Elementary Mechanisms of Deformation in Amorphous Solids: From Zero to Low Temperature

Anaël Lemaître

Rhéophysique

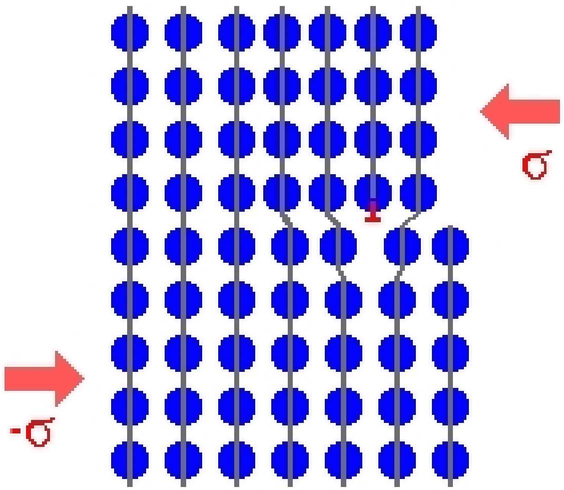
Université Paris-Est

Christiane Caroli, Joyjit Chattoraj

What are the elementary mechanisms of deformation?

<u>In crystals</u>

defects = dislocations
(Volterra, 1930; SEM, 1960)



Interaction and motion understoon
(Peierls, Nabarro, Friedel, 1950's)

Dislocation dynamics in computer codes since the 1980's

<u>In disordered materials</u>

No topological order => defects?

What are the elementary mechanisms of deformation?

How can we up-scale the dynamics?

Amorphous materials

Hard glasses

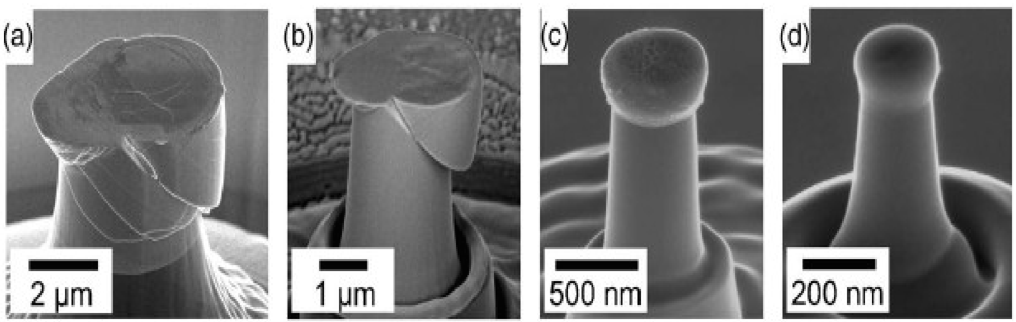
Soft glasses

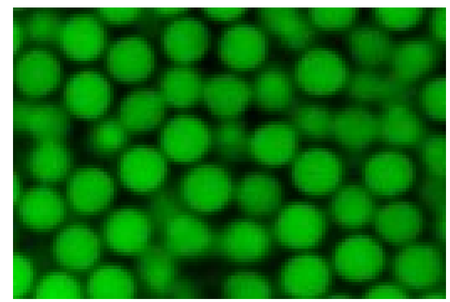
Metallic/oxyde glasses

Polymers

Colloids

Foams





Length scales ⩽ nm's

Energies ~ 0.1—1 eV

Stresses ~ GPa

Length scales ⩾ 0.1 μm's

Energies ~ kT = 1/40 eV

Stresses ~ Pa—kPa

Can we identify some mechanisms of deformation,
at least for broad classes of materials, or time-, energy-, length-scales?

Plasticity

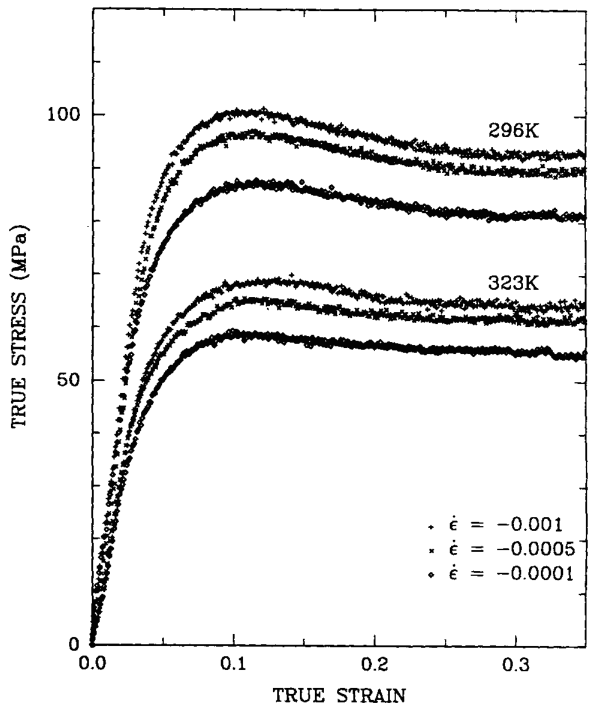
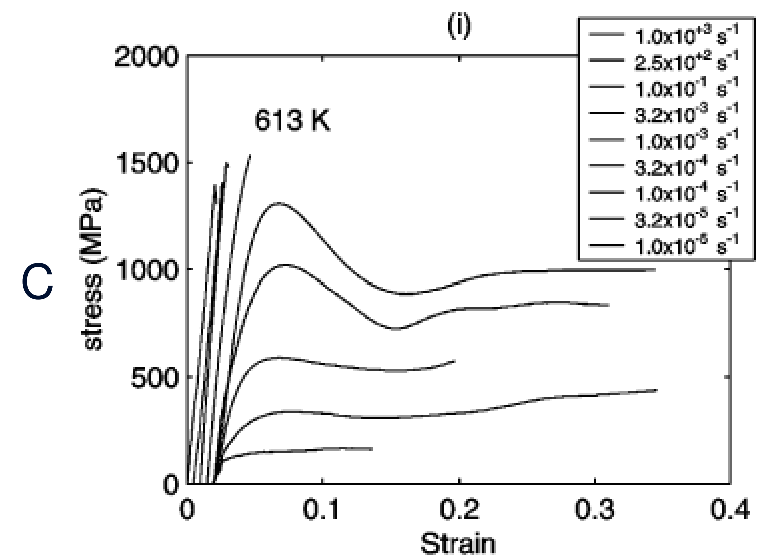


Fig. 2. Constant true strain rate tests on quenched PMMA at 296 and 323 K.

PMMA, uniaxial tension
(Hasan & Boyce, 1997)

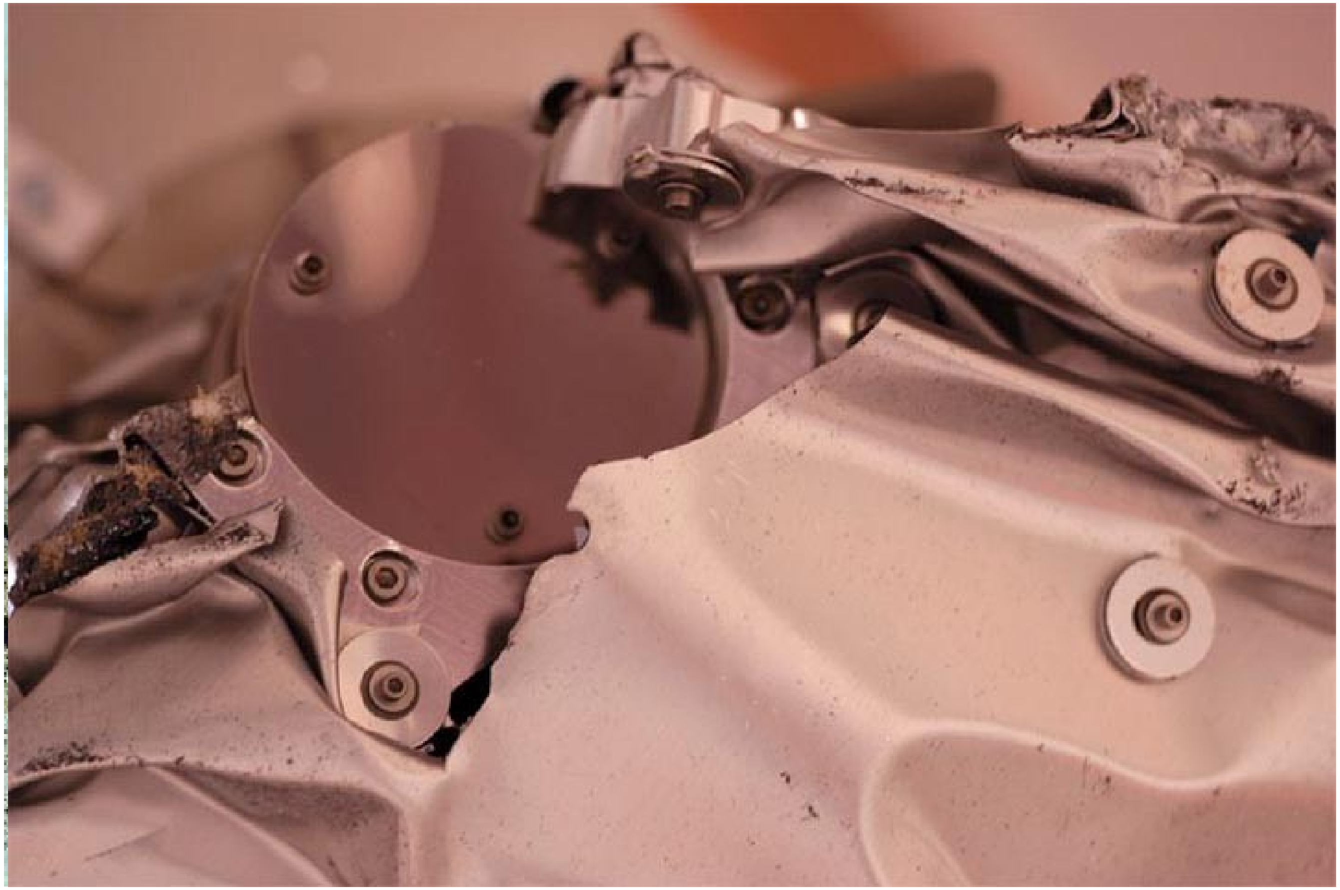


Metallic glass
(Johnson, Caltech)

Depending on loading conditions:

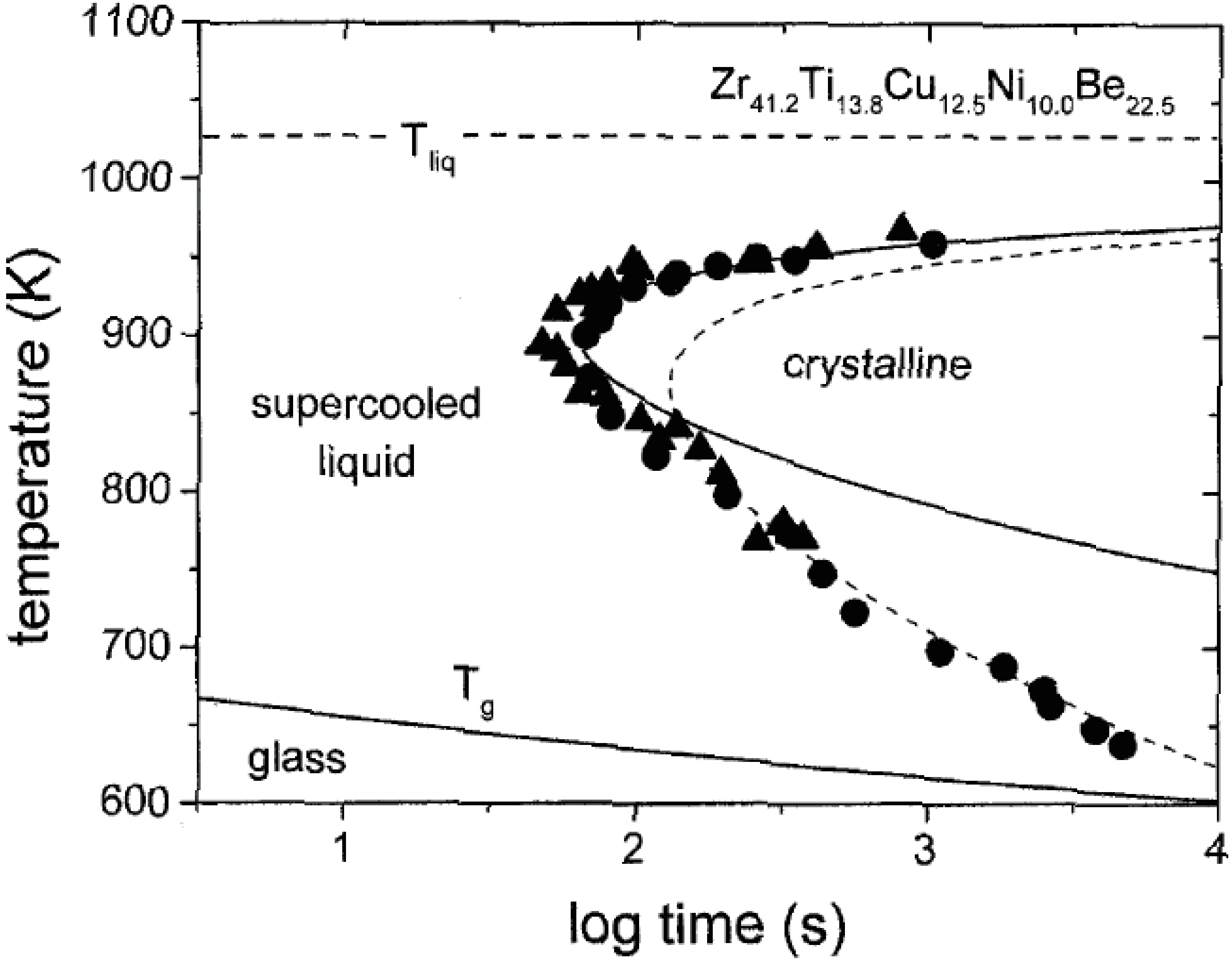
- Fracture
- Localization
- Homogeneous flow

Metallic glasses

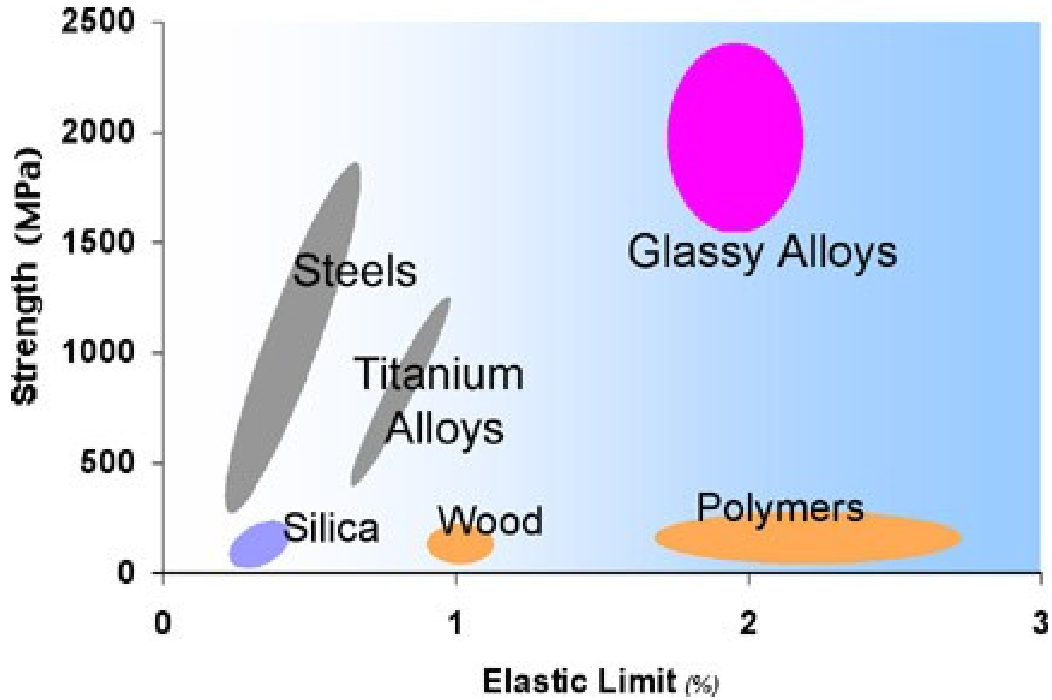


Cassini spacecraft, after landing

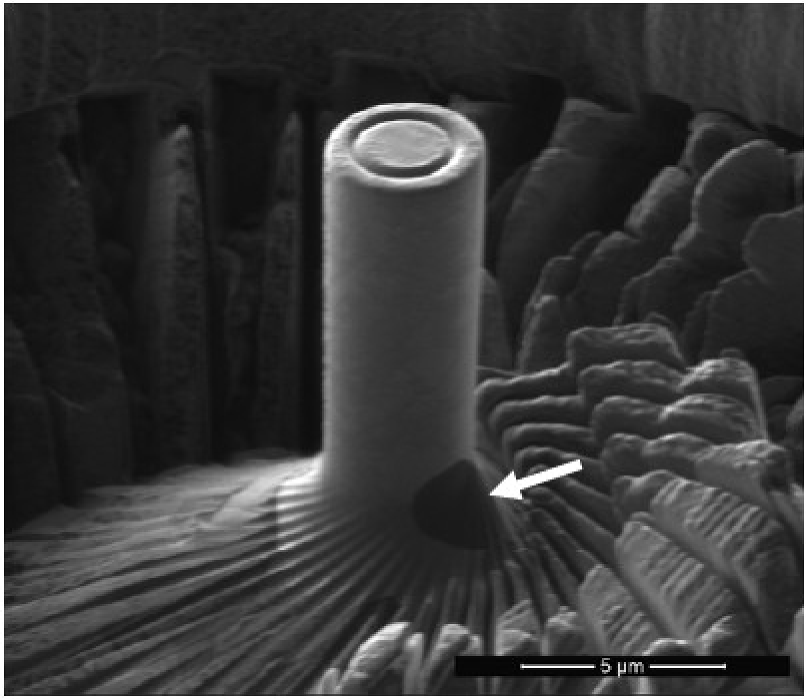
Metallic glasses

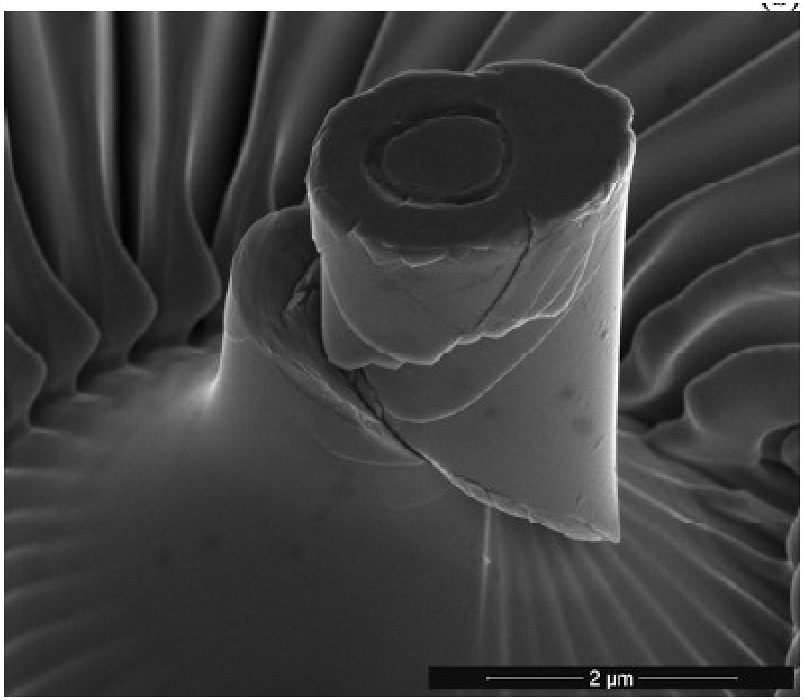


Metallic glasses



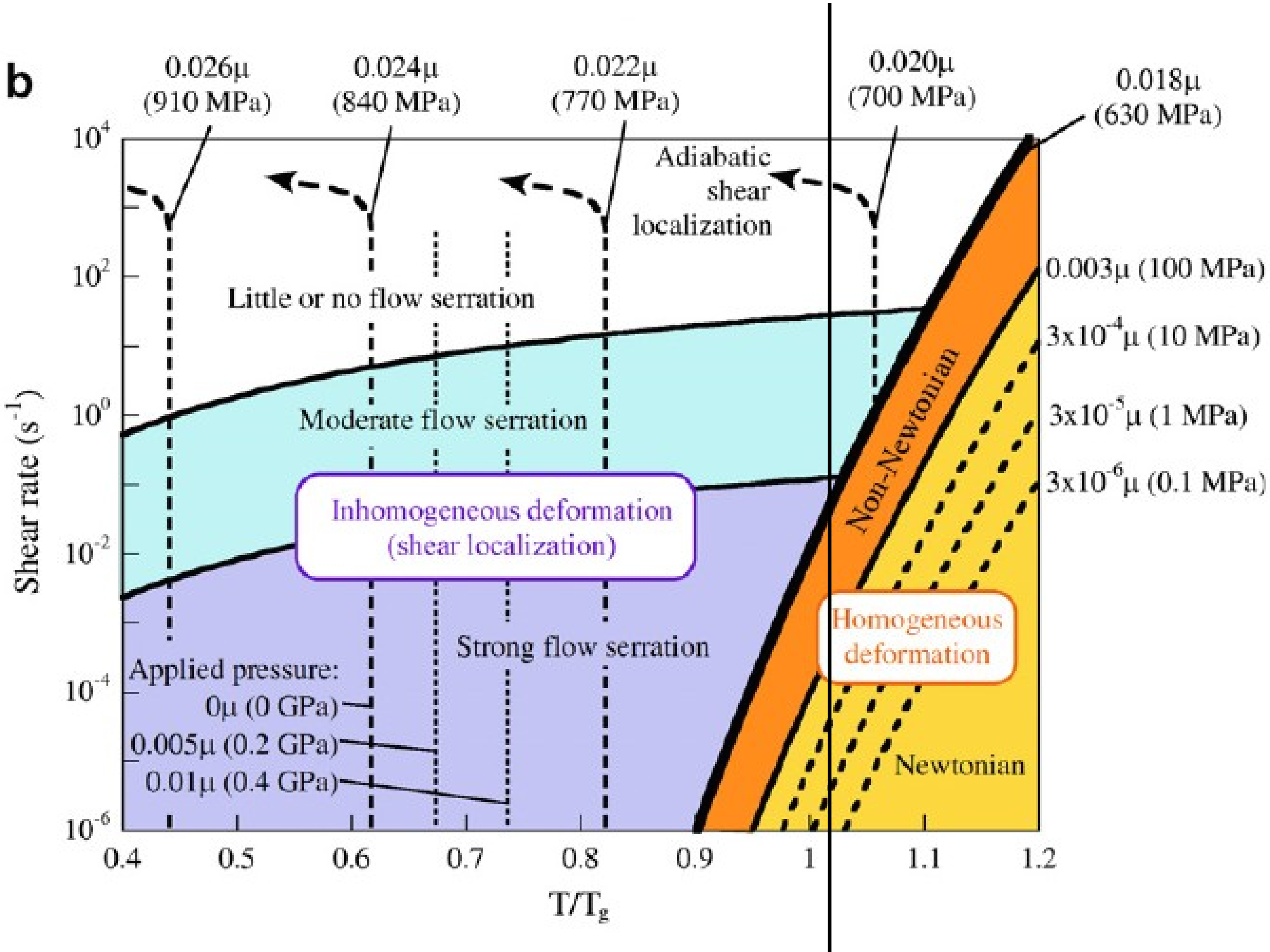
Johnson



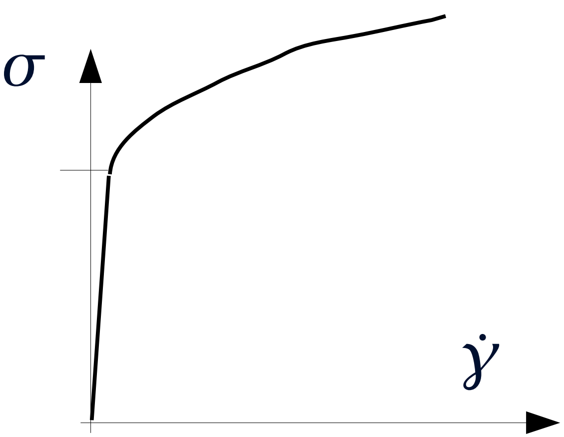


Schuster el al (2007)

