On some aspects of price competition in a homogeneous product market: Evolution of cooperation, existence of equilibrium and other issues

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Consider a homogeneous product duopoly.

Demand $D(p)$ with $D'(p) < 0$.

Two firms. Identical costs. $C(q_i) = cq_i$.

No fixed costs. Constant marginal and average costs.

Firms simultaneously and independently quote prices.

The firm which quotes the lower price gets the entire market, the firm quoting the higher price gets no demand. If they quote the same price the demand is split equally.

We will also assume that a firm always supplies the demand it faces.

The above assumption can be rationalised by assuming that when a firm sets a price p_i this represents a commitment to supply the forthcoming demand.

This may be the case in regulated industries (for example, in the supply of electricity or telephone) or the result of consumer protection laws. For example, it is typical of "common carrier" regulation to require firms to meet all demand at the set prices. In case that the supply of a product is exhausted the customer may take a "rain check" (a coupon to purchase the good at the posted price at a later date).

This is also plausible when there are large costs of turning customers away.

Firms choose prices as decision variables.

Let $D_i(p_i, p_j)$ be the demand accruing to firm i when it quotes p_i and its rival quotes p_j.

$$D_i(p_i, p_j) = \begin{cases} D(p_i) \text{ if } p_i < p_j \\ \frac{1}{2} D(p_i) \text{ if } p_i = p_j \\ 0 \text{ if } p_i > 0. \end{cases}$$

In this price choice game

1. the players are the firms

2. the strategies are prices i.e. $p_i \in [0, \infty)$

3. payoff functions (profits)

$$\pi_i(p_i, p_j) = p_i D_i(p_i, p_j) - cD_i(p_i, p_j)$$

Relevant equilibrium concept: Nash equilibrium

Unique pure strategy Nash equilibrium.

$p_1^* = p_2^* = c$.

Both firms get zero profit in equilibrium.

Phenomenon - Bertrand Paradox.

Suppose there are n firms.

If there are no fixed costs and if the marginal costs are constant (and equal to c), then the price vector $(P_1^*, P_2^* \ldots P_n^*)$ is a Bertrand equilibrium if and only if $P_i^* \geq c$ for all i and at least two firms set the price equal to c.

If the number of firms is greater than two then the pure strategy equilibrium is not unique.

However, in all such equilibria firms earn zero profits. (Bertrand paradox).

Introducing fixed costs

Let cost functions be

$$C(q_i) = \begin{cases} 0 \text{ if } q_i = 0 \\ F + cq_i \end{cases}$$

In the same price game there is no Nash equilibrium (*not even in mixed strategies*).

Surprisingly, this has been proved only recently.

Some definitions:

A cost function $C : [0,\infty) \rightarrow [0,\infty)$ is strictly subadditive on $[0,\infty)$ if and only if $C(x+y) < \; C(x)+C(y)$ for all $x, y \in (0,\infty)$.

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Some general results

1. If costs are strictly subadditive on $[0,\infty)$ then there exists *no* equilibrium either in pure strategies or in mixed strategies.

   The above is a generalisation of the non-existence result with fixed costs.

2. If costs are strictly superadditive on $[0,\infty)$ then there exists a pure strategy Bertrand equilibrium. The Bertrand equilibrium in this case is always non-unique.

