Statistics of Kolkata Paise Restaurant Problem

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Main Publication:

- **The kolkata paise restaurant problem and resource utilization, Physica A 388 (2009)**
- **Kolkata paise restaurant problem ,Wolfram Demonstration (http://demonstrations.wolfram.com/KolkataPaiseRestaurantKPRProblem/)**

Plan Of The Talk

- What is KPR ?
- Random choice case.
- Avoiding-past-crowd choice

  a) Deterministic b) Probabilistic
- Strict-rank-dependent choice.
- Summary and discussion.

What is KPR ?

- There are N agents and N equally priced but differently ranked (agreed by all) restaurants, each capable of serving lunch to only one agent each evening.
- If , on any evening, more than one person arrive at the same restaurant, one of them, chosen randomly, gets food; rest do not get food that evening.
- No interaction (parallel discussion among the agents).
- A repeated game.
- Information regarding the agent distributions for previous evenings are available to everyone.
- Each agent wants to get food most evenings , and that too at the higher ranked restaurants!

The strategy chosen by each agent…..

Let the probability to arrive at the k^{th} ranked restaurant be:

$$p_k(t) = \frac{1}{z}\left[k^{\alpha} \exp\left(-\frac{n_k(t-1)}{T}\right)\right] \qquad z = \sum_{k=1}^{N} k^{\alpha} \exp\left(-\frac{n_k(t-1)}{T}\right)$$

$\alpha, T > 0$;

- $\alpha = 0, T \to \infty$ corresponds to random choice
- $\alpha = 1, T \to \infty$ corresponds to strict rank-dependent choice
- $\alpha = 0, T \to 0$ corresponds to avoiding-past-crowd choice

Random - choice…..

- Here we take λN agents and N restaurants.
- Any agent selects any restaurant with probability $\mathrm{P} = \frac{1}{\mathrm{N}}$.
- Probability that a single restaurant is chosen by m agents is given by

$$D(m) = \binom{\lambda N}{m} p^m (1-p)^{\lambda N - m}; \; p = \frac{1}{N}$$

$$= \frac{\lambda^m}{m!} \exp(-\lambda) \qquad N \to \infty$$

continued.....

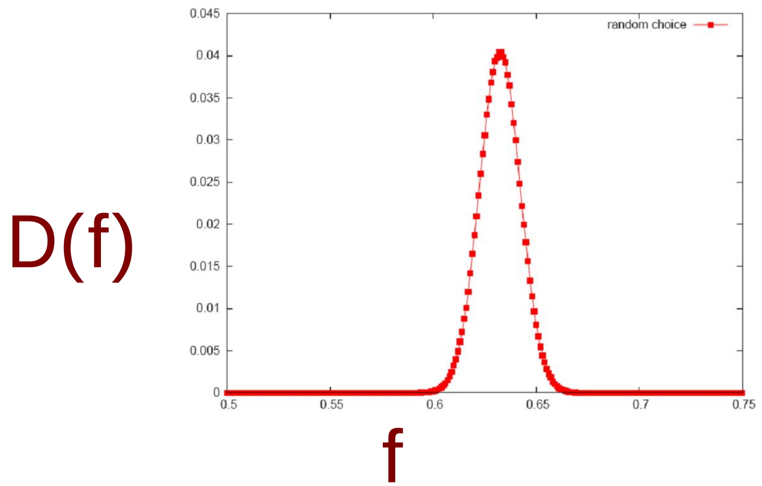
- Fraction of restaurants not chosen by any agents is given by

$$\tilde{f} = 1 - \exp(-\lambda)$$

$\simeq 0.63$ for $\lambda = 1$

- The fraction of utilization will be Gaussian around this average.

Distribution D of customers getting dinner any evening



f: fraction of customers

Strict-rank-dependent choice

- An agent goes to the k-th restaurant with probability

$$p_k(t) = \frac{k^{\alpha}}{\sum k^{\alpha}} \quad [\quad T \to \infty \quad]$$

- In this case, fraction of successful customers or restaurants is around 0.58 for $\alpha = 1$.

Analytical result for strict-rank-dependent choice.....

- We make $N/2$ pairs of restaurants and each pair has restaurant ranked k and $N+1-k$ where $1 \le k \le N/2$.
- An agent chooses any pair with probability $p = 2/N$, giving $\lambda = 2$
- The fraction of pairs selected by the agents

$$f_0 = 1 - \exp(-\lambda) \simeq 0.86$$

continued….

