

Quantum Physics in one dimension

T. Giamarchi

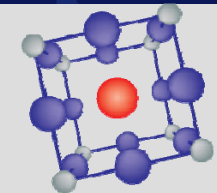
http://dpmc.unige.ch/gr_giamarchi/



**UNIVERSITÉ
DE GENÈVE**

FNSNF

FONDS NATIONAL SUISSE
SCHWEIZERISCHER NATIONALFONDS
FONDO NAZIONALE SVIZZERO
SWISS NATIONAL SCIENCE FOUNDATION



MaNEP
SWITZERLAND

A. Lobos (Geneva)

C. Kollath (Polytechnique)

M. Zvonarev (LPTMS)

P. Bouillot (Geneva)

C. Berthod (Geneva)



- Spin ladders:

A. Tsvelik (BNL), R. Chitra (Jussieu)

E. Orignac (ENS-Lyon), R. Citro (Salerno U.)

M. Klanjsek + group C. Berthier (Grenoble)

B. Thielemann + group C. Rugg (LCN/PSI) +LANL Group

D. Poilblanc , S Capponi (Toulouse), A. Läuchli (Dresden) , B. Normand

- Deconfinement:

A. Georges, S. Bierman (Polytechnique), A. Lichtenstein (Hamburg U.)

- Noise and bath:

M. Muller (ICTP), E. Demler (Harvard), E. Dalla Torre, E. Atman (Weizman)

- Ferromagnetic liquid:

V. Cheianov (Lancaster U.)

Questions

Questions

- Understood: free electrons

Questions

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- Real systems : Coulomb interaction

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- Properties of realistic systems

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- Understood: free electrons
- Real systems : Coulomb interaction
- Properties of realistic systems
- Free electron theory works quite well

Effect of interactions

Effect of interactions

- Landau Fermi liquid

Effect of interactions

- Landau Fermi liquid



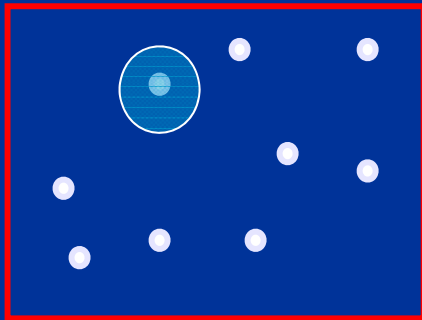
Effect of interactions

- Landau Fermi liquid
- Individual fermionic excitations exist (quasiparticles)



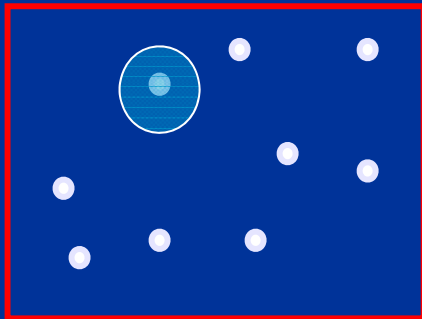
Effect of interactions

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$$m \rightarrow m^*$$

Fermi liquid theory

Fermi liquid theory

- Shown perturbatively in U

Fermi liquid theory

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- Much more general and robust

Fermi liquid theory

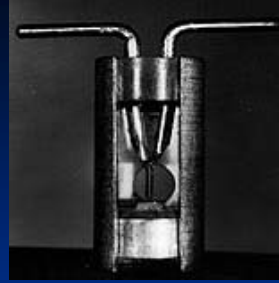
- Shown perturbatively in U
- Much more general and robust

Element	m^*/m	χ/χ_0
Nb	2	1
3He	6	20
Heavy fermion	100	100

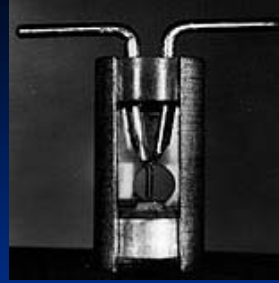


• Transistor

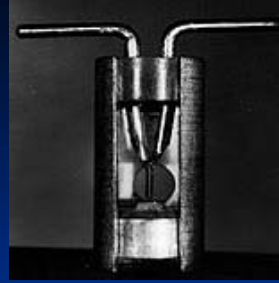
• Transistor



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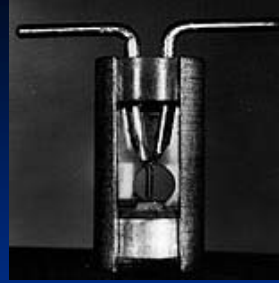


• Transistor



1956

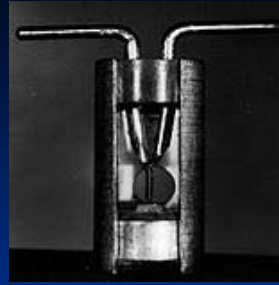
• Transistor



1956

• Supraconductivité

• Transistor



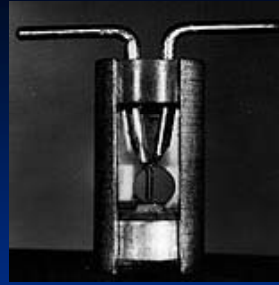
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• Supraconductivité



• Magnéto-résistance
géante

• Transistor



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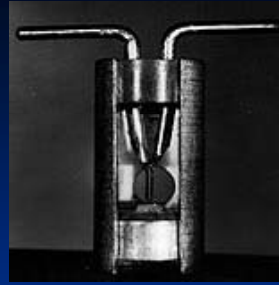
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(1913), 1972,
1973, 1987, 2003

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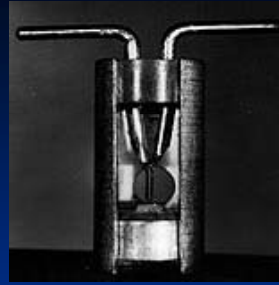
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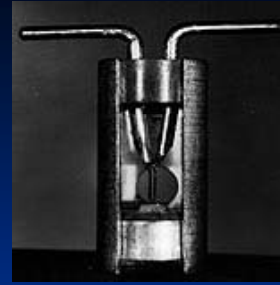
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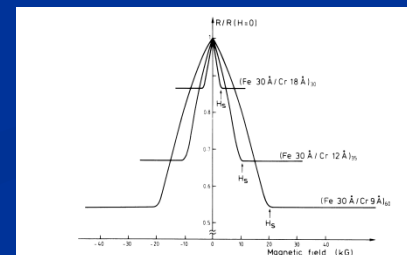
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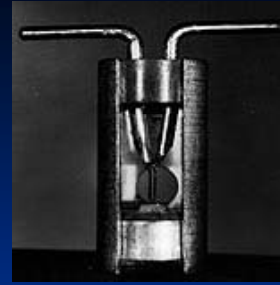


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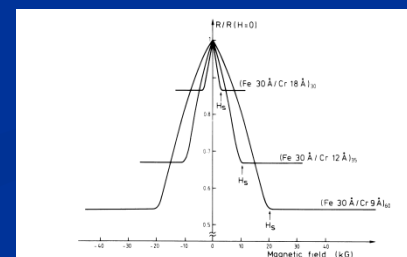
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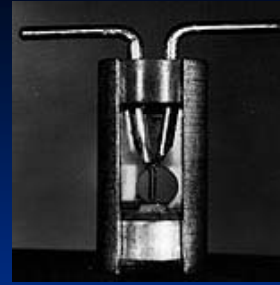


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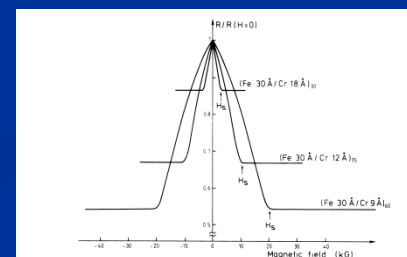
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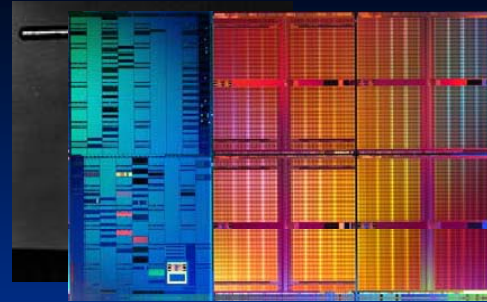
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2007

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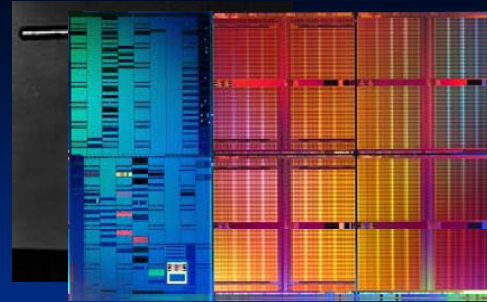
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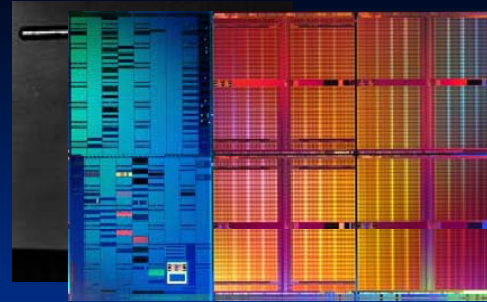
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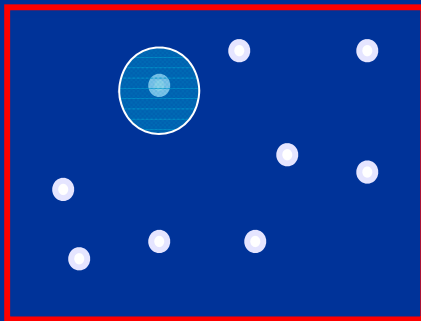


2007

One dimension is different

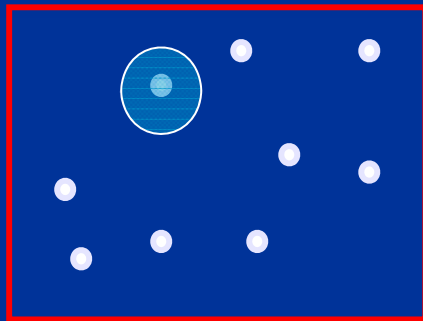
One dimension is different

- No individual excitation can exist (only collective ones)



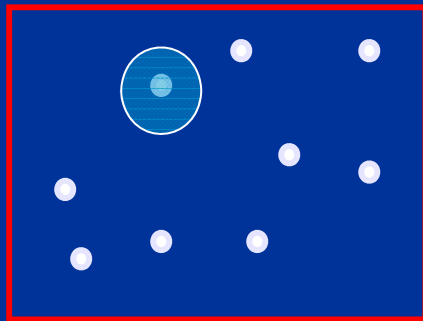
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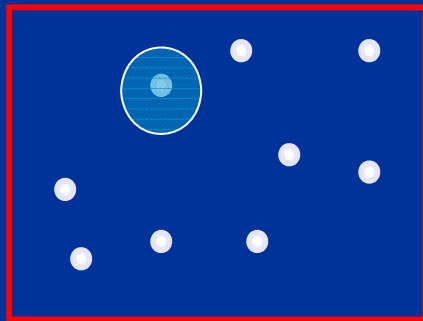
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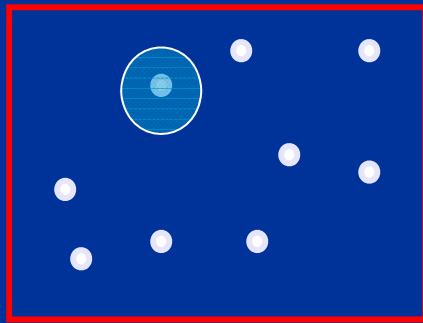
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- Strong quantum fluctuations

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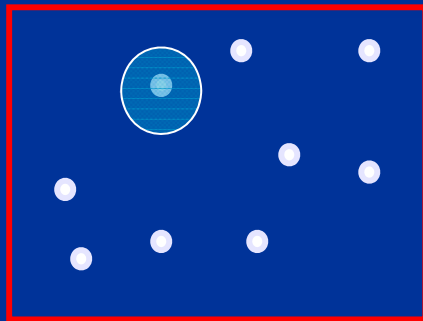


- Strong quantum fluctuations

$$\psi \rightarrow \psi e^{i\theta}$$

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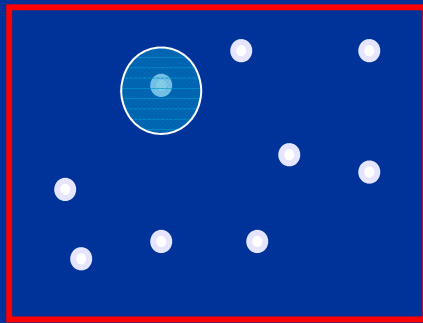
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Continuous symmetry

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Continuous symmetry

An exciting theoretical problem

How to make a theory

How to make a theory



How to make a theory



Tomonaga



How to make a theory



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Luttinger

60': A. Larkin, I. Dzialoshinskii, L. Gorkov, Bichkov

E. Lieb, D. Mathis

70': A. Luther, V.J. Emery

80': F.D.M. Haldane

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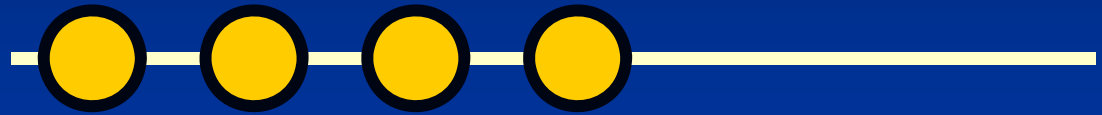
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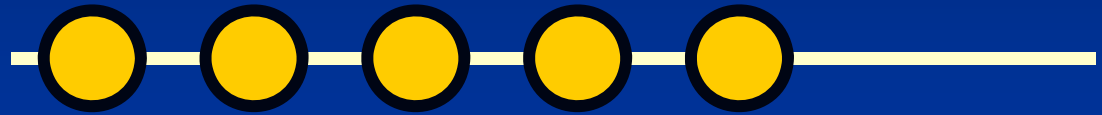
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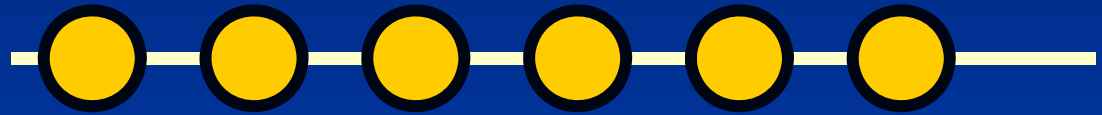
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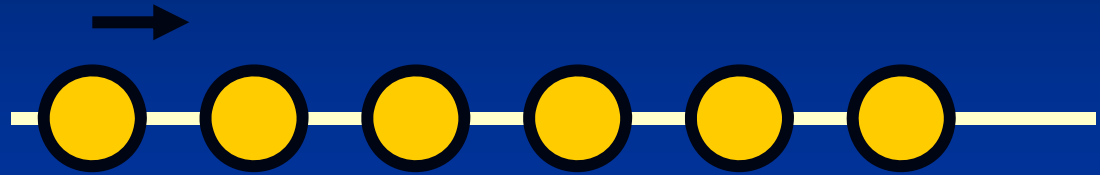
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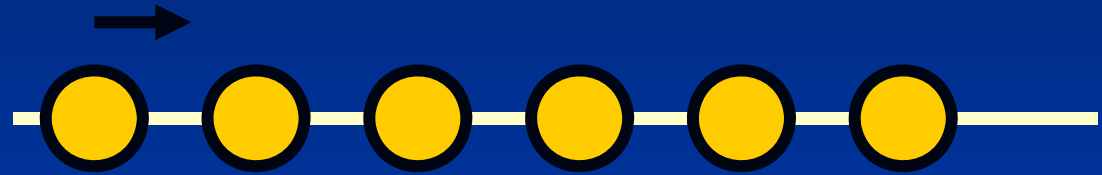
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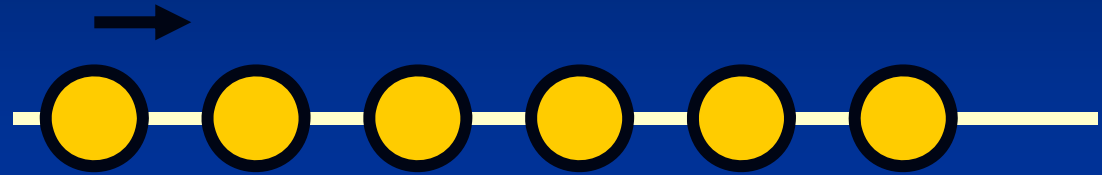
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Good excitations: collective ones

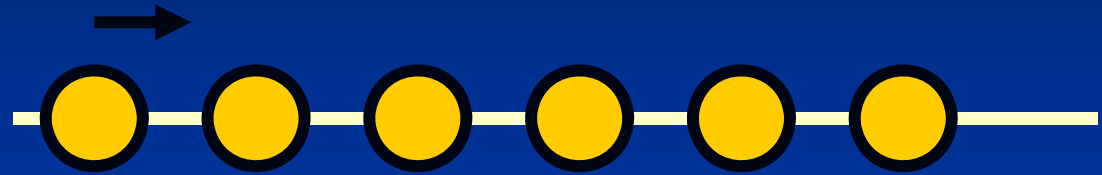
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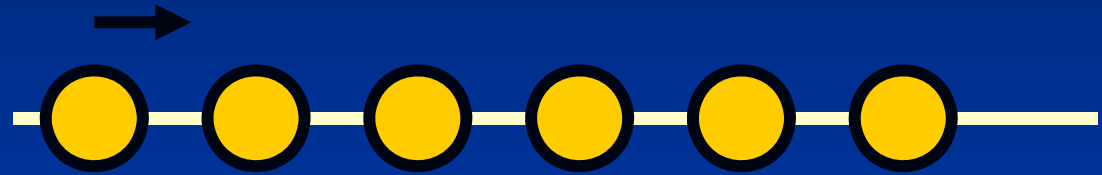
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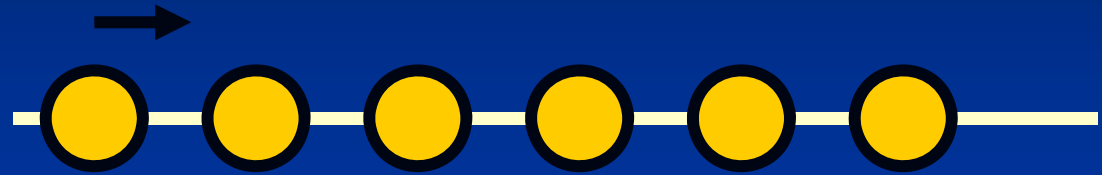
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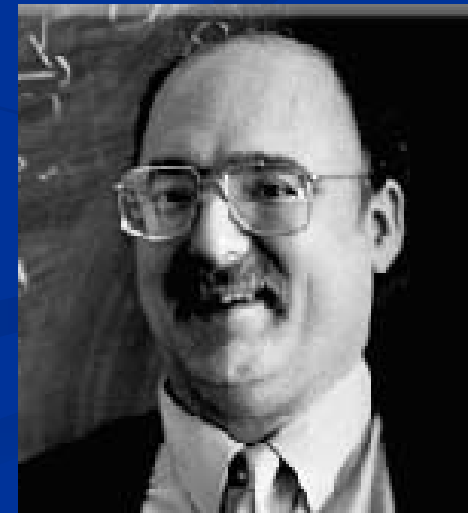
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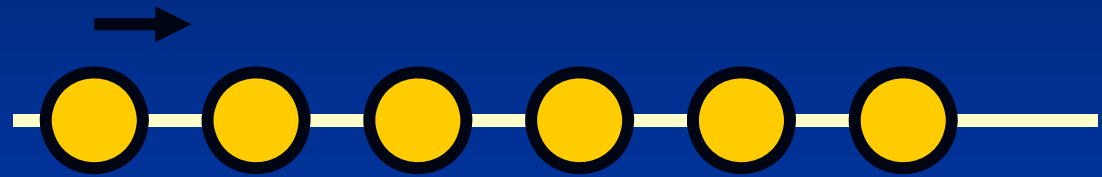
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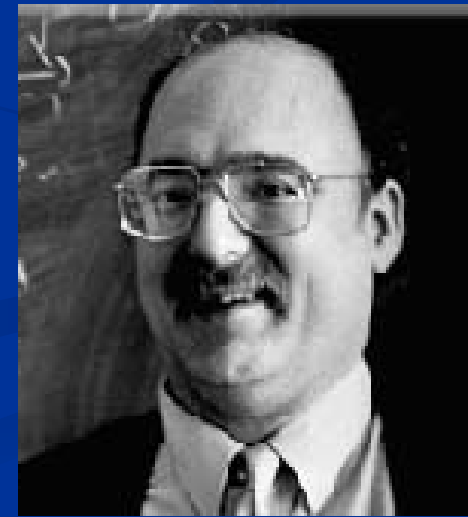
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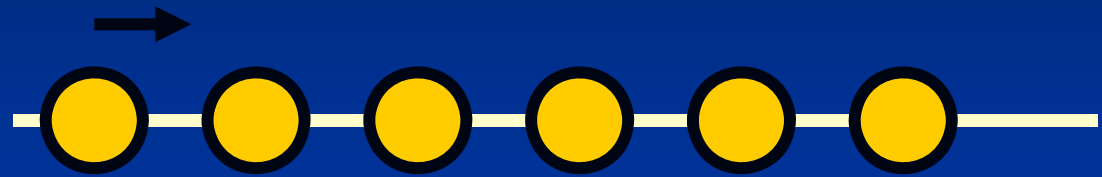
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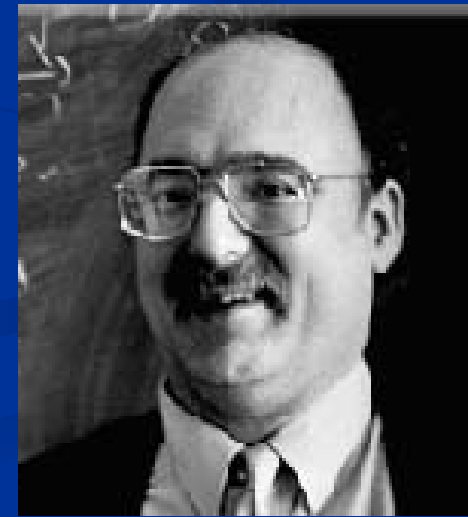
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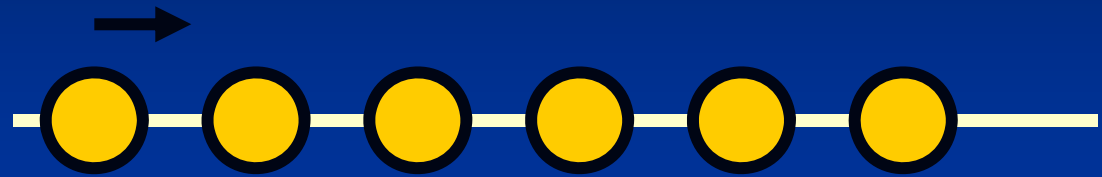
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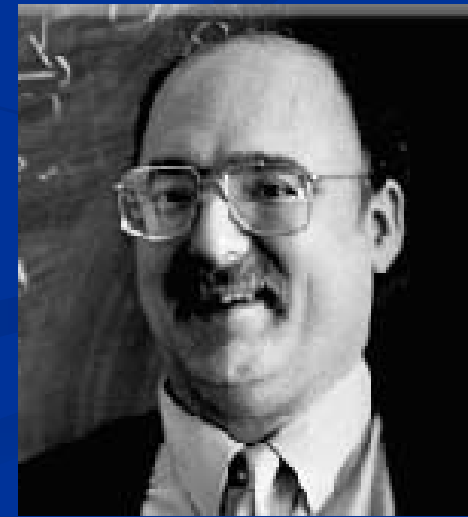
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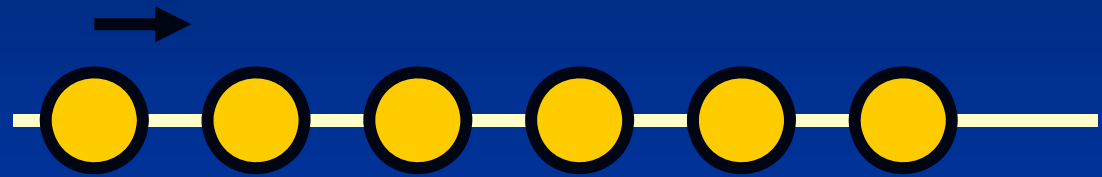
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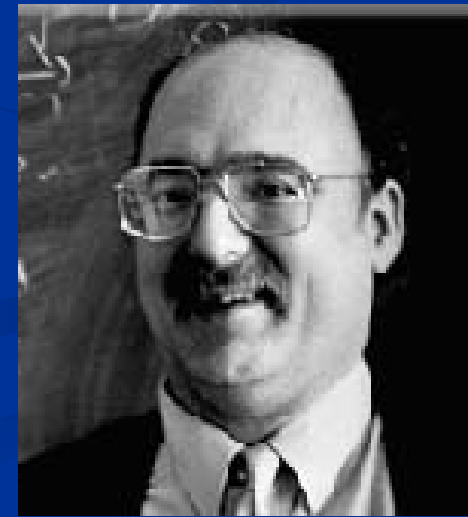
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Universal description: Luttinger liquid

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- *All* effects described by *two* parameters:
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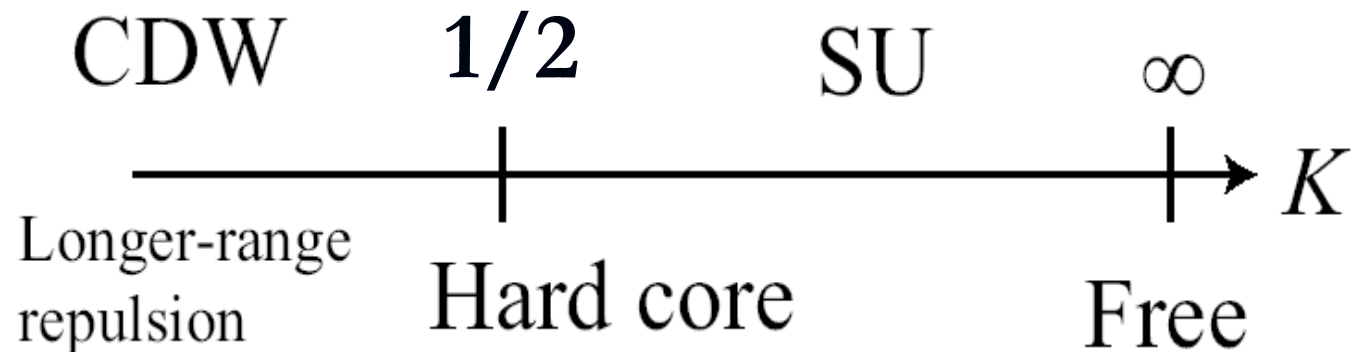
Universal description: Luttinger liquid

- Low energy effective description
- **All** effects described by **two** parameters:
u and K (Luttinger liquid parameters)
- Power law correlation functions

Power-law correlations

$$\langle \psi(r)\psi^\dagger(0) \rangle = A_1 \left(\frac{\alpha}{r} \right)^{\frac{1}{2K}} + \dots$$

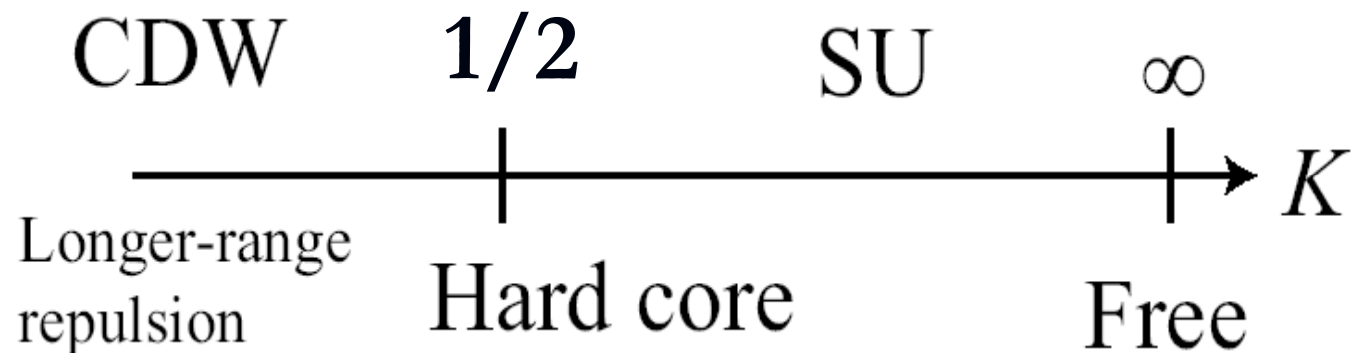
$$\langle \rho(r)\rho(0) \rangle = \rho_0^2 + \frac{K}{2\pi^2} \frac{y_\alpha^2 - x^2}{(y_\alpha^2 + x^2)^2} + A_3 \cos(2\pi\rho_0 x) \left(\frac{1}{r} \right)^{2K} + \dots$$



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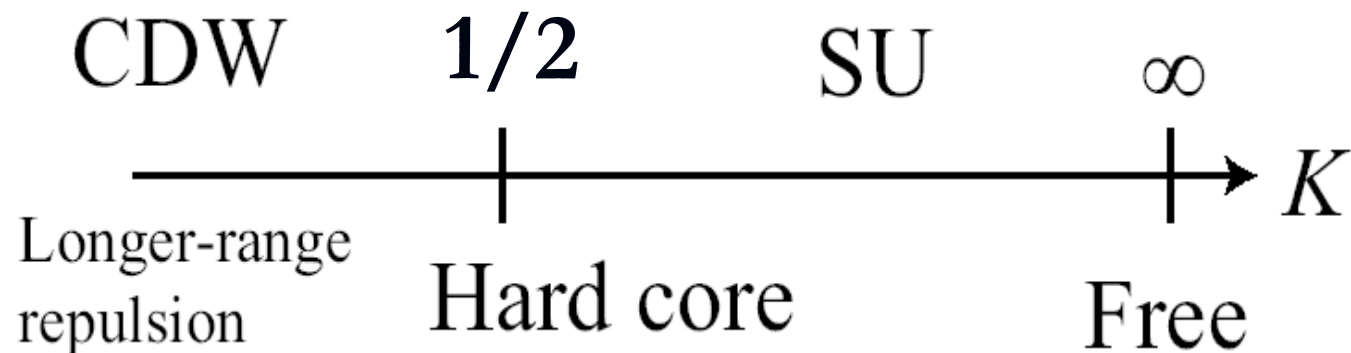
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Luttinger parameters

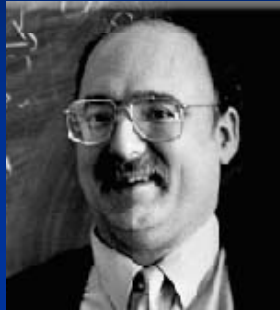
**General Relation of Correlation Exponents and Spectral Properties of
One-Dimensional Fermi Systems: Application to the Anisotropic**

$S = \frac{1}{2}$ Heisenberg Chain

F. D. M. Haldane

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Luttinger parameters

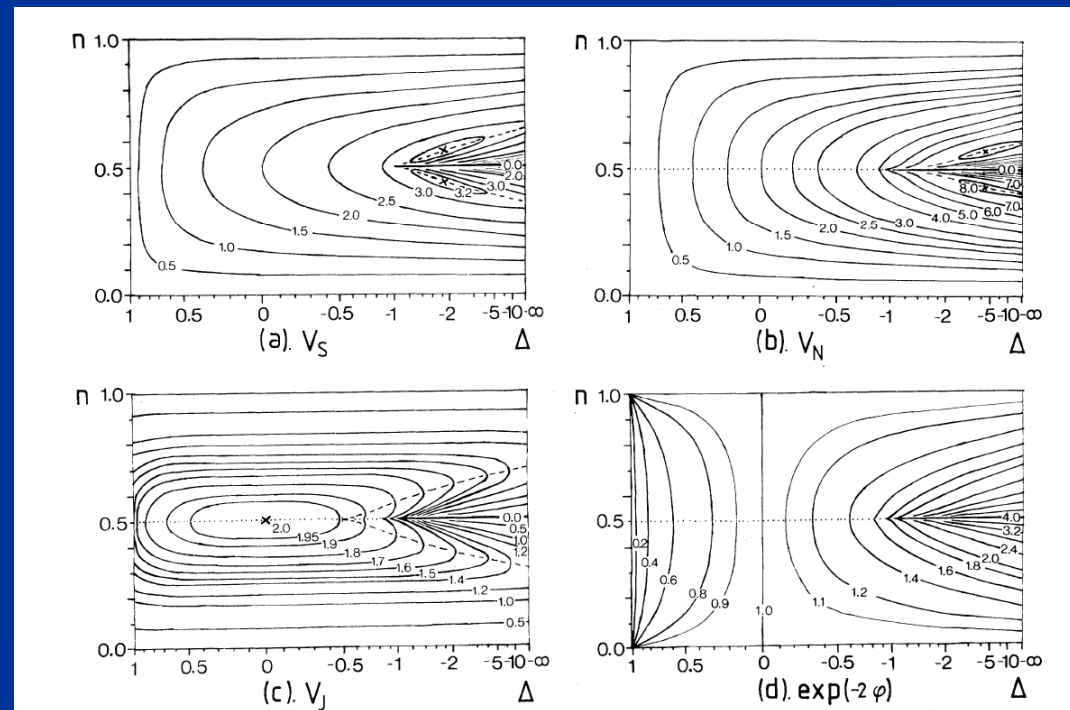


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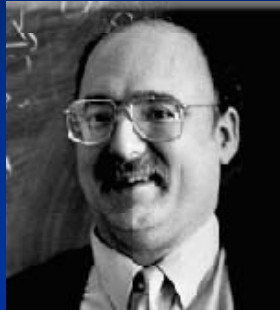
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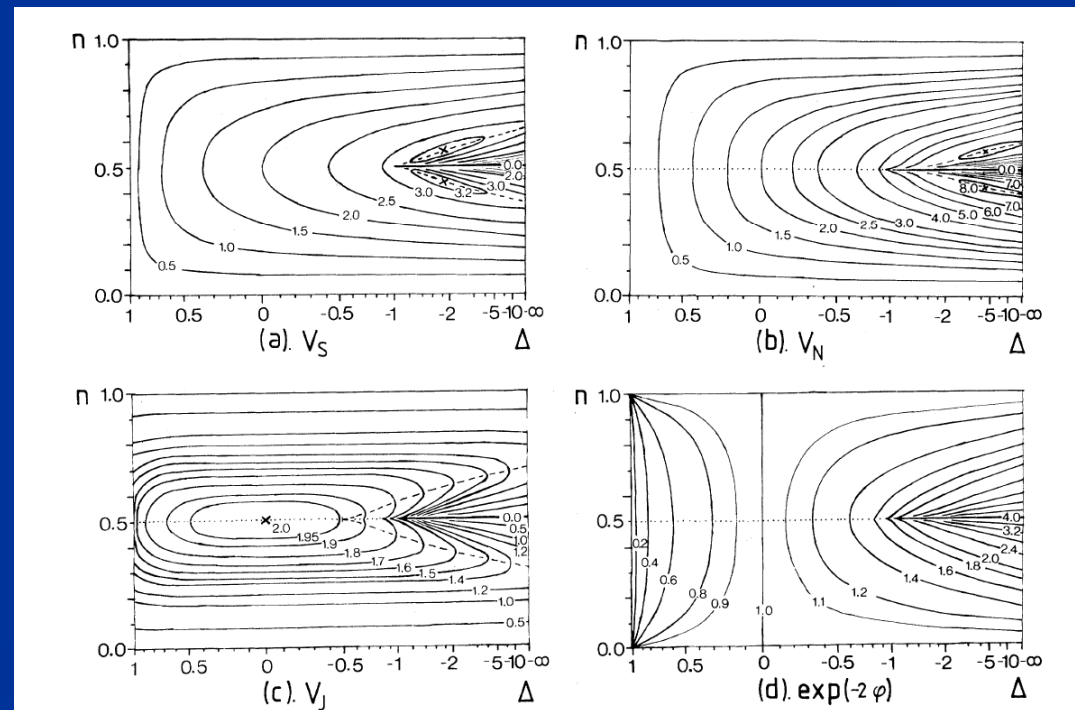


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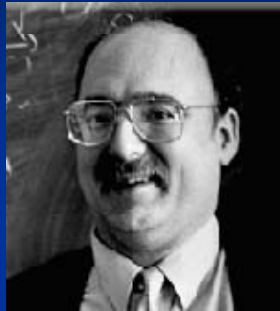
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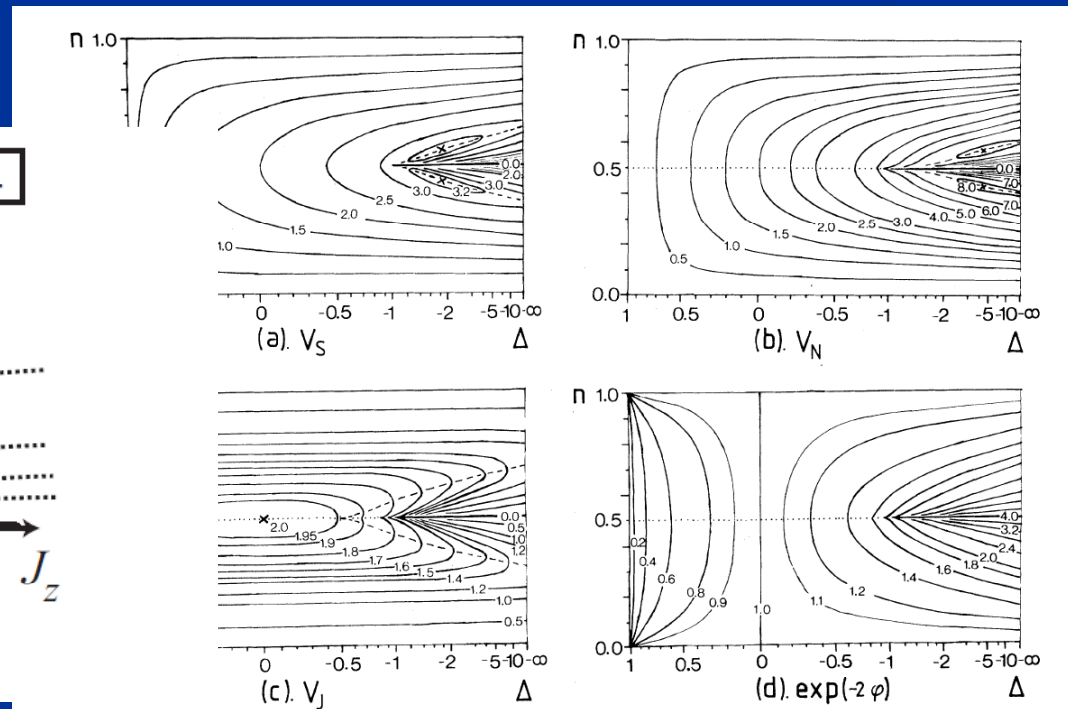
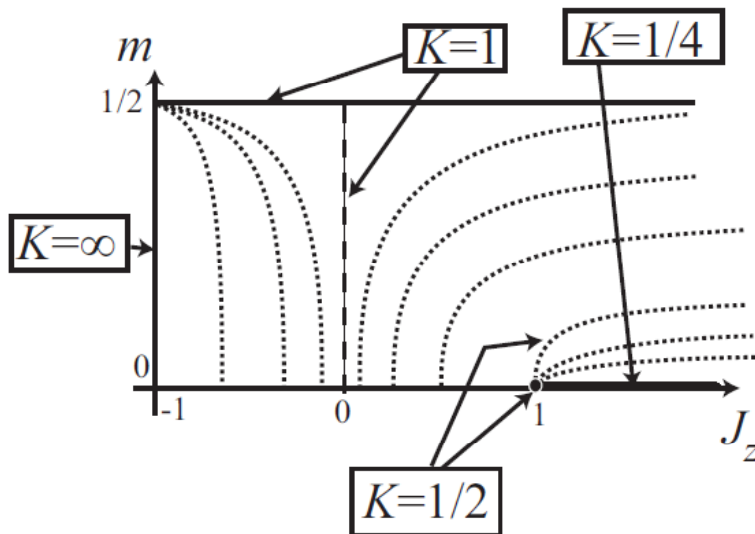
Luttinger parameters



General Relation of Correlation Exponents and Spectral Properties of One-Dimensional Fermi Systems: Application to the Anisotropic $S = \frac{1}{2}$ Heisenberg Chain

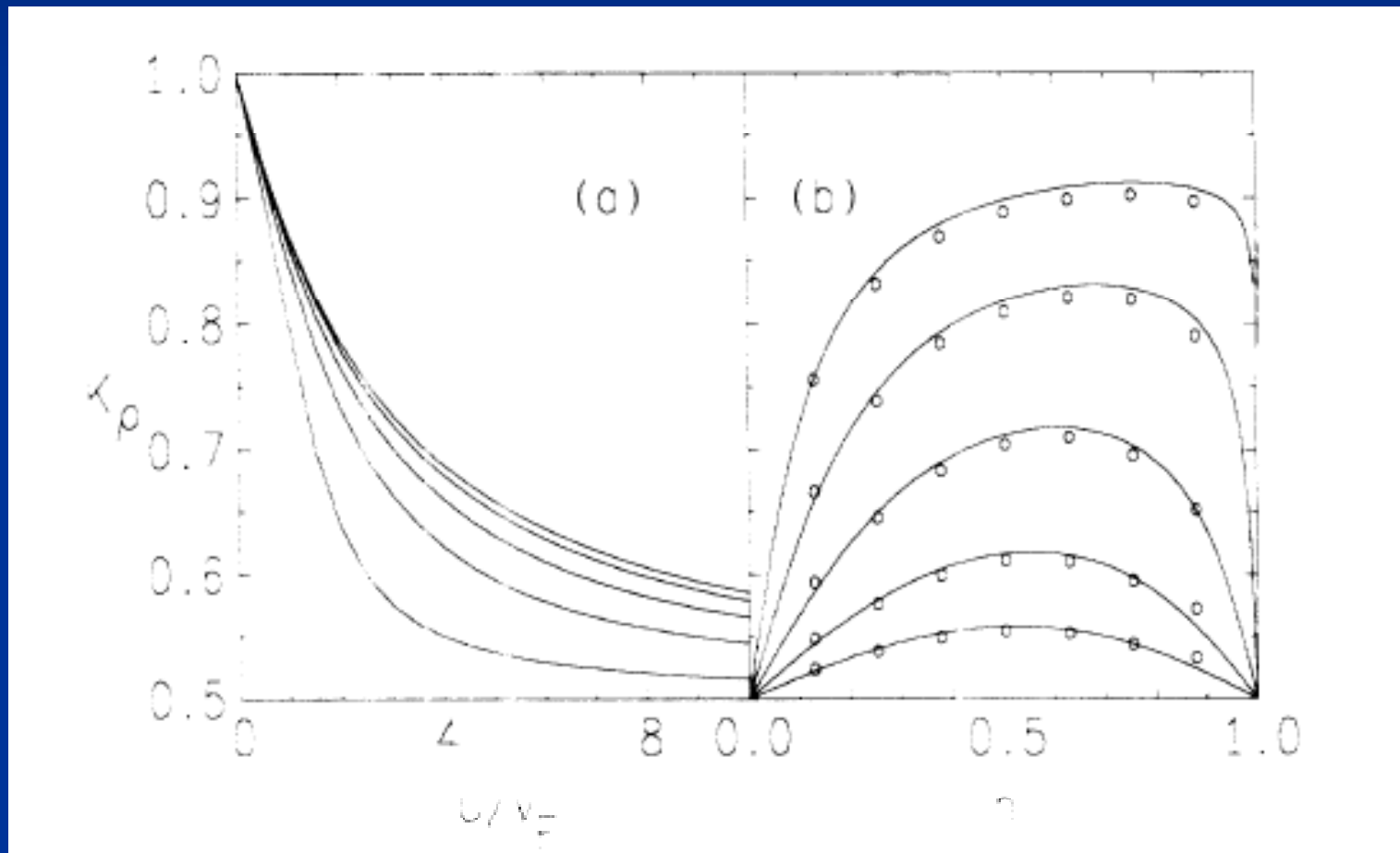
F. D. M. Haldane

Haldane, F. D. M. (1980). *Phys. Rev. Lett.*, **45**, 1358.