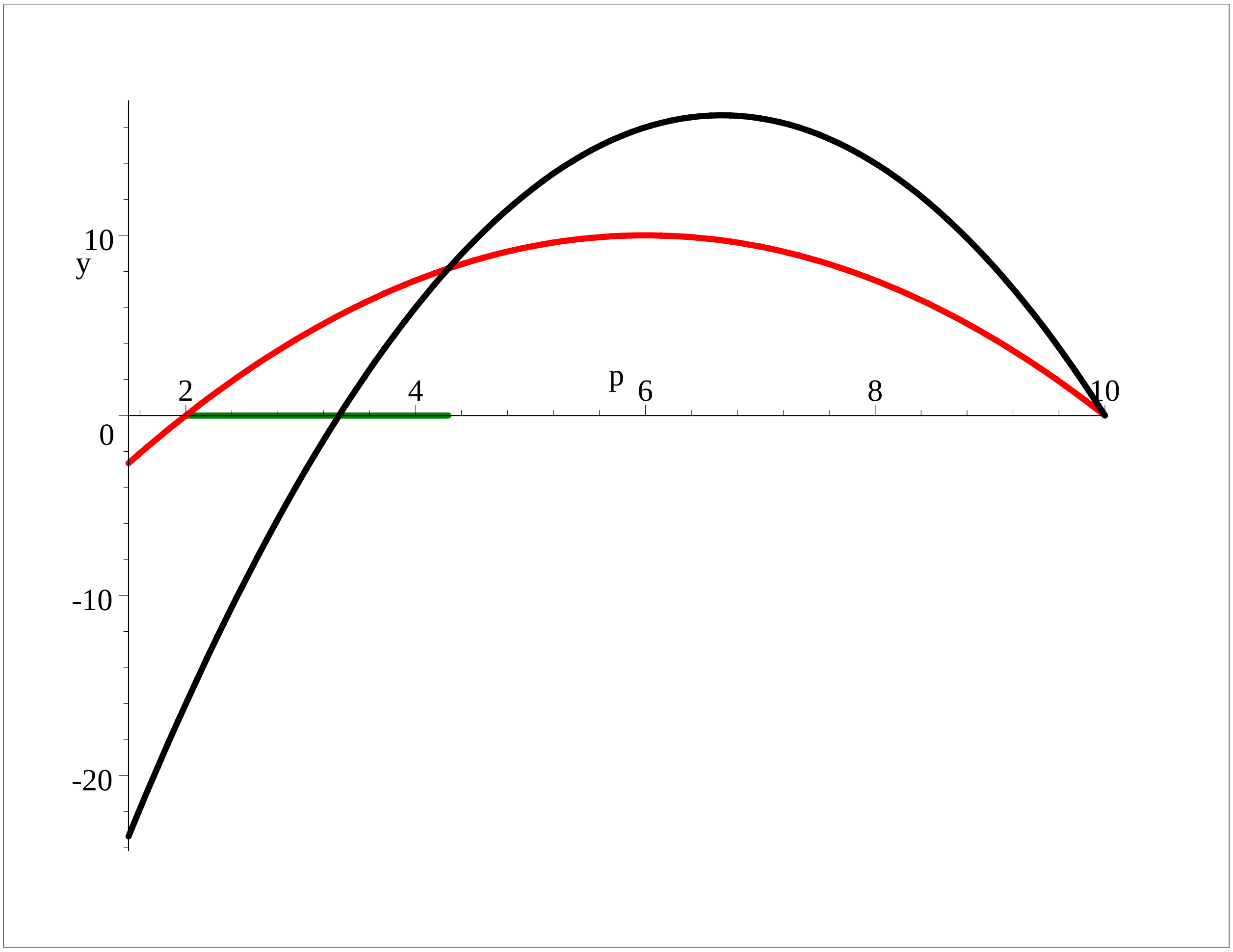
One example with superadditive costs:

Let $n = 2,\ D(p) = 10 - p$ and $c_i(q_i) = \frac{1}{2}q_i^2$.

Let $\pi(P) = PD(P) - C_i(D(P))$

and $\hat{\pi}(P) = \frac{1}{2}PD(P) - C_i(\frac{1}{2}D(P))$

Both firms quoting any price $p^* \in \left[2, \frac{30}{7}\right]$ is a Bertrand-Nash equilibrium.



Red: $\hat{\pi}(p)$, Black: $\pi(p)$

Green Line: Bertrand-Nash equilibrium range.

Digression: Incomplete information games

There are n firms who quote prices at which they want to sell in a market characterised by a negatively sloped demand curve $D(p)$.

The firm quoting the lowest price wins the market.

Each firm has a constant marginal cost c_i.

We list the assumptions of our model below.

1. Seller $i's$ marginal cost c_i is private knowledge to the seller. Seller i knows c_i but does not know c_j $(j \neq i)$.

2. Independent types: $c_1 \ldots . c_n$ are independently distributed. Seller i believes that

   $c_1, c_2 \ldots c_{i-1}, c_{i+1}, \ldots c_n$ are random variables to which he can attribute a joint probability distribution.

3. Symmetry: Each random variable $c_i \in [\alpha, \ \beta]$ has the same distribution function $F(.)$ and associated density $f(.)$. That is, each seller i believes that competitors' MCs are given by $c_j \in [\alpha, \ \beta]$ with distribution function $F(.)$ and density function $f(.)$. We assume $\alpha > 0$.

4. The sellers are all risk neutral.

5. $\forall c_i,$ the function $\pi_i = (p - c_i)D(p)$ is strictly concave in p.

Strategies are prices which are functions of types (costs)

$$p_i(c_i) : \left[\alpha,\ \beta\right] \rightarrow [0, \infty)$$

We look for a profile of strategies that constitute a Bayesian- Nash equilibrium.

There exists a symmetric Bayesian Nash equilibrium with strategies $p(c_i)$ which is strictly increasing in c_i.

