Ballistic transport beyond linear response in graphene and emergence of the electron – hole plasma

Baruch Rosenstein

Nat. Chiao Tung U, Hsinchu, Taiwan, R.O.C.

Collaborators:

Hsien-Chong Kao (Nat. Normal U, Taiwan, R.O.C.) ,

Meir Lewkowicz (Ariel UC, Israel) ,

Evgeniy Kornijenko (NCTU)

Kanpur, February 4, 2010

Outline

1. Graphene – a truly 2D electron system

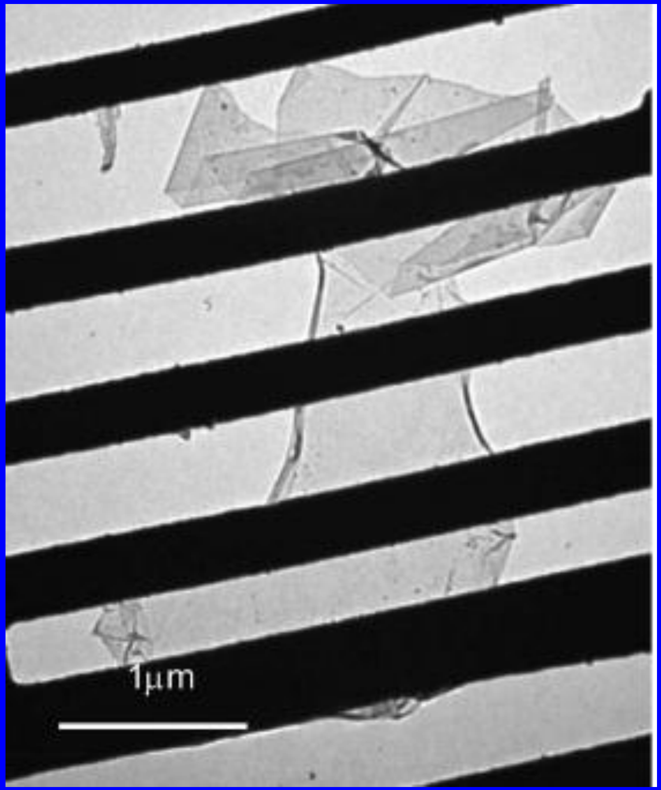
2. Tight binding model of the graphene sheet. Dirac points and pseudo – diffusive “resistivity” without either impurities or carriers.

3. Dynamical approach: pair creation by weak electric fields and the chiral (parity) anomaly. Role of electrons far from the Dirac points.

4. Beyond the linear response. Schwinger’s pair creation rate.

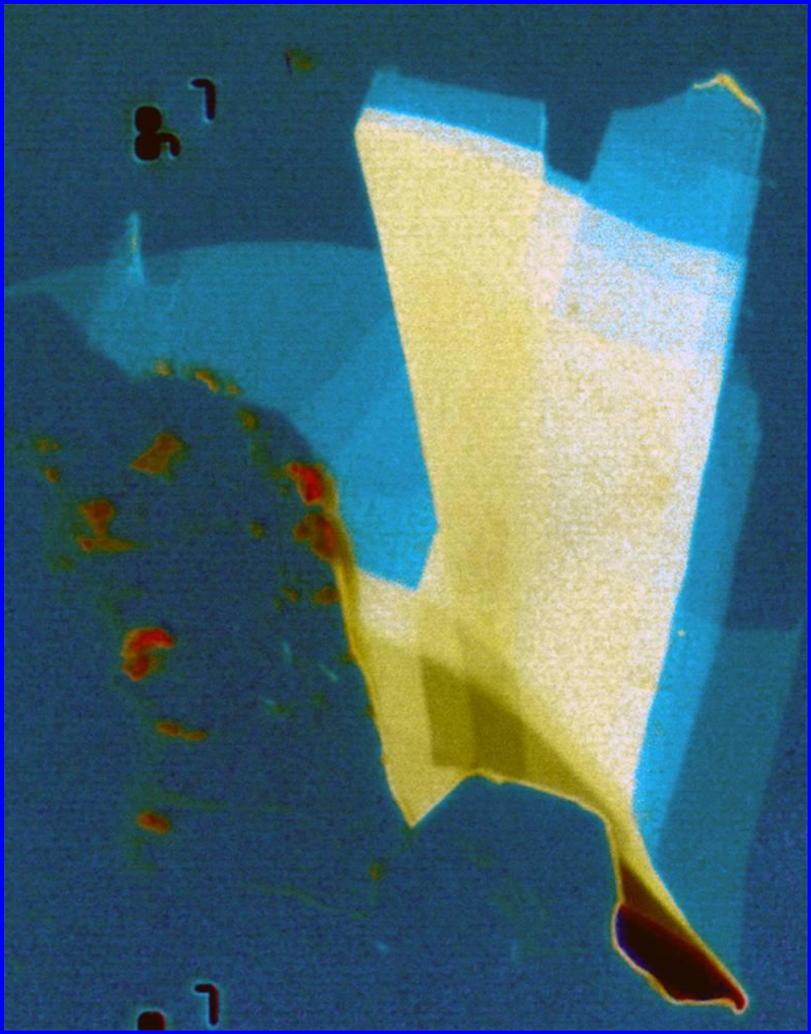
5. Where the energy and the pairs go? The electron – hole plasma and radiation friction.

1. Graphene – a truly 2D electron system



Novoselov et al, *Proc. Nat. Acad. Sci. USA 102 10451 (05)*

Single graphene sheet (one atom thick) on a metal (b) as seen by atomic force microscopy (left) and optically (right). No domains visible.

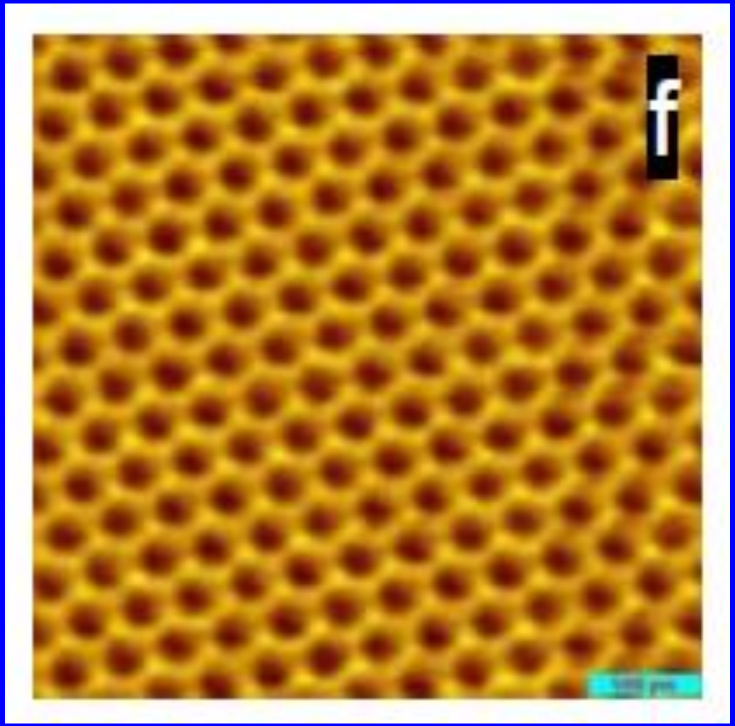


Novoselov, Geim et al, *Nature 438, 197 (05)*

Carbon atoms are arranged in honeycomb crystal. Covalent bonds are mostly between nearest neighbours.

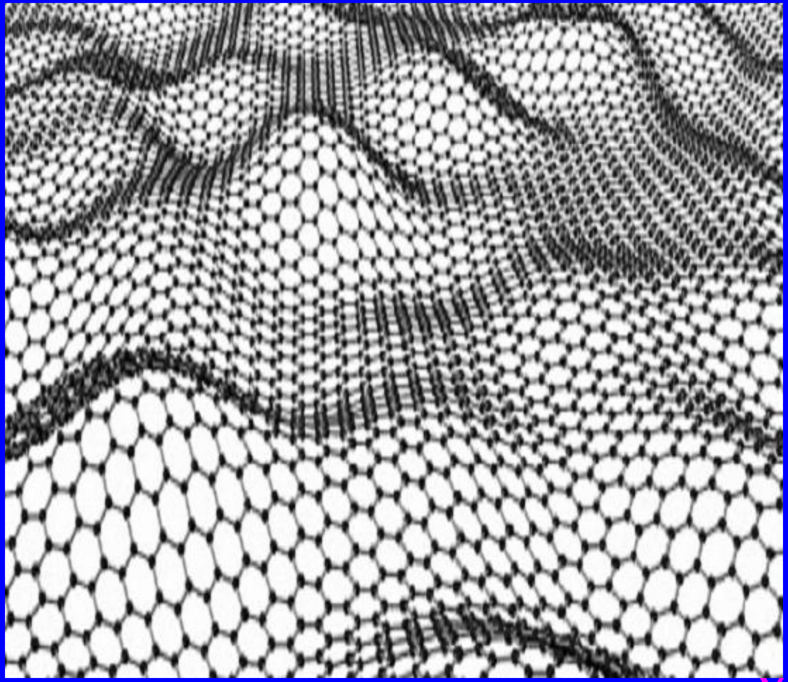
E. Andrei et al, Nature Nano 3, 491 *(08)*