Correlation Exponents and the Metal-Insulator Transition in the One-Dimensional Hubbard Model

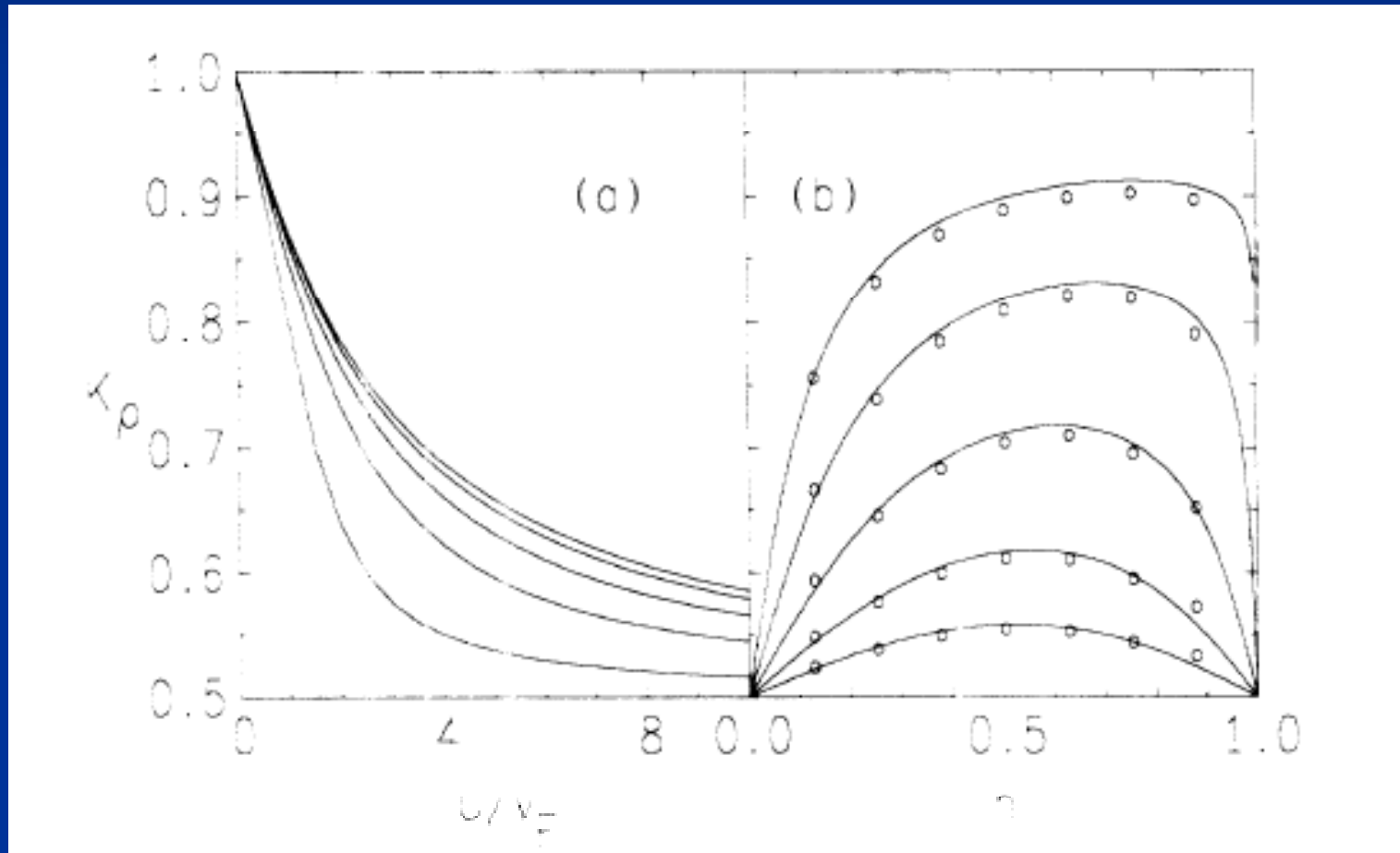
H. J. Schulz





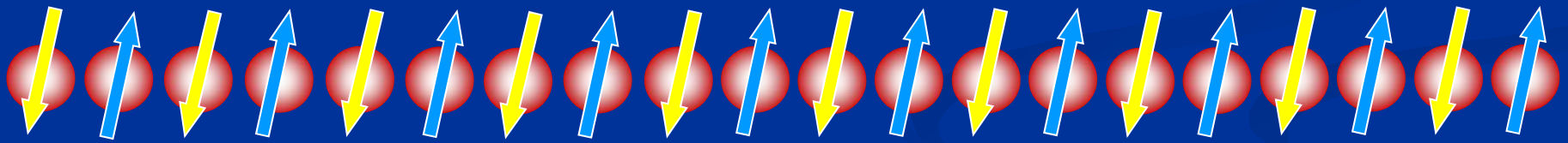
Correlation Exponents and the Metal-Insulator Transition in the One-Dimensional Hubbard Model

H. J. Schulz



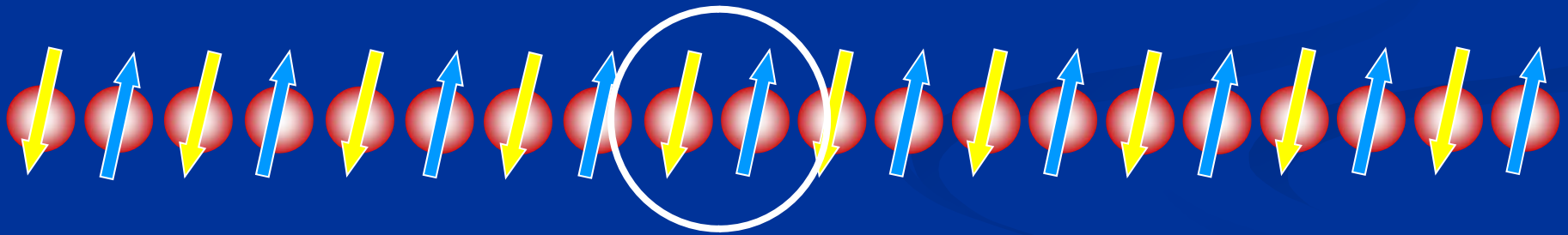
Fractional excitations

Spin



Fractional excitations

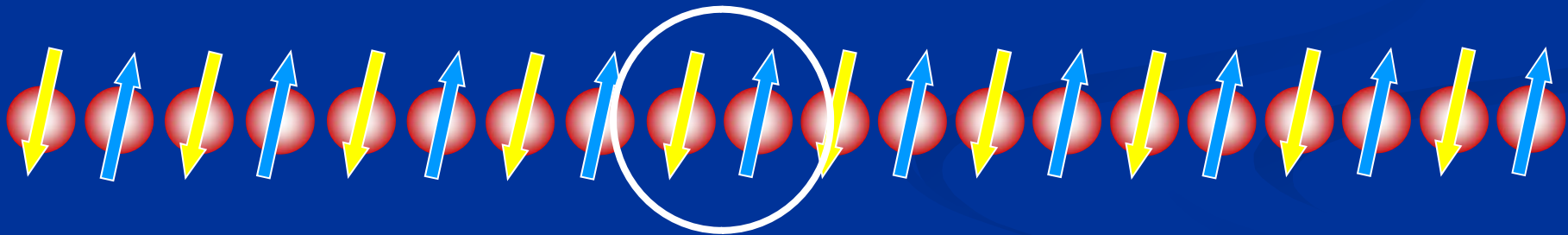
Spin



Fractional excitations

Spin

Charge

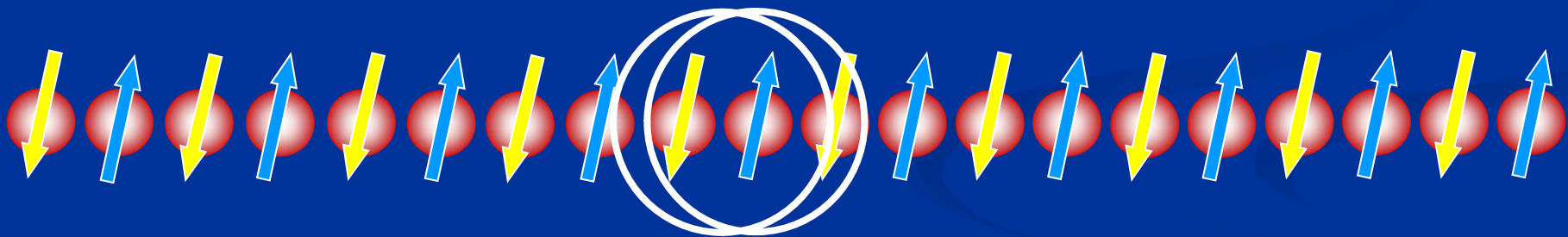


Spinon

Fractional excitations

Spin

Charge

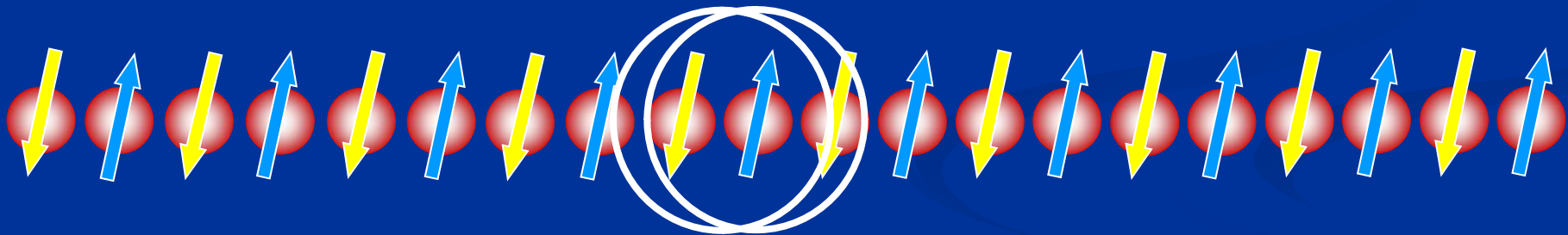


Spinon

Fractional excitations

Spin

Charge

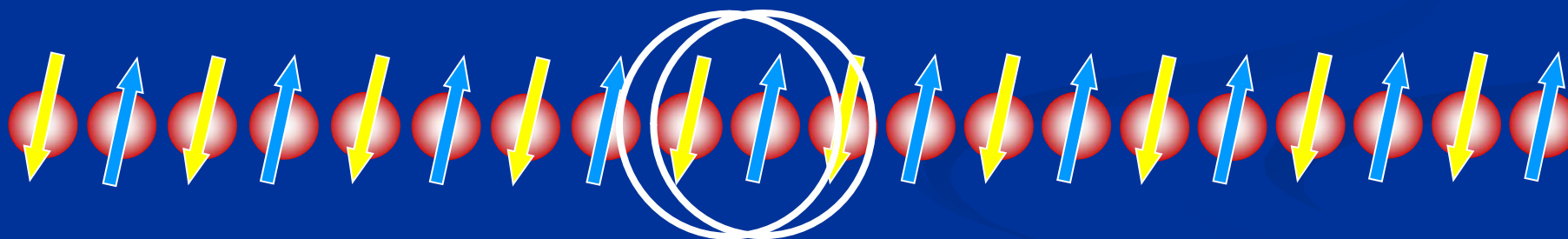


Spinon

Fractional excitations

Spin

Charge

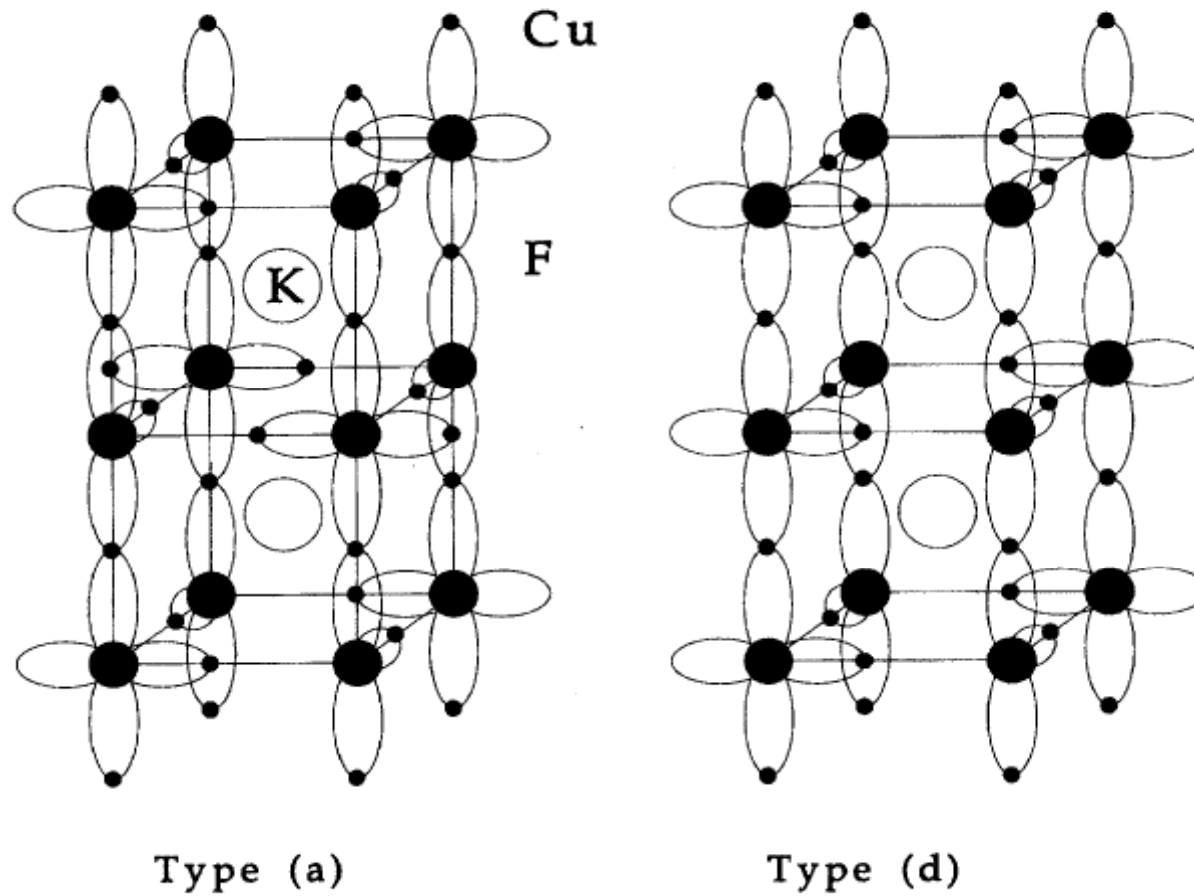


Spinon

Holon

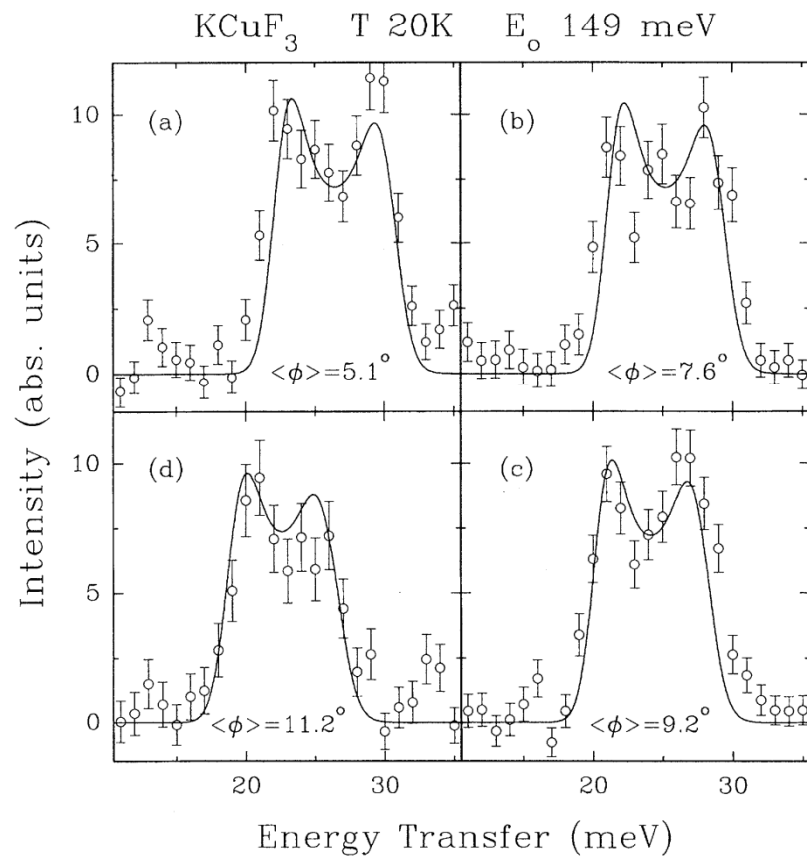
Does Luttinger Liquid exists ?

Spin chains

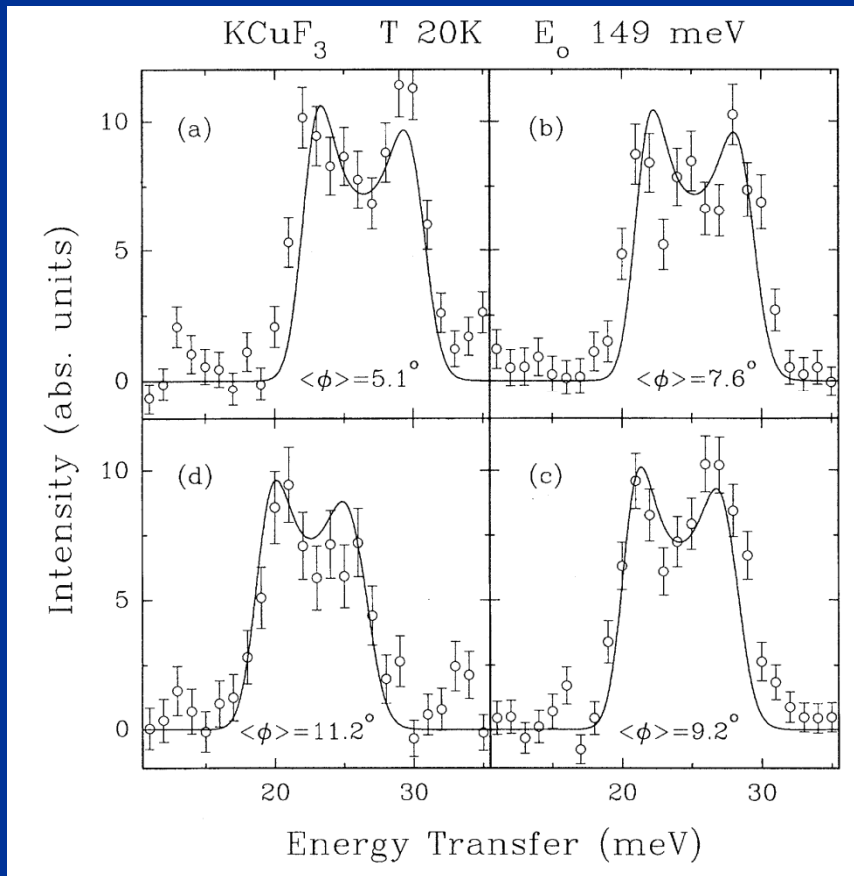


Hint in spin systems

Hint in spin systems

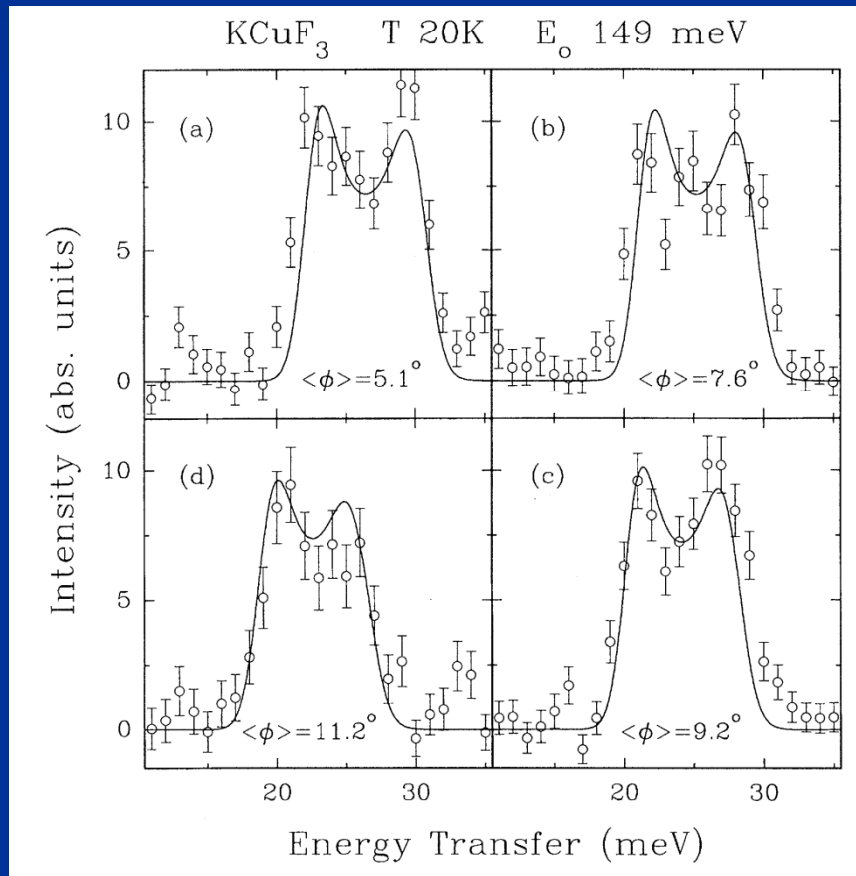


Hint in spin systems



Tennent et al. PRB 52 13368 (1995)

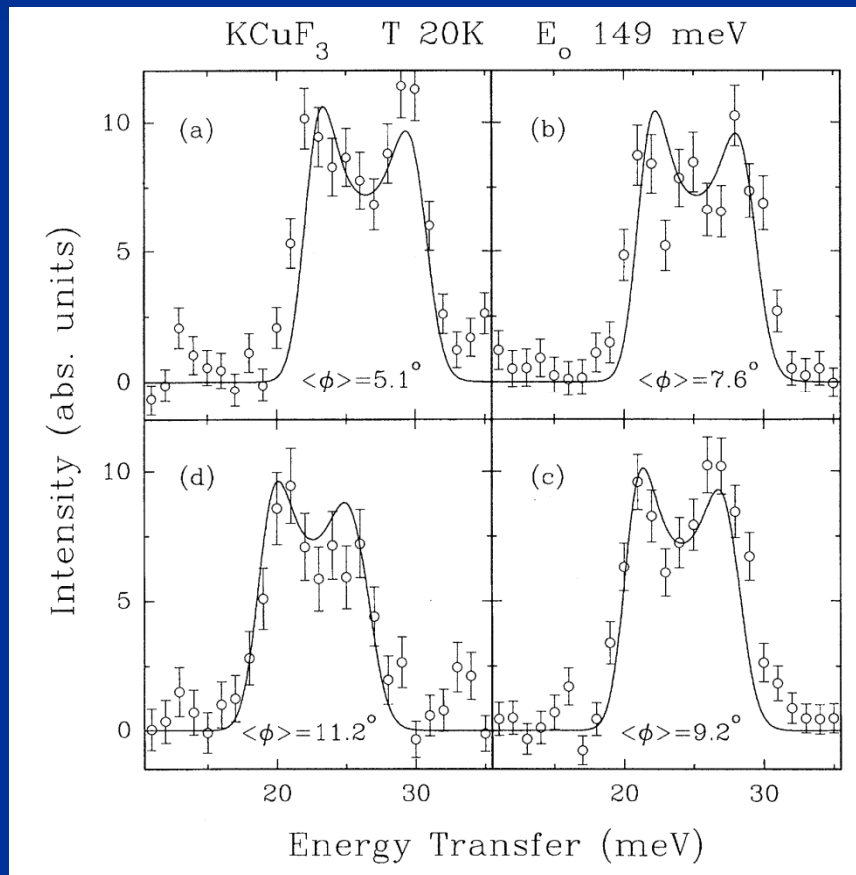
Hint in spin systems



H. J. Schulz PRB 34 6372 (1986)

Tennent et al. PRB 52 13368 (1995)

Hint in spin systems



H. J. Schulz PRB 34 6372 (1986)

Lineshape:

FT of power law better

than two

gaussian/Lorentzian peaks

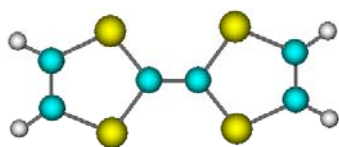
Tennent et al. PRB 52 13368 (1995)

Organic conductors

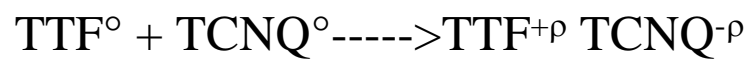
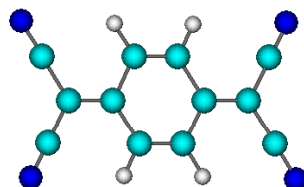
1973: F.Wudl, D.Cowan, A.Garito, A.Heeger

Organic conductors

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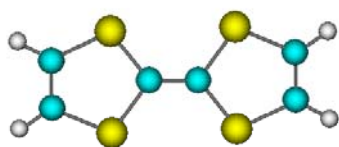


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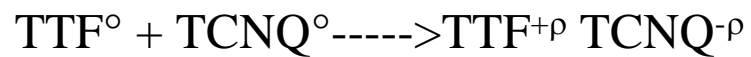
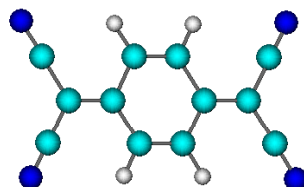


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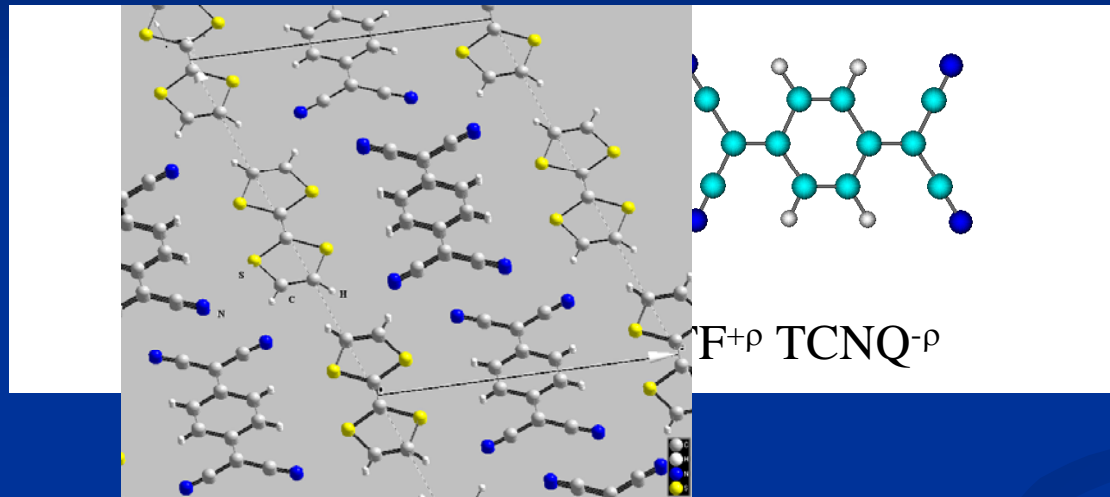


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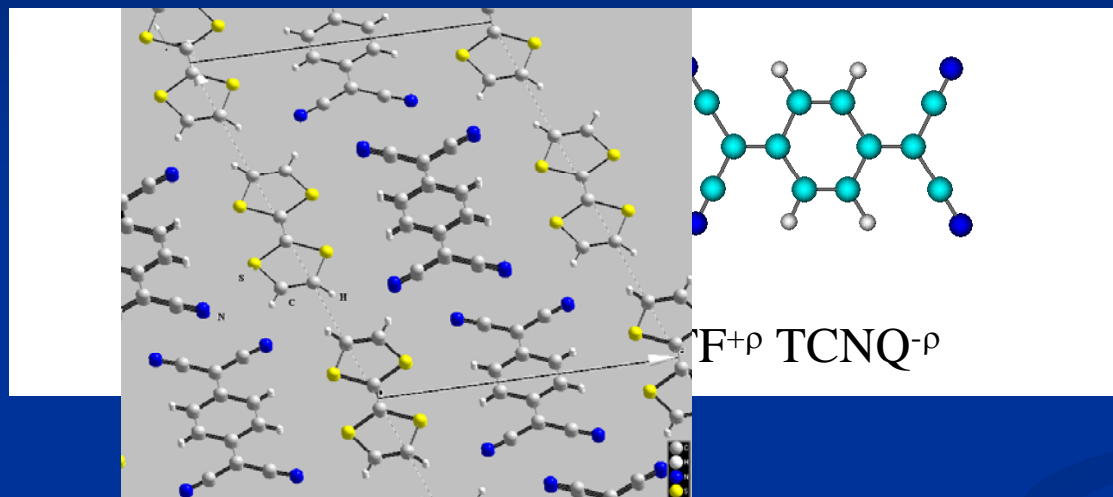
Organic conductors

1973: F.Wudl, D.Cowan, A.Garito, A.Heeger



Organic conductors

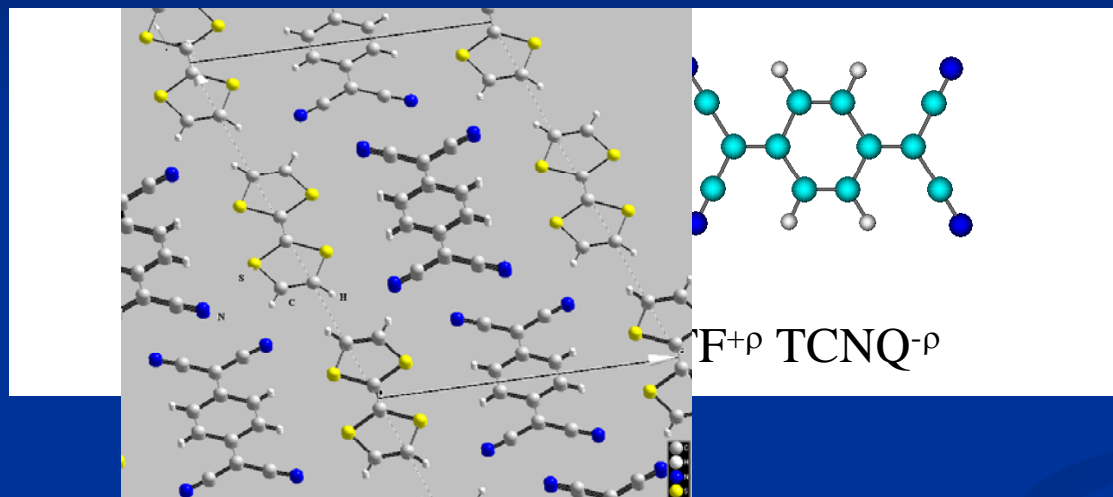
1973: F.Wudl, D.Cowan, A.Garito, A.Heeger



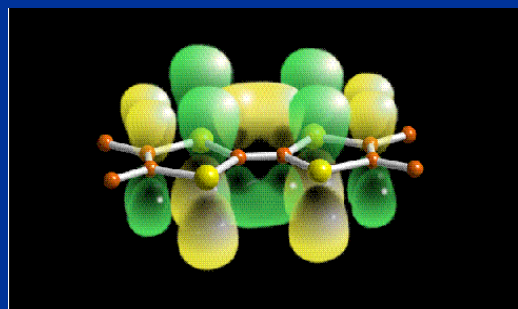
1979: K. Bechgaard

Organic conductors

1973: F.Wudl, D.Cowan, A.Garito, A.Heeger

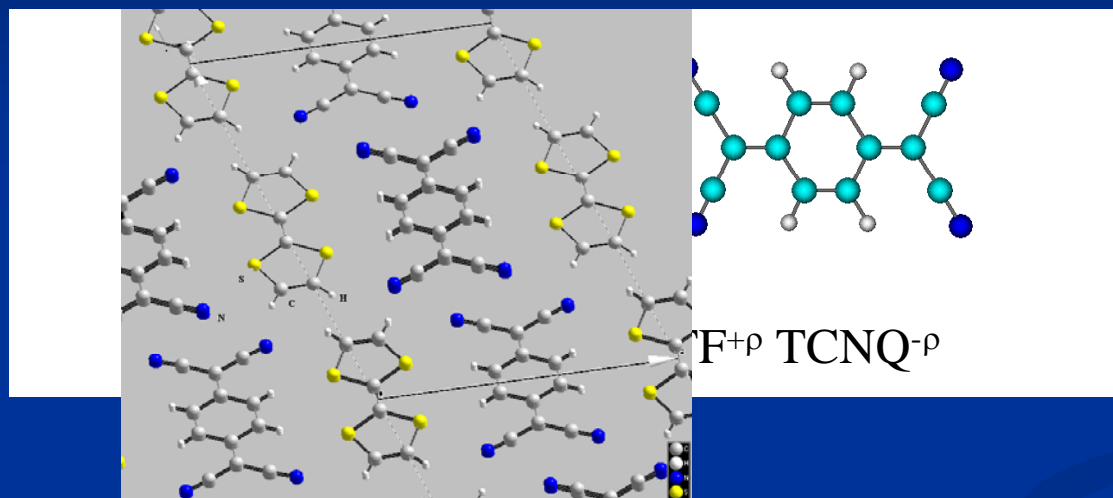


1979: K. Bechgaard

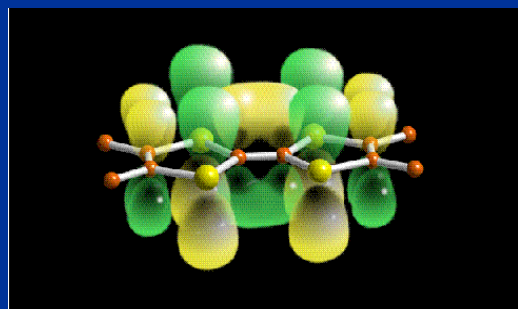


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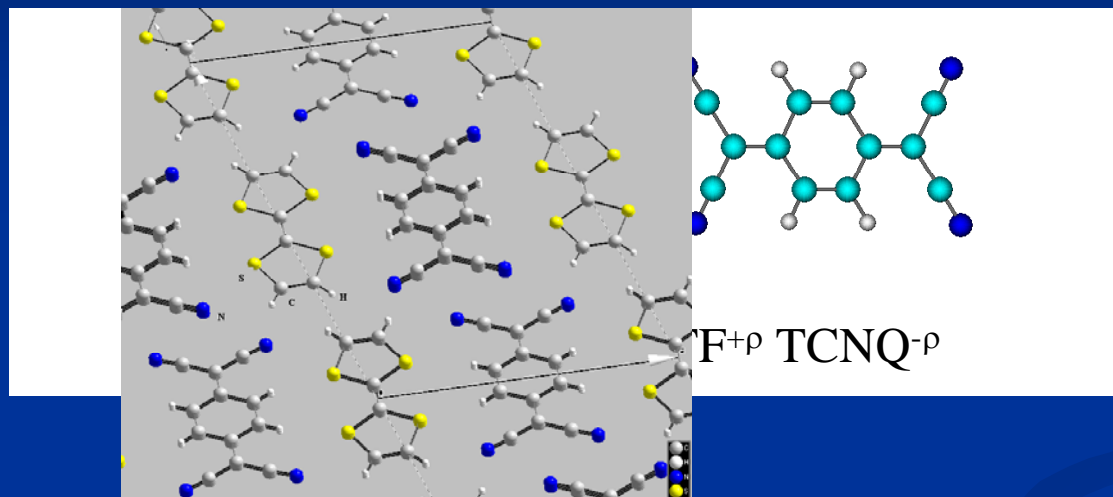
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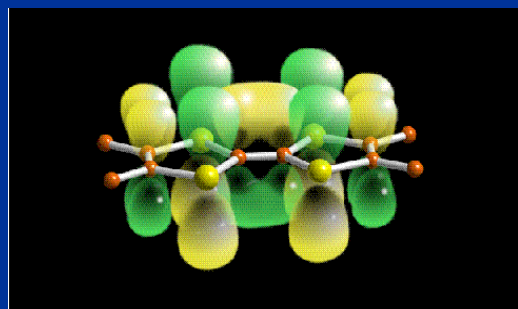
TMTSF

Organic conductors

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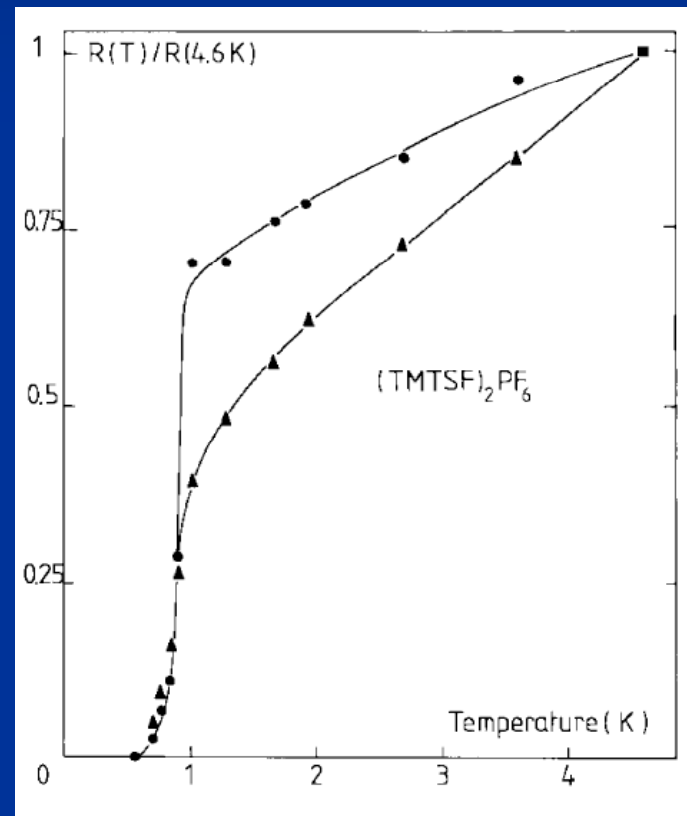
TMTSF



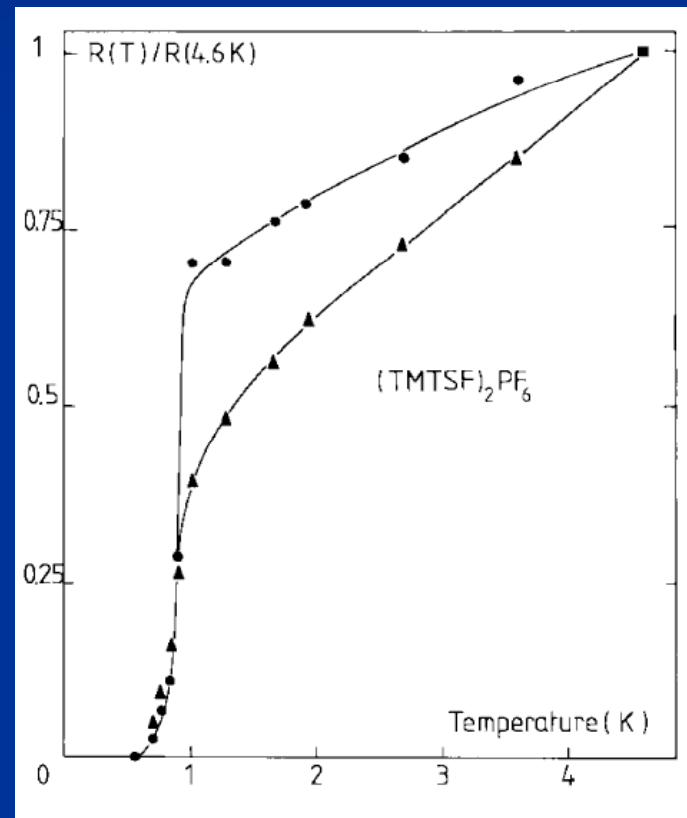
Superconductivity !



Superconductivity !



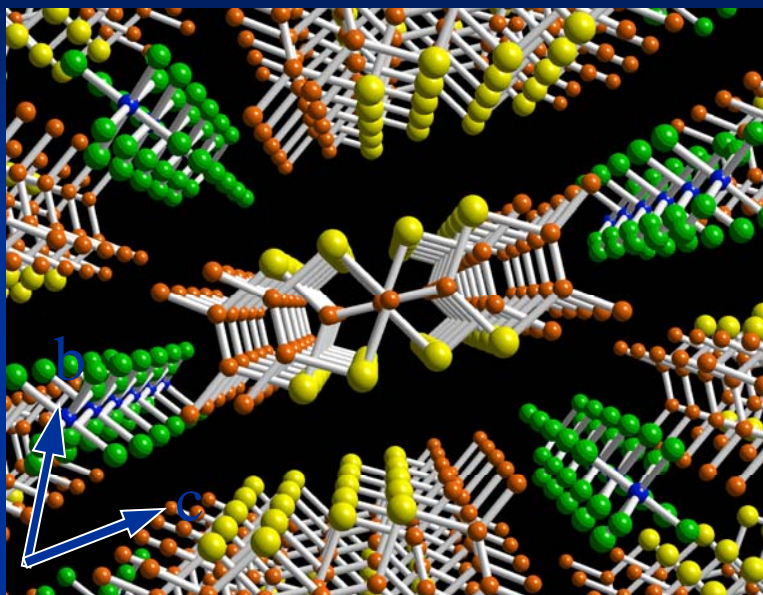
Superconductivity !



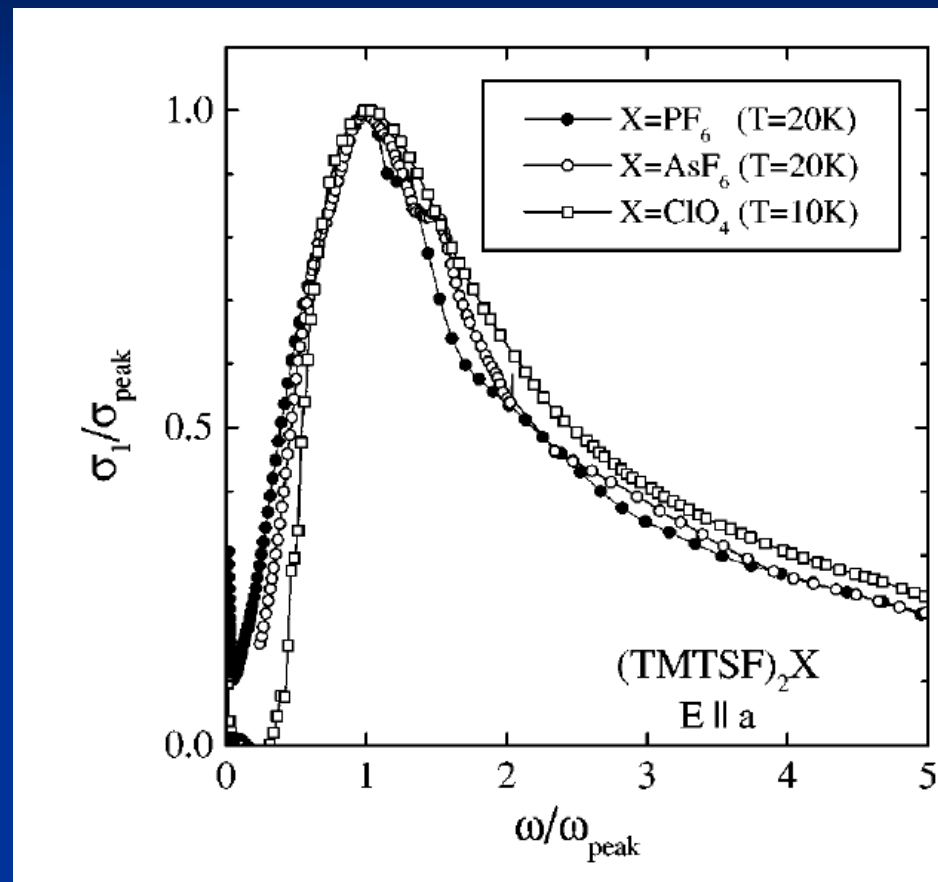
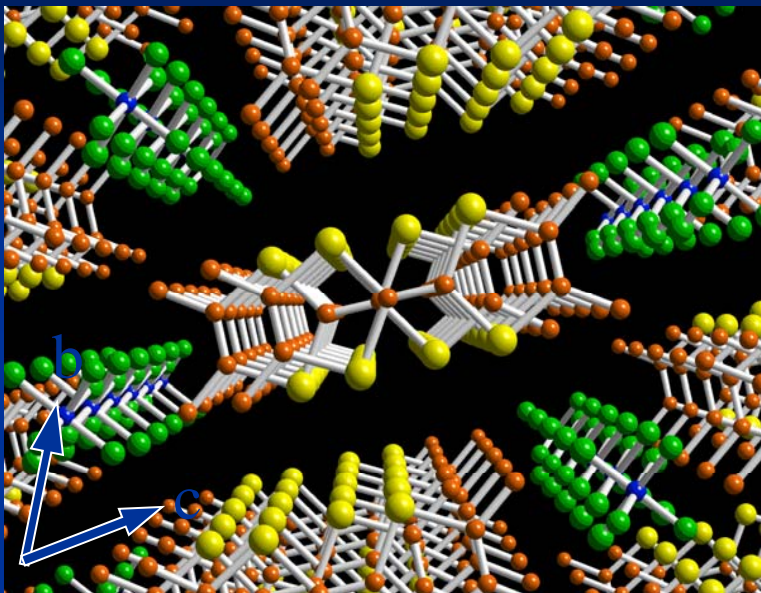
D. Jerome, M. Ribault, J. Mazaud and K. Bechgaard (1980)

Organic conductors

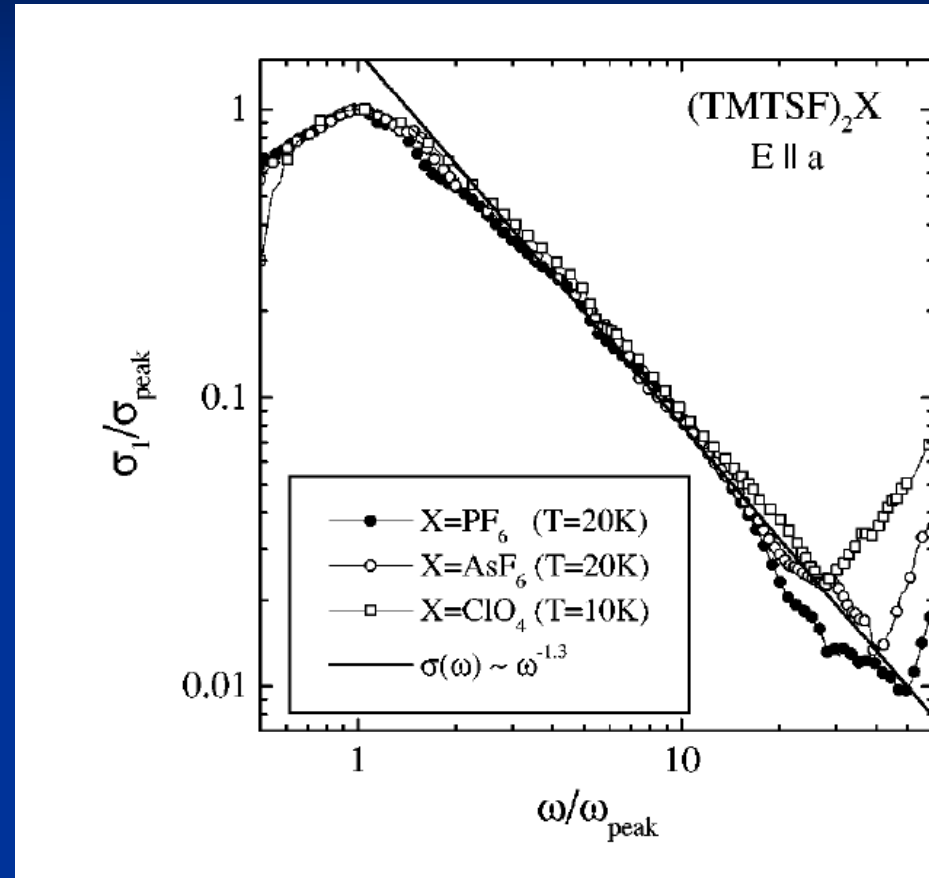
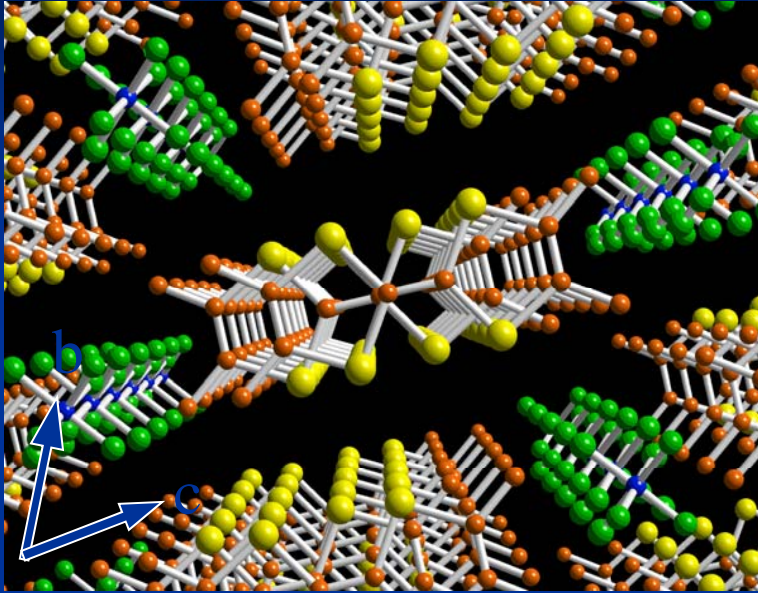
Organic conductors



Organic conductors

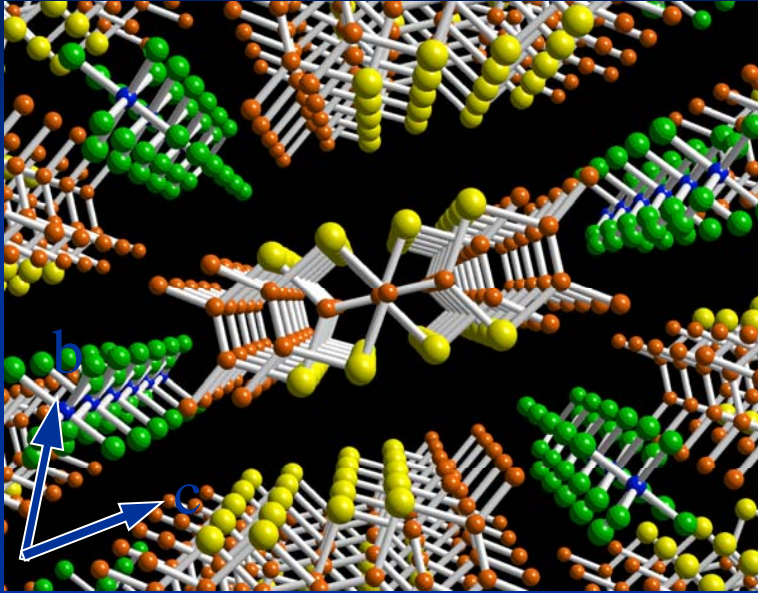


Organic conductors

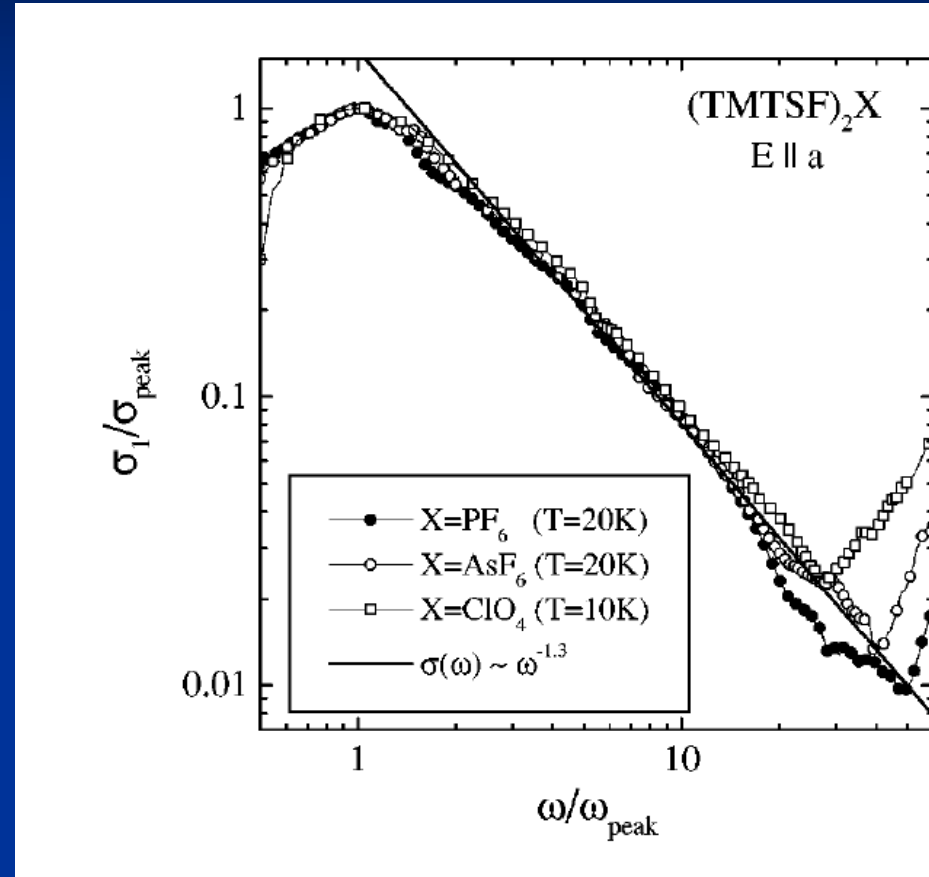


A. Schwartz et al. PRB 58
1261 (1998)

Organic conductors

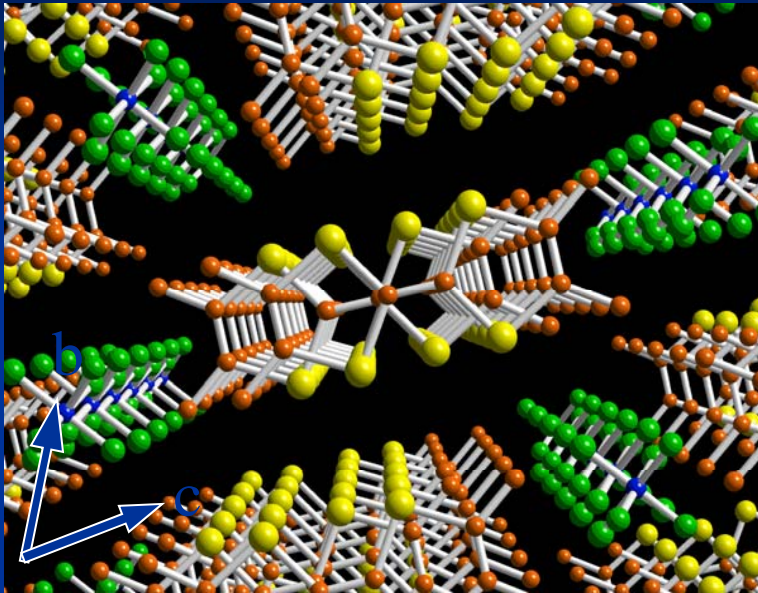


$$\sigma(\omega) \sim \omega^\nu$$



A. Schwartz et al. PRB 58
1261 (1998)

Organic conductors

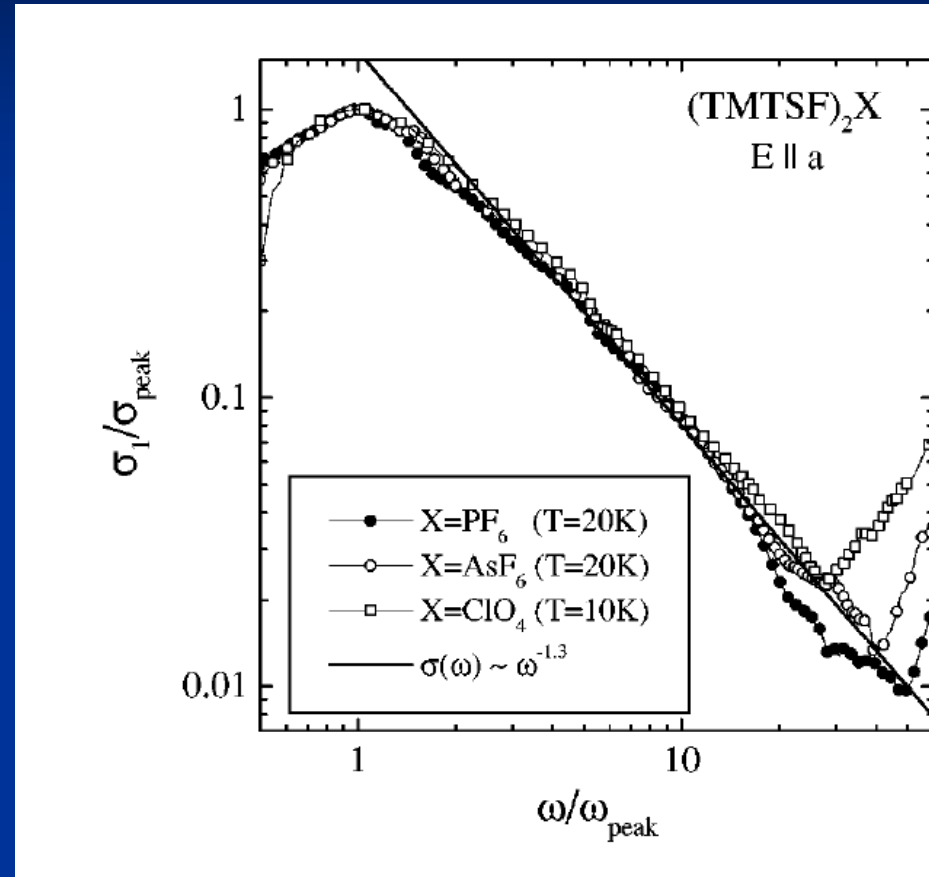


$$\sigma(\omega) \sim \omega^\nu$$

TG PRB (91) :

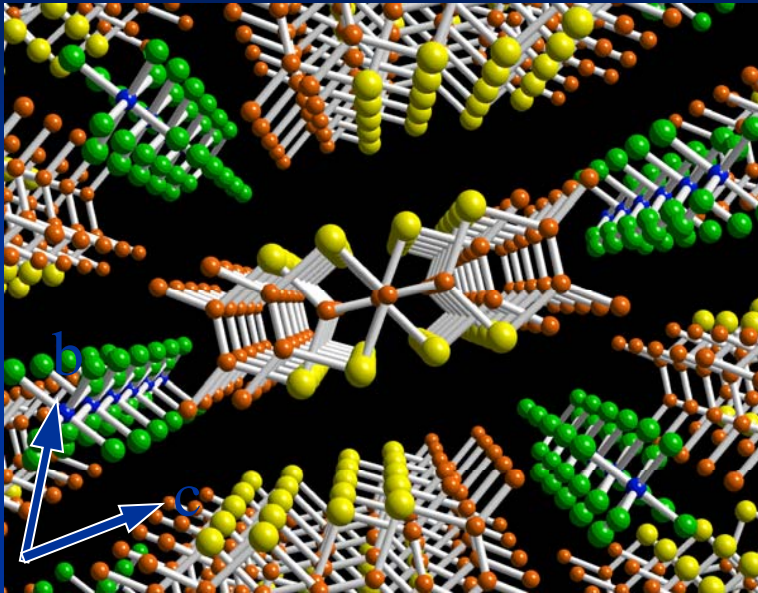
Physica B 230 (1996)

First observation of LL !!