Deformation map for a metallic glass

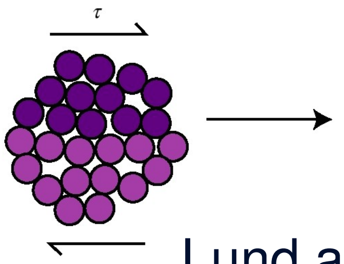


Schuh *et al*, Acta Mat. 55, 4067 (2007)

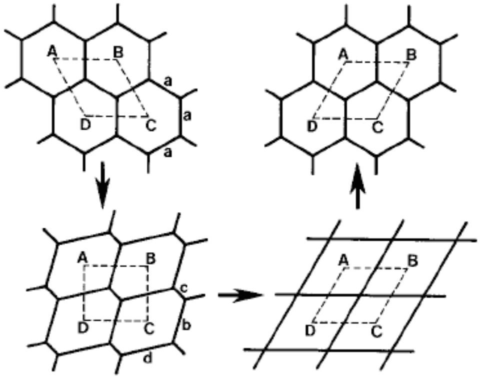


Shear transformations

Ali Argon (1979):
Deformation due to local
"shear transformations"



Lund and Schuh (2004)



Princen (1981)

Mean-field theories

Free-volume theory (Spaepen, 1977)

$$\dot{\gamma} = \Delta \epsilon_0 e^{-v_0/v_f} (R_+ - R_-) \qquad v_f = \text{ free-volume, varies with time}$$

STZ theory (Falk & Langer, 1998)

$$\dot{\gamma} = \Delta \epsilon_0 (R_+ n_+ - R_- n_-) \qquad n_\pm = \text{ “zone” densities, varies with time}$$

density of defects $\propto e^{-1/\chi}$ with χ a dynamical “effective temperature”

SGR theory (Sollich, 1998)

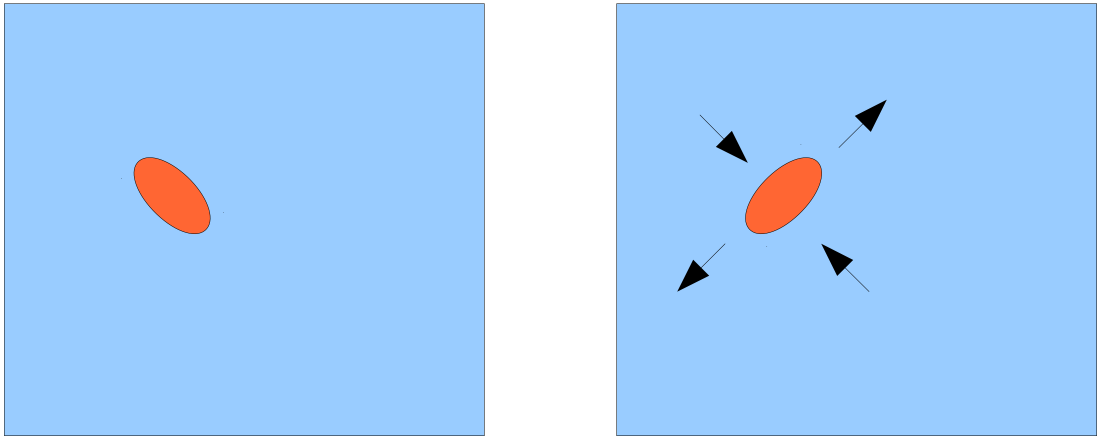
$$\frac{\partial P}{\partial t} = -\dot{\gamma} \frac{\partial P}{\partial l} - \Gamma_0 e^{-(E - \frac{1}{2} k l^2)/x} P + \Gamma(t) \rho(E) \delta(l)$$

$P =$ distribution of barrier energies (from Bouchaud's trap model)

$x =$ effective temperature, fixed, interpreted as due to “noise”

$\sigma = k \langle l \rangle_P$

Martensitic transformations (Eshelby)

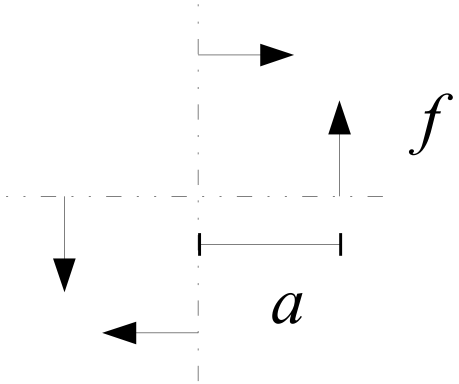


Eshelby (1957): the effect of the transformation can be modeled by forces lying at the boundary

Picard et al (2004):

$$\mu \Delta \vec{u} - \nabla p = \sum_i \vec{f}_i \delta(\vec{r} - \vec{r}_i)$$

$$\nabla . \vec{u} = 0$$



$$\vec{u} = \frac{2 a^2 \Delta \epsilon_0}{\pi} \frac{x y}{r^4} \vec{r}$$

$$\sigma_{xy} = \frac{2 \mu a^2 \Delta \epsilon_0}{\pi} \frac{\cos(4\theta)}{r^2}$$

Dipolar strength: $f a = \mu a^2 \Delta \epsilon_0$

A transforming zone is embedded in an elastic medium

Numerics are really cool at T=0

The problem of timescales:

The typical LJ time unit:

$$\tau_{LJ} \sim 10^{-13}\,\mathrm{s}$$

Simulations can access:

$$10^{5}\,\tau_{LJ} \sim 10^{-8}\,\mathrm{s}$$

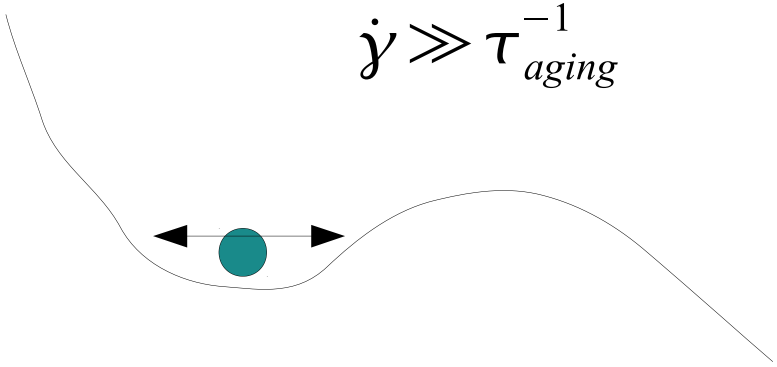
To reach 100% strain:

$$\dot{\gamma} \sim 10^{8}\,\mathrm{s}^{-1}$$

Low temperature glass:

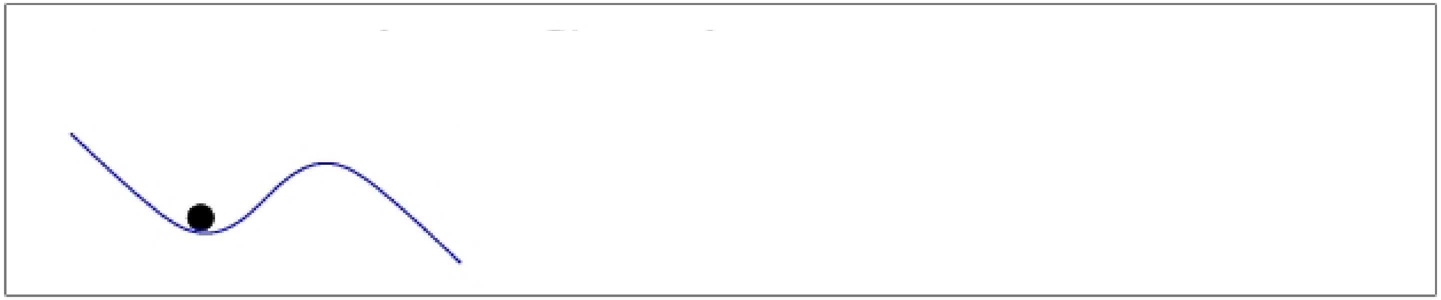
- As soon as $T < T_g$ a glass usually lies near local minima (inherent structures):

$$\dot{\gamma} \gg \tau_{aging}^{-1}$$



Potential Energy Perspective

- Strain biases landscape (Malandro and Lacks 98):



Elastic segment: system follows an energy minimum; reversible.

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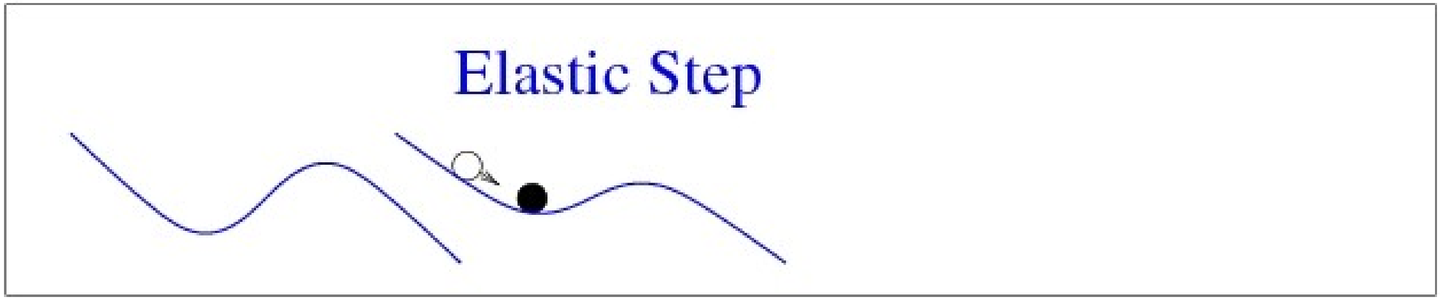

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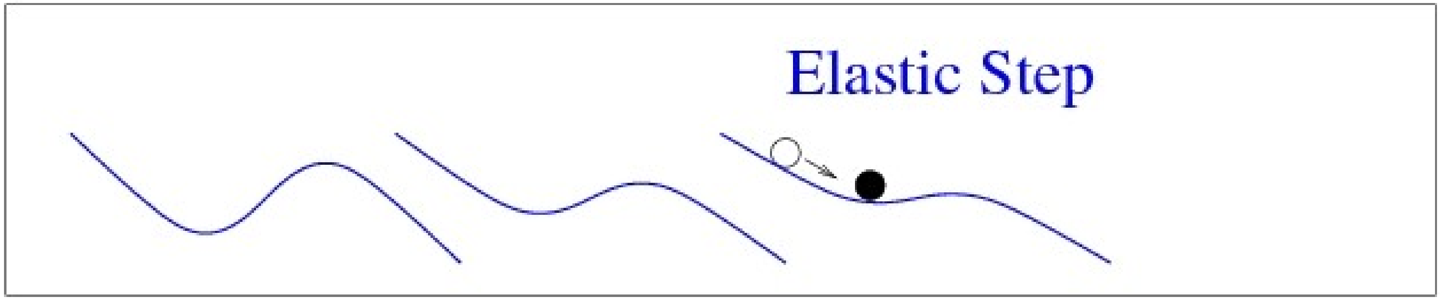

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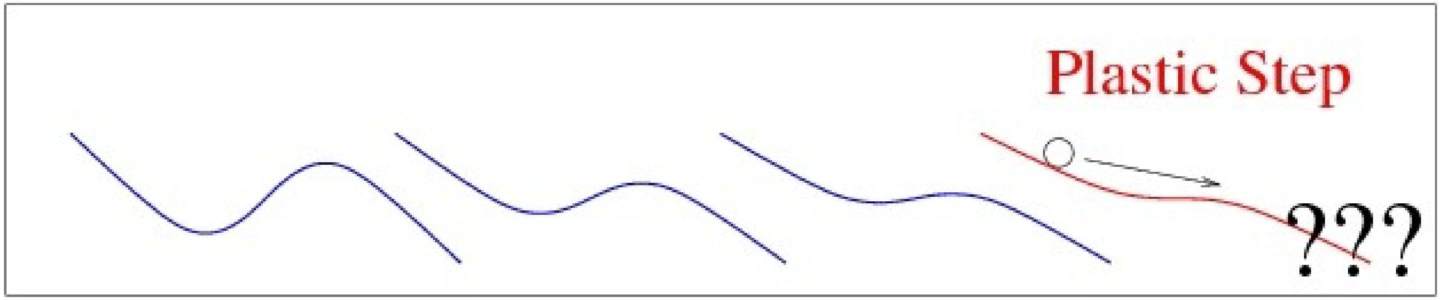

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Plastic event: local minimum annhilates; irreversible.

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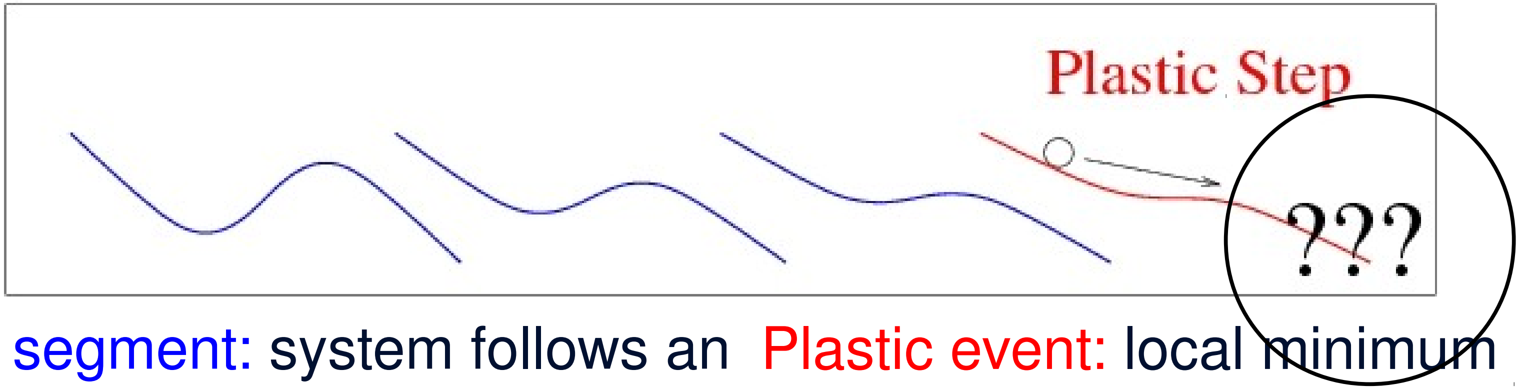


Elastic segment: system follows an energy minimum; reversible.

Plastic event: local minimum annhilates; irreversible.

Brillouin 1904:

Finite yield stress $\Rightarrow$ finite dissipation for vanishing $\dot{\gamma}$

As $\gamma \nearrow$, periods of adiabatic adaption (shifted equilibrium) interspeded with fast instability events

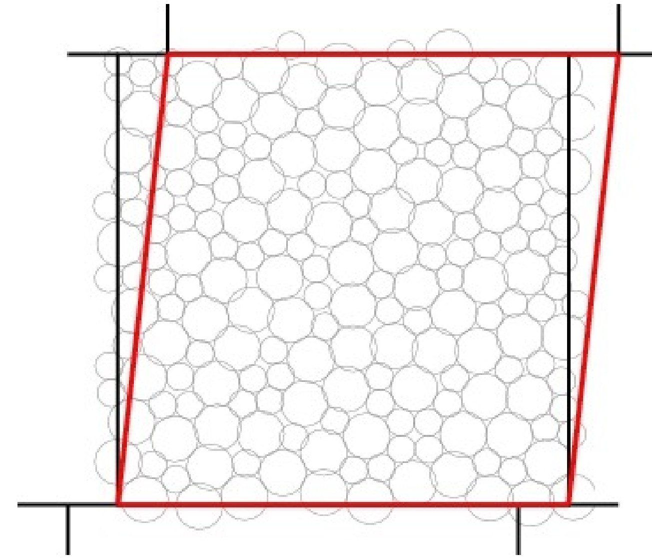
Yield stress = signature of multistability

Events occur at spinodals in stress-biased landscape

What is the nature of elementary events?

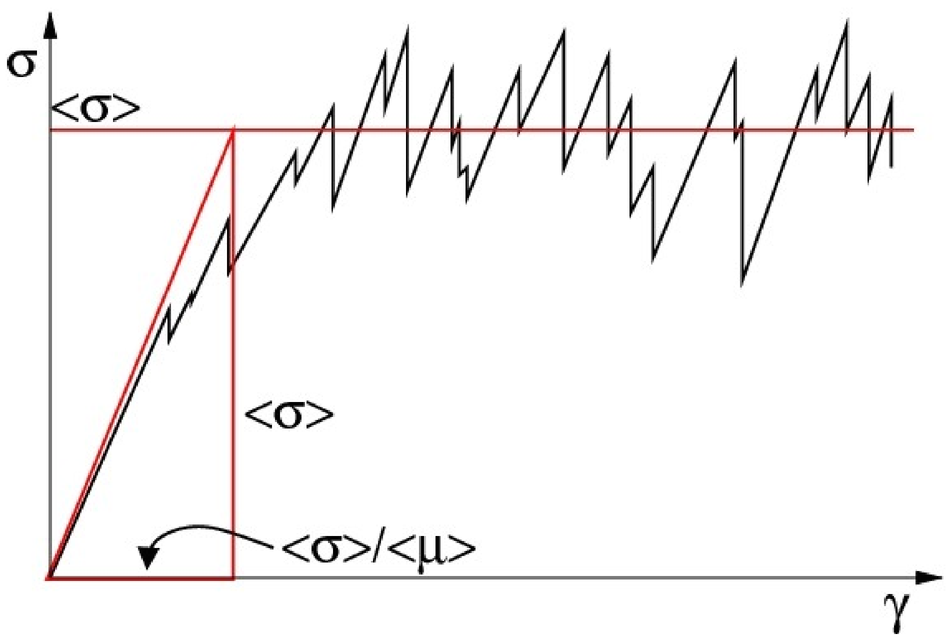
AQS simulations

$$\tau_{aging}^{-1} \ll \dot{\epsilon} \ll \tau_{relax}^{-1}$$



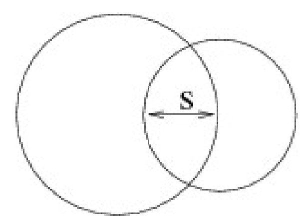
Athermal, quasi-static protocol: $T = 0 \quad \dot{\gamma} \to 0$

- Minimize energy
- Apply a small increment of strain (homogeneously)
- Repeat



Focus on steady state

Interparticular forces:



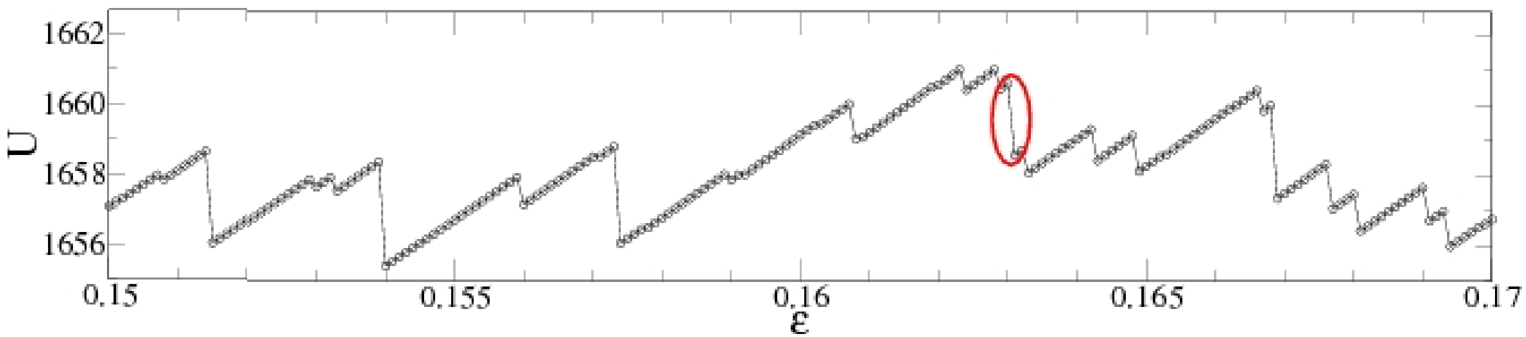
$$U = \frac{1}{2} k\, s^{5/2}$$ grains

$$U = \frac{1}{2} k\, s^{2}$$ bubbles

$$U = k\left(r^{-12} - 2 r^{-6}\right)$$ atoms, particles

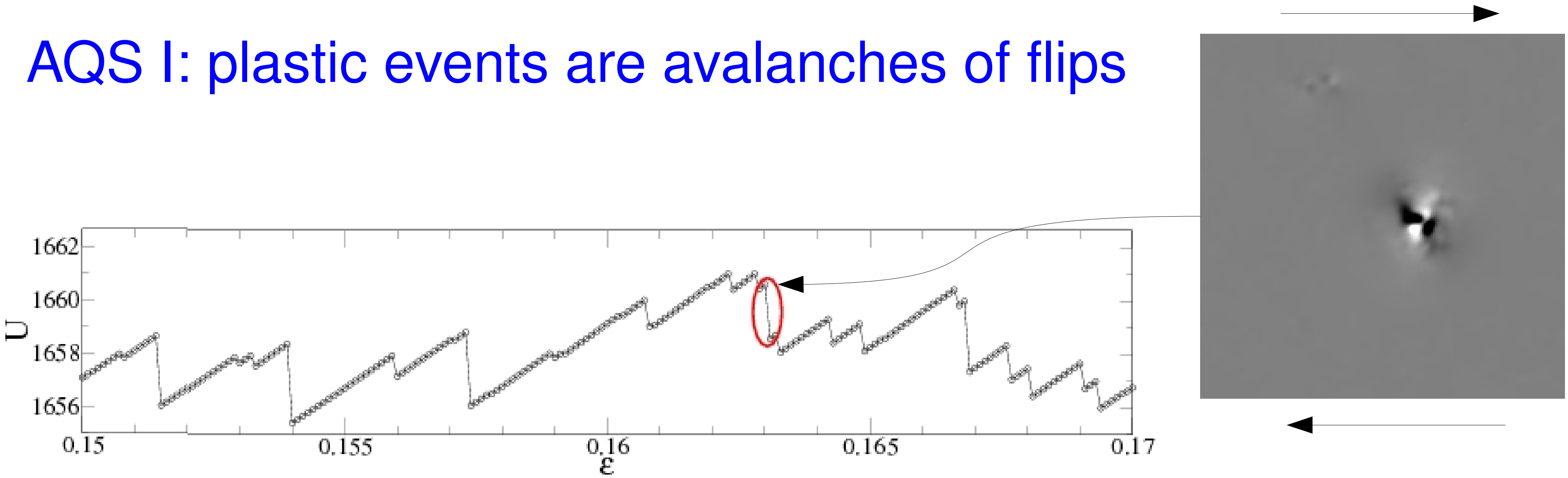
C. Maloney and AL, PRL 93, 016001 (2004); PRE 74, 016118 (2006)

