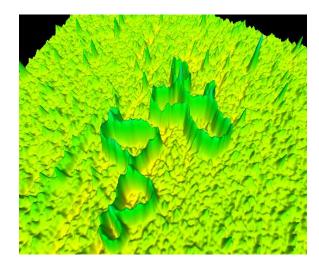
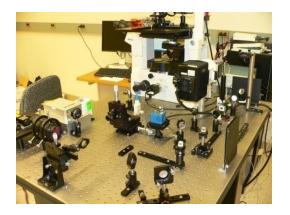
Motor Proteins Investigation and Modelling

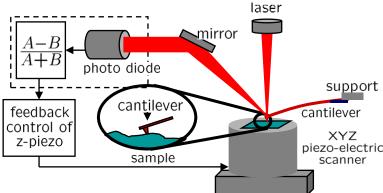
Murti V. Salapaka murtis@umn.edu

NanoDynamics Systems Lab Electrical Engineering Department University of Minnesota, Minneapolis







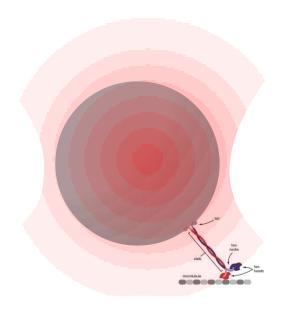


NanoDynamics Tyrbemr Liboratory

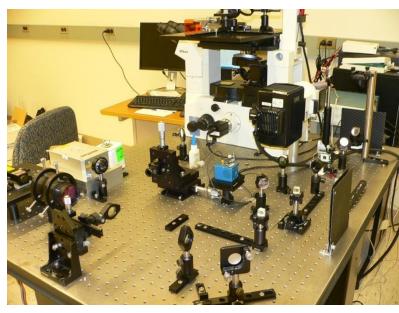
Outline

- Motor Proteins
 - ★ What are they?
 - ⋆ Role in human physiology
 - ⋆ Single molecule studies
 - Why single molecule and comparison with ensemble studies
 - Challenges
 - Modes of Investigation
 - Optical Tweezers for Single Molecule Experiments
 - * A new Step detection method
 - Application to Kinesin walk detection
 - ⋆ Cooperative behavior amongst molecular motors
 - Understanding cargo transport in cells with multiple motors

Motor Proteins

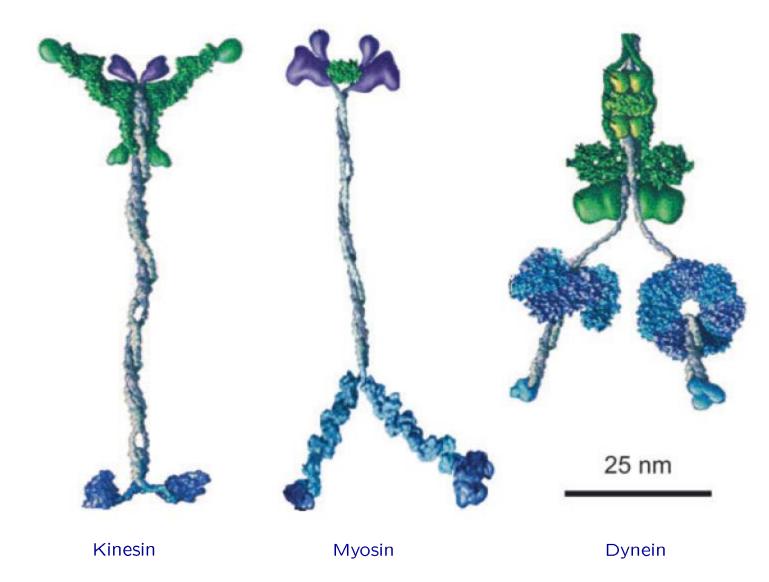








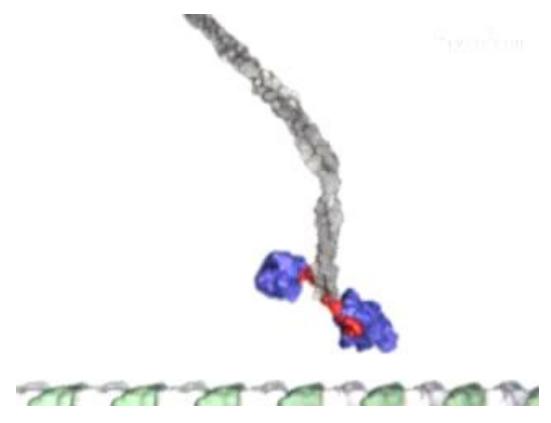
What are they



Pictures from: Kolomeisky, A. B., Fisher, M. "Molecular Motors: A Theorist's Perspective", Anuual Review of Phys. Chem, 58, 675-695, 2007



Transport in cells



Kinesin Motor

- * Processive motor
 - Two heads
 - walks over microtubule
 - Length of Kinesin molecule is about 100 nm
 - Cargo can be about a μm in dimension

Animation

This video is from: Essential Cell Biology, 3rd Edition Alberts, Bray, Hopkin, Johnson, Lewis, Raff, Roberts, Walter ISBN: 978-0-8153-4129-1



What are they



Motor Proteins

- Enzymatic molecules that convert chemical to mechanical work and motion
- * Processive motor proteins
 - Kinesin, Dynein and Myosin
 - Step unidirectionally along linear tracks carrying a cargo

Gliding Assay: Microtubules transported by Kinesin on substrate (work at NDSL)



Physiological Relevance (Examples)

Kinesin

- ★ Left-right body determination (KIF3) (body axis)
- * Suppression of tumorogenesis
- KIF17 and KIF5 involved in higher brain functionality (long term memory)
- ⋆ Tau factor regulation implicated in Alzheimers

NanoDynamics Systems Liporatory

Challenges

- Are there substeps
- Backward steps; are they real
 - * Rare event detection
- Is the biocehmical pathway altered when an assitive force is provided
- Browninan ratchet or power stroke
 - ⋆ Modeling to identify parameters to control
- How do multiple Kinesin coordinate motion intransport of the same cargo
 - ⋆ Typically less than five kinesin involved.
- How do multiple motor protein types (Dynein and Kinesin) achieved tasks (like mitosis)
 - ★ Typically six to seven dynein Vs 1 kinesin.