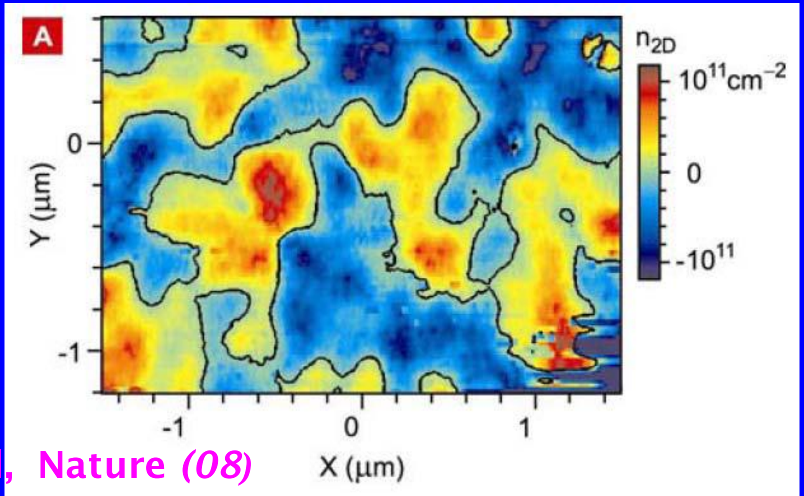
Single graphene sheet on STM



Even suspended graphene is not entirely homogeneous and flat

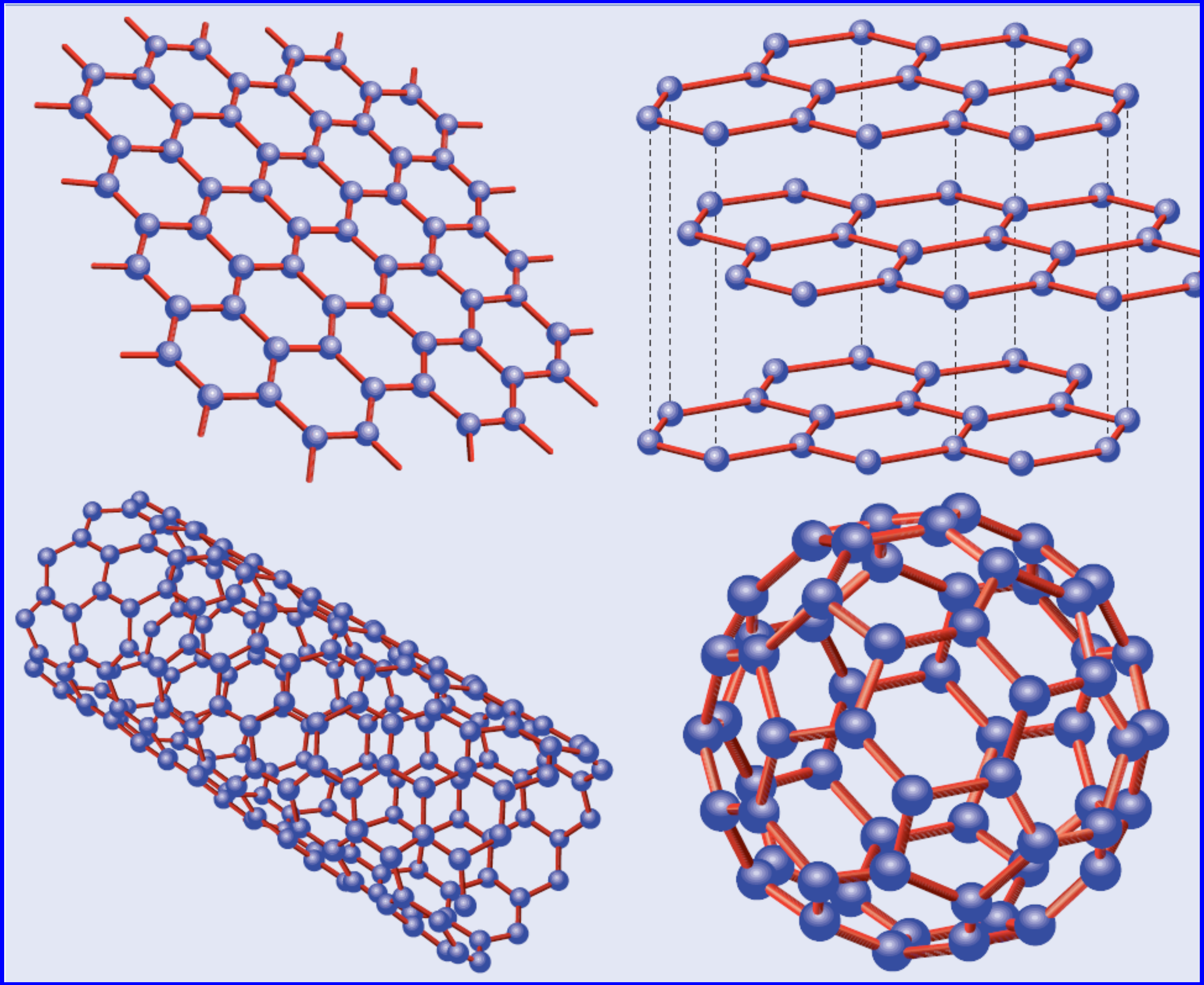
Meyer et al, *(09)*





Yacoby et al, Nature *(08)*

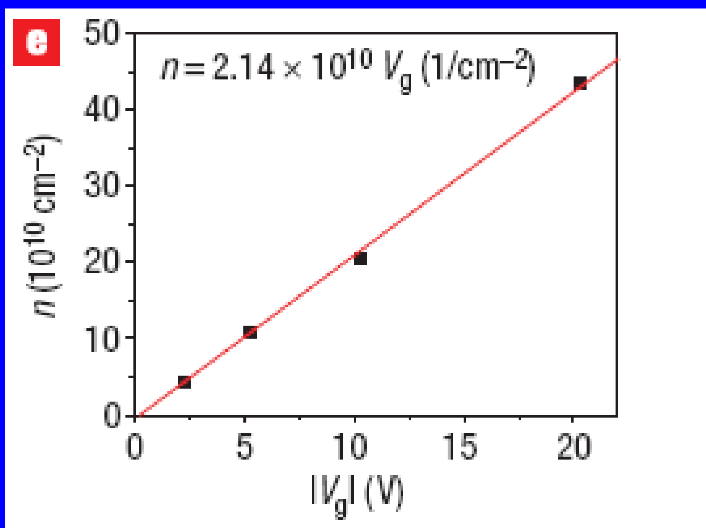
Single graphene sheet can be viewed as a prototype of various 0D-3D structures based on the similar bonds.

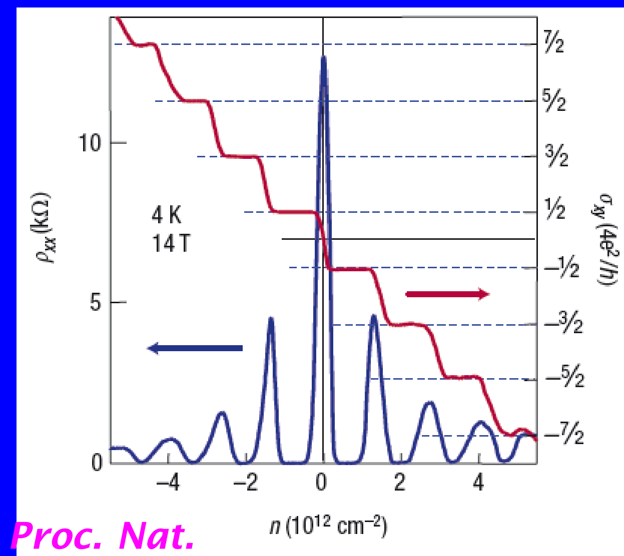


Castro Neto, Peres, Guinea, Physics Today. *10451 (06)*

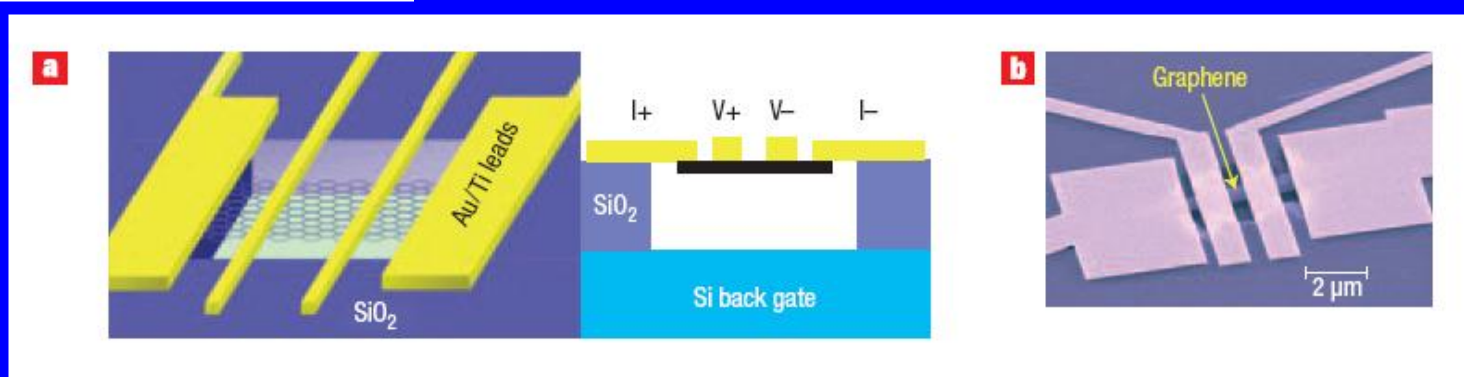
Very high mobility in graphene on substrate and suspended graphene