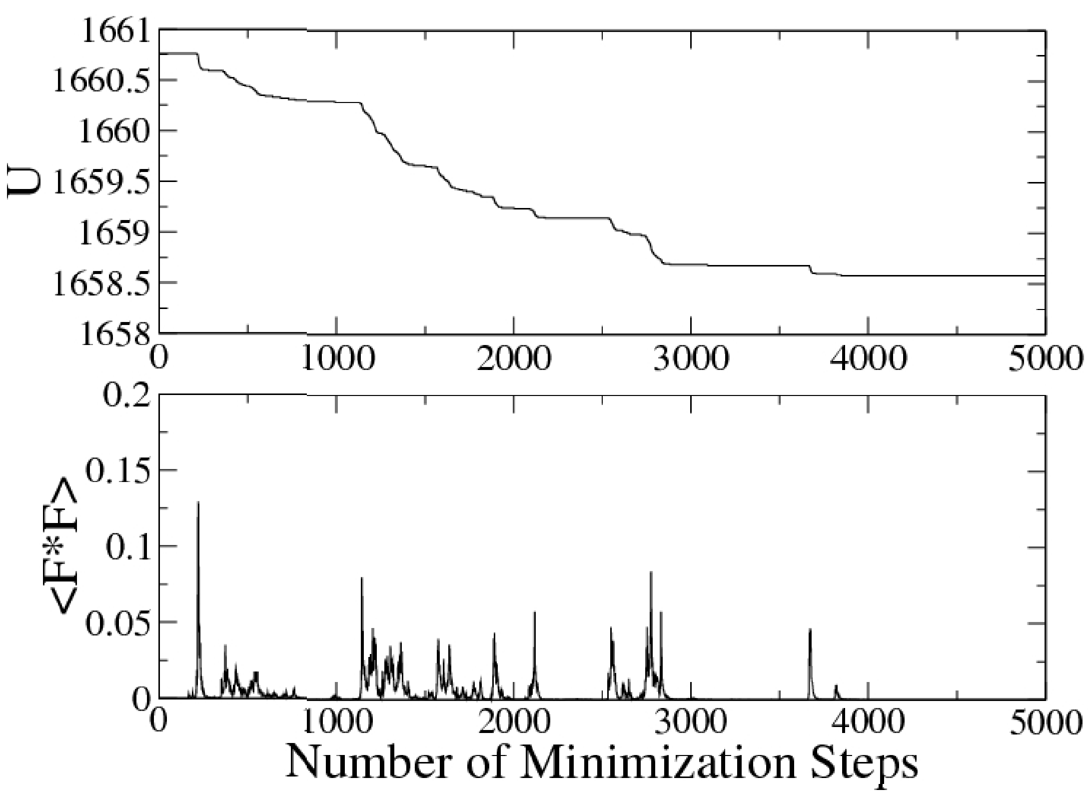
AQS simulations



- Very stringent convergence criterion
- Very small steps to guarantee convergence to QS limit
- Be patient... (patience is bitter but its fruits are sweet)
- Look at the spatial organization of elementary events, hoping to observe elementary shear transformations

AQS I: plastic events are avalanches of flips





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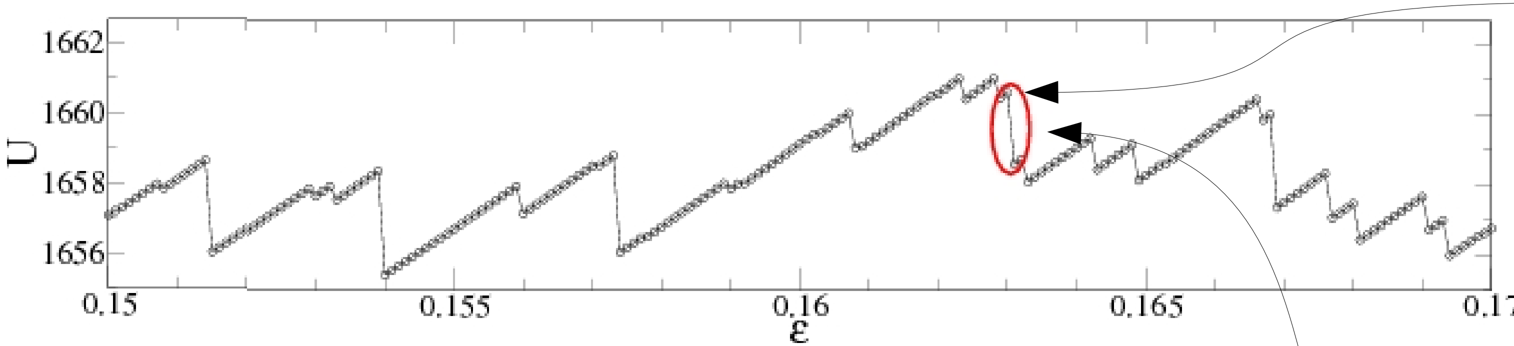

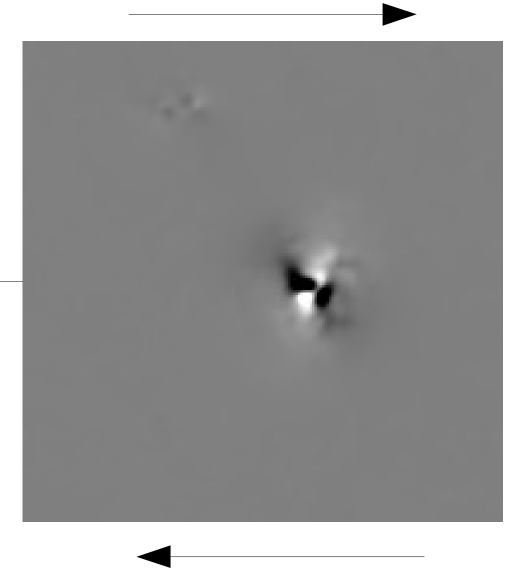

<u>In 2D</u>

C. Maloney and AL,
PRL 93, 016001 (2004);
PRE 74, 016118 (2006)

$$\Delta E \sim L$$

E. Lerner and I. Procaccia,
PRE 79, 066109 (2009)

$$\Delta E \sim L^{\beta}, \beta = 0.74$$

<u>In 3D</u>

N. Bailey et al
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$$\Delta E \sim L^{1.4}$$

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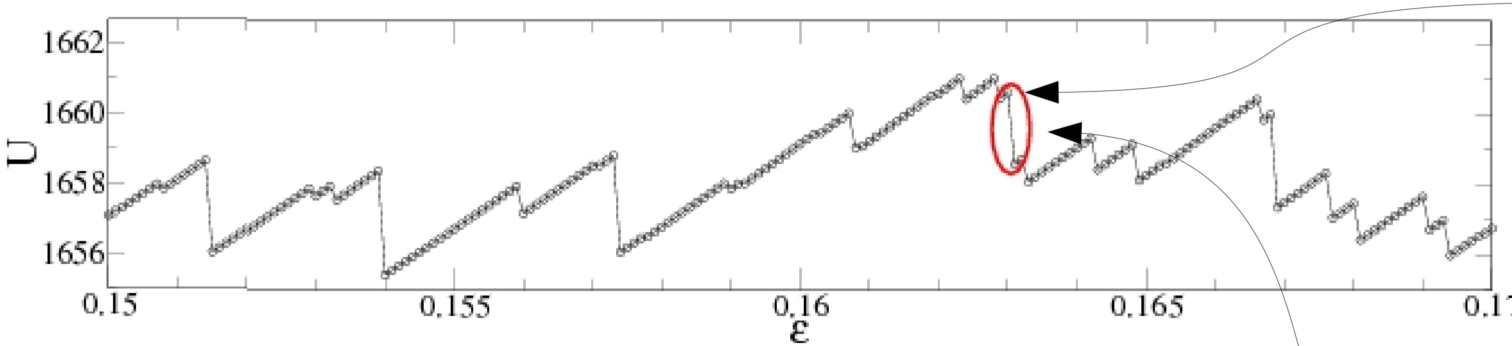

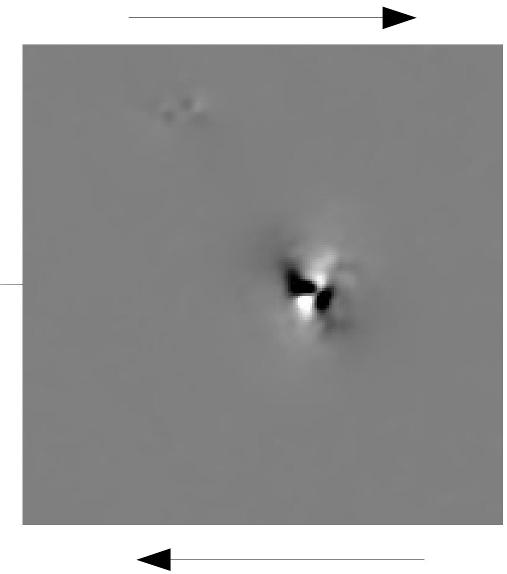

L

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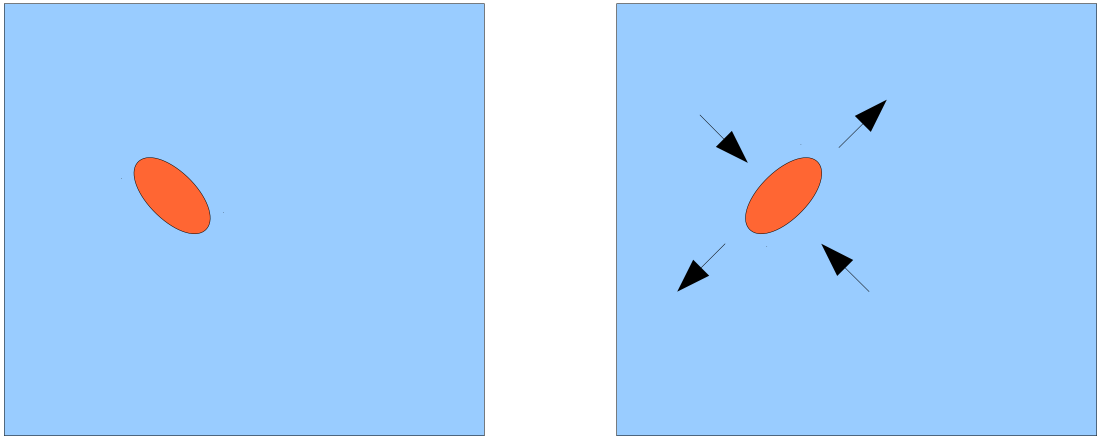

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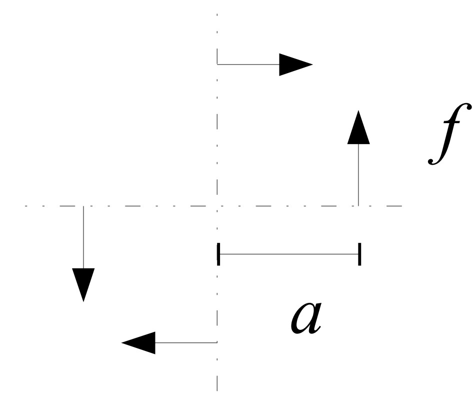


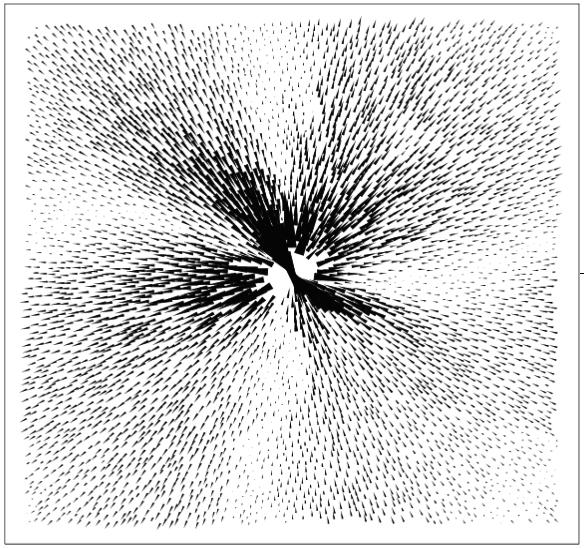
Dipolar strength: $f\, a = \mu\, a^2 \Delta \epsilon_0$

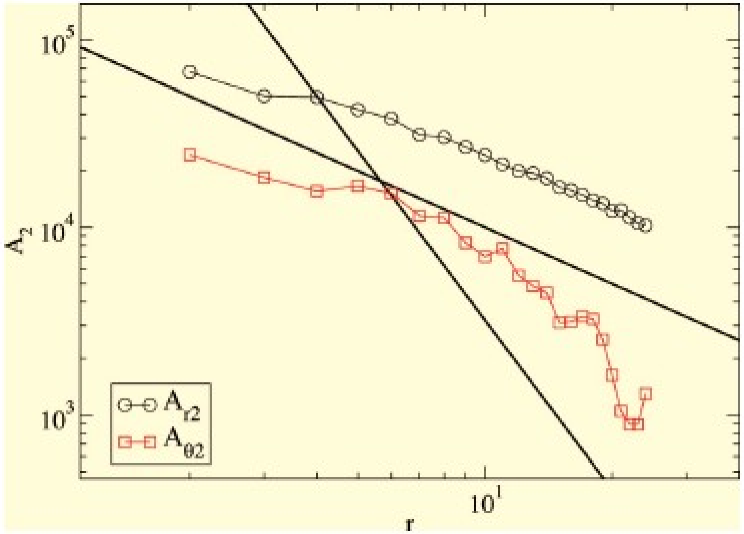
$$\vec{u} = \frac{2 a^2 \Delta \epsilon_0}{\pi} \frac{x\, y}{r^4} \vec{r}$$

$$\sigma_{xy} = \frac{2 \mu\, a^2 \Delta \epsilon_0}{\pi} \frac{\cos(4\theta)}{r^2}$$

A transforming zone is embedded in an elastic medium

AQS II: flips are analogous to Eshelby transformations





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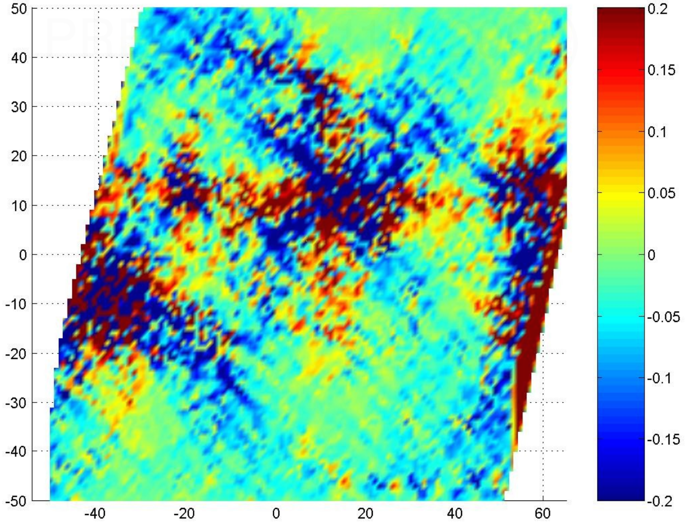

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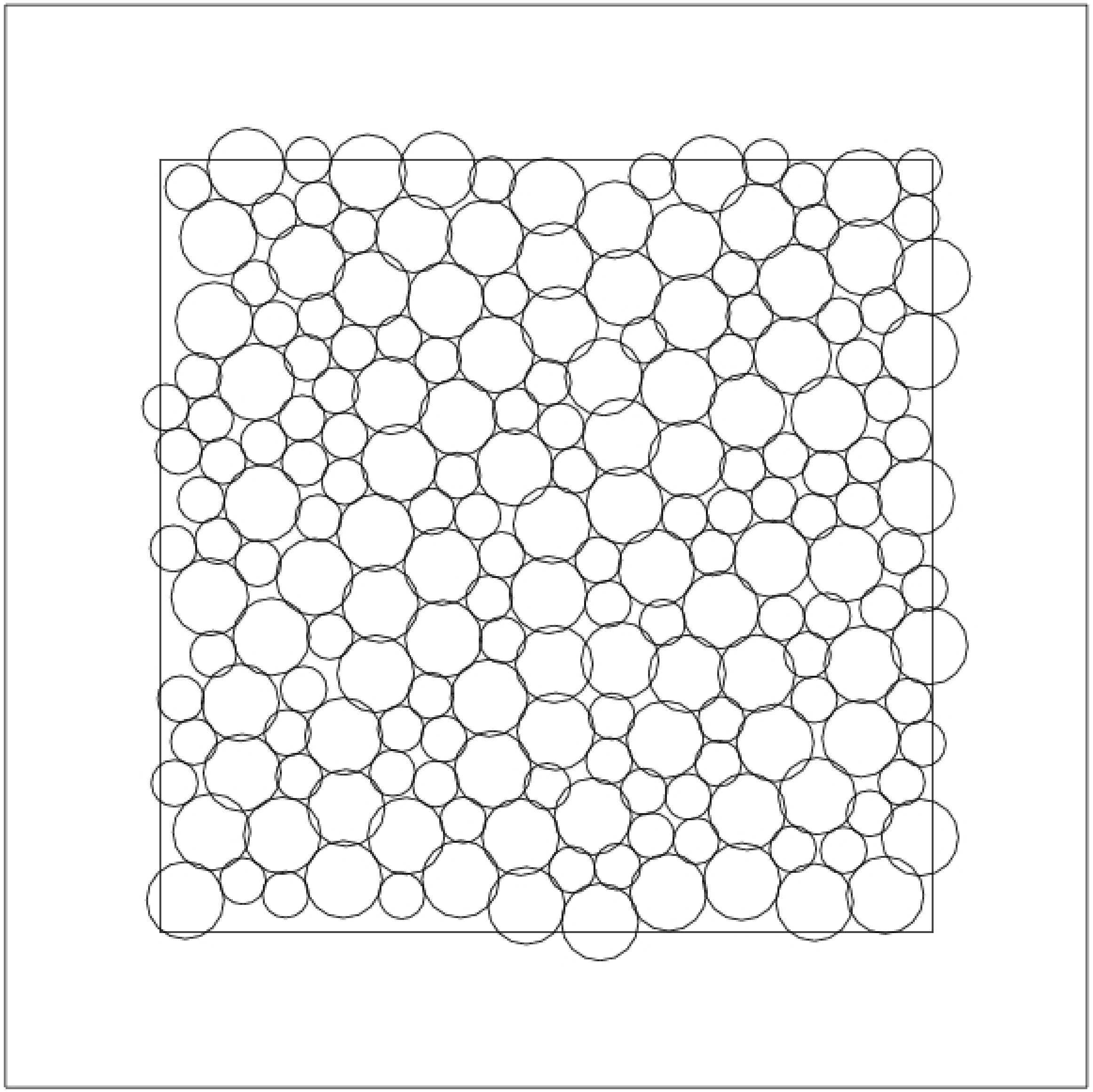
A. Tanguy *et al,* EPJE 20, 355 (2006)