Single Molecule Vs Ensemble Studies

- Biological systems form is inextricably linked to function
 - Corollary of Darwins evolution theory
 - Huge effort in determining static structure of molecules
- Study of dynamics of molecules
 - * Ensemble methods
 - Need to synchronize all elements of ensemble to the same initial state
 - Initial state memory lasts only for a short time with thermal forces lead to decoherence
 - Studies can be done for millisecond time range after synchronization
 - Processes have time scales in much larger time scales; protein folding in the minutes time scale
 - Rare events cannot be studied
 - Multimodal behavior cannot be unravelled

- Single Molecule methods
 - * Advantages
 - No synchronization issues
 - Rare occurences can be studied by tailoring the effect
 - Effects of forces and torques can be studied
 - Larger time scales are accesible
 - Internal noise effects can be better understood
 - Challenges
 - Signal is twenty orders smaller
 - Dynamics can be quite complex

Modes of investigating Single molecule physics

Optical Tweezer Atomic Force Microscope FRET





Optical Tweezer

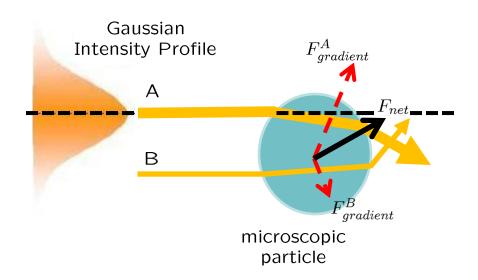
Harnessing the momentum of Light







Physics

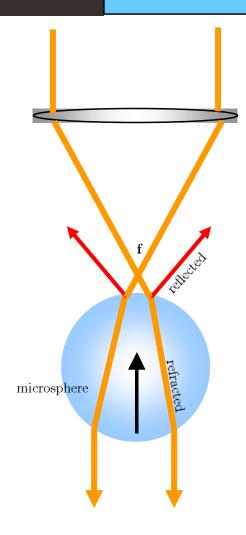


- Optical trap
 - * Radiation pressure of light
- Scattering force
 - In the direction of beam propagation
 - A destabilizing effect
- Gradient Force
 - * Stabilizing influence
- Numerical Aperture Large
 - Leads to a stable trap with a Single Laser beam

A. Ashkin, J. M. Dziedzic, J. E. Bjorkholm, and Steven Chu. Observation of a single-beam gradient force optical trap for dielectric particles. Opt. Lett., 11(5):288, 1986



Optical Trapping

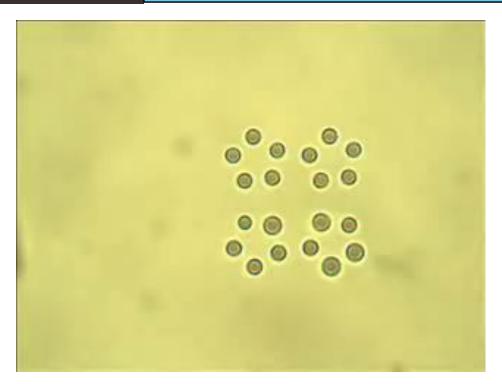


- Optical trap
 - ⋆ Maximum force: 100pN
 - ⋆ Hookean spring constant
 - 0.01pN/nm to 0.5pN/nm
- Manipulation range
 - **★** 50 μm
- Bead size
 - * on the order of micron

A. Ashkin, J. M. Dziedzic, J. E. Bjorkholm, and Steven Chu. Observation of a single-beam gradient force optical trap for dielectric particles. Opt. Lett., 11(5):288, 1986



Single Beam Time Multiplexed traps



Achieved at NDSL

Dynamic Patterns

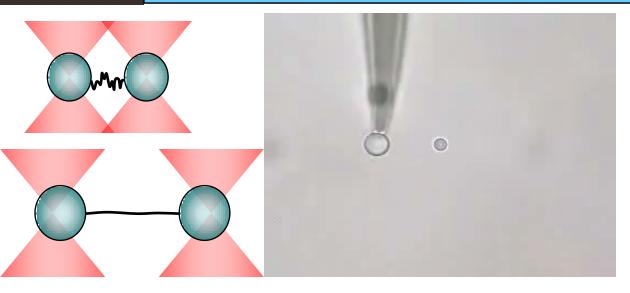
- * Generate MicroFluidic flows
 - Realized experimentally at NDSL
- ⋆ Projects
 - Using CCD output as measurement maintain a robust pattern

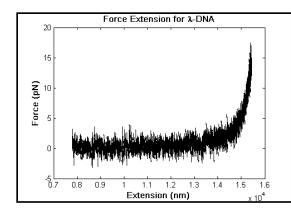


Achieved at NDSL



Single Molecule Force Spectroscopy





Achieved at NDSL

- Force Spectroscopy
 - * Identify domains
 - Protein Folding
 - ⋆ Projects
 - Identification of events

Coordinated Motion of Molecular Motors: Modeling and Analysis



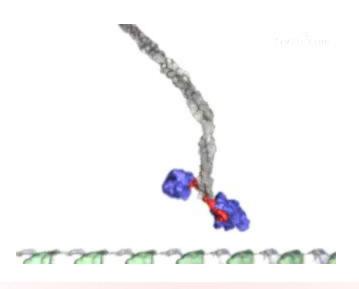
Dr. Donatello Materassi

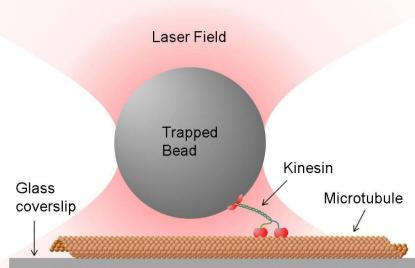






The Individual Motor model





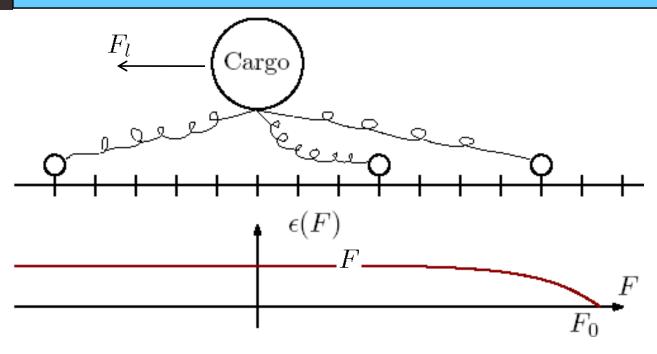
Stepping Probability

- * The Michaelis-Menten dynamics leads to Hydrolysis rate $=\frac{k_{cat}[ATP]}{[ATP]+k_m}$ where $k_m=(k_{cat}+k_{off})/k_{on}$ is the Michaelis-Menten constant
- * A step is successful with rate ϵ (efficiency of a step)
- * The step size is d. We have $P_{step} = \frac{k_{cat}[ATP]}{[ATP] + k_m} \epsilon$

