

ICM Satellite Conference on
Geometry, Topology and Dynamics
in Negative Curvature

Venue: Raman Research Institute, 2–7 August, 2010.

Titles and Abstracts

Monday, 2nd August, 2010

10:30 - 11:30 Plenary lecture 1

Martin Bridson

Title: Rigidity for $\text{Out}(F)$, and actions of higher-rank lattices on free groups

Abstract: Following a discussion of rigidity for mapping class groups and automorphism groups of free groups, I shall outline the proof of a recent result with Ric Wade: every homomorphism from a higher-rank lattice to the outer automorphism group of a free group has finite image.

12:00 - 13:00 Plenary lecture 2

Emmanuel Breuillard

Title: Approximate groups

Abstract: I will describe a recent joint work with Ben Green and Terence Tao in which we study large finite subsets of an ambient group that are almost closed under multiplication, i.e. the so-called approximate groups. I will first survey the basic definitions and properties regarding approximate groups in a general context. Then, bringing tools from combinatorics and making key use of the geometry of simple algebraic groups I will give a classification of their approximate subgroups. These results bring a new twist to the standard results on the growth of groups in geometric group theory by taking into account the size of the generating set. I will also describe applications of these results to showing that random Cayley graphs of finite simple groups of Lie type are expanders graphs (joint also with Bob Guralnick).

14:30 - 15:30 Parallel session 1

Pallavi Dani

Title: Filling invariants at infinity

Abstract: The k -dimensional Dehn function of a group captures the difficulty of filling k -spheres with $(k + 1)$ -balls in a suitable space associated with the group. On the other hand, divergence is an invariant

that measures the spread of geodesics in such a space. These two ideas are united by “higher divergence functions”, which measure rates of filling spheres by balls “at infinity”, i.e., far from a basepoint. It turns out that these filling rates can detect some geometric properties of the space.

After giving the basic definitions and motivation, I will describe recent joint work with A. Abrams, N. Brady, M. Duchin, and R. Young on higher divergence in the class of right-angled Artin groups.

14:30 - 15:30 Parallel session 1

Anne Thomas

Title: Lattices in complete Kac-Moody groups

Abstract: A complete Kac-Moody group over a finite field is a totally disconnected, locally compact group, which may be thought of as an “infinite-dimensional Lie group”. An example is $G = SL(n, K)$ with K the field of formal Laurent series over a finite field. We study uniform and nonuniform lattices in such G of rank 2, where the associated Bruhat-Tits building is a tree. We use finite group theory and the dynamics of the group action on the tree and its boundary. This is joint work with Inna (Korchagina) Capdeboscq

16:00 - 17:00 Parallel session 2

Marc Bourdon

Title: Quasiconformal geometry and Coxeter groups (joint work with Bruce Kleiner)

Abstract: Every hyperbolic group has a canonical action on its boundary at infinity; with respect to any visual metric, this action is by uniformly quasi-Moebius homeomorphisms. This structure has a central role in the proofs of Mostow’s rigidity theorem and numerous other results in the same vein, which are based on the analytic theory of quasiconformal homeomorphisms of the boundary. With the aim of extending these rigidity results to a larger class of hyperbolic groups, we study a quasi-Moebius invariant property of the boundary, called the Combinatorial Loewner Property.

16:00 - 17:00 Parallel session 2

Indira Chatterji

Title: Median spaces and property (T)

Abstract: We will explain the notion of median spaces as well as the notion of property (T), and show that a group has property (T) if and

only if any isometric action on a median space has a bounded orbit. This is joint work with Drutu and Haglund.

Tuesday, 3rd August, 2010

09:00 - 10:00 Plenary lecture 3

Etienne Ghys

Title: On cutting cloth, according to Chebyshev

Abstract: On August 28th, 1878, Chebyshev gave a talk with the same title in Paris (seventh meeting of the Association for the Progress of Science). Given a shape, like an elbow for example, how should we cut a piece of cloth to cover it, avoiding folds... Chebyshev gave several concrete examples. I would like to revisit this kind of questions and I'll discuss in particular a good way to cloth a sphere or a large domain in the Poincaré disc.