A. Schwartz et al. PRB 58
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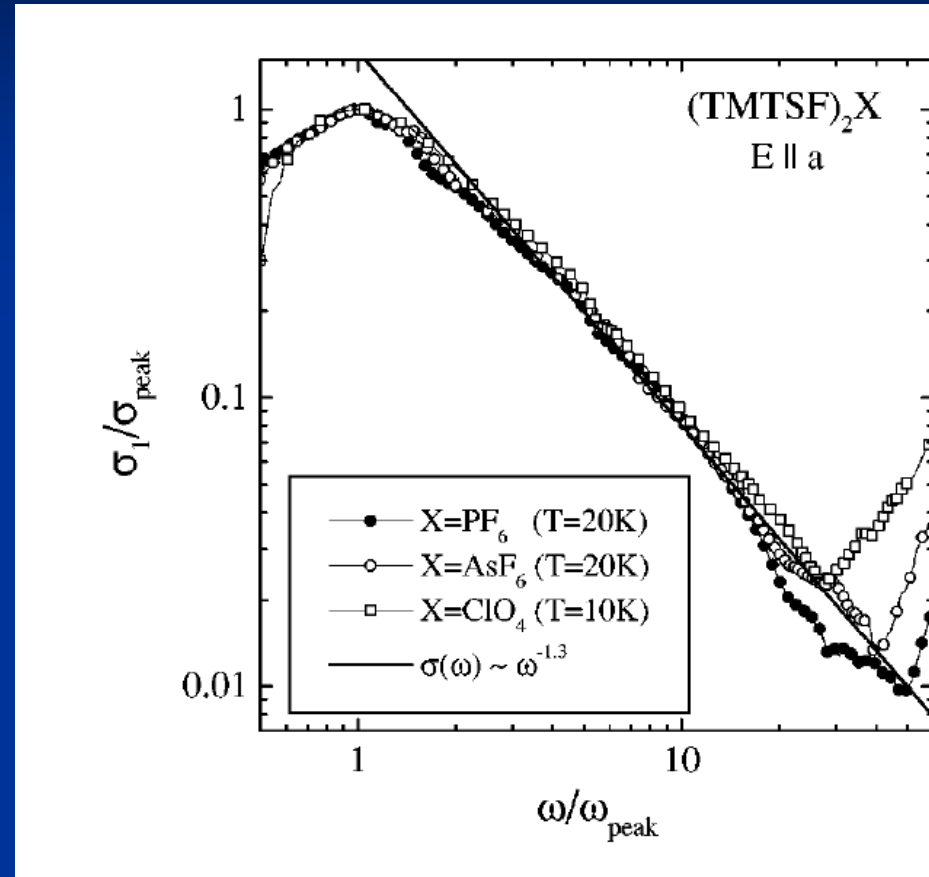


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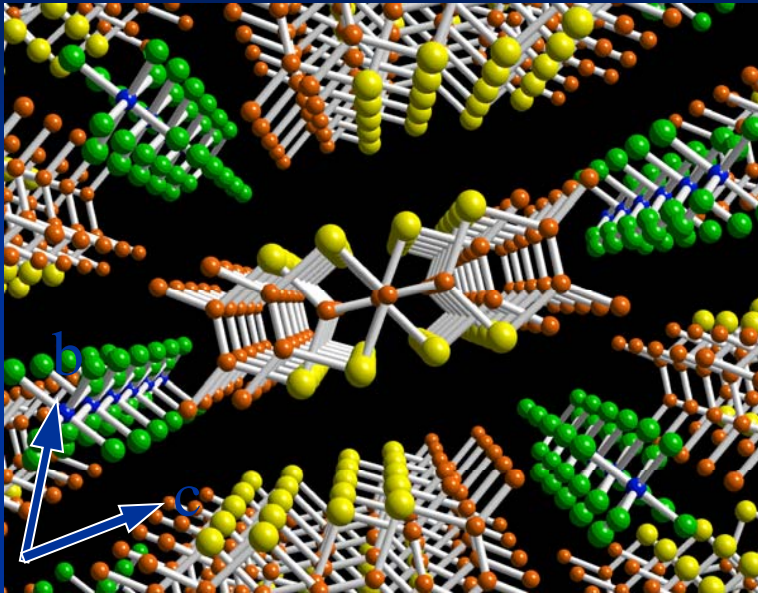
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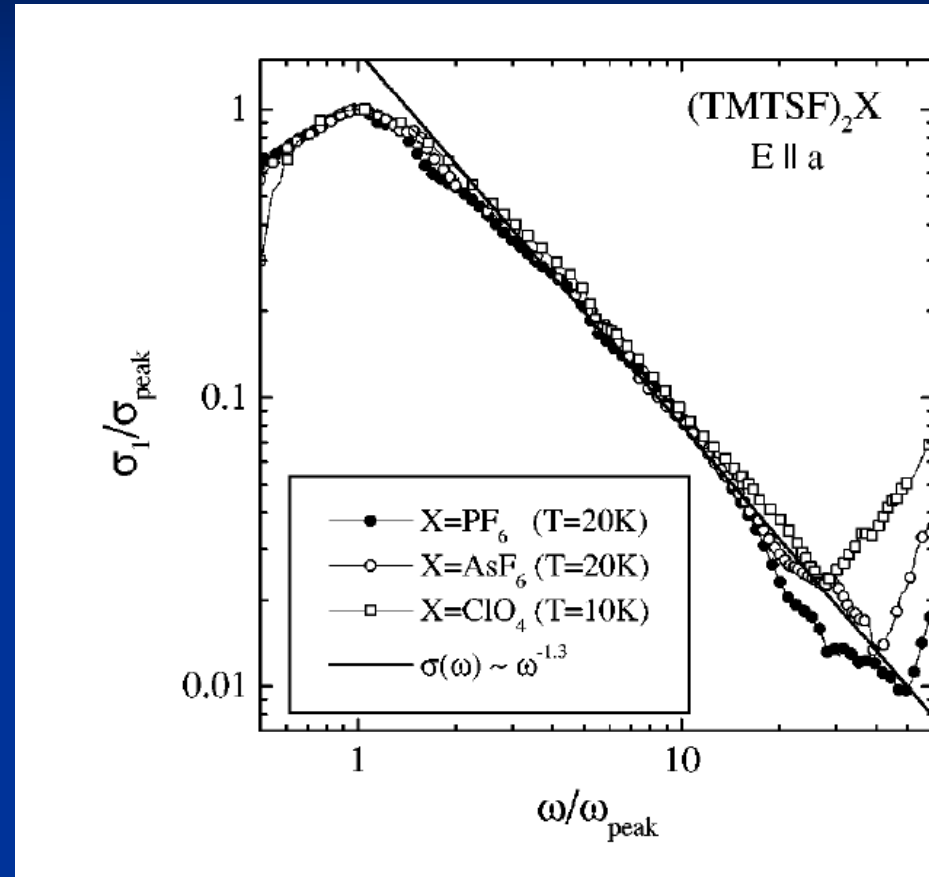


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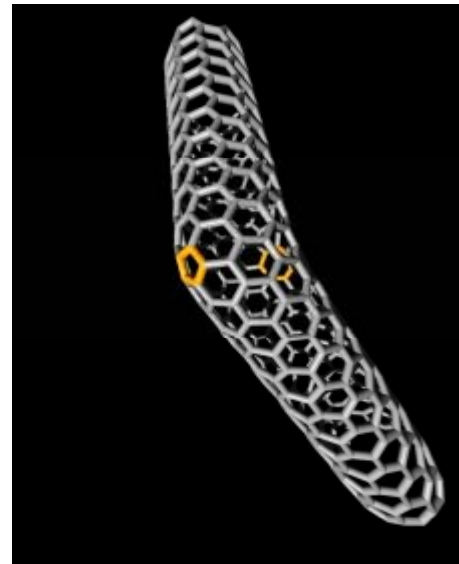
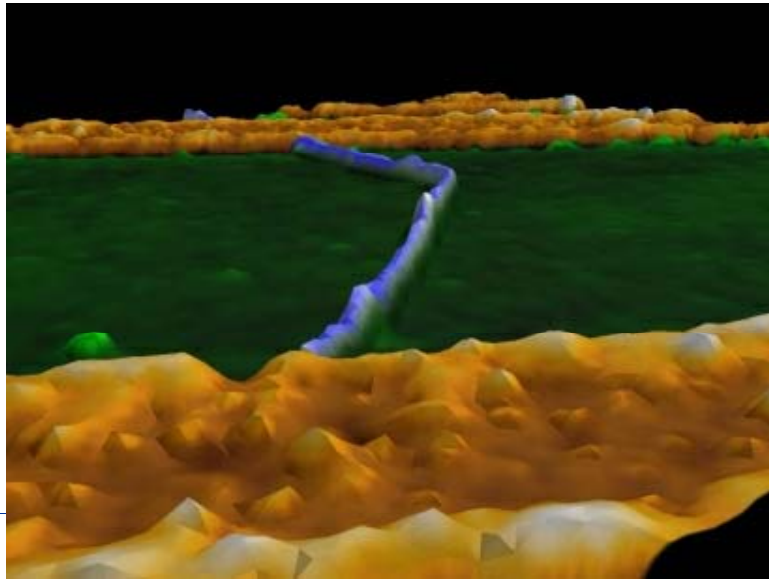
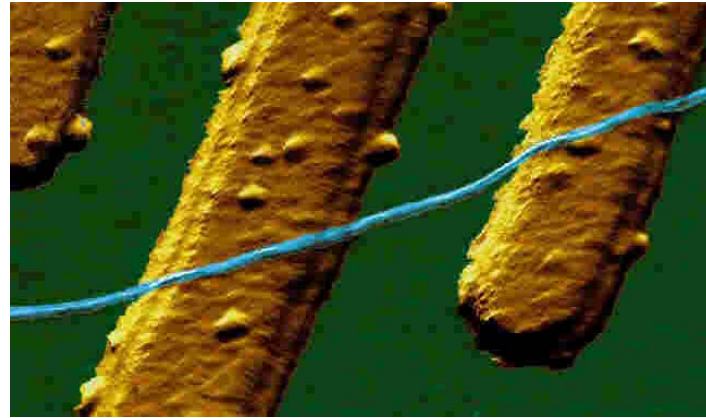


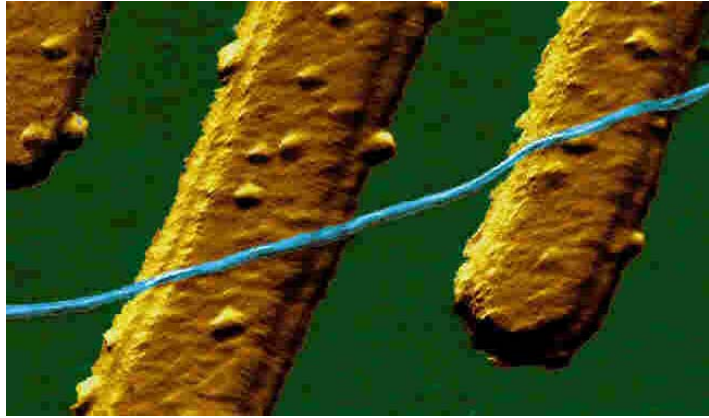
A. Schwartz et al. PRB 58
1261 (1998)

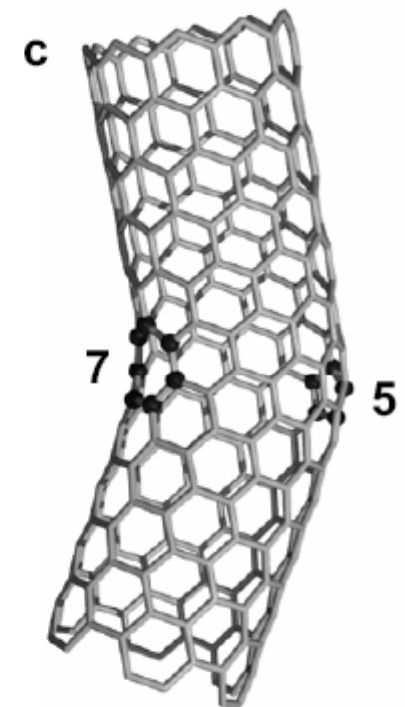
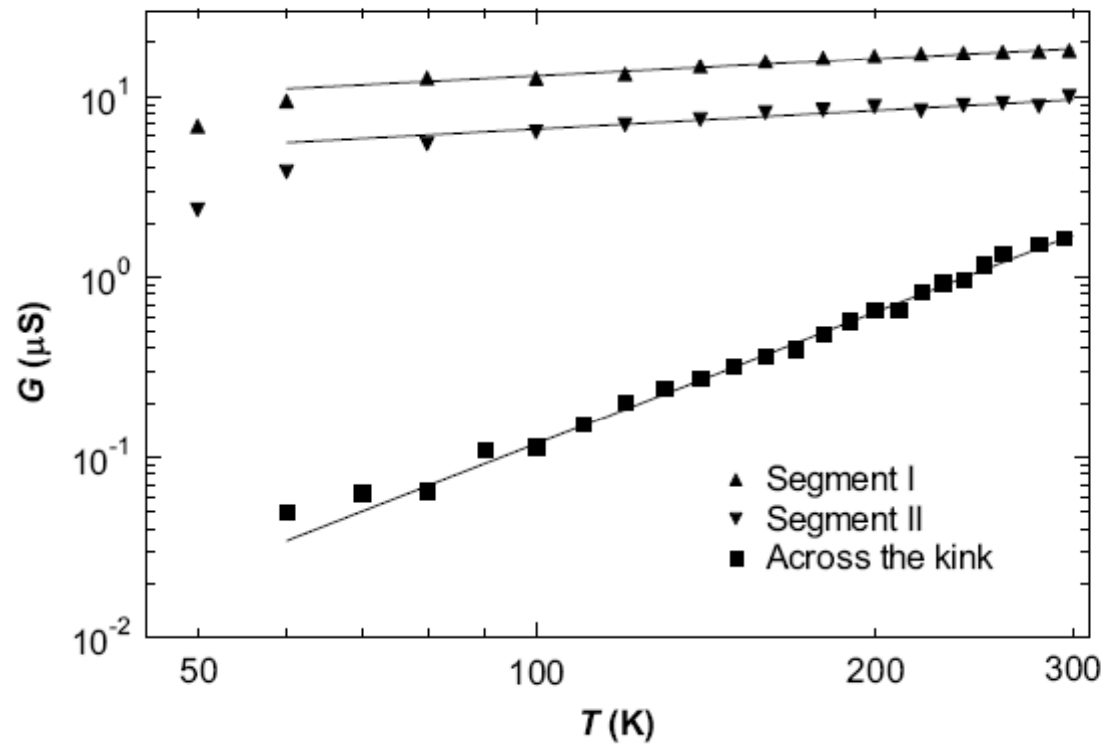
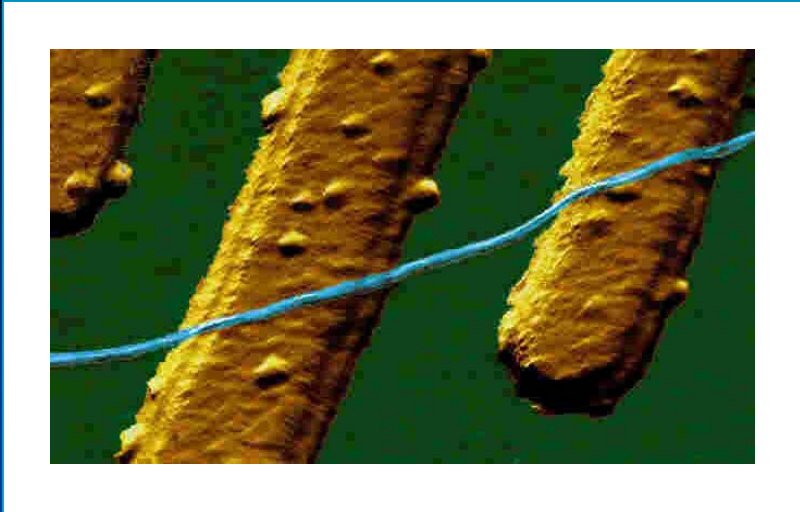
CARBON NANOTUBES

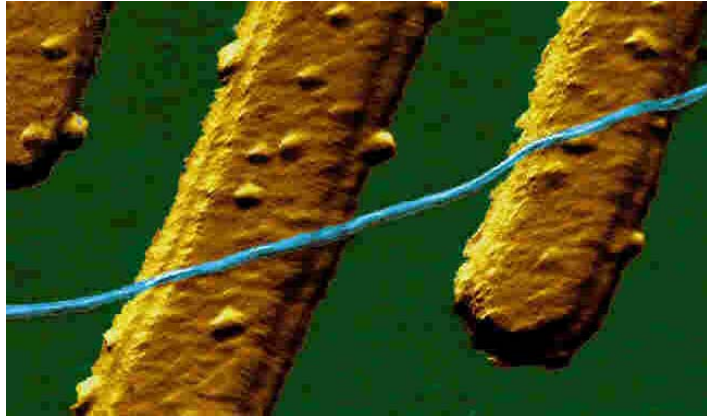


Cees Dekker

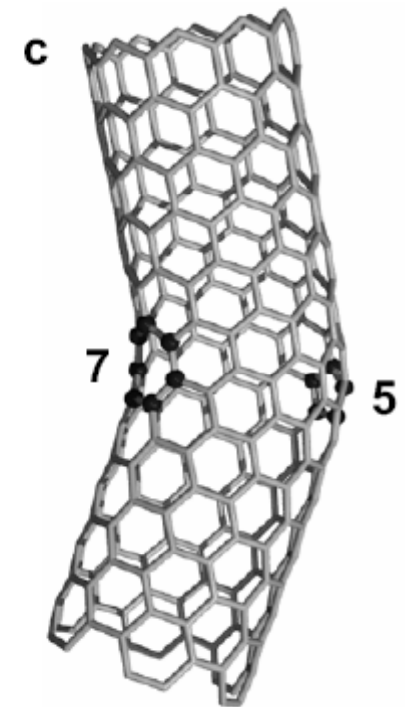
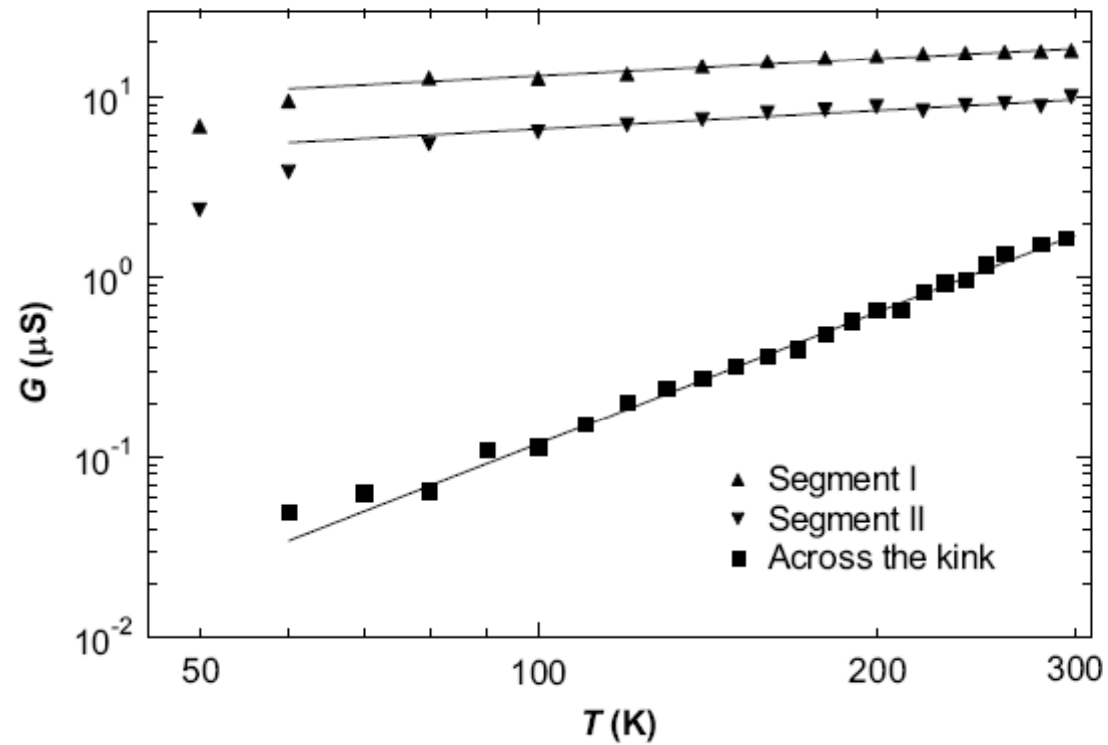




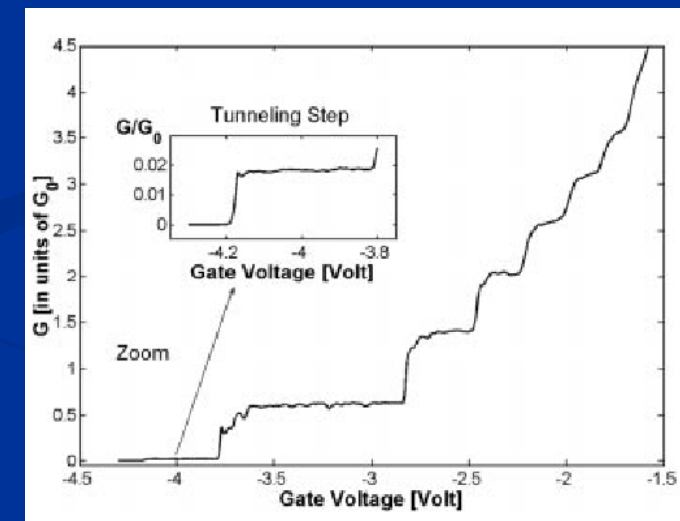
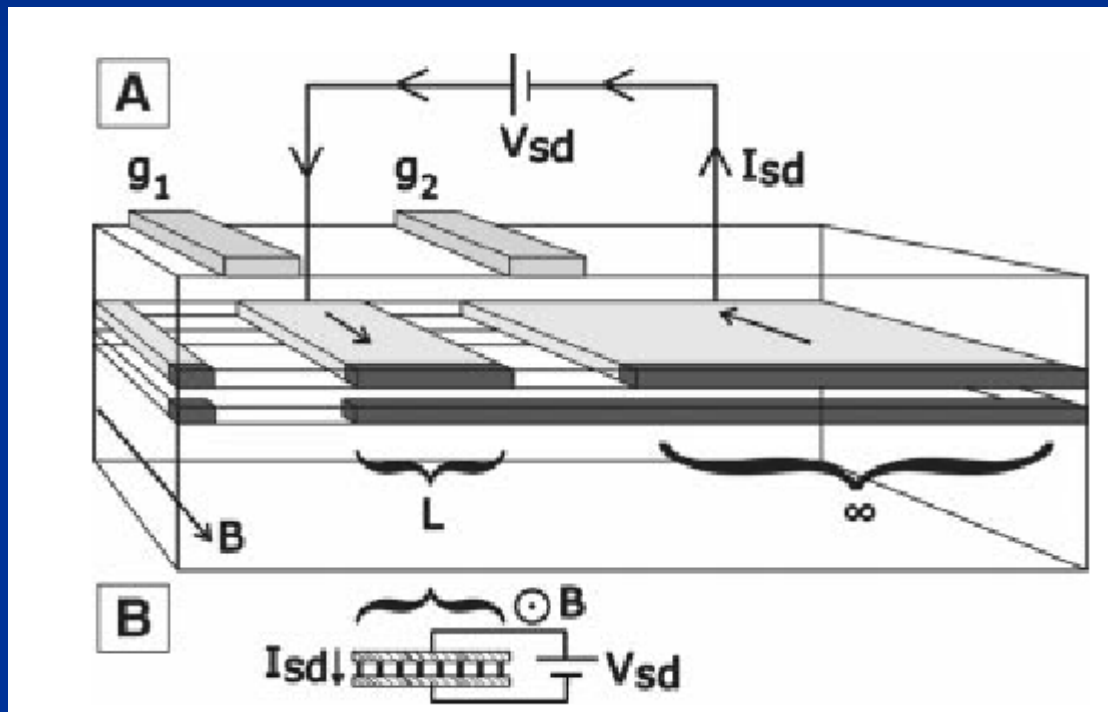




Z. Yao et al. Nature 402
273 (1999)

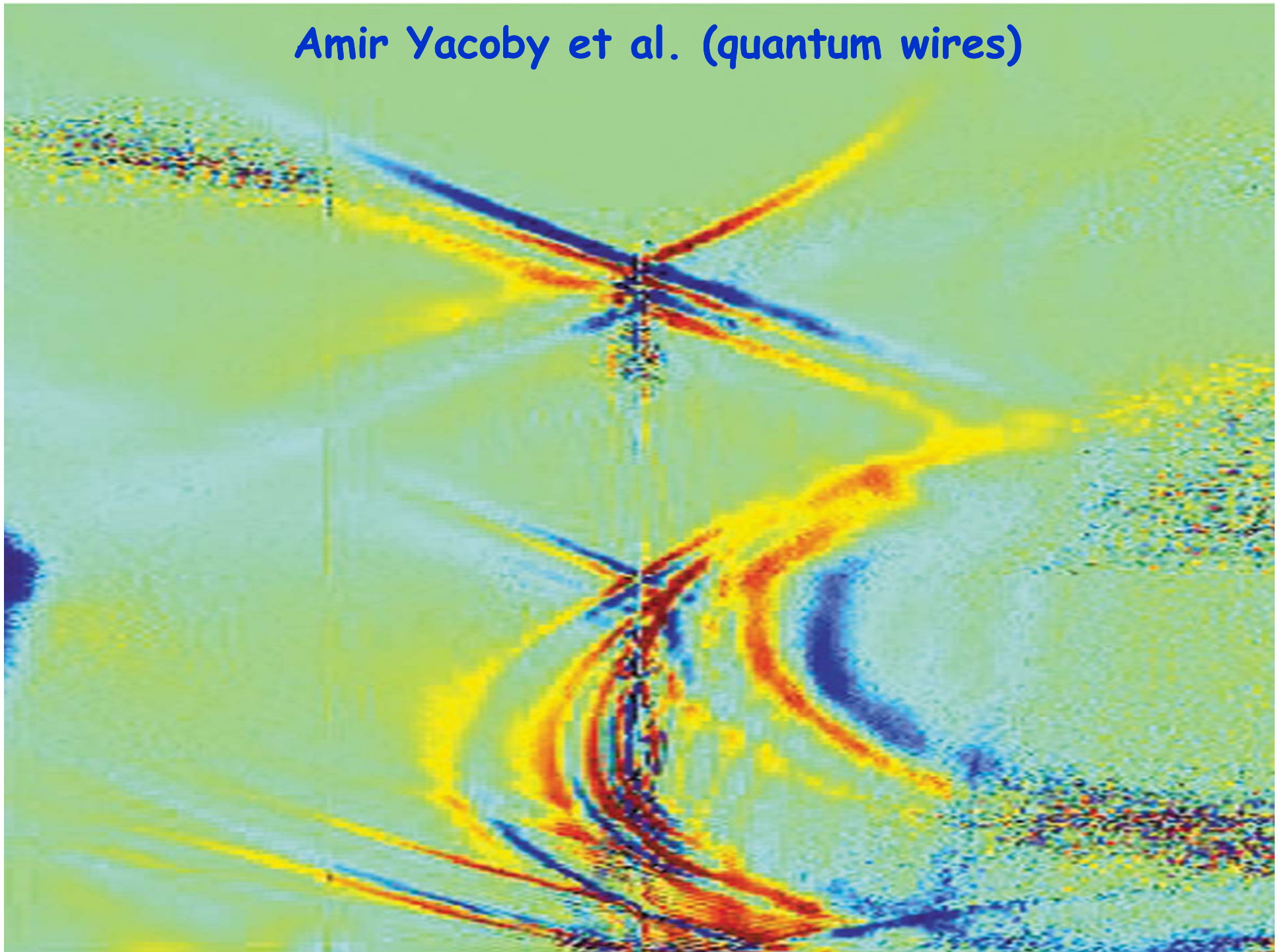


Quantum wires

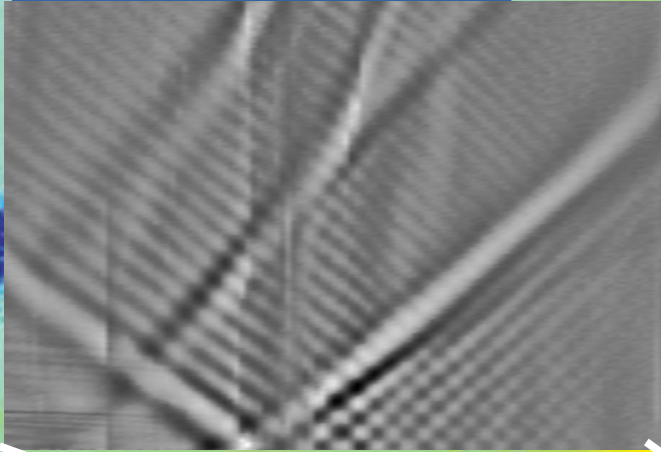
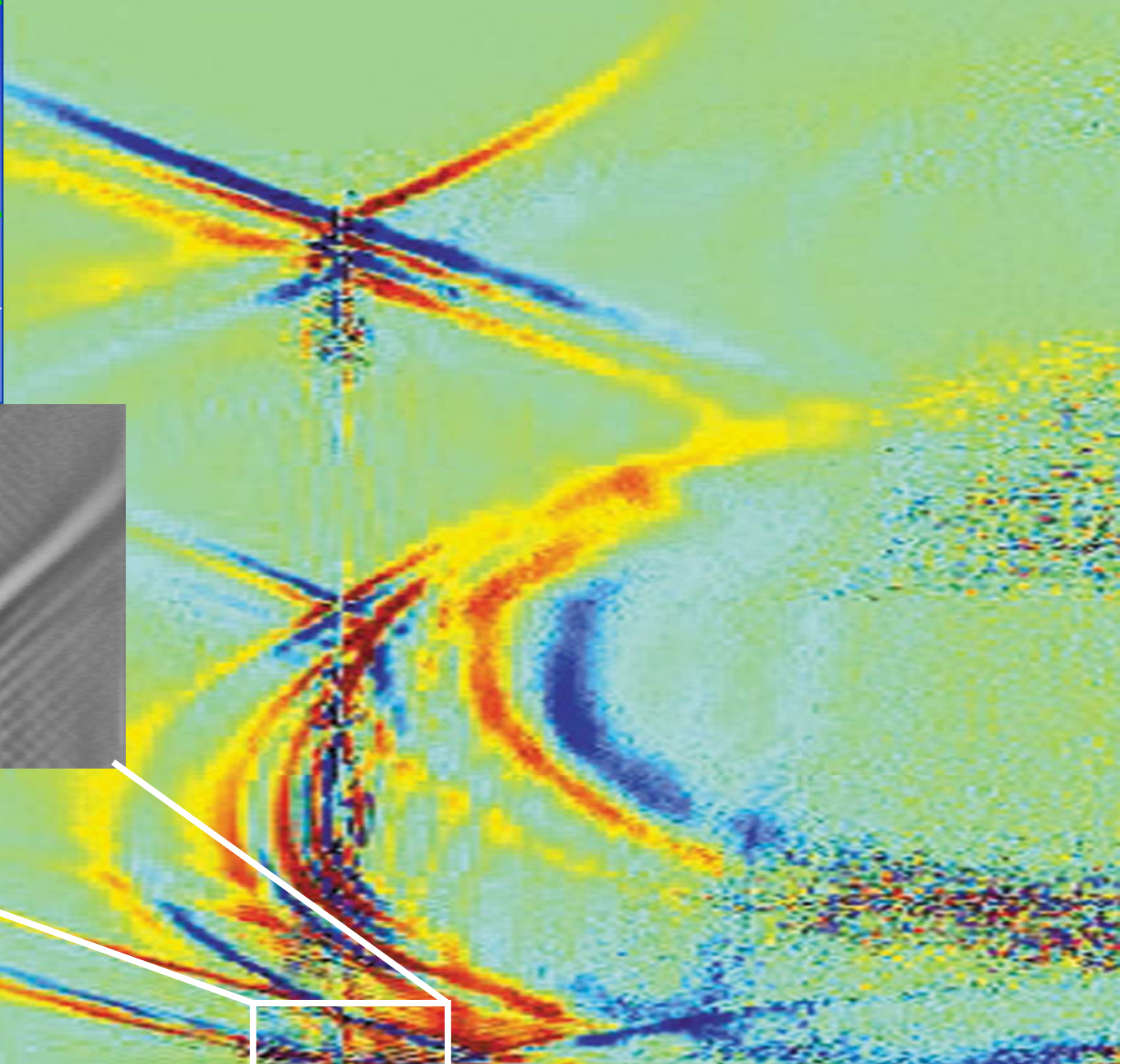
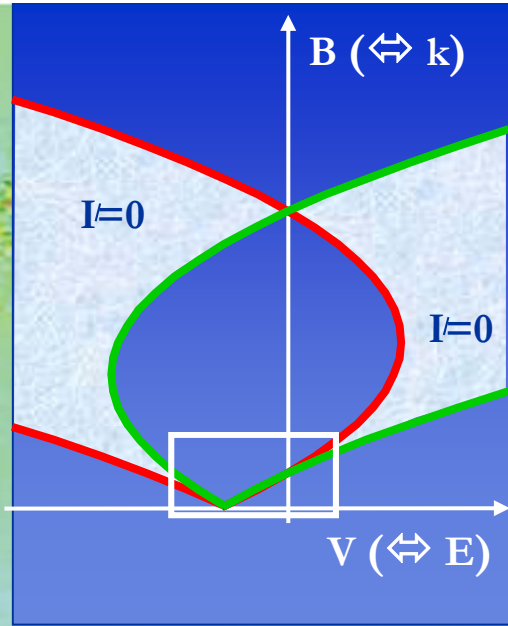


O.M Auslander et al., Science **298** 1354 (2001)
Y. Tserkovnyak et al., PRL **89** 136805 (2002)
Y. Tserkovnyak et al., PRB **68** 125312 (2003)

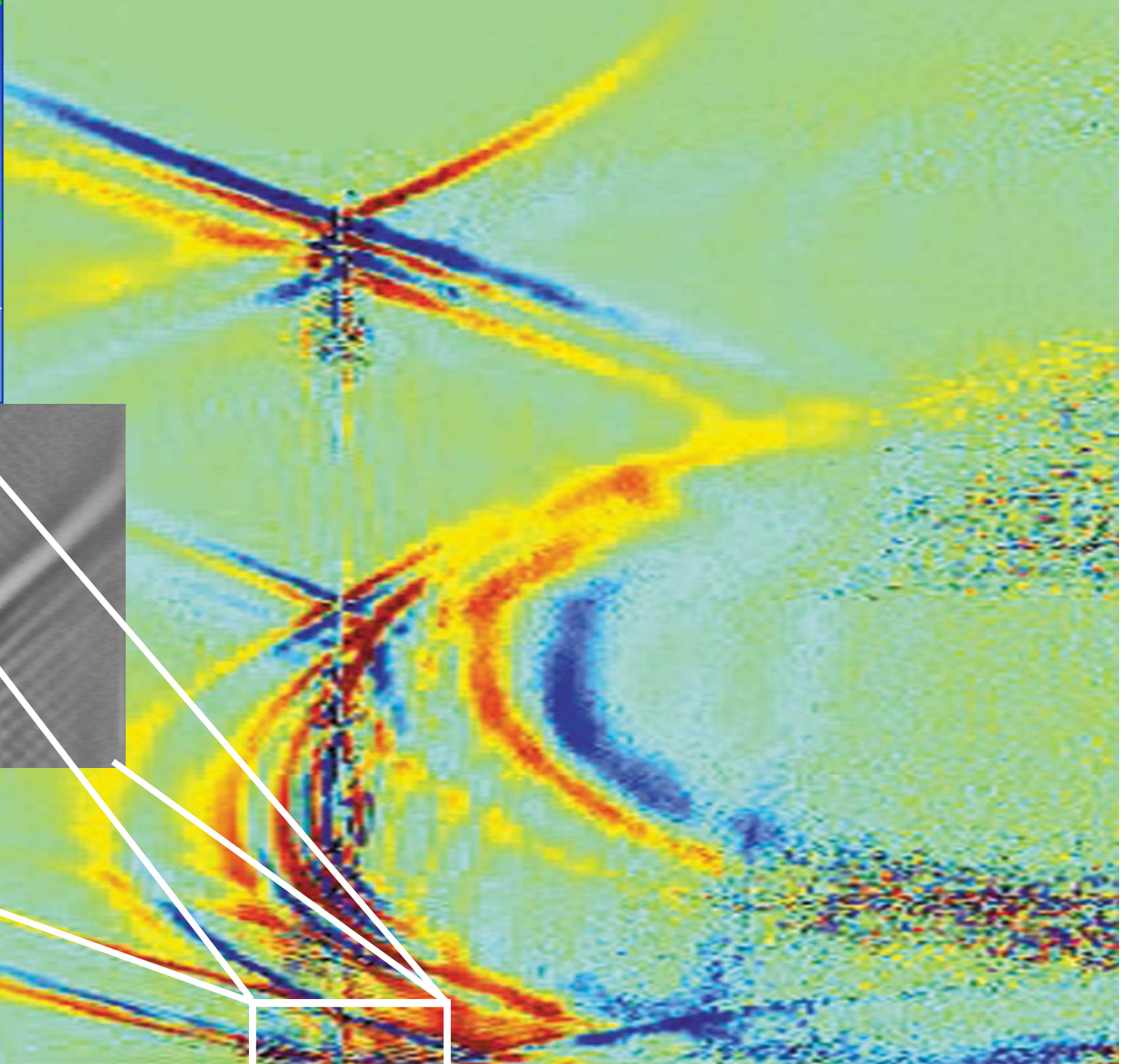
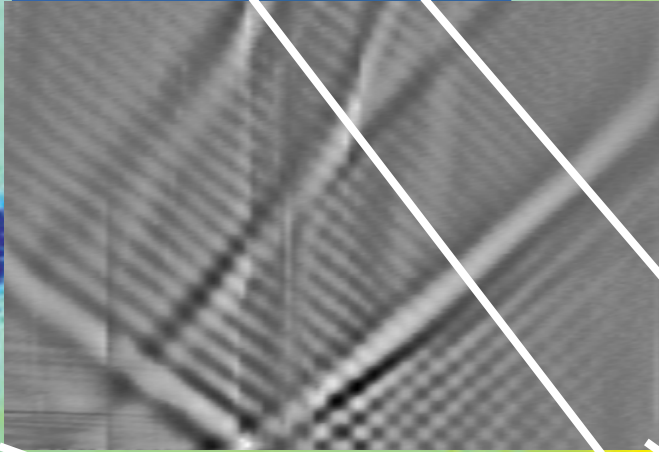
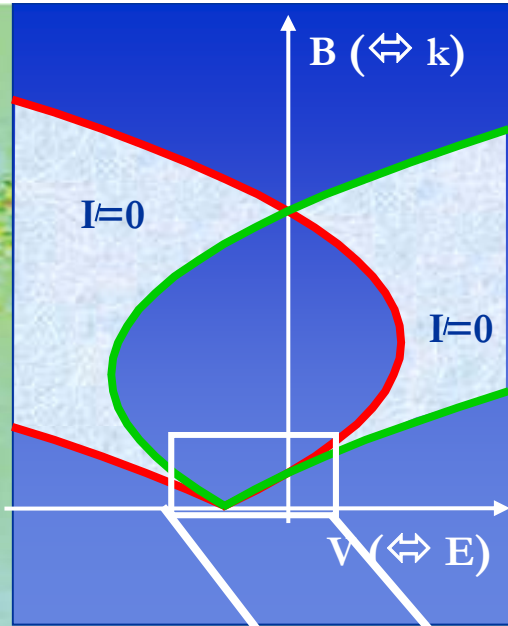
Amir Yacoby et al. (quantum wires)



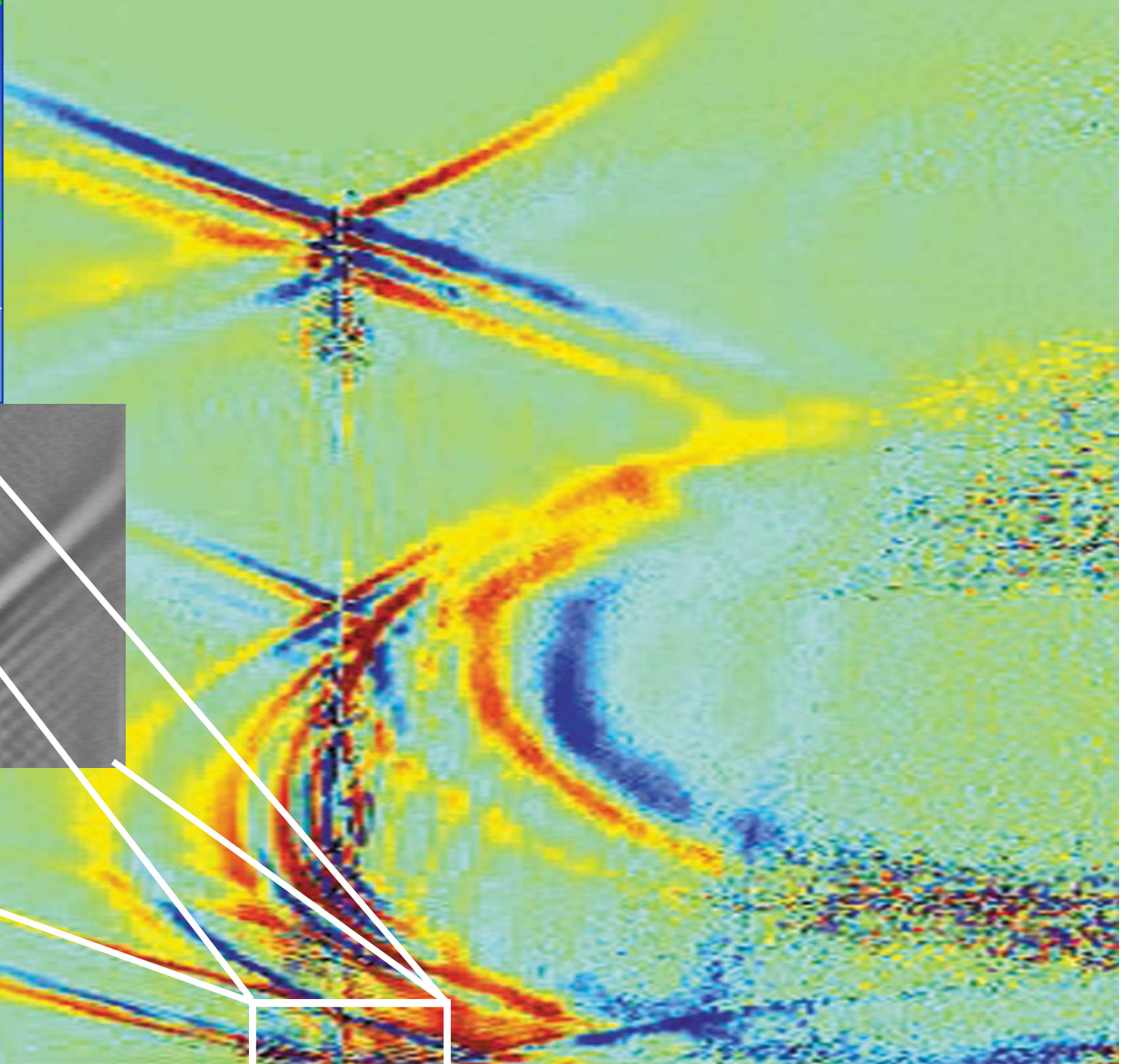
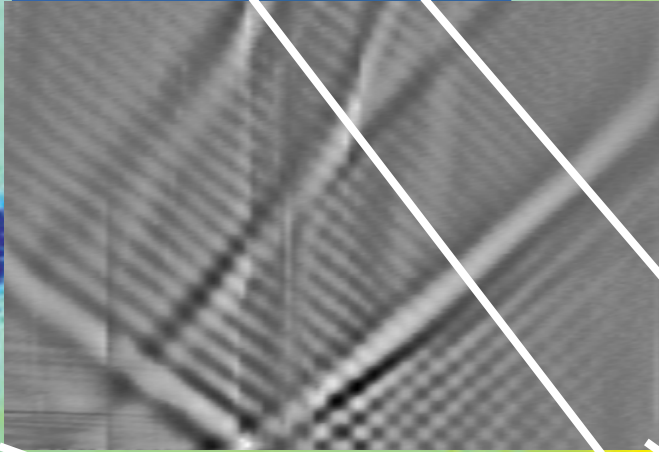
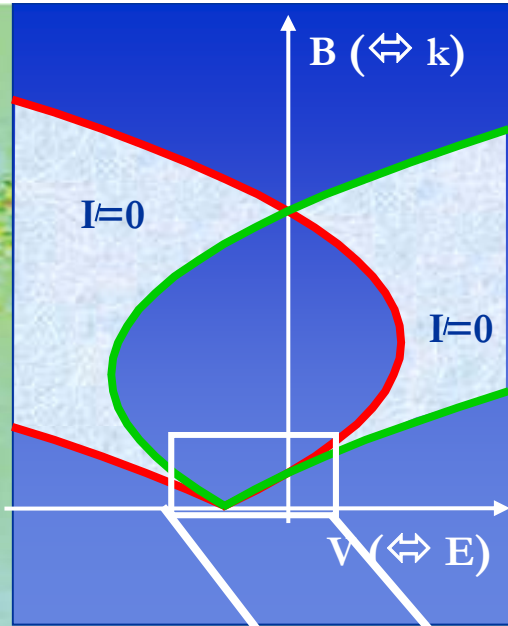
Yacoby et al. (quantum wires)



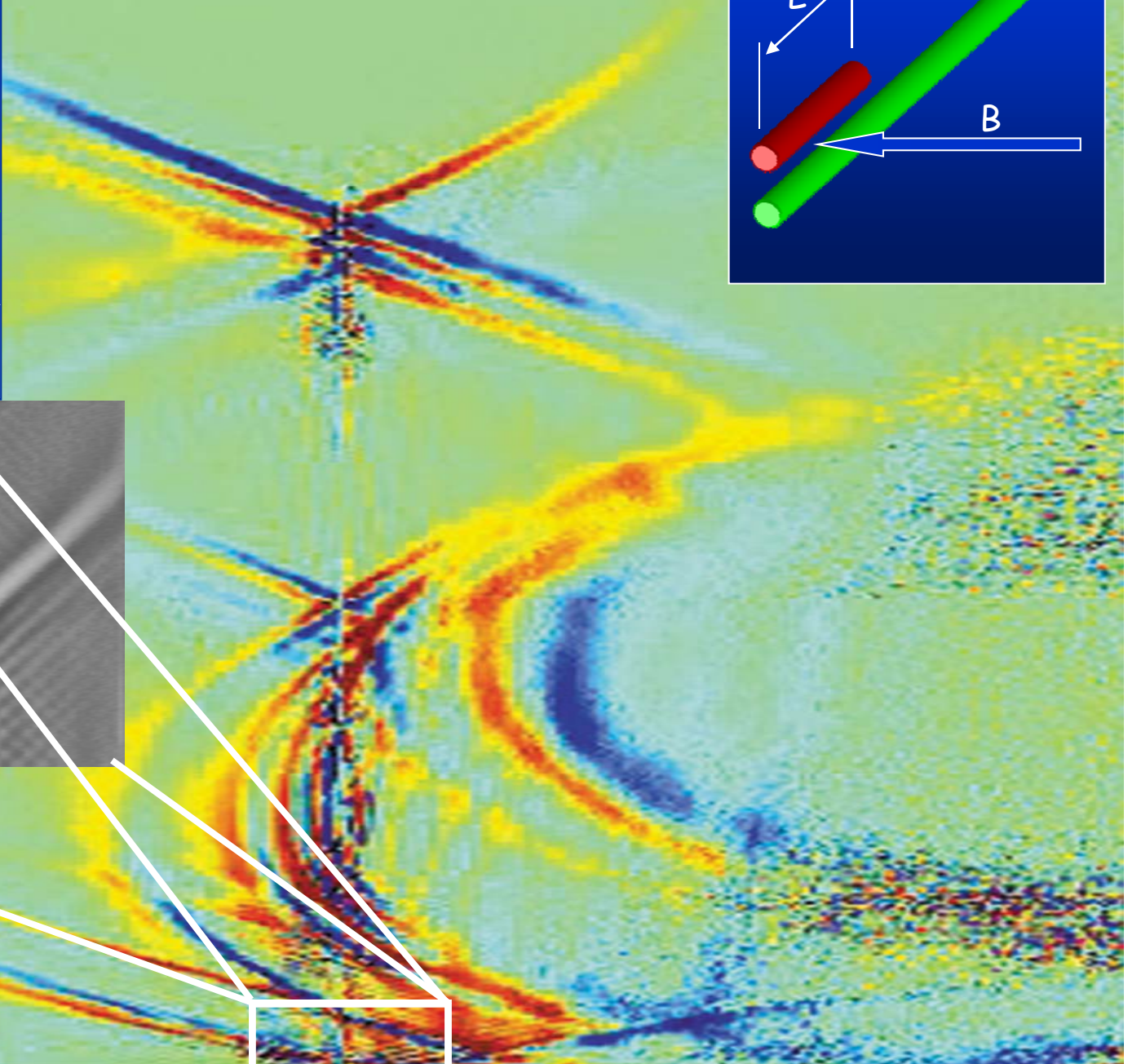
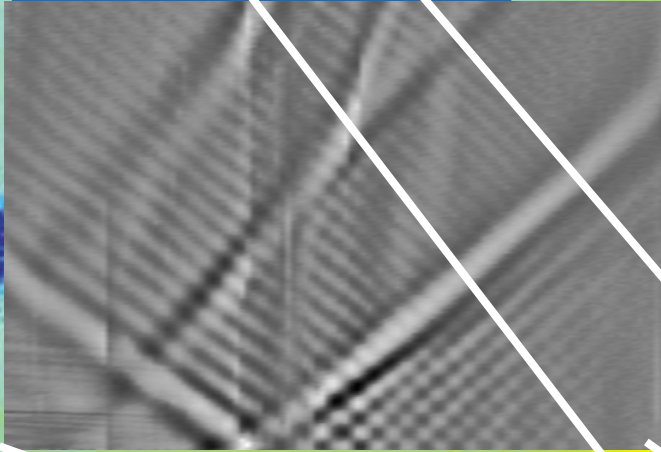
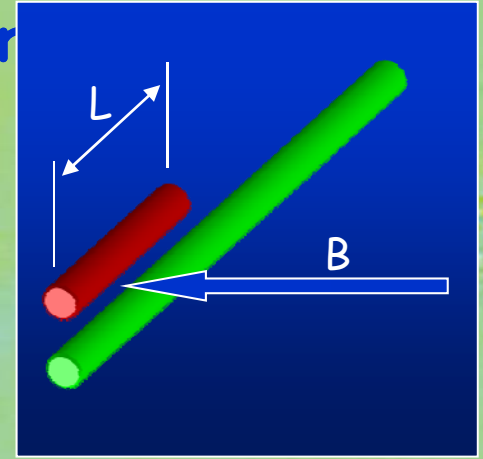
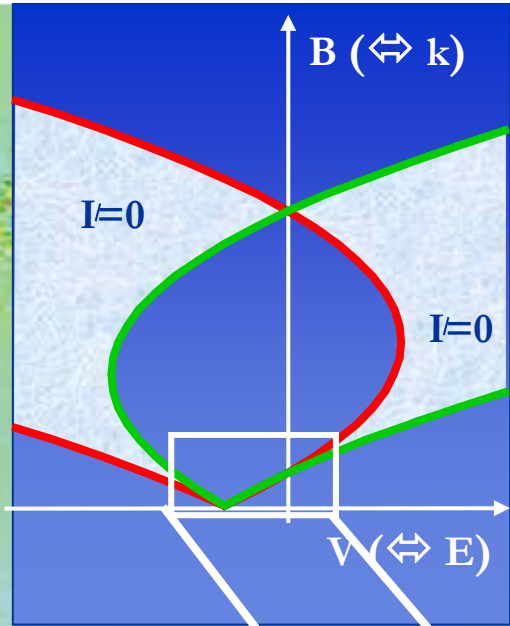
Yacoby et al. (quantum wires)