- Expected number of restaurants occupied in a pair of restaurants with rank k and $N+1-k$ by a pair of agents is

$$E_k = 1\times\frac{k^2}{(N+1)^2} + 1\times\frac{(N+1-k)^2}{(N+1)^2} + 2\times 2\times\frac{k(N+1-k)}{(N+1)^2}\ .$$

continued….

- Therefore, the fraction of restaurants occupied by pairs of agents is

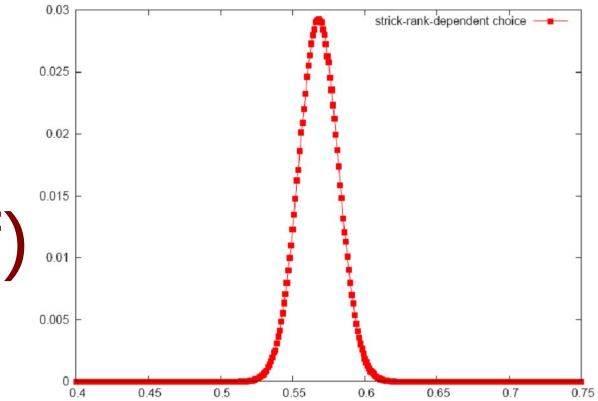
$$f_1 = \frac{1}{N} \sum_{k=1,....N/2} E_k \simeq 0.67$$

- Actual fraction of restaurant occupied by the agent is

$$\tilde{f} = f_0 . f_1 \simeq 0.58$$

Distribution D of customers getting dinner any evening

D(f)



f

f: fraction of customers

Avoiding-past-crowd choice: a) Strict avoidance

- In this case an agent completely avoids the past crowd.
- Therefore an agent only chooses from the vacant restaurants in previous evening.
- The fraction of restaurants visited by the agents in the last evening $(\widetilde{f})$ is avoided by the agents this evening.

Continued….

- Therefore in this case $N(1-\tilde{f})$ restaurants remain to choose from.
- Agents choose randomly among those restaurants with $\lambda = 1/(1-\tilde{f})$
- Therefore we get the equation for $\tilde{f}$ as

$$(1-\tilde{f})\left[1-\exp\left(-\frac{1}{1-\tilde{f}}\right)\right] = \tilde{f}$$

Solution of this equation gives $\tilde{f} \simeq 0.46$

continued...

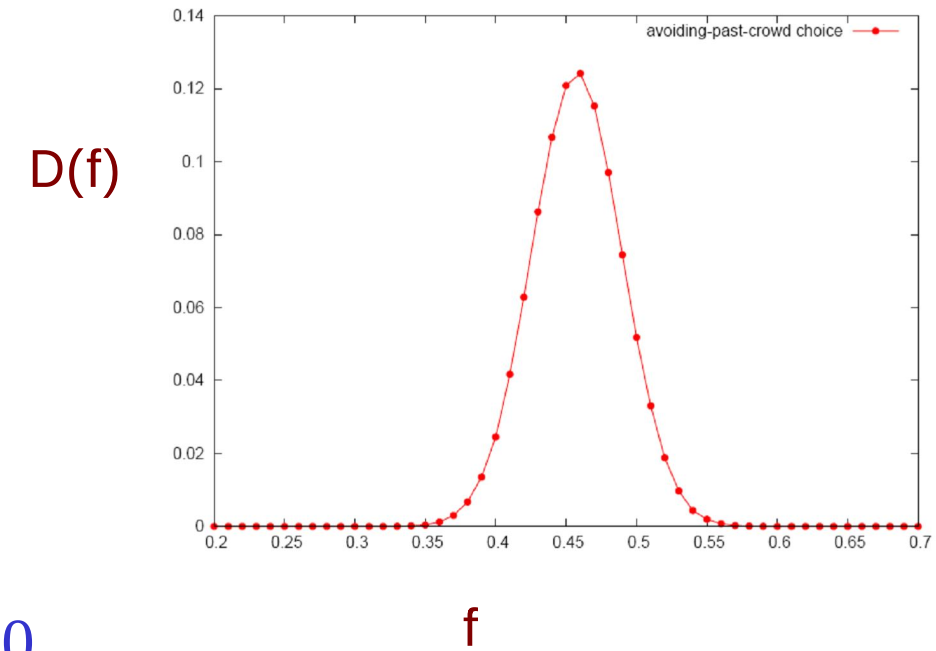
Avoiding-past-crowd choice

- In this case $T \to 0$ and $\alpha = 0$

  .