10:30 - 11:30 Plenary lecture 4

Marc Burger

Title: Higher Teichmueller Spaces: from $SL(2, R)$ to other Lie Groups

Abstract: In this talk we present various viewpoints on classical Teichmueller theory and show how they lead to different ways of studying the space of representations of the fundamental group of a surface into a Lie group. This will lead to the notion of positive representation in the case of a real split simple Lie group, (work of Hitchin, Fock-Goncharov, Labourie, Guichard, Wienhard), to maximal, and more generally, causal representations, in the case where the group is of hermitian type. We will particularly insist on causal representations, and present some recent works concerning their geometric significance (work with Ben Simon, Hartnick, Iozzi, Wienhard)

12:00 - 13:00 Plenary lecture 5

Tom Farrell

Title: Space of negatively curved metrics; bundles with negatively curved fibers

Abstract: This is a report on joint work with Pedro Ontaneda. Let R, G and T denote the spaces of all negatively curved Riemannian metrics, geometries and marked geometries (respectively) on an n -dimensional closed smooth manifold M ; G and T are quotient spaces of R where isometric and marked isometric metrics (respectively) are identified. We focus attention on the case where n is large instead of the classical setting $n = 2$. And obtain results on the homotopy

and homology of R, G and T ; e.g. R has infinitely many components when $n > 9$. And if M supports a real hyperbolic metric (and $n > 9$) then G is also disconnected for sufficiently large finite sheeted covers of M . These results relate to studying bundles equipped with negatively curved fibers.

14:30 - 15:30 Parallel session 3

Uri Shapira

Title: Dynamics and continued fractions.

Abstract: We will present some recent results regarding the continued fraction expansion of certain sequences of numbers. In particular, we will focus on the following result (joint with Menny Aka): Let x be a quadratic irrational and p a prime. Then the statistics of the period of the continued fraction expansion of $p^n x$ converges to the "right" statistics; i.e. to the one given by the Gauss measure.

14:30 - 15:30 Parallel session 3

Mahan Mj

Title: On Discreteness of Commensurators

Abstract: We begin by showing that commensurators of Zariski dense subgroups of isometry groups of symmetric spaces of non-compact type are discrete provided that the limit set on the Furstenberg boundary is not invariant under the action of a (virtual) simple factor. In particular for rank one or simple Lie groups, Zariski dense subgroups with non-empty domain of discontinuity have discrete commensurators.

This generalizes a Theorem of Greenberg for Kleinian groups. We then prove that for all finitely generated, Zariski dense, infinite covolume discrete subgroups of $\text{Isom}(\mathbf{H}^3)$, commensurators are discrete. Together these prove discreteness of commensurators for all known examples of finitely generated, Zariski dense, infinite covolume discrete subgroups of $\text{Isom}(X)$ for X an irreducible symmetric space of non-compact type.

16:00 - 17:00 Parallel session 4

Jayadev Athreya

Title: Cusp Excursions on Parameter Spaces

Abstract: We describe an axiomatic approach to studying statistical behavior of cusp excursions for diagonal and unipotent flows (arising from an ambient Lie group action) on parameter spaces. Our approach applies in particular to the space of unimodular lattices and the space

of quadratic differentials. Some of the results described are joint work with Prof. G. Margulis.

16:00 - 17:00 Parallel session 4

Jens Heber

Title: Cocompact isometry groups in nonpositive curvature

Wednesday, 4th August, 2010

09:00 - 10:00 Plenary lecture 6

Mladen Bestvina

Title: Groups acting on quasi-trees

Abstract: If G is the fundamental group of a closed non-positively curved rank 1 manifold, then G admits many actions on quasi-trees (spaces quasi-isometric to trees). In fact, groups like mapping class groups and $Out(F_n)$, which are not non-positively curved but exhibit rank 1 behavior, also admit actions on quasi-trees. I will present a construction of these actions (joint work with Bromberg and Fujiwara). If time permits, I will describe an application, that mapping class groups have finite asymptotic dimension.