Quantum Hall effect at room temperature clearly shows high mobility. In suspended sample electron density vanishes at Dirac point





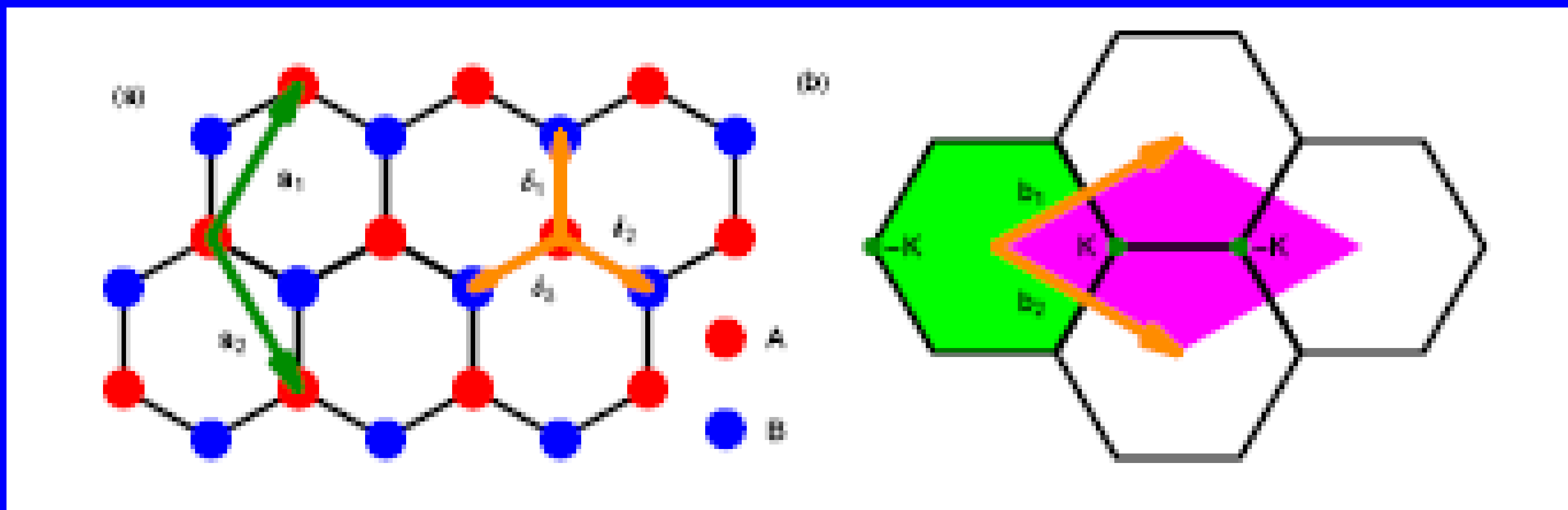
Novoselov et al, *Proc. Nat. Acad. Sci. USA 102 10451 (05)*



E. Andrei et al, Nature *(08)*

2. Tight binding nearest neighbour model

There are two sublattices and consequently Hamiltonian is an off diagonal matrix with several symmetries.



$$a_1 = \frac{a}{2}\left(1, \sqrt{3}\right); \quad a_2 = \frac{a}{2}\left(1, -\sqrt{3}\right)$$

Wilson links

$$\delta_1 = \frac{1}{3}\left(a_1 - a_2\right),$$

$$\delta_2 = \frac{1}{3}\left(a_1 + 2a_2\right),$$

$$\delta_3 = -\frac{1}{3}\left(2a_1 + a_2\right)$$

$$\hat{\ } = \gamma \sum_{\alpha} \hat{a}_A^{+}(r)\, e^{i\frac{e}{\hbar c}A_t\cdot\delta_\alpha}\, \hat{a}_B\left(r+\delta_\alpha\right) + c\,c$$

r s e t i s

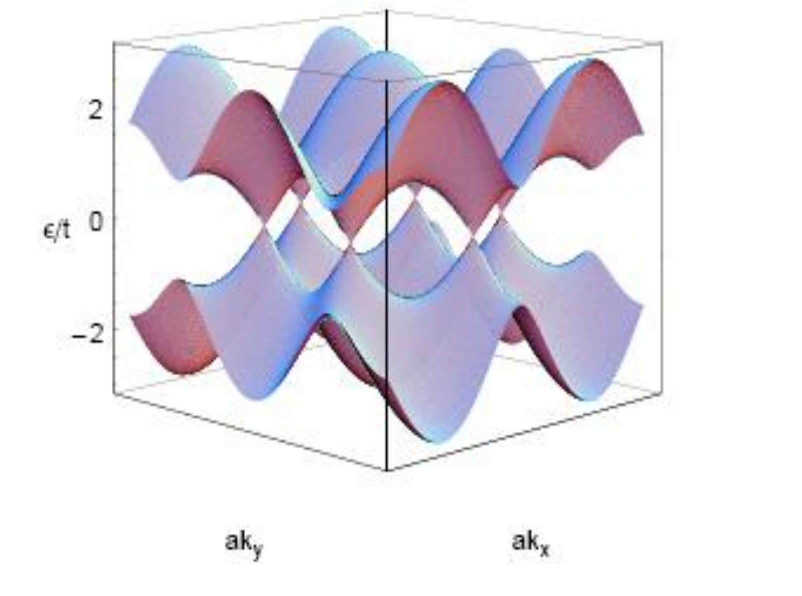
The off diagonal couplings include three neighbours. In momentum space one writes

$$\hat{\mathrm{H}} = \sum_{\mathrm{BZ}} \begin{pmatrix} \hat{a}_A^+(p) & \hat{a}_B^+(p) \end{pmatrix} \begin{pmatrix} 0 & h(p) \\ h^*(p) & 0 \end{pmatrix} \begin{pmatrix} \hat{a}_A(p) \\ \hat{a}_B(p) \end{pmatrix}$$

$$h(p) = -\gamma \sum\nolimits_\alpha e^{ip\cdot\delta_\alpha}$$

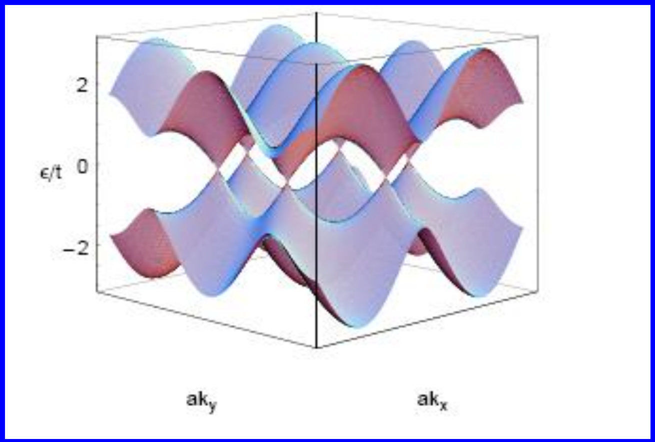
$$= -\gamma\left[\exp\left(i\frac{ap_y}{\sqrt{3}}\right) + 2\cos\left(-i\frac{ap_y}{2\sqrt{3}}\right)\left(\frac{ap_x}{2}\right)\right]$$

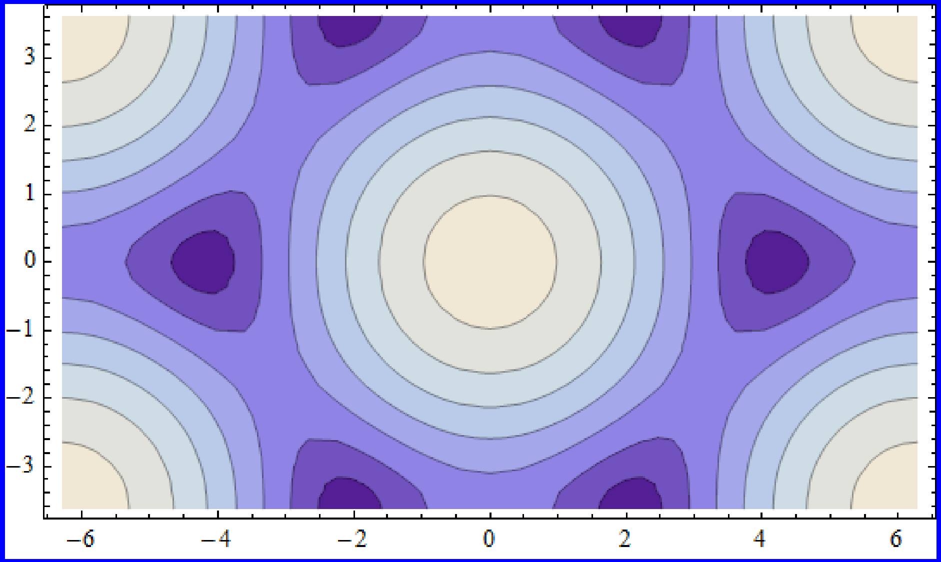
Which has the following spectrum



$$\varepsilon(p) = |h(p)|$$

There are two points representing "species doubling".





