Nonequilibrium current carrying states in type II superconductors in magnetic field

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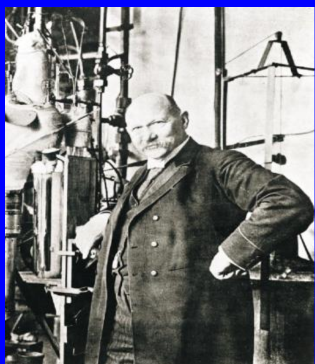
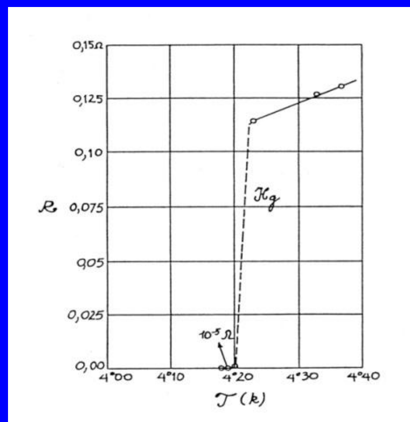
Kanpur, February 1, 2010

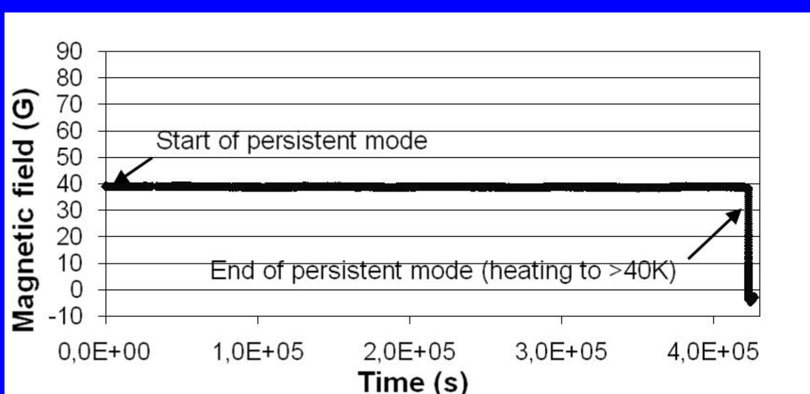
Two defining electromagnetic properties of a superconductor:

1. Zero resistivity



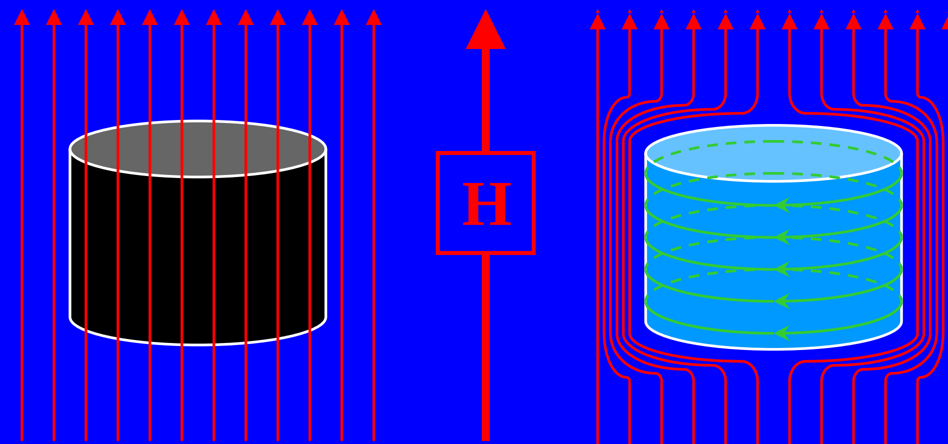


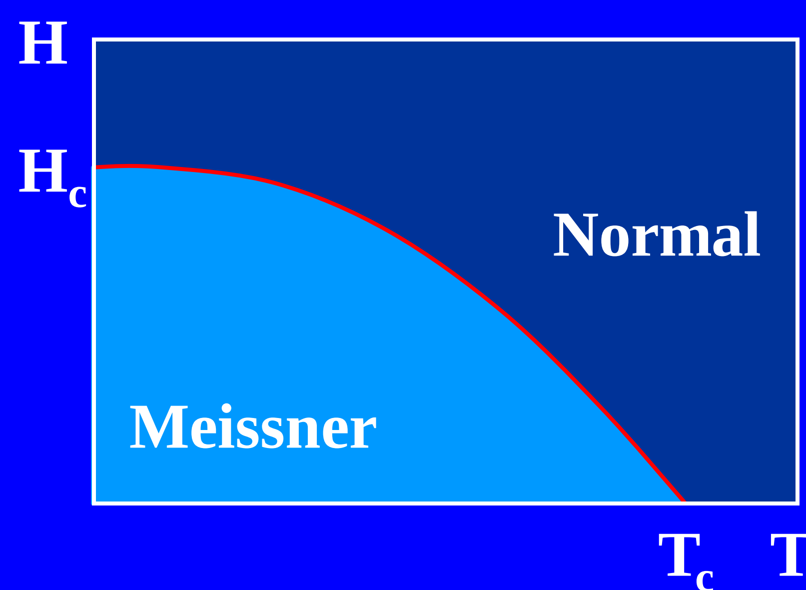
Kamerlingh Onnes, (1911)



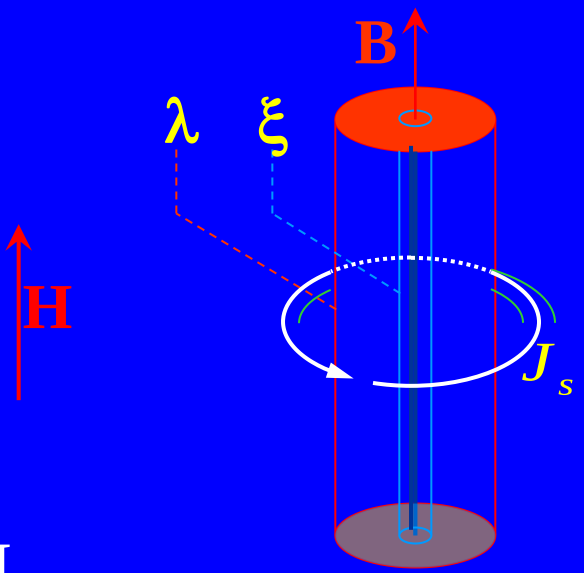
Persistent current flows in superconductors for years

2. Perfect diamagnetism





Magnetic (Abrikosov) vortices in a type II superconductor



$\lambda >> \xi$

$$\Phi_0 = \frac{hc}{e^*}$$

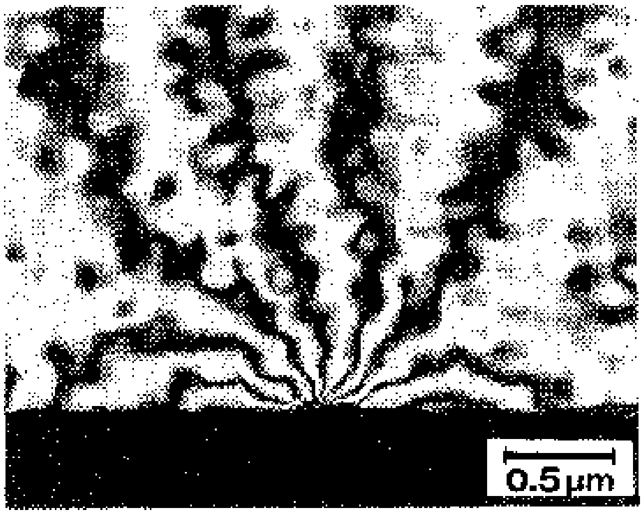