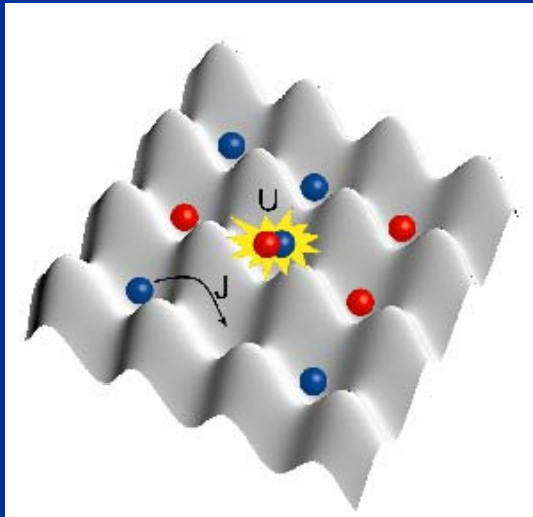
Yacoby et al. (quantum wires)



Yacoby et al. (quantum wire)

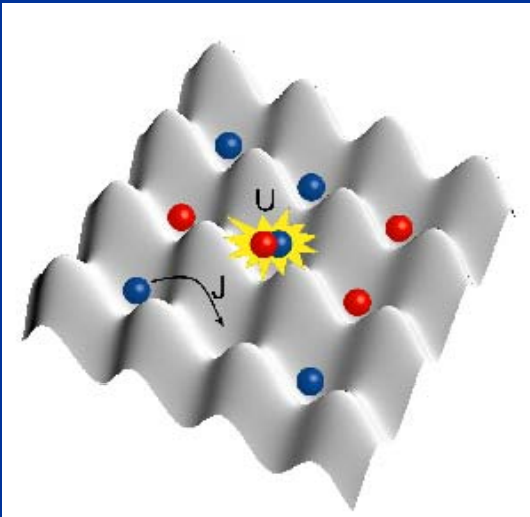


Cold atomic gases



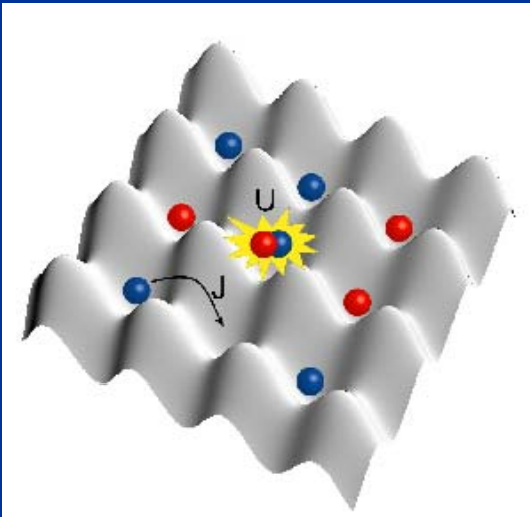
Cold atomic gases

- Novel “materials” to address condensed matter issues
- Extreme control and versatility



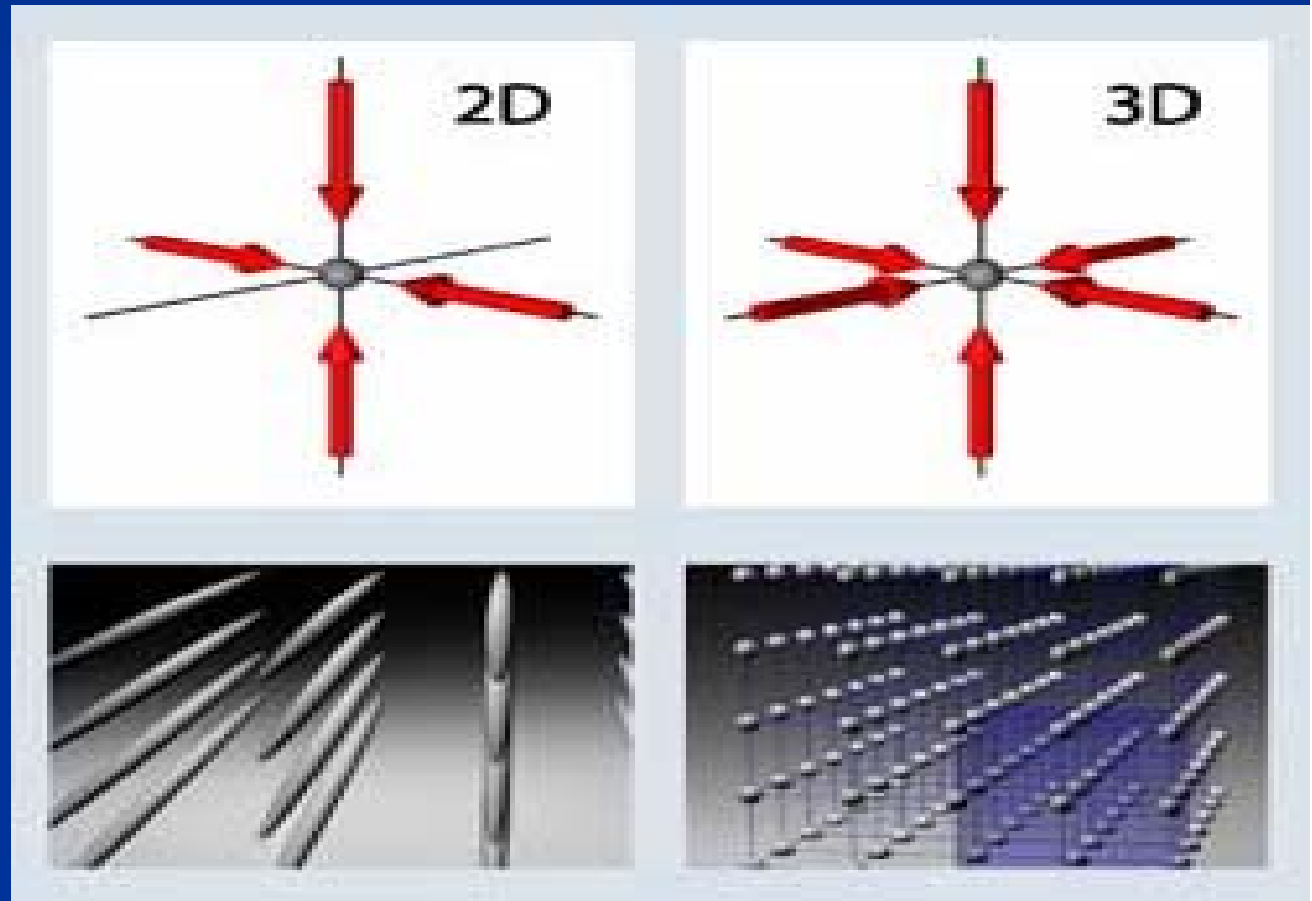
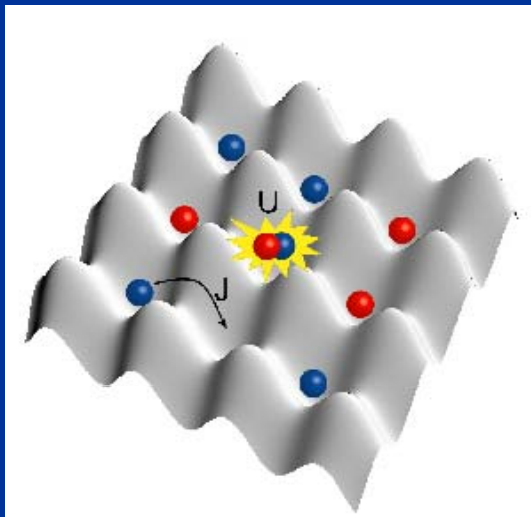
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Cold atomic gases

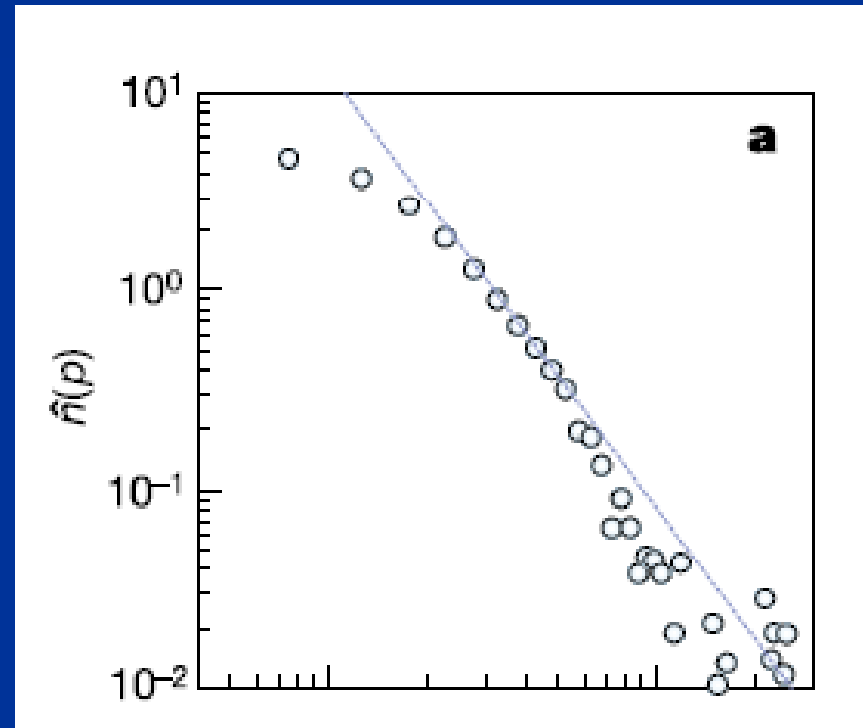
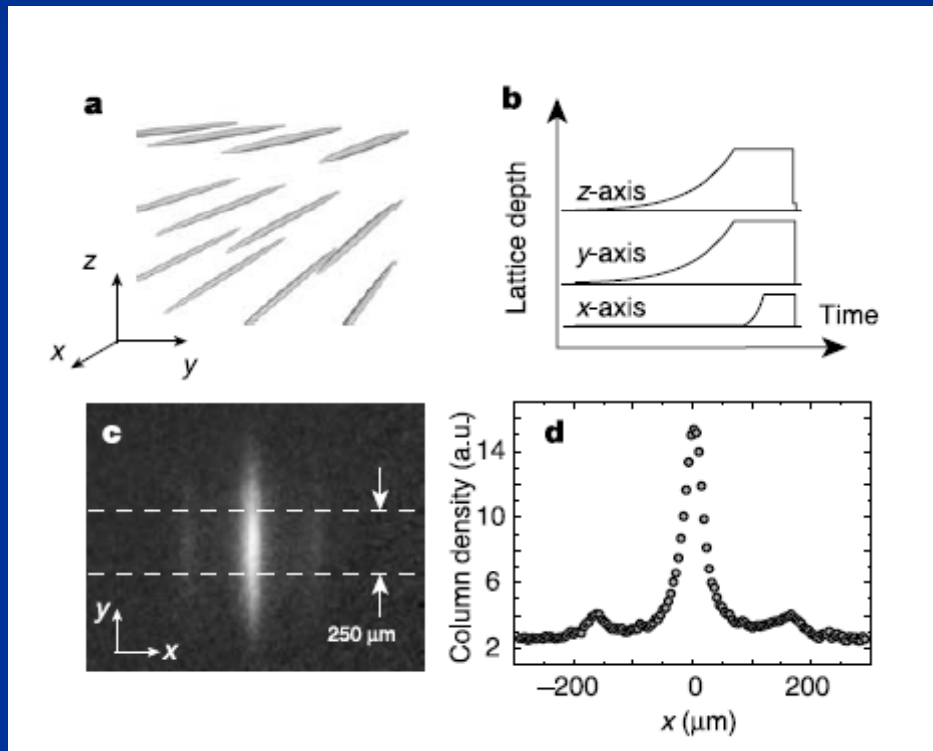
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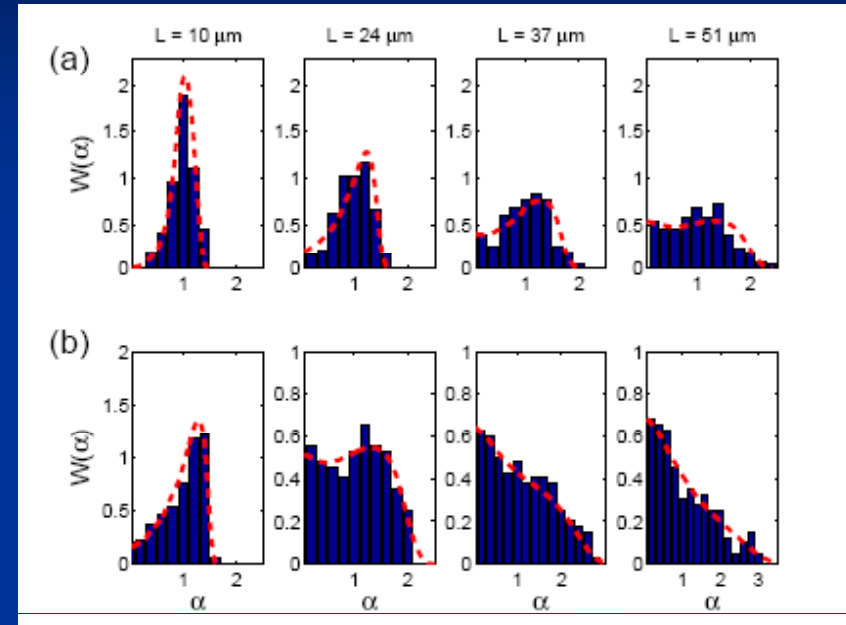
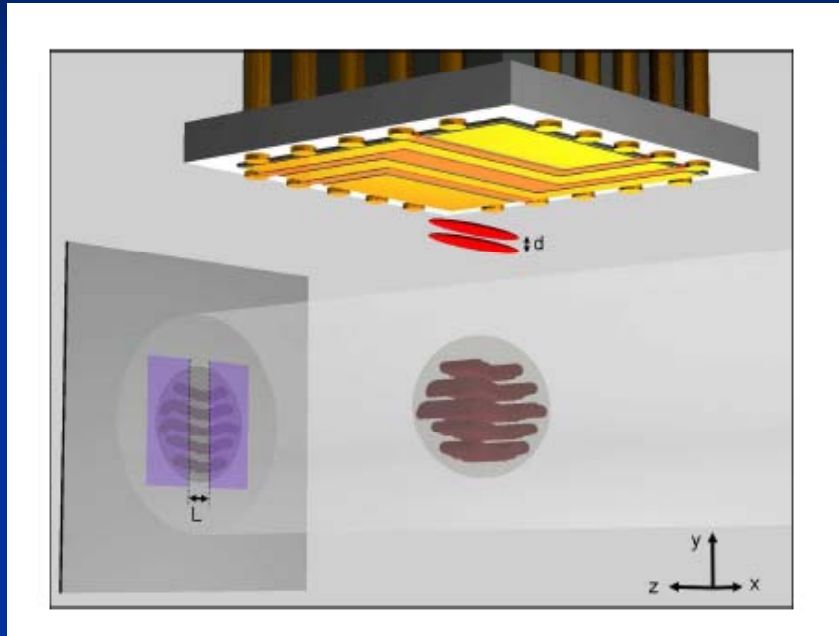
Cold atoms (Tonks limit)



B. Paredes et al., Nature 429 277 (2004)



Cold atoms: Interferences



S. Hofferberth et al. Nat. Phys 4
489 (2008)

$$\int_0^L \langle \psi^\dagger(\mathbf{r}) \psi(0) \rangle$$

but K large (42) $\rho \sim \rho_0$ hydrodynamic ?

But.....

But.....

- Exponent is an adjustable parameter

But.....

- Exponent is an adjustable parameter
- Universality ? Different correlation functions ?

But.....

- Exponent is an adjustable parameter
- Universality ? Different correlation functions ?
- Variation with a control parameter ?

But.....

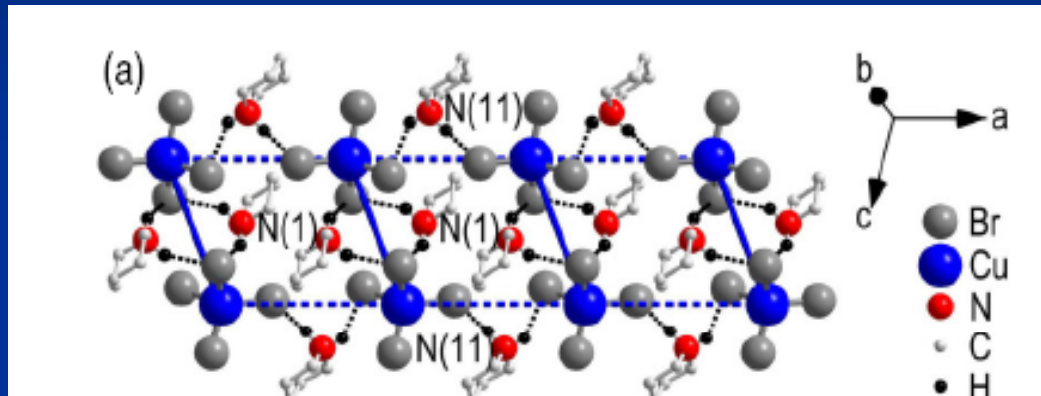
- Exponent is an adjustable parameter
- Universality ? Different correlation functions ?
- Variation with a control parameter ?

can one do a quantitative test ????

Spin ladders

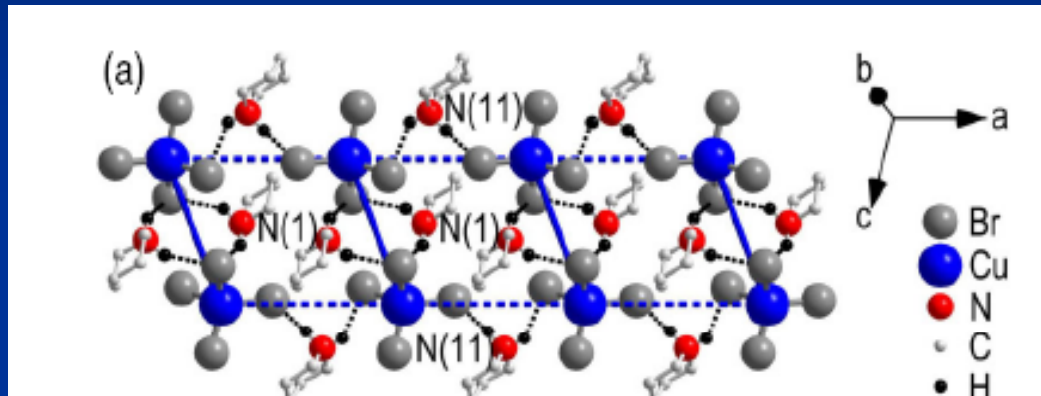
Spin ladders

B. C. Watson et al., PRL 86 5168 (2001)



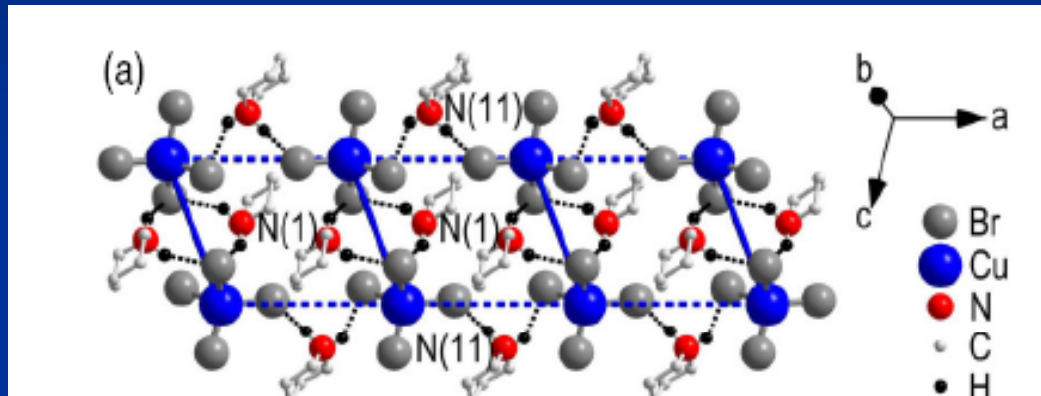
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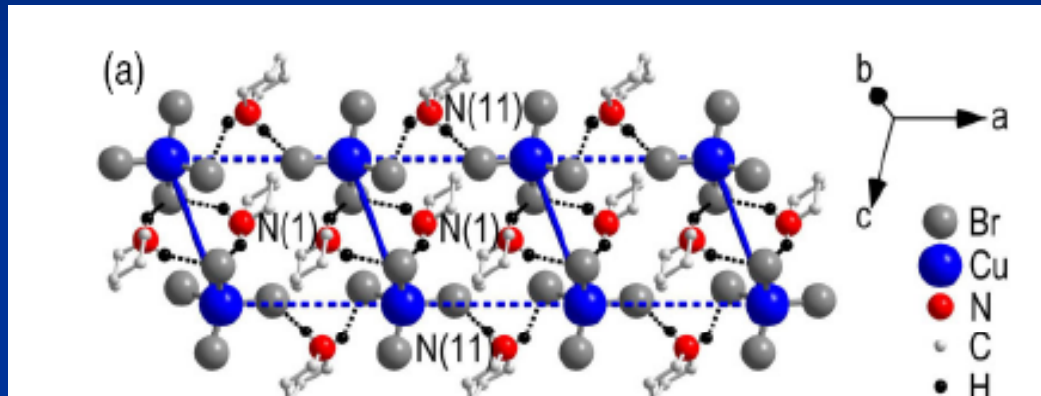


M. Klanjsek et al.,

PRL 101 137207 (2008)

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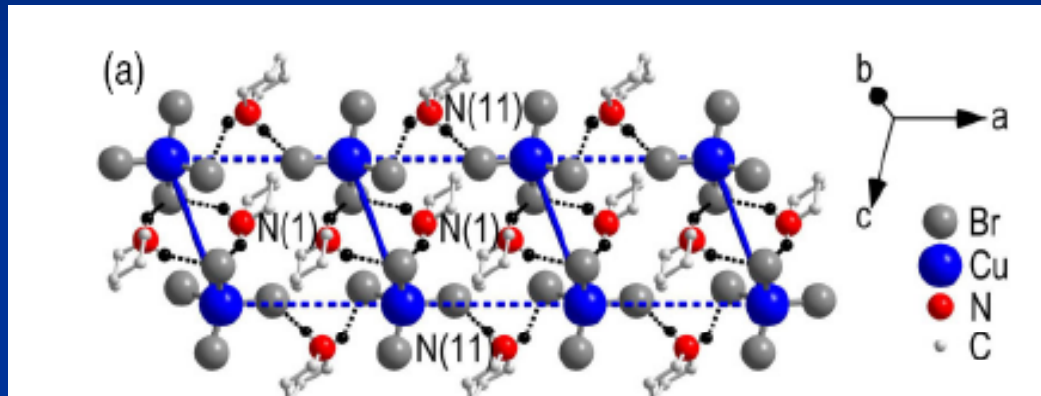
PRL 101 137207 (2008)

B. Thielemann et al.,

PRB 79, 020408(R) (2009)

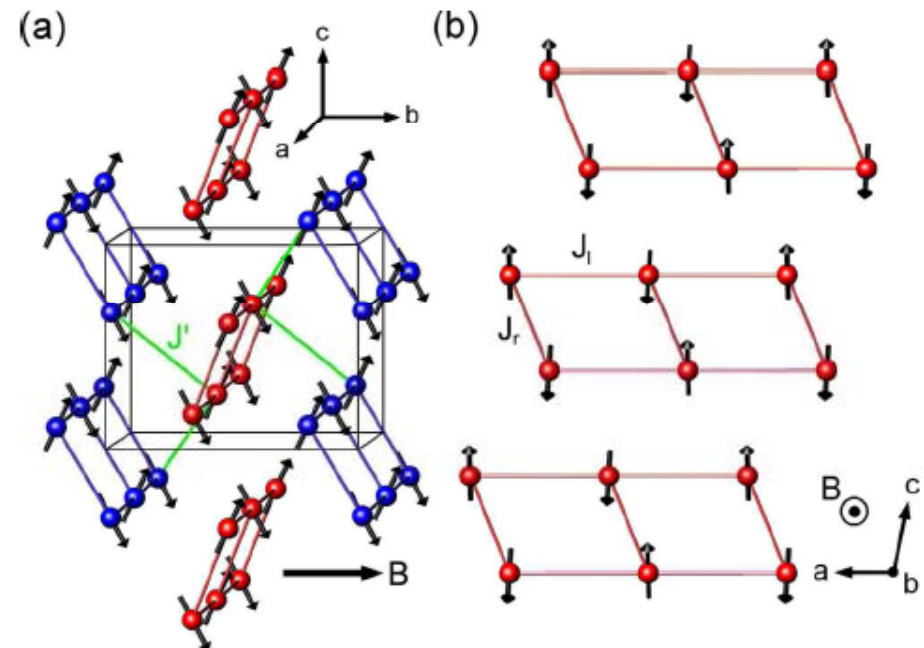
Spin ladders

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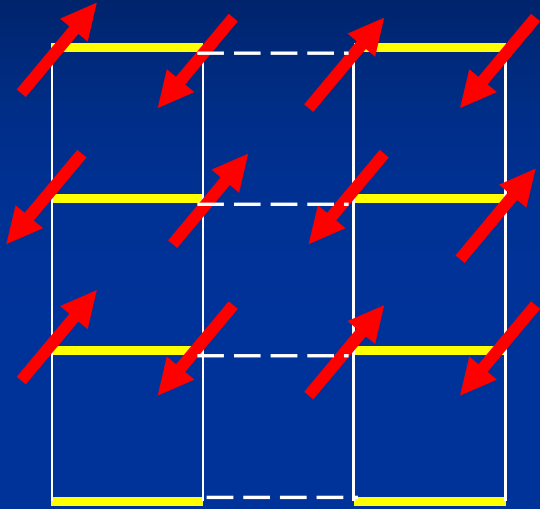


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PRL 101 137207 (2008)

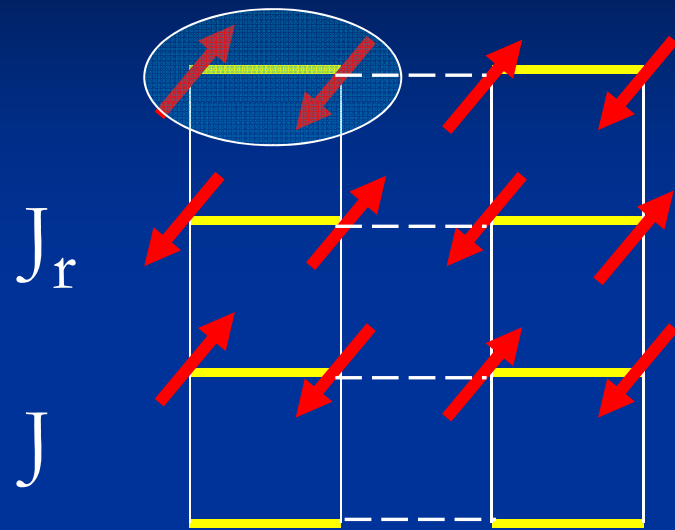
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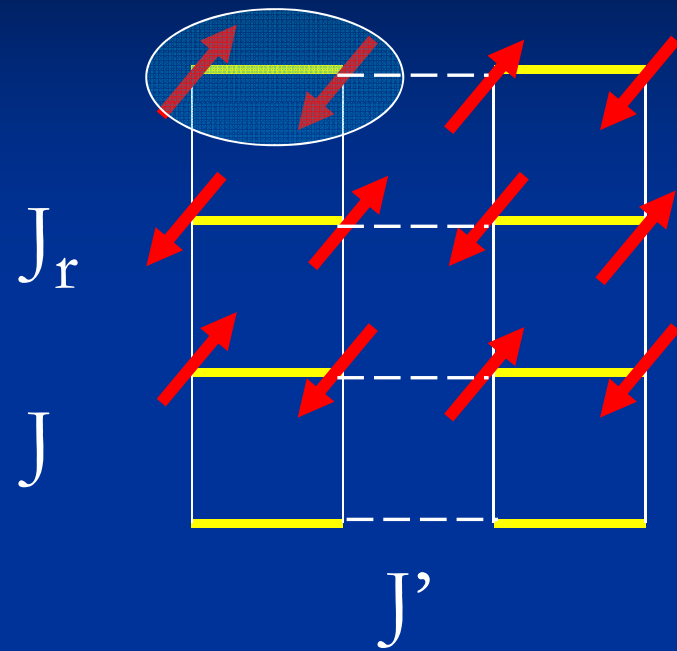
Spin Ladders



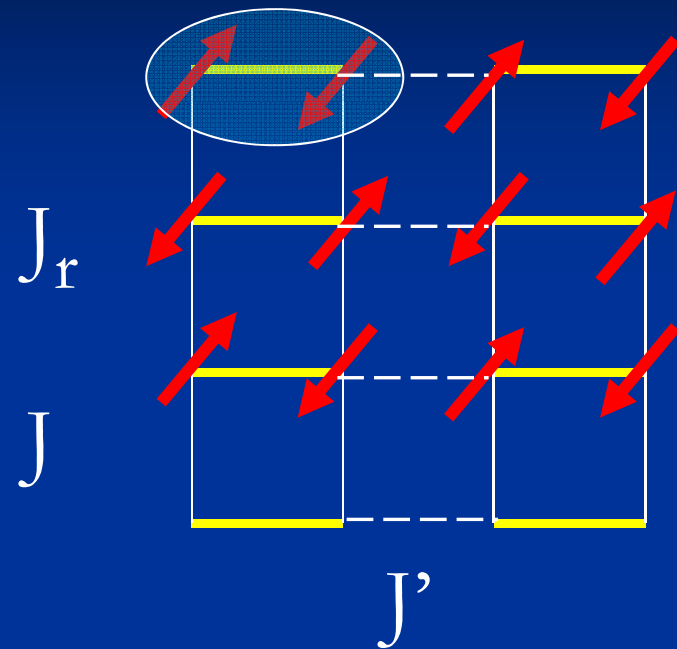
Spin Ladders



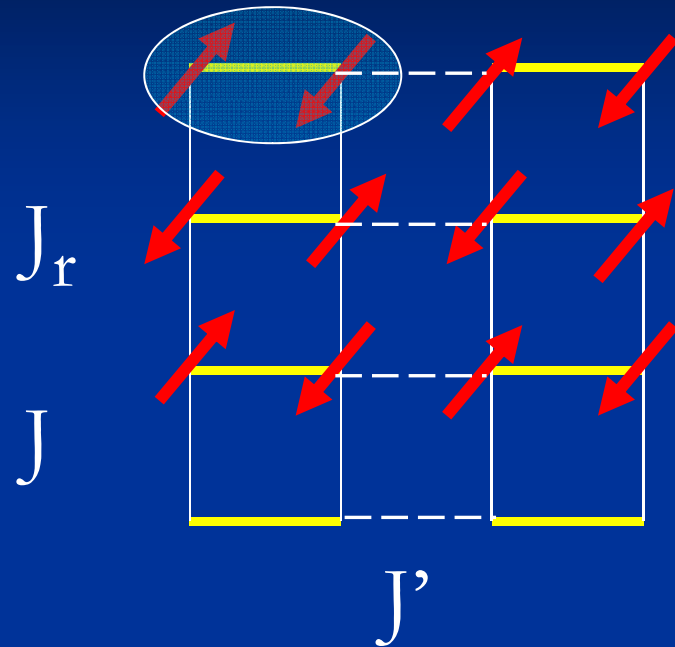
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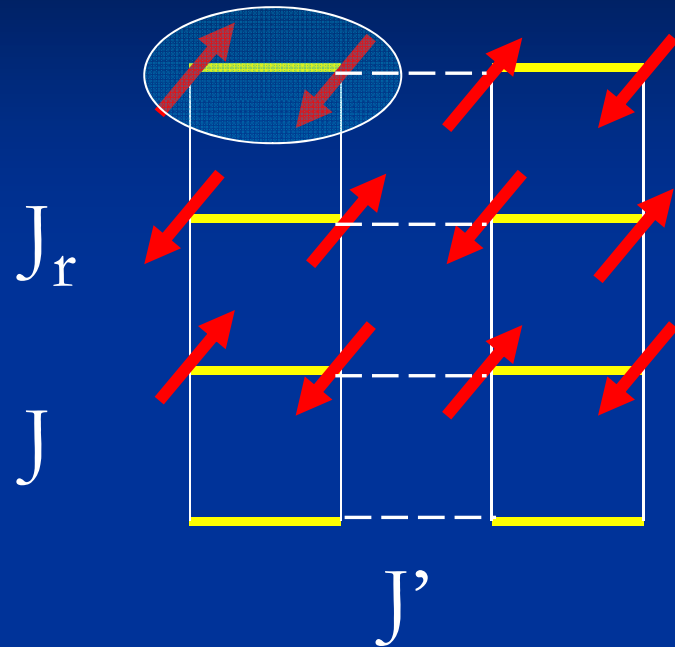
3 scales :

J_r : rungs

J : legs

J' : interladder exchange

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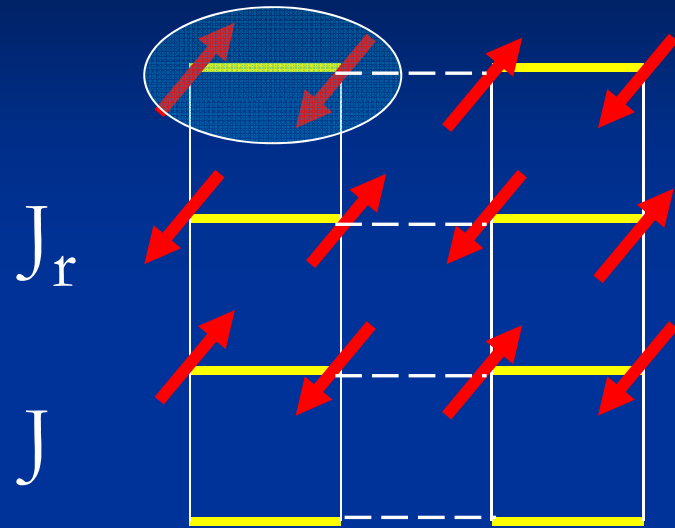
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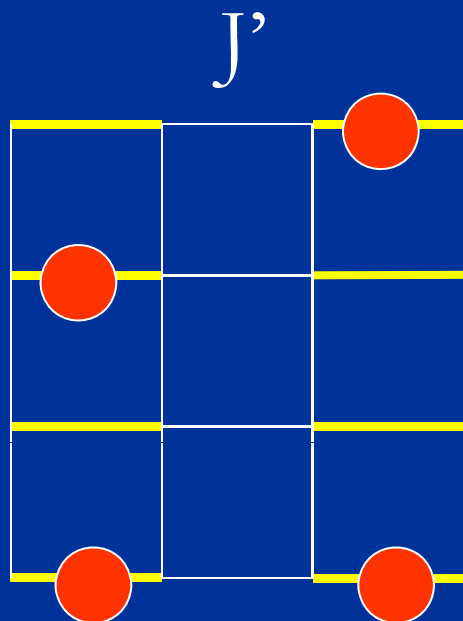


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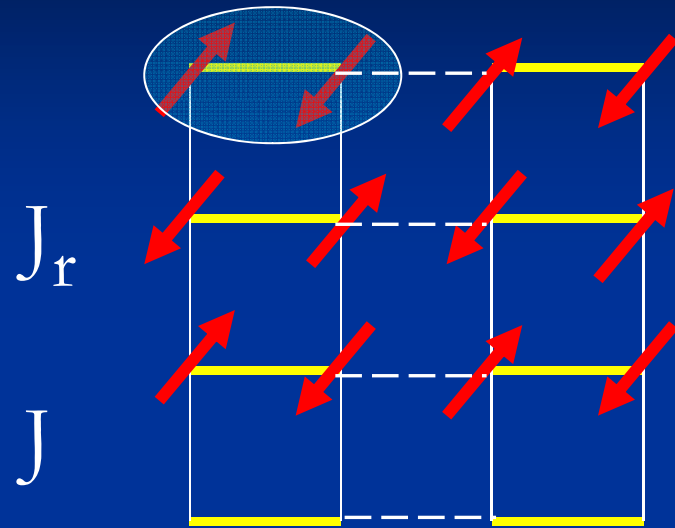
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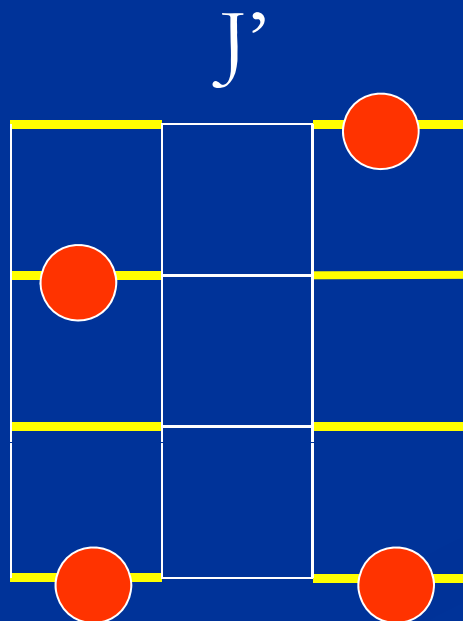


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Triplons:

hard core bosons

1D : spinless fermions

Effect of magnetic field



Effect of magnetic field



Effect of magnetic field



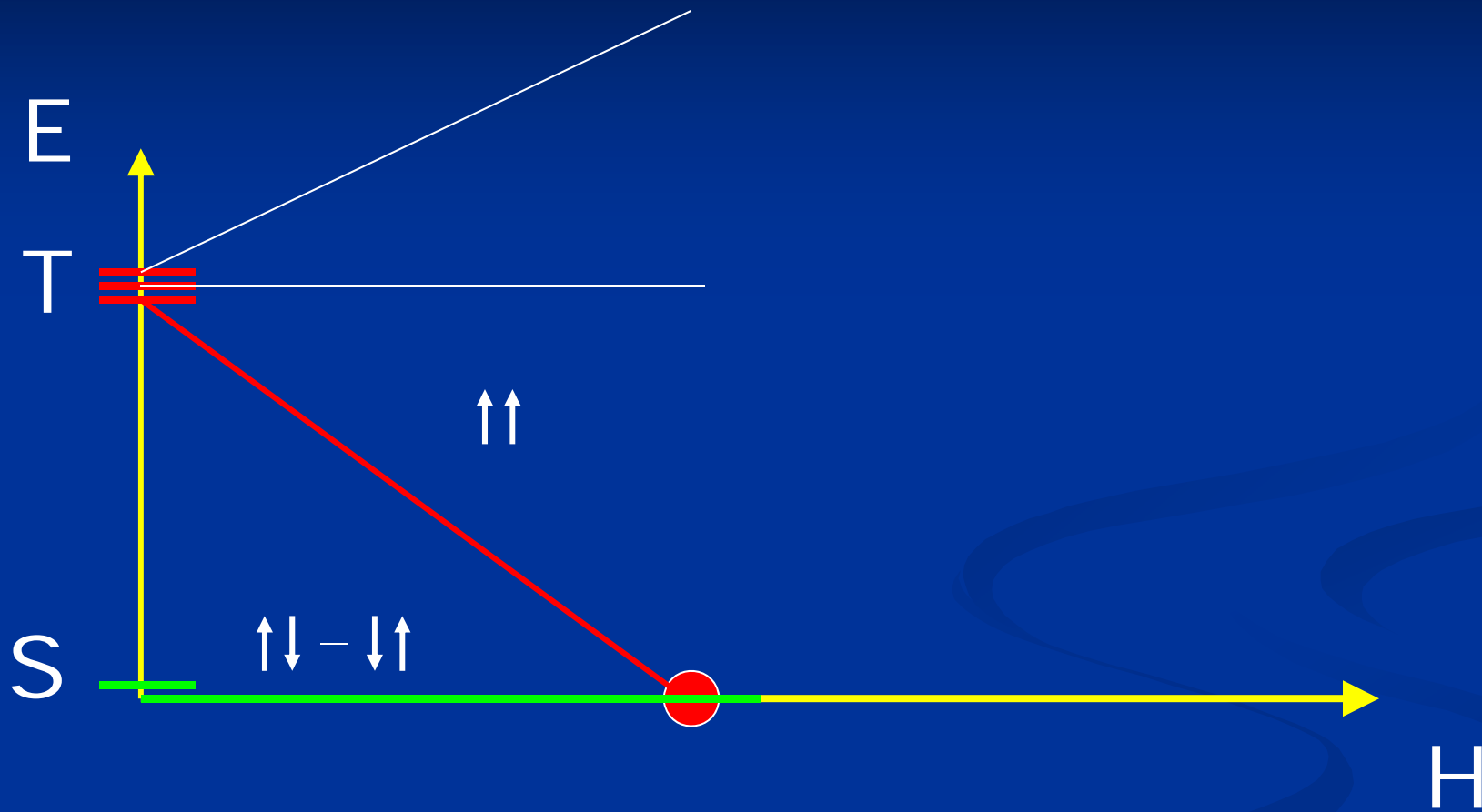
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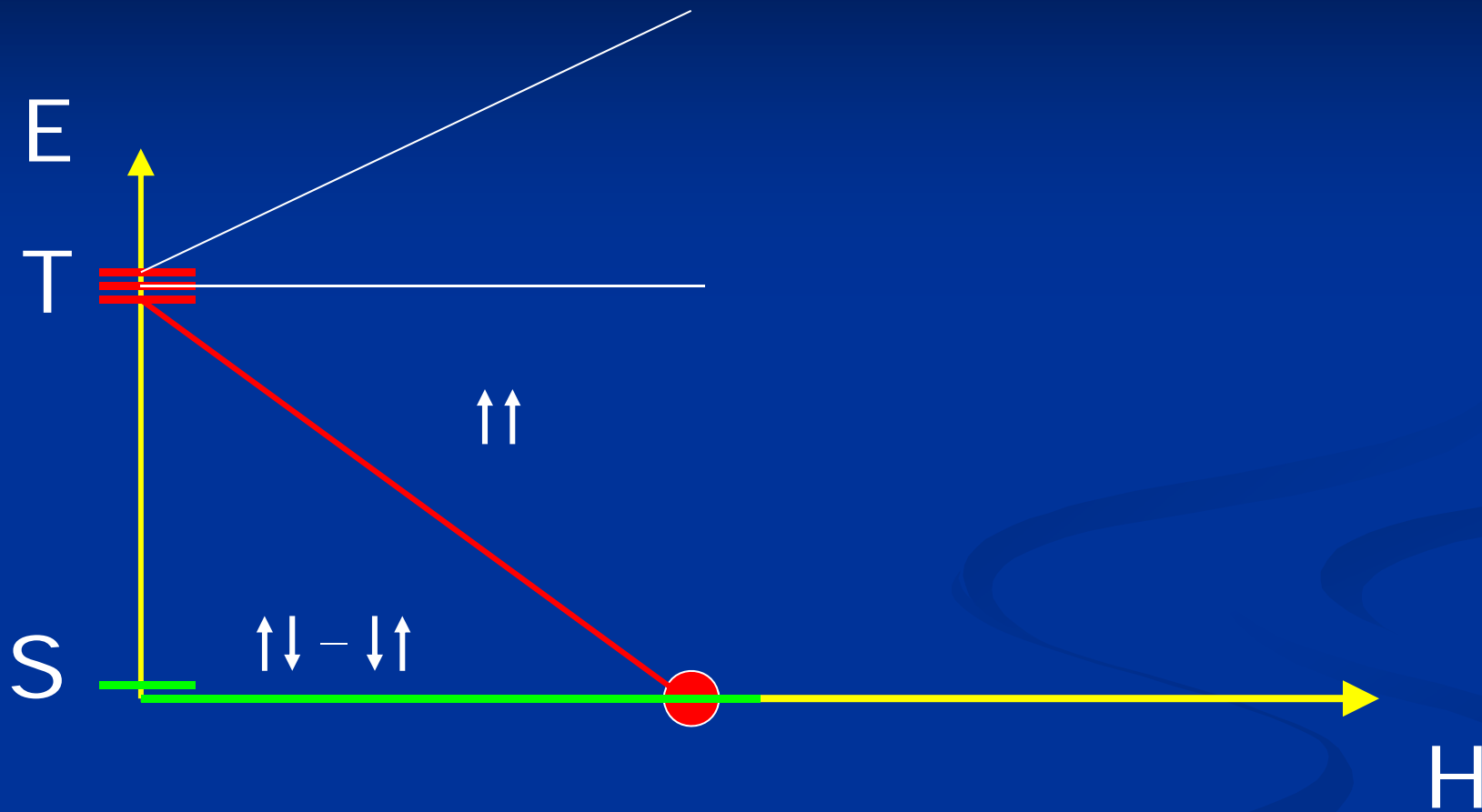
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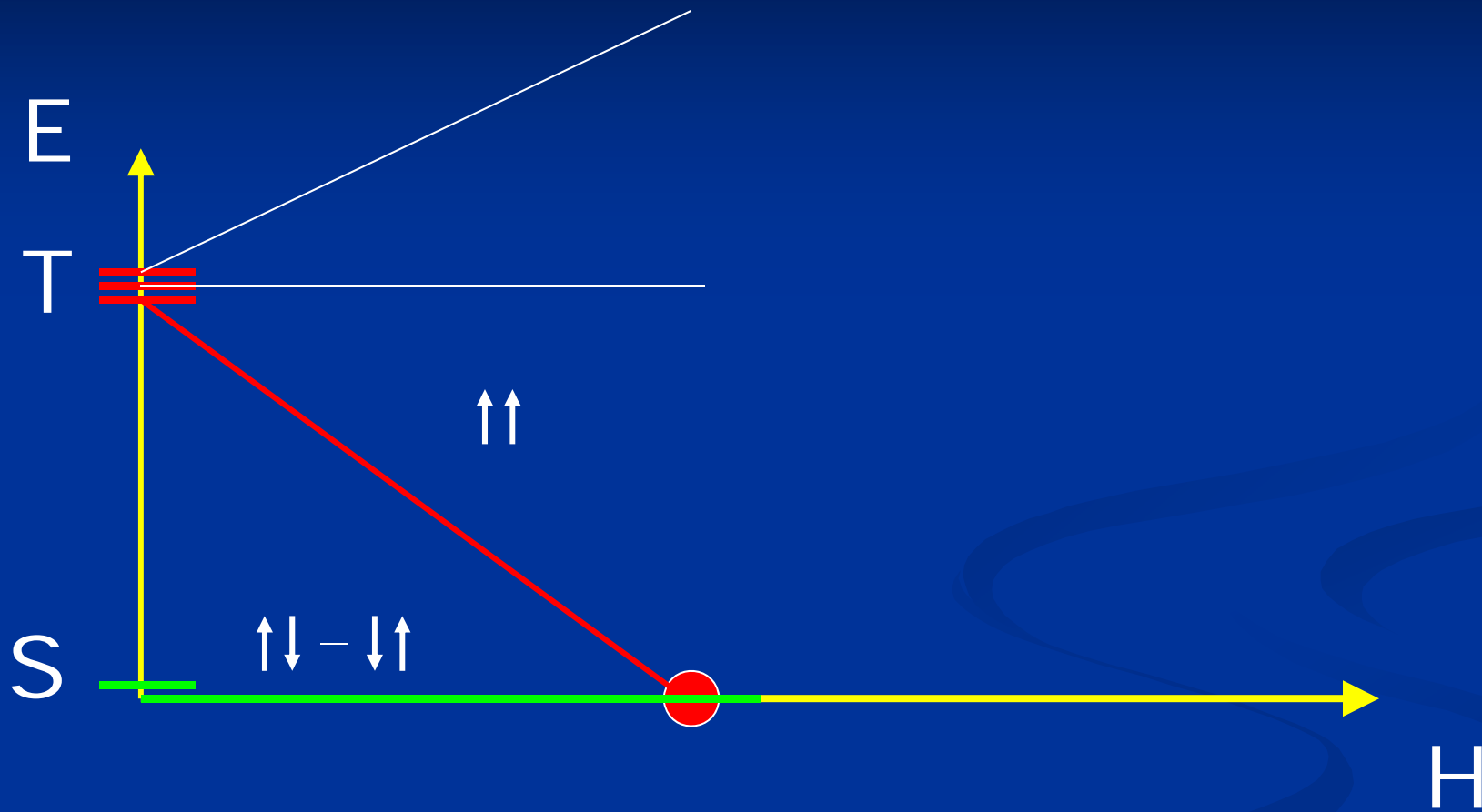
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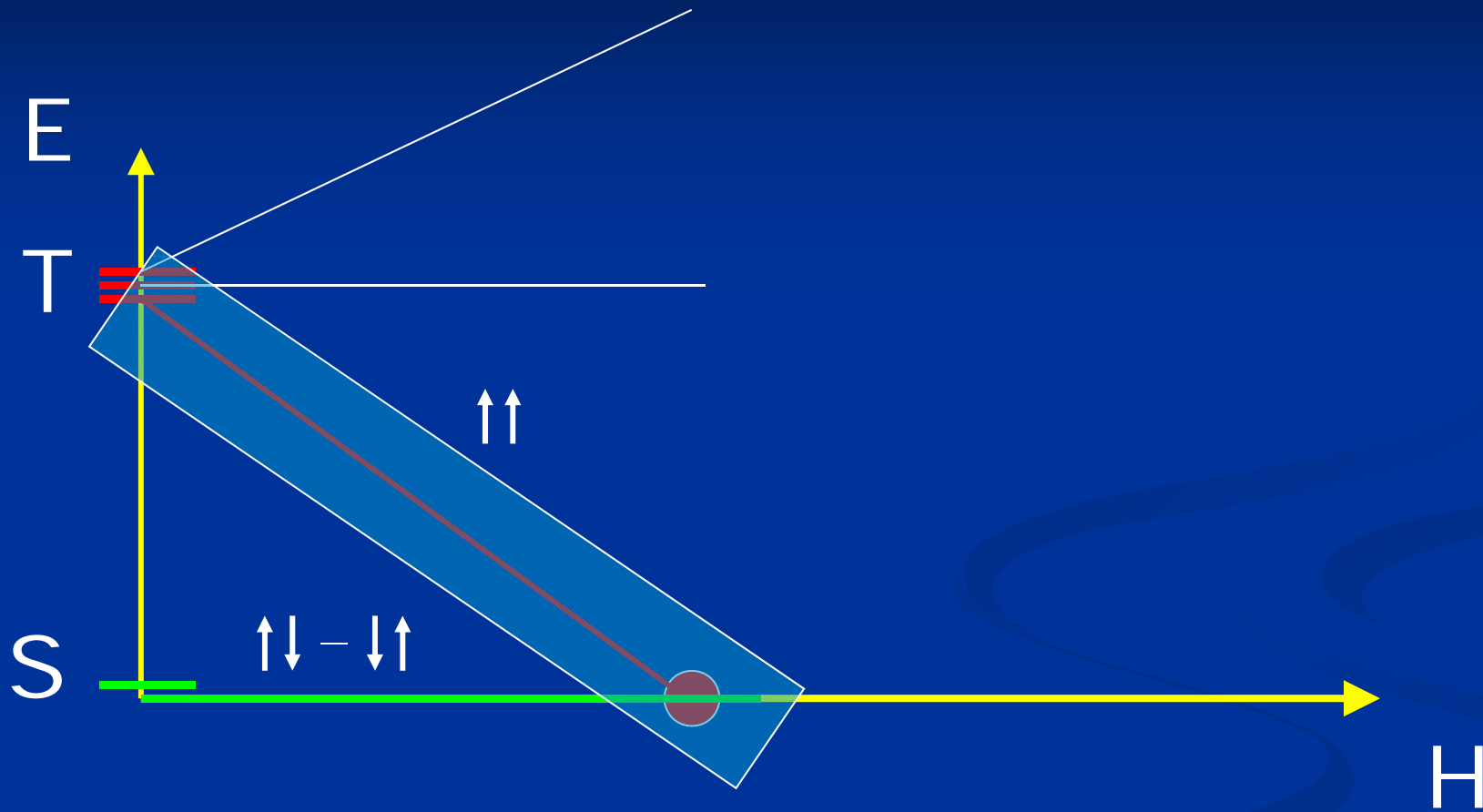
Effect of magnetic field



