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# Formalizing the notion of ‘innovation’ in an evolutionary model

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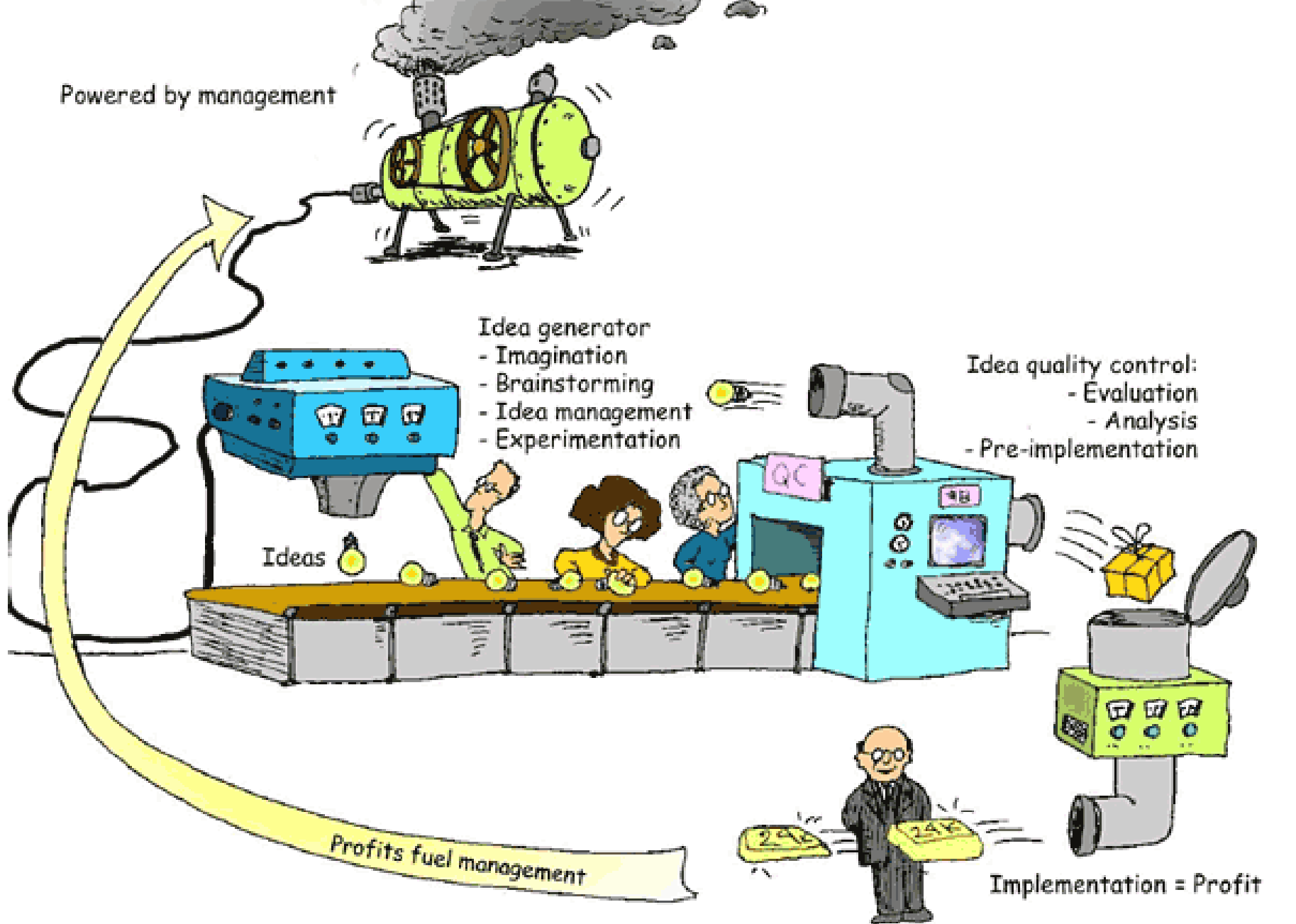
## References:

S. J. and Sandeep Krishna

Proc. Nat. Acad. Sci. (USA) 2002; Phys. Rev. E 2002

Handbook of Graphs and Networks (eds. Bornholdt and Schuster) 2003

Econophysics and Sociophysics (eds. Chakrabarti et al) 2006



## The Corporate Innovation Machine

# Innovation (everyday usage)

“Something new that brings about a change”

## Examples

- artifacts that humans build (wheel, steam engine, computer)
- processes (agriculture, manufacture of steel)
- the world of ideas (discovery of zero, law of gravitation)
- social organization (money, parliamentary democracy)

In biology, “evolutionary innovation”

- photosynthesis
- multicellularity
- eye

Birth of a star, origin of the earth are not considered innovations.  
“Innovation” seems to presuppose an evolutionary context.

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Innovation can have both constructive and destructive consequences.

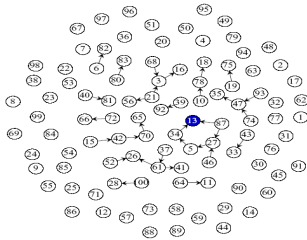
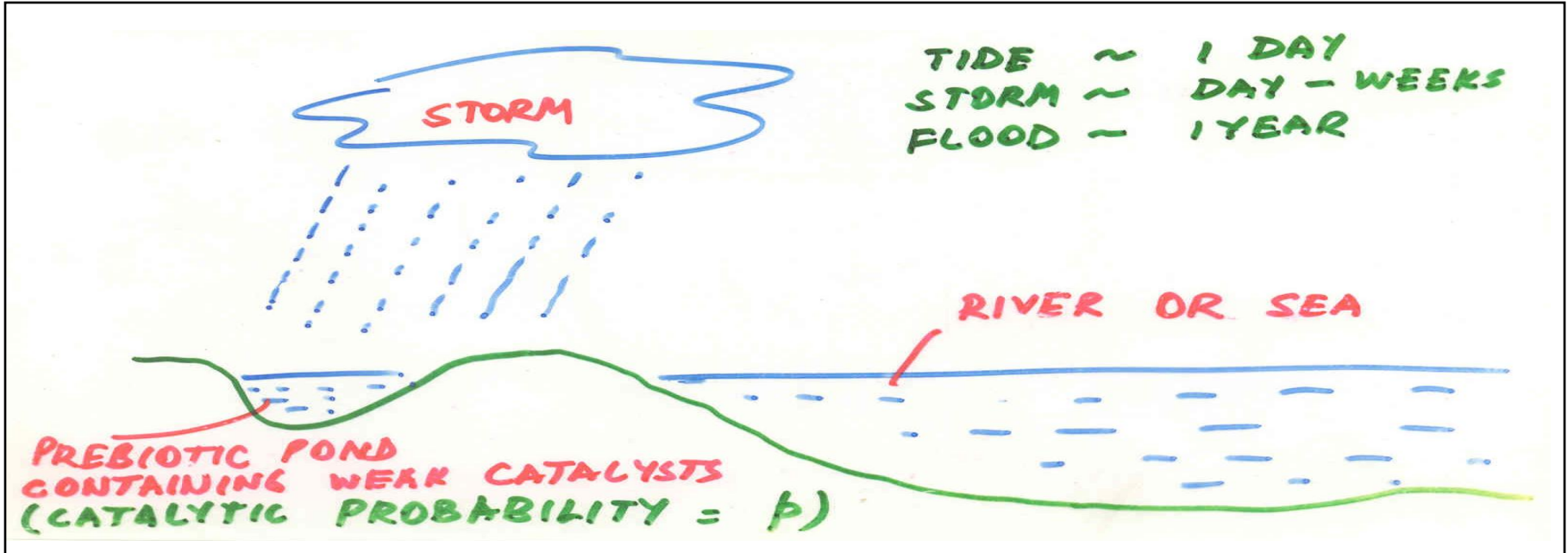
- Automobile destroyed the horse drawn carriage industry.
- Aerobic organisms out competed anaerobic organisms.

Success of innovation depends on the context.

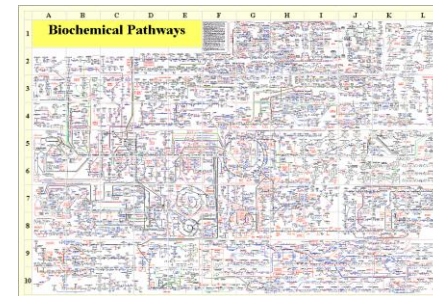
Successful innovation then changes the context.

Will describe a **mathematical model** that captures this two faced nature of innovation and its dynamical relationship with the context.

# The model (based on the origin of life problem -- emergence of pre-biotic chemical organization)

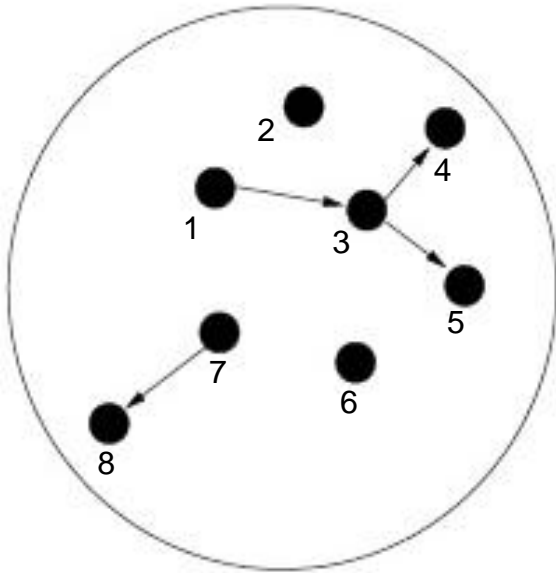


100-500 million years



# Variables of the model

## A graph of interacting molecular species



An arrow from node  $j$  to  $i$  implies that  $j$  is a catalyst for the production of  $i$ , and then

$$c_{ij} = 1$$

The absence of an arrow from  $j$  to  $i$  implies that

$$c_{ij} = 0$$

The  $s \times s$  matrix  $C = (c_{ij})$  is the adjacency matrix of the graph

$s$  is the number of molecular species

Each species  $i$  has a population  $y_i$  or a relative population  $x_i$

The variables  $x$  and  $C$  characterize the chemical organization in the pond and they change with time.

