaxation time of the underlying RRN, we can predict the breakdown-time in the large L limit.

Conclusion on the RE-COGNITION of Informations:

- The steady state configuration is uniquely recognized even for a random initial configuration. The exponents of the dynamics do depend on the initial configurationtion. But, the final configuration is very ROBUST. It is like the strong memory in the protein-folding problem (Leventhal)!!

- Their origin is not related to any self-organized criticality, but due to the nonlinearity and the perfect statistical correlation, in the placement of the t-bonds, in-built in the network (RRTN).

- The information content is *algorithmically compressible*. This is required! [See P.C.W. Davies, Why the physical world is so comprehensible? In "Complexity, Entropy and the Physics of Information?" Ed. W.H. Zurek, pp 61-70 (Addison-Wesley, Redwood city CA, 1990]. The information to be kept between some Bob and an Alice is minimal: just the random no. generator and the initial seed. Then V generates the picture'!

- One then notes that this property of the network is very useful for fault-tolerant cryptography with public key crypto-syetems.

THANK YOU