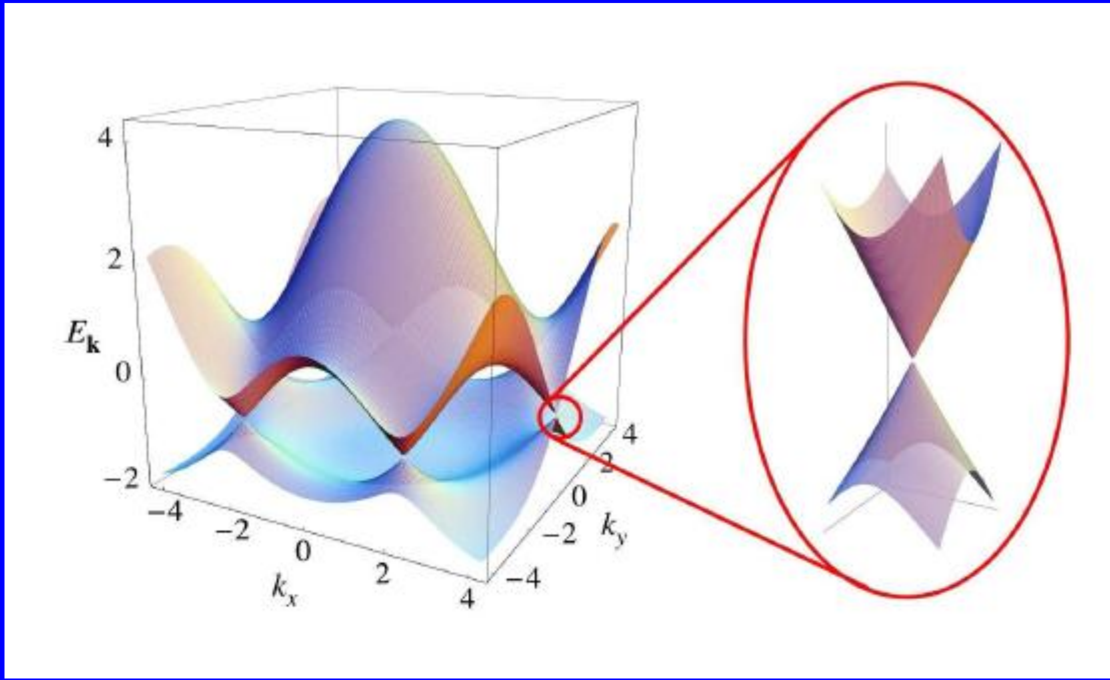
This corresponds to (rarely used in particle physics) Hamiltonian version of the staggered fermions formulation of the lattice gauge theory. Two components of fields "live" on different sublattices. It has much higher degree of rotation invariance than square lattice.

Let's zero onto the continuum limit points

Fermi surface of undoped graphene: Dirac points

Fermi surface is located exactly at two non - equivalent points of the Brillouin zone

$$K = \frac{2\pi}{a}\left(\frac{1}{3}, \frac{1}{\sqrt{3}}\right), K' = \frac{2\pi}{a}\left(\frac{2}{3}, 0\right)$$



Around which the spectrum becomes "ultra - relativistic" with velocity

$$\varepsilon = v_g |k|$$

with velocity

$$v_g = \frac{\sqrt{3}}{2}\frac{a\gamma}{\hbar} \approx \frac{c}{300}$$

Wallace PR *71 622 (1949)*

The first quantized function (in k space) obeys

$$\Psi = e^{ik\cdot r}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}; \quad i\hbar\partial_t\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = \begin{pmatrix}0 & h\\ h^* & 0\end{pmatrix}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

Expansion around K and K' and redefinition of fields gives Weyl equations for left and rigt chirality massless fermions respectively

$$h\left(K+\Delta p\right) = v_g\left(\Delta p_x + i\Delta p_y\right)e^{-i\pi/3};\ \psi_1^L = \psi_1 \quad \psi_2^L = e^{i\pi/3}\psi_2$$
$$h\left(K'+\Delta p\right) = v_g\left(\Delta p_x - i\Delta p_y\right); \quad \psi_1^R = \psi_1 \quad \psi_2^R = \psi_2$$

The lattice gauge theory obeys Nielsen – Ninomiya theorem due to two Dirac points and correct "matching" of two massless "species". Parity anomaly is avoided only when both species are included.

The “missing π” problem of the minimal DC conductivity

Theory predicted a value far below the experimental in graphene on substrate.

$$\sigma_1 = \frac{4}{\pi}\frac{e^2}{h}$$

Fradkin, PRB33, 3257 (1986)
Lee, PRL71, 1887 (1993)
Ludwig et al, PRB *(1994)*
Ando et al, J. P. S. Jap. 71, 1318 (02)
Gusynin, Sharapov, PRB73, 245411 (06)
Peres et al, PRB73, 125411 (06)…
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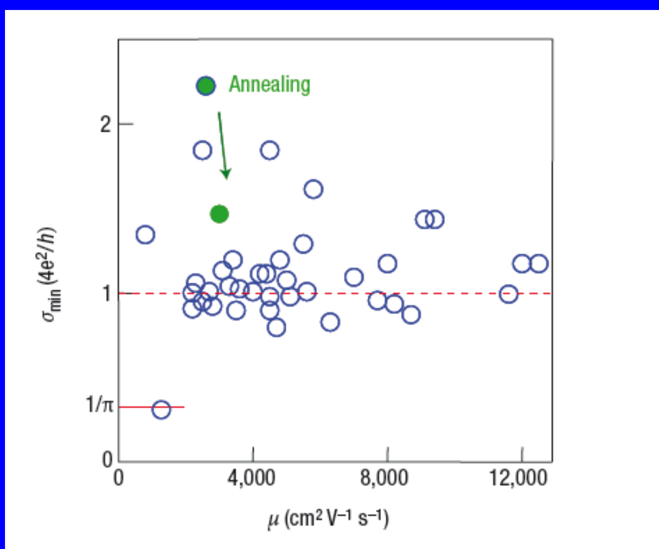
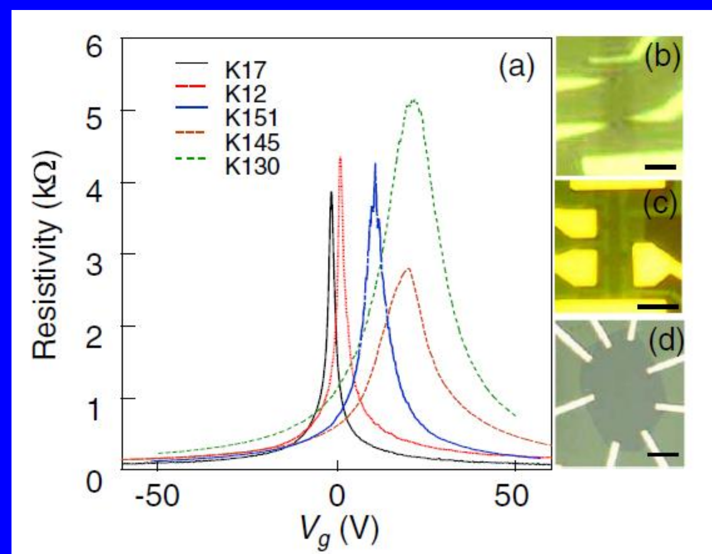




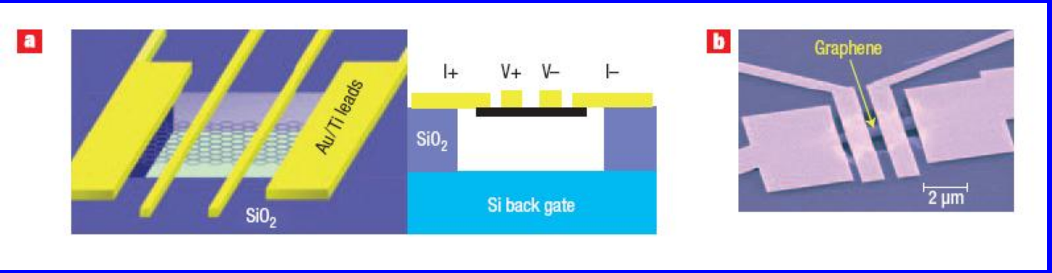
Geim et al, *Nature Mat. 6, 183 (07)*

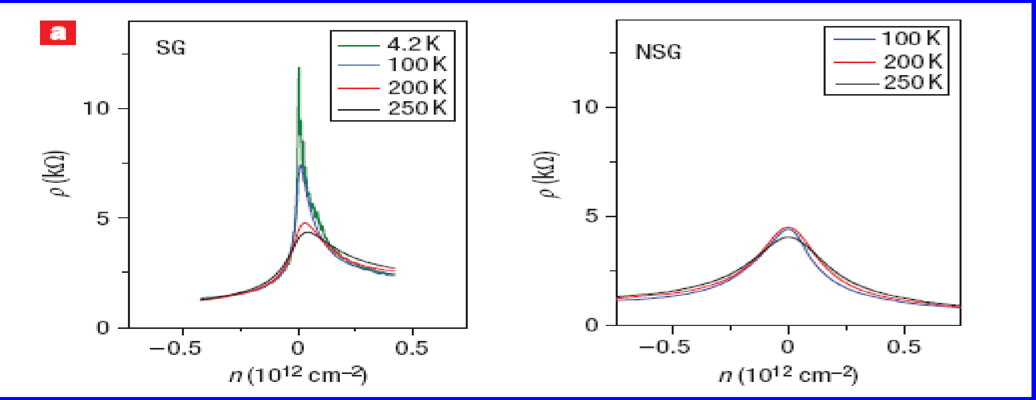
Moreover the theoretical value depends on the regularization used

Ziegler, *PRB75, 233407 (07)*; Beneventano et al, *arXiv 0901.0396 (09) …*

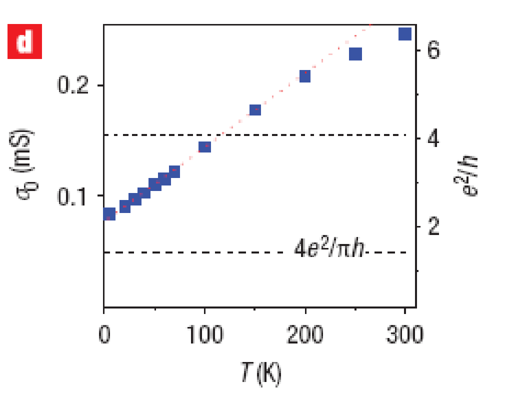
Very recent advance – suspended graphene (SG).