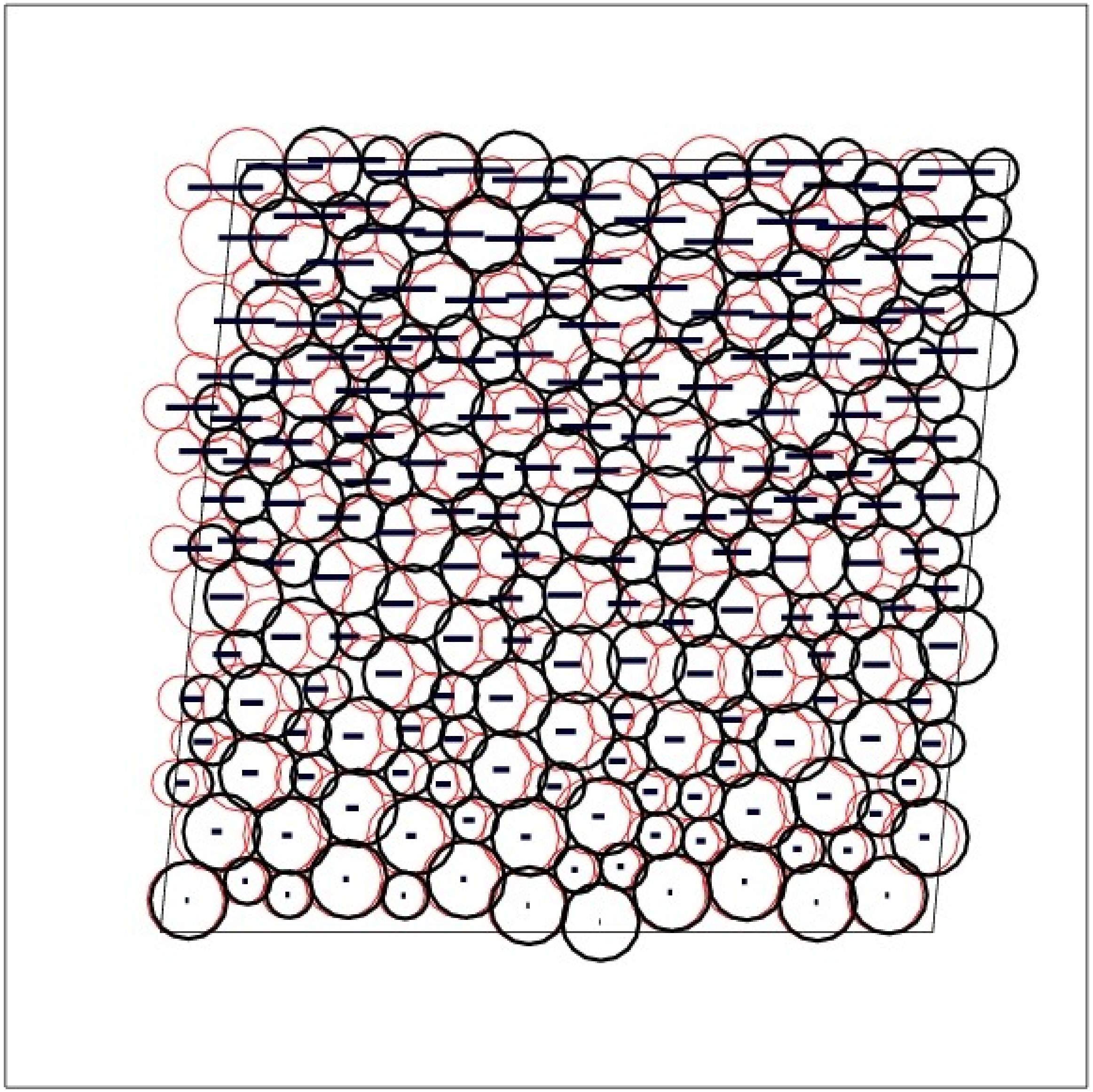
Non-affine displacement field



γ_0

$\underline{r}_i(\gamma_0)$

Non-affine displacement field



γ_1

$\underline{\underline{F}}.\underline{r}_i(\gamma_0)$

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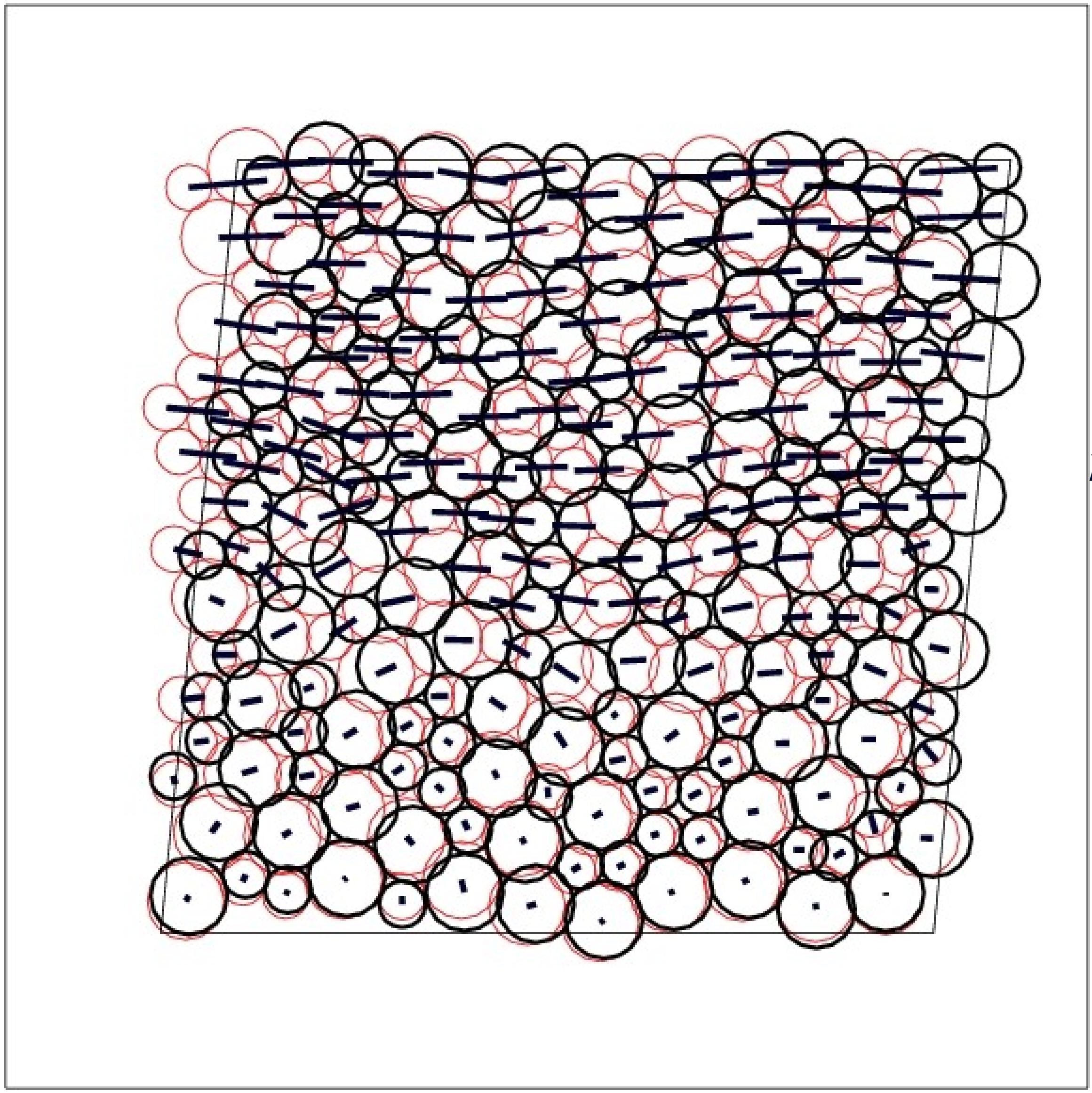
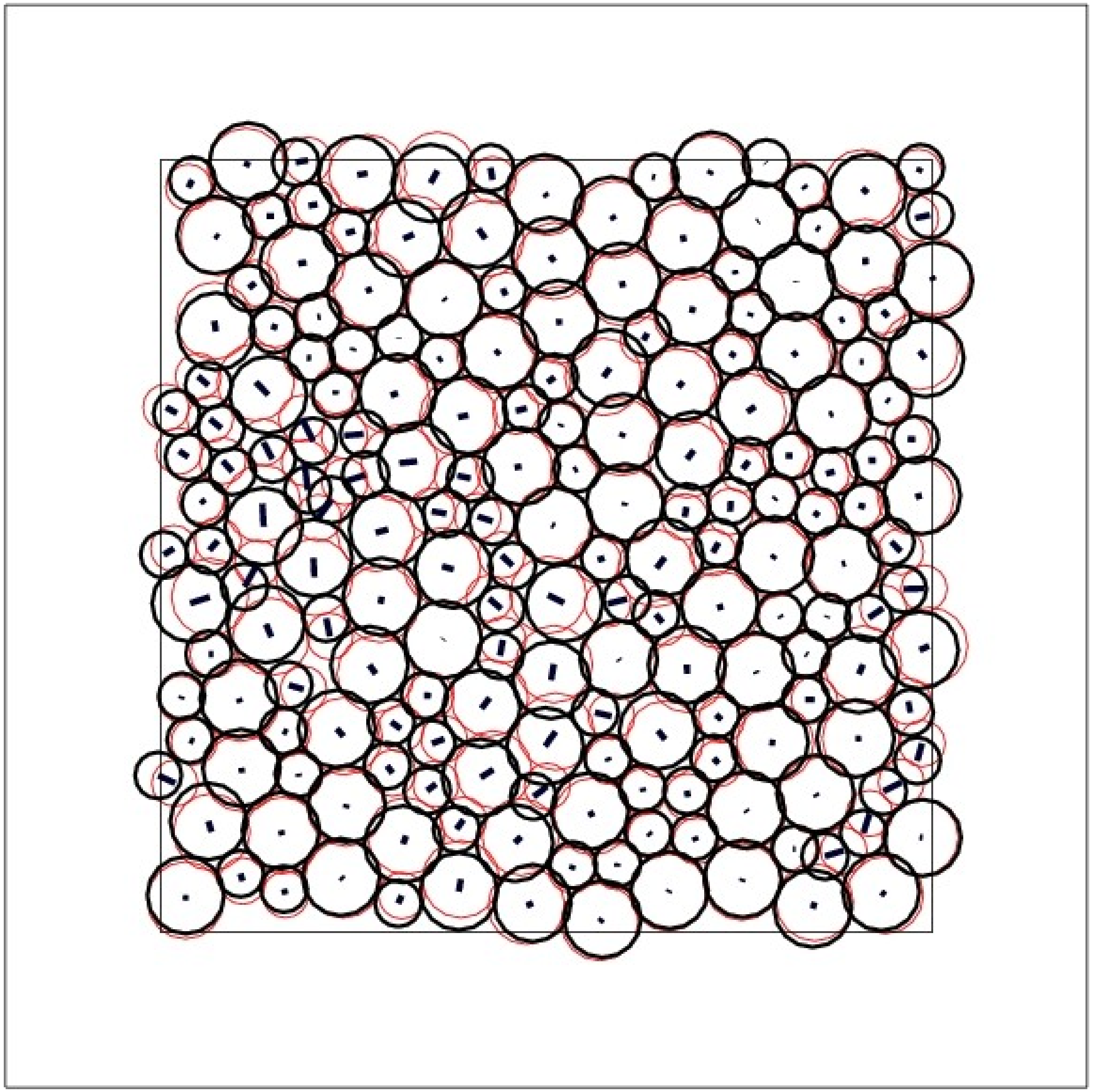


γ_1

$$\underline{r}_i(\gamma_1) \neq \underline{\underline{F}}.\underline{r}_i(\gamma_0)$$

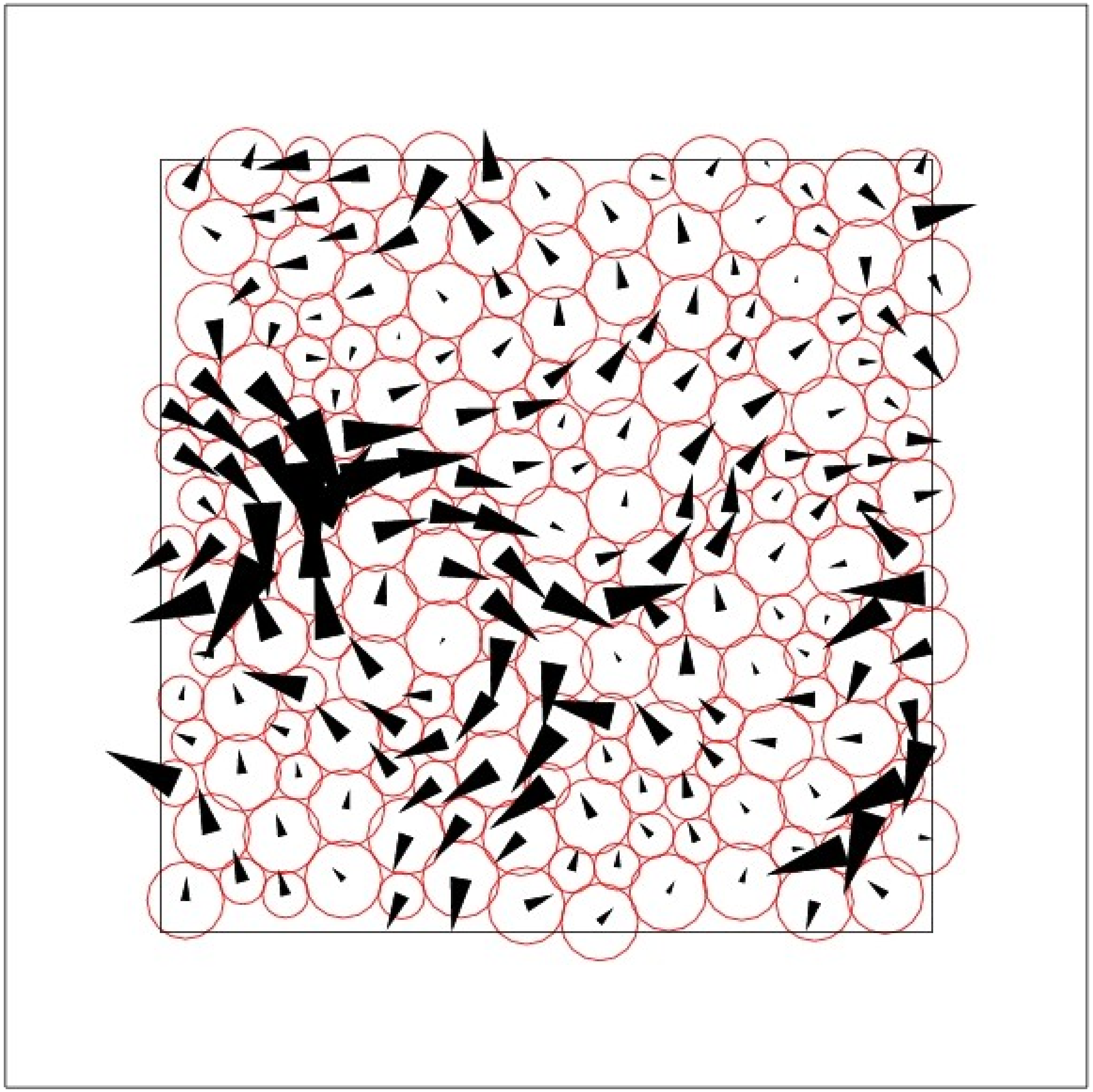
Non-affine displacement field



γ_1

$\underline{\underline{F}}^{-1} \cdot \underline{r}_i(\gamma_1)$

Non-affine displacement field



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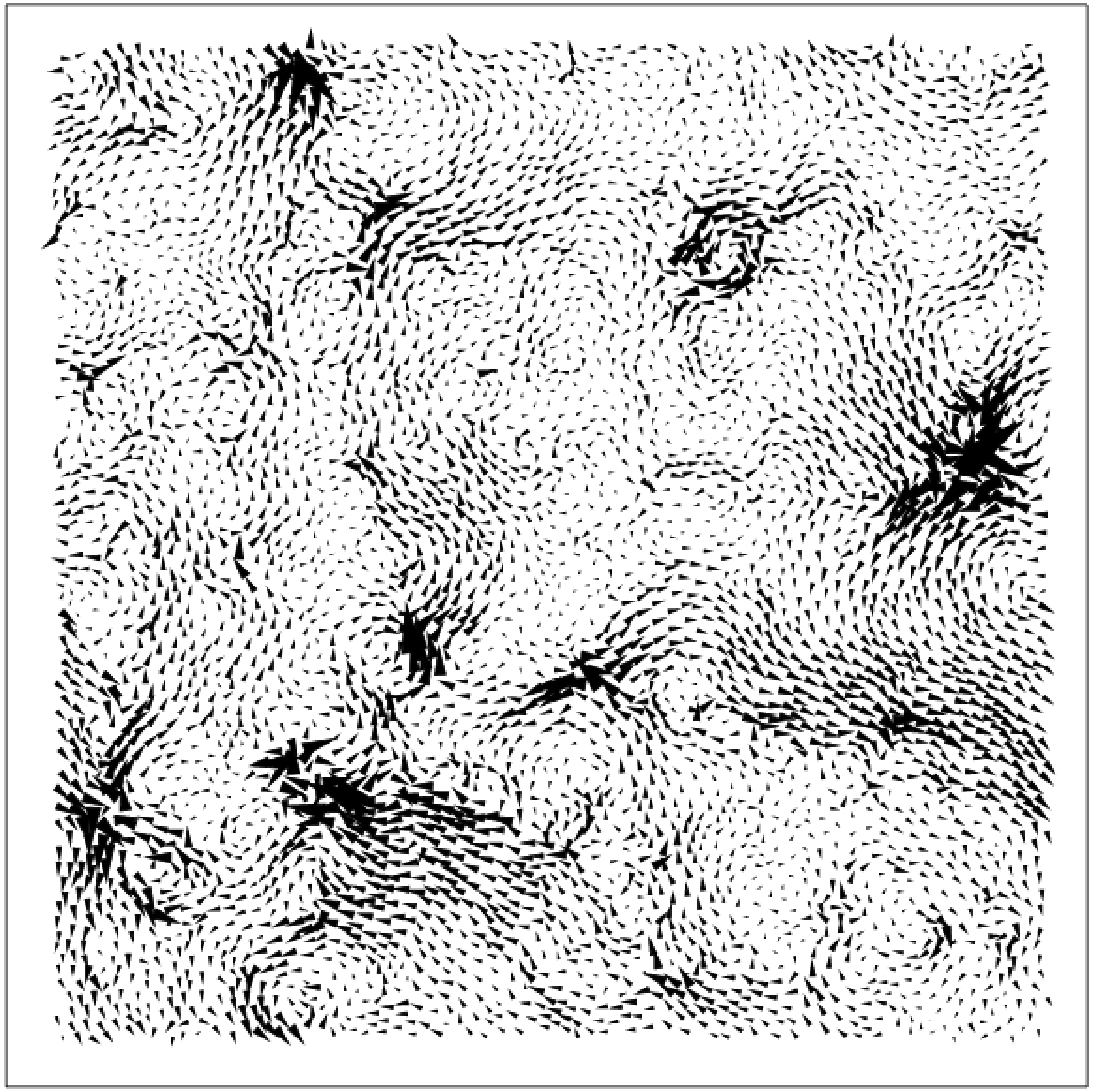
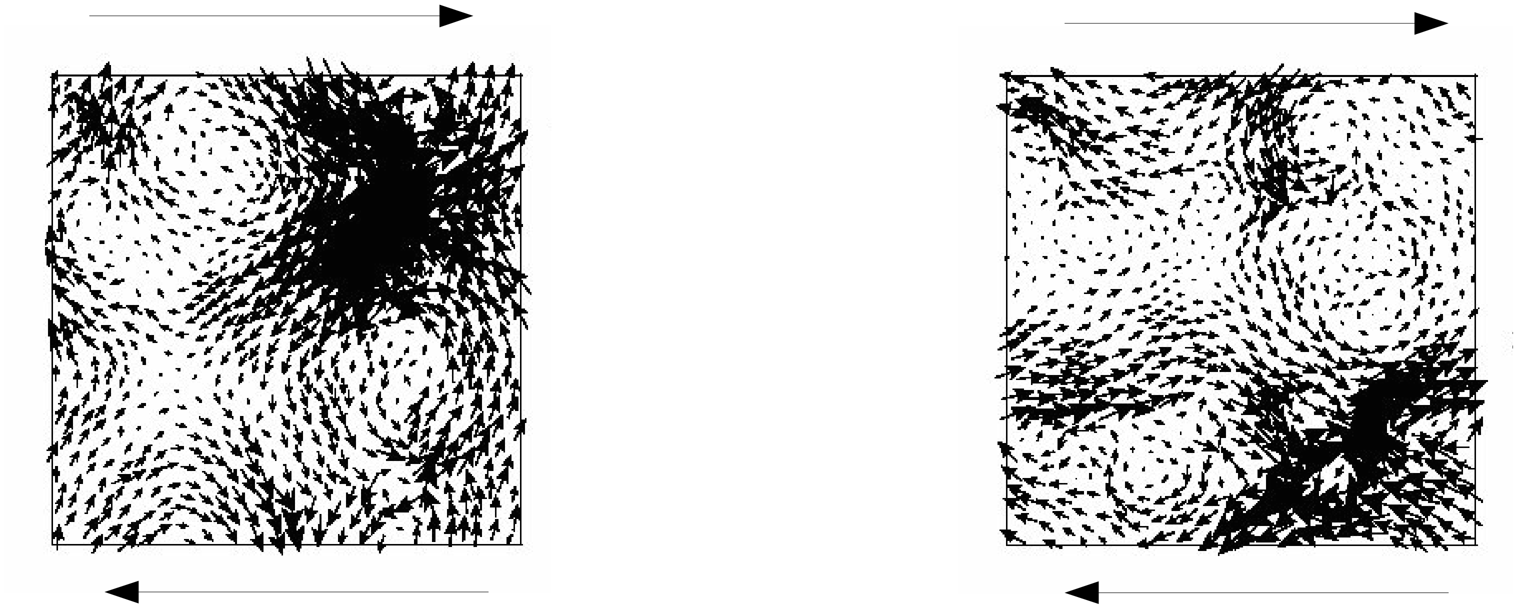


AQS III: zones are progressively loaded and interact

$$\gamma = 5.56 \,\&\, 5.58\,\%$$



PRE 76, 036104 (2007)

<u>AQS phenomenology:</u>

- Progressive convection of shear zones towards instability
- Each flip produces an Eshelby-like field likely to trigger secondary flips

⇒ system-spanning avalanches

Conclusion (partial)

- AQS simulations support the following phenomenology:
  - Plasticity results from local shear transformations
  - Occurring as zones are progressively convected towards spinodals
  - Each flip produces an Eshelby-like field likely to trigger secondary flips

  This shows up as system-spanning avalanches

What happens at finite strain-rate?

Athermal systems $T = 0$

Near QS regime $\dot{\gamma} \neq 0$

$$\lambda_c = 5\,d$$
$$\tau = 0.2\,\tau_{LJ}$$

<u>Pair potential:</u>

$$U = k\left(r^{-12} - 2\,r^{-6}\right)$$ Lennard-Jones

<u>Dissipative forces:</u>

$$f_{ij} = \frac{m}{\tau}\,\phi(r)\left(\vec{v}_j - \vec{v}_i\right)$$ Viscous drag

$$\phi(r) = 1 - 2\,(r/2)^4 + (r/2)^8$$

<u>This form of dissipation guarantees that:</u>

- long wavelength are not damped
- short wavelength are, for:

$$\lambda < \lambda_c = \frac{\pi\,d^2}{\tau\,c_s}$$

Non-affine velocity

$\vec{v}_i - \dot{\gamma}\, y_i \vec{e}_x$

$L = 160$

$\dot{\gamma} = 5.10^{-5}$

The strain field

Dynamics of non-affine velocity field show:

Flips retain same nature (progressive softening)

Acoustic propagation of long range signals observable

But:

- Are flips correlated?
- Are there avalanches?

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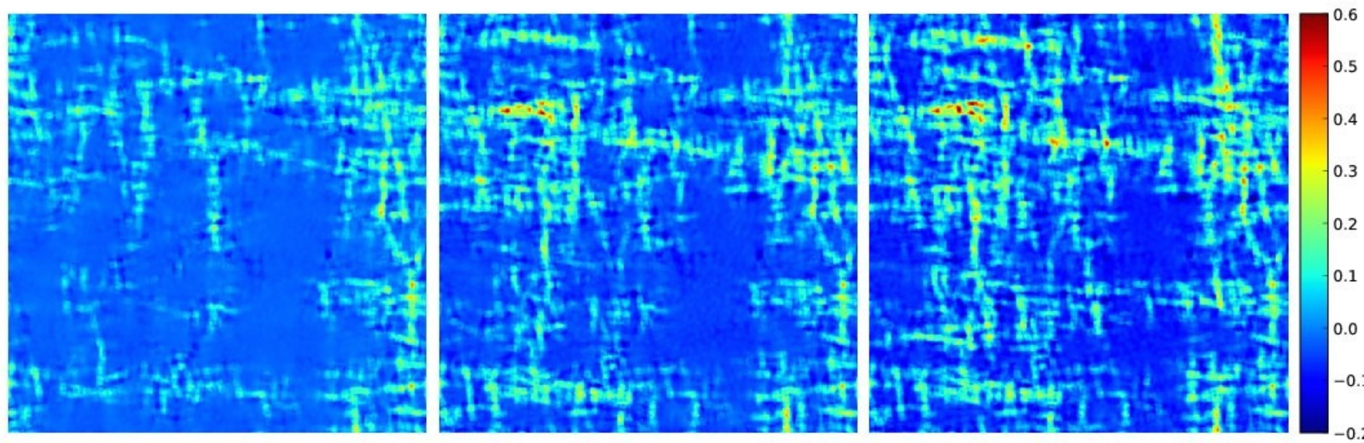
Deformation maps

$\epsilon_{xy}(\vec{r})$



$\gamma = 1\%$ $\gamma = 5\%$ $\gamma = 10\%$

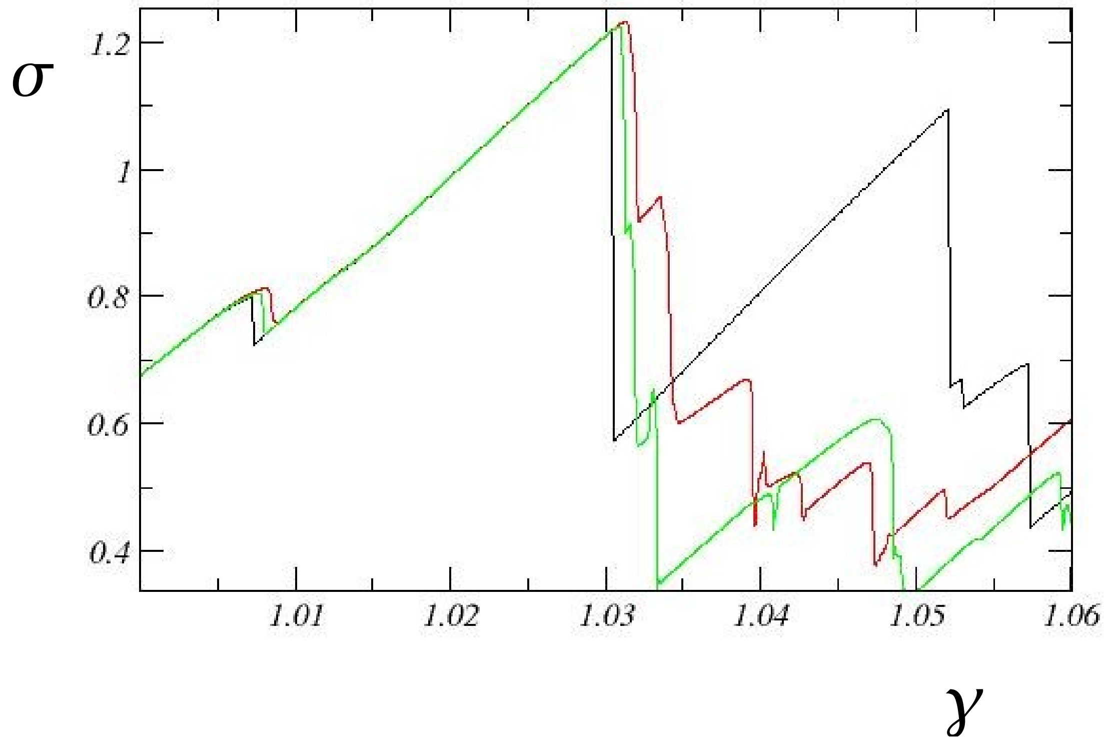
Looking for robust observables

<u>In AQS limit:</u>

Plastic events = discontinuous drops
= avalanches

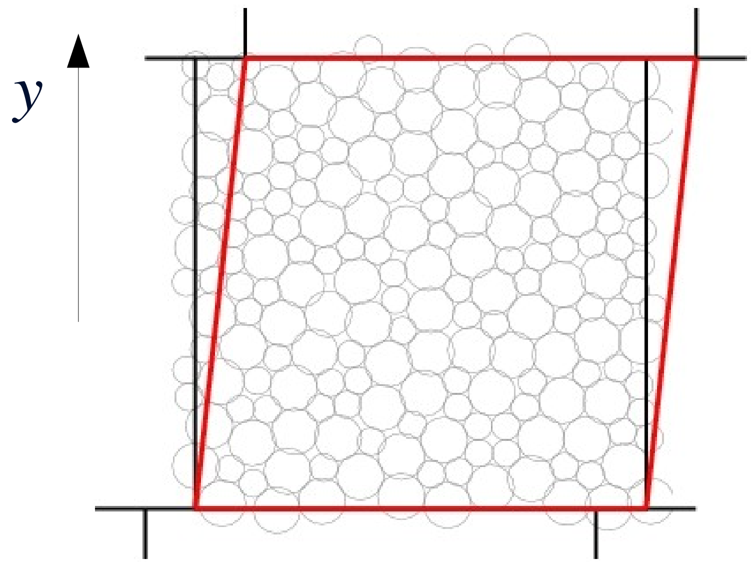
<u>At finite strain rates:</u>

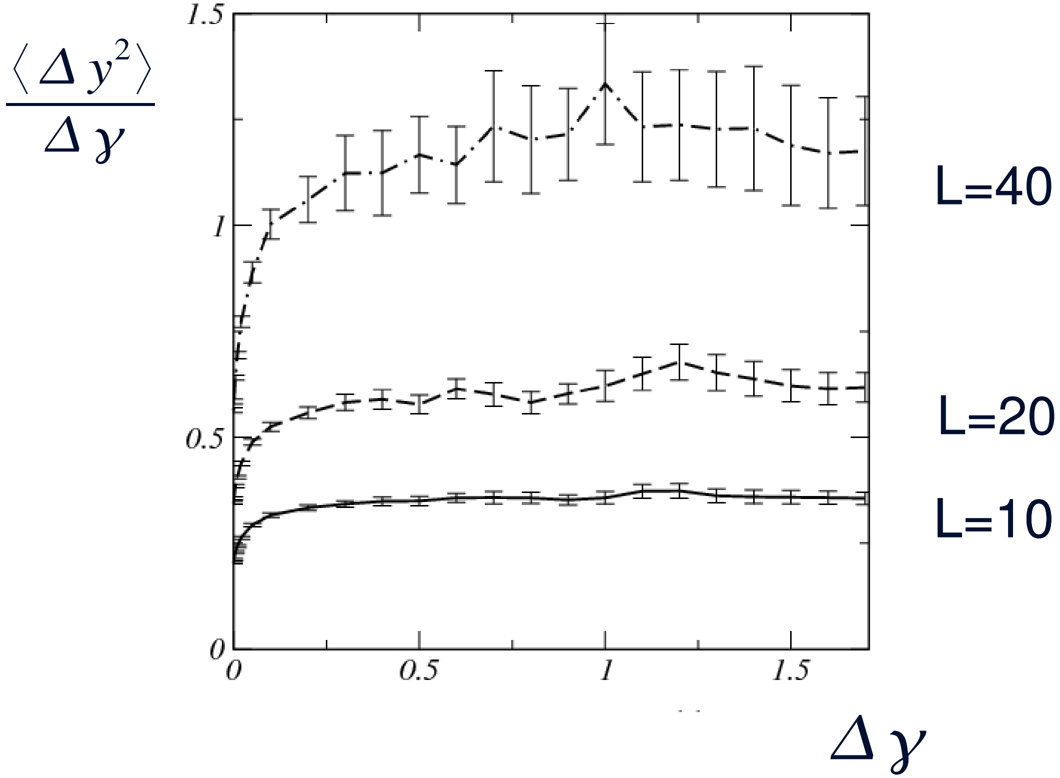
Flips and avalanches have a finite duration



What observables should we look at to characterize flip-flip correlations?