**Initialization:**  $C_{ij} = 1$  with probability  $p$ ,  
 $= 0$  with probability  $1-p$

$p$  is the “catalytic probability”

$m = ps =$  average connectivity

$x_i$  are chosen randomly

# Dynamical rules

1. Keeping  $C$  fixed, let the relative populations  $\mathbf{x}$  change with time according to

$$\frac{dx_i}{dt} = \sum_{j=1}^s c_{ij} x_j - x_i \sum_{j,k=1}^s c_{jk} x_k \quad (\text{catalytic dynamics})$$

$c_{ij} = 1$  if molecule  $j$  is a catalyst for the production of molecule  $i$   
 $= 0$  otherwise

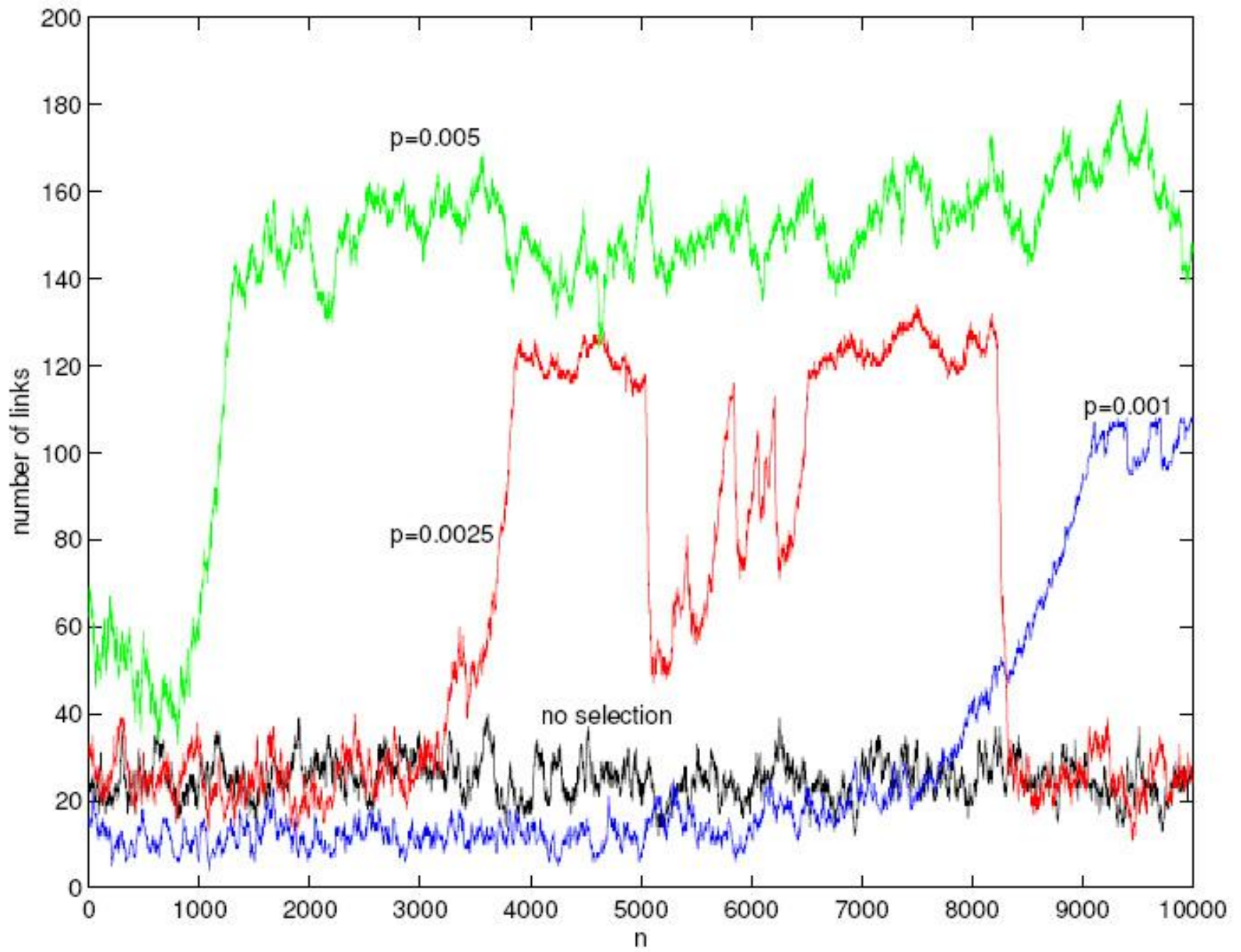
Let the populations reach a steady state (attractor is a fixed point, and is an eigenvector of  $C$  corresponding to its largest eigenvalue – Perron Frobenius eigenvalue)

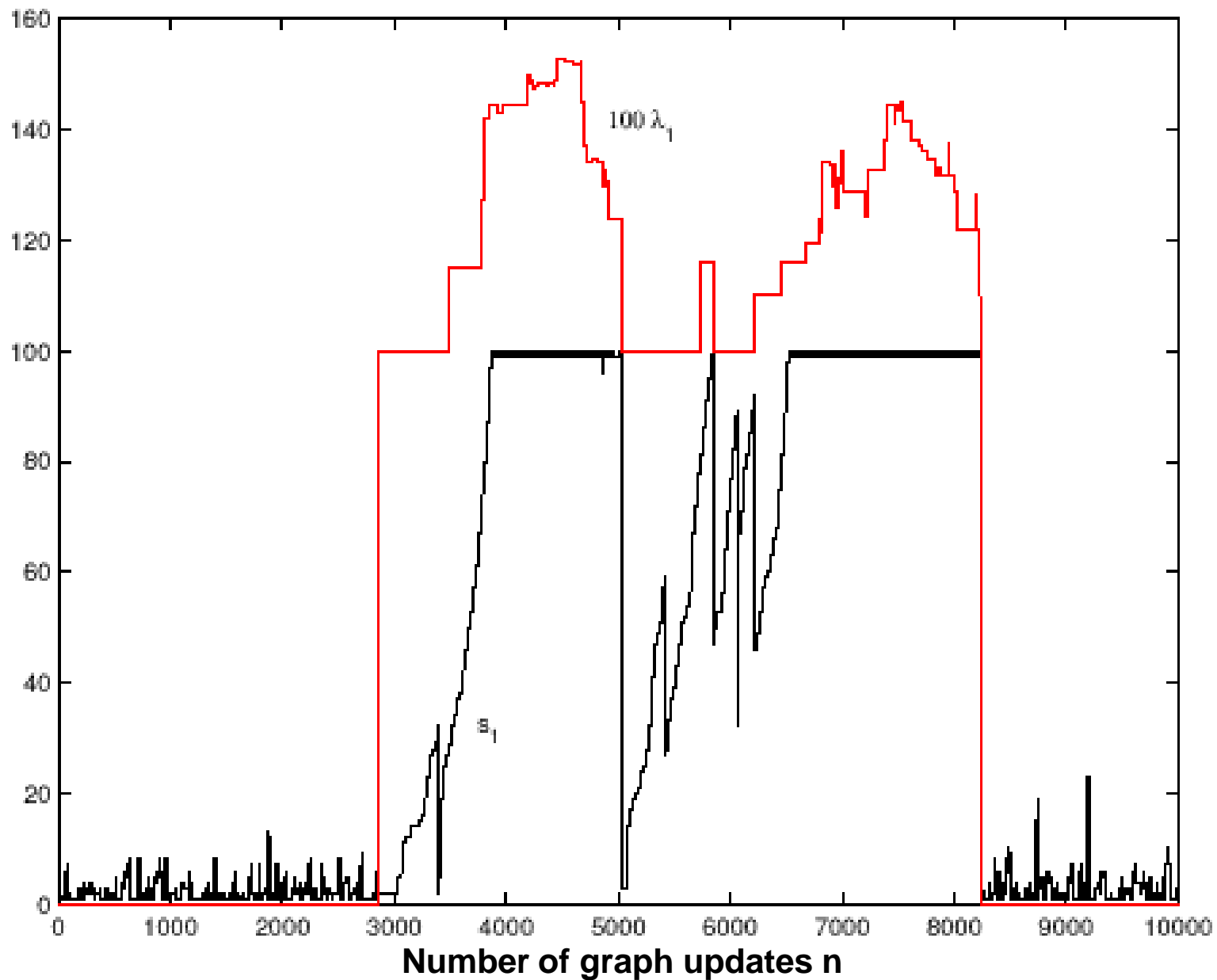
2. Now change  $C$

- (a) Remove the node with the least population along with all its links (selection; tide washes out the least populated molecular species)
- (b) Introduce a new node whose connections to the existing nodes are made randomly with prob  $p$  (introduction of novelty; tide brings in a new molecule)

