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Title: Pattern Rigidity

Abstract: We initiate a study of the topological group $PPQI(G,H)$ of pattern-preserving quasi-isometries for G a hyperbolic Poincare duality group and H an infinite quasiconvex subgroup of infinite index in G. Suppose ∂G admits a visual metric d with $dim_H < dim_t +2$, where dim_H is the Hausdorff dimension and dim_t is the topological dimension of $(\partial G,d)$. \\
a) If Q_u is a group of pattern-preserving uniform quasi-isometries (or more generally any locally compact group of pattern-preserving quasi-isometries) containing G, then G is of finite index in Q_u. \\
b) If instead, H is a codimension one filling subgroup, and Q is any group of pattern-preserving quasi-isometries containing G, then G is of finite index in Q.

Moreover,
(Topological Pattern Rigidity)} if L is the limit set of H, $\LL$ is the collection of translates of L under G, and Q is any pattern-preserving group of homeomorphisms of ∂G preserving $\LL$ and containing G, then the index of G in Q is finite.

An important ingredient of the proof is a version of the Hilbert-Smith conjecture for certain metric measure spaces, which uses the full strength of Yang's theorem on actions of the p-adic integers on homology manifolds. This might be of independent interest.