AQS V: Transverse diffusion is system size dependent





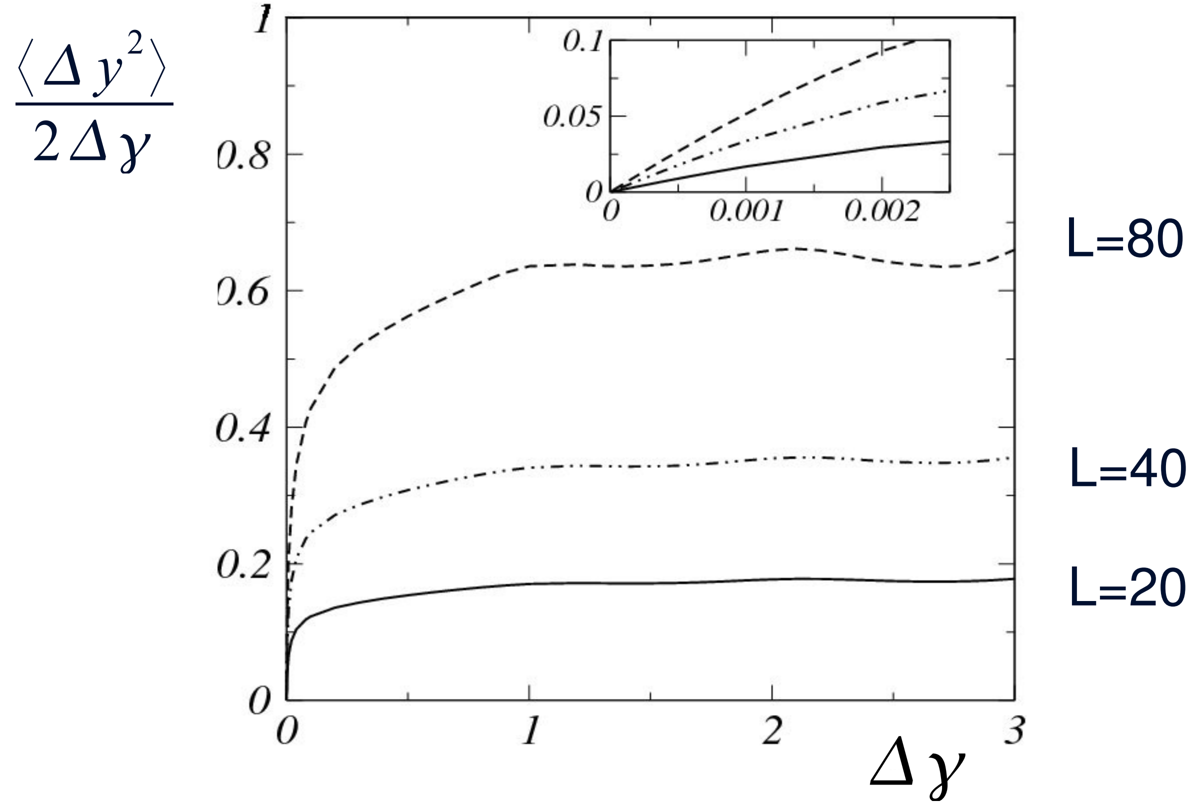
$$\frac{\langle \Delta y^2 \rangle}{\Delta \gamma} \longrightarrow \hat{D} \qquad (\Delta \gamma > 0.5)$$

with L

PRE 76, 036104 (2007)

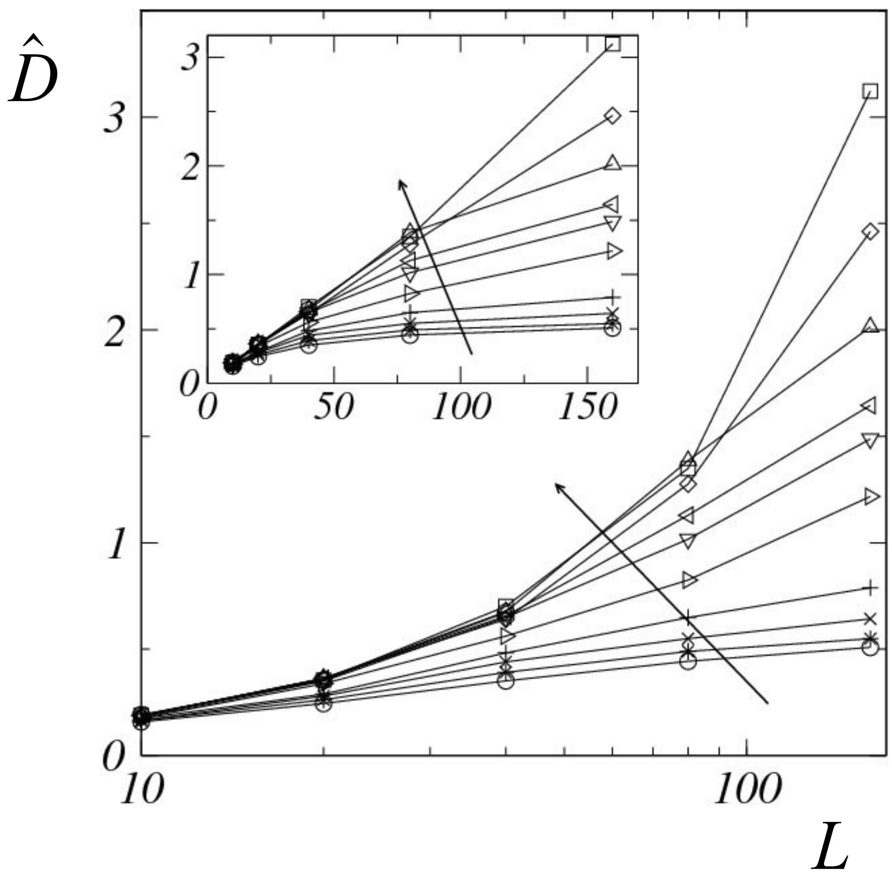
Transverse diffusion at finite strain rate

$$\dot{\gamma} = 10^{-3}$$



$$\frac{\langle \Delta y^2 \rangle}{2\Delta\gamma} \rightarrow \hat{D} = D / \dot{\gamma}$$

Transverse diffusion at finite strain rate



$$\widehat{D}(\dot{\gamma}, L) \sim \ln L \qquad (\dot{\gamma}=10^{-2})$$

$$\sim L \qquad (\dot{\gamma}=10^{-4})$$

Decomposing the plastic response in terms of flips

Characteristic flip strain: $\Delta\epsilon_0$

Characteristic flip size: a^2

Eshelby: $\Delta\overline{\sigma_{xy}} = \frac{2\mu a^2 \Delta\epsilon_0}{L^2}$

Over a long strain interval $\Delta\gamma$:

the average number of flips verifies: $N_f(\Delta\gamma)\Delta\overline{\sigma_{xy}} = 2\mu\Delta\gamma$

$$N_f(\Delta\gamma) = \frac{L^2\Delta\gamma}{a^2\Delta\epsilon_0}$$

$$R_f = \frac{L^2\dot{\gamma}}{a^2\Delta\epsilon_0}$$

Assume flips independent

Over a large strain interval: $\Delta y_i = \sum_f u_y(\vec{r}_i - \vec{r}_f)$

$$\Rightarrow \langle \Delta y^2 \rangle = N_f(\Delta\gamma)\ \langle u_y^2 \rangle$$

$$N_f(\Delta\gamma) = \frac{L^2 \Delta\gamma}{a^2 \Delta\epsilon_0}$$

Eshelby: $\vec{u} = \frac{2a^2 \Delta\epsilon_0}{\pi} \frac{x\,y}{r^4} \vec{r}$

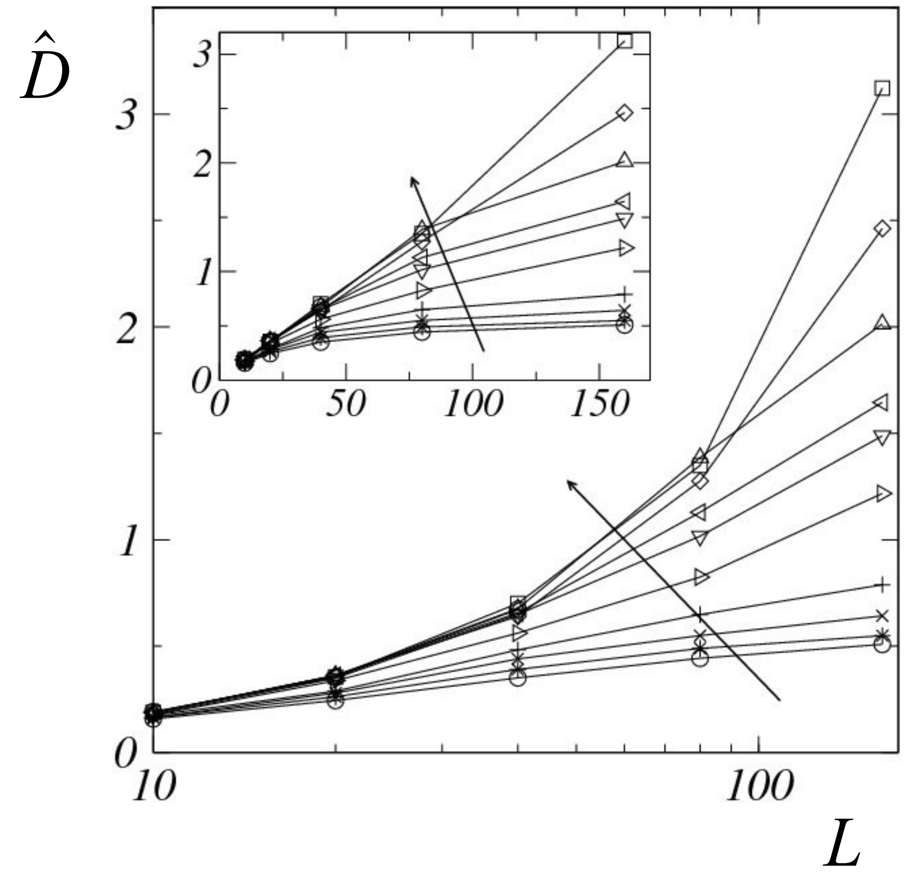
$$\langle u_y^2 \rangle = \frac{1}{L^2} \int_a^L u_y^2 \mathrm{d}\vec{r} = \frac{a^4 \Delta\epsilon_0^2}{4\pi L^2} \ln(L/a)$$

$$\boxed{\frac{\langle \Delta y^2 \rangle}{\Delta\gamma} = \frac{a^2 \Delta\epsilon_0}{4\pi} \ln(L/a)}$$

Transverse diffusion at finite strain rate

$$\frac{\langle \Delta y^2 \rangle}{\Delta \gamma} = \frac{a^2 \Delta \epsilon_0}{4\pi} \ln(L/a)$$

$$a^2 \Delta \epsilon_0 \sim 1 \quad \begin{cases} \Delta \epsilon_0 \sim 4\% \\ a \sim 5 \end{cases}$$



Very high strain rates $(\dot{\gamma} = 10^{-2})$

~ no flip correlations

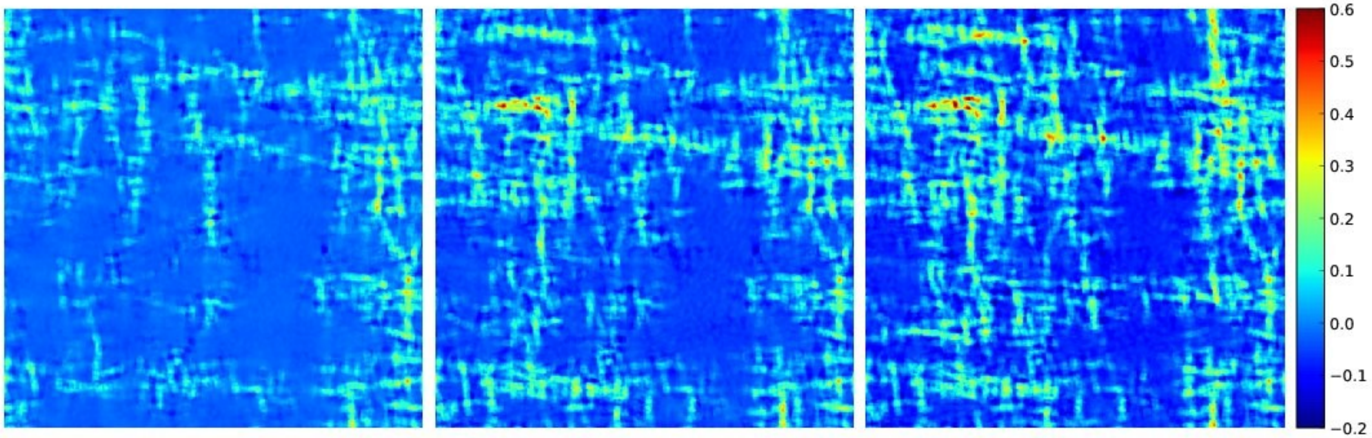
Decreasing $\dot{\gamma}$: growing departure from ln$L$

= growing correlation length (avalanche size)?

Assume events = correlated flips = linear avalanches

Deformation maps

$\epsilon_{xy}(\vec{r})$

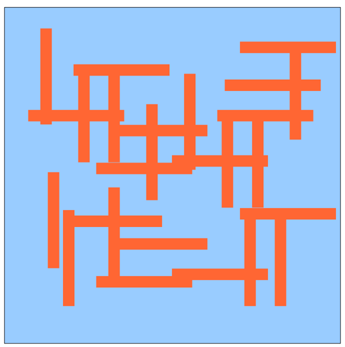


$\gamma = 1\%$ $\gamma = 5\%$ $\gamma = 10\%$

Assume events = correlated flips = linear avalanches

Avalanches:
- linear extension: l (= flip correlation length)
- density of flips in an avalanche, ν constant

$$N_{\mathrm{av}}(\Delta\gamma) = N_f(\Delta\gamma)/\nu\, l$$



$$\langle \Delta y^2 \rangle_{\mathrm{av}} = \nu^2 \int_0^l \int_0^l \mathrm{d}s\, \mathrm{d}s' \langle u_y(\vec{r}-\vec{r}_s)\, u_y(\vec{r}-\vec{r}_{s'}) \rangle$$

$$= \frac{a^4 \Delta\epsilon_0^2 \nu^2}{2\pi} \left(\frac{l}{L}\right)^2 \ln(L/l)$$

$$\equiv \langle u_y^2 \rangle_A$$

Particles diffusion

$$\Delta y_i = \sum_A u_y^A(\vec{r}_i - \vec{r}_A)$$

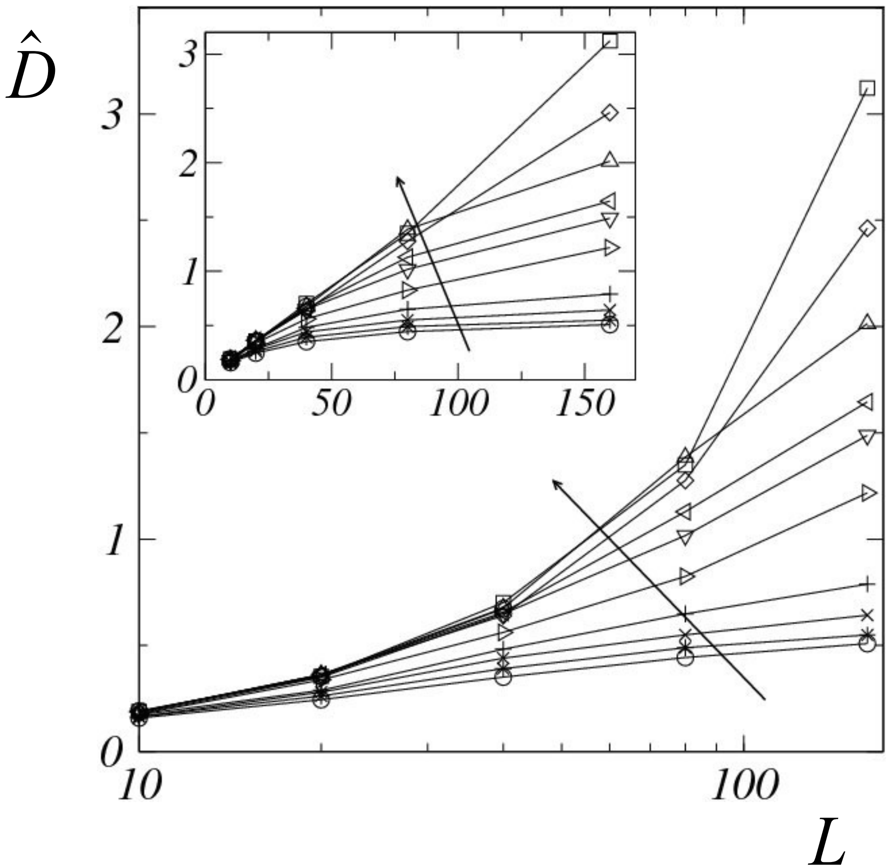
Assuming avalanches independent:

$$\langle \Delta y^2 \rangle = N_A(\Delta\gamma) \langle u_y^2 \rangle_A$$

$$\frac{\langle \Delta y^2 \rangle}{\Delta\gamma} = \frac{a^2 \Delta\epsilon_0}{4\pi} \nu\, l \ln(L/l)$$

Transverse diffusion at finite strain rate

$$\frac{\langle \Delta y^2 \rangle}{\Delta \gamma} = \frac{a^2 \Delta \epsilon_0}{4\pi} \nu \, l \ln(L/l)$$



Large $\dot{\gamma} \Rightarrow l \sim a \qquad \hat{D} \sim \ln L$

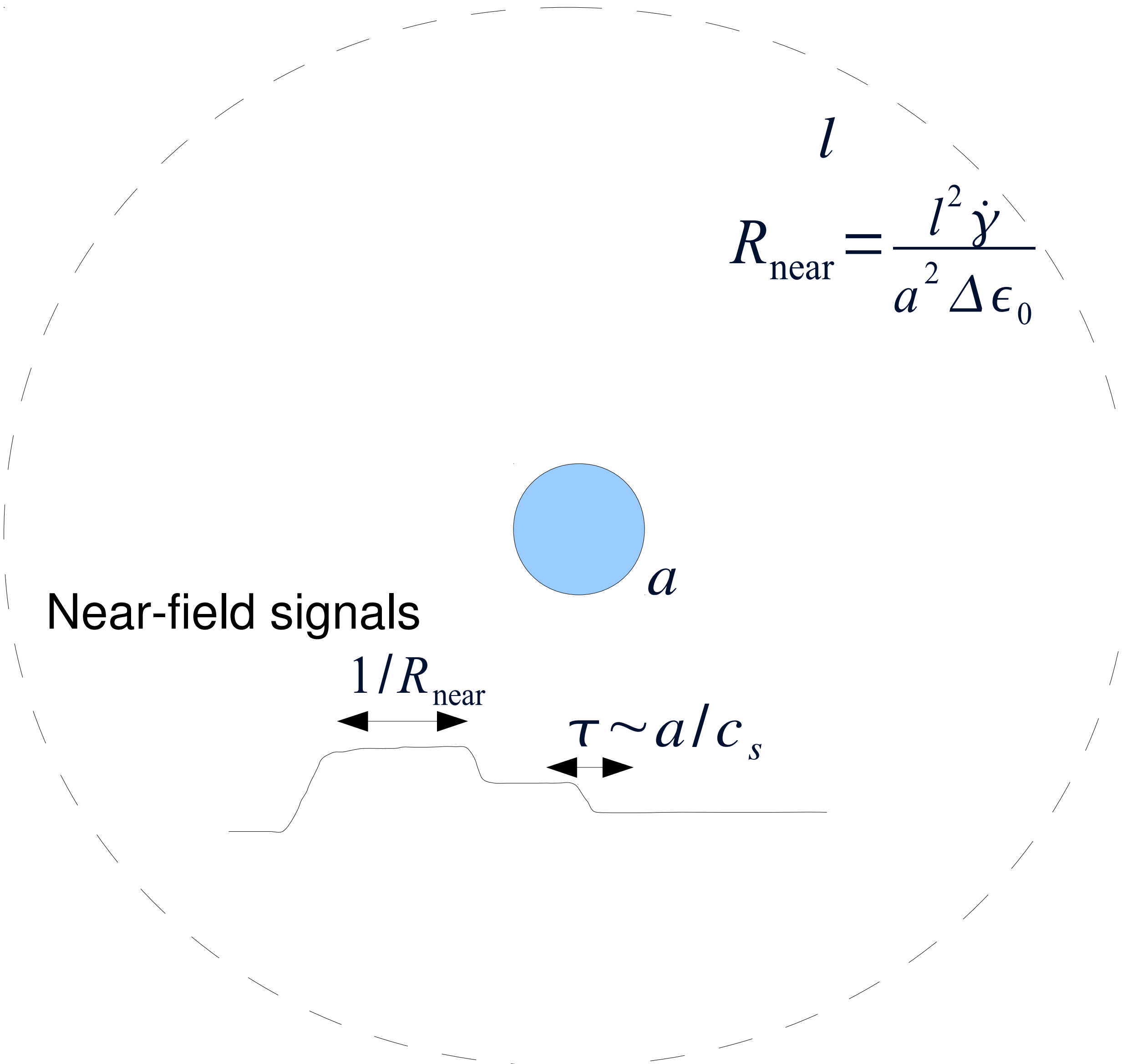
$\dot{\gamma} \to 0 \Rightarrow l \sim L \qquad \hat{D} \sim L$ *QS regime*

In between, evaluate $l(\dot{\gamma})$ *?*

What is the noise received by a zone?