Graphene on substrate (NSG) exhibits a network of positive and negative paddles and therefore does not probe directly the Dirac point.





Suspended graphene (SG) is closer and indeed conductivity drops to "missing 1.7 instead of 3 at zero temperature.

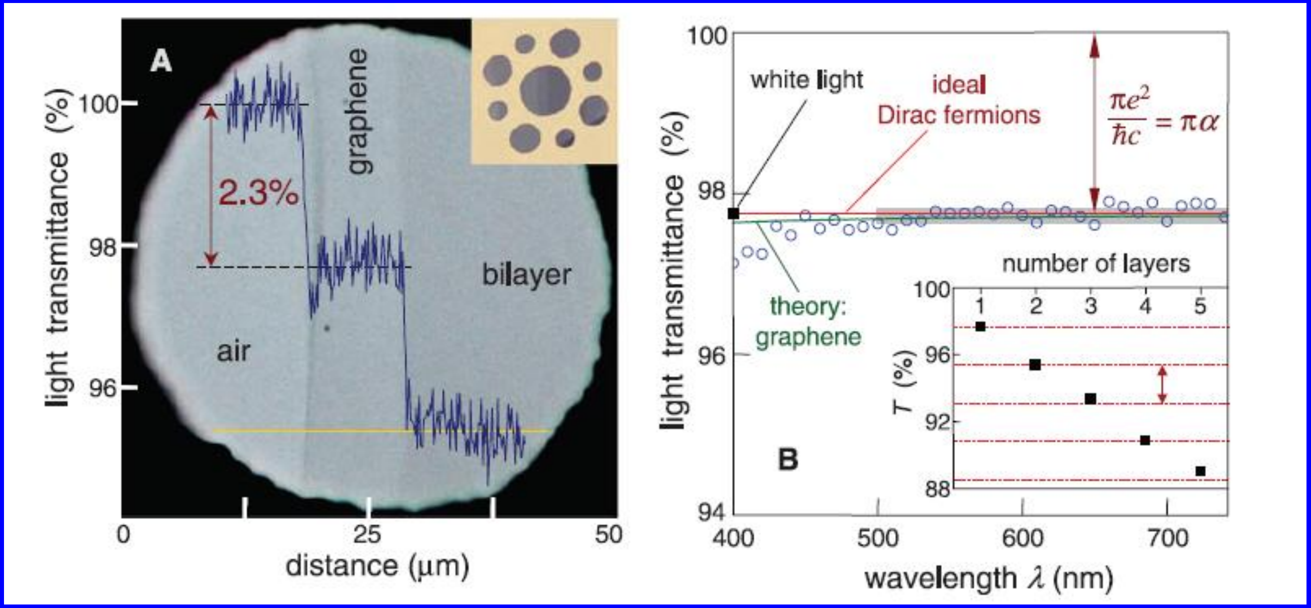


$$\sigma_0 = 84\ \mu S = 1.7 \frac{4e^2}{\pi h}$$

E. Andrei et al, Nature Nano 3, 491 *(08)*

Transparency at optical frequences.

Optical frequencies conductivity is much lower and was measured recently to accuracy of 1%. Agrees with theoretical value

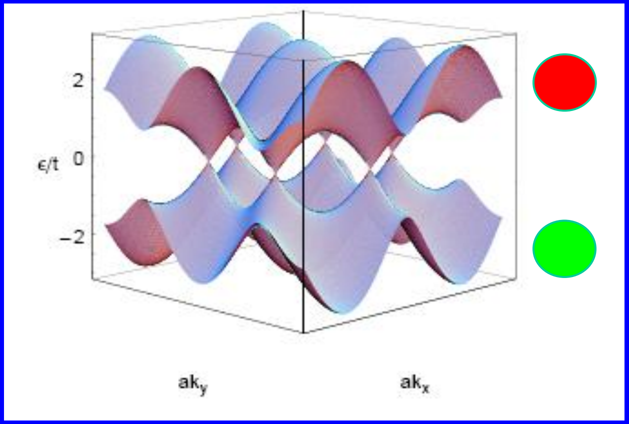


$$\sigma_2 = \frac{\pi}{2}\frac{e^2}{h}$$

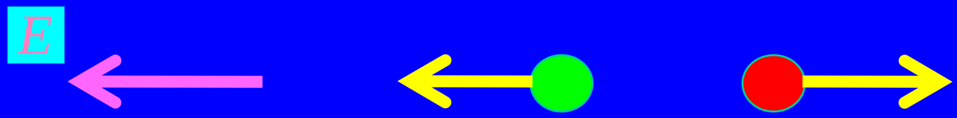
Geim, Novoselov et al, *Science 102 10451 (08)*

This value corresponds to the high frequency limit which is also universal. Note however that the only time scale for pure graphene is $t_\gamma = \hbar/\gamma = 10^{-16}\,\mathrm{sec}$ so it is not clear why it is different from the DC one .

3. The dynamical approach to ballistic transport in graphene.



The basic picture of the pseudo – diffusive resistivity in pure graphene is the creation of the electron – hole pairs by electric field. The pairs carry current, **without carrying net momentum**



Therefore we use a method which does not require "regularizations" and can be applied directly to DC at Dirac point $E_F = 0$, T=0, bypassing the Kubo formula. The first quantized function obeys

Fradkin, Gitman, Shvarzman, "QED with unstable vacuum"

$$\Psi = e^{ik\cdot r}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix};\quad i\hbar\partial_t\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = \begin{pmatrix}0 & h(p)\\ h^*(p) & 0\end{pmatrix}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

Electric field is switched on at t=0

$$p_x = k_x \, , \, p_y = k_y + \frac{e}{\hbar c} A_t$$

$$\vec{A}_t = -c\ \theta(t)(0, -E \cdot t)$$

Initially the first quantized field describes filled valence band

$$\psi(t=0) = u_k = \begin{pmatrix} 1 \\ -h^* / \varepsilon \end{pmatrix}$$

The current density therefore is

$$\mathrm{J} = \sum_{B.Z.} \begin{pmatrix} \psi_1{}^* & \psi_2{}^* \end{pmatrix} \begin{pmatrix} 0 & j \\ j^* & 0 \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}; \quad j = -c \frac{e}{\hbar c} h'(p)$$

Expanding the electric current to first order, one obtains two contributions:

Lewkowicz, B.R., PRL102, 106802 *(09)*

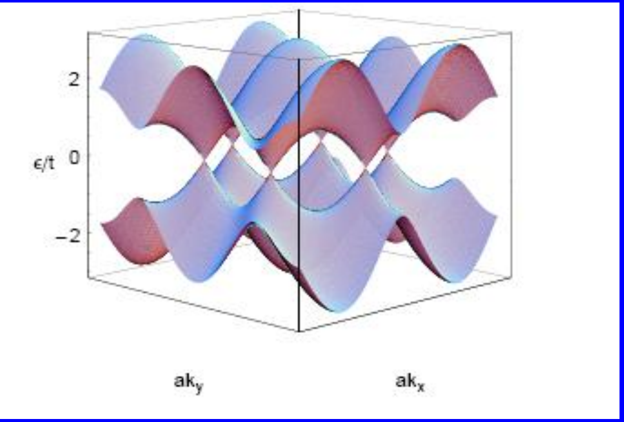
$$\sigma = \frac{J}{E} = -\frac{e^2}{2\hbar} \sum_{BZ} \left\{ t \frac{d^2 \varepsilon}{dk_y^2} + \left(\frac{h^* h' - h^*{}' h}{2\varepsilon^2} \right)^2 \sin\left(\frac{2\varepsilon}{\hbar} t \right) \right\}$$

The divergent part cancellation and contributions from regions between Dirac points

The term proportional to time has vanishing coefficient upon integration over BZ, since it can be chosen to exhibit periodicity in the field direction.

$$\sigma_{div} = -\frac{e^2}{2\hbar}\sum_{BZ} t \frac{d^2\varepsilon}{dk_y^2}$$

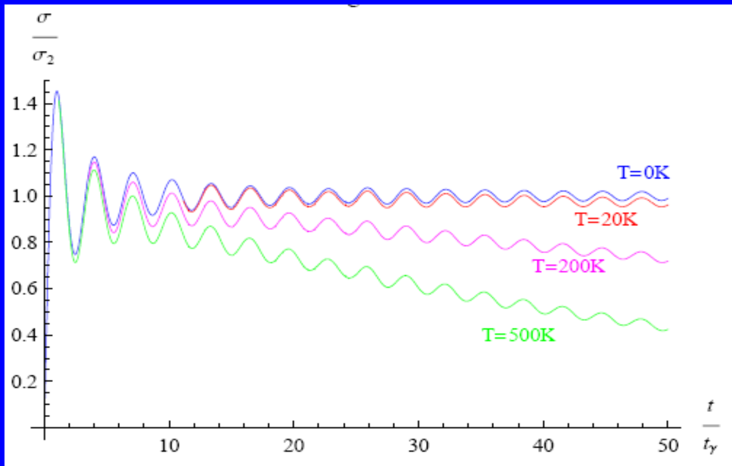
If one uses the Dirac model instead both Dirac points contribution diverge in UV and have the same sign.