$$V = P_{step}d = \frac{k_{cat}[ATP]}{[ATP] + k_m}d\epsilon$$

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The Individual Motor model



- Influence of force on the motor
 - * The efficiency $\epsilon = \epsilon(F)$ $\epsilon(F) = \begin{cases} 1 & \text{if } F \leq 0 \\ 1 \left(\frac{F}{F_0}\right)^2 & \text{if } 0 < F < F0 \\ 0 & \text{otherwise} \end{cases}$
 - \star F_0 is the "stall force"



The Individual Motor model

- Modeling detachment
 - * k_{off} increases with increasing forces [Gross et al, 2005] $k_{off} = k_{0off} e^{Fd_l/K_bT}$
 - We have the probability of a successful step under a force
 F

$$P_{step} = \frac{k_{cat}[ATP]}{[ATP] + \frac{k_{on} + k_{off}(F)}{k_{cat}}} \epsilon(F)$$

$$= \frac{k_{cat}[ATP]}{[ATP] + \frac{k_{on} + k_{0off}e^{Fd_l/K_bT}}{k_{cat}}} \left[1 - \left(\frac{F}{F_0}\right)^2\right]$$

ManoDynamics Systems Urboratory

The Individual Motor model

- Modeling detachment
 - \star From [Schnitzer et al. 2000], the processivity L is

$$L = \frac{d[ATP]Ae^{-F\delta_l K_b T}}{[ATP] + B(1+A)e^{-F\delta_l K_b T}}$$

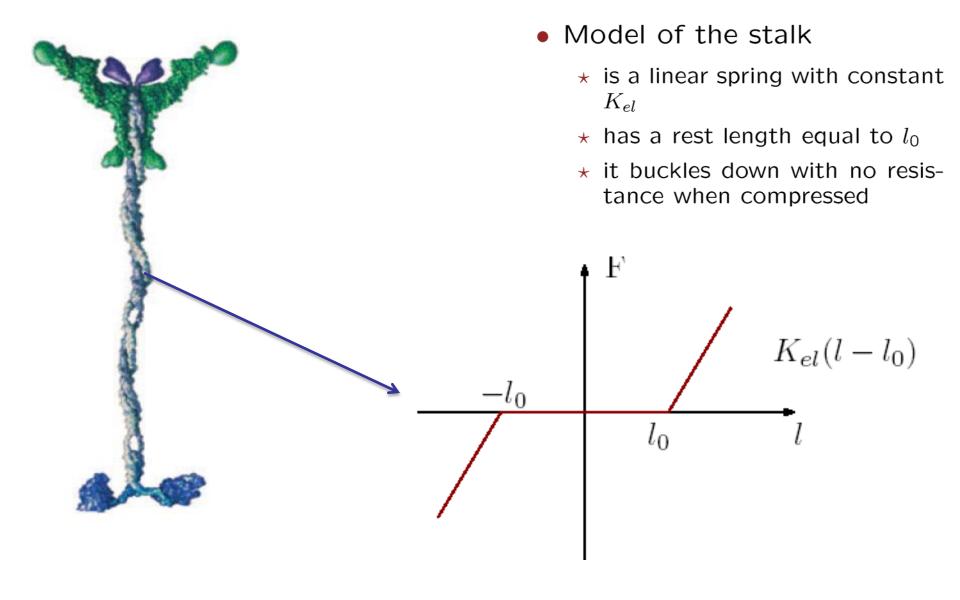
* Then

$$\frac{P_{step}(F)}{P_{detach}(F)} = \frac{L}{d} = \frac{[ATP]Ae^{-F\delta_l K_b T}}{[ATP] + B(1+A)e^{-F\delta_l K_b T}}$$

- The probability of reattachment is $P_a = 5s^{-1}$.
- A motor can only reattach at a location whose distance from the cargo is less than l_0 .

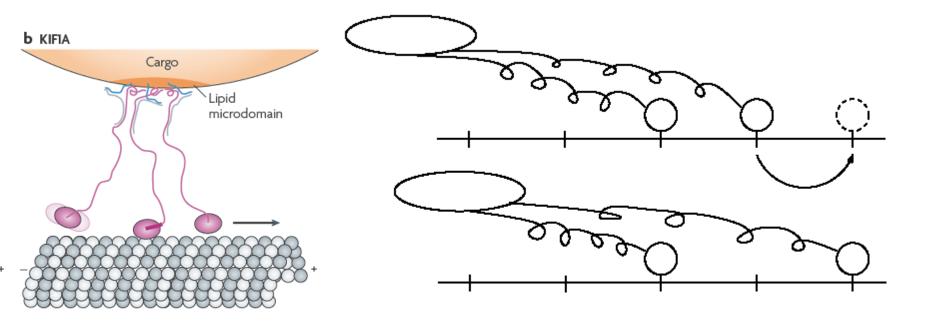


The Individual Motor model





Cargo Model



- Cargo position determined entirely by force balance of forces exerted by motors and Force applied
 - If a motor steps, the cargo reaches the new position instantaneously
 - If there are m_a "active" motors we observe a step in the cargo with length d/m_a
 - A shift of the configuration is equivalent