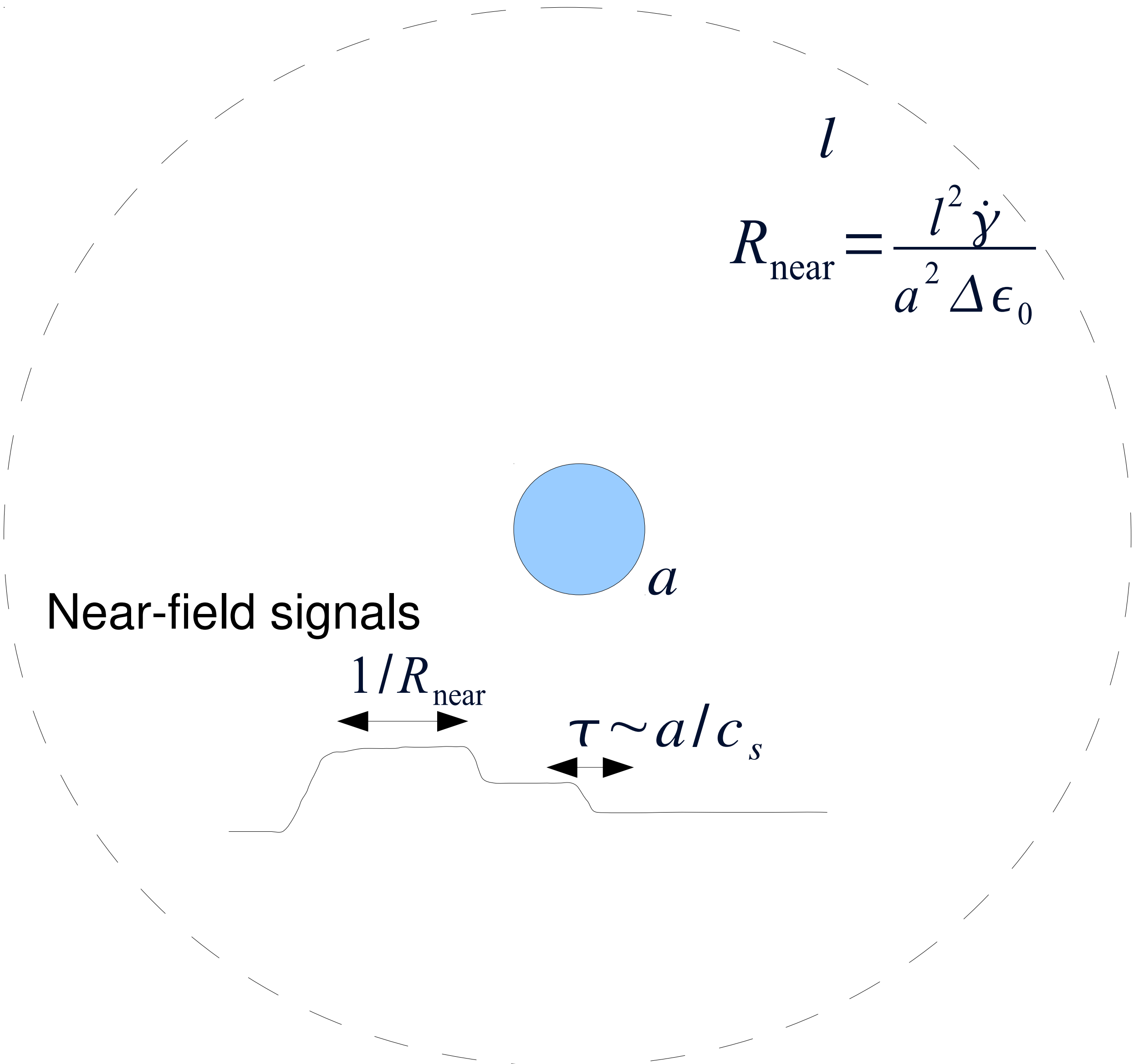
$$R_{\text{flip}} = \frac{L^2 \dot{\gamma}}{a^2 \Delta \epsilon_0}$$

L



Near-field signals are separated iff: $1/R_{\text{near}} \gg \tau \;\Leftrightarrow\; l \ll \sqrt{a^2 \Delta \epsilon_0 / \dot{\gamma}\, \tau_{\text{flip}}}$

What is the noise received by a zone?



L

$$R_{\text{flip}} = \frac{L^2 \dot{\gamma}}{a^2 \Delta \epsilon_0}$$

Background noise

$$R_{\text{back}} = \frac{(L^2 - l^2)\dot{\gamma}}{a^2 \Delta \epsilon_0}$$

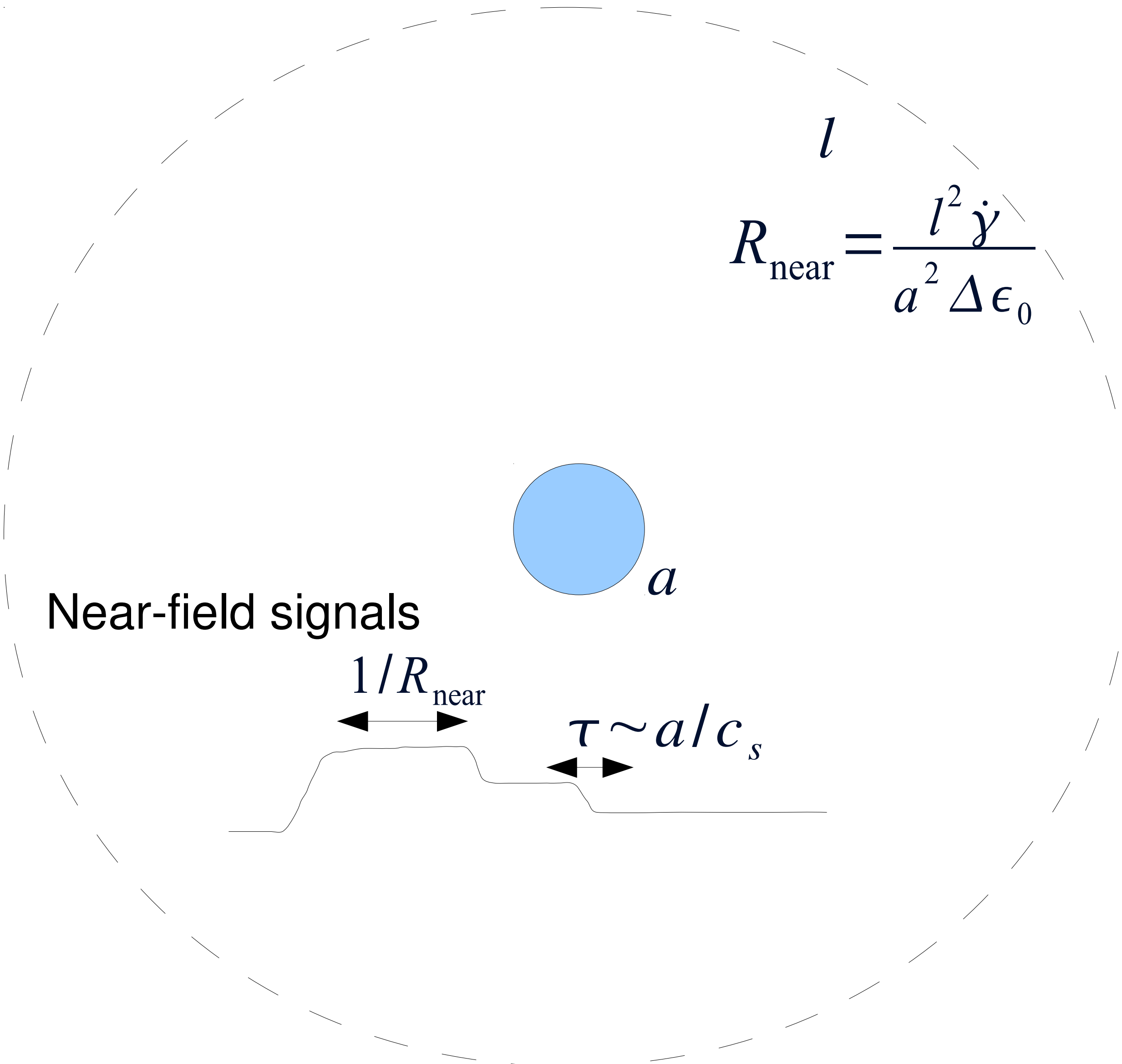
During time τ , stress diffuses:

$$\langle \Delta \sigma^2 \rangle \sim R_{\text{back}} \tau \langle \Delta \sigma^2 \rangle_{\text{Esh.}}$$

$$\langle \Delta \sigma^2 \rangle \sim \dot{\gamma} \tau (\mu^2 a^2 \Delta \epsilon_0 / l^2)$$

Near-field signals are separated iff: $1/R_{\text{near}} \gg \tau \Leftrightarrow l \ll \sqrt{a^2 \Delta \epsilon_0 / \dot{\gamma} \tau_{\text{flip}}}$

What is the noise received by a zone?



$$R_{\text{flip}} = \frac{L^2 \dot{\gamma}}{a^2 \Delta \epsilon_0}$$

$$R_{\text{near}} = \frac{l^2 \dot{\gamma}}{a^2 \Delta \epsilon_0}$$

Background noise

$$R_{\text{back}} = \frac{(L^2 - l^2)\dot{\gamma}}{a^2 \Delta \epsilon_0}$$

During time τ , stress diffuses:

$$\langle \Delta \sigma^2 \rangle \sim R_{\text{back}} \tau \langle \Delta \sigma^2 \rangle_{\text{Esh.}}$$

$$\langle \Delta \sigma^2 \rangle \sim \dot{\gamma} \tau (\mu^2 a^2 \Delta \epsilon_0 / l^2)$$

$$\sqrt{\langle \Delta \sigma^2 \rangle} \ll \mu (a^2 \Delta \epsilon_0 / l^2)$$

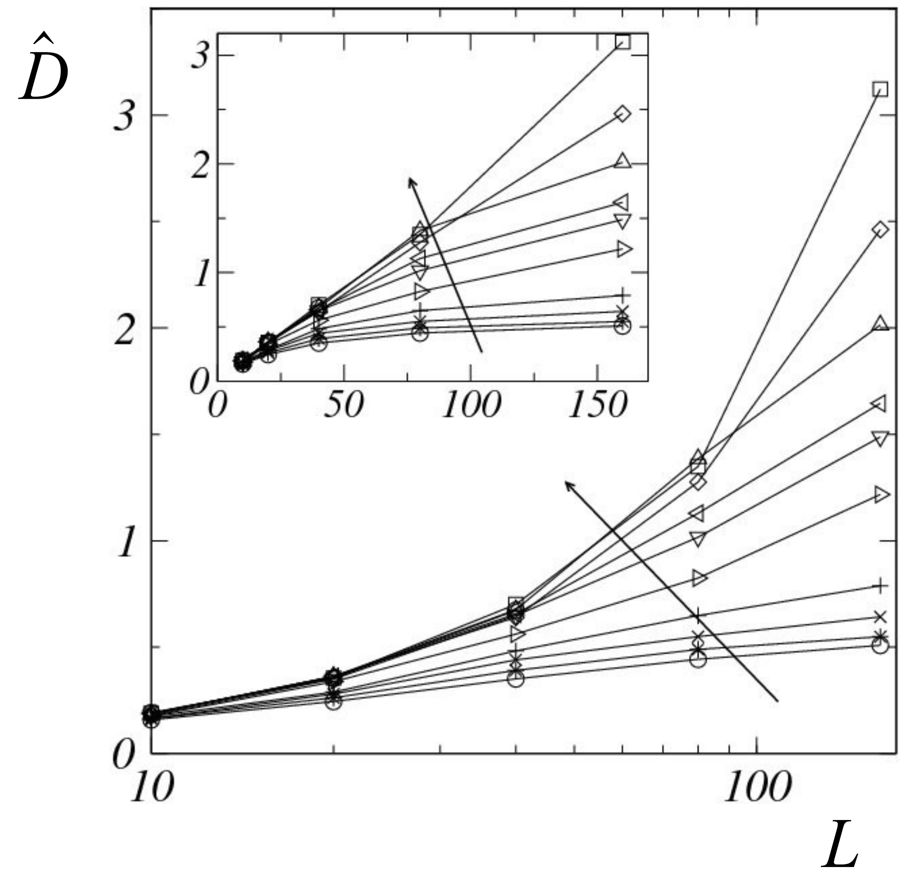
Near-field signals are separated iff: $1/R_{\text{near}} \gg \tau \Leftrightarrow l \ll \sqrt{a^2 \Delta \epsilon_0 / \dot{\gamma} \tau_{\text{flip}}}$

Back to diffusion data

PRL 103, 065501 (2009)

$$\begin{cases} \hat{D} = \dfrac{a^2 \Delta \epsilon_0^2}{2\pi} \nu \, l \ln(L/l) \\ l \sim \sqrt{a^2 \Delta \epsilon_0 / \dot{\gamma} \tau_{\text{flip}}} \end{cases}$$

$$\hat{D}/L \sim f(L\sqrt{\dot{\gamma}})$$



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PRL 103, 065501 (2009)

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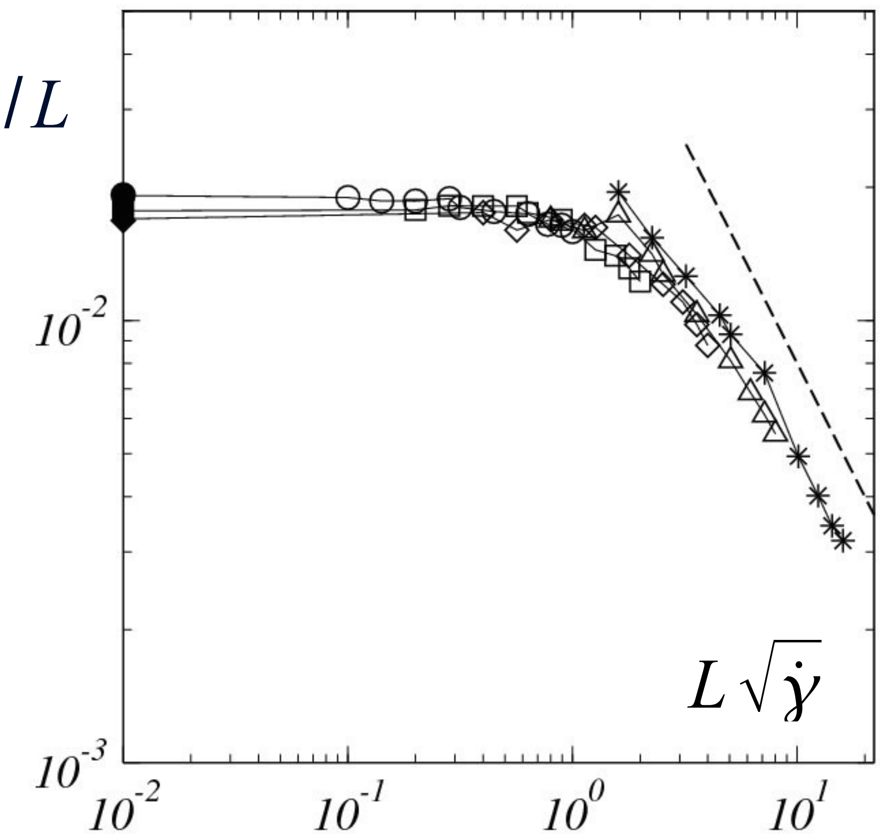


Crossover from dynamically controlled correlation length $l \sim \dot{\gamma}^{-1/2}$

to QS regime $l \sim L$

For

$$\dot{\gamma} \sim \dot{\gamma}_{\text{co}} \approx a^2 \Delta \epsilon_0 / \tau_{\text{flip}} L^2$$

Inferences

- Extension to 3D $l(\dot{\gamma}) \sim a(\Delta\epsilon_0 / \dot{\gamma}\,\tau_{\text{flip}})^{1/3}$

  $\Rightarrow$ For atomic glass, with $\tau_{\text{LJ}} \sim 10^{-13}\,\text{sec}$, $a \sim 1\,\text{nm}$, $\Delta\epsilon_0 \sim 5\,\%$

For $\dot{\gamma} \leq 1\,\text{sec}^{-1}$, $l \geq 10\,\mu\text{m}$

- 2D flow curve $\sigma(\dot{\gamma})$

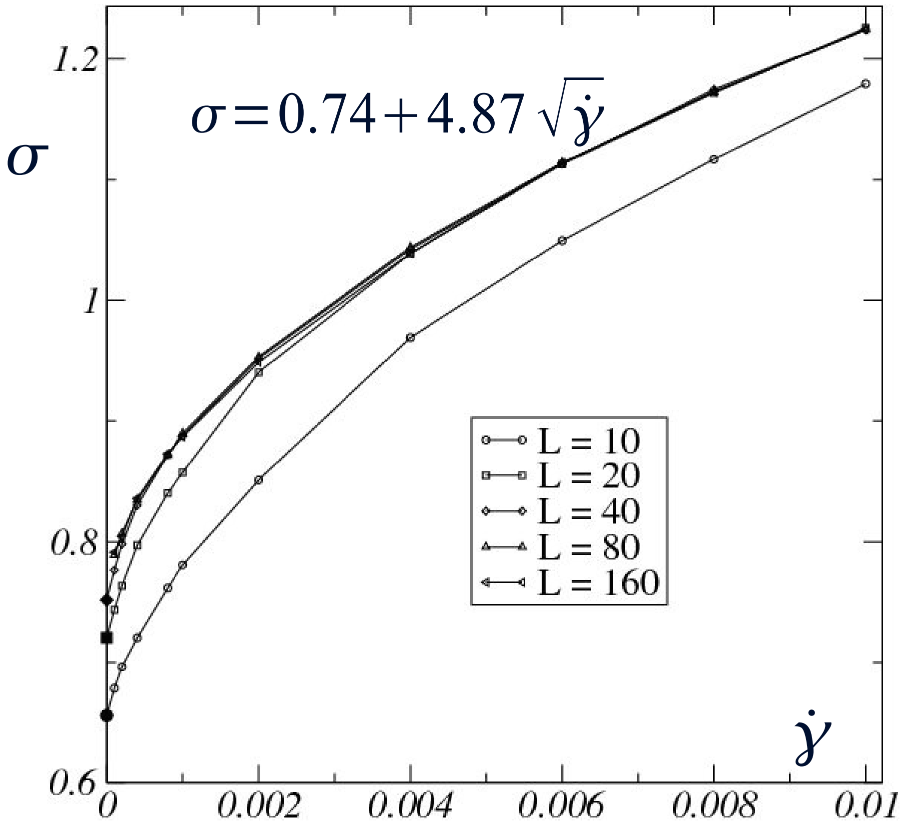
guess: $\sigma - \sigma_y \approx \mu\,\dot{\gamma}\,\tau_{\text{av}}$

event duration: $\tau_{\text{av}} \sim l / c_s$

(domino-like avalanches)

$\Rightarrow$ $\boxed{\sigma = \sigma_y + C\sqrt{\dot{\gamma}}}$

$$C = \frac{\mu}{c_s} a^2 \frac{\Delta\epsilon_0}{\tau} \approx 13$$



Conclusion (partial)

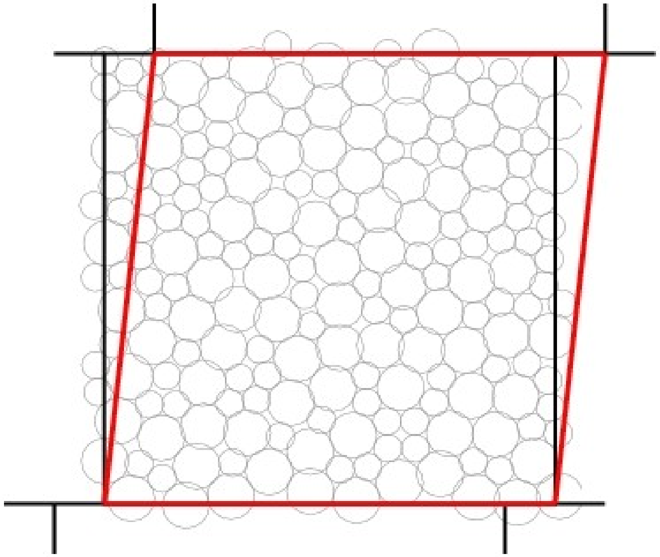
- AQS simulations support the following phenomenology:
  - Plasticity results from local shear transformations
  - Zones are progressively convected towards instability
  - Each flip produces an Eshelby-like field likely to trigger secondary flips

  This shows up as system-spanning avalanches

- At usual finite $\dot{\gamma}$, the same phenomenology continues to govern plasticity
  - The size of avalanches $l \sim \dot{\gamma}^{-1/D}$
  - With normal cross-over behavior when $l \sim L$
  - We propose these changes govern stress/strain-rate relation

What happens at finite temperature?