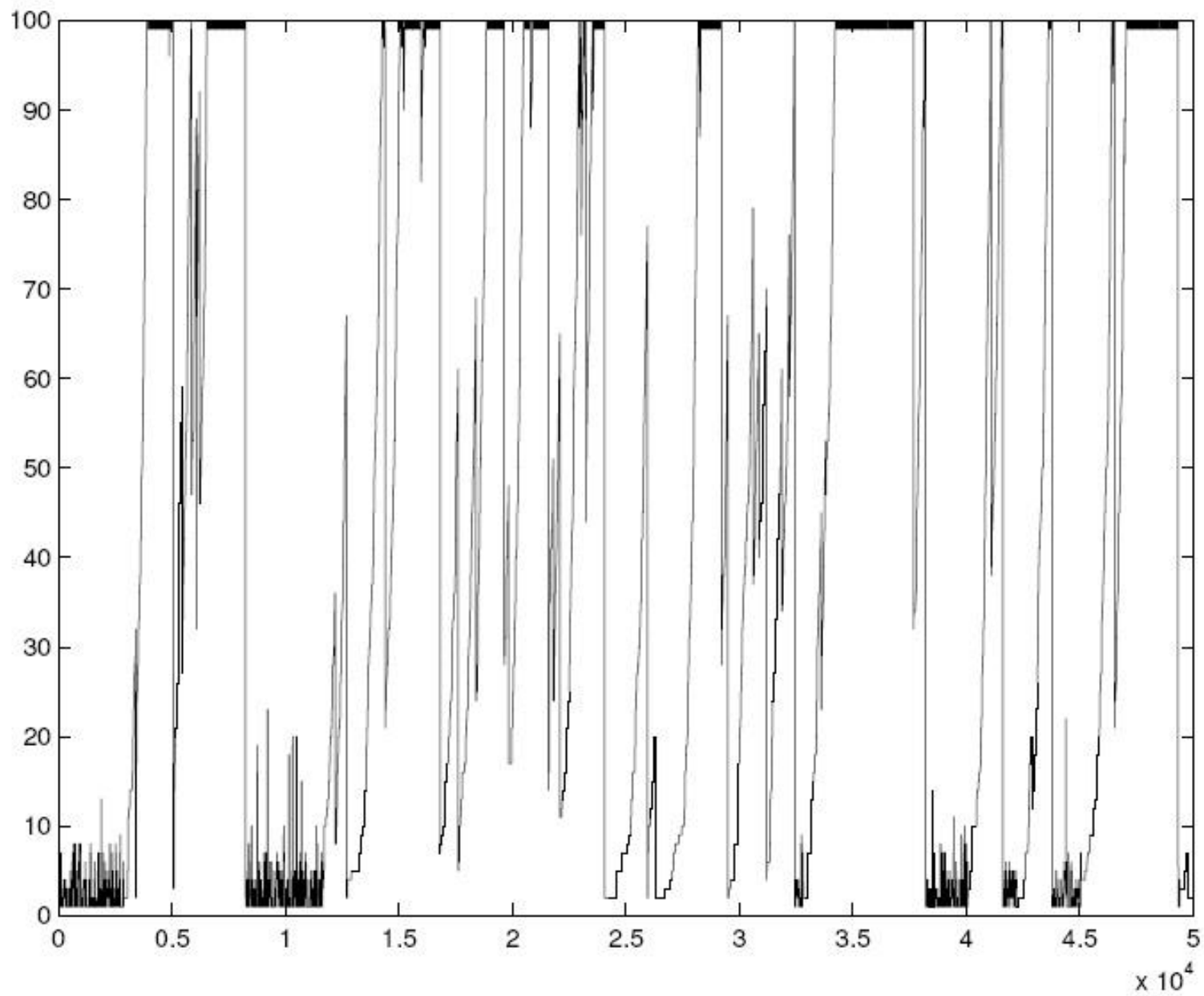
3. Iterate steps 1 and 2



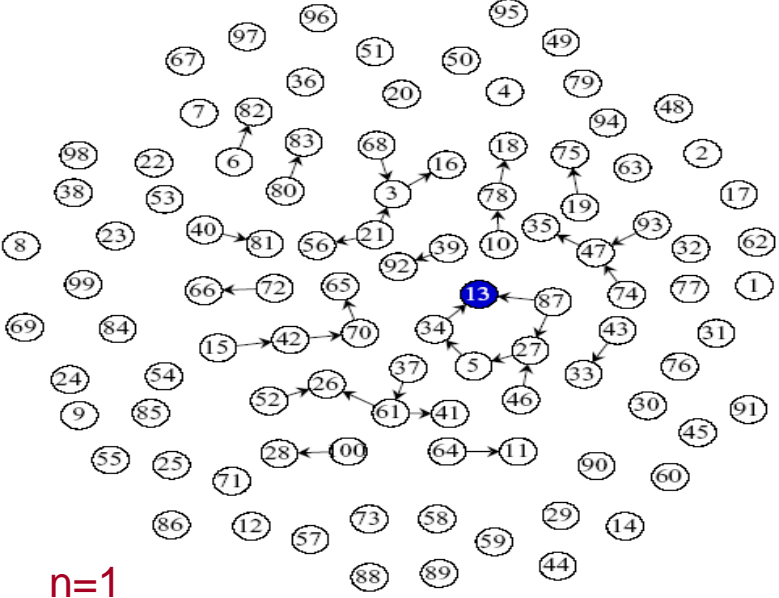




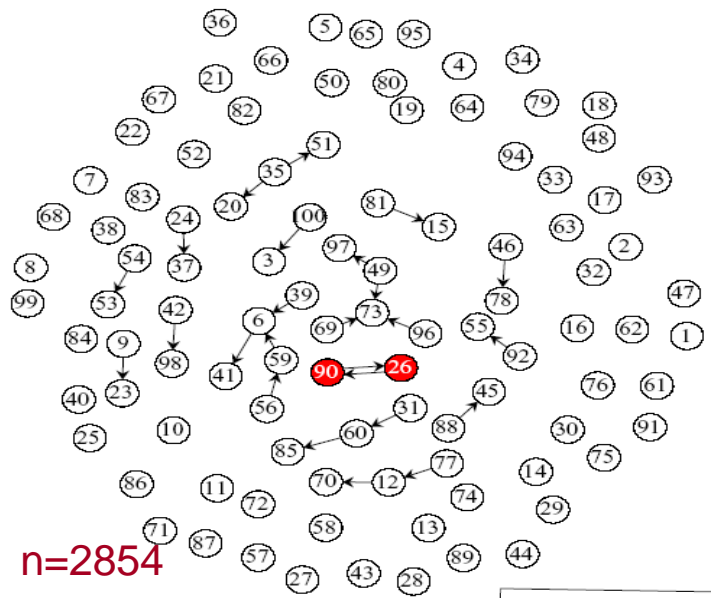
Black curve: Number of nodes with relative populations > 0  
Red curve: Perron-Frobenius eigenvalue of the graph



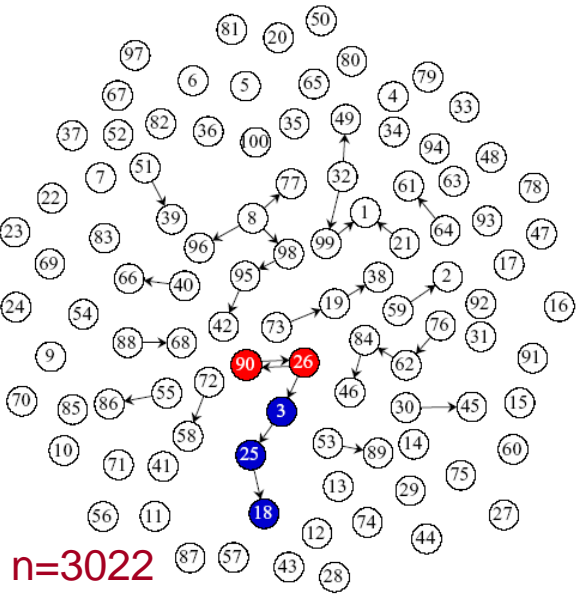
A small autocatalytic set (ACS) appears by chance. Its cooperative structure means that both its nodes do well (the ACS is a replicator), hence both survive. .



n=1

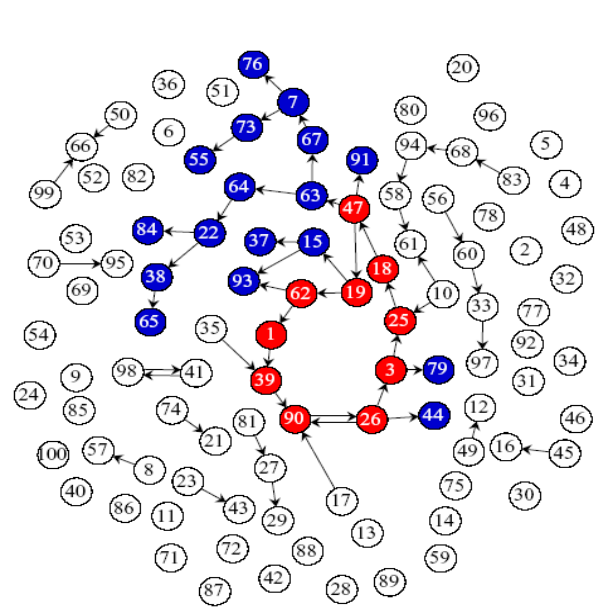


n=2854

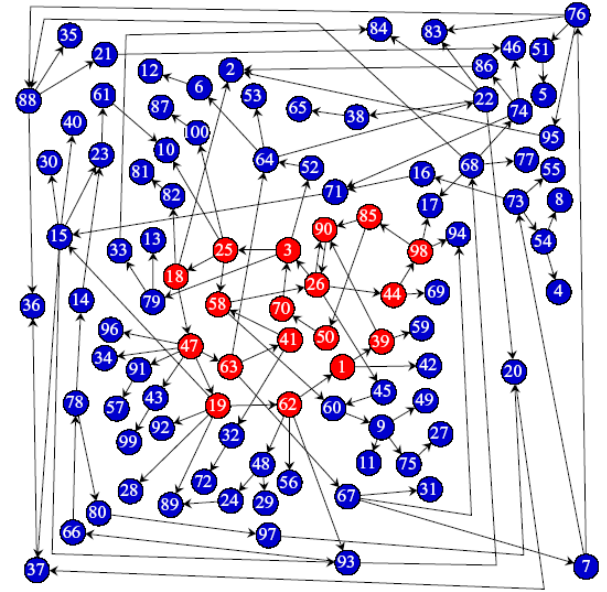


n=3022

The ACS is the seed from which growth of complexity and structure occurs



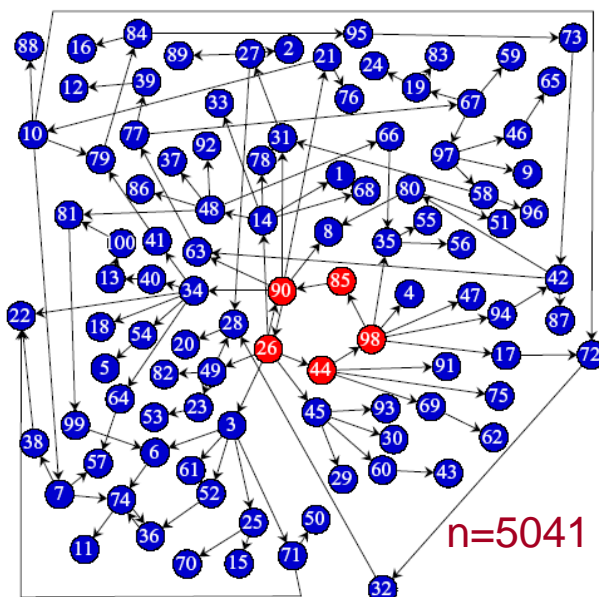
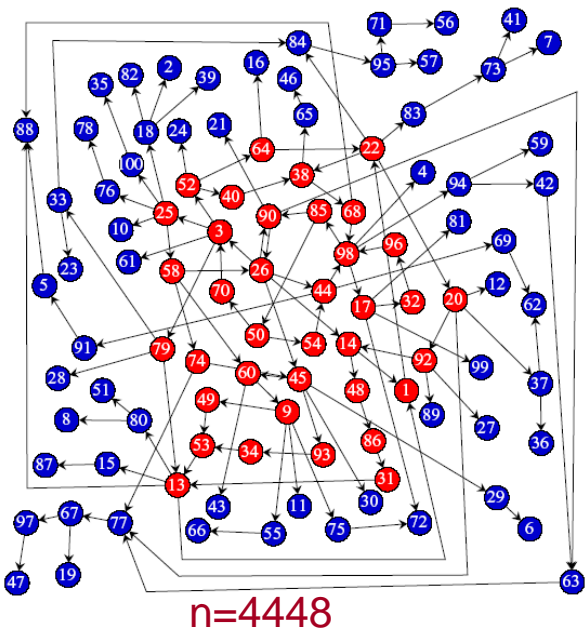
n=3489