A Equivalent Model

Elements

- * bi-infinite "Microtubule" with equally spaced locations $\{a_k\}_{k\in\mathcal{Z}}$ $a_k=x^{(0)}+kd_s$
- * m engaged motors on the Microtubule
- \star the *i*-th motor position $x_i \in \{a_k\}_{k \in \mathcal{Z}}$
- \star z_k : number of motors in the location a_k
- ⋆ a configuration is the bi-infinite sequence

$$Z = (..., z_{-1}, z_0, ..., z_k, ...)$$

A Equivalent Model

Transitions

- \star $\lambda(Z_2,Z_1)$ is the rate of transition $Z_1 \to Z_2$
- \star A step from a_k to a_{k+1}

$$Z = \begin{pmatrix} \vdots \\ z_k \\ z_{k+1} \\ \vdots \end{pmatrix} \xrightarrow{\lambda(Z + R_k^{(s)}, Z) = P_{step}} \begin{pmatrix} \vdots \\ z_k \\ z_{k+1} \\ \vdots \end{pmatrix} + \begin{pmatrix} \vdots \\ -1 \\ +1 \\ \vdots \end{pmatrix} = Z + R_k^{(s)}$$

 \star A detachment/attachment at location a_k

$$Z = \begin{pmatrix} \vdots \\ z_k \\ z_{k+1} \\ \vdots \end{pmatrix} \xrightarrow{\lambda(Z \pm R_k^{(a)}, Z) = P_{det}/P_{att}} \begin{pmatrix} \vdots \\ z_k \\ z_{k+1} \\ \vdots \end{pmatrix} + \begin{pmatrix} \vdots \\ \pm 1 \\ 0 \\ \vdots \end{pmatrix} = Z \pm R_k^{(a)}$$

* A translation in space preserves the rates

A Equivalent Model

- An equivalent model
 - \star Transitions $Z \to Z + R$ define a Gillespie Model
 - \star P(Z,t) (probability of configuration Z at time t) satisfies the Master Equation

$$\frac{\partial}{\partial t}P(Z,t) = -P(Z,t)\sum_{Z'\in\mathcal{Z}}\lambda(Z',Z) + \sum_{Z'\in\mathcal{Z}}\lambda(Z,Z')P(Z',t),$$

- * Enumerate the configurations $Z_1,...,Z_j,...$ and define $\mathcal{P}(t):=(P(Z_1,t),...,P(Z_j,t),...)^T$
- ⋆ The Master Equation becomes

$$\frac{\partial}{\partial t} \mathcal{P}(t) = \mathcal{A} \mathcal{P}(t)$$



Spread is finite

Spread

* For a configuration
$$Z=(...,z_k,...)$$
, define the $spread$ $s(Z):=\max_{z_k\neq 0}\{a_k\}-\min_{z_k\neq 0}\{a_k\}$ Also define $s^{(max)}:=\max\left\{\frac{\overline{m}F_s-F_{load}}{K}+d_s,\frac{F_{load}}{K}\right\}+2l_0.$ Under the KVXG rules for any $S\geq s^{(max)}$, if $s(Z(t_0))\leq S$ $Pr\{s(Z(t))\leq S\}=1$ for $t>t_0$.

Ensemble Representation

- Tracking number of motors at each location relative to spread
 - * Define the regular dimension of the ensemble

$$n := \left\lceil \frac{s^{(max)}}{d_s} \right\rceil$$

Given $Z = \{z_k\}_{k \in \mathcal{Z}}$, define its *ensemble representation* as the n-vector

$$Q = (z_{k_1}, ... z_{k_1+n-1})^T$$

where

$$k_1 := \inf\{k \in \mathcal{Z} | z_k \neq 0\}$$

* We write $Q := \Pi^{(e)}(Z)$. If $k_1 = -\infty$, Q is $(0, 0, ..., 0)^T$.

Ensemble Representation

Projection

- * The operator $\Pi^{(e)}: Z \to Q$ is a projection
- * Information of the absolute position is lost
- ⋆ Relative distances are preserved
- ⋆ This projection is still Markov

The following holds

* Define

$$\lambda_{Q}(Q',Q) := \sum_{\substack{\Pi^{(e)}(Z') = Q' \\ s(Z) \le n}} \lambda(Z',Z).$$

If $s(Z(t_0)) < n$ the dynamics of the projection space follows

$$\begin{split} & \frac{\partial}{\partial t} P_Q(Q,t) = -P_Q(Q,t) \sum_{Q' \in \mathcal{Q}} \lambda(Q',Q) + \sum_{Q' \in \mathcal{Q}} \lambda(Q,Q') P_Q(Q',t) \end{split}$$



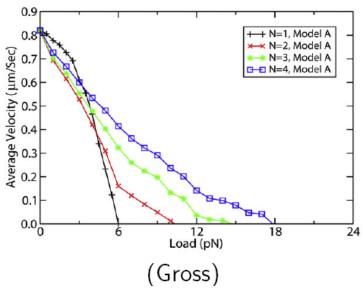
Achievements

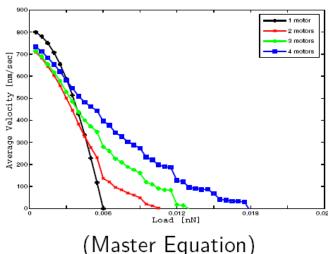
Projection

- * The dynamics of the system has no steady state
- ★ The dynamics of the projection has a steady state
- * The expected velocity of the ensemble can be determined
- The average runlength can be determined
- ★ The number of engaged/active motors can be determined
- Rare events are captured



Unexpected Behavior





Unexpected behavior

* Issue

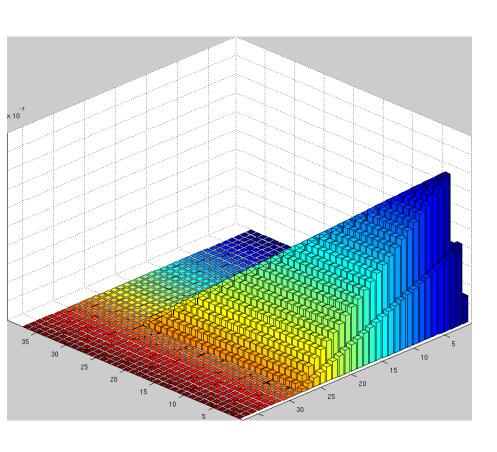
- If two motors or motors are attached their center of mass is always faster (in average) than a single motor
- If all but one motor detach, the one still attached is still as fast as a single motor
- Why, for low loads, more motors are slower than one single motor?