Effect of magnetic field



Effect of magnetic field



Magnetic field: like a chemical potential (hc_1 , hc_2)

m



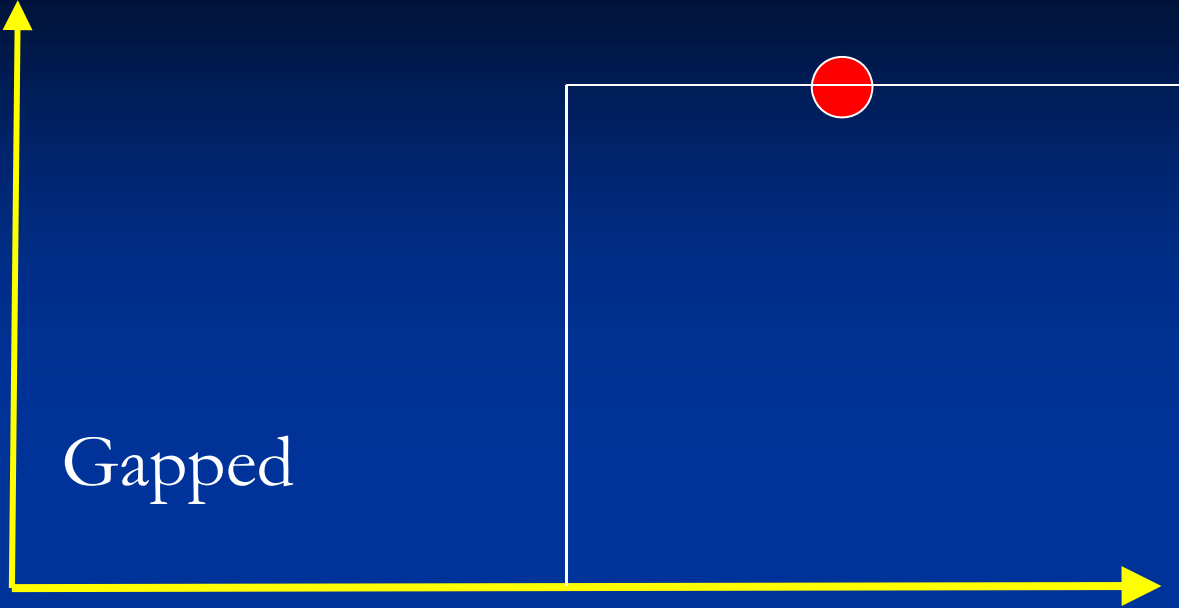
Gapped



h



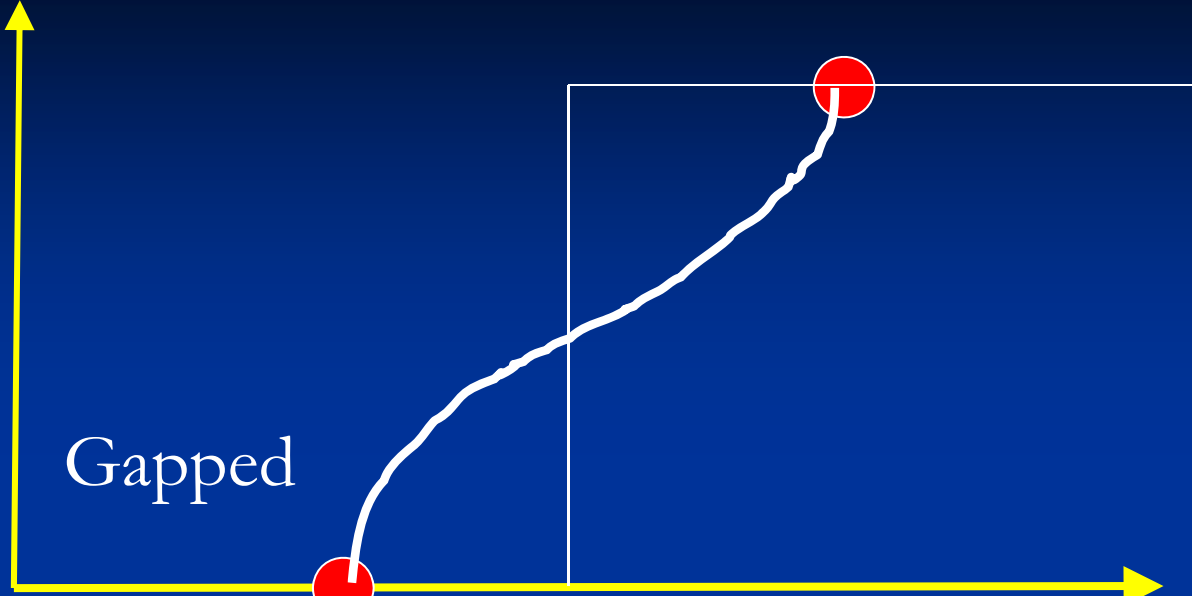
m



Gapped

h

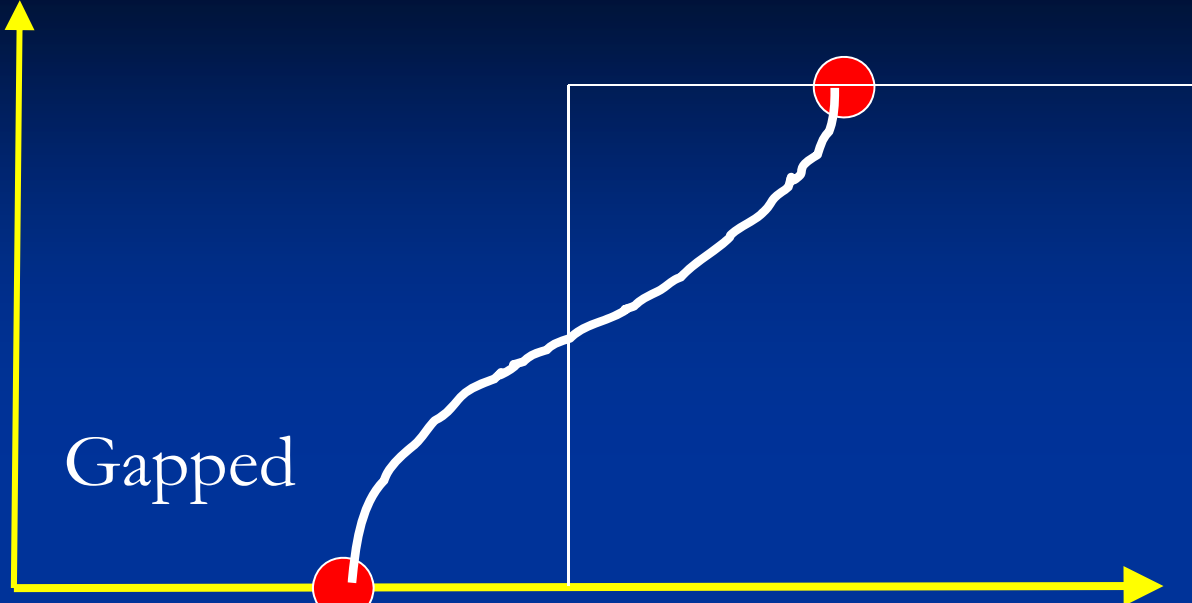
m



Gapped

h

m

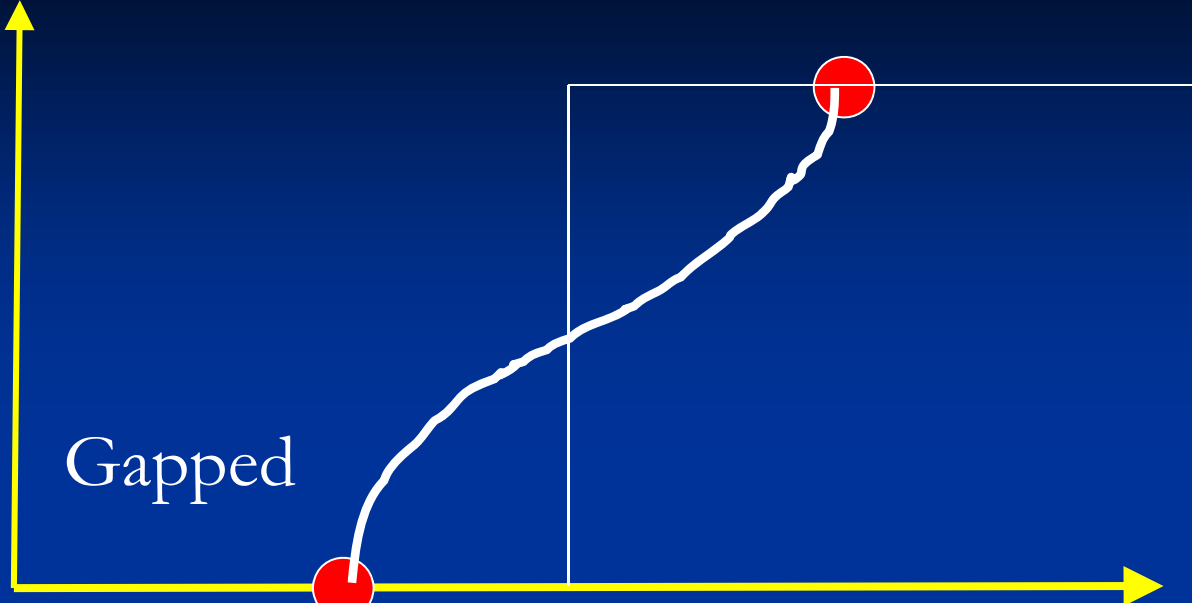


Gapped

h_{c1}

h

m



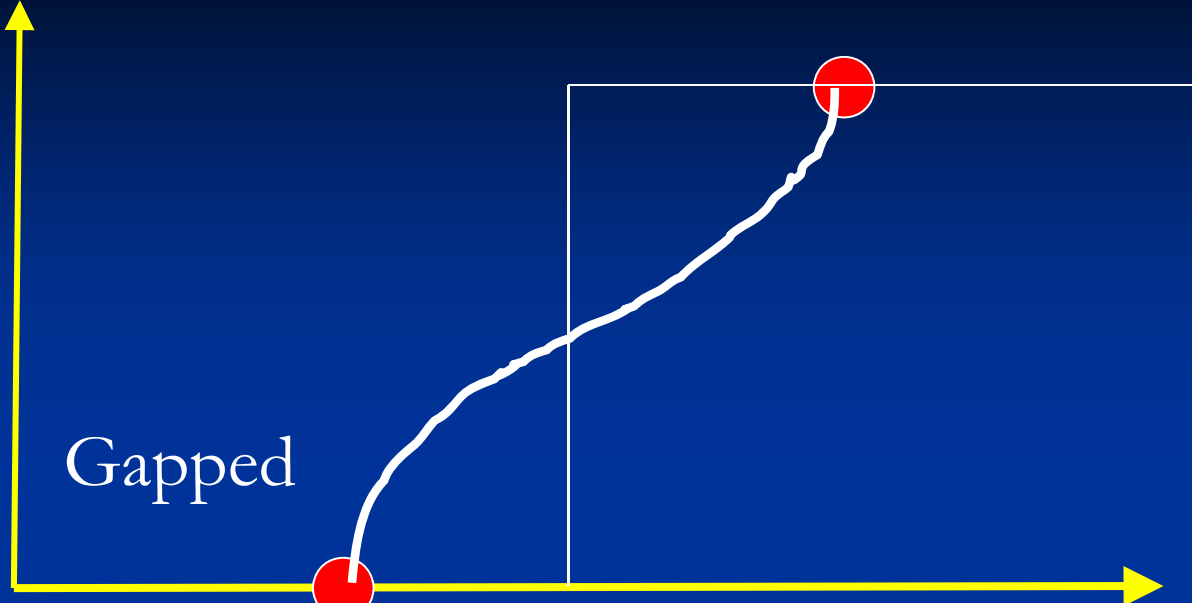
Gapped

h_{c1}

h_{c2}

h

m



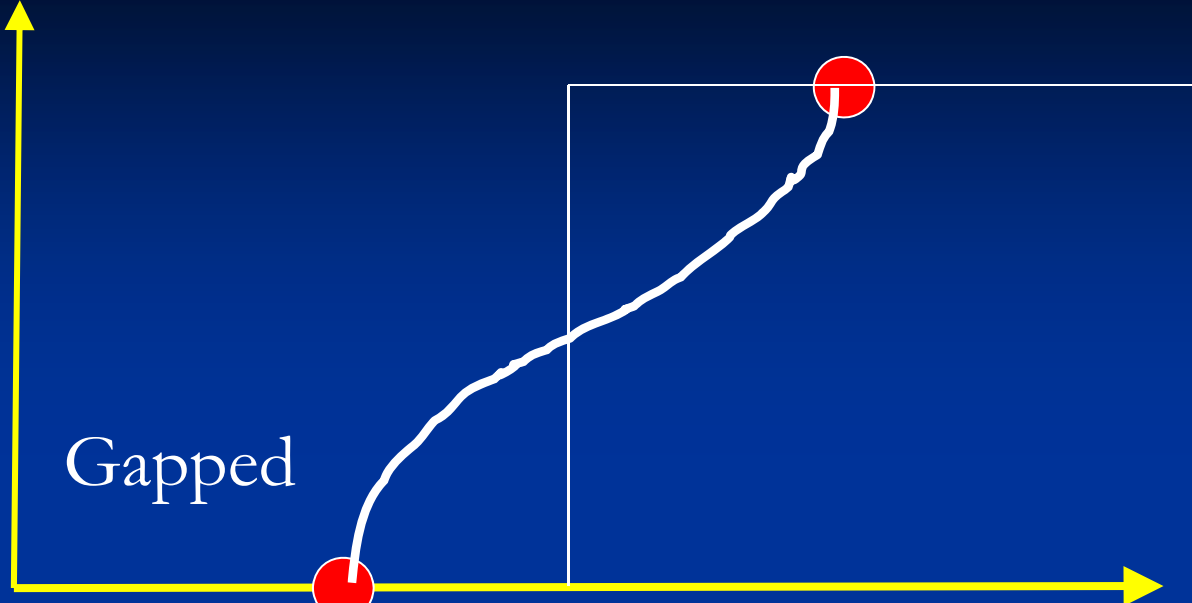
Gapped

h_{c1}

h_{c2}

h

m

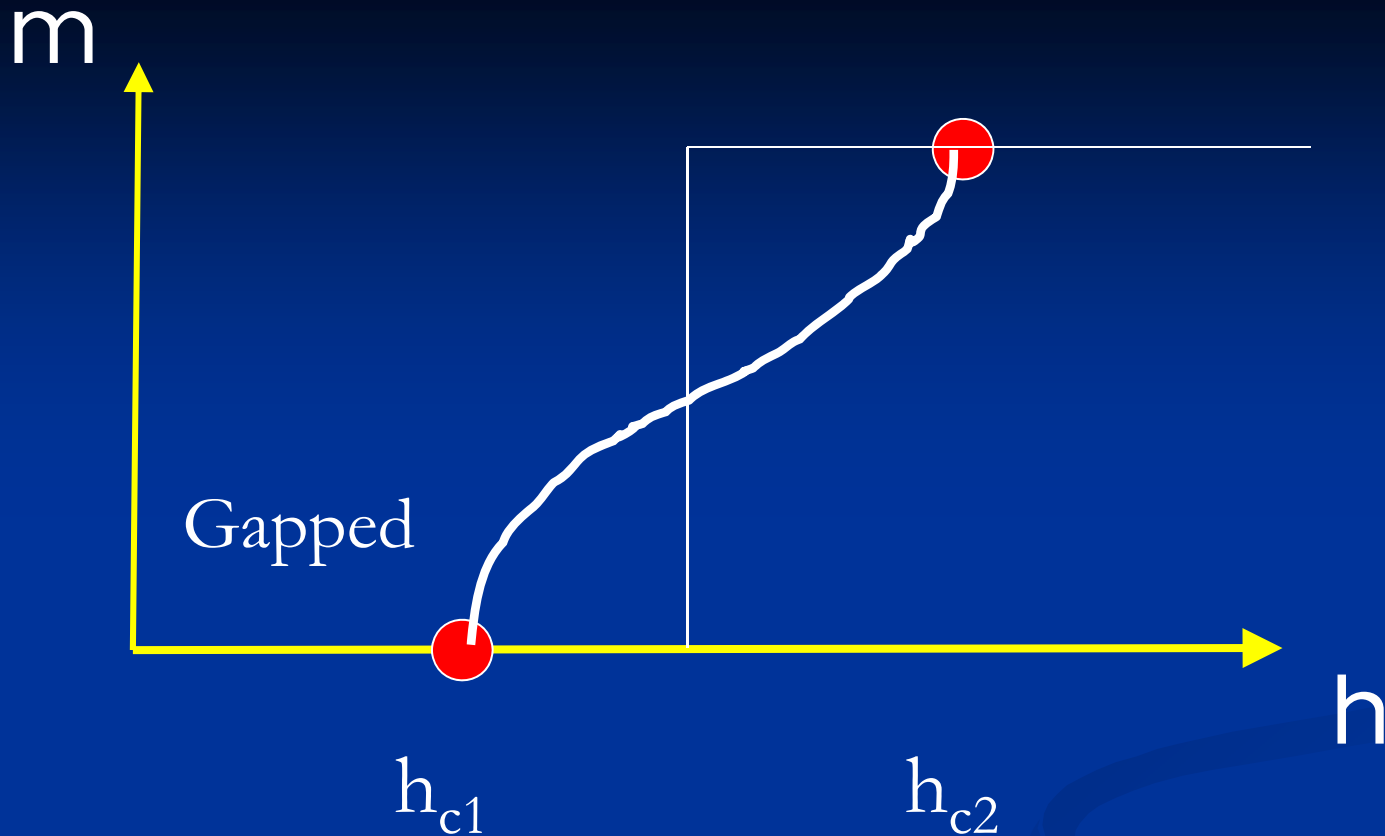


Gapped

h_{c1}

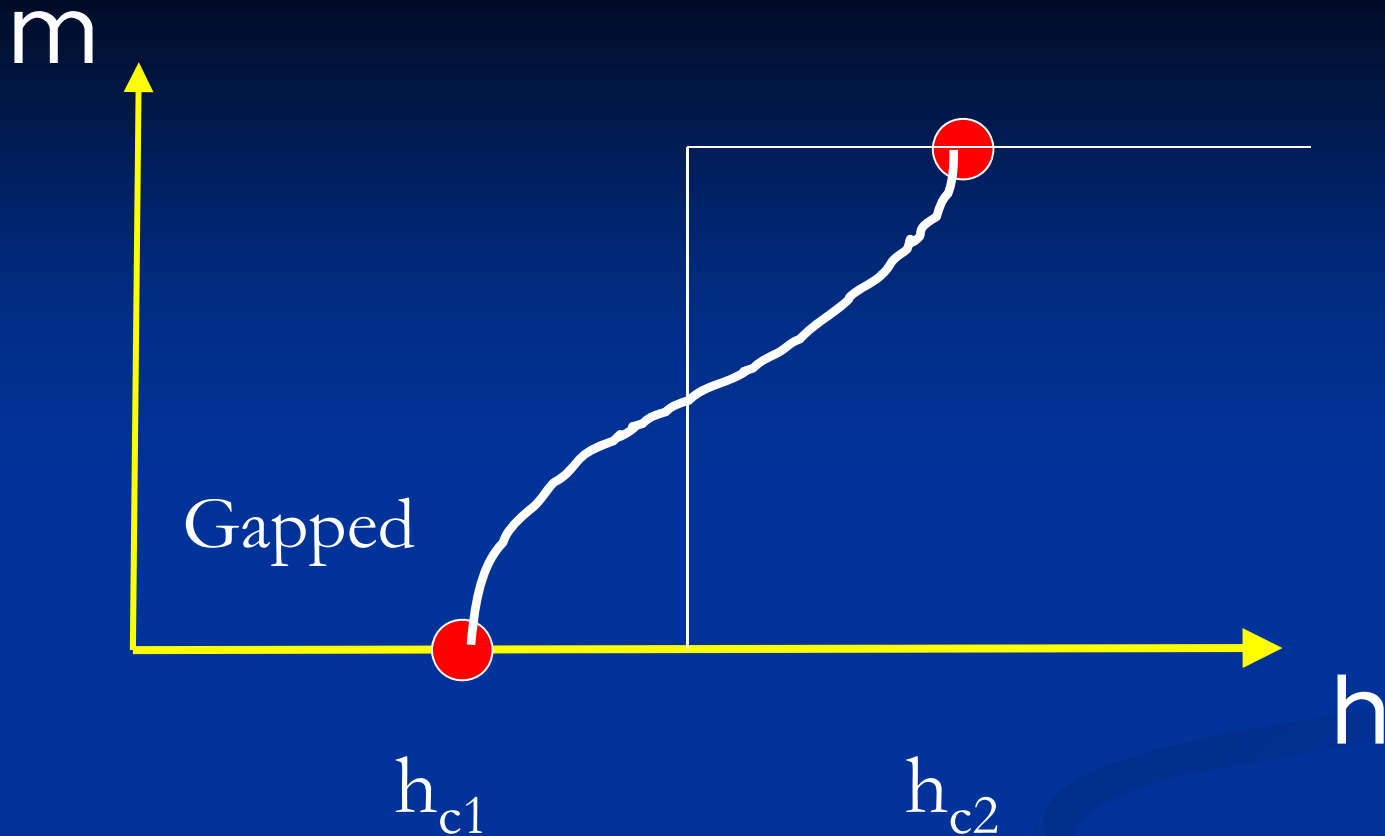
h_{c2}

h



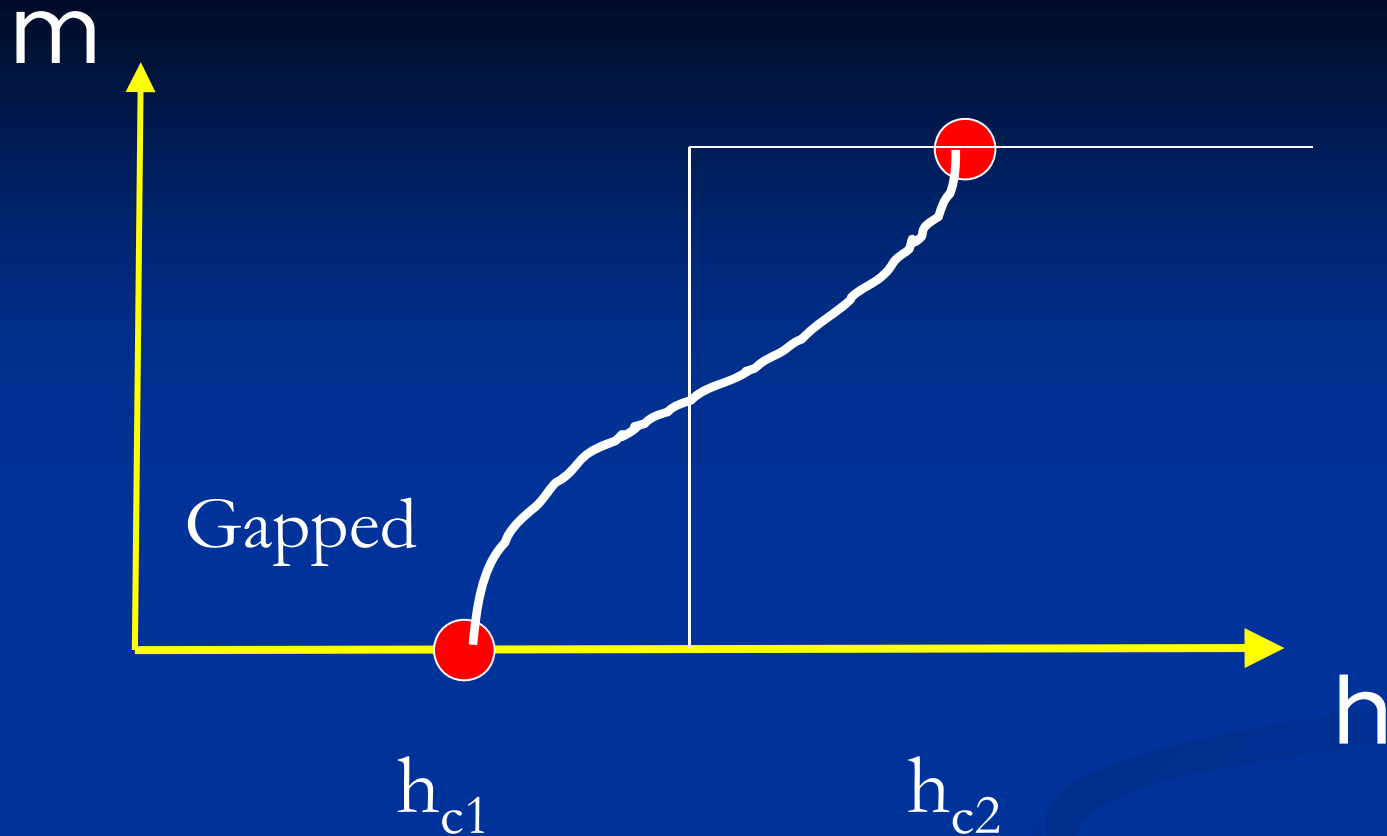
$$h_{c1} \sim J_r ; h_{c2} \sim J_r + J$$

A way to study interacting spinless fermions !



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A way to study interacting spinless fermions !

All microscopic parameters are known !

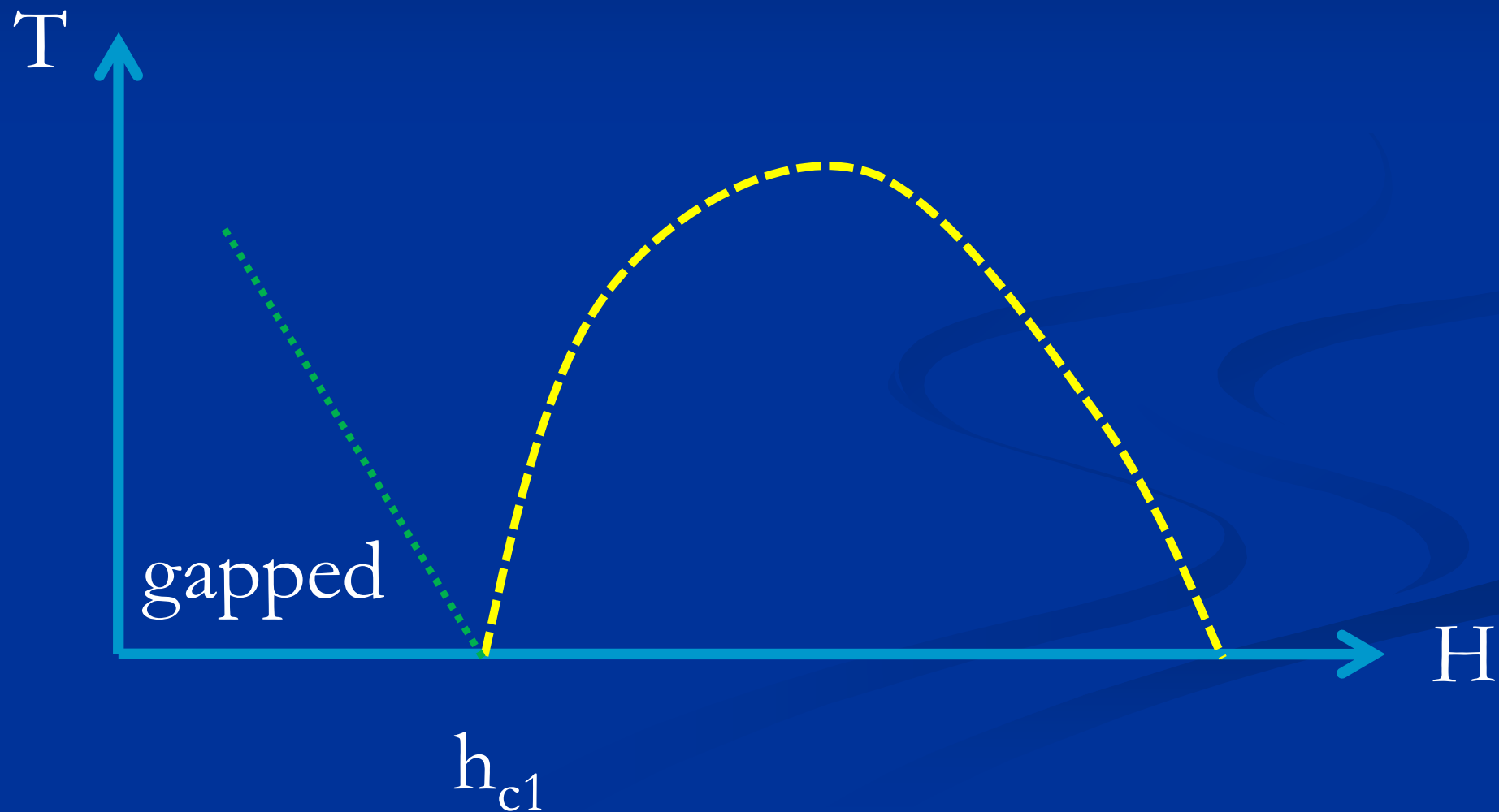
Expected phase diagram

S. Sachdev , T. Senthil, R. Shankar PRB 50 258 (94); R. Chitra, TG PRB 55 5816 (97); TG,
A.M. Tsvelik PRB 59 11398 (99)



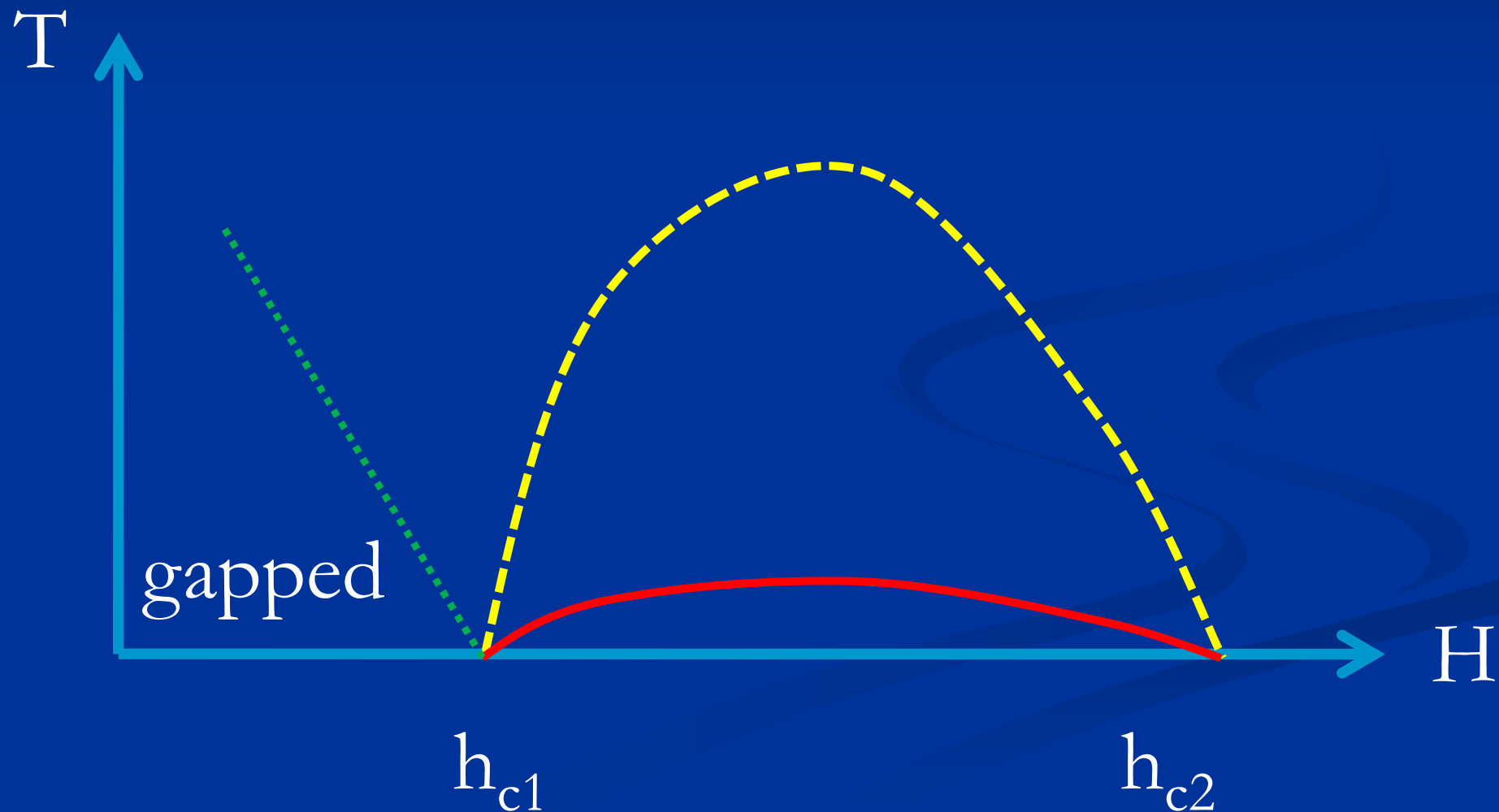
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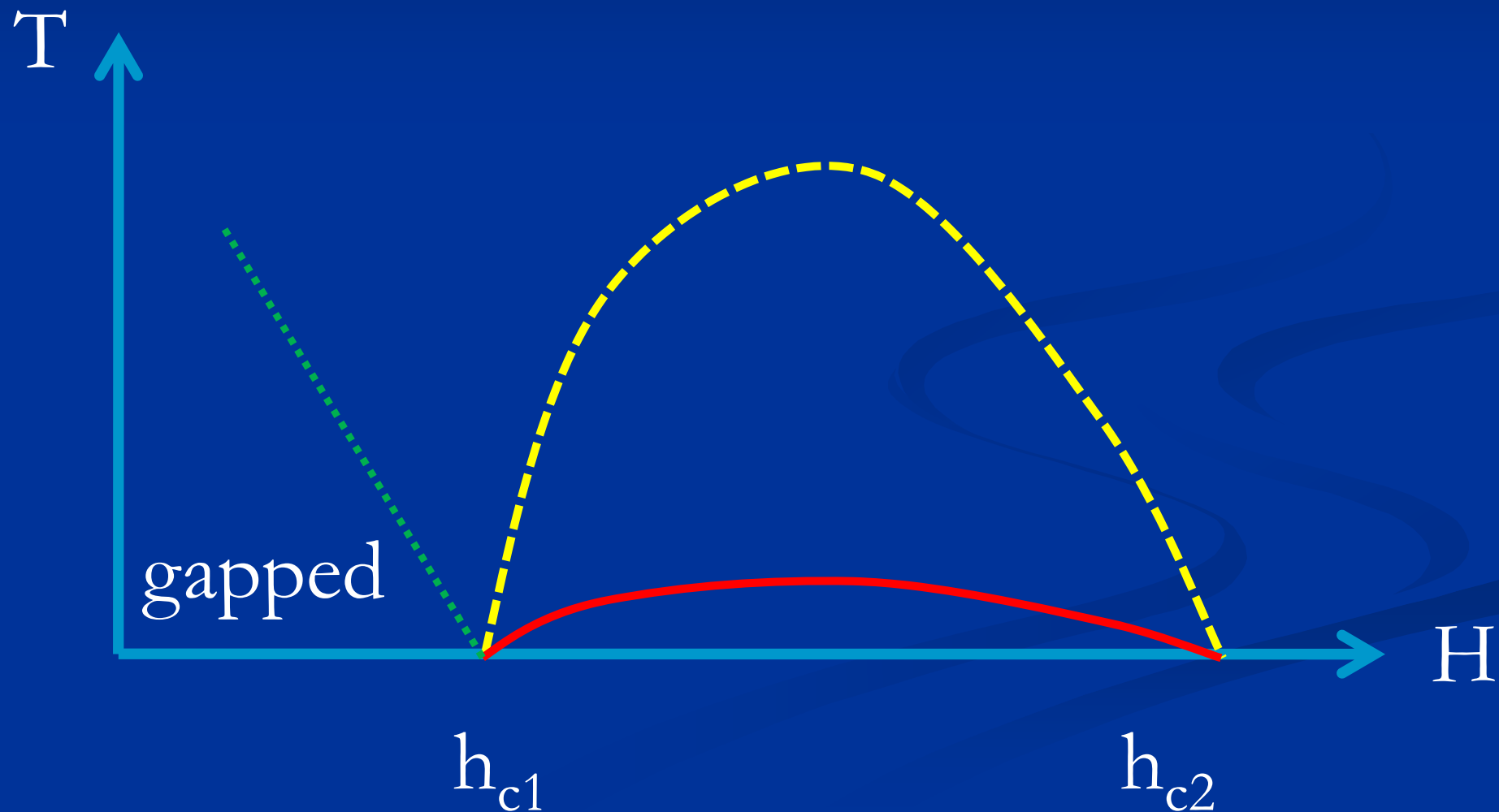
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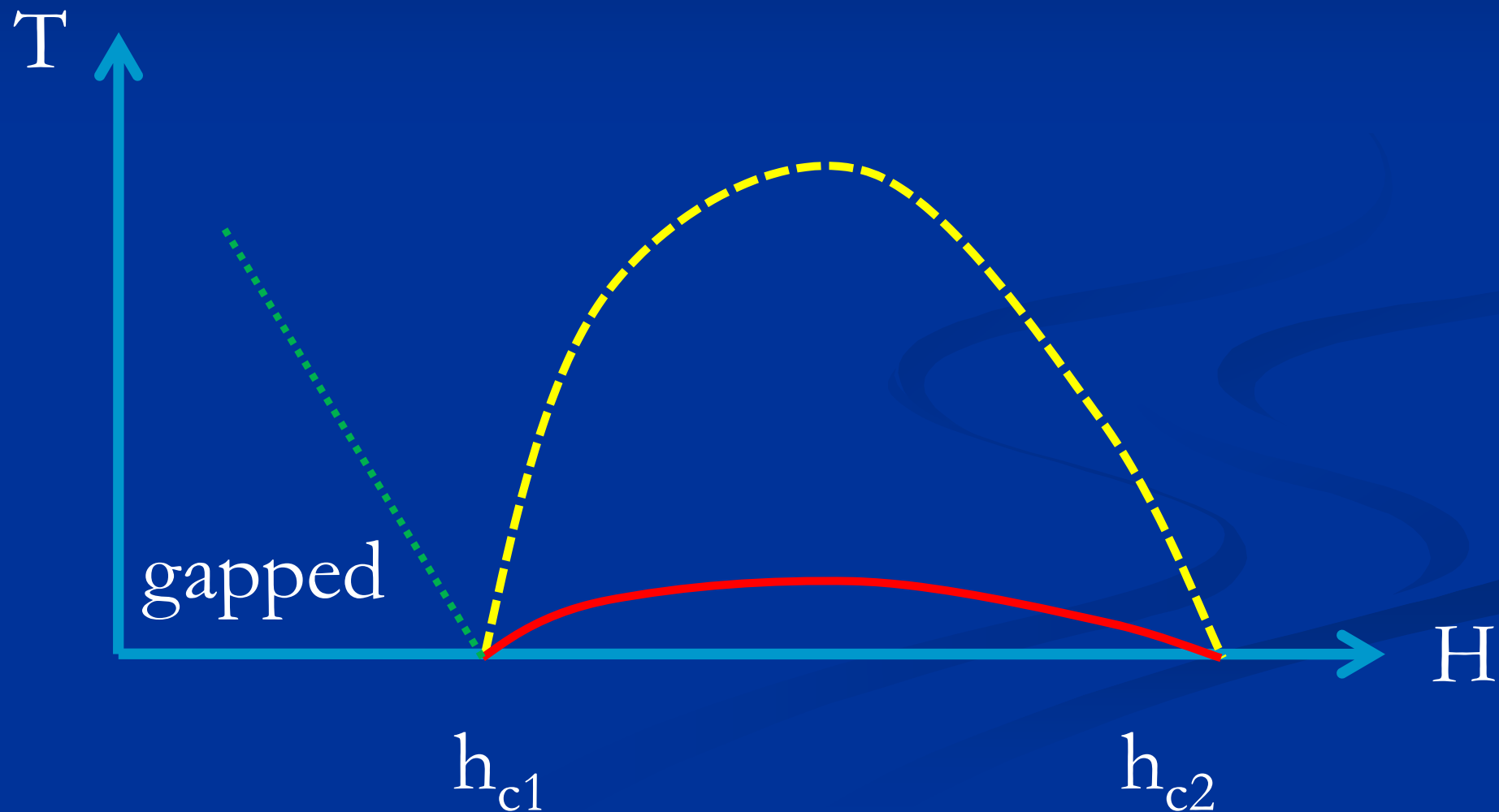
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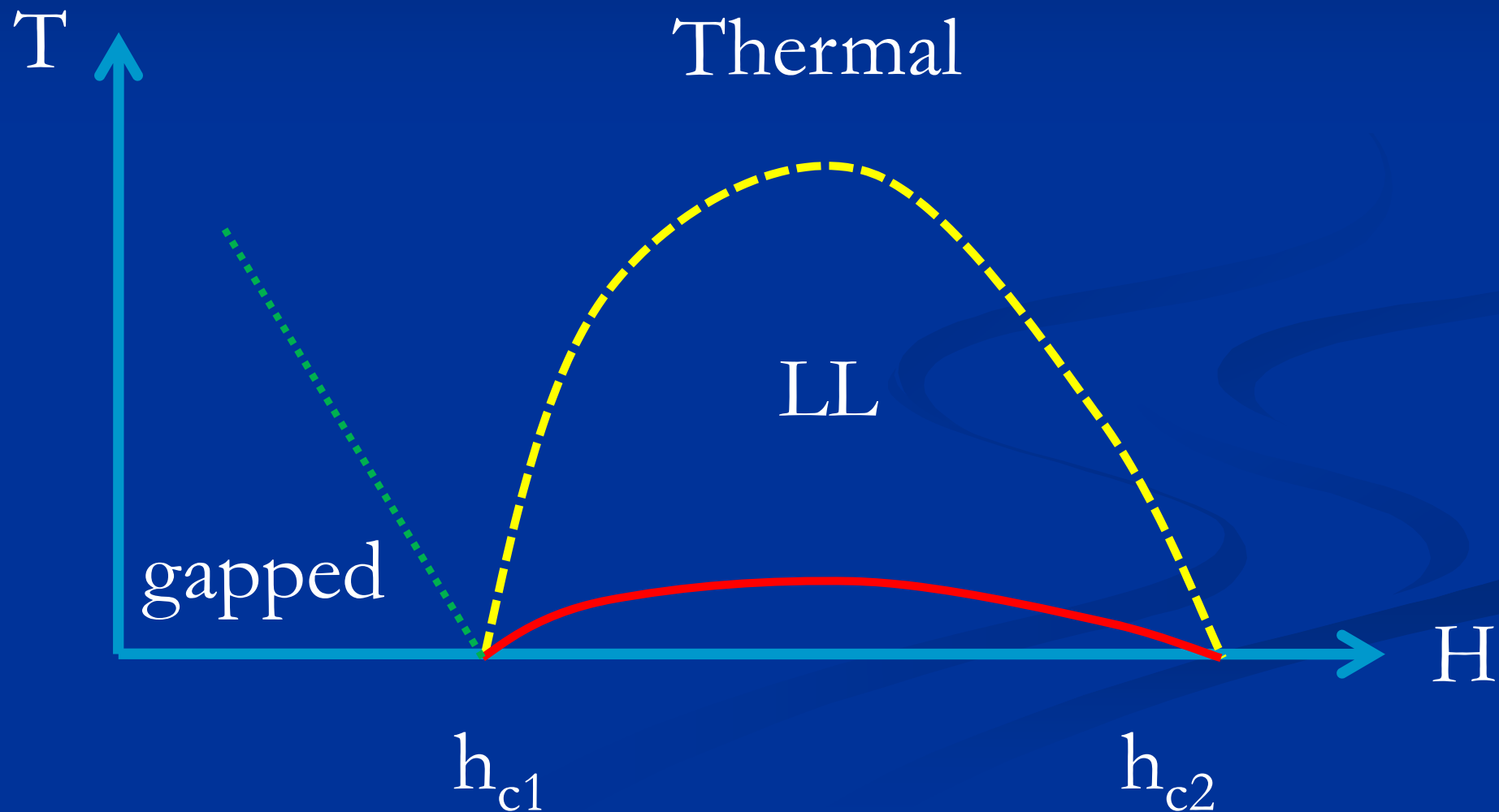
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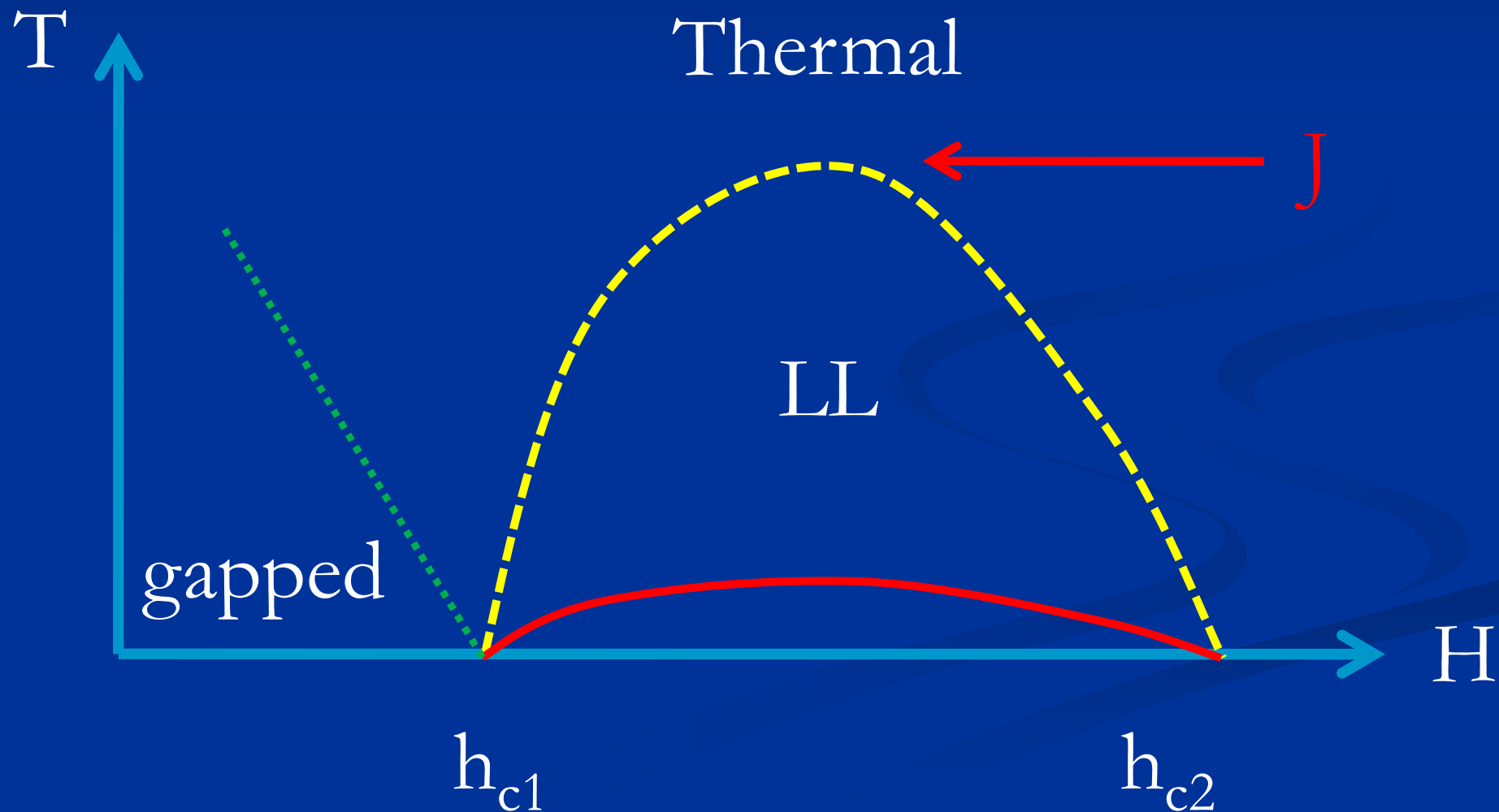
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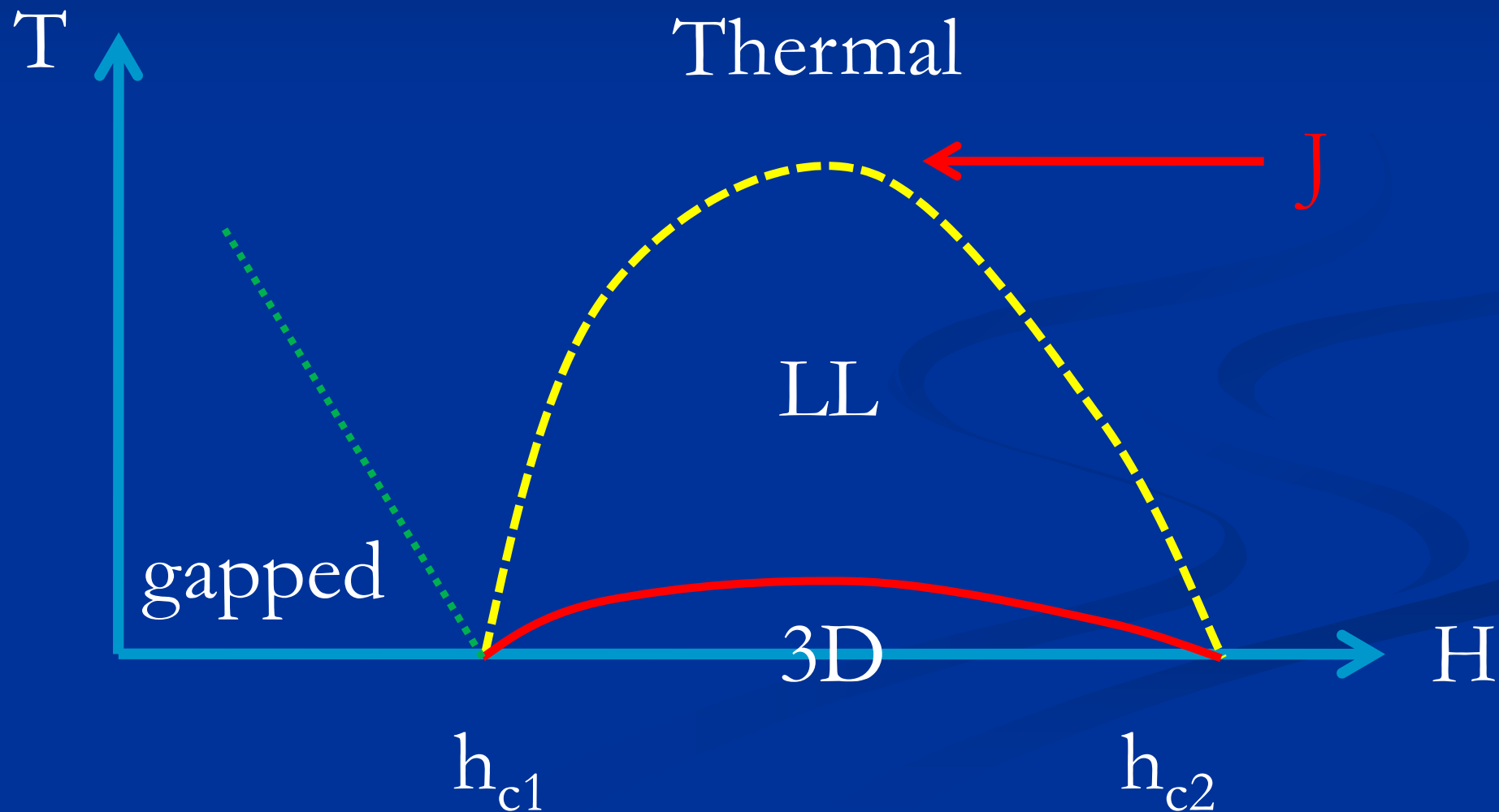
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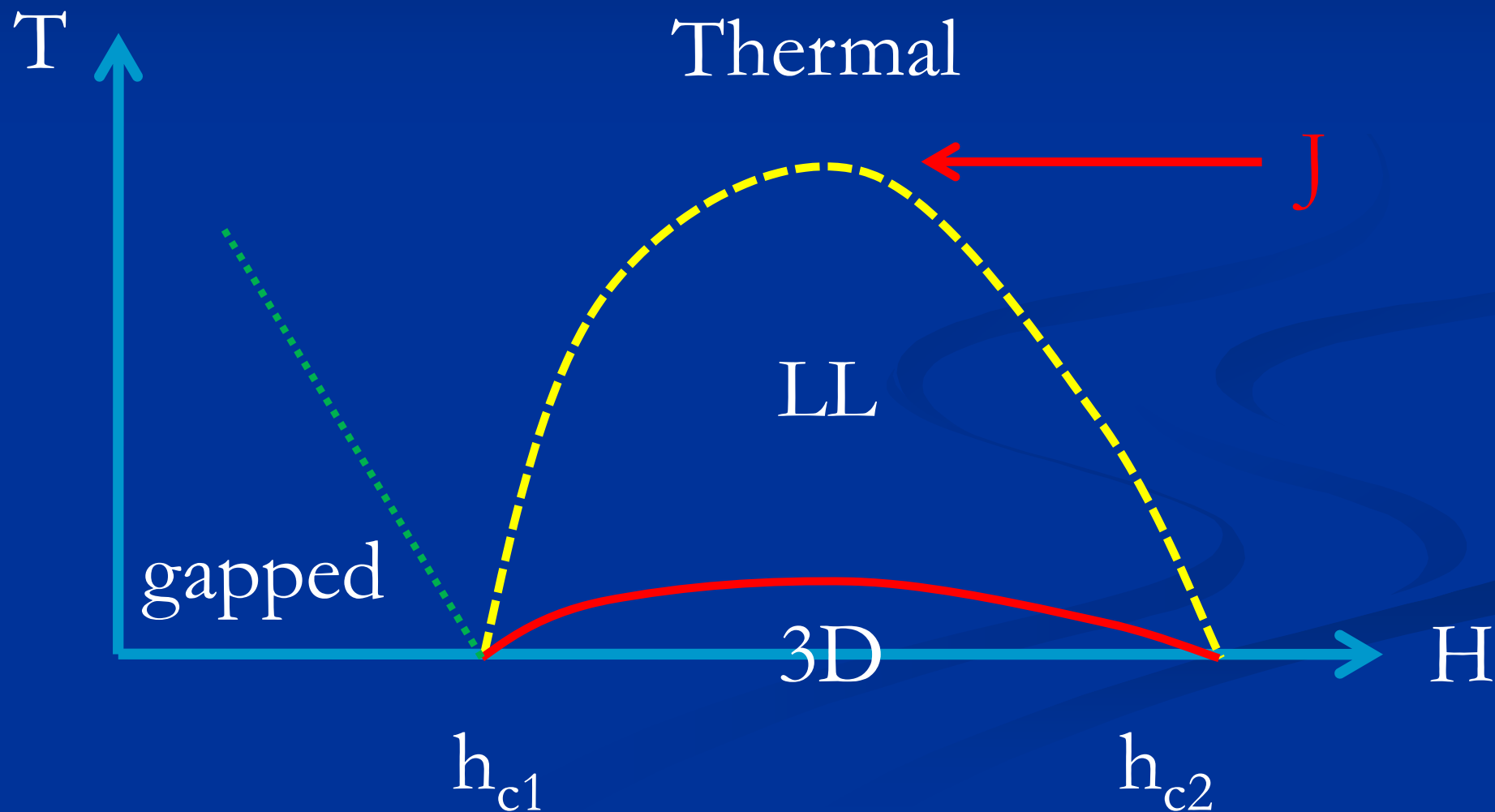
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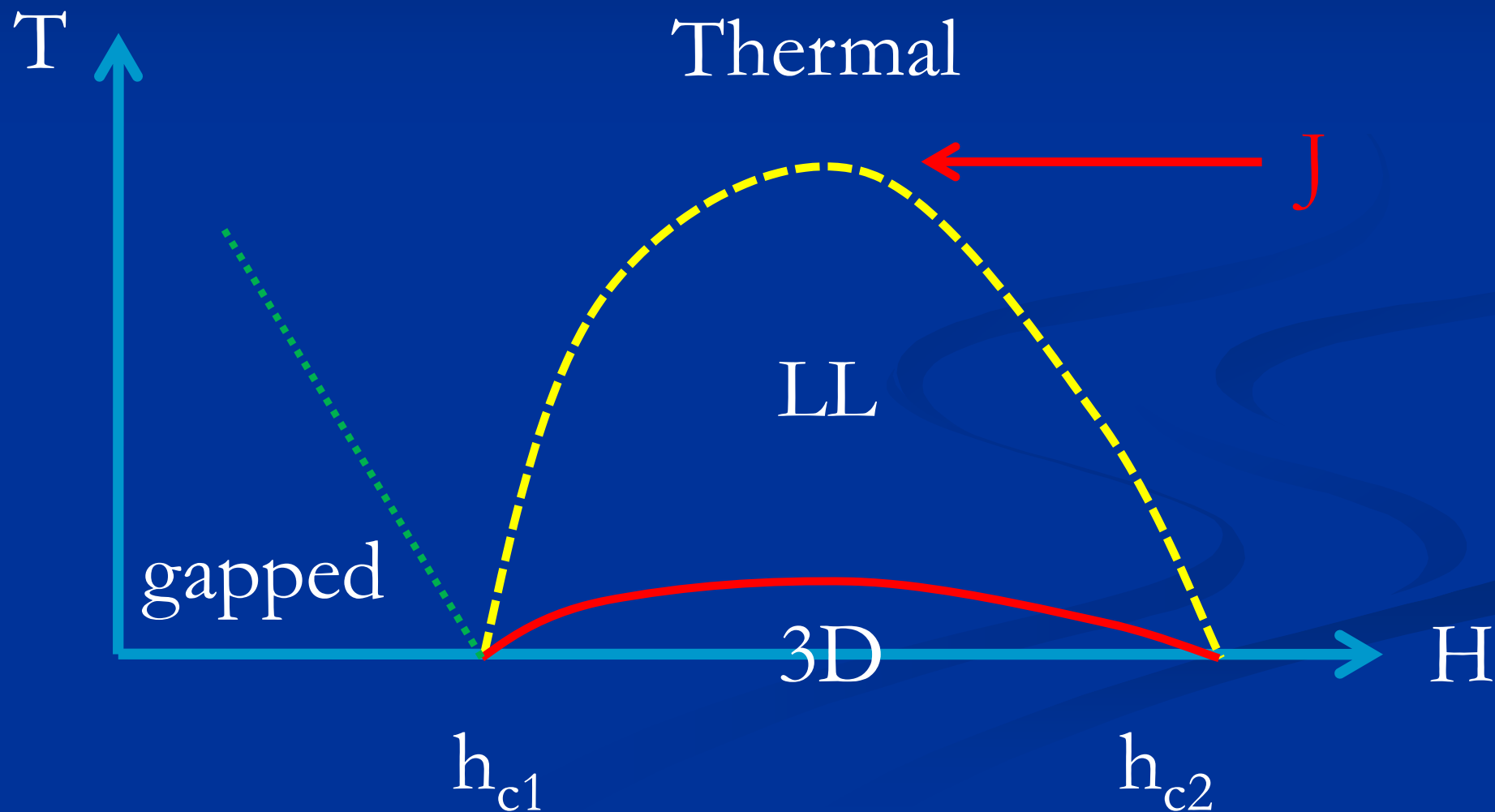
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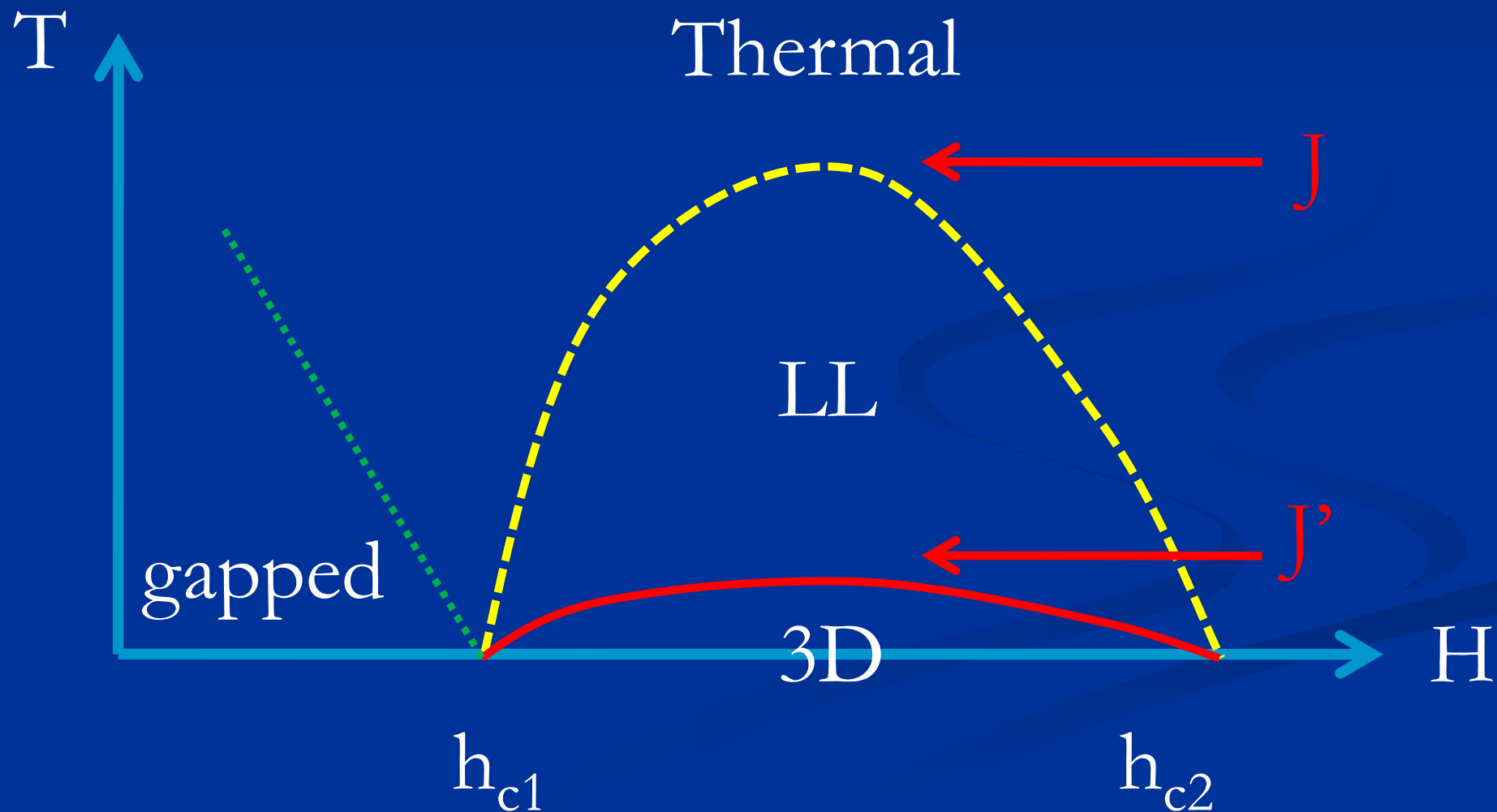
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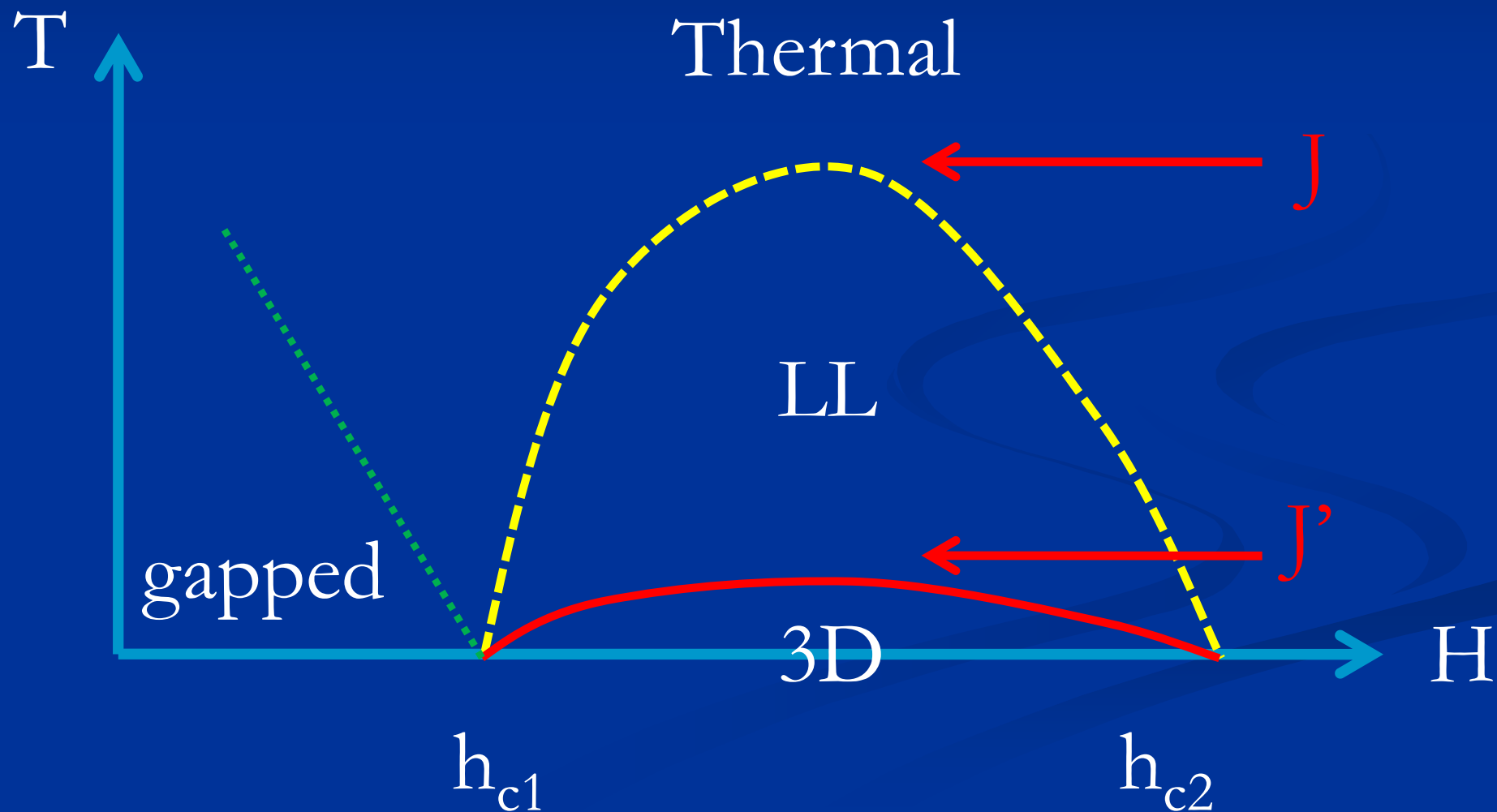
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- Analytical calculations (Luttinger liquid + BA)