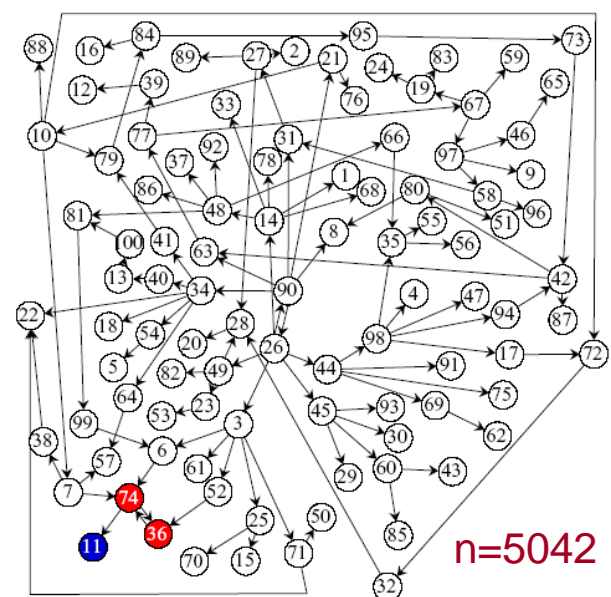
n=3880

ACS eventually takes over the whole pond

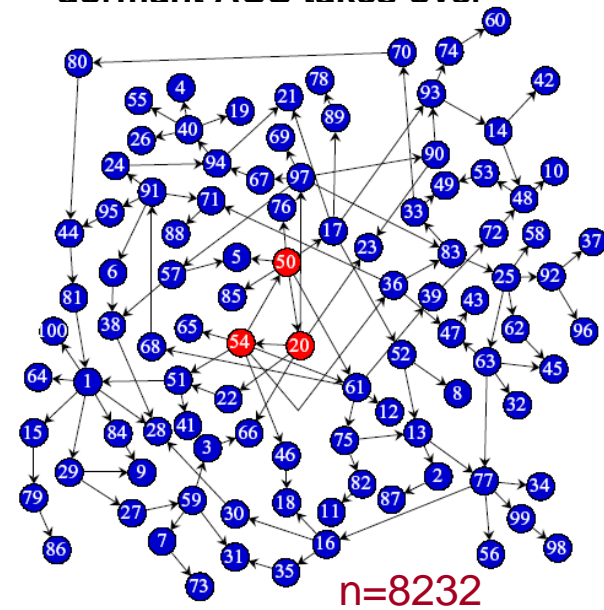
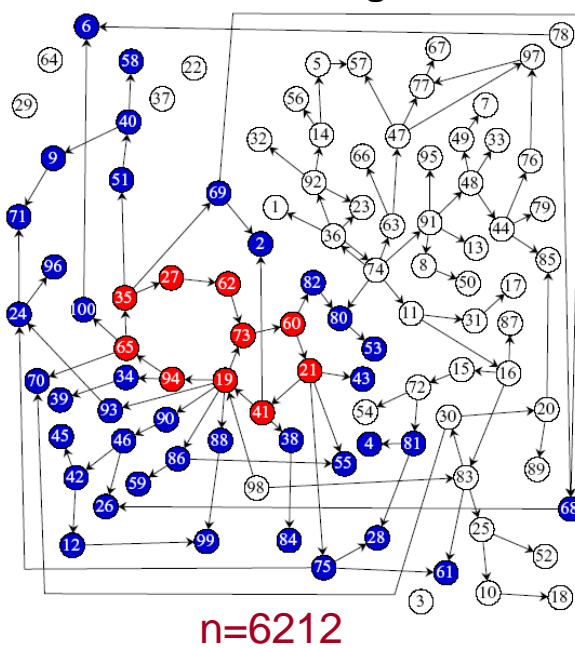
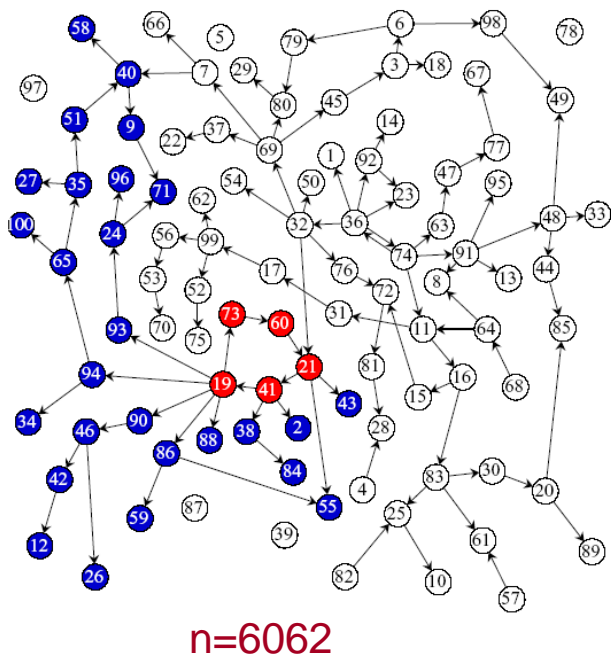
Evolution of a prebiotic chemical organization



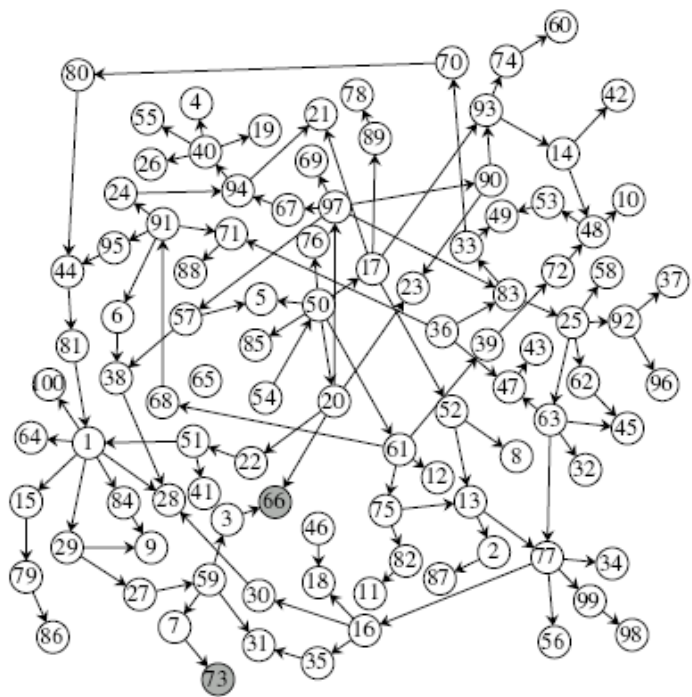
**Internal competition makes the core of the ACS fragile**



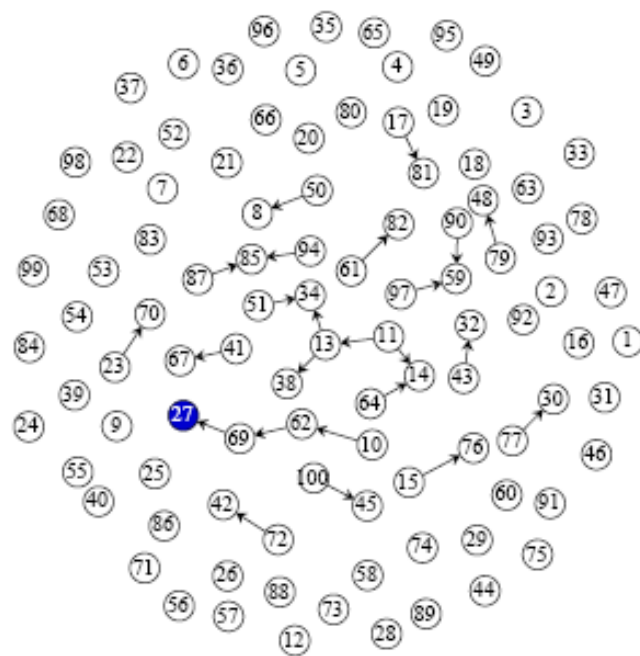
**The ACS crashes and a small dormant ACS takes over**



**Another fragile core state. Results in a complete crash. ACS is completely destroyed**



$n = 8233$



$n = 10000$

# Innovation

‘Novelty’ enters the system at one point in the dynamics: when the new node is brought in and its links with the existing nodes are chosen randomly

Not every new thing qualifies as an ‘innovation’

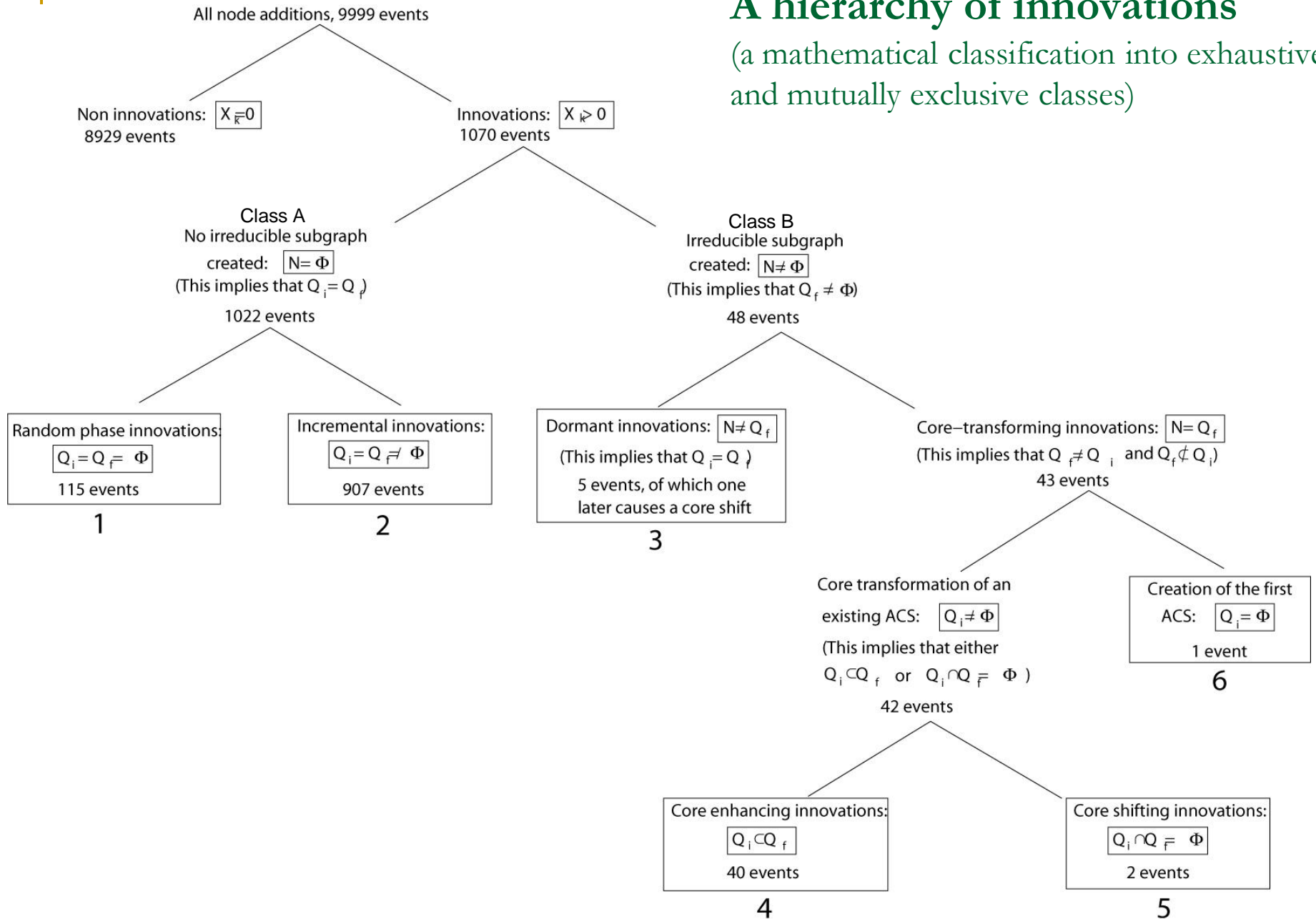
A node addition will be called an **innovation** if the new node

has a **nonzero relative population** in the next steady state

(There needs to be some ‘performance’ criterion for something to qualify as ‘innovation’)