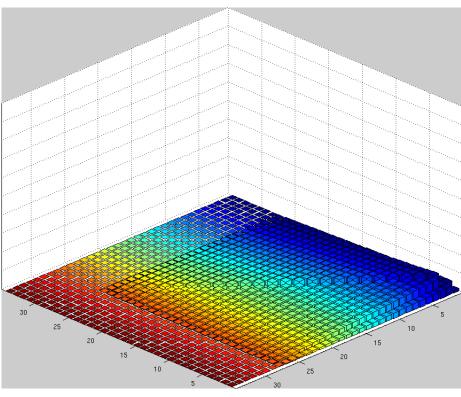
⋆ Possible explanation

- This can be explained by backward reattachments
- The vanguard motors are relatively more likely to detach
- The phenomenon does not occur at high loads because at high loads motors tend to clusterize and share the load



Marching or a stroll



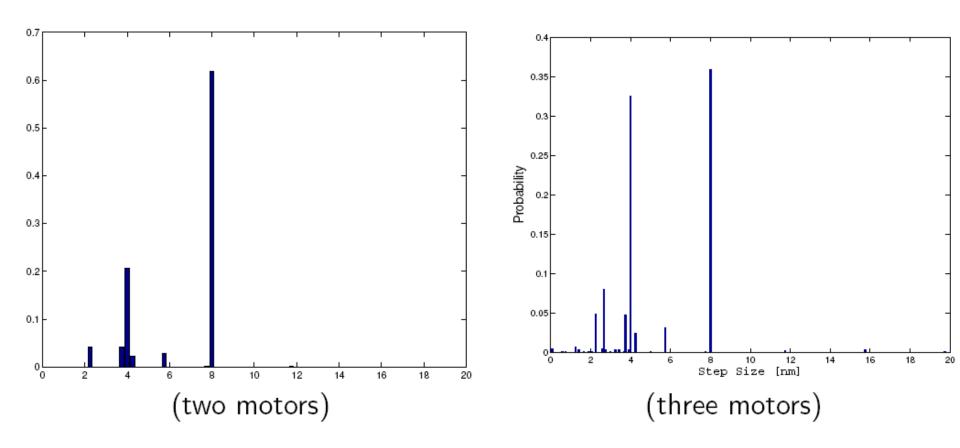


Low Load

High load



PDF of Step Size



- Small probability of a 11.5nm step
- Detachment of a reargard motor



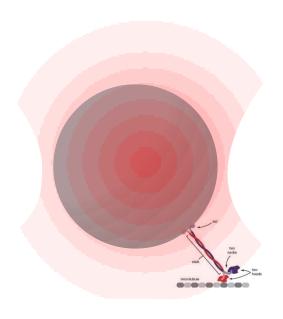
New Insights

- ▶ We can obtain insights about the transient dynamics
 - How fast can the motors reach steady state?
 - Attachment/Detachment are less frequent events
 - Is it true that the number of engaged motors defines at each time the behaviour of the system?
- We can obtain insights on rare events
 - Rare steps can be found
 - Possible explanation of jumps in [Leduc et. al]?
 - Better runlength when close to stall? Verifiable?

High Bandwidth Step Detection Method



Tanuj Aggarwal

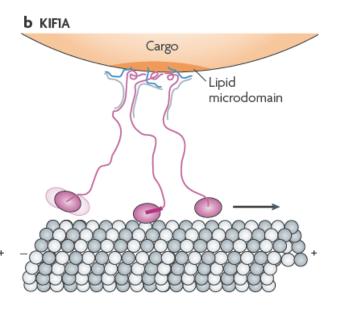






Kinesin Walk





- Kinesin moves in discrete steps
 - ⋆ Steps of 8nm
- Multiple Kinesin can carry the same cargo
 - sychronous vs asynchronous behavior
 - * Phase Transitions
 - * Rare modalities
 - Mutated Kinesin (disease causing)



Challenges

Higher bandwidth operation

- ⋆ Motivation
 - Kinesin walk occurs in vivo often over 1 $\mu m/s$
 - Kinesin walk delibrately slowed to approximately 25nm/s by lower ATP concentration
 - Kinesin dynamics remains elusive at higher speeds

Resolution

- ★ A typical setup achieves < 5 nm at 40-50 Hz</p>
 - Multiple Kinesin molecules driving a load can take smaller steps (<4 nm) at 1 $\mu m/s$



Challenges

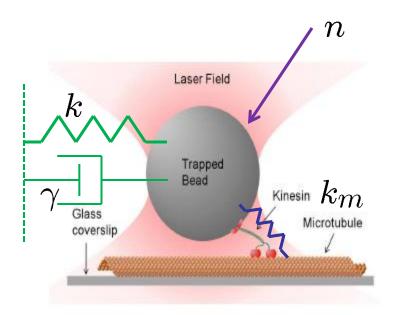
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Model

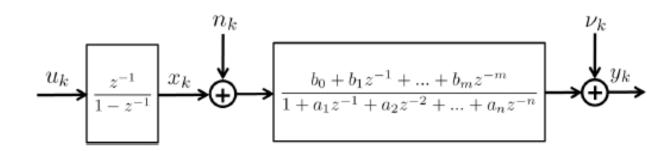
- * Process noise is thermal
- Optical trap modeled as a Hookean Spring
- ⋆ Damping is substantial; inertial effects negligible
- * Kinesin motor modeled as a spring
- * Bead position is measured
 - Measurment noise is ν .
- * Dynamics