10:30 - 11:30 Plenary lecture 7

Yves Benoist

Title: Invariant subsets of finite volume homogeneous spaces

Abstract: Let X be a finite volume quotient of a connected semisimple Lie group G with no compact factor and H be a Zariski dense subgroup of G . We prove that every H -orbit closure is an orbit under a larger group. For that we classify the probability measures on X which are stationary under a probability measure on G whose support is compact and spans H : the ergodic ones are homogeneous under a larger group. This is joint work with Jean-Francois Quint.

12:00 - 13:00 Parallel session 5

Gerhard Knieper

Title: New results on harmonic manifolds

Abstract: The Lichnerowicz conjecture asserts that all harmonic manifolds are either flat or locally symmetric spaces of rank 1. This conjecture has been proved by Z. Szabó for harmonic manifolds with compact universal cover. Furthermore, the conjecture was obtained by Besson,

Courtois and Gallot for compact manifolds of strictly negative curvature as an application of their entropy rigidity theorem in combination with the rigidity theorems by Benoist, Foulon and Labourie on stable and unstable foliations.

On the other hand, E. Damek and F. Ricci provided examples showing that in the noncompact case the conjecture is wrong. However, such manifolds do not admit a compact quotient.

In this talk we will show that for all simply connected, noncompact and nonflat harmonic spaces X the following properties are equivalent:

- X has rank 1
- X has purely exponential volume growth
- X is Gromov hyperbolic
- The geodesic flow on X is Anosov with respect to the Sasaki metric

Furthermore we obtain, that no focal points imply the above properties. Combining those results with the above mentioned rigidity theorems shows that the Lichnerowicz conjecture is true for all compact harmonic manifolds without focal points or with Gromov hyperbolic fundamental groups. There is some evidence to believe that all non compact harmonic manifolds have no focal points.

12:00 - 13:00 Parallel session 5

Igor Mineyev

Title: Systems of complexes and the Hanna Neumann Conjecture.

Abstract: The Hanna Neumann Conjecture is a question about ranks of subgroups in a free group Γ ; it has been open for more than 50 years. In order to generalize the statement of the Strengthened Hanna Neumann Conjecture (SHNC) we use cell complexes and the Murray-von Neumann dimension of Hilbert modules. We define systems of cell complexes which are certain multiple pull-back diagrams and observe that SHNC can be restated in terms of ℓ^2 -Betti numbers for such systems consisting of graphs.

The next interesting case is systems consisting of hyperbolic surfaces. We state an analog of SHNC for this situation; it is a (conjectured) inequality in terms of the first ℓ^2 -Betti numbers of subgroups of a surface group. This conjecture for surfaces implies the original SHNC. This surface case is of particular interest because it gives still another, geometric, restatement on SHNC: in terms of the areas of surfaces.

We provide several approaches to proving SHNC and its generalizations. They use Hilbert modules and Linnell's result that free groups satisfy the Atiyah conjecture.

Some questions will be posed. For example, we would like to know whether the geometry of surfaces can be directly used to prove the conjecture. Another question that might be appropriate for a conference

with "dynamics" in its title: Can one obtain the inequality in SHNC using Γ -actions on probability spaces and Gaboriau's definition of ℓ^2 -Betti numbers for equivalence relations?

14:30 - 15:30 Parallel session 6

Enrico Leuzinger

Title: The asymptotic Schottky problem

Abstract: Let M_g denote the moduli space of compact Riemann surfaces of genus g and let A_g be the moduli space of principally polarized abelian varieties of dimension g . The map $J : M_g \rightarrow A_g$ which associates to a Riemann surface its Jacobian is injective and the image $J_g := J(M_g)$ is contained in a proper subvariety of A_g when $g \geq 4$. The classical and longstudied Schottky problem is to characterize the Jacobian locus J_g in A_g . In the talk we address a large scale version of this problem posed by B. Farb: What does J_g look like "from far away", or how dense is J_g in the sense of coarse geometry?