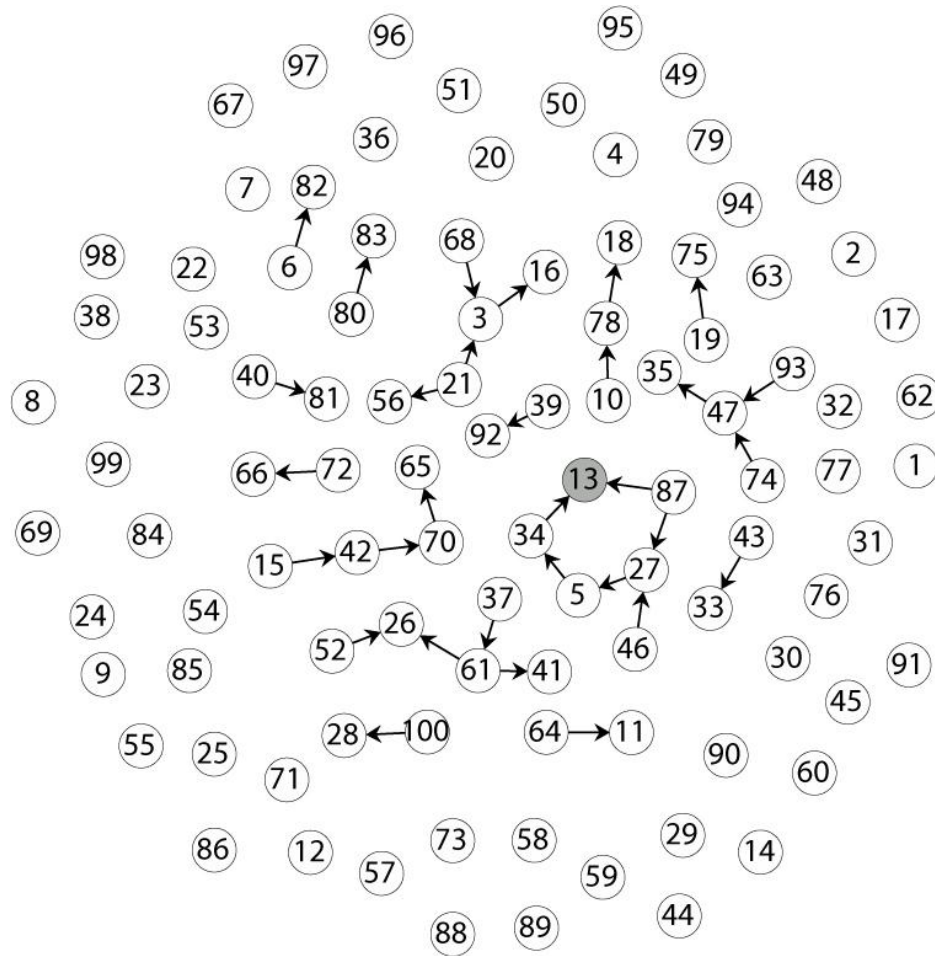
# A hierarchy of innovations

(a mathematical classification into exhaustive and mutually exclusive classes)