$$- \gamma \dot{x}_b + kx_b = k_m(x_m - x_b) + n$$

* Measurement

$$- y = x_b + \nu.$$

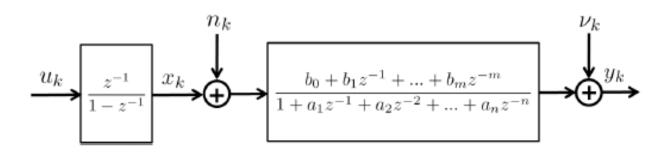


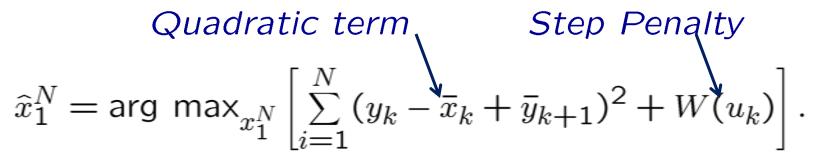


- Model
 - $\star x_{k+1} = x_k + u_k$. (u_k denotes a step; a prior for u_k is assumed)
 - $\star x_1^N = \{x_1, \dots, x_N\}$
- Problem: Solve for $\hat{x}_1^N = \text{arg max}_{x_1^N} \quad [p_{x_1^N | y_1^N}(x_1^N | y_1^N)].$
- One can show that $\widehat{x}_{1}^{N} = \arg\max_{x_{1}^{N}} \left[\sum_{i=1}^{N} (y_{k} - \bar{x}_{k} + \bar{y}_{k+1})^{2} + W(u_{k}) \right] \cdot \underset{\star}{\text{Prior}}$ $\star \ \bar{x}_{k} := \sum_{i=1}^{m} b_{i} x_{k-i}, \ \bar{y}_{k} = \sum_{j=1}^{l} a_{j} y_{k-j}. \ W(u_{k}) = -2\bar{\sigma}_{n}^{2} \log p(u_{k}).$

$$\star \bar{x}_k := \sum_{i=1}^m b_i x_{k-i}, \bar{y}_k = \sum_{j=1}^l a_j y_{k-j}. W(u_k) = -2\bar{\sigma}_n^2 \log p(u_k)$$

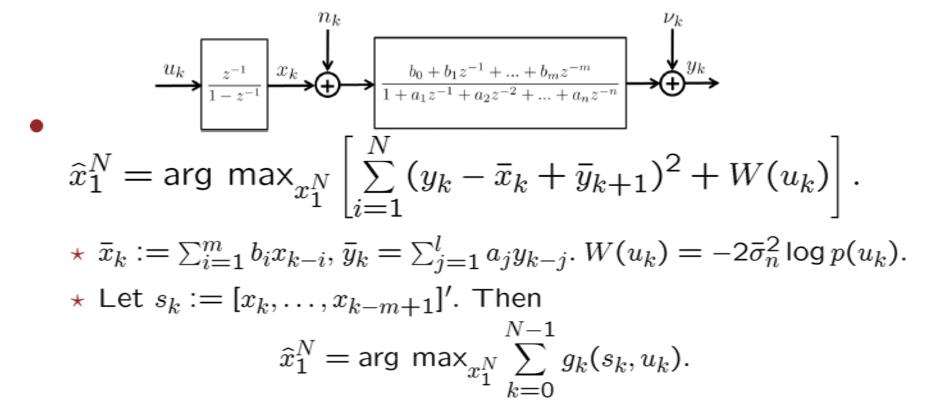






- * $\bar{x}_k := \sum_{i=1}^m b_i x_{k-i}, \, \bar{y}_k = \sum_{j=1}^l a_j y_{k-j}. \, W(u_k) = -2\bar{\sigma}_n^2 \log p(u_k).$
- Problem is equivalent to minimizing a quadratic cost with penalty for steps
- ⋆ Relation to "compressive sensing" issues



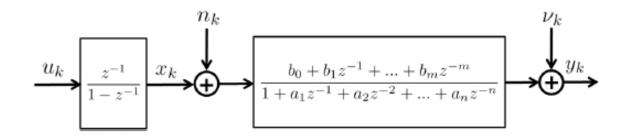


All pole model then

$$g_k(s_k, u_k) = (y_{k+1} - b_0 x_k - b_0 u_k + \sum_{j=1}^{\ell} y_{k+1-j})^2$$

* Tractable Dynamic Programming Problem

Pdf Updation using Priors



$$\begin{split} \widehat{x}_1^N &= \text{arg max}_{x_1^N} \left[\sum_{i=1}^N (y_k - \bar{x}_k + \bar{y}_{k+1})^2 + W(u_k) \right]. \\ \star \ \bar{x}_k &:= \sum_{i=1}^m b_i x_{k-i}, \ \bar{y}_k = \sum_{i=1}^l a_j y_{k-j}. \end{split}$$

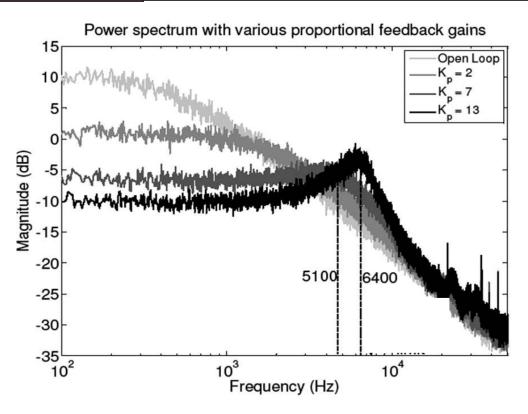
All pole model then

$$g_k(s_k, u_k) = (y_{k+1} - b_0 x_k - b_0 u_k + \sum_{j=1}^{\ell} y_{k+1-j})^2$$

Update Prior based on previous iteration

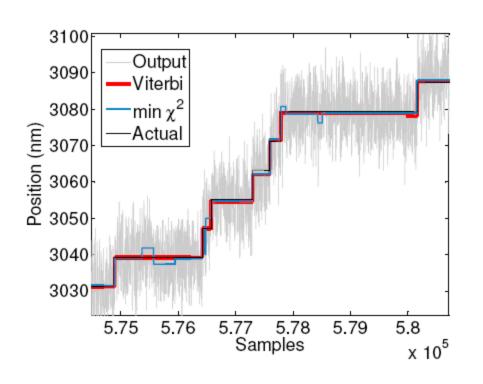
$$\star W(u_k) = -2\bar{\sigma}_n^2 \log p(u_k).$$

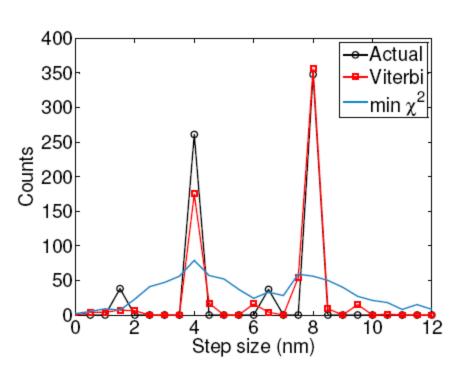
Anomalous Behavior



- Model when implemented experimentally does not work
- * Systems model
- Assess the input-output model
 - Not first principles based