$p(c_i)$ solves the following differential equation (1) with the boundary condition (1a).

$$\frac{dp}{dc} = \frac{(n-1)f(c_i)}{1-F(c_i)}\left[\frac{(p(c_i)-c_i)D(p(c_i))}{D(p(c_i))+(p(c_i)-c_i)D'(p(c_i))}\right] \quad --1$$

and $p(\beta) = \beta$ $\quad --1a$

It may be noted that $p(c_i) > c_i$ for all $c_i \in [\alpha, \beta)$.

Equation (1a) suggests that the worst-off seller - the seller with highest possible cost, β - has zero expected profit.

Others have positive expected profit.

That is, *No Bertrand paradox*.

We now go back to complete information games.

Cooperative outcomes

Let p^m be the monopoly price.

That is,

$$p^m = \arg\max_{p \geq 0} pD(p) - C(D(p)).$$

We denote monopoly profits by π^m. That is,

$$\pi^m = p^m D(p^m) - C(D(p^m)).$$

Let p^J be the joint profit maximising price.

That is,

$$p^J = \arg\max_{p \geq 0} p \frac{D(p)}{n} - C\left(\frac{D(p)}{n} \right).$$

$$\text{Let } \pi^J = p^J \frac{D(p^J)}{n} - C\left(\frac{D(p^J)}{n} \right)$$

Note: If cost functions are of the form $C(q_i) = cq_i$ (constant marginal costs) then $p^J = p^m$.

We restrict our attention to such cost functions only (where marginal costs are constant and there are no fixed costs) .

Question: how does one get cooperative outcomes (cartels)?

That is, is it possible to have an equilibrium outcome with all firms quoting $p^J = p^m$ and sharing the market and monopoly payoff equally?

Answer: In a one shot game where firms meet only once, collusive outcomes are not possible in equilibrium. The only possible equilibrium outcome is the Bertrand Paradox.

What about repeated interactions?

We consider a repeated price choice game where firms discount the future.

The initial period is the 0^{th} period. That is $t = 0, 1, 2$

Let $\pi_t =$ payoff to a firm in time period t.

The present discounted value of a stream of payoffs $\pi_0, \pi_1, \pi_2 \ldots \pi_t$ is

$$V = \sum_{k=0}^{t} \delta^k \pi_k$$

where $\delta \in (0, 1)$ is the discount rate.

Relevant equilibrium concept for repeated games:

<u>Subgame Perfect Nash Equilibrium (SPNE)</u>

Now consider a homogeneous product duopoly with constant marginal costs (equal to c). The rules of the price choice game are same as before.

If the price choice game is repeated finite number of times then the only SPNE possible is where firms choose prices (c, c) in every stage and get zero payoffs.

If we have an infinitely repeated price choice game then it opens up a lot of possibilities. For example, consider the following strategy profile (called trigger strategy).

Period 0: choose p^m.

Period t: choose p^m if in each of the previous periods the outcome has been (p^m, p^m). Otherwise choose (c, c).

The above strategy profile constitutes a SPNE provided $\delta \geq \frac{1}{2}$.

The outcome is that in each period firms choose (p^m, p^m) and get $\left(\frac{1}{2}\pi^m, \frac{1}{2}\pi^m\right)$.

That is, we get collusive outcomes through non-cooperative interactions.

Infinitely repeated games give us insights into collusive behaviour.

It can also be used to theoretically explain certain empirically observed phenomenon like ”price war during booms”. For example, it has been observed that cartels tend to break down when sudden large orders are placed in the market.

One can show that at a SPNE of an infinitely repeated price choice game with demand shocks (i.e. demand can be high or low) firms behave collusively in low demand periods but such collusions cannot be sustained during high demand periods (boom times).

Can we at all have collusive outcomes in finitely repeated games?

Answer: At times yes - provided the Nash equilibrium in the one shot version of the price choice game is *not unique*.

In fact, it is possible to have collusive outcomes in a one shot price choice game with strictly convex costs. We provide an example to illustrate this.

Let the demand be given by $D(p) = 10 - p$ and costs by $C(q_i) = 3q_i^2$ (where $i = 1, 2$).

Routine calculations show that the joint-profit-maximizing price $p^J = 8$.

We can show that both firms quoting the same price in the range $[6,\ 8.1818]$ is a pure-strategy Bertrand equilibrium.

Since $p^J \in [6,\ 8.1818]$, our example shows that it is possible to sustain collusive outcomes in a single-shot game of price competition.

Moral of the story:

Existence and nature of Bertrand-Nash equilibrium in single shot games depends on the nature of the cost functions.

Whether collusive outcome can be observed in equilibrium depends upon

(a) whether the game is repeated infinitely or not

(b) on the nature of cost functions.