Chattoraj et al,
arxiv/1005.1179



- Binary LJ, radii 0.3 and 0.5
- Velocity rescaling
- Tg = 0.27

Data Accumulation:

T = 0.025, 0.05, 0.1, 0.2, 0.25, 0.3, 0.35, 0.4, 0.5

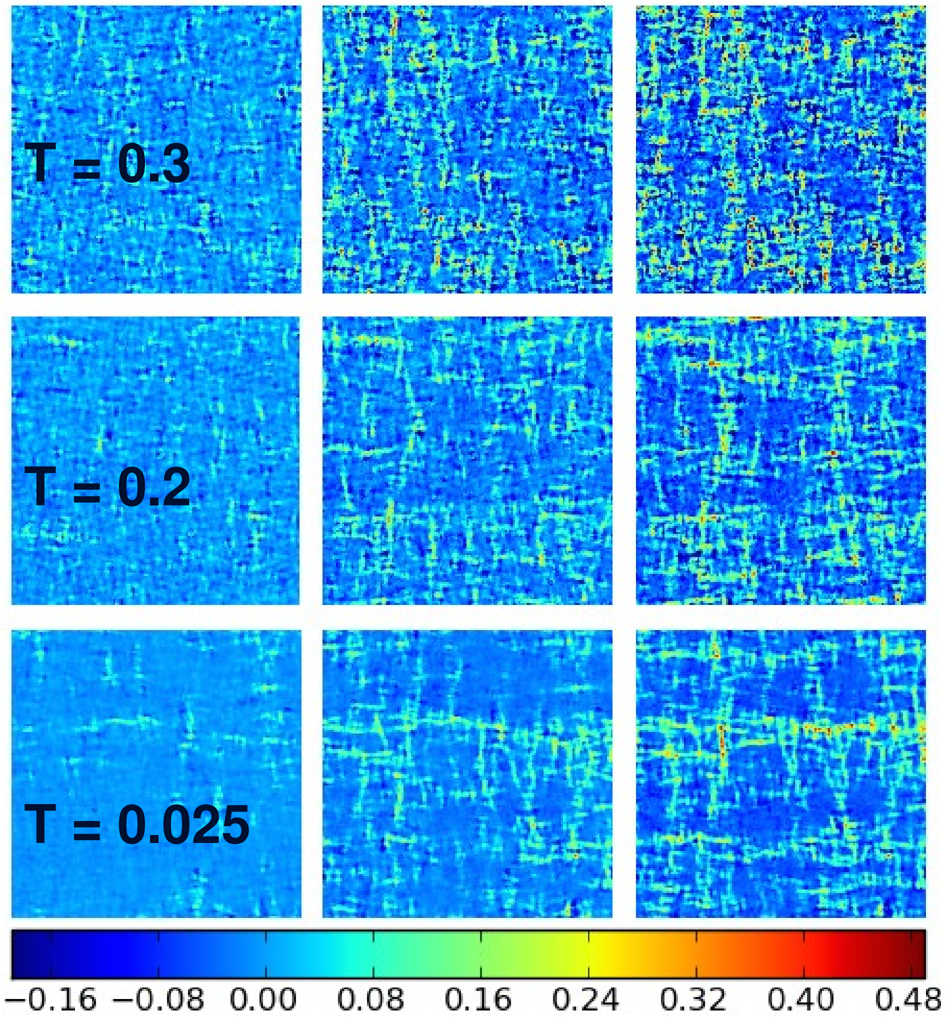
$\dot{\gamma}$ = 4e-05, 0.0001, 0.0004, 0.001, 0.004, 0.01

L = 10, 20, 40, 80, 160

Deformation Maps at T≠0

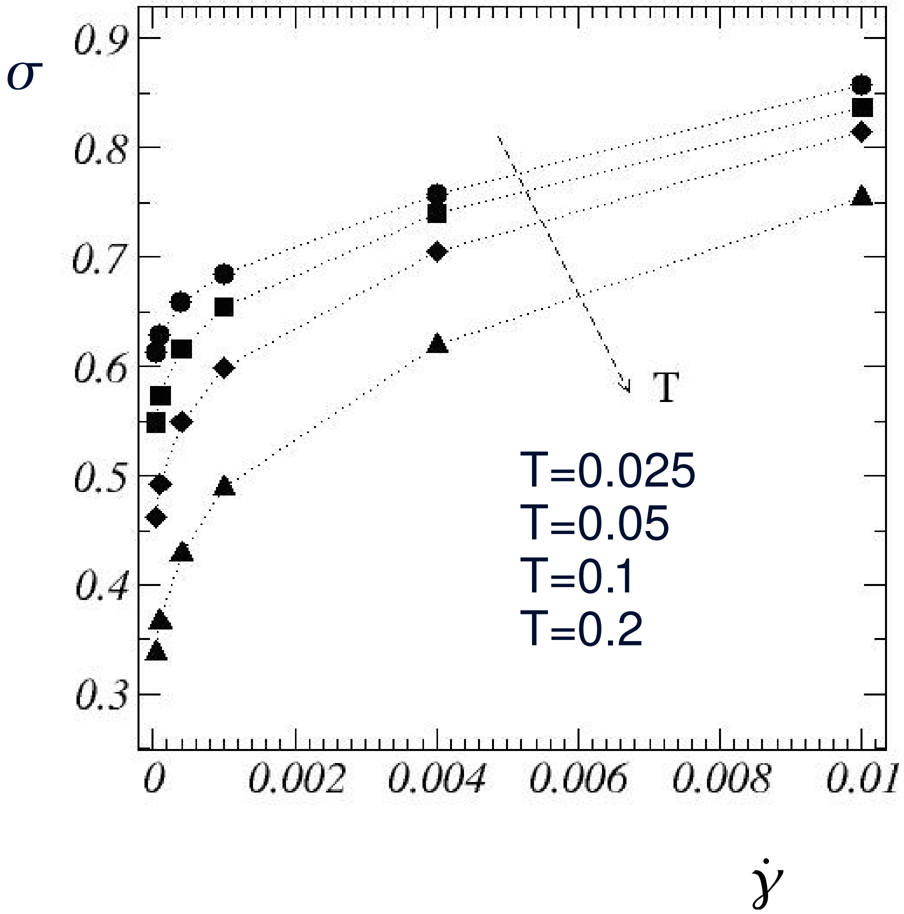
Tg=0.27

Avalanches
are clearly visible
below Tg



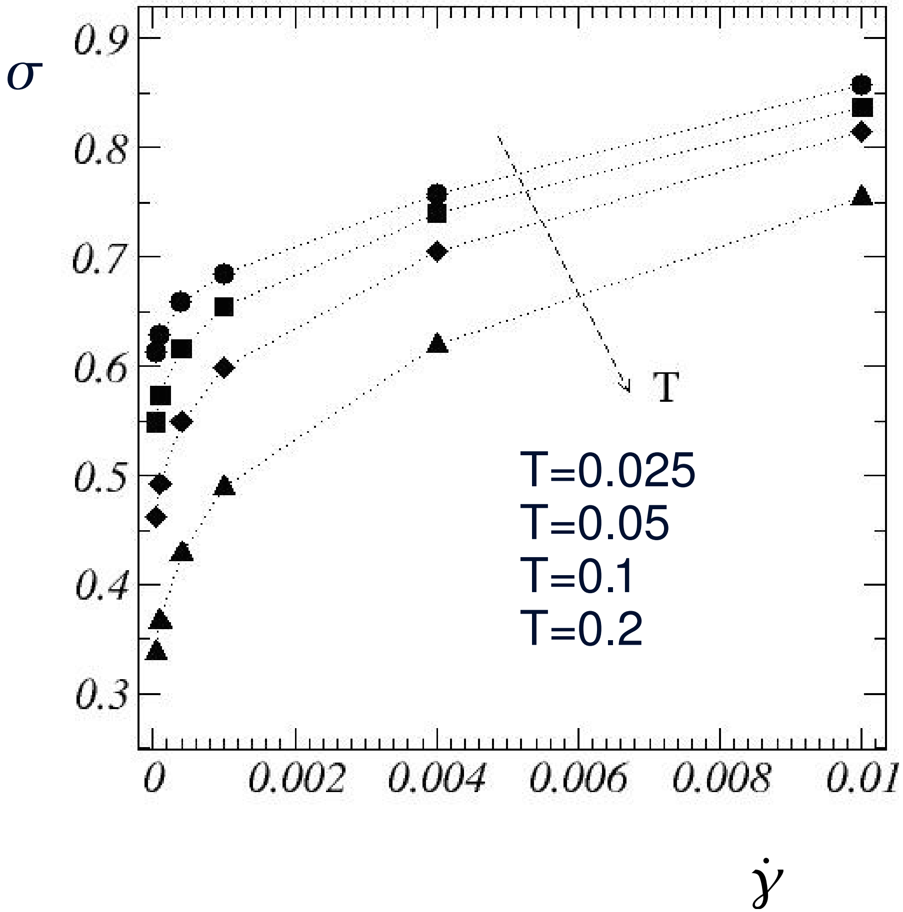
$\delta\gamma = 1\%$ $\delta\gamma = 5\%$ $\delta\gamma = 10\%$

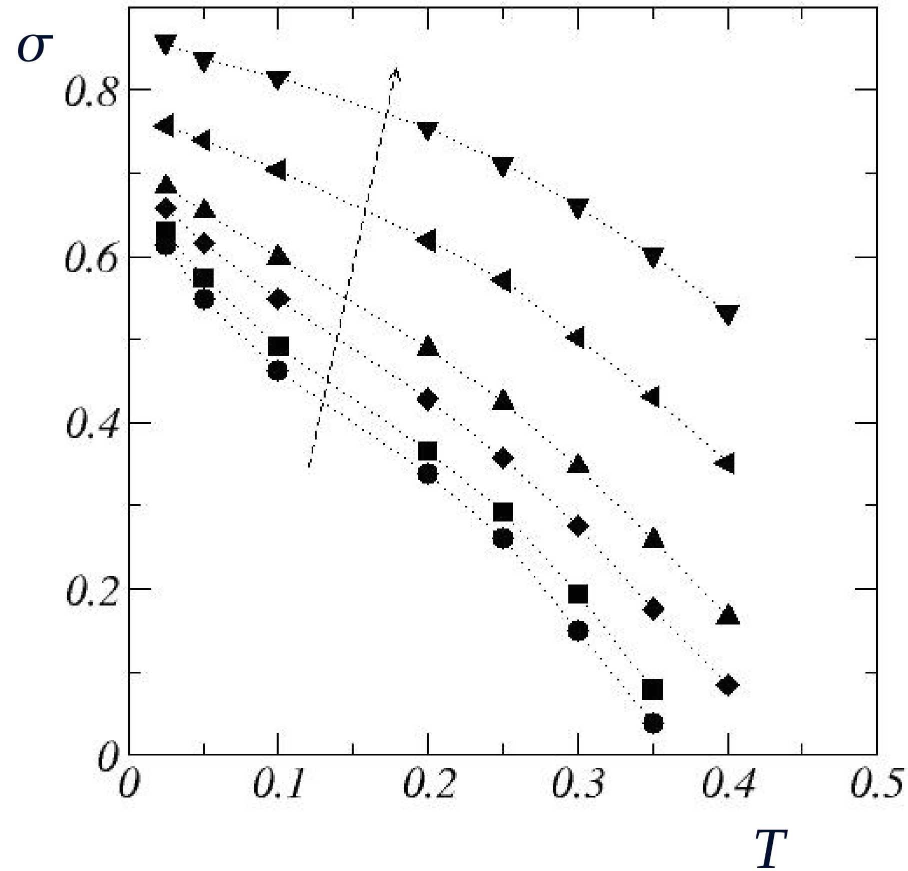
Stress data



σ
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0
0.002
0.004
0.006
0.008
0.01
T
T=0.025
T=0.05
T=0.1
T=0.2
γ̇

Stress data

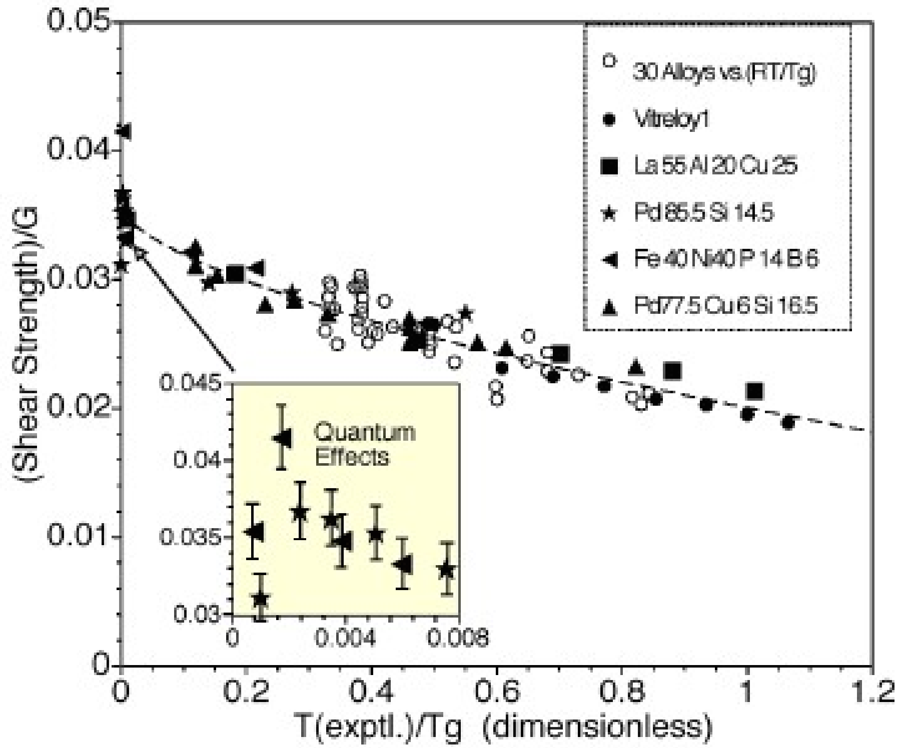

σ
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0
0.002
0.004
0.006
0.008
0.01
T
T=0.025
T=0.05
T=0.1
T=0.2
$\dot{\gamma}$


σ
0.8
0.6
0.4
0.2
0
0
0.1
0.2
0.3
0.4
0.5
T

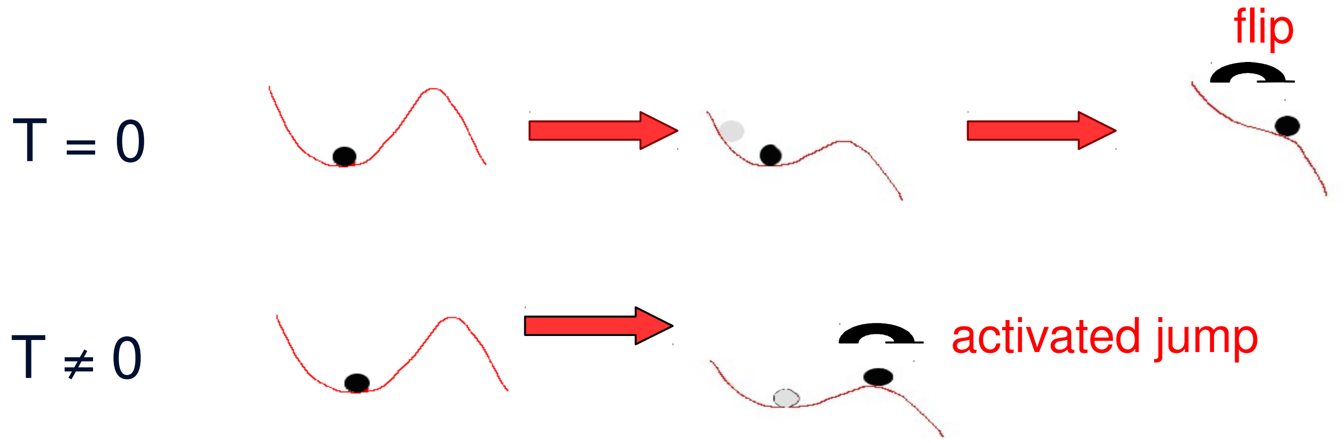
Metallic glass yield stress

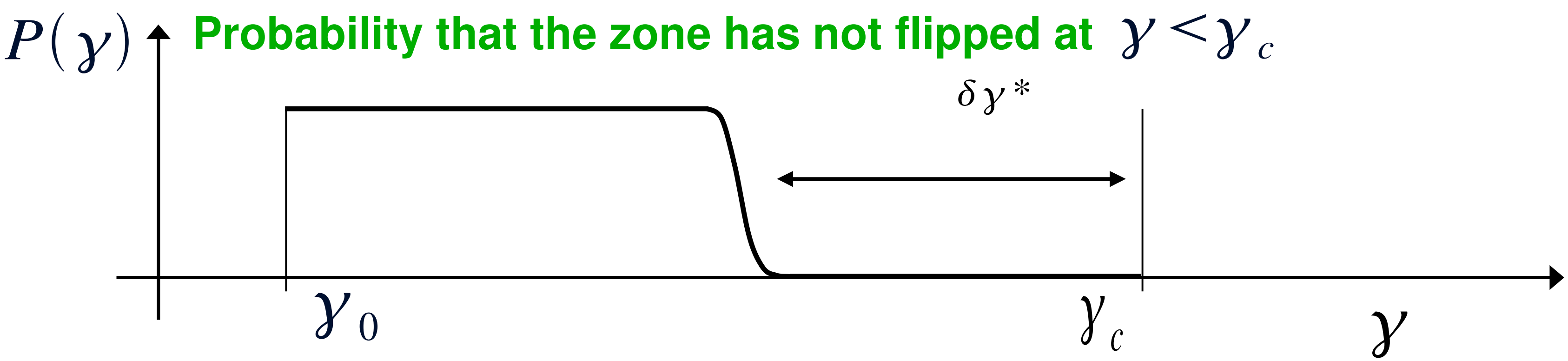
Johnson & Samwer 95, 195501 (2005)

$$\sigma - \sigma_Y \propto T^{2/3}$$



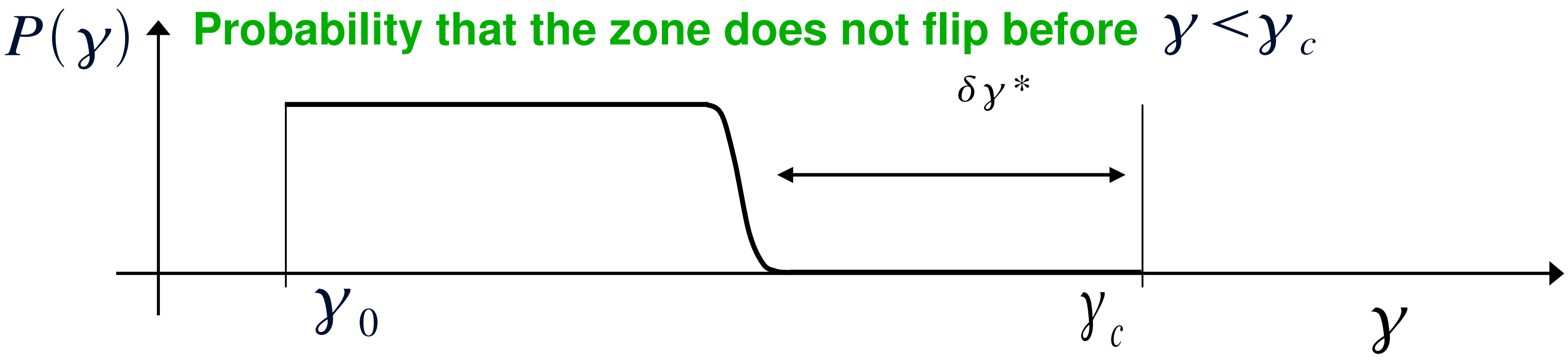
Effect of finite T on a single zone driven towards threshold





$$\sigma(\dot{\gamma};T) = \sigma(\dot{\gamma};T=0) - 2\mu\delta\gamma^*$$

Effect of finite T on a single zone driven towards threshold



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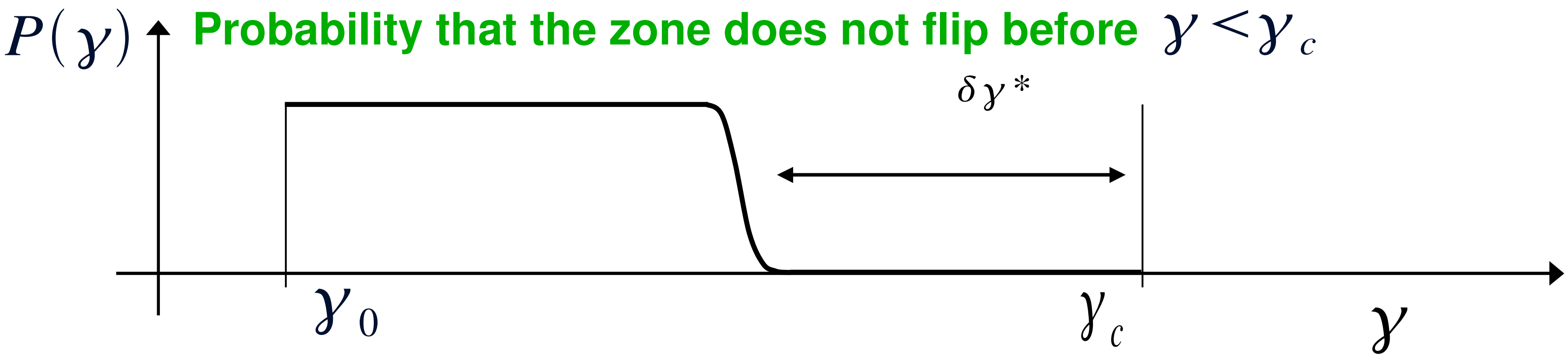
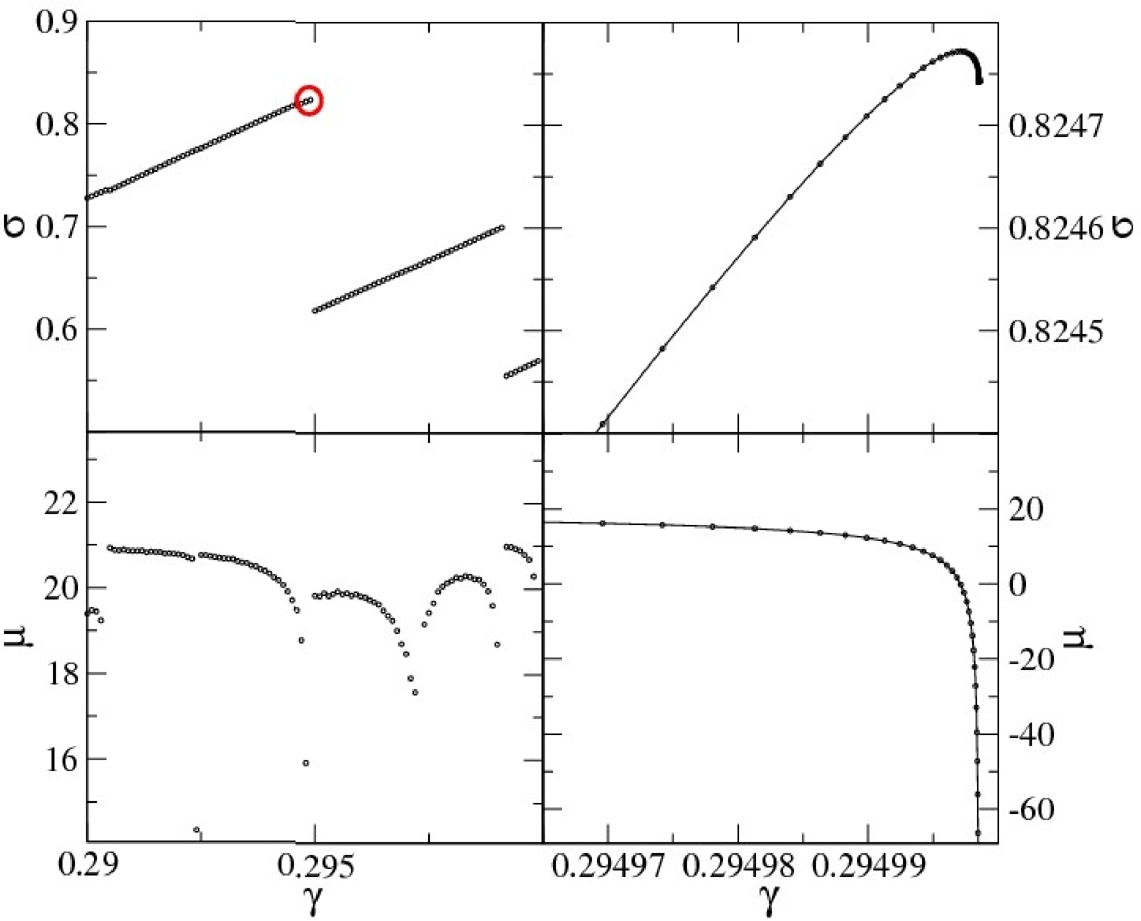


$$\frac{\partial P}{\partial \gamma} = -\frac{1}{\dot{\gamma}} P(\gamma) R(\gamma)$$