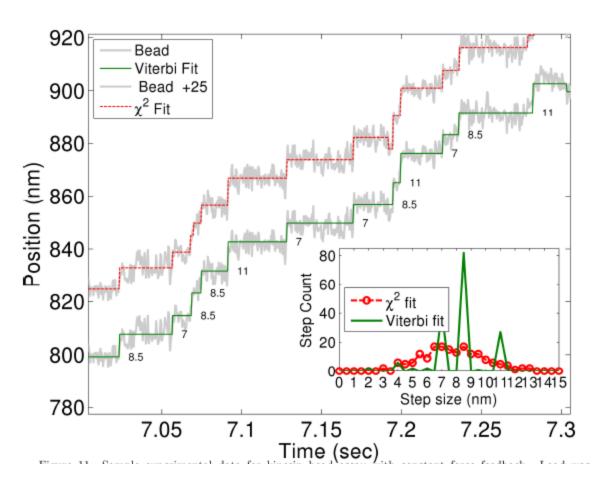




- Advanced step detection in place to hone into 4nm substeps
- Work to asses error probailities being undertaken



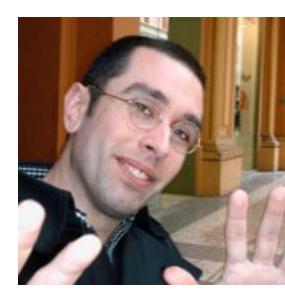
Experimental data



- Advanced step detection in place to hone into 4nm substeps
- Work to asses error probailities being undertaken

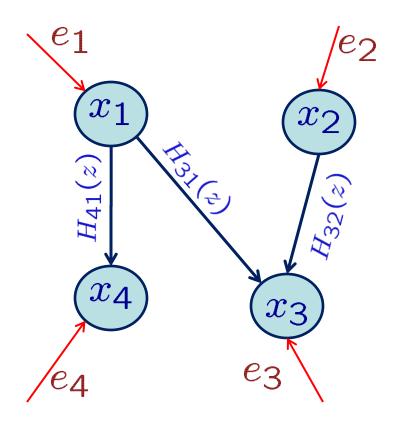
Network Topology Reconstruction





Dr. Donatello Materassi

Framework



$$x_3 = H_{32}x_2 + H_{31}x_1 + e_3$$

Assumed Underlying Model

$$\star x_j = \sum_{i=1}^N H_{ji}(z)x_i + e_j.$$

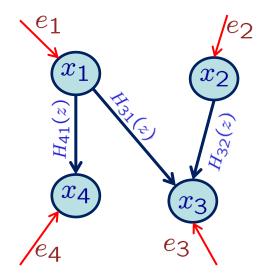
- $-x_i$ is the node value,
- e_j is noise affecting each node
- Given the time series data $x_j(t), t \in R$ for all j = 1, ..., N identify if $H_{ji} = H_{ij} = 0$.
 - Reconstruction of the undirected graph
- No assumptions on the graph's edges
 - * Loops are allowed

Strategy

- \star Project x_j on the $M_j := \mathsf{tf}\text{-span}\{x_1, x_2, \dots, x_{j-1}, x_{j+1}, \dots, x_N\} \subset H.$
 - * Find
 - $\hat{x}_j = \sum_{i \neq j} W_{ji}(z) x_i$
 - $-(x_j-\hat{x}_j)\perp M_j$
 - \star It turns out that $W_{ji} \neq 0$ only if i is a kin of j



Kins



$$x_3 = H_{32}x_2 + H_{31}x_1 + e_3$$

Assumed Underlying Model

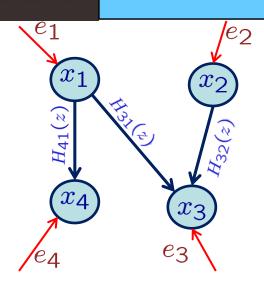
$$\star x_j = \sum_{i=1}^N H_{ji}(z)x_i + e_j.$$

- x_j is the node value,
- e_i is noise affecting each node

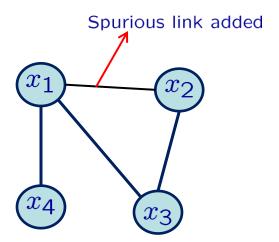
Kins

- Consists of parents, children and co-parents
 - x_1 is a parent of x_3 , x_4 is a child of x_1 .
 - x_2 is a co-parent of x_1 (with child x_3).
 - Kins of x_1 are x_3 , x_1 and x_2 .

Kins have all the information



$$x_3 = H_{32}x_2 + H_{31}x_1 + e_3$$



Projection on Kins

- \star Let \mathcal{K}_j be the tf-span of kins of j and let \widehat{x}_j' be the projection of x_j on \mathcal{K}_j
 - $(x_j \hat{x}_j') \perp \mathcal{K}_j$
- \star \hat{x}_j is the projection on tf-span $\{x_i\}_{i\neq j}$.
- Theorem: $\hat{x}'_j = \hat{x}_j$

$$\star \ \hat{x}_j = \sum_{i \neq i} W_{ji} x_i = \sum_{i \in K_i} W_{ji} x_i.$$

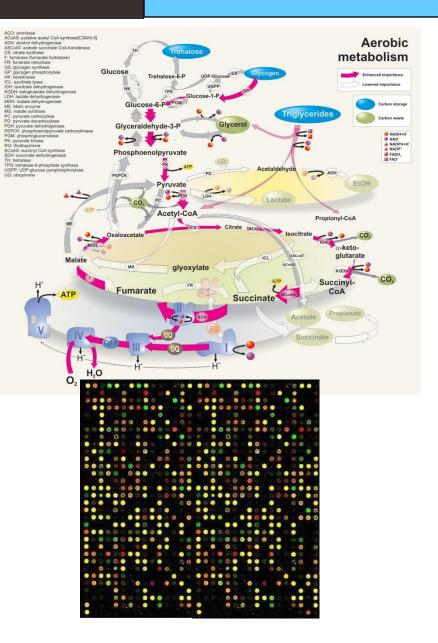
- Projection on all nodal values=Projection on Kins
- Can be used to reconstruct topologies

$$\overset{\star}{W}_{j}(\omega) = \phi_{x_{j}x_{j^{c}}}(\omega)\phi_{x_{j^{c}}x_{j^{c}}}^{-1}(\omega).$$

- Can be constructed using timeseries data
- * Spurious Links remain local



Model is Good!