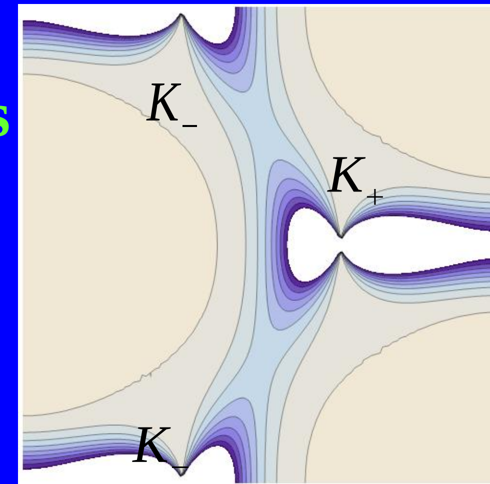
The use of effective Dirac model therefore hinges on using chirally invariant UV regularization and one cannot just multiply the result by number of valleys.

This is related to chiral (parity in 2+1) anomaly which predicts that certain relations between observable quantities in quantum field theory are markedly different from naively expected in classical field theory.

The finite part



The second term is dominated by the two Dirac points

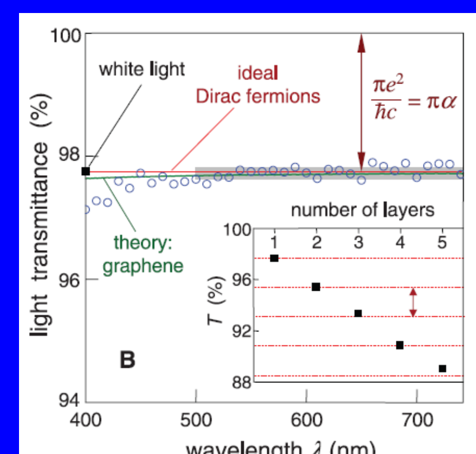


and approaches the dynamical value

$$\sigma = -\frac{e^2}{2\hbar}\sum_{BZ}\left(\frac{h^*h' - h^*{}'h}{2\varepsilon^2}\right)^2 \sin\left(\frac{2\varepsilon}{\hbar}t\right) \to \sigma_2 = \frac{\pi}{2}\frac{e^2}{h}$$

rather than the standard DC one $\sigma_1 = \frac{4}{\pi}\frac{e^2}{h}$ **obtained using disorder strength as a regularization**

Geim, Novoselov et al, *Science 102 10451 (08)*

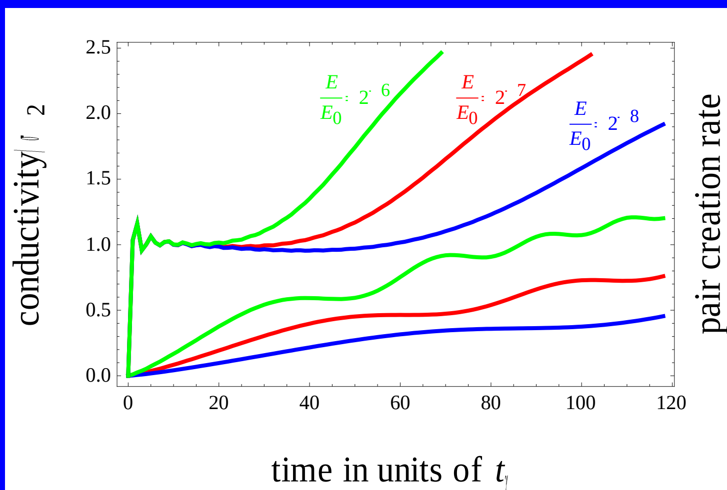


It turns our that it is independent of frequency for in the whole range

$$\omega << 1/t_\gamma$$

4. Beyond linear response.

The tight binding model can be solved exactly.

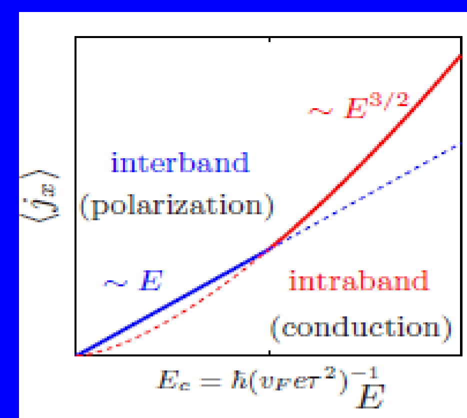


Electric field is given in microscopic units of $E_0 = \gamma / ea = 10^{10} V / m$

The crossover time from the linear response into a linear dependence

$$J(t) = \frac{2}{\pi^2} e v_g^{1/2} \left(\frac{eE}{\hbar} \right)^{3/2} t$$

is $t_{nl} = \sqrt{\hbar / eEv_g}$

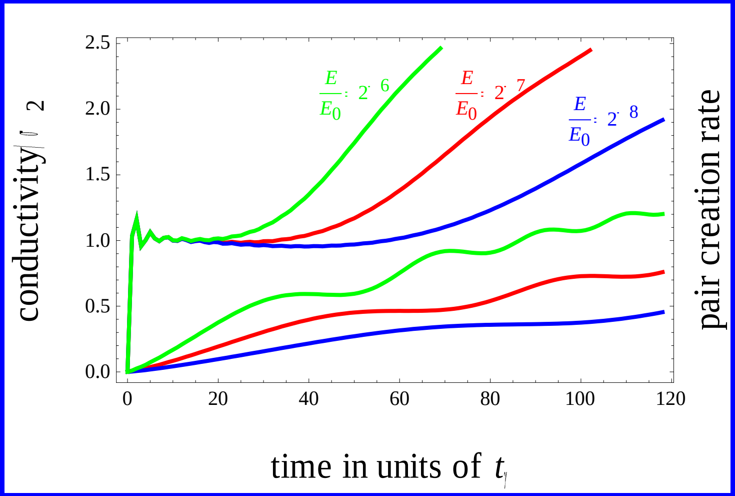


This is consistent with the third order calculation

$$\sigma = \sigma_2 \left[1 + \frac{3}{64} \left(\frac{Et^2}{t_\gamma} \right)^2 \right]$$

B.R., H. C. Kao, Lewkowicz, Kornijenko, PRB81, R041416 *(2010)*

Pair creation rate and Schwinger's formula



Vicinity of Dirac points dominate the pair creation rate and follows the 2D generalization of the Schwinger's asymptotic rate at zero mass limit

Schwinger, *PR82, 664 (1951)*

$$N(t) = \sum_{BZ} \left|\langle \psi | v_k \rangle\right|^2 ; \quad v_k = \begin{pmatrix} 1 \\ -h^* / \varepsilon \end{pmatrix}$$

$$\frac{dN}{dt} = \frac{1}{\pi^2} \frac{1}{v_g^{1/2}} \left(\frac{eE}{\hbar} \right)^{3/2}$$

This would lead to electron – hole plasma. Are these fields and ballistic times feasible?

$$t_{nl} = t_b = L / v_g = 2000 t_\gamma$$

$$L = 0.5 \mu m, W = 1.5 \mu m \Rightarrow E_{pl} = 10^4 V / m, I = mA$$

A simple WKB derivation

At each k_x one solves, using for example WKB approximation, the Klein tunneling problem similar to that in the Landau – Zener transition.

Gavrilov, Gitman, *PRD53, 7162 (1986)*

The interband transition probability at large times is

$$n_k = \theta\left(k_y\right)\theta\left(eEt - k_y\right)\exp\left[-\frac{\pi v_g k_x^2}{\hbar eE}\right]$$

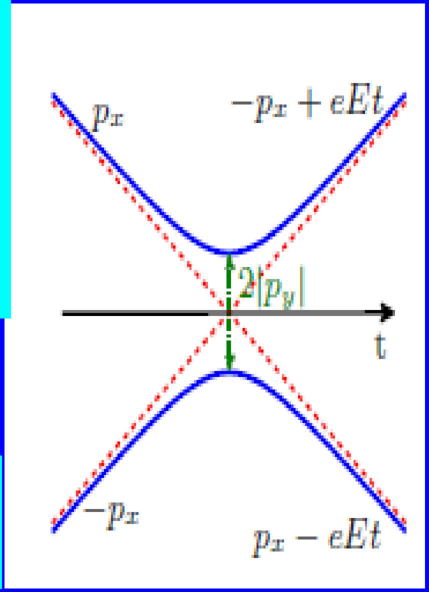
$$N(t) = \frac{4}{(2\pi)^2}\int_{k_x}\theta\left(k_y\right)\theta\left(eEt/\hbar - k_y\right)$$