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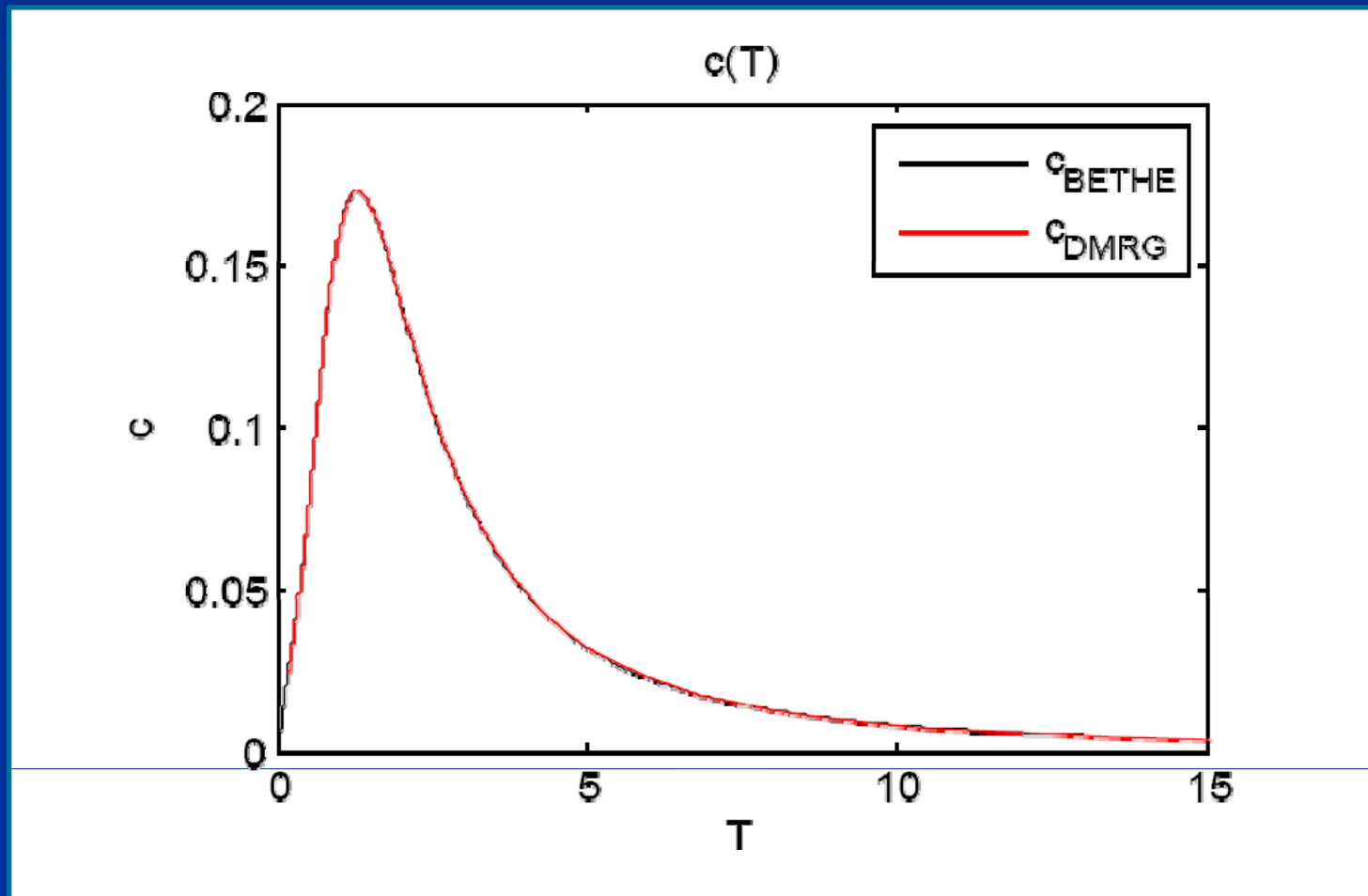
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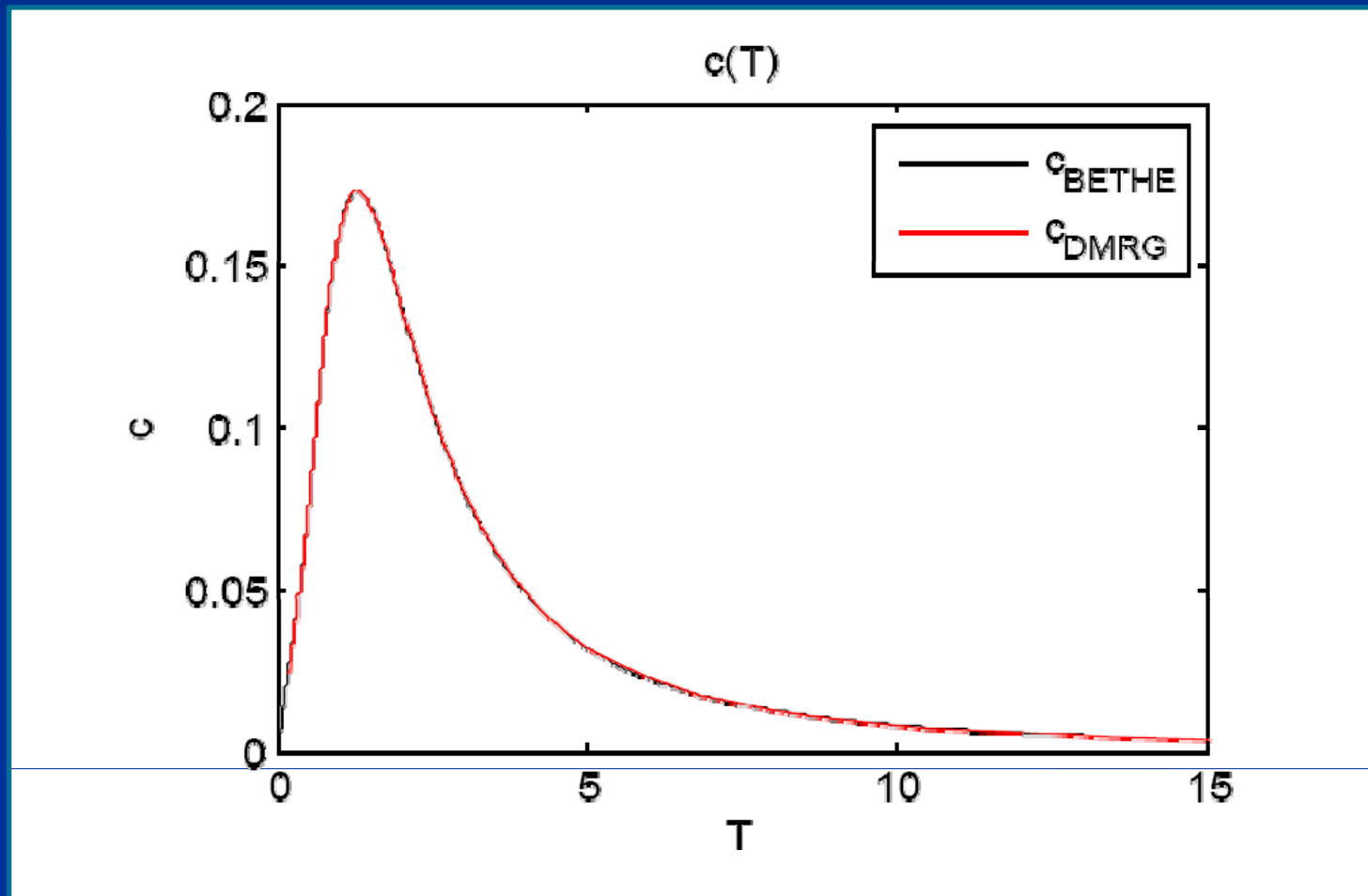
S. White

Finite temperature DMRG



Finite temperature DMRG

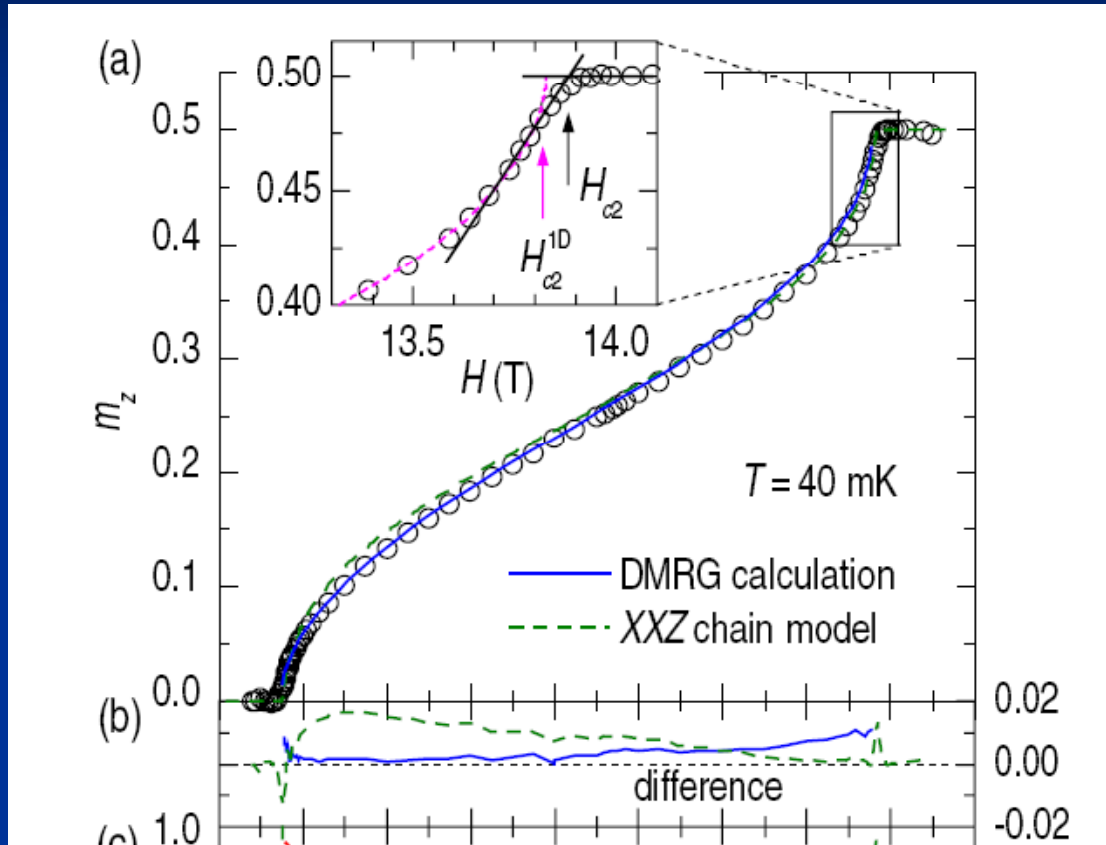
Specific heat of spin chain



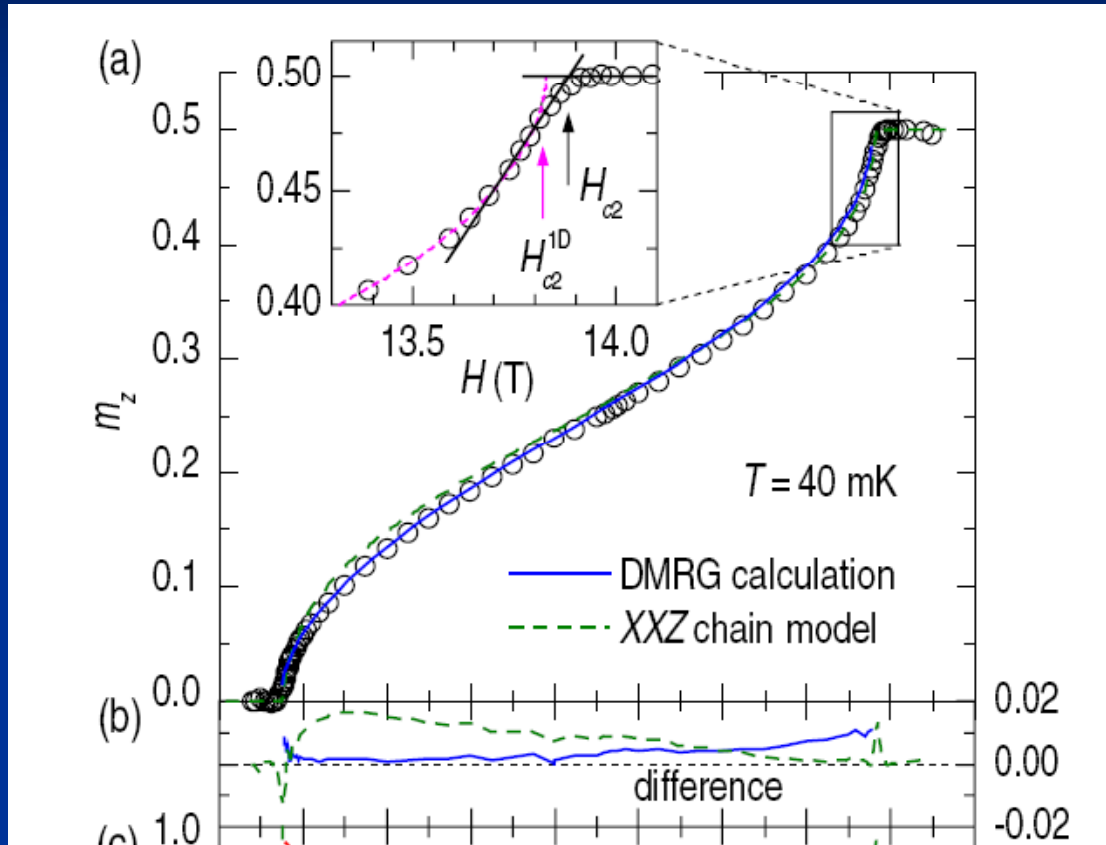
Magnetization



Magnetization

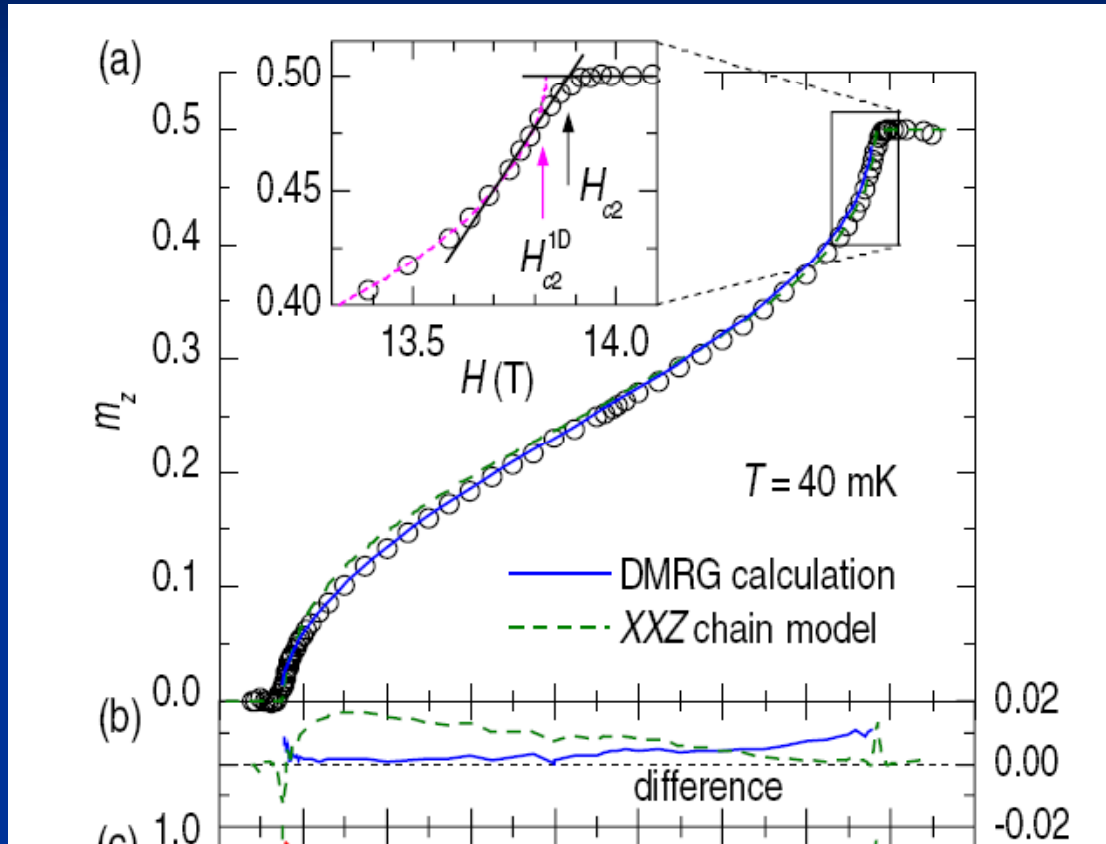


Magnetization



M. Klanjsek et al., PRL 101 137207 (2008)

Magnetization



Fixes:

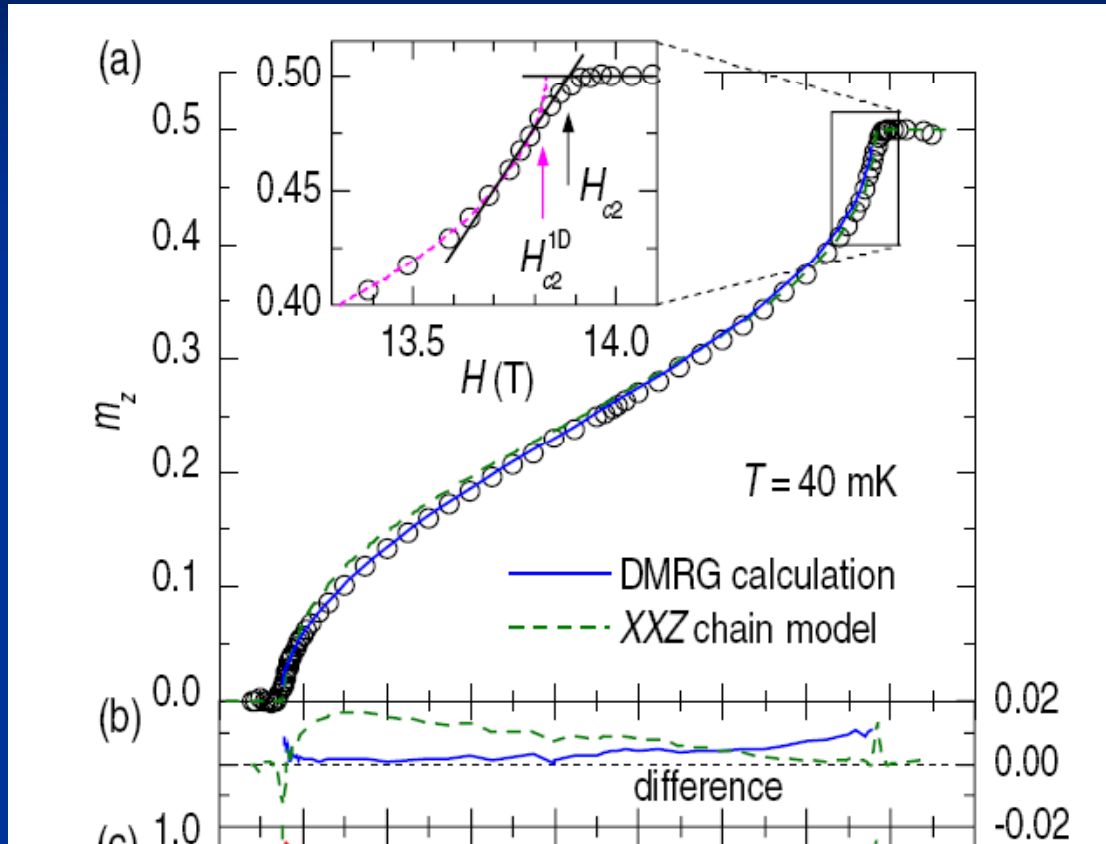
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M. Klanjsek et al., PRL 101 137207 (2008)

Magnetization



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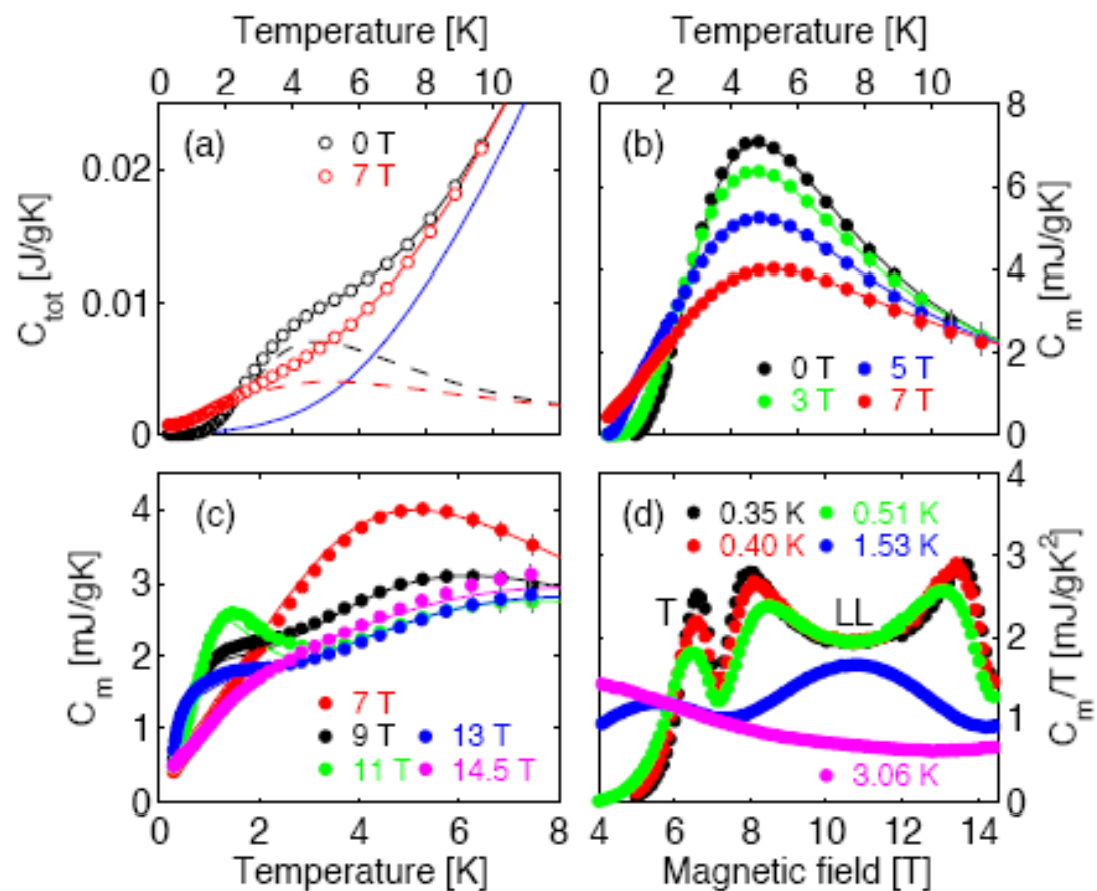
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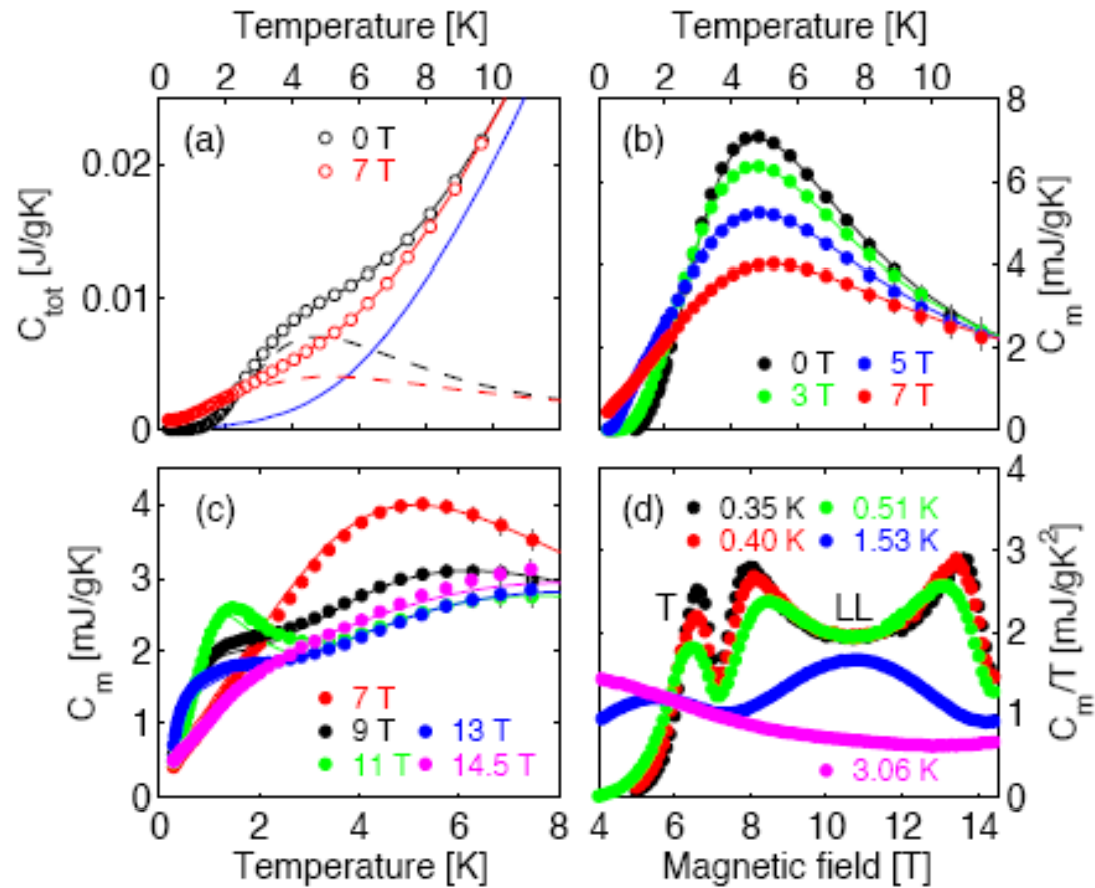
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Specific heat

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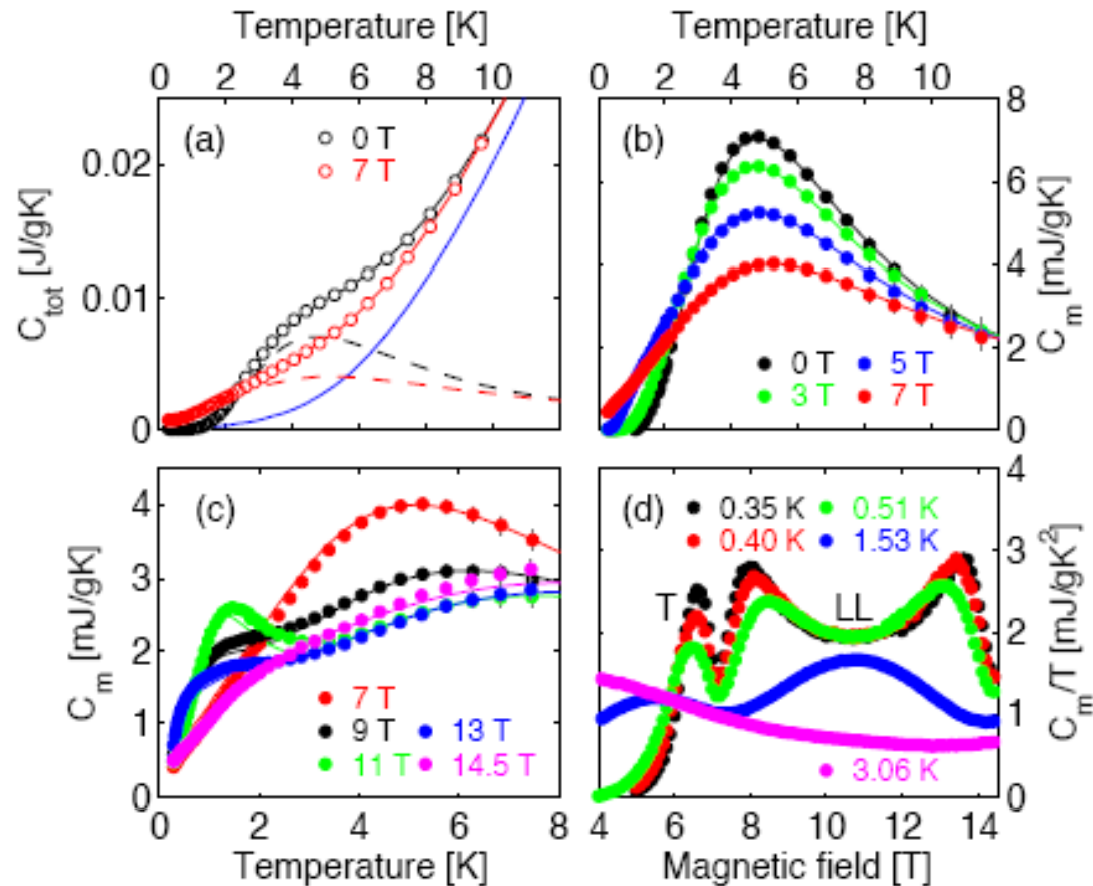


Specific heat



Ch. Rüegg et al.,
PRL 101, 247202 (2008)

Specific heat

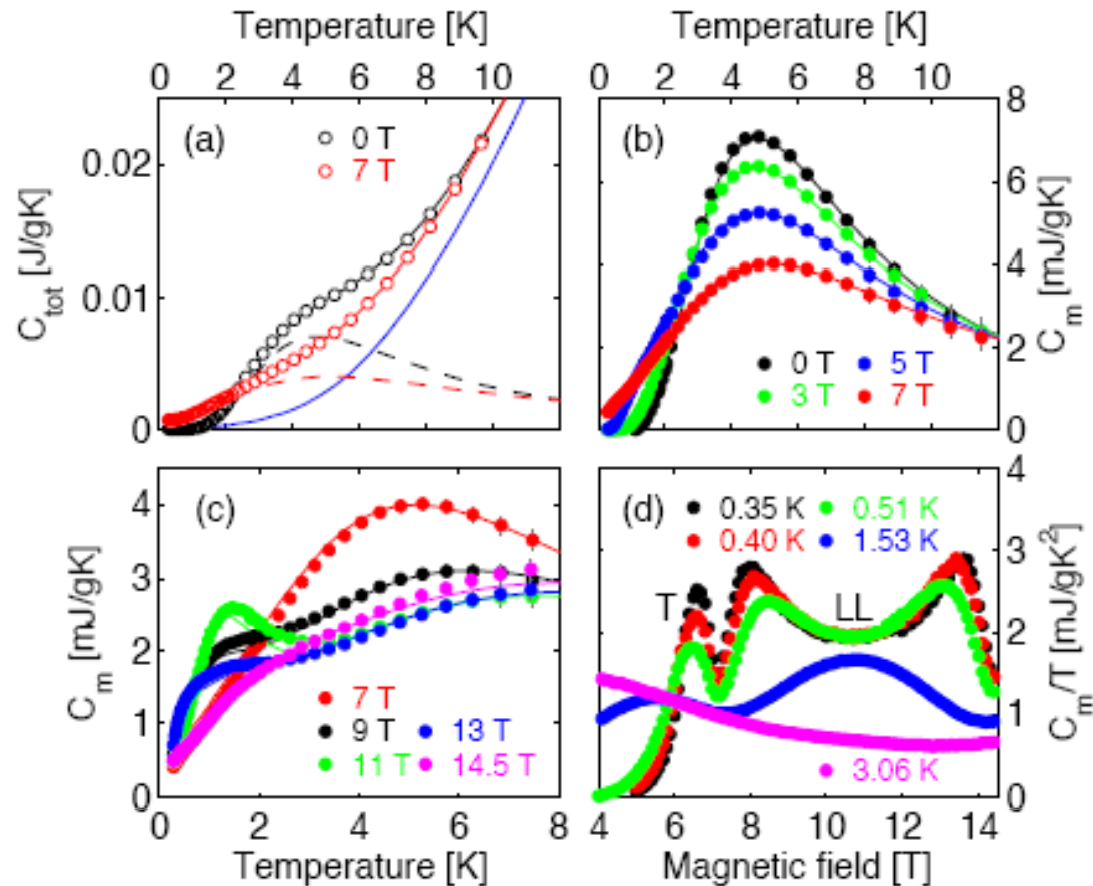


Ch. Rüegg et al.,
PRL 101, 247202 (2008)

Luttinger liquid: $C_v \propto \gamma T$

Peak : end of LL

Specific heat

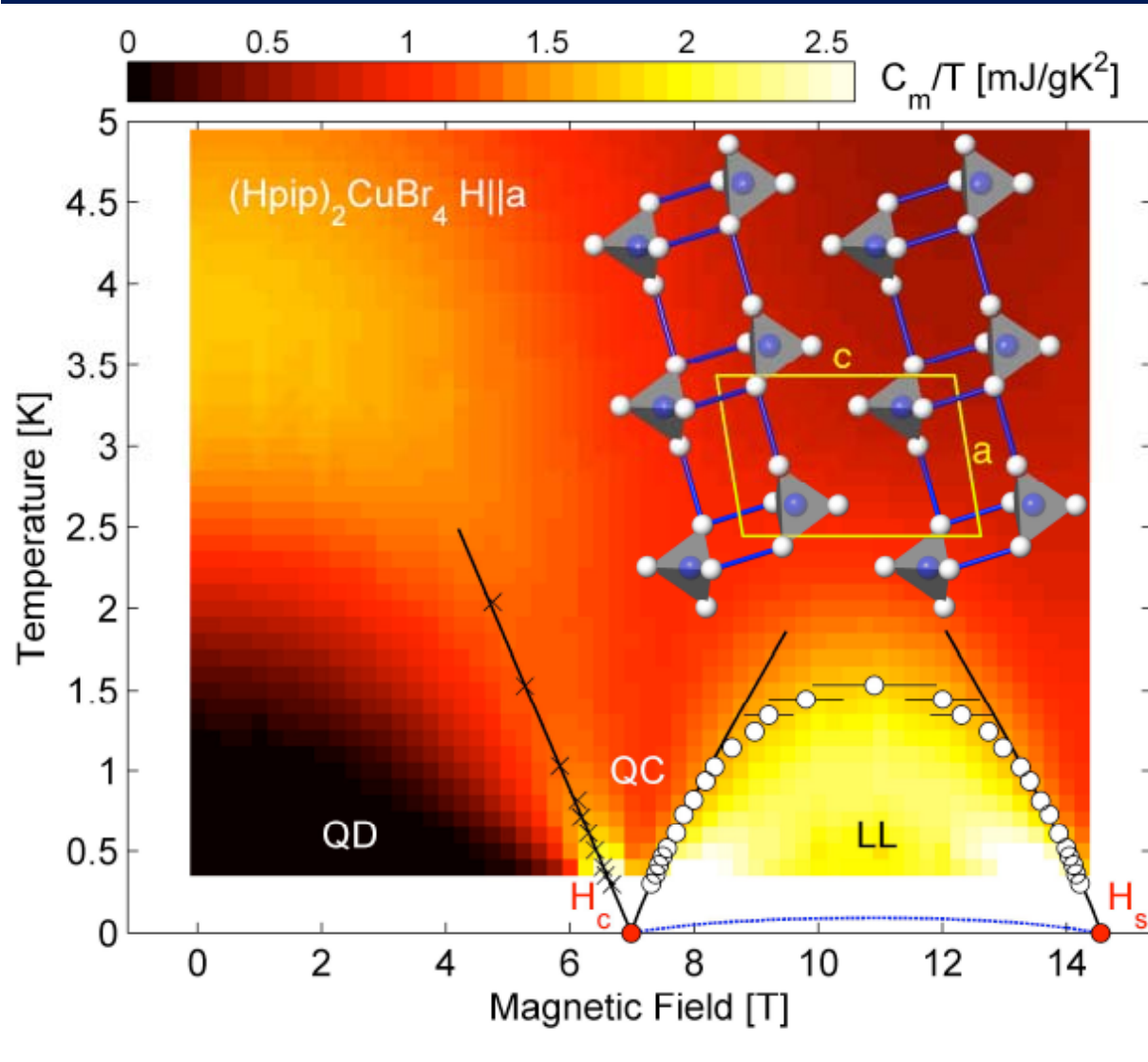


Ch. Rüegg et al.,
PRL 101, 247202 (2008)

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Ch. Rüegg et al., PRL
101, 247202 (2008)

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- Compute $u(h)$ and $K(h)$ from (J_r, J, h)

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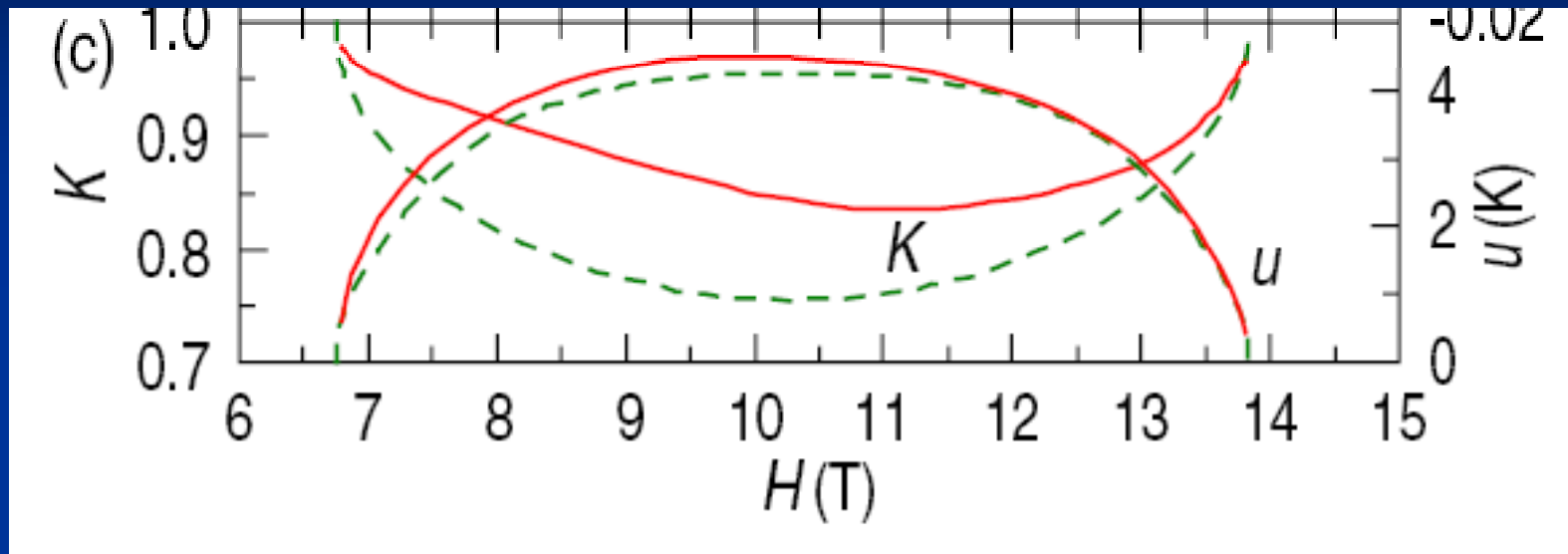
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Allows to test for Luttinger Liquid !