Assumed Underlying Model

$$\star x_j = \sum_{i=1}^N H_{ji}(z)x_i + e_j.$$

- $-x_i$ is the node value,
- e_j is noise affecting each node
- Above model underlying many systems
 - Correctly reconstructs topology in genenetworks, financial data etc



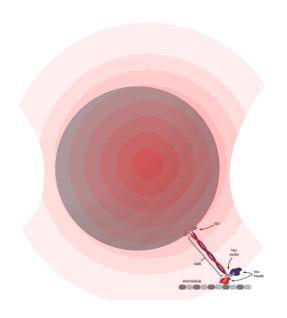
Conclusions

- Exciting times as nanoscale biological functioning units now accessible for probing and engineering
 - Huge opportunities to further open up newer temporal and spatial regimes
 - Robust Control techniques are in place
- Experiments need to be driven by hypothesis to be provided by simulation engines
 - Need for innovaton for simulation capabilities of realistic cases
 - Five to six motor configurations can be simulated in "closed-form" on a PC

Experiments

- ⋆ Quote
 - Do not think too much. Do the experiment. JUST DO IT
- * A new technique to probe sub 5 nm steps developed

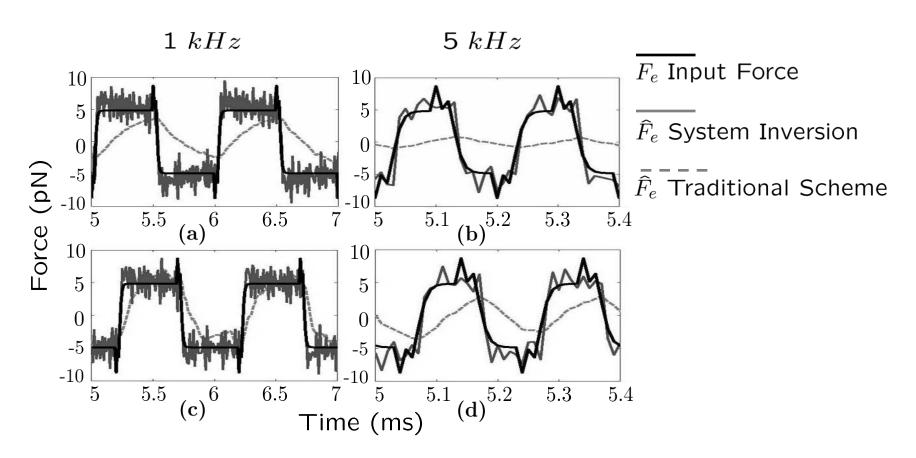
Systems model and robust control design for isometric clamp





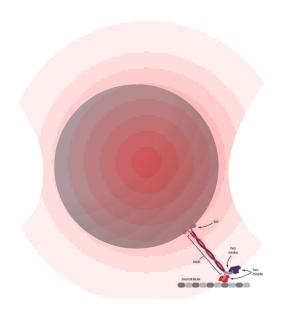


Experimental Results



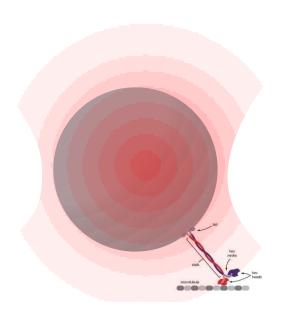
- Open Loop results in (a) and (b). Closed Loop results in (c) and (d).
- More than an order in magnitude of bandwidth increase in force estimation

Stochastic Simulation Engine





Future Objectives







Instrumentation Objectives

- Instrumentation objectives
 - Robust control techniques for isotonic clamps of optical tweezers to achieve an order increase in estimation of motor movement
 - ★ Simultaneous x,y,z control of the cargo
 - ★ Resolution enhancements to detect 1nm steps at 1 Khz bandwidth or more; this will enable experimental corroboration for four kinesin motors attached to the same cargo
 - A enhanced FPGA based hardware platform to implement controllers

- Instrumentation objectives
 - * Enhanced CCD camera
 - ⋆ DIC optics
 - Microfluidic channels to induce flows (nutrient supplies)
 - ★ Enhanced nanopositioning stage



Biological Objectives

Modeling

- Enhanced Stochastic Simulation engine and integration with the supercomputing facility
 - Study of cooperative behavior amongst similar motors arraying cargo as well as different species carrying cargo
 - Study of changing parameters of a fraction of attached motors to simulate diseased motor
 - Study of transient behavior and related time scales
 - Links with equilibrium statistical mechanics
 - Study of control actions based on modalities of cargo movement
 - Integration with experimental results
 - Thermal ratchet studies

Experimental objectives

- * Higher bandwidth instrumentation for studying Kinesin stepping for realistic ATP concentrations and other parameters.
- Methods to attach and discern mutiple Kinesin motors
- Methods for mutant Kinesin molecules to be made and to study their processivity
- ⋆ Develop dynein model system
- Develop methods to attach Dynein and Kinesin to the same cargo
- ★ Study tug-of-war issues on how cargo is transported
- Discern ways to see if the thermal ratchet is the means of movement or is it a power stroke or both
- ★ Controls to increase efficiency of energy conversion

Ensemble Representation

Projection

- ★ The way of coding the configurations is critical
- ★ Examples with a trivial code
 - -n = 20, m = 4, detach, then N = 160,000
- ★ Examples with a "smart" code
 - -n = 20, m = 4, no detach, then N = 1,540
 - -n = 20, m = 4, detach, then N = 1,771
 - -n = 30, m = 4, no detach then N = 4,960
 - n = 40, m = 4, no detach, then N = 11,480
 - -n = 40, m = 4, detach, then N = 12,302
- \star The number m of motor is a smaller concern
- ★ Low stiffness requires high n
- There are techniques to tackle the problem reducing the dimensionality at low cost for the accuracy