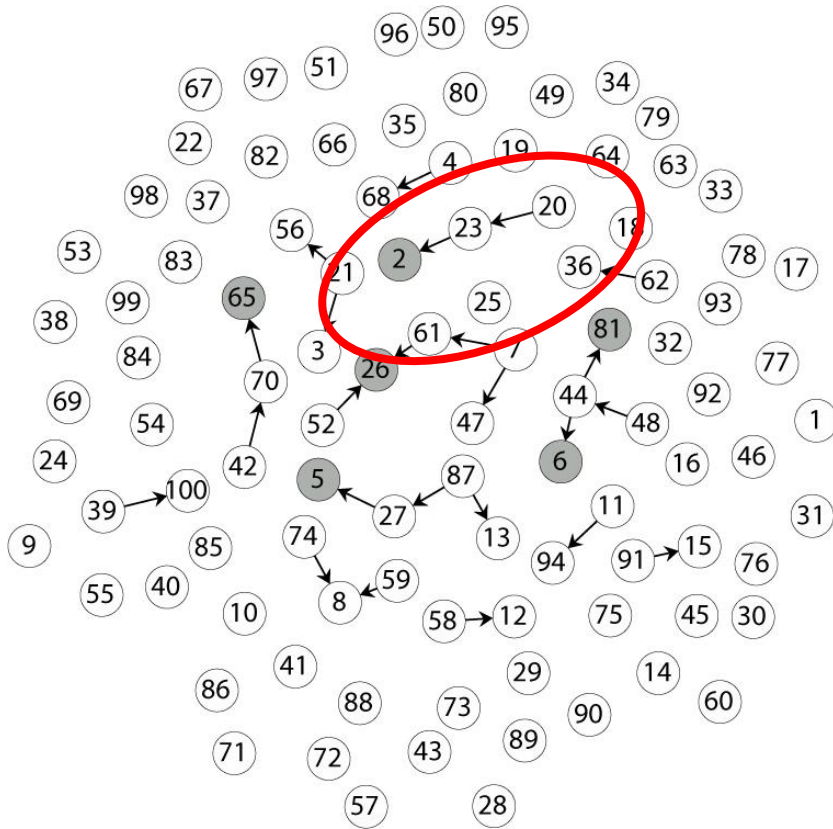


# n = 1

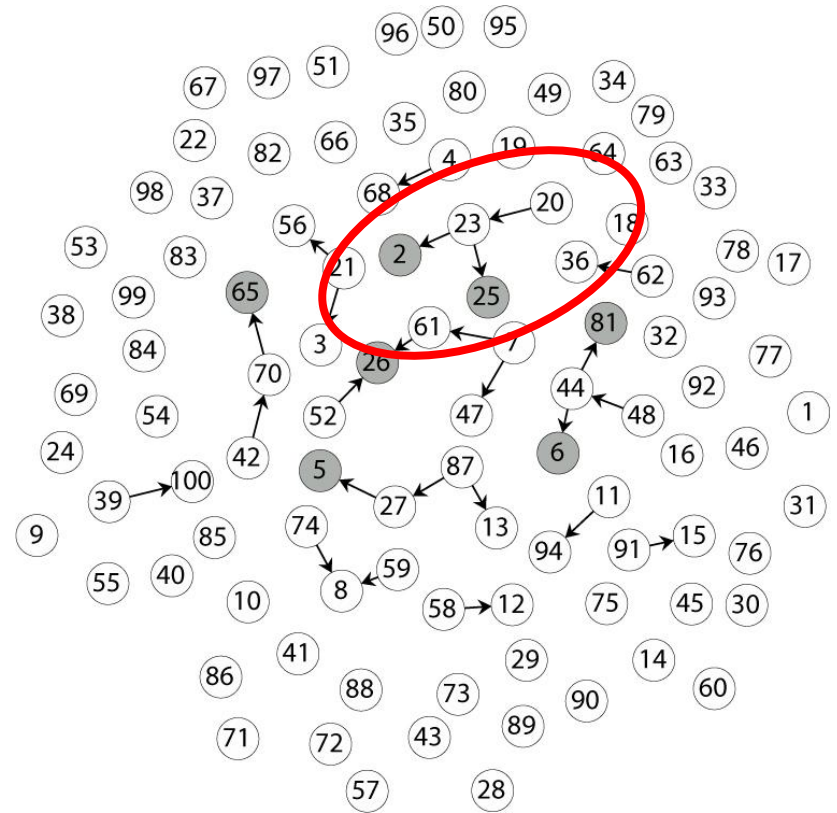


Initially, a random, sparse, graph

n = 78

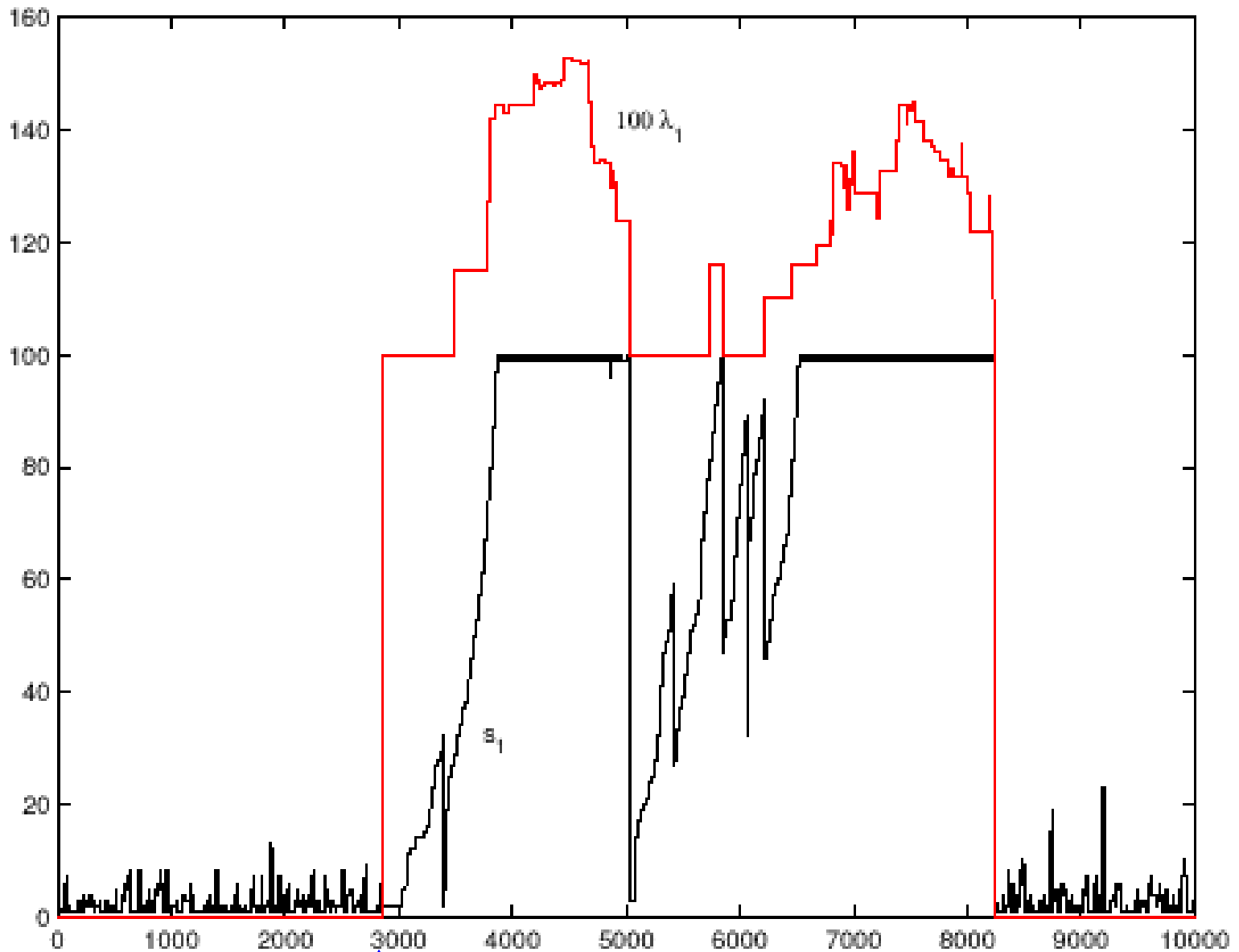


n = 79



A random phase innovation: Uncaring and unviable (shortlived) winners

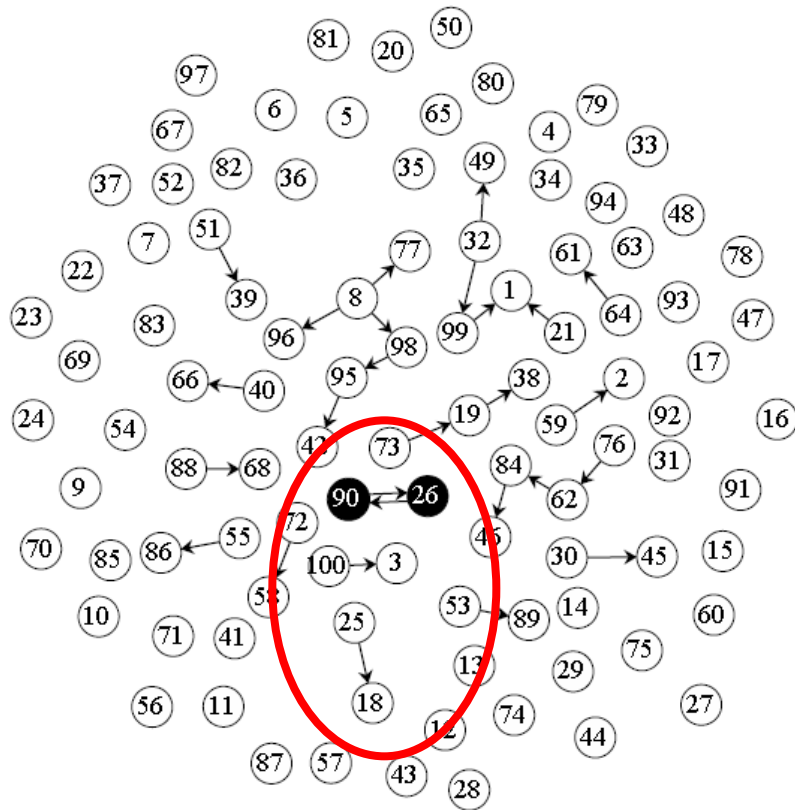




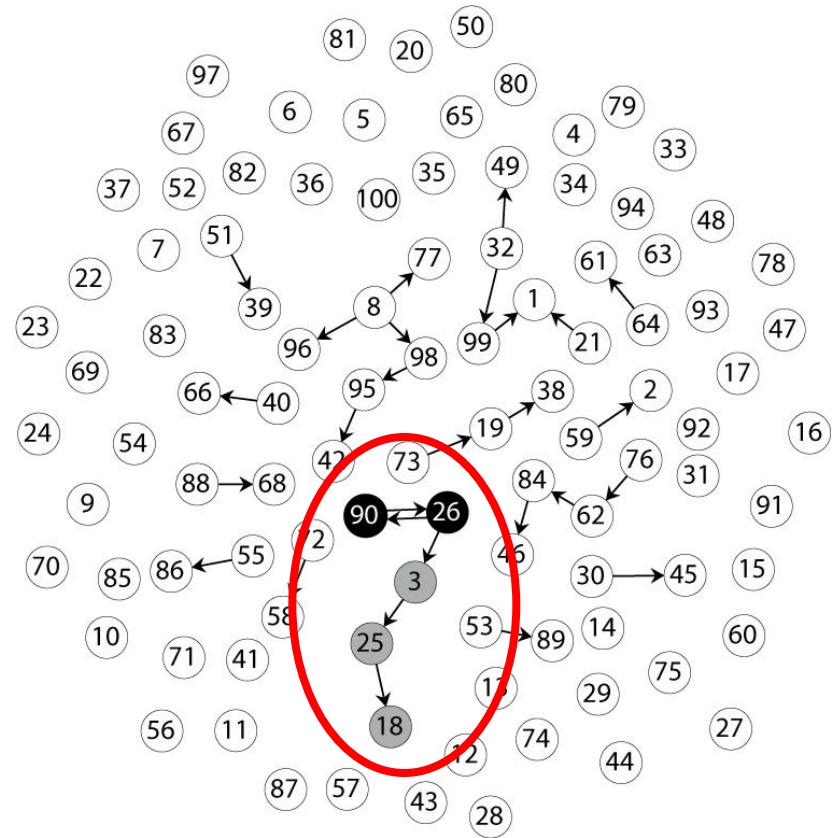
↑ Number of graph updates n

Black curve: Number of nodes with relative populations > 0  
Red curve: Perron-Frobenius eigenvalue of the graph