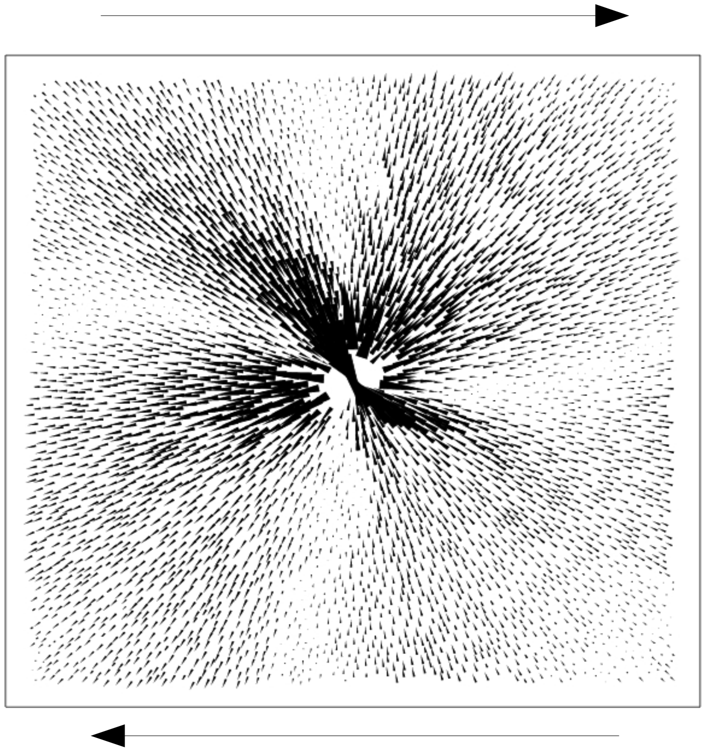
rate of activated jumps: $R = \omega \exp\left(-\frac{E(\gamma)}{T}\right)$

AQS III: plastic events occur via saddle-node bifurcations



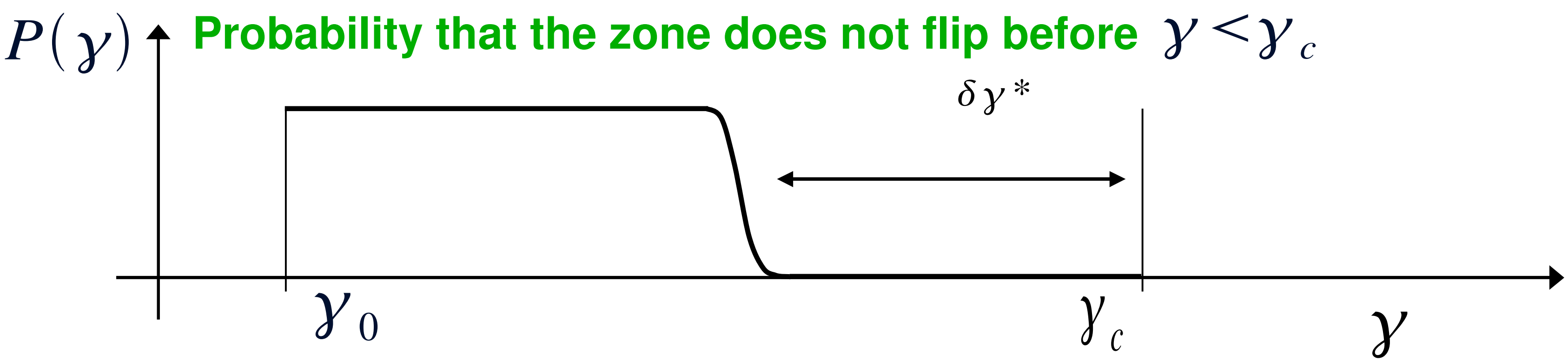
$$\sigma \sim \mu_\infty (\gamma - \gamma_0) - A\sqrt{\gamma_c - \gamma}$$

Zero mode



PRL 93, 195501 (2004)

Effect of finite T on a single zone driven towards threshold

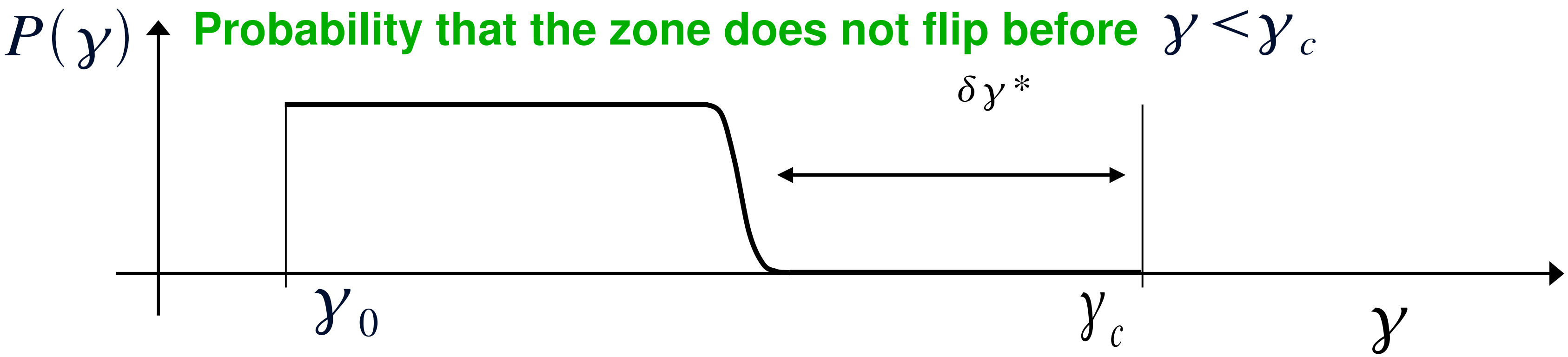


$$\frac{\partial P}{\partial \gamma} = -\frac{1}{\dot{\gamma}} P(\gamma) R(\gamma)$$

rate of activated jumps: $R = \omega \exp\left(-\frac{E(\gamma)}{T}\right)$

$$\begin{cases} \omega = \nu\left(\gamma_c - \gamma\right)^{1/4} \\ E = B\left(\gamma_c - \gamma\right)^{3/2} \end{cases}$$

Effect of finite T on a single zone driven towards threshold



$$\frac{\partial P}{\partial \gamma} = -\frac{1}{\dot{\gamma}} P(\gamma) R(\gamma) \quad \Rightarrow P(\gamma;\gamma_0) = \exp\left(-\frac{1}{\dot{\gamma}} \int_{\gamma_0}^{\gamma} R(\gamma')\,\mathrm{d}\gamma'\right)$$

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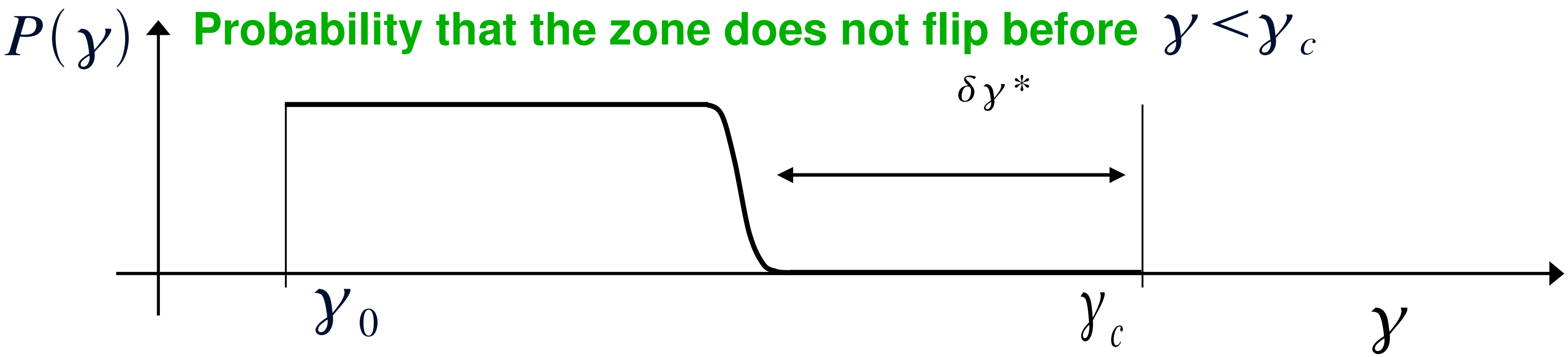

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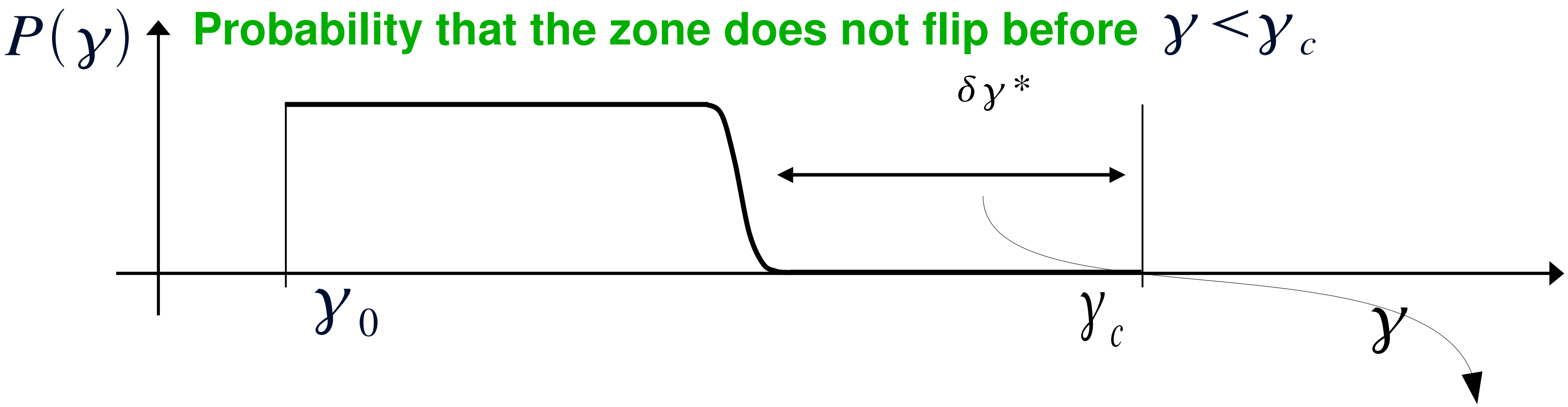
rate of activated jumps: $R = \omega \exp\left(-\frac{E(\gamma)}{T}\right)$

$$\begin{cases} \omega = \nu\left(\gamma_c - \gamma\right)^{1/4} \\ E = B\left(\gamma_c - \gamma\right)^{3/2} \end{cases}$$

$$P(\gamma) = \exp\left(-\frac{2}{3}\frac{\nu}{\dot{\gamma}}\left(\frac{T}{B}\right)^{5/6}\left(Q(\delta\gamma) - Q(\delta\gamma_0)\right)\right)$$

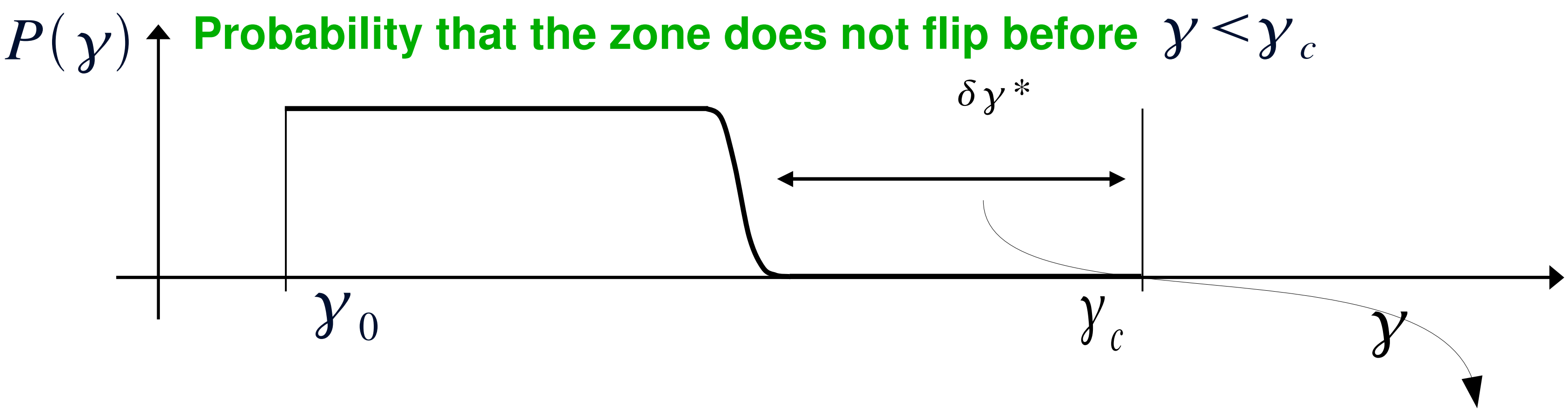
$$Q(\delta\gamma) = \Gamma\left(\frac{5}{6}; \frac{B}{T}\delta\gamma^{3/2}\right)$$

Effect of finite T on a single zone driven towards threshold



$$\delta\gamma^* \sim \left[\frac{T}{B}\ln\left(\frac{2}{3}\frac{\nu}{\dot{\gamma}}\left(\frac{T}{B}\right)^{5/6}\right)\right]^{2/3}$$

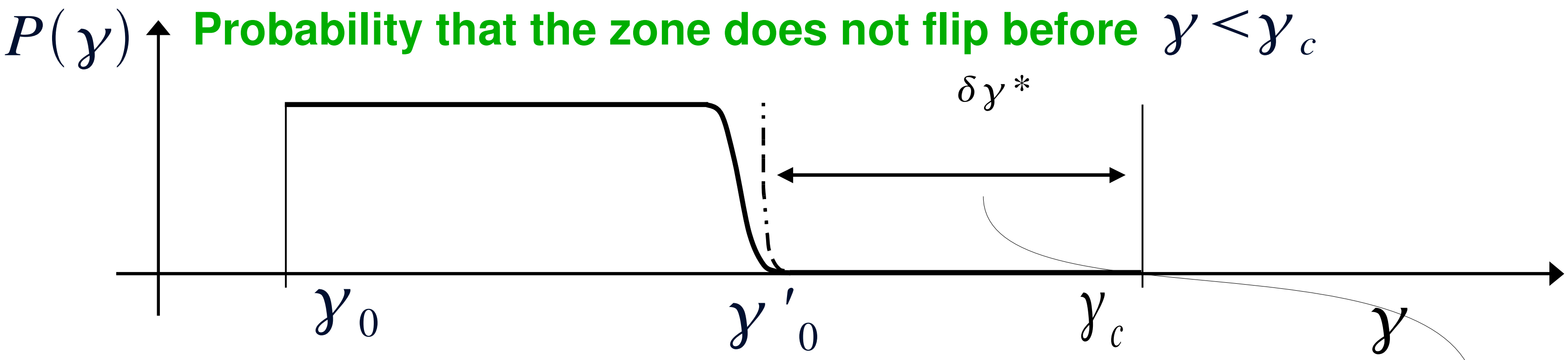
Effect of finite T on a single zone driven towards threshold



$$\delta\gamma^* \sim \left[\frac{T}{B}\ln\left(\frac{2}{3}\frac{\nu}{\dot{\gamma}}\left(\frac{T}{B}\right)^{5/6}\right)\right]^{2/3}$$

Effect of noise?

Effect of finite T on a single zone driven towards threshold



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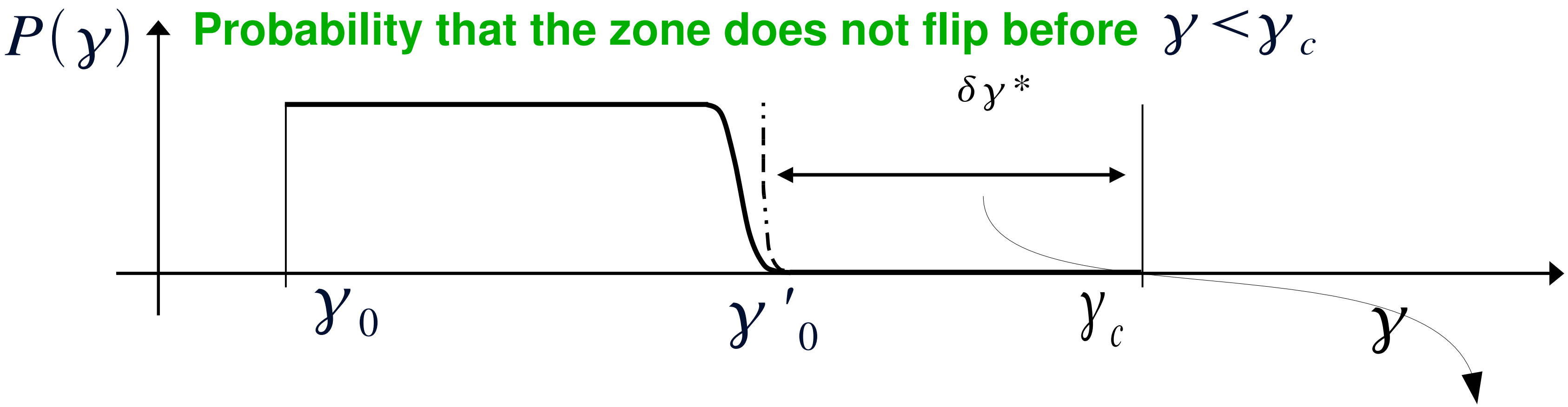
<u>Effect of noise?</u>

Effect of finite T on a single zone driven towards threshold



$$\delta\gamma^* \sim \left[\frac{T}{B}\ln\left(\frac{2}{3}\frac{\nu}{\dot{\gamma}}\left(\frac{T}{B}\right)^{5/6}\right)\right]^{2/3}$$

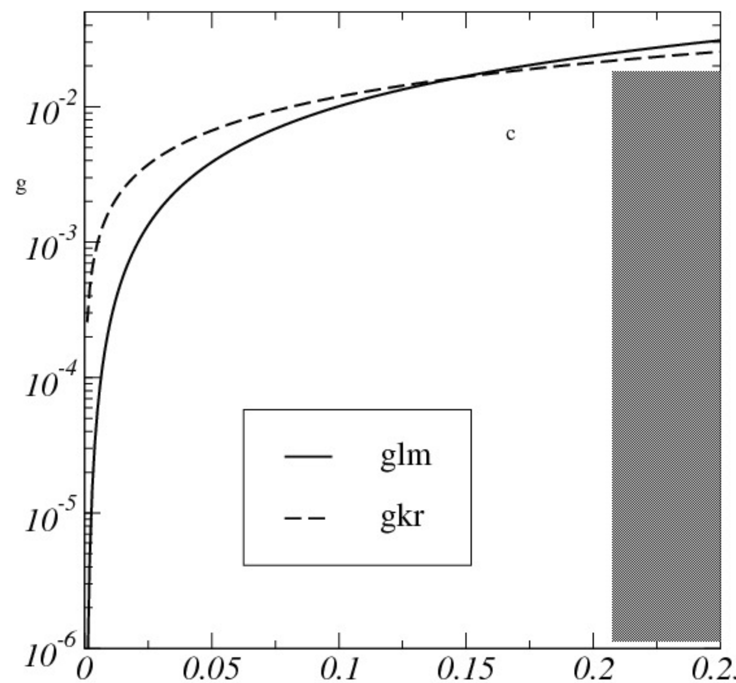
<u>Effect of noise?</u>

Can be separated if:

$$\frac{\Delta E}{E} \ll 1$$ during activation time

<u>Applicability of Kramers expression:</u>

$$\frac{E}{T} \gg 1$$



Consequences for stress

$$\sigma(\dot{\gamma};T)=\sigma(\dot{\gamma};T=0)-2\mu\,\delta\gamma^{*}$$

$$\delta\gamma^{*}=\left[\frac{T}{B}\ln\left(\frac{2}{3}\frac{\nu}{\dot{\gamma}}\left(\frac{T}{B}\right)^{5/6}\right)\right]^{2/3}$$

$$\sigma=A+B\sqrt{\dot{\gamma}}-C\left[T\ln\left(\frac{DT^{5/6}}{\dot{\gamma}}\right)\right]^{2/3}$$

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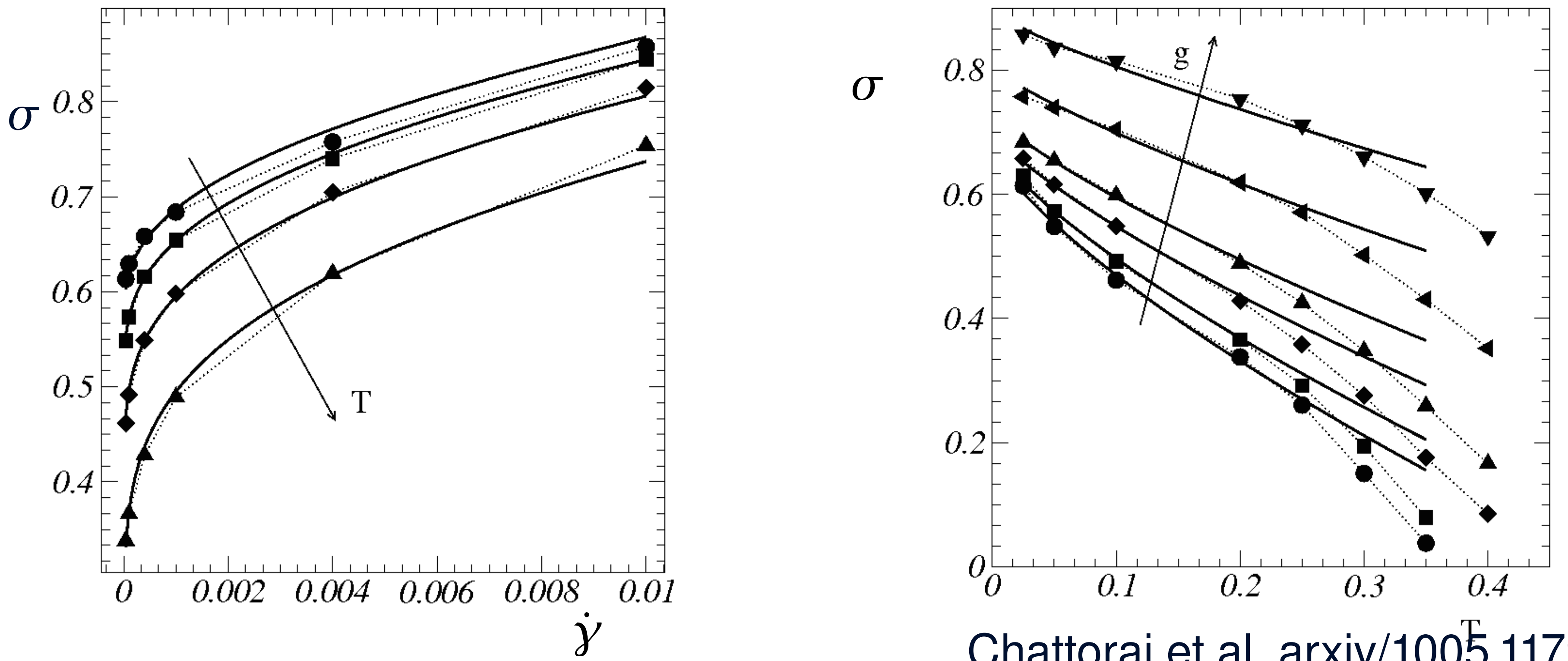
$$\sigma(\dot{\gamma};T) = \sigma(\dot{\gamma};T=0) - 2\mu\,\delta\gamma^*$$

$$\delta\gamma^* = \left[\frac{T}{B}\ln\left(\frac{2}{3}\frac{\nu}{\dot{\gamma}}\left(\frac{T}{B}\right)^{5/6}\right)\right]^{2/3}$$

$$\sigma = A + B\sqrt{\dot{\gamma}} - C\left[T\ln\left(\frac{DT^{5/6}}{\dot{\gamma}}\right)\right]^{2/3}$$



Chattoraj et al, arxiv/1005.1179

Conclusion

- AQS simulations support the following phenomenology:
  - Plasticity results from local shear transformations
  - Zones are progressively convected towards instability
  - Each flip produces an Eshelby-like field likely to trigger secondary flips

  This shows up as system-spanning avalanches

- At usual finite $\dot{\gamma}$, the same phenomenology continues to govern plasticity
  - The size of avalanches $l \sim \dot{\gamma}^{-1/D}$
  - With normal cross-over behavior when $l \sim L$
  - We propose these changes govern stress/strain-rate relation

- At finite T < Tg, the same phenomenology continues to hold
  - Activations of driven zones leads to global lowering of the flow stress consistent with a $T^{2/3}$ stress correction
  - Meanwhile, avalanche continues to be present and are only progressively blurred when approaching Tg