

## Gerhard Knieper

Title: New results on harmonic manifolds

Abstract: The Lichnerowicz conjecture asserts that all harmonic manifolds are either flat or locally symmetric spaces of rank 1. This conjecture has been proved by Z. Szabó for harmonic manifolds with compact universal cover. Furthermore, the conjecture was obtained by Besson, Courtois and Gallot for compact manifolds of strictly negative curvature as an application of their entropy rigidity theorem in combination with the rigidity theorems by Benoist, Foulon and Labourie on stable and unstable foliations.

On the other hand, E. Damek and F. Ricci provided examples showing that in the noncompact case the conjecture is wrong. However, such manifolds do not admit a compact quotient.

In this talk we will show that for all simply connected, noncompact and nonflat harmonic spaces  $X$  the following properties are equivalent:

- $X$  has rank 1
- $X$  has purely exponential volume growth
- $X$  is Gromov hyperbolic
- The geodesic flow on  $X$  is Anosov with respect to the Sasaki metric

Furthermore we obtain, that no focal points imply the above properties. Combining those results with the above mentioned rigidity theorems shows that the Lichnerowicz conjecture is true for all compact harmonic manifolds without focal points or with Gromov hyperbolic fundamental groups. There is some evidence to believe that all non compact harmonic manifolds have no focal points.