- Simulations have been done for $N = 1000$ and averaging over 10^6 time steps

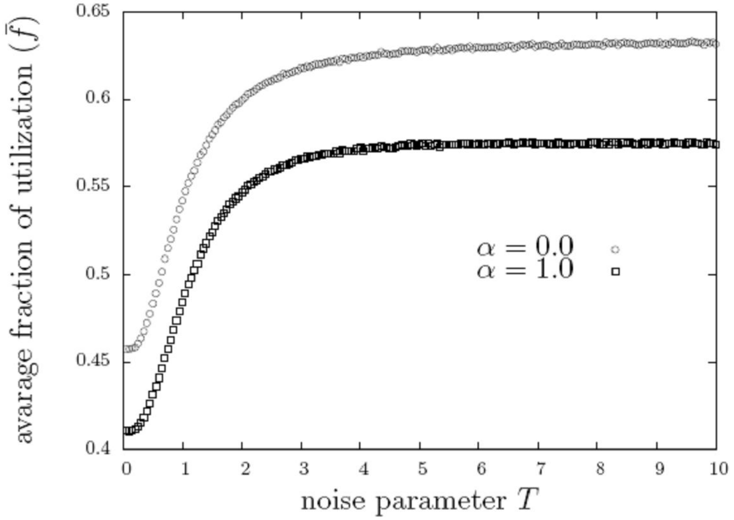
Distribution D of customers getting dinner any evening



f: fraction of customers

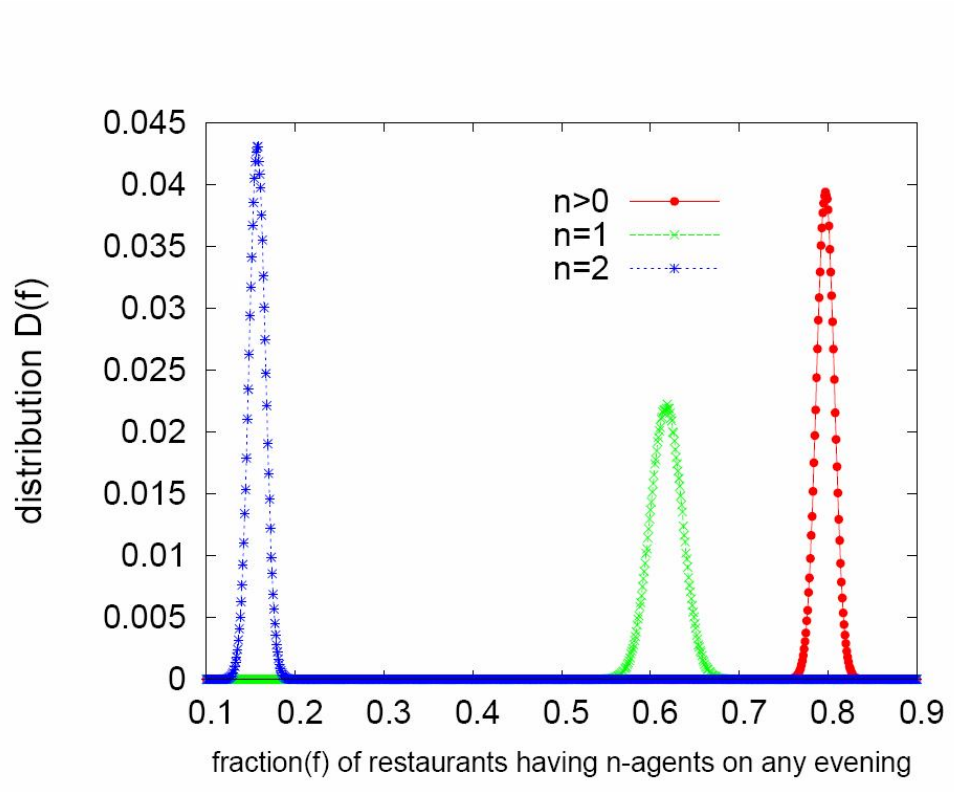
continued...

Utilization against the noise parameter.



Avoiding-Past-Crowd: b) Probabilistic Choice

- Typical prospective customer (agent) distribution on any evening



Discussion...

- Here we consider N agents who, on every evening (t), attempt to choose form N equally priced restaurants having well defined ranking.
- The KPR problem has a trivial solution (dictated from out side) where each agent gets into one of the respective restaurants.
- In KPR game, it can be extremely difficult.
- Social Utilization fraction is seen to be more for relatively stupid stochastic strategies (e.g. random choice or probabilistic past-crowd avoidance case) compared to all the smarter strategies considered here!

THANK YOU