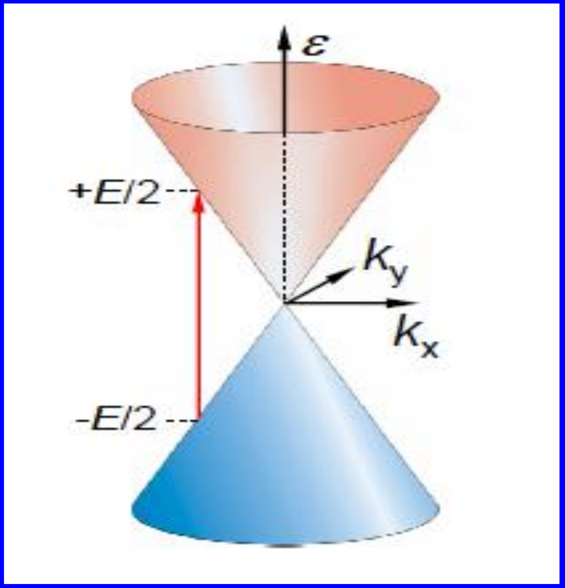
$$\int_{k_y}\exp\left[-\frac{\pi v_g k_x^2}{\hbar eE}\right]$$

$$= \frac{1}{\pi^2}\frac{eEt}{\hbar}\sqrt{\frac{\hbar eE}{v_g}} = \frac{1}{\pi^2}\left(\hbar v_g\right)^{-1/2}\left(eE\right)^{3/2} t$$

The current density:

$$J(t) = 2ev_g N(t) = \frac{2e}{\pi^2}\sqrt{v_g/\hbar}\left(eE\right)^{3/2} t$$



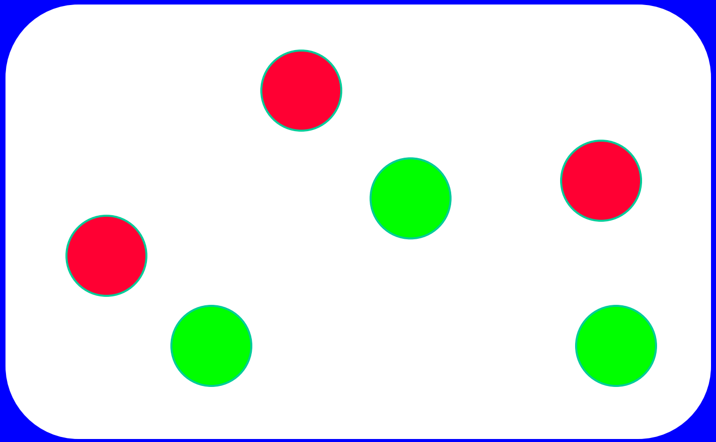


5. Where the energy and the pairs go in suspended graphene?

The universal conductivity value is obtained assuming no contributions from scattering off impurities, phonons or electron – electron interactions. Therefore the energy is released in the leads.

$$P = \frac{d\varepsilon_{tot}}{dt} = 2i\left\langle \psi \,\middle|\, \frac{dH}{dt} \,\middle|\, \psi \right\rangle = EJ$$

The number of pairs also continuously increases beyond the crossover time till some annihilation channel is open when density of pairs becomes large enough.



Following are some of the options for the energy and pairs before the ballistic time runs out:

I. Impurity scattering effectively absorbing energy without affecting electric conductivity. Experiments suggest that this possibility is not realized

Cheianov, Falco, Altshuler ,Aleiner, PRL *(2006)*

2. Electron – electron interactions in suspended graphene is not screened, but current calculations give just correction of order 1% to conductivity due to poorly understood cancellations

Mishchenko, cond-mat *(2010)*.

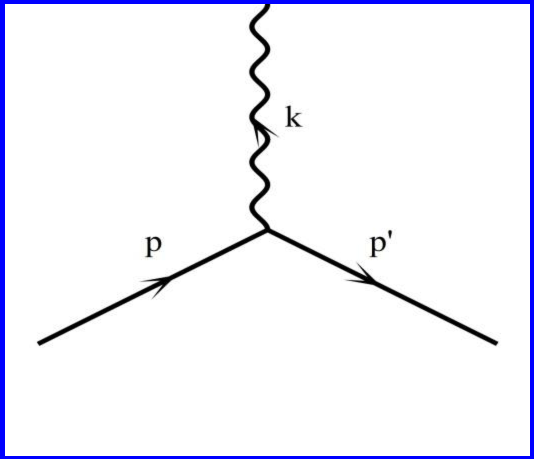
3. Phonon or ripplon scattering.

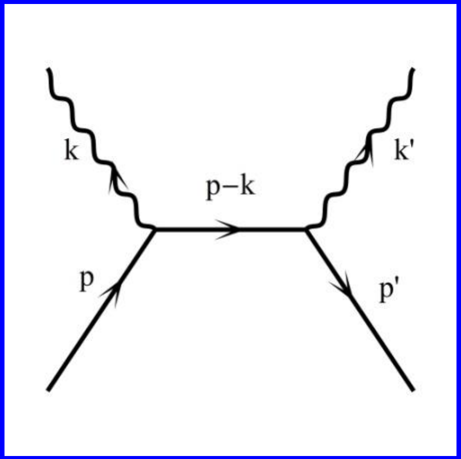
Morozov et al, *PRL100, 016602 (2008)*

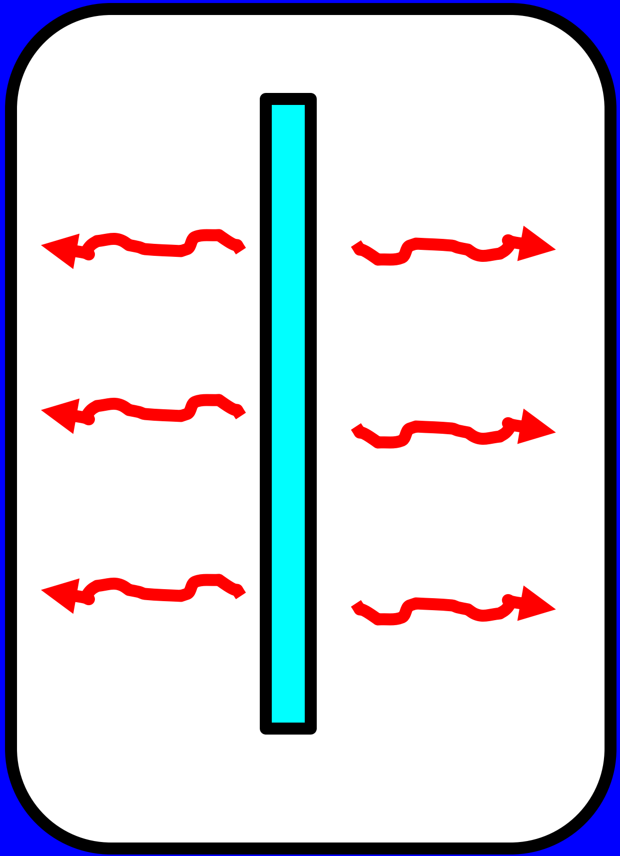
4. Electromagnetic radiation friction. Energy is radiated away and the elecron – hole plasma state is formed.

Radiation friction.

To write Boltzmann equations for the electron – hole plasma one calculates the anihillation rate. The leading diagram in QED would be emission of two photons:







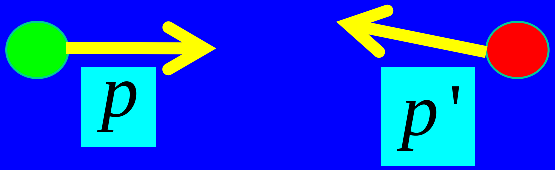
The one photon emission would be forbidden by the energy and momentum conservation.
However the momentum should not be conserved in perpendicular directions (just creation of ripples)

In addition graphene velocity is much smaller than velocity of light and there is an additional small parameter

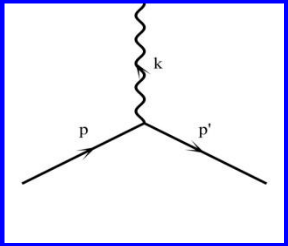
$$v/c = 1/300 \sim \alpha_{QED}$$

This leads to the phase space restriction on the one photon processses

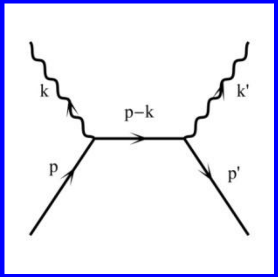
$$1 - 2\frac{v}{c} < \frac{p'}{p} < 1 + 2\frac{v}{c}$$



Still the one photon process is dominant:



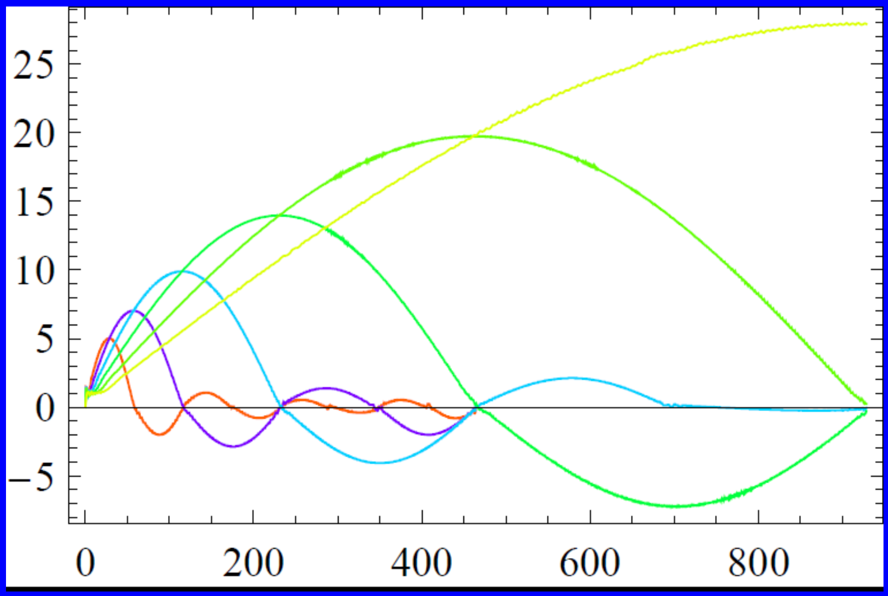
$$\frac{dN(p)}{dt} = \frac{16\pi^2 \alpha_{QED}}{3} p \left(\frac{v_g}{c}\right)^3 N^2$$