14:30 - 15:30 Parallel session 6

Igor Belegradek

Title: Moduli spaces of metrics of nonnegative curvature

Abstract: There are analogies between open complete simply-connected manifolds of nonnegative and nonpositive curvature: in either case the structure of flats and the ideal boundary plays a prominent role, and methods of comparison geometry are fruitful. In the talk I will survey what is known on moduli spaces of metrics of nonnegative curvature on open manifolds, and relate it to cancellation phenomena in topology.

16:00 - 17:00 PUBLIC LECTURE

Speaker: Patrick Eberlein

Title: Ergodic behavior in Negative curvature

Thursday, 5th August, 2010

09:00 - 10:00 Plenary lecture 8

Francois Labourie

Title: An algebra of observables for cross ratios

Abstract: We define a Poisson Algebra called the swapping algebra using the intersection of curves in the disk. We interpret a subalgebra of the fraction swapping algebra – called the algebra of multifractions – as an algebra of functions on the space of cross ratios and thus as

an algebra of functions on the Hitchin component as well as on the space of $SL(n; R)$ -opers with trivial holonomy. We finally relate our Poisson structure to the Drinfel'd-Sokolov structure and to the Atiyah-Bott-Goldman symplectic structure for classical Teichmüller spaces and Hitchin components

10:30 - 11:30 Plenary lecture 9

Shahar Mozes

Title: Stationary measures, stiffness and equidistribution on the torus

Abstract: In a joint work with Jean Bourgain, Alex Furman and Elon Lindenstrauss we study stationary measures and equidistribution on the d -dimensional torus. Let ν be a probability measure supported on a (finite) subset of $SL_d(\mathbb{Z})$. We show that if the group generated by the support of ν is large enough, in particular if this group is Zariski dense in SL_d , for any irrational point x in the torus the probability measures $\nu^{*n} * \delta_x$ tend to the uniform measure on T^d . If in addition x is Diophantine generic, we show this convergence is exponentially fast.

12:00 - 13:00 Plenary lecture 10

Nimish Shah

Title: Counting points or circles in orbits of geometrically finite hyperbolic groups

Abstract: In this joint work with Hee Oh, we extend some of the earlier results about counting points on discrete orbits of lattices, to the case of counting on orbits of geometrically finite hyperbolic groups. We also consider the problem of counting circles in circle packings invariant under Kleinian groups.

14:30 - 15:30 Parallel session 7

Chris Connell

Title: Rigidity for Maps and Measures on Foliated Spaces

Abstract: I will present some entropy and volume rigidity statement for general foliated maps between compact foliated spaces in the sense of Besson, Courtois and Gallot. In particular, we establish an isentropic inequality with respect to a transverse (quasi-)invariant measure which is optimized when all leaves are locally symmetric. We will give some applications of this, and indicate how it relates to the entropy rigidity conjecture for higher rank spaces together with a classification of the harmonic measures. Respective portions of this work are joint with either Zhenyu Li or Matilde Martinez.

14:30 - 15:30 Parallel session 7

Viktor Schroeder

Title:

Abstract:

16:00 - 17:00 Parallel session 8

Dave Constantine

Title: Rank-rigidity and frame flow in non-positive curvature

Abstract: I will discuss how a condition on curvature, higher hyperbolic rank, and study of the dynamics of the frame flow for a non-positively curved, rank one manifold can be combined to prove that a manifold has constant negative curvature. The result holds when the manifold is of odd dimension, or is of even dimension and has sufficiently pinched curvature. I will mention a few other questions related to the hyperbolic rank condition and what sort of information it and frame-flow dynamics can give us about the curvature.

16:00 - 17:00 Parallel session 8

Frederic Paulin

Title: Equidistribution, counting and arithmetic applications in hyperbolic manifolds

Abstract: Let M be a finite volume hyperbolic manifold. We show the equidistribution in M of the equidistant hypersurfaces to a finite volume totally geodesic submanifold C . We prove a precise asymptotic on the number of geodesic arcs of lengths at most t , that are perpendicular to C and to the boundary of a cuspidal neighbourhood of M . We deduce from it counting results for quadratic irrationals over \mathbb{Q} or over imaginary quadratic extensions of \mathbb{Q} , in given orbits of congruence subgroups of the modular groups, as well as counting results for the number of representations of integers by integral indefinite binary Hermitian forms. This is joint work with Jouni Parkkonen.