n = 3021

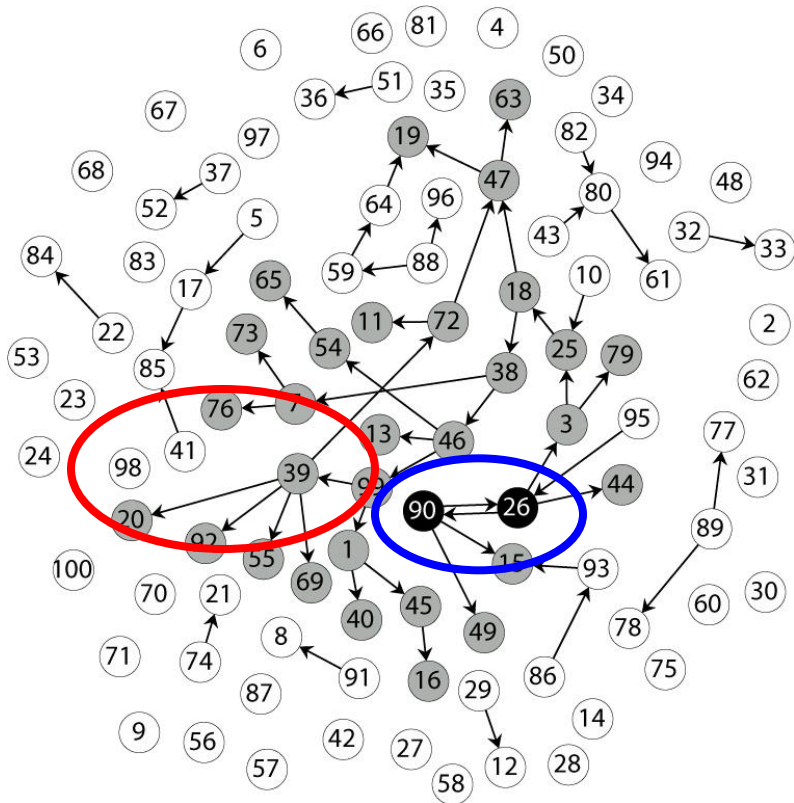


n = 3022

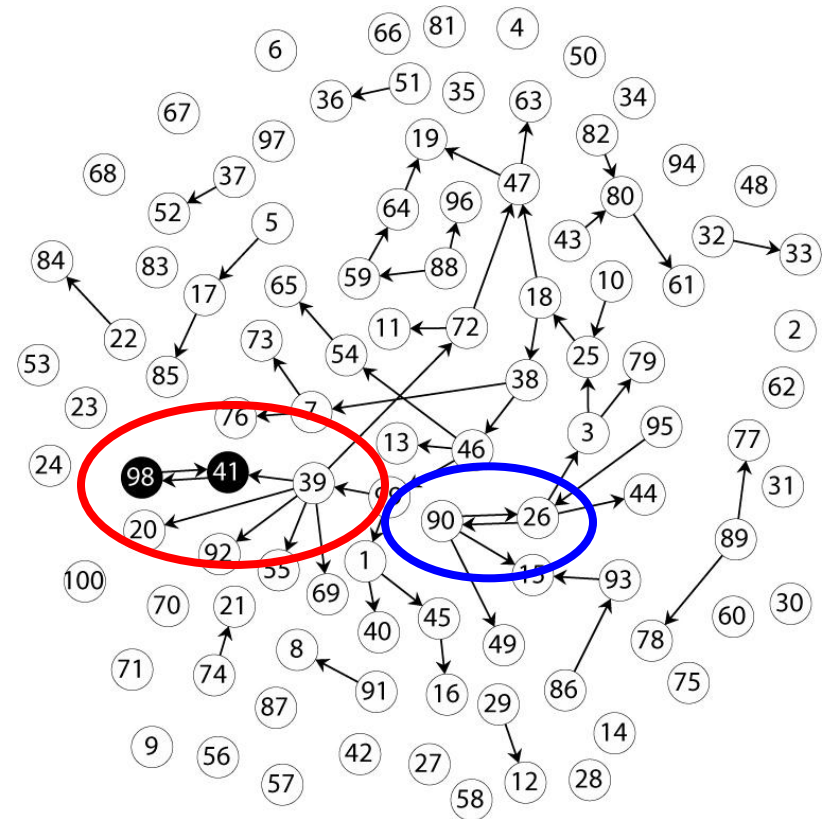


Expansion of the organization at its periphery: Incremental innovation

n = 3386

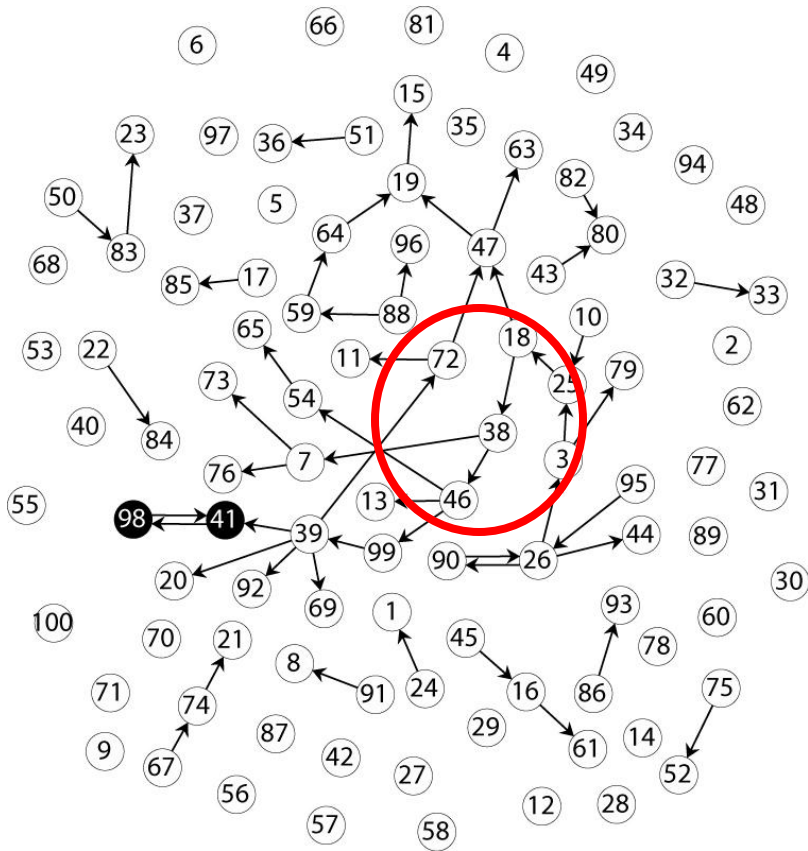


n = 3387

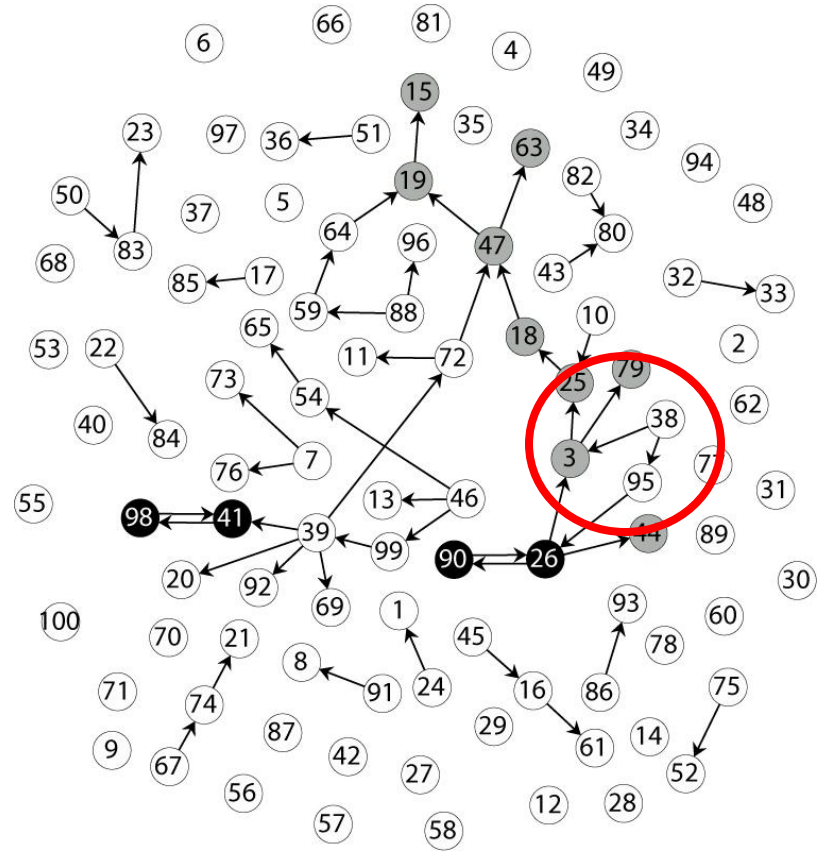


Core shift 1: Takeover by a new competitor

n = 3402



n = 3403

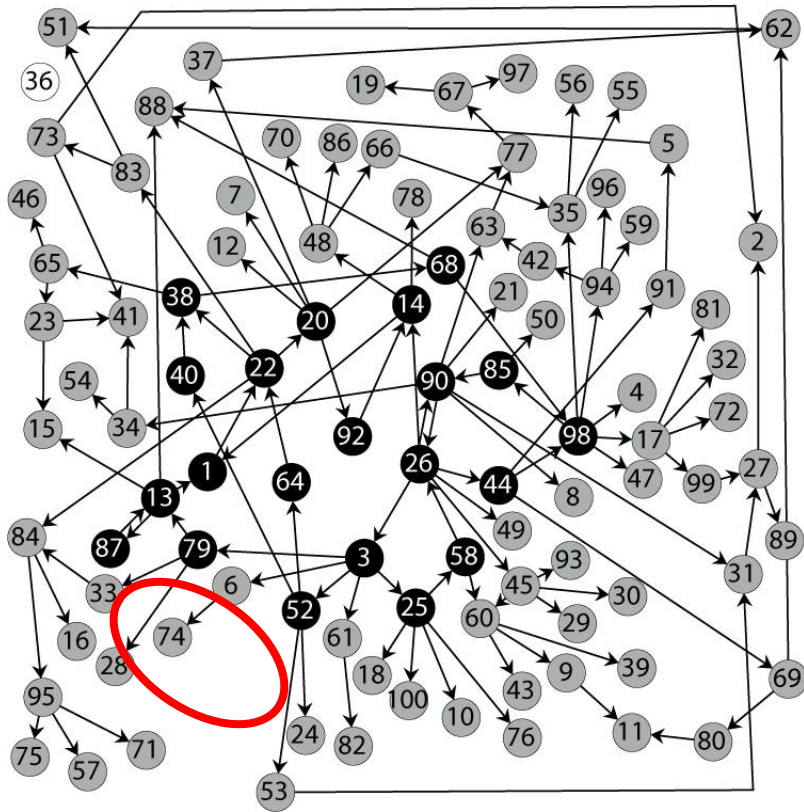


Revival of the old ACS

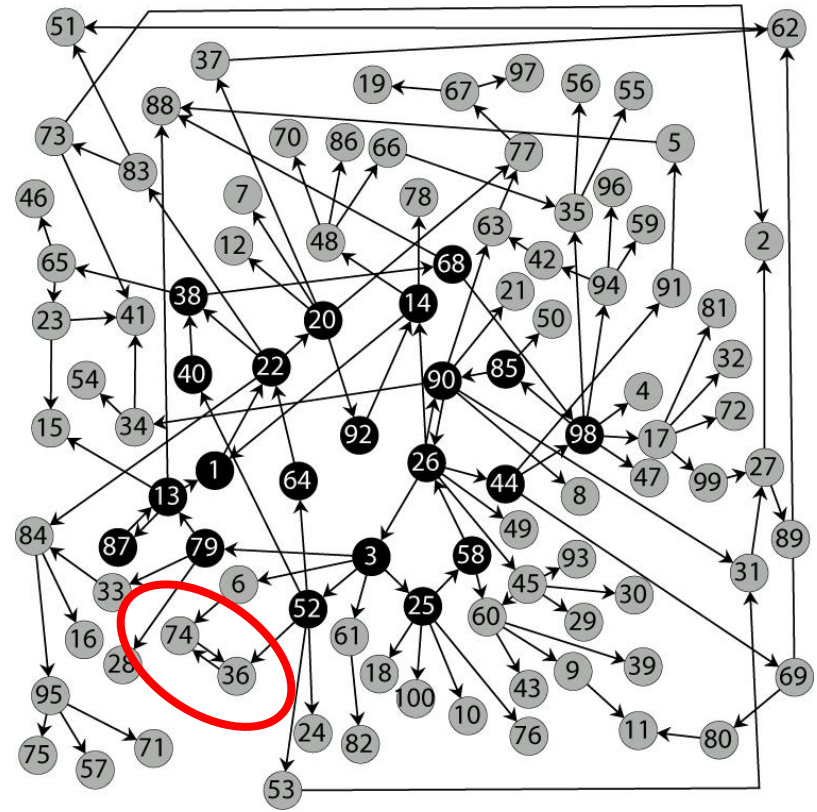




n = 4695

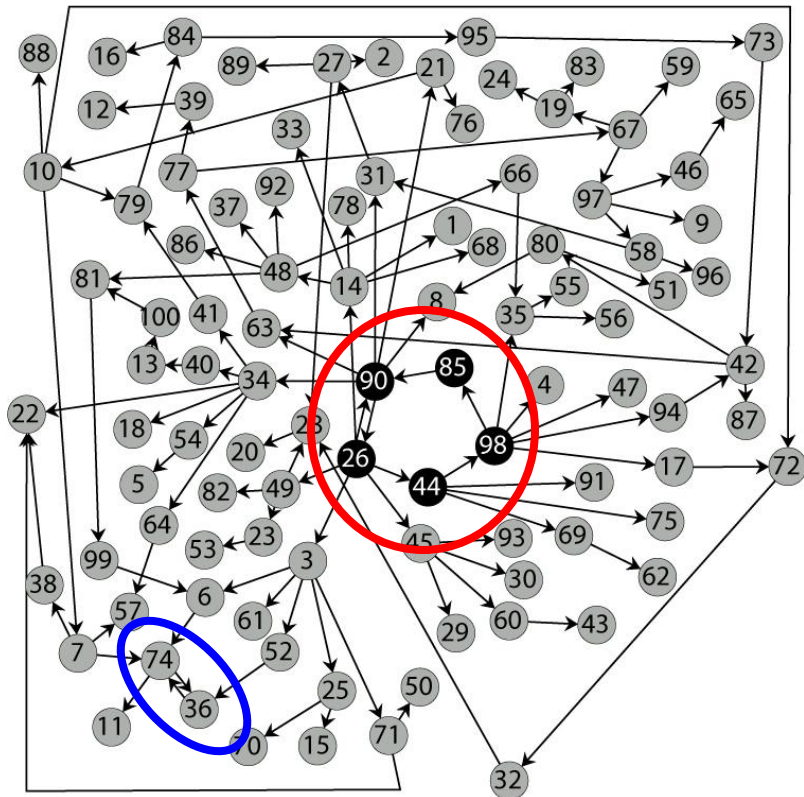


n = 4696

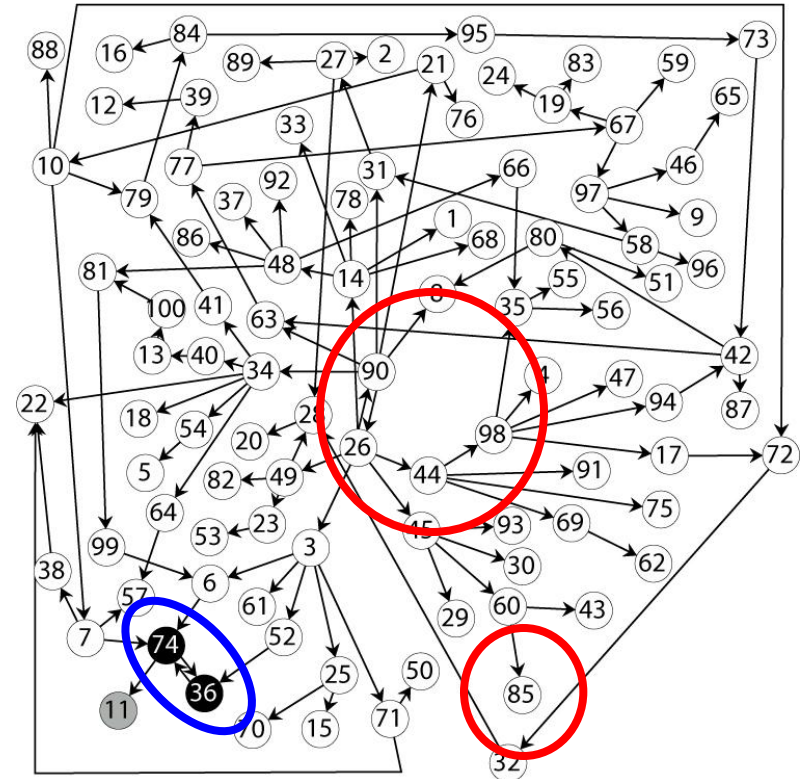


A small ACS appears in the periphery  
(dormant innovation)