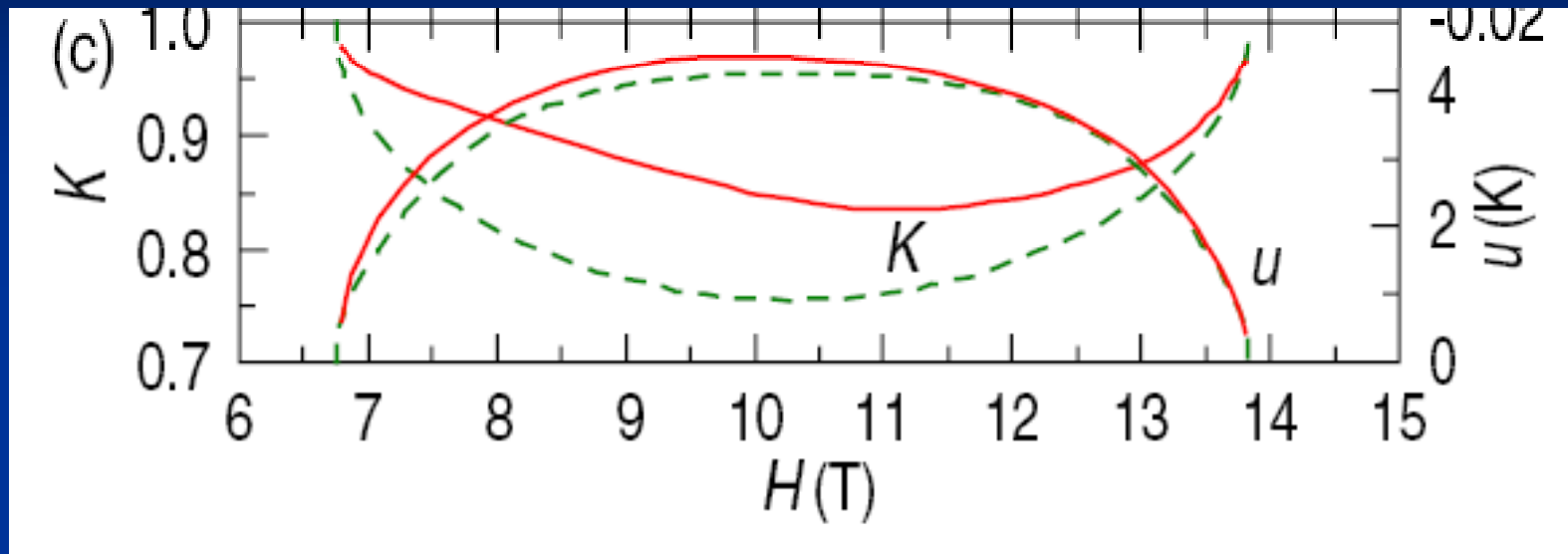
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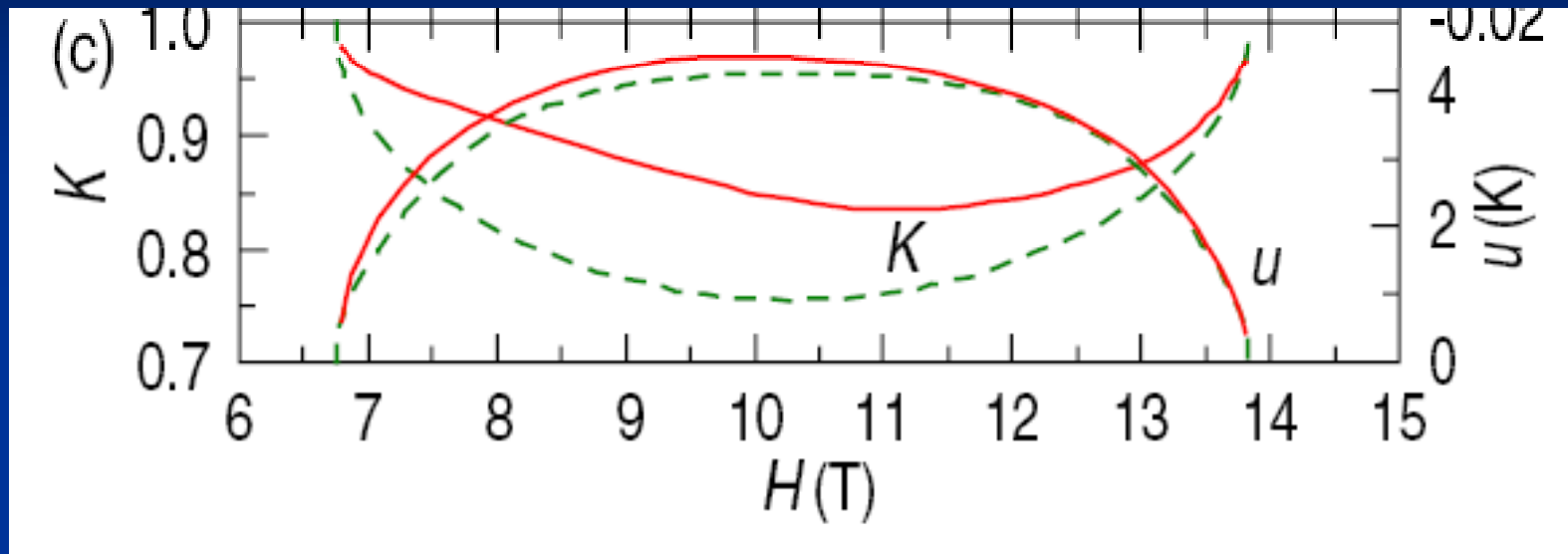
M. Klanjsek et al., PRL 101 137207 (2008)

Luttinger parameters



M. Klanjsek et al., PRL 101 137207 (2008)

Luttinger parameters



M. Klanjsek et al., PRL 101 137207 (2008)

Red : Ladder (DMRG)

Green: Strong coupling ($J_r \rightarrow \infty$) (BA)

Correlation functions

Correlation functions

M. Klanjsek et al., PRL 101 137207 (2008)

- NMR relaxation rate:

Correlation functions

M. Klanjsek et al., PRL 101 137207 (2008)

R. Chitra, TG PRB 55 5816 (97); TG, AM Tsvelik PRB 59 11398 (99)

■ NMR relaxation rate:

$$T_1^{-1} = \frac{\hbar \gamma^2 A_{\perp}^2 A_0^x}{k_B u} \cos\left(\frac{\pi}{4K}\right) B\left(\frac{1}{4K}, 1 - \frac{1}{2K}\right) \left(\frac{2\pi T}{u}\right)^{(1/2K)-1},$$

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- Order parameter at $T=0$

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Ordered phase

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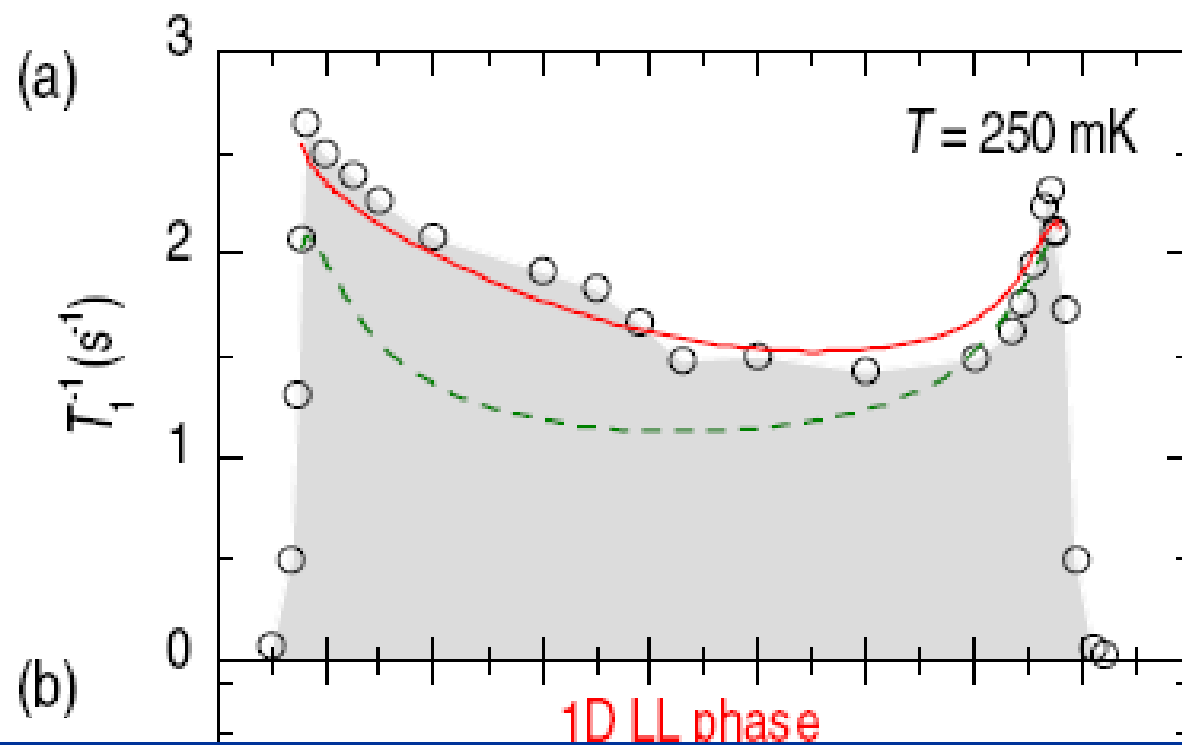
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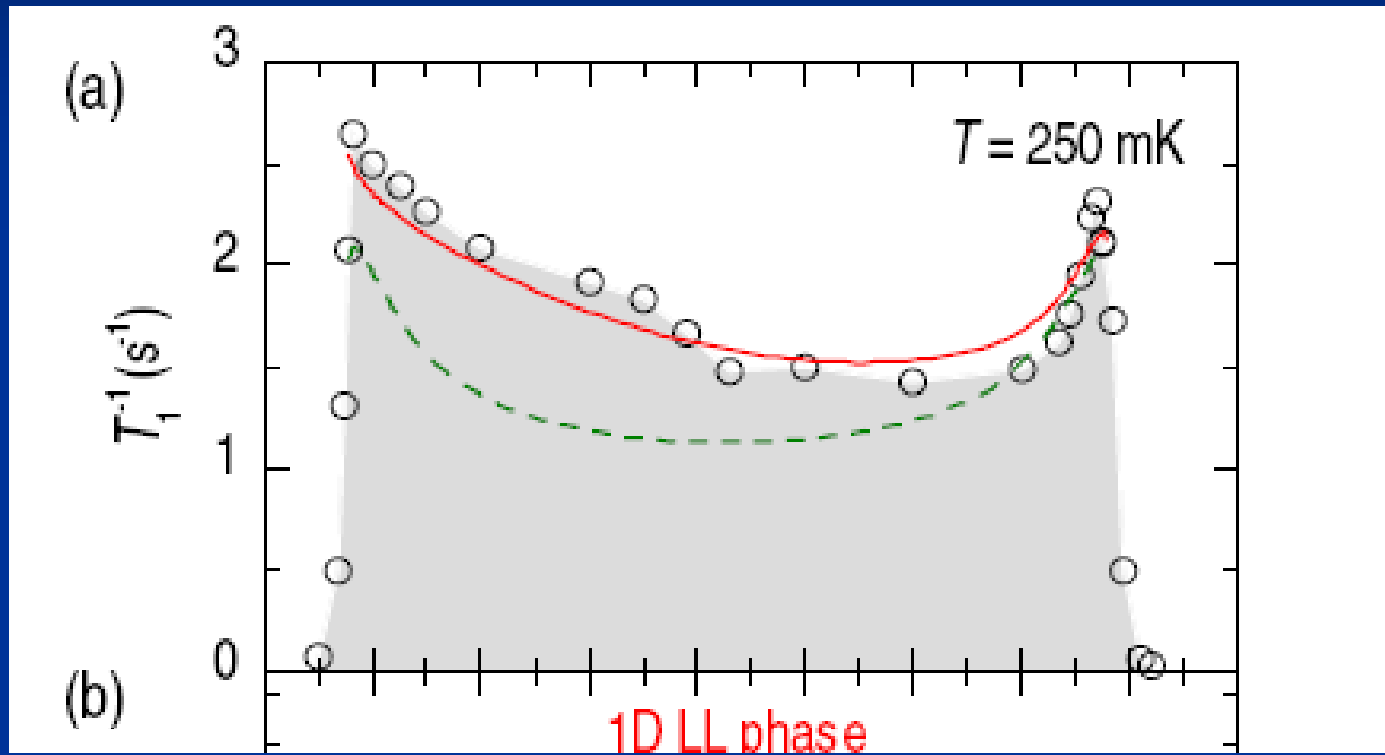
NMR



NMR



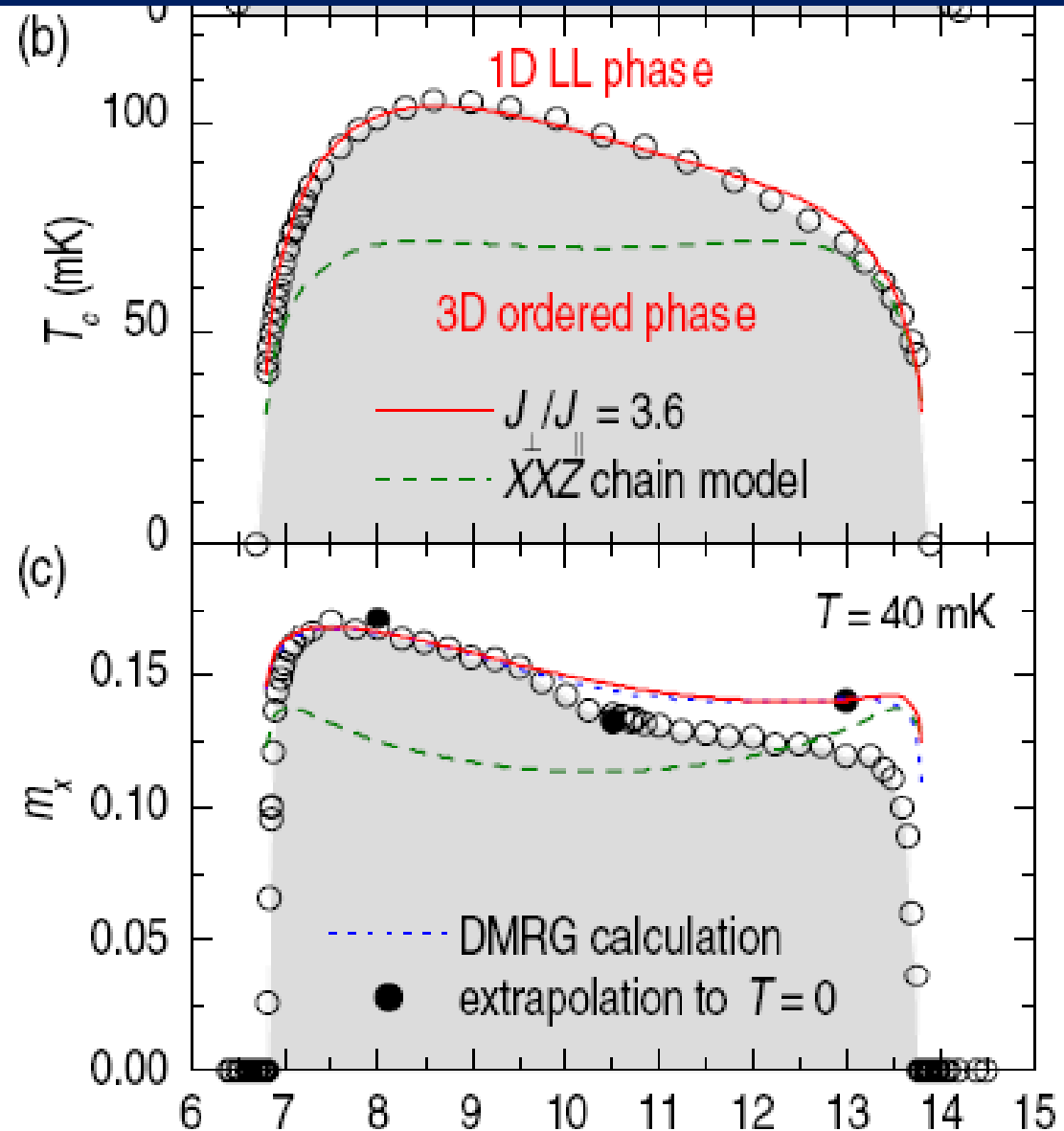
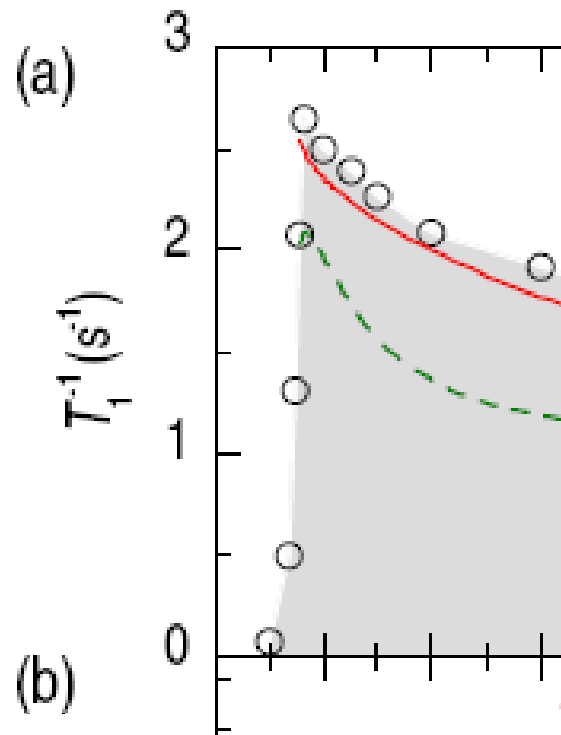
NMR



M. Klanjsek et al.,

PRL 101 137207 (2008)

NMR

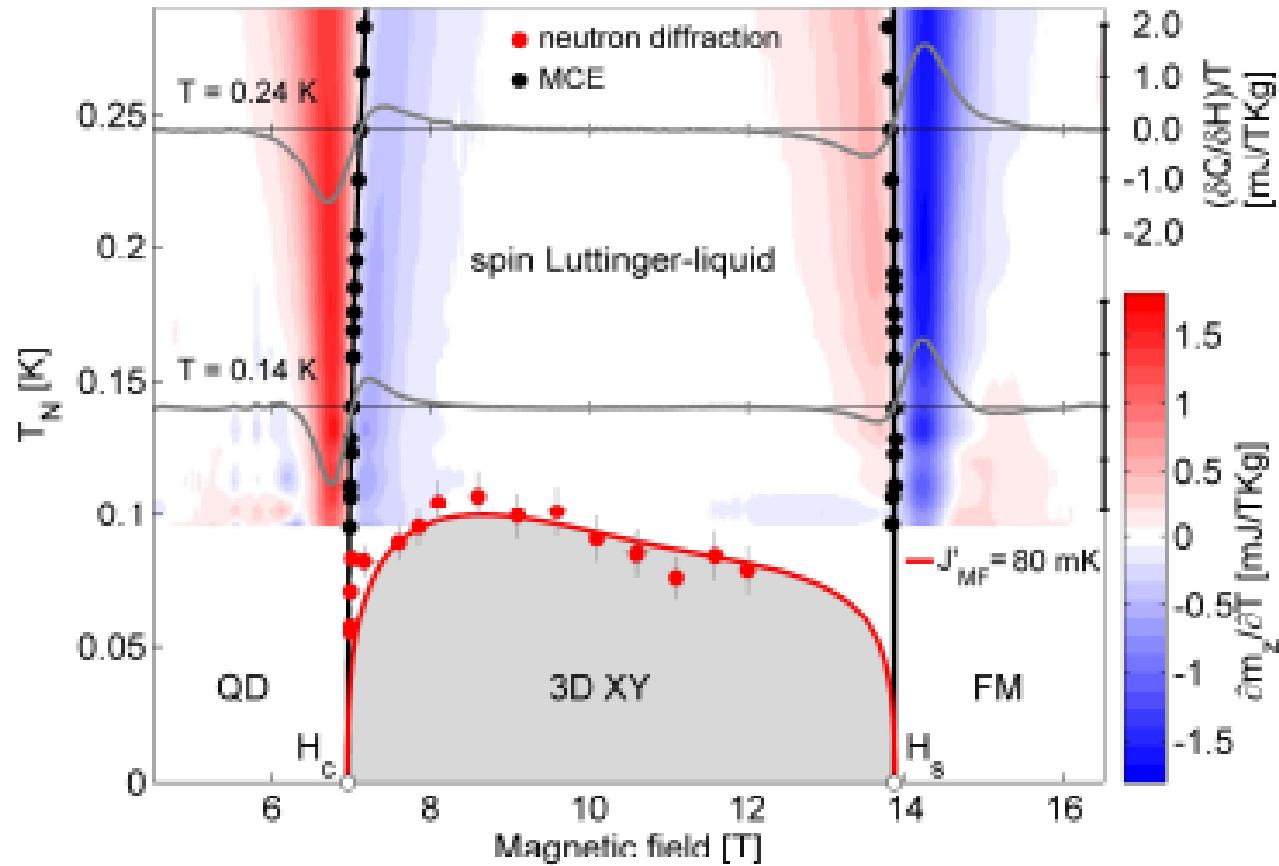


M. Klanjsek et al.,

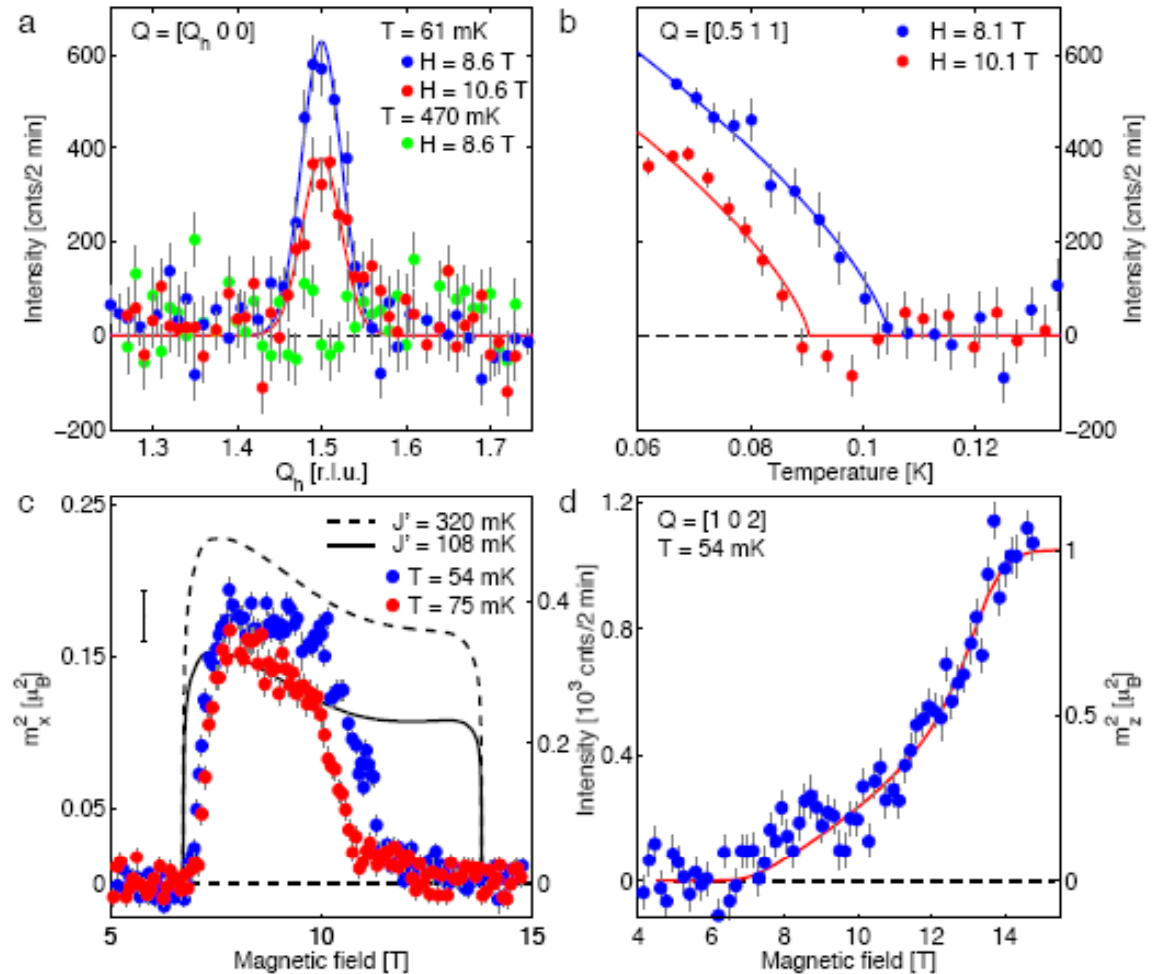
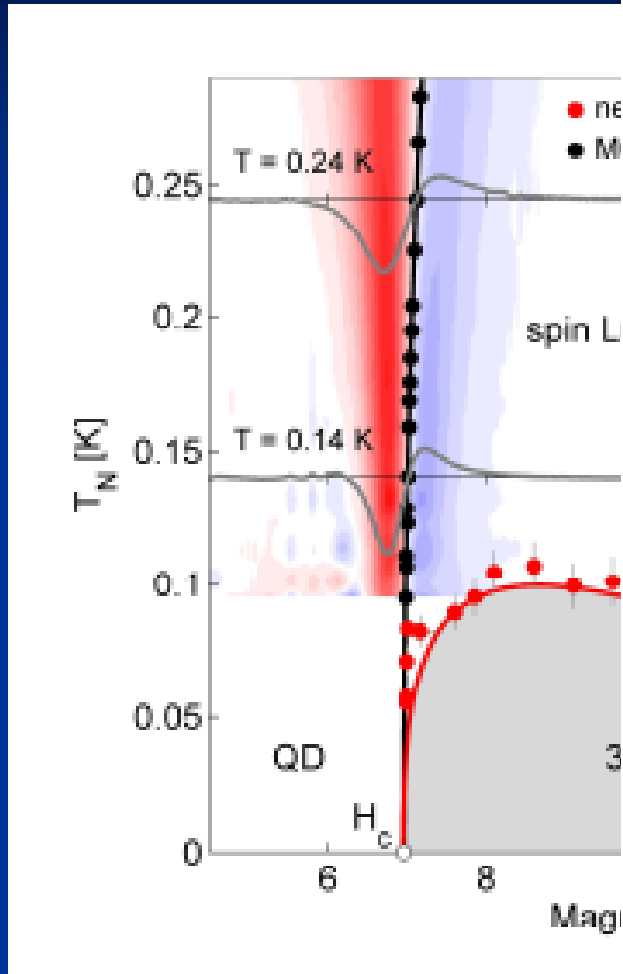
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Neutrons

Neutrons



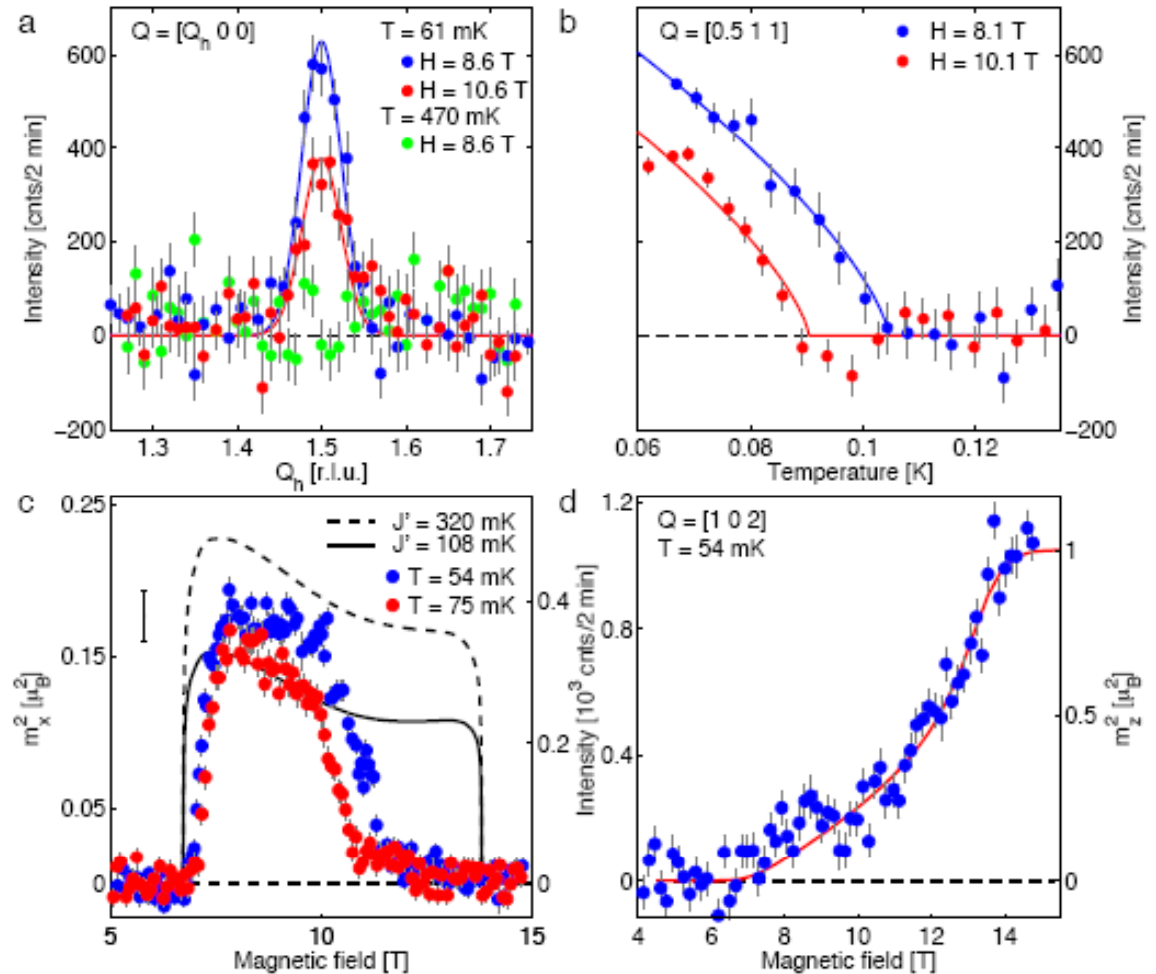
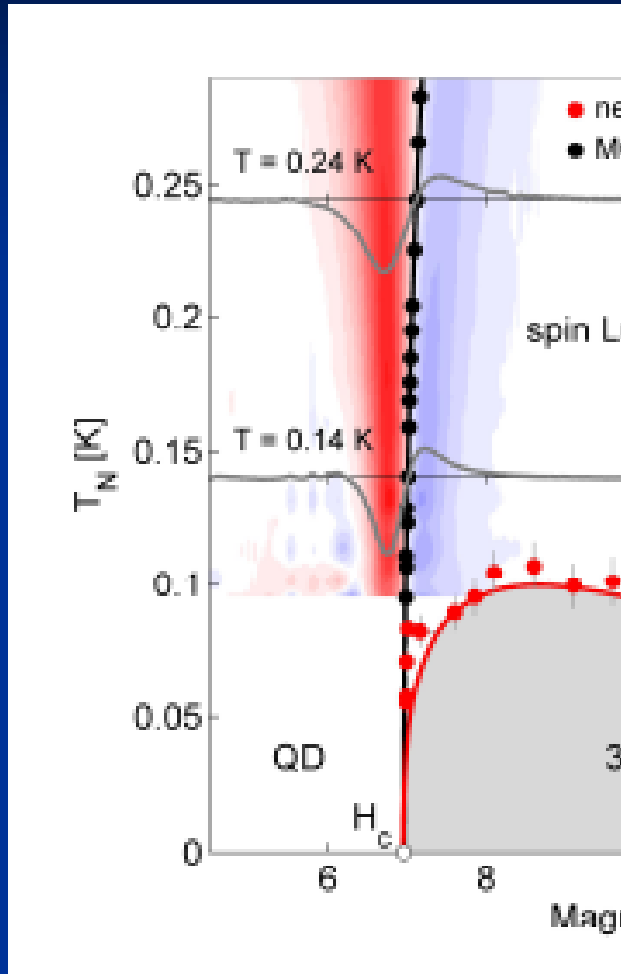
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B. Thielemann et al.,
PRB 79, 020408(R) (2009)

$$J' = 27 \text{ mK}$$

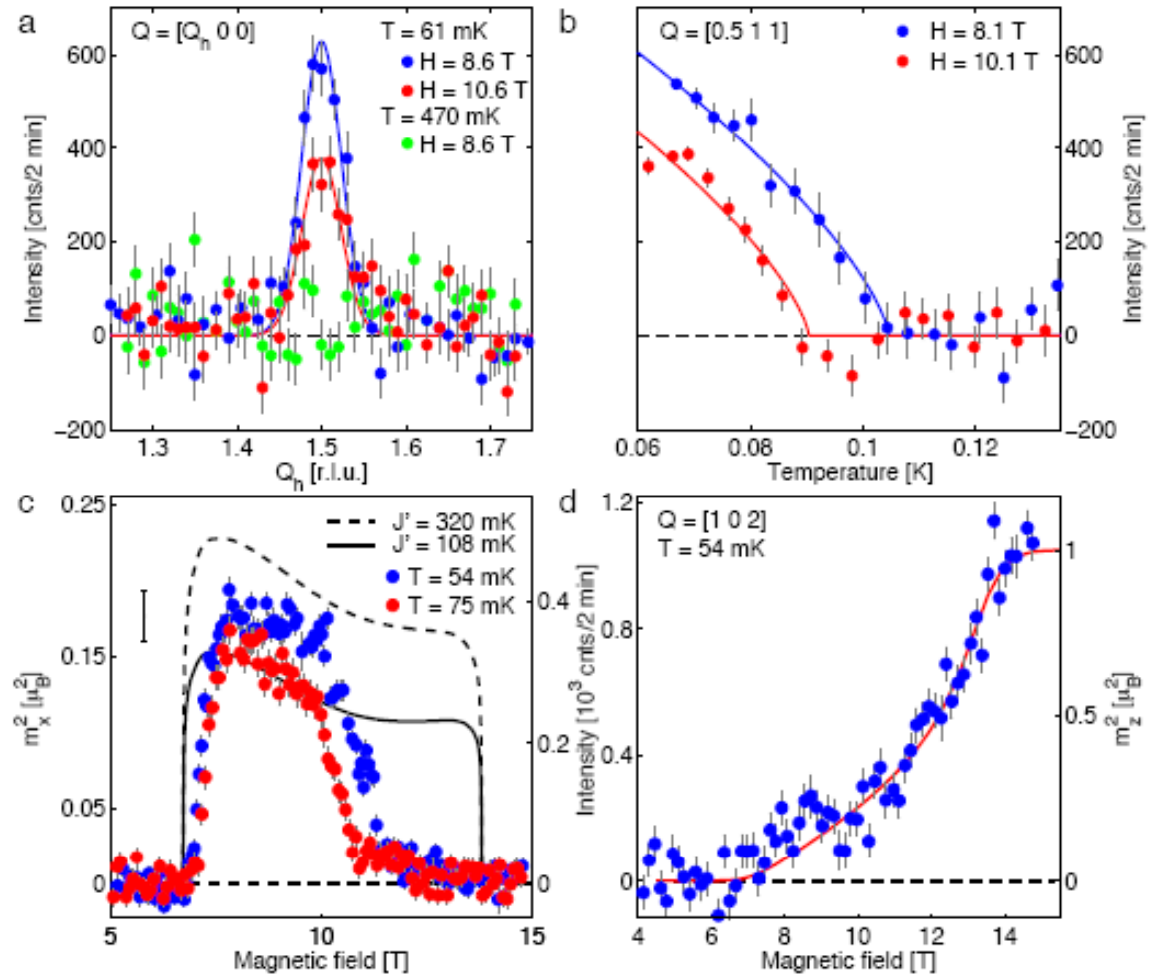
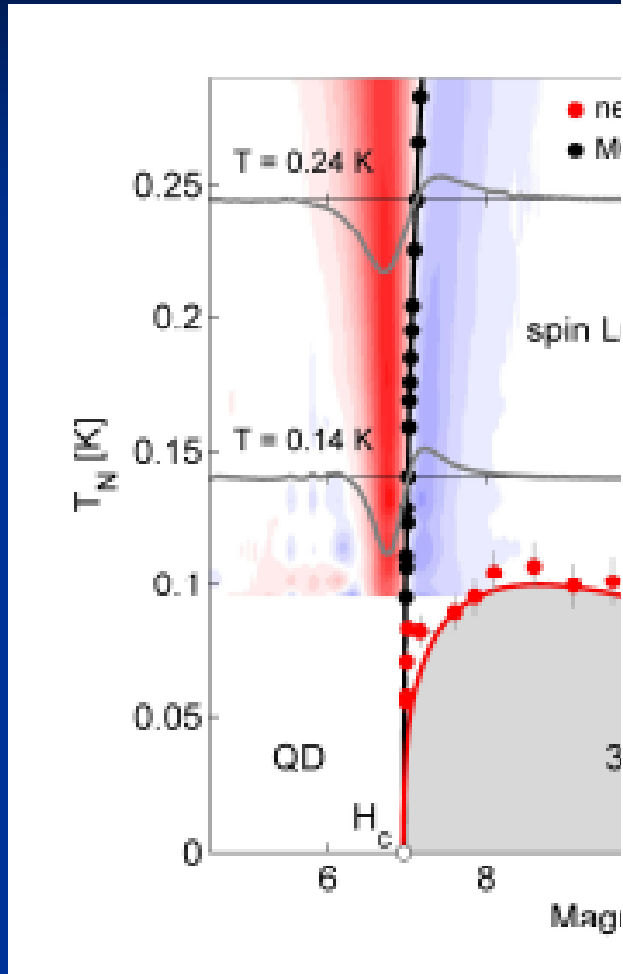
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End of the story ????????

Beyond Luttinger liquids

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- Luttinger liquids: allows to treat additional effects

Beyond Luttinger liquids

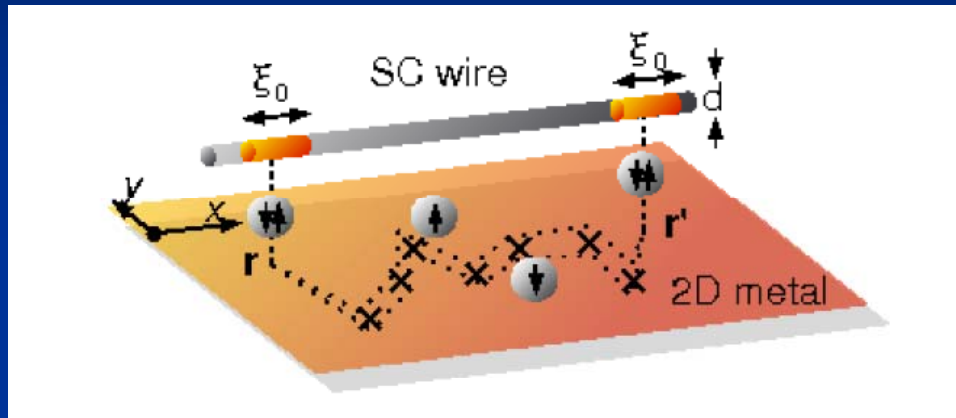
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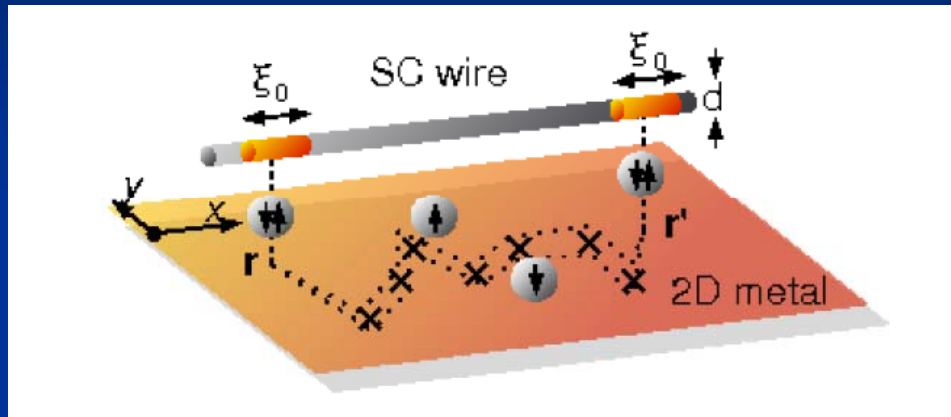
Beyond Luttinger liquids

- Luttinger liquids: allows to treat additional effects
- 1D simple perturbation: Lattice (Mott transition), disorder (Bose glass) etc.
- Major deviations from Luttinger liquids
(hard problems 😊)

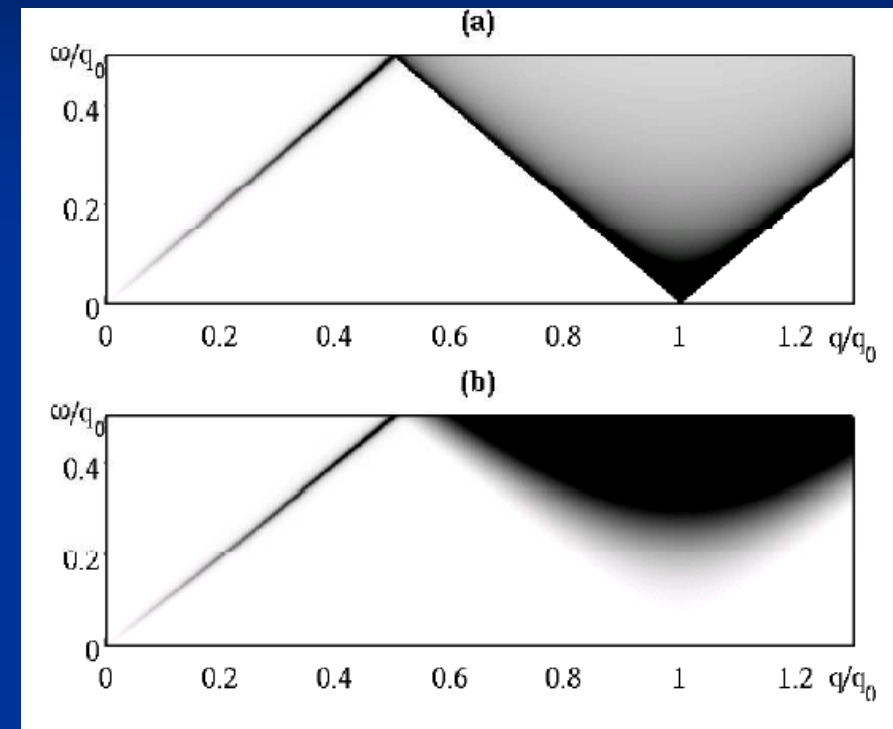
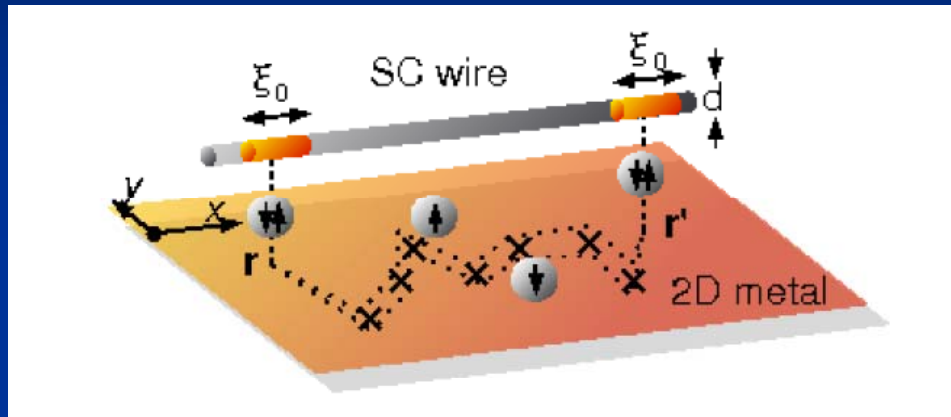
Luttinger liquid + bath or noise



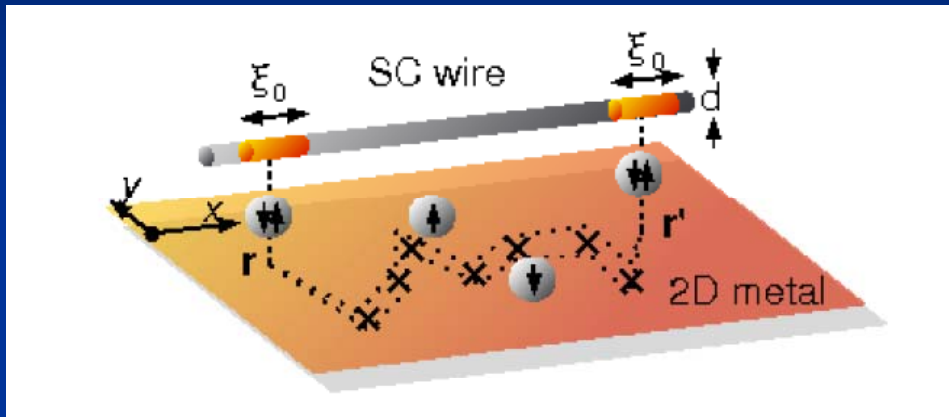
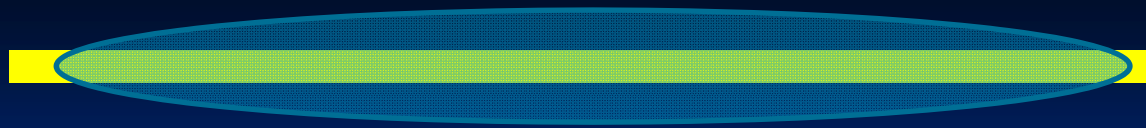




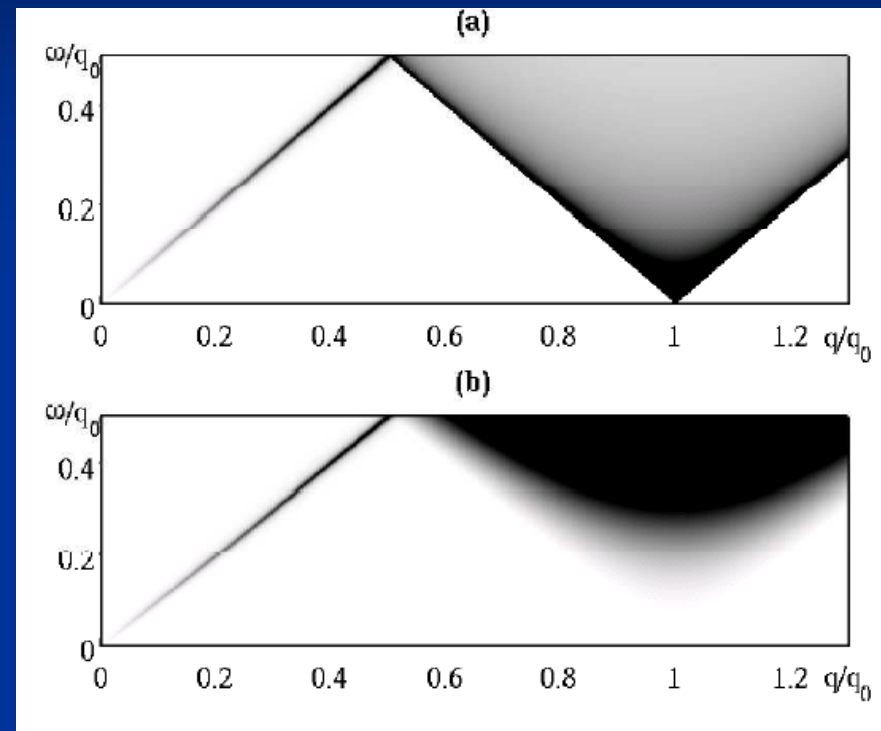
A. Lobos, A. Iucci, M. Muler,
TG PRB 80 214515 (09)