Friday, 6th August, 2010

09:00 - 10:00 Plenary lecture 11

Keith Burns

Title: Ergodicity of the Weil-Petersson geodesic flow

Abstract: I will talk about recent joint work with Howie Masur and

Amie Wilkinson. We have shown that the geodesic flow for the Weil Petersson metric on the moduli space of a Riemann surface is ergodic.

12:00 - 13:00 Parallel session 9

Peter Linnell

Title: Approximating Betti numbers over arbitrary fields

Abstract: Back in 1994, Lueck proved some beautiful theorems for approximating L^2 -Betti numbers in the case the coefficient field is the rational numbers. This has since been extended to the case where the coefficient field is the algebraic numbers. In this talk, extensions of this result to arbitrary fields, or even arbitrary skew fields, will be discussed. Among the classes of groups considered will be amenable groups and pro- p groups. Applications to ultraproducts and the Atiyah conjecture over \mathbb{C} will also be discussed.

This is joint work with Wolfgang Lueck, Roman Sauer and Thomas Schick.

12:00 - 13:00 Parallel session 9

Darren Long

Title: Small subgroups of $SL(3, \mathbb{Z})$

Abstract: The finitely generated infinite index subgroups of $SL(3, \mathbb{Z})$ are somewhat mysterious. We exhibit some interesting examples and nonexamples, including infinite families of Zariski dense surface groups.

14:30 - 15:30 Parallel session 10

Alex Gorodnik

Title: Diophantine approximation and automorphic spectrum

Abstract: We discuss the problem of Diophantine approximation on algebraic varieties, which was considered by Waldschmidt in the setting of abelian varieties. We investigate this problem for varieties equipped with actions of semisimple groups. Using techniques from the theory of automorphic representations, we establish estimates on the exponents of Diophantine approximation that are optimal in a number of cases. It turns out that the quality of the exponents is very closely related to the Ramanujan conjecture and its generalisations.

14:30 - 15:30 Parallel session 10

Jean-Francois Lafont

Title: 4-dimensional locally CAT(0)-manifold with no Riemannian smoothing

Abstract: I will describe the construction of smooth 4-dimensional manifolds that support locally CAT(0) metrics, but which cannot support any Riemannian metrics of non-positive sectional curvature. This is joint work with Mike Davis and Tadeusz Januszkiewicz.

16:00 - 17:00 *OPEN PROBLEMS SESSION* Chaired by:
Mike Davis, Pedro Ontaneda

Saturday, 7th August, 2010

09:00 - 10:00 Plenary lecture 13
Gerard Besson

Title: Collapsing irreducible 3-manifolds with nontrivial fundamental group

Abstract: We shall describe the main ideas of an alternative approach for the last step in Perelmans proof of Thurstons Geometrization Conjecture. This is a joint work with L. Bessieres, M. Boileau, S. Maillot and J. Porti. Using two covering arguments we reduce the problem to Thurston's proof of his conjecture for Haken manifolds. One of the argument uses Gromov's simplicial volume.

10:30 - 11:30 Plenary lecture 14
Tadeusz Januskiewicz

Title: Complex links in Kahler polyhedra

Abstract: Recently Dmitri Panov introduced and studied the concept of Kähler polyhedra. I will describe this and discuss its relation to nonpositive curvature. Briefly: Kähler polyhedron has both real links, which are piecewise spherical and complex links which are piecewise complex projective.

One can express CAT0 property of a Kähler polyhedron both in terms of real links (CAT1) and complex links (perhaps somewhat surprisingly CAT4). This gives many interesting examples, already in dimension 2, as described by Panov, and many promising higher dimensional possibilities.

12:00 - 12:30 Valedictory address by **Ravi Kulkarni**