n = 5041

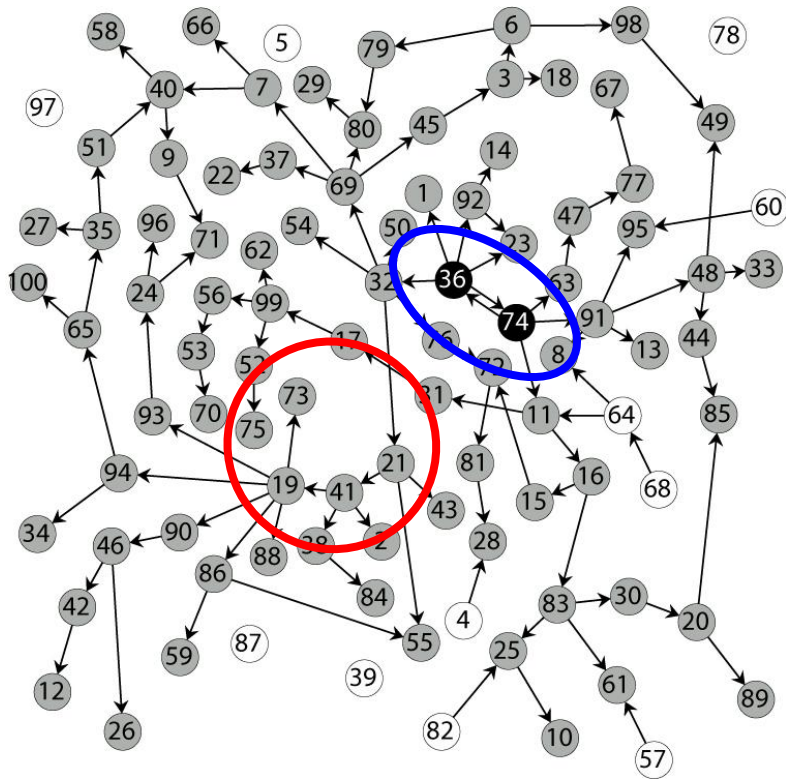


n = 5042

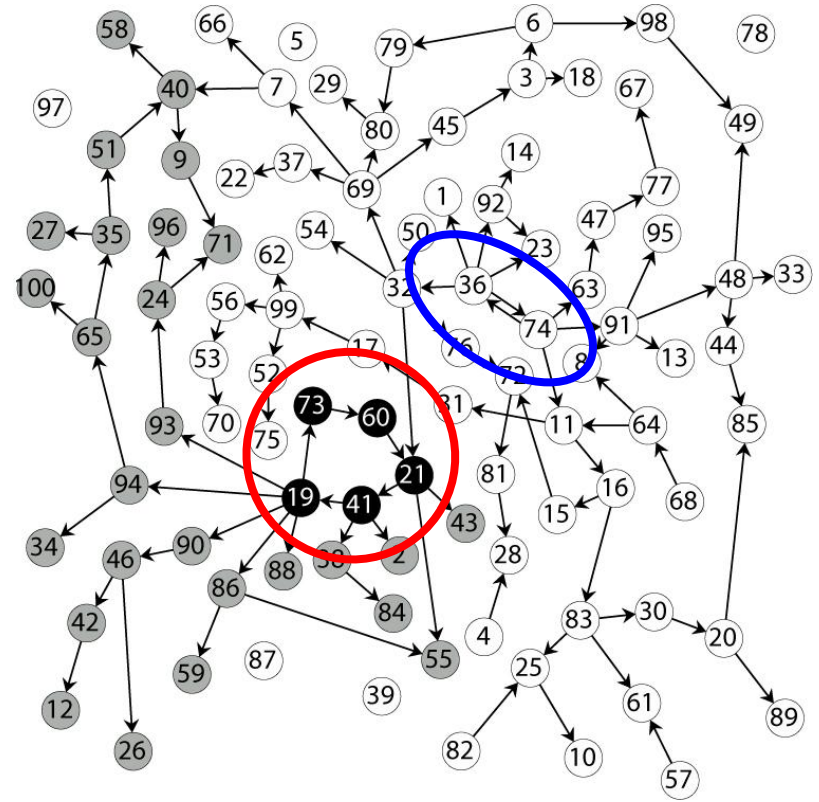


Core shift 2: Takeover by a dormant innovation  
The flourishing of dormant phyla after the Permian extinction.

n = 6061



n = 6062

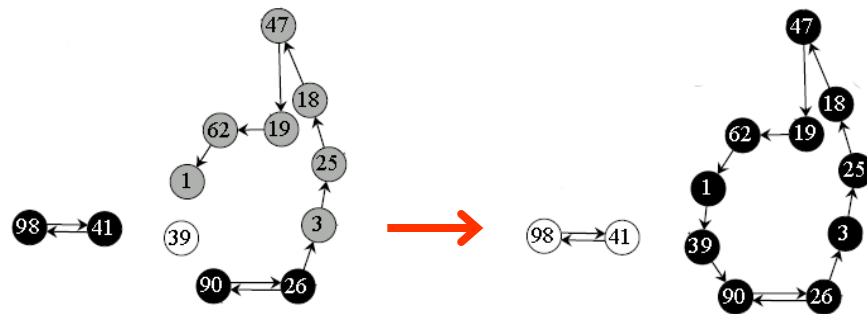


Another example of a core shifting innovation

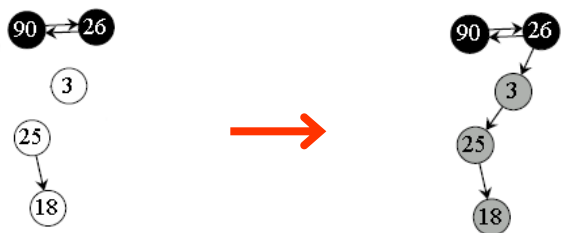
The automobile causes the demise of the horse drawn carriage industry



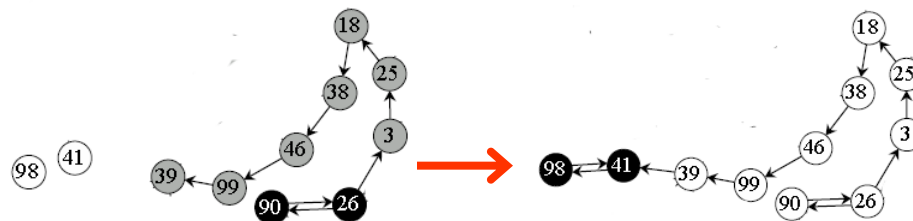
**1:** Random phase innovation



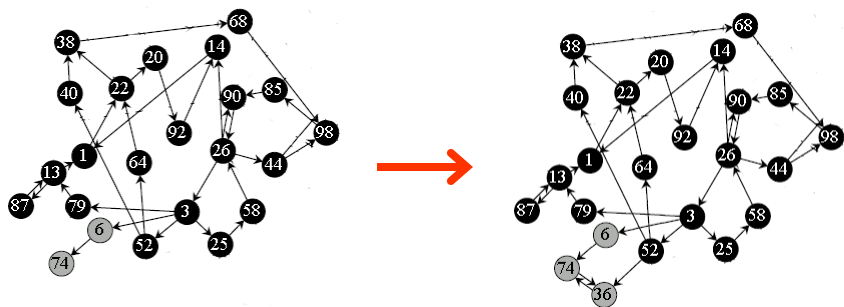
**4:** Core enhancing innovation



**2:** Incremental innovation



**5:** Core shifting innovation



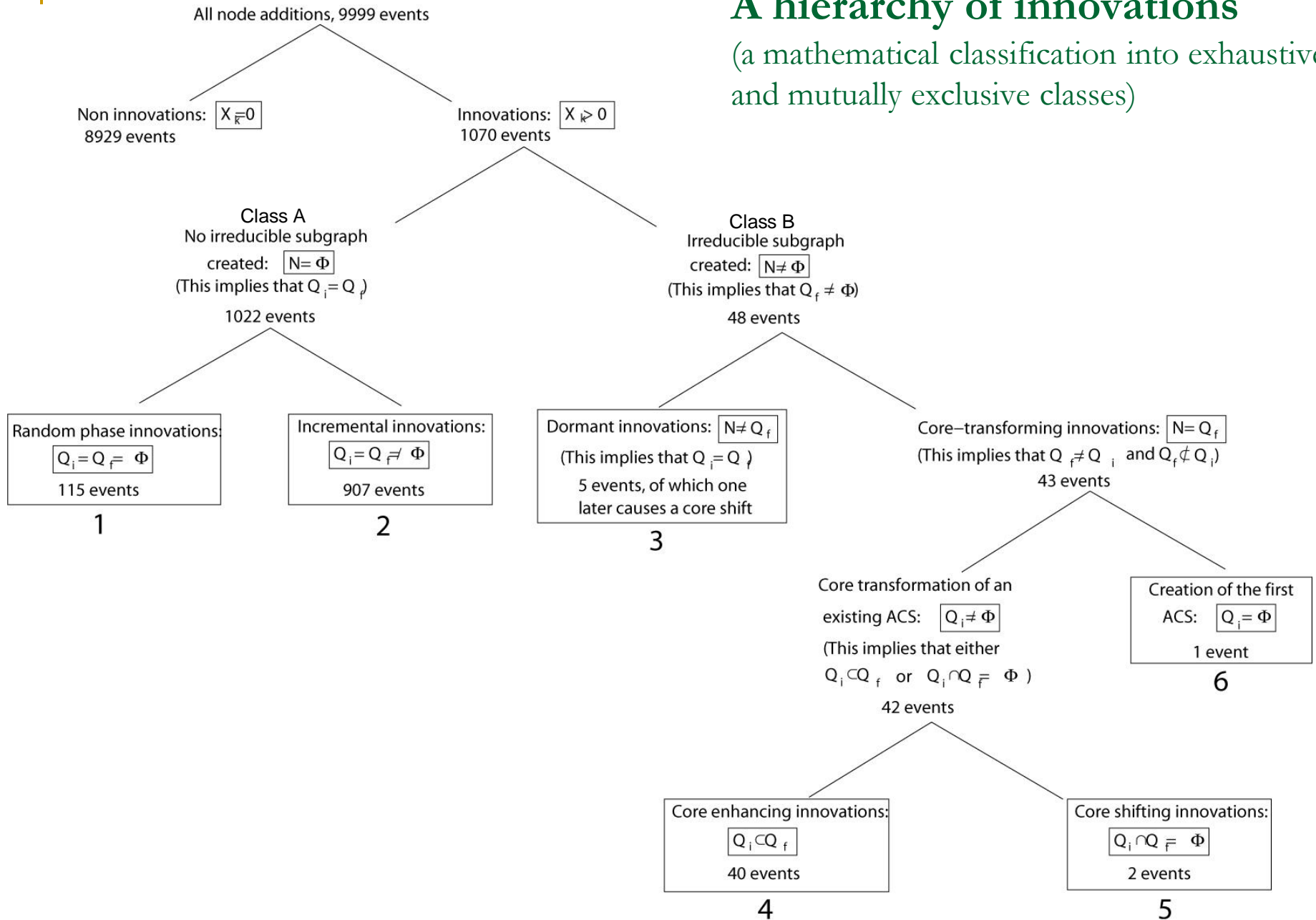
**3:** Dormant innovation



**6:** Creation of the first ACS

# A hierarchy of innovations

(a mathematical classification into exhaustive and mutually exclusive classes)



# Conclusions

- We have defined an innovation in structural-dynamical terms.
- The innovation depends on how the new node is linked to the existing nodes. Thus, it depends not only on the new links, but also on the 'context' in which the new node is embedded.
- Through a knowledge of how the node is linked to the existing graph we can qualitatively estimate the impact of this innovation.
- Innovations that modify the dynamics (flow patterns) on the network have the maximum impact. Typically these are those innovations that create new feedback loops.