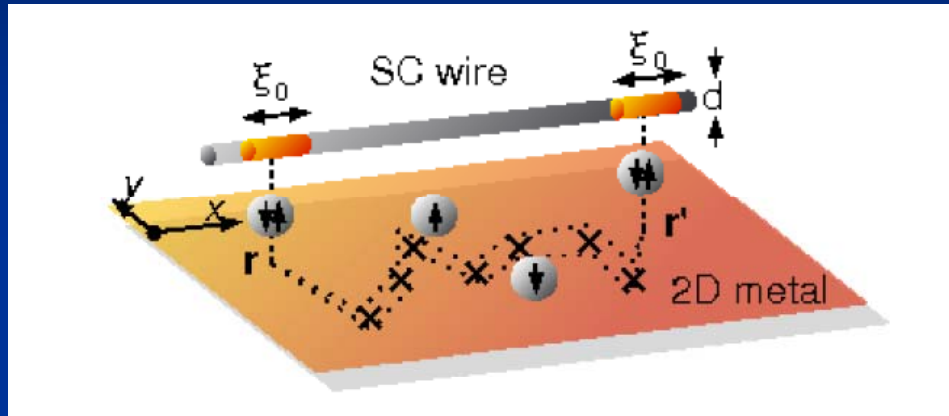
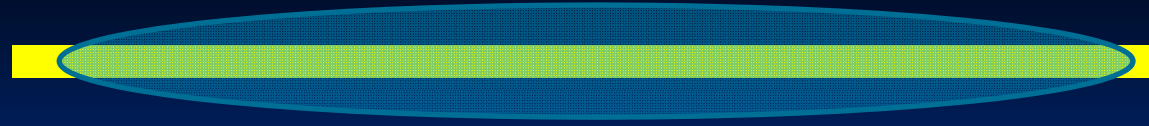
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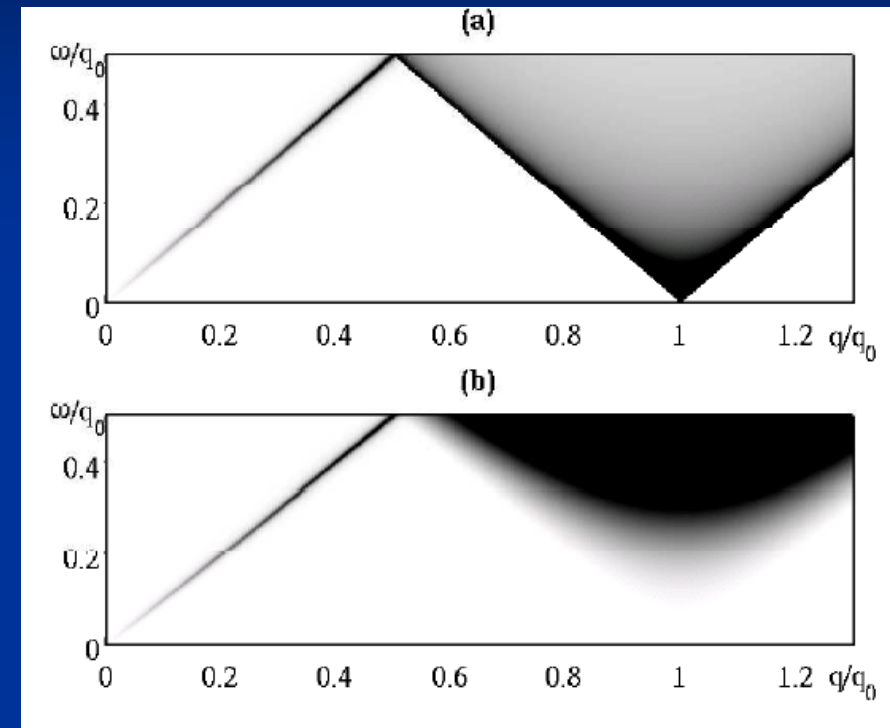
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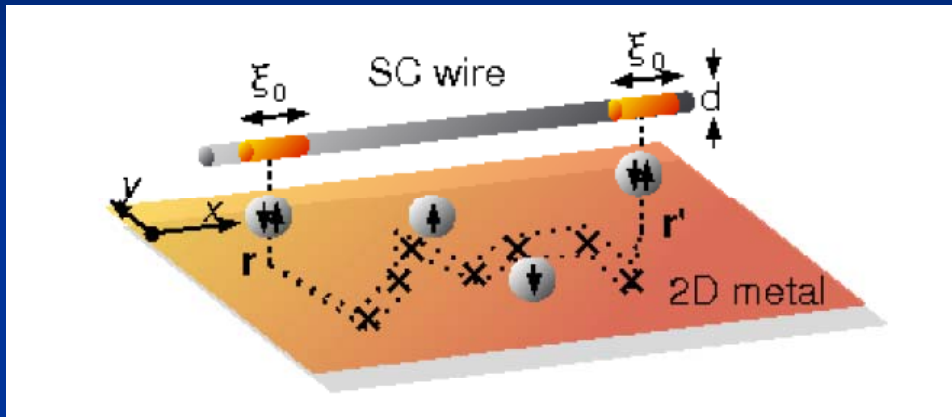
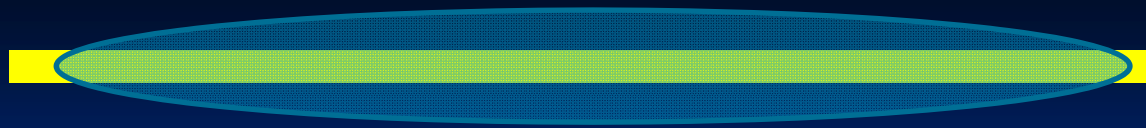
E. Dalla Torre, E. Demler, TG,
 E. Altman, Arxiv/0908.3345



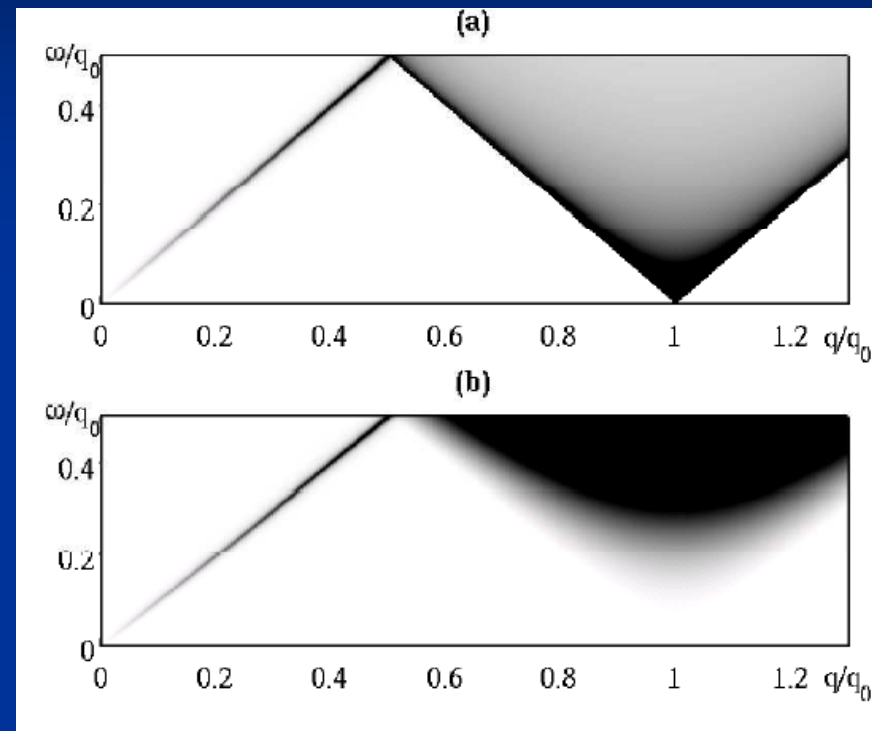
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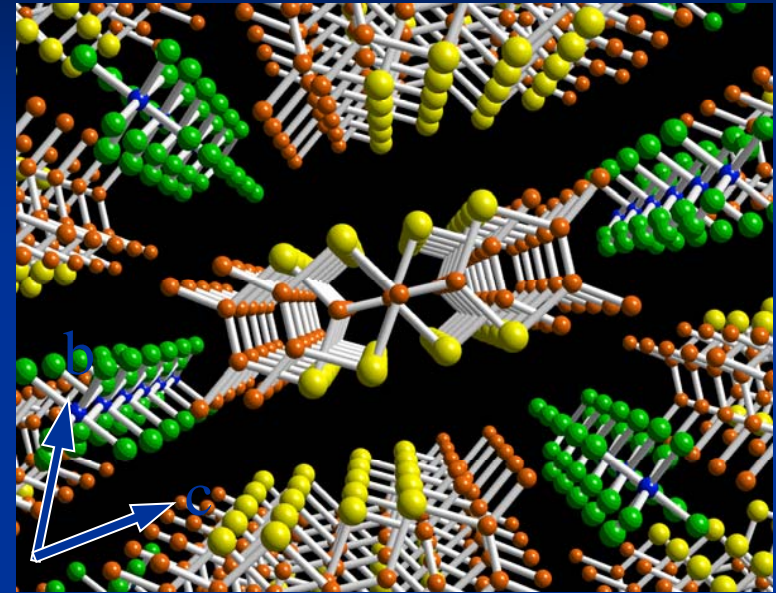
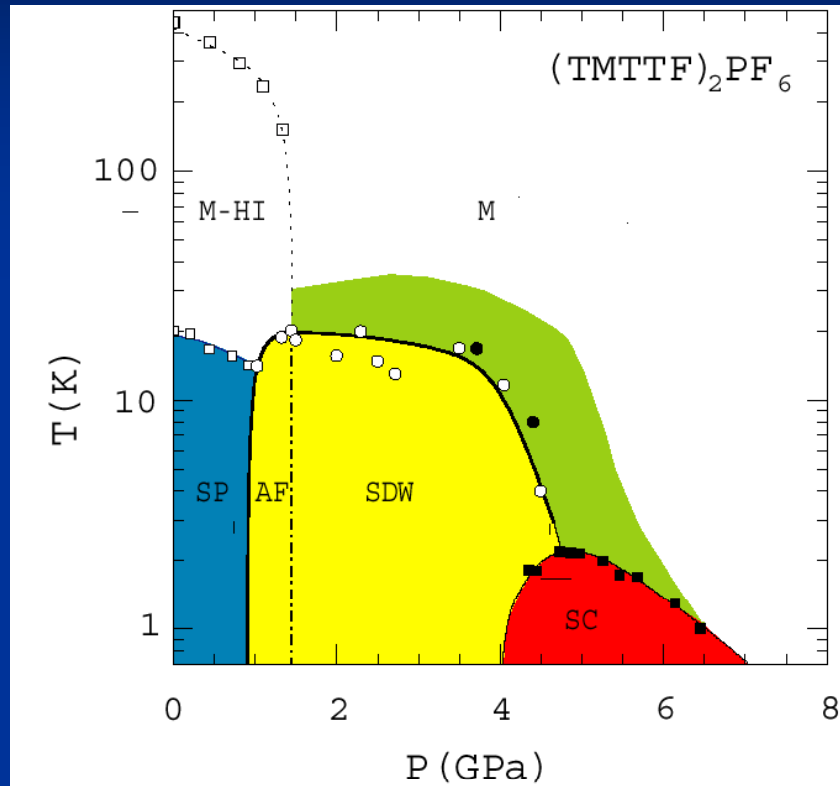
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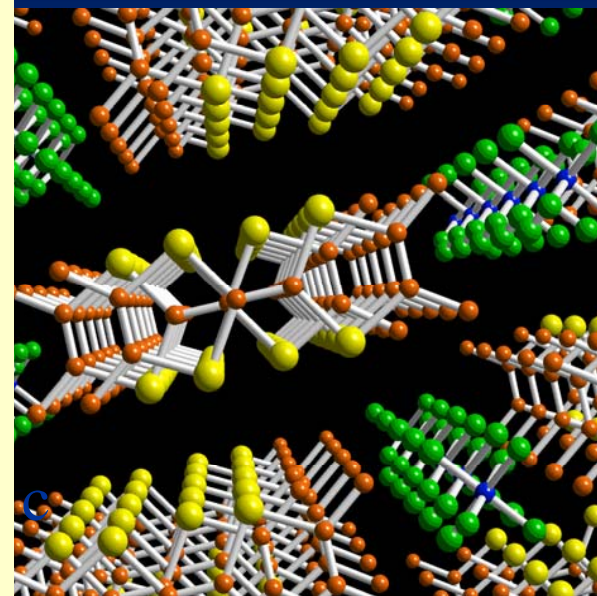
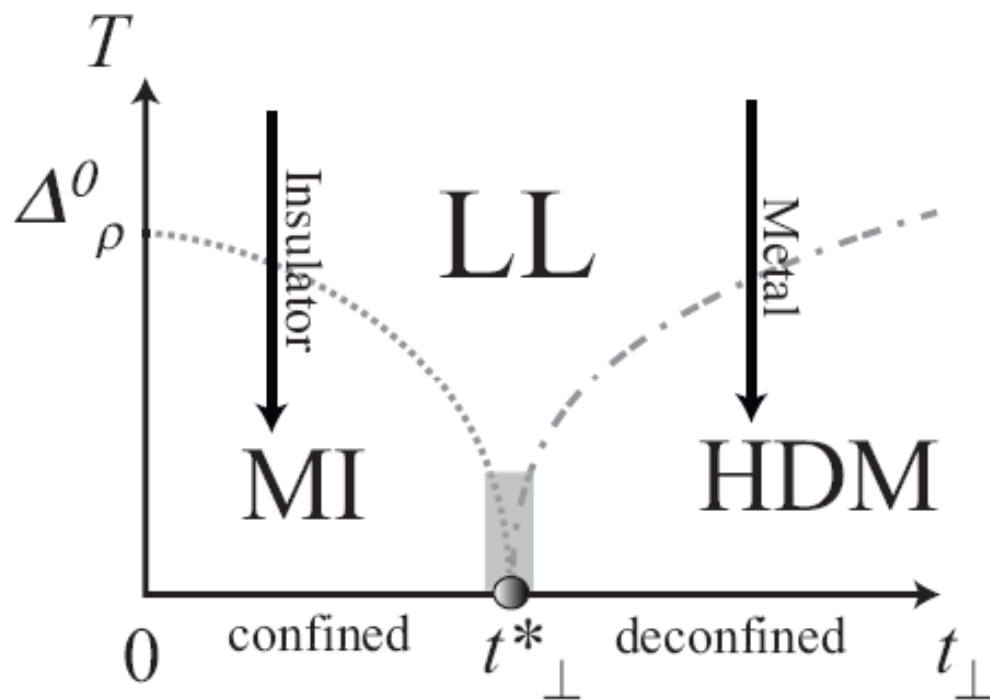
Coupled 1D chains: deconfinement

Deconfinement



D. Jaccard et al., J. Phys. C, 13 L89 (2001)

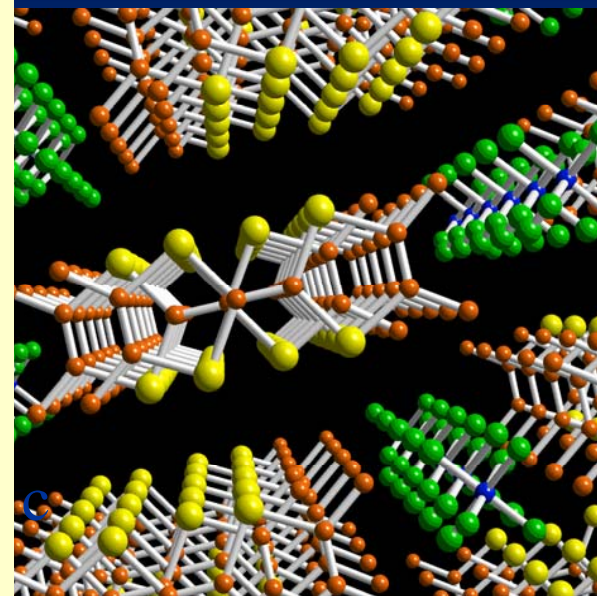
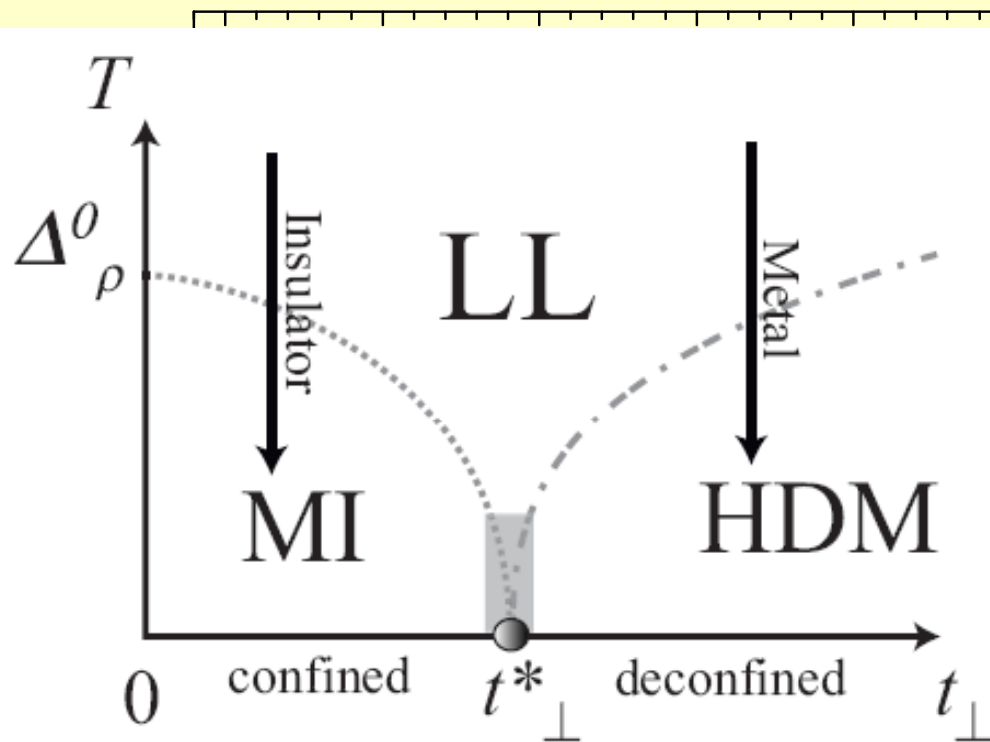
Deconfinement



TG Chemical
Review 104 5037
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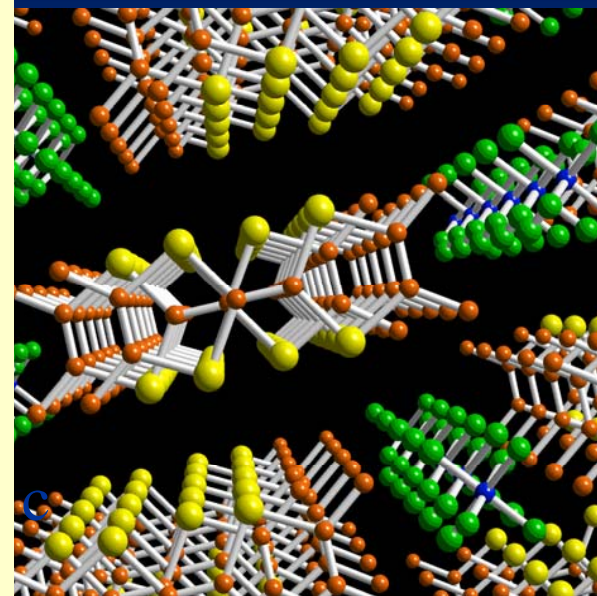
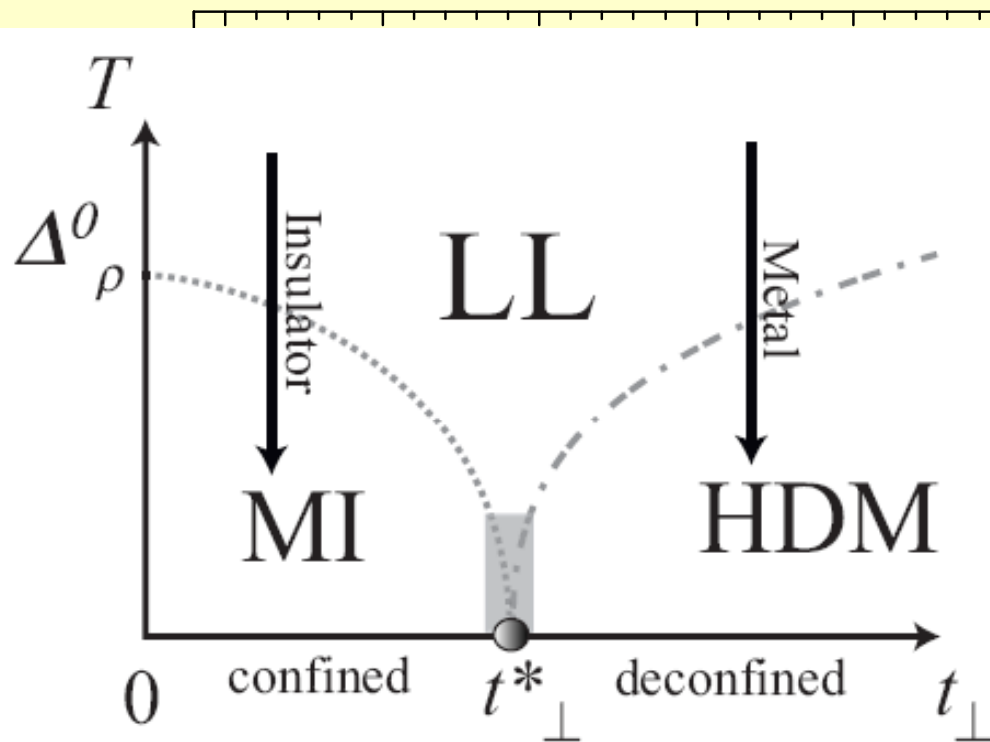
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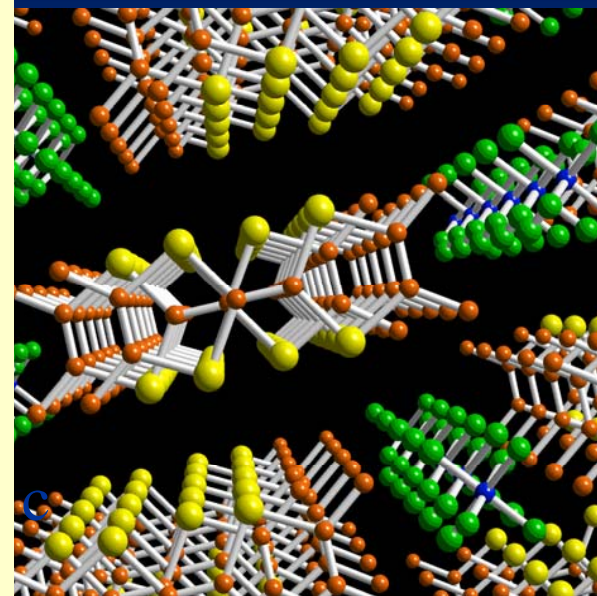
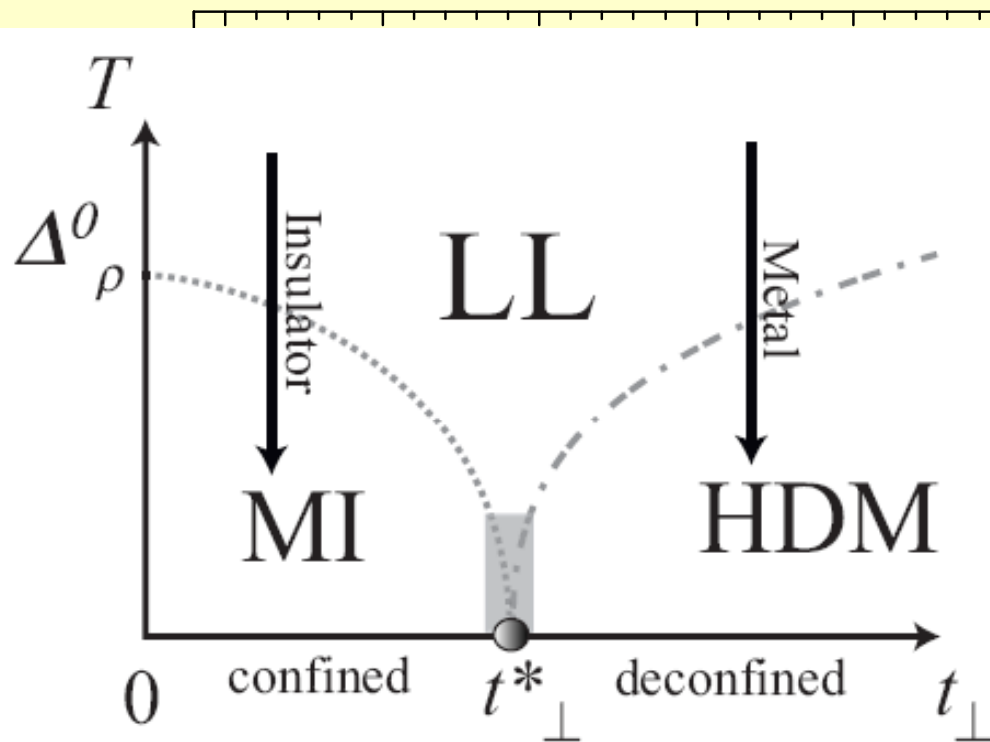
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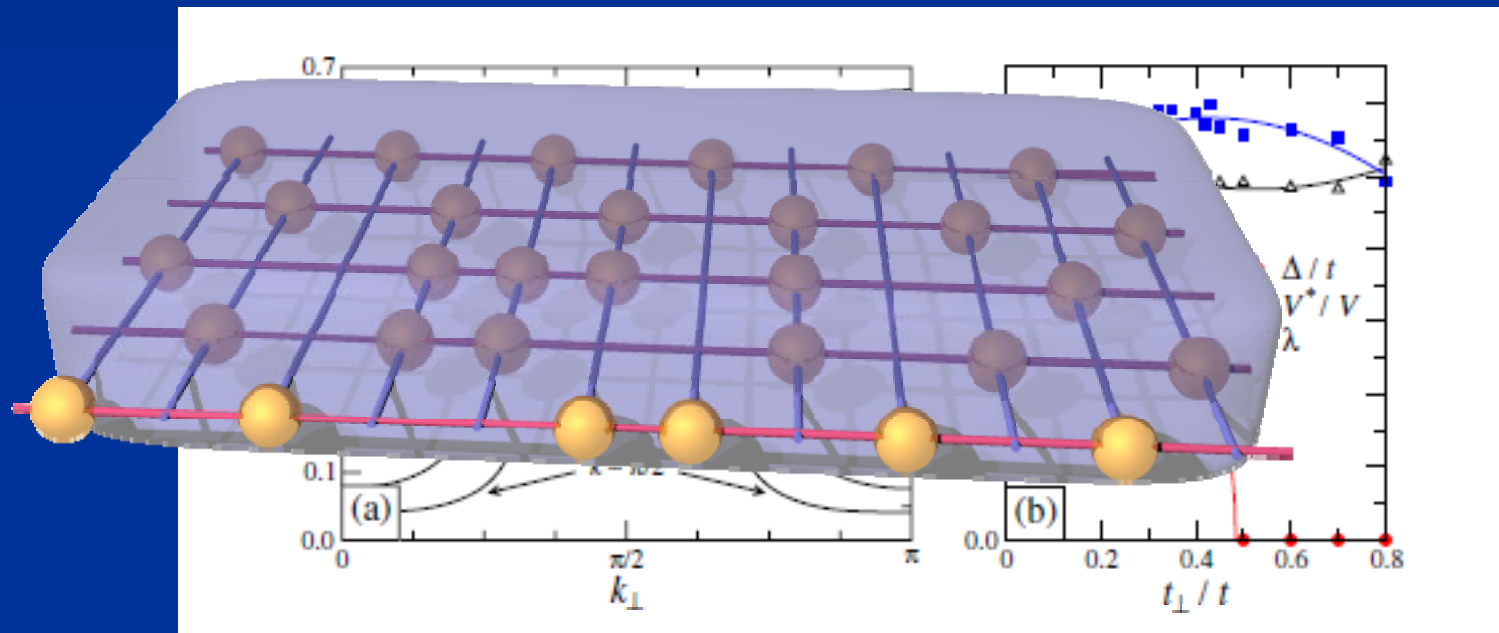
S. Biermann, A. Georges, A. Lichtenstein, TG, PRL 87 276405 (2001)

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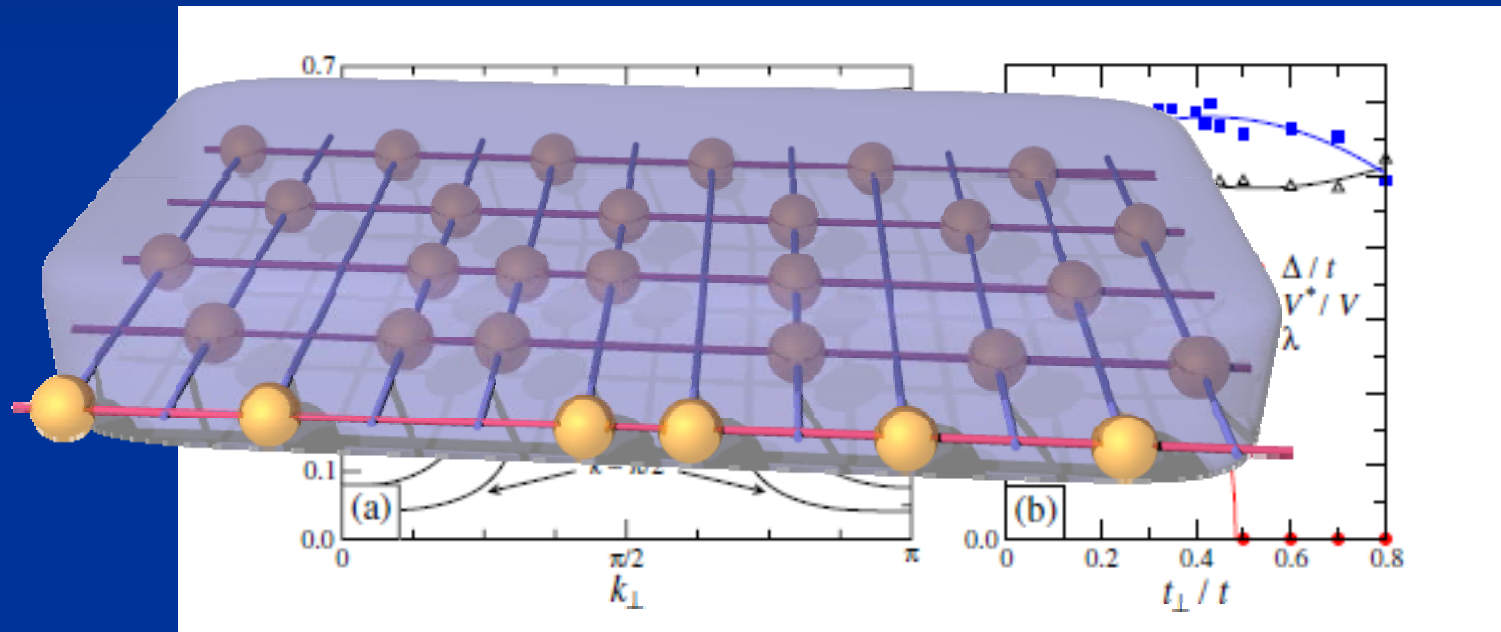
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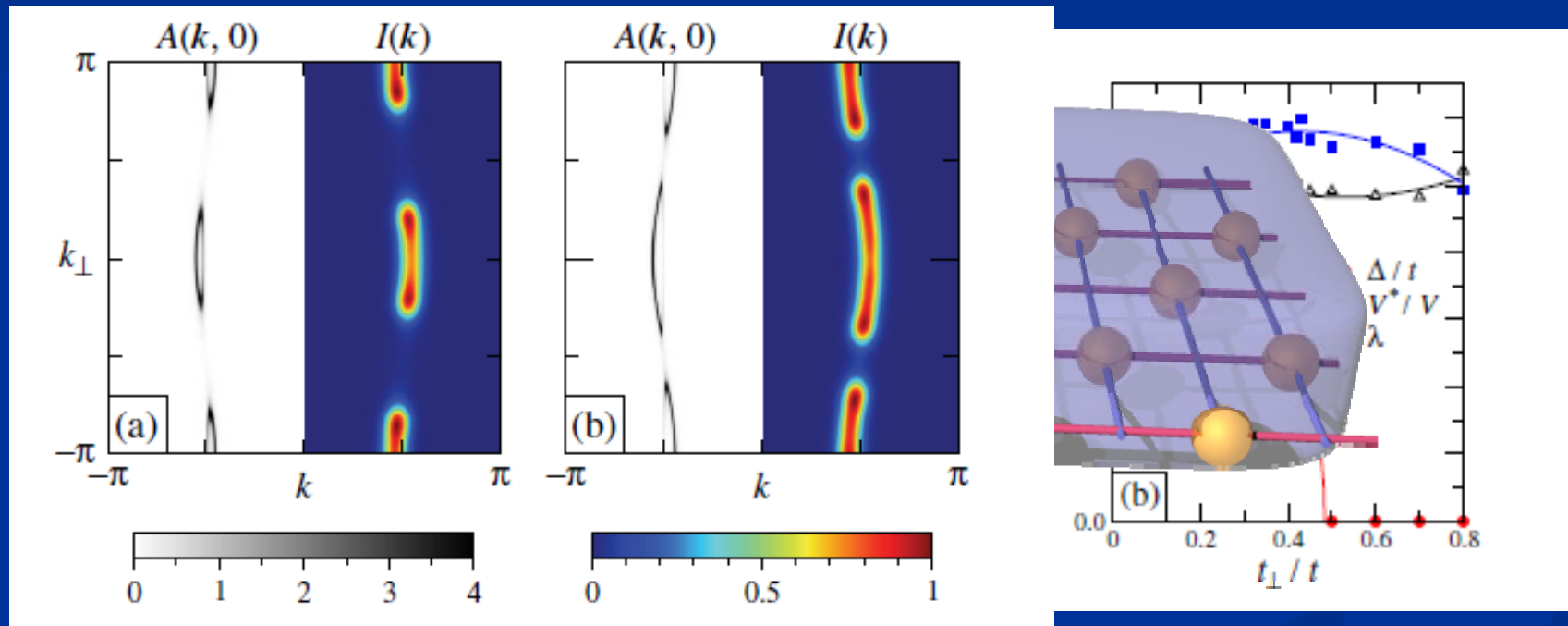
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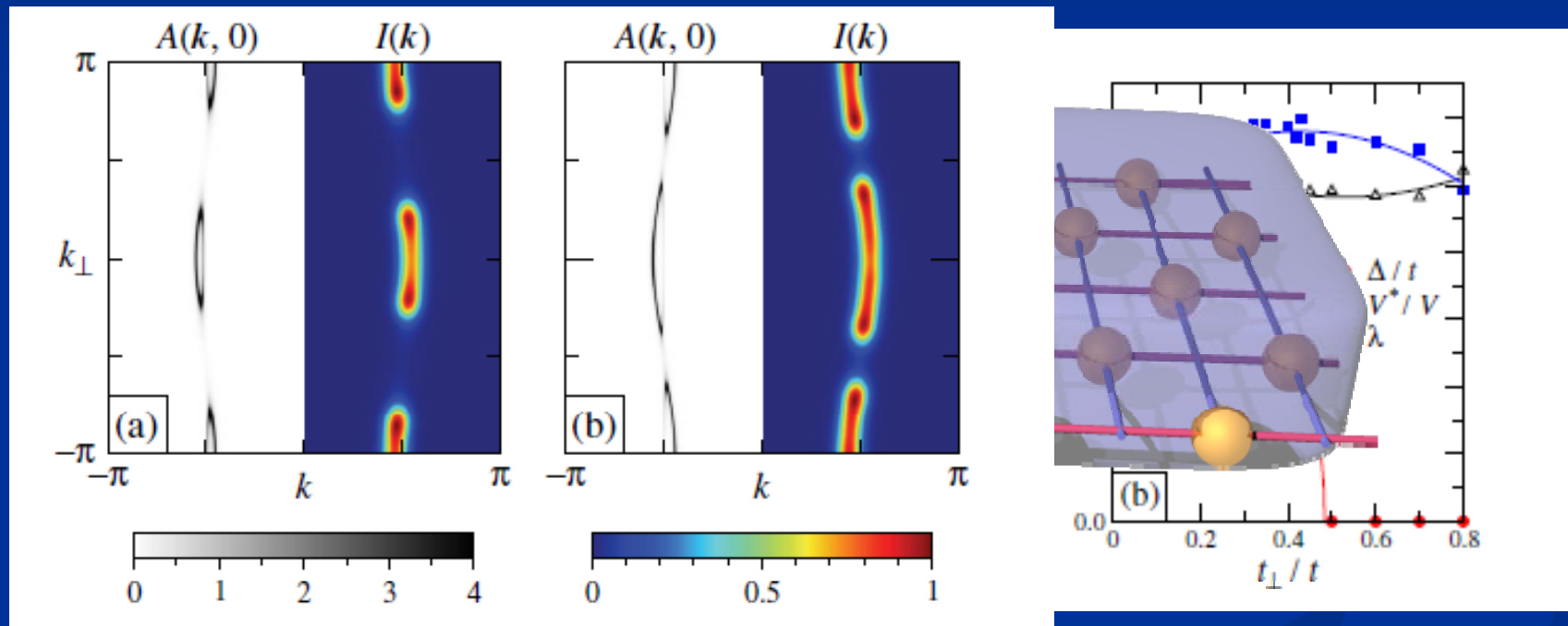
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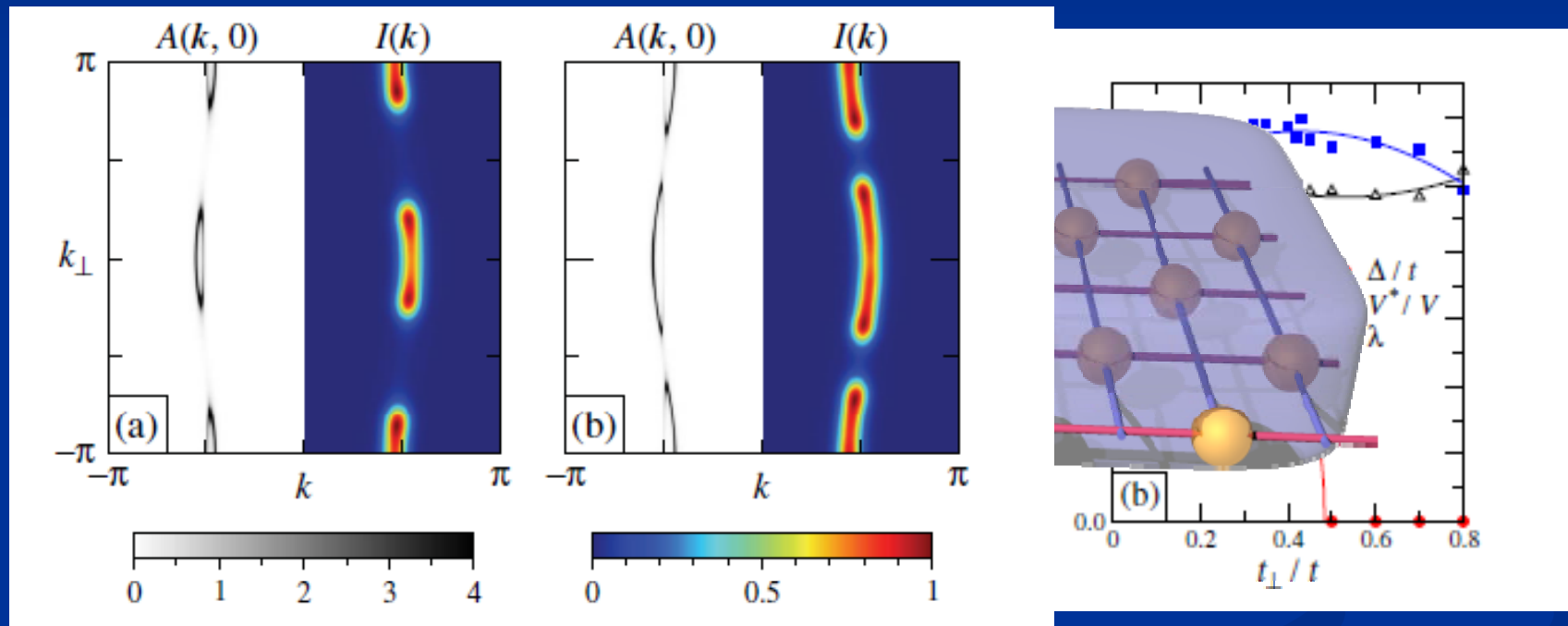
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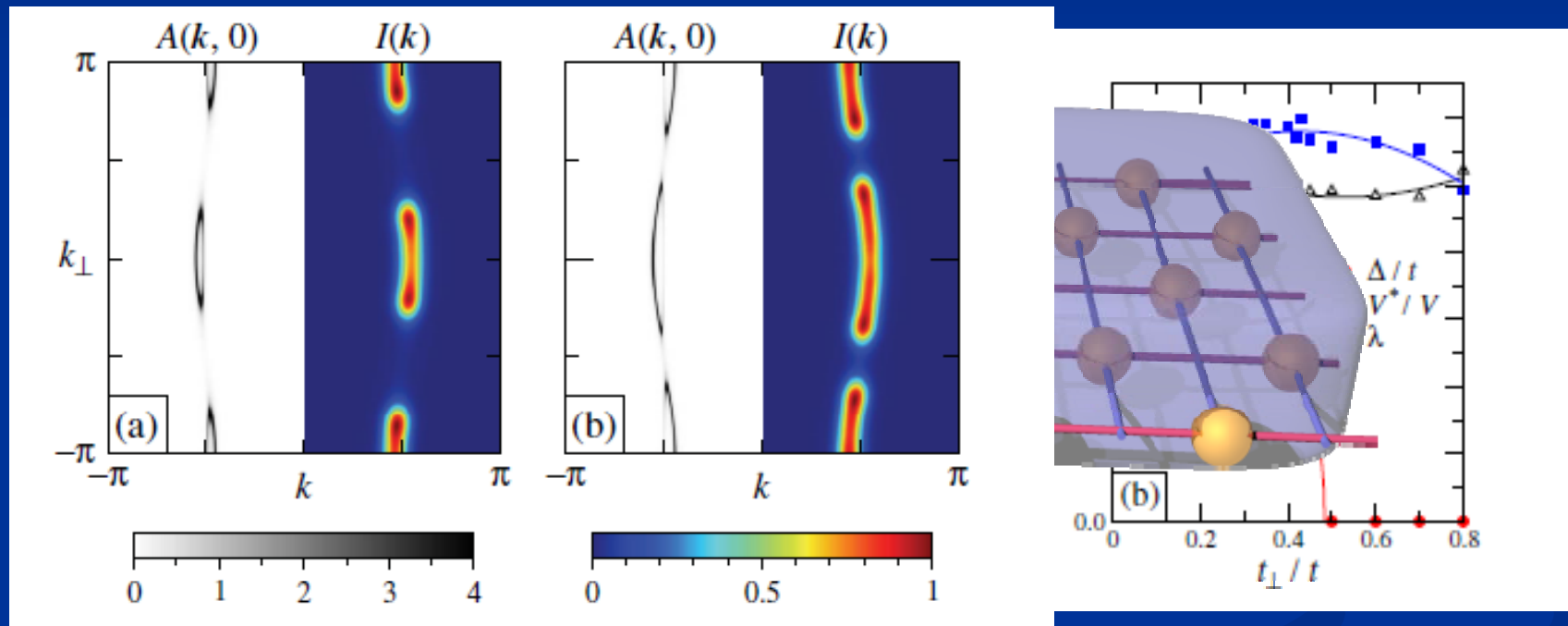
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Non Luttinger liquids





- Interacting 1D system (Luttinger liquid):
e.g. bosons -- “spin up”



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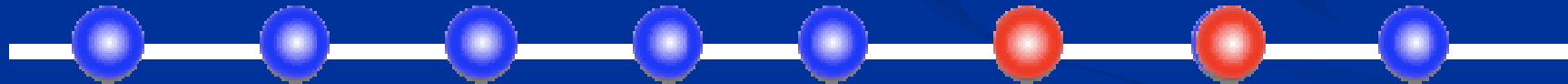


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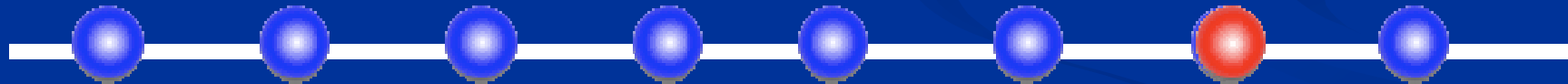


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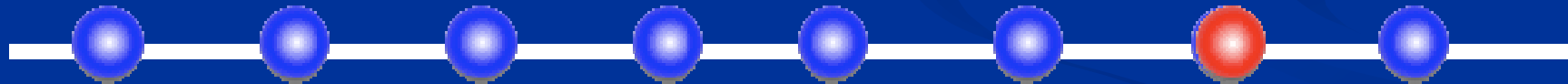


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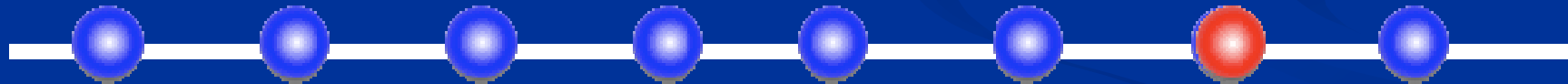
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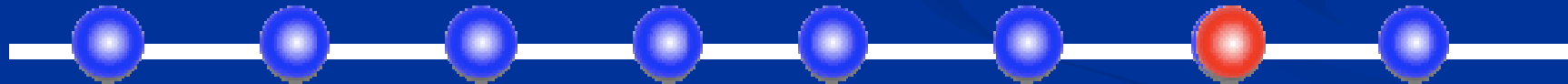
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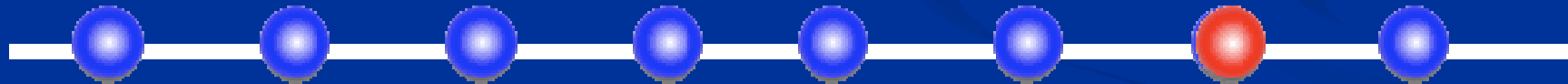
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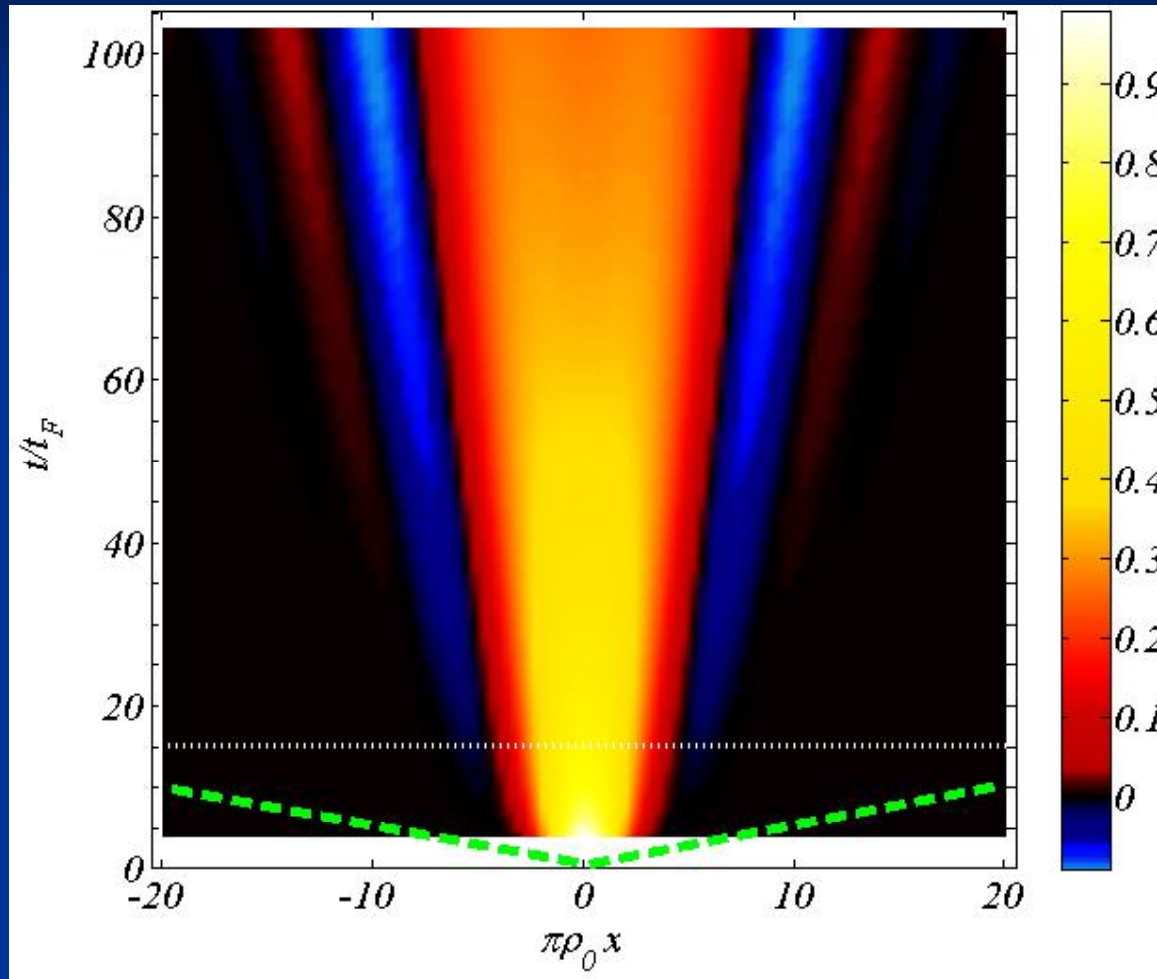
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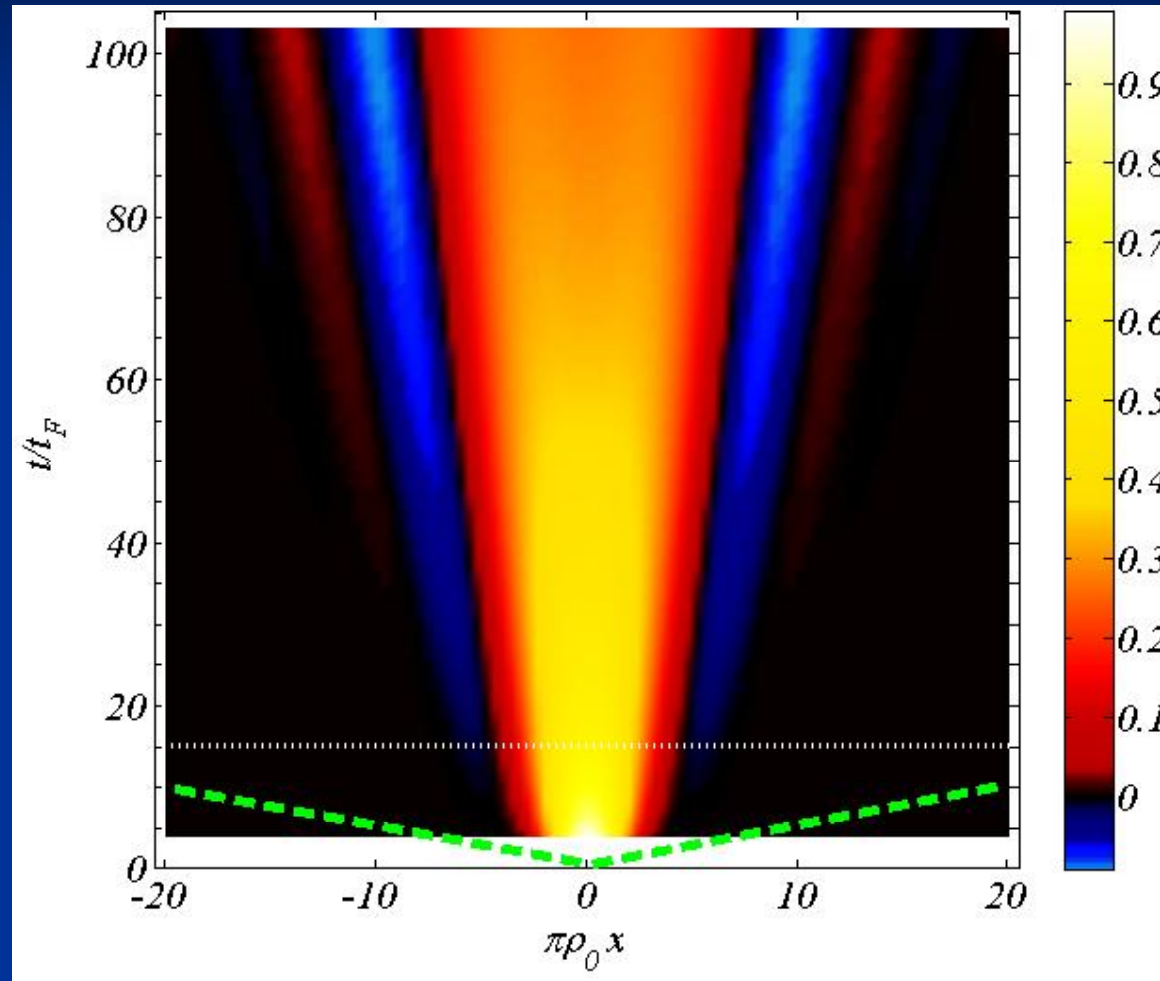
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Transverse spin excitations



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M. B. Zvonarev, V. V. Cheianov, TG, PRL 99 240404 (2007)

Interest

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