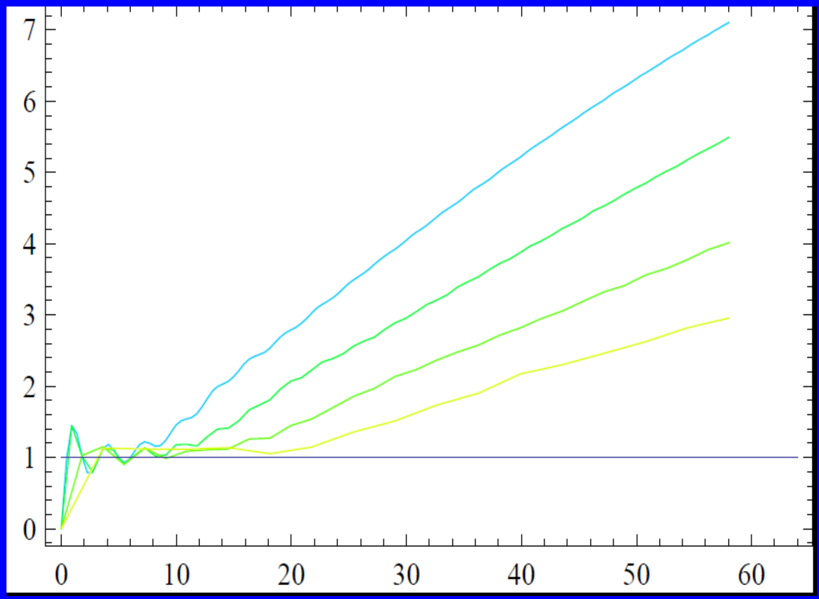


$$\frac{dN(p)}{dt} \propto \alpha_{QED}^2 p \left(\frac{v_g}{c}\right)^3 N^2$$

What if ballistic motion continues?

The tight binding model can be solved exactly using Floquet theory.





It exhibits Bloch oscillations at time scale.

$$t_B = \frac{8\pi}{\sqrt{3}}\frac{E_0}{E}t_\gamma$$

It is much larger than the scale t_{nl} and is perhaps not achievable.

Time scales for purely ballistic motion

1. Basic microscopic time scale is

$$t_\gamma = \hbar / \gamma = 10^{-16}\,\text{sec}$$

2. The crossover time from the linear response into a linear dependence is

$$t_{nl} \sim \sqrt{\frac{E_0}{E} t_\gamma}$$

Where electric field is given in units of microscopic field

$$E_0 = \gamma / ea = 10^{10} V / m$$

3. Period of Bloch oscillations is

$$t_B \sim \frac{E_0}{E} t_\gamma$$

Extentions and open questions.

1. Finite samples. Within the Buttiker - Landauer formalism, using the “zitterbewegung” phenomena aspect ratio dependent conductivity was obtained. Can this be verified within the dynamical approach?

Katzenelson, Eur. Phys. J. B51, 157 (06)
Tworzydlo et al PRL 96, 246802 (06)

While the noise experiments in NSG show remarkable agreement with this theory, transport on suspended graphene apparently does not

Danneauet al, PRL100, 196802 *(08)*

2. Double – layered graphene. Dynamical approach was extended to the double – layered graphine in the the pair creation is even more effective

Y.-X. Wang, L.-P. Shi, S.-J. Xiong, EPL 87, 57002 *(09)*

Conclusions

1. **DC conductivity is equal to its “dynamical” value.**
2. **It is independent of frequency (at zero temperature) all the way up to UV.**
3. **There exists a time thresholds beyond which the linear response crosses over into a linear conductivity rise and fast Schwinger’s pair creation phase sets in leading to formation of electron – hole plasma.**
4. **Mechanism of energy release in the plasma is not clear, the radiation friction is an option.**

Linear response to DC field switched on at t=0

The current generally has two contributions

$$j = -\frac{e}{\hbar} h'(k) - \left(\frac{e}{\hbar}\right)^2 \frac{1}{c} h''(k) A_t \equiv j_p + j_d$$

where $$\vec{A}_t = -c\left(0, E \cdot t \cdot \theta(t)\right)$$

Expanding to first order $$\psi_k = u_k + E\xi_k; \qquad j_k = j_k^0 + E\sigma_k$$

Zero order vanishes upon integration

$$j_k^0 = -\frac{ec}{2\hbar}\begin{pmatrix} 1 & -h/\varepsilon \end{pmatrix}\begin{pmatrix} 0 & h' \\ h^{*}{}' & 0 \end{pmatrix}\begin{pmatrix} 1 \\ -h^{*}/\varepsilon \end{pmatrix} = \frac{ec}{2\hbar}\frac{h^{*}h' + h^{*}{}'h}{\varepsilon} = \frac{ec}{\hbar}\varepsilon'$$

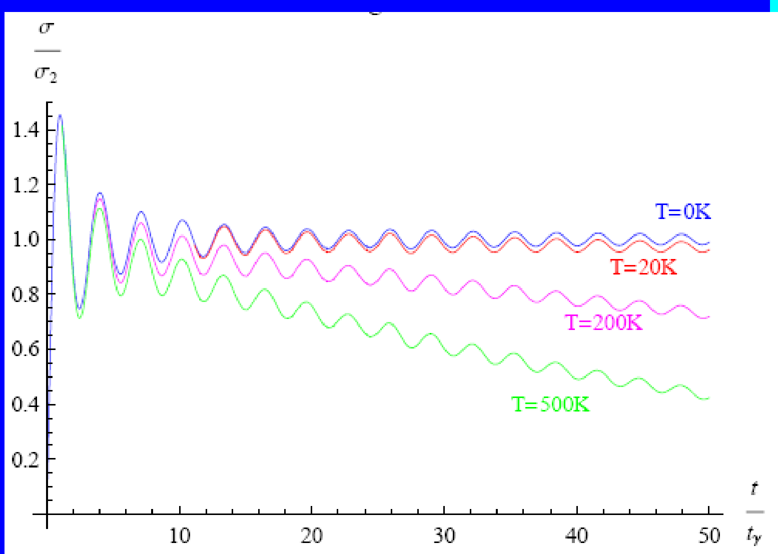
$$J^0 = \int_{B.z.} j_k^0 = \frac{ec}{\hbar}\left[\varepsilon\left(k_y^{\max}\right) - \varepsilon\left(k_y^{\min}\right)\right] = 0$$

First order can be presented as

$$\sigma_k^p = -\frac{e}{\hbar} u_k^+ \begin{pmatrix} 0 & h' \\ h^{*}{}' & 0 \end{pmatrix} \xi_k - \frac{e}{\hbar} \xi_k^+ \begin{pmatrix} 0 & h' \\ h^{*}{}' & 0 \end{pmatrix} u_k$$

$$\sigma_k^d = \left(\frac{e}{\hbar}\right)^2 t \cdot u_k^+ \begin{pmatrix} 0 & h'' \\ h^{*}{}'' & 0 \end{pmatrix} u_k$$

$$\sigma = -\frac{e^2}{4\hbar} \sum_{B.Z.} \frac{\left(h^* h' - h^*{}' h\right)^2}{\varepsilon^4} \frac{1}{2} \sin\left(\frac{2\varepsilon}{\hbar} t\right)$$



Approaches the "dynamical" value!

$$\sigma_2 = \frac{\pi}{2} \frac{e^2}{h}$$

Which is larger than the standard DC value

$$\sigma_1 = \frac{4}{\pi} \frac{e^2}{h}$$

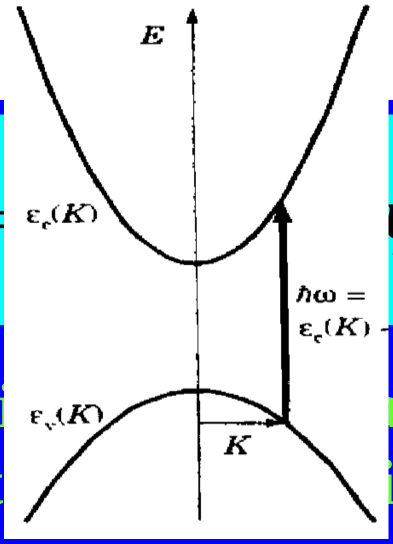
Lewkowicz, B.R., PRL102, 106802 (09)

6. Where does the energy go?

The universal conductivity is finite for zero electron density due to Dirac cones with no dissipation mechanism.

$$\varepsilon_{tot} = 2\int_{BZ} \psi_k^{\dagger}(t)\, H \psi_k(t) = 2\left(h\psi_1^*\psi_2 + h^*\psi_2^*\psi_1\right) = 2\left\langle \psi \left| H \right| \psi \right\rangle$$

$$P = \frac{d\varepsilon_{tot}}{dt} = \ \ldots\ \psi \Big\rangle = EJ$$



It is therefore linearly raisi ... r response time scale and even raises faster beyond t ... ime till the Bloch oscillations set in.

The universal conductivity value is obtained assuming no contributions from scattering off impurities , phonons or electron – electron interactions. Therefore the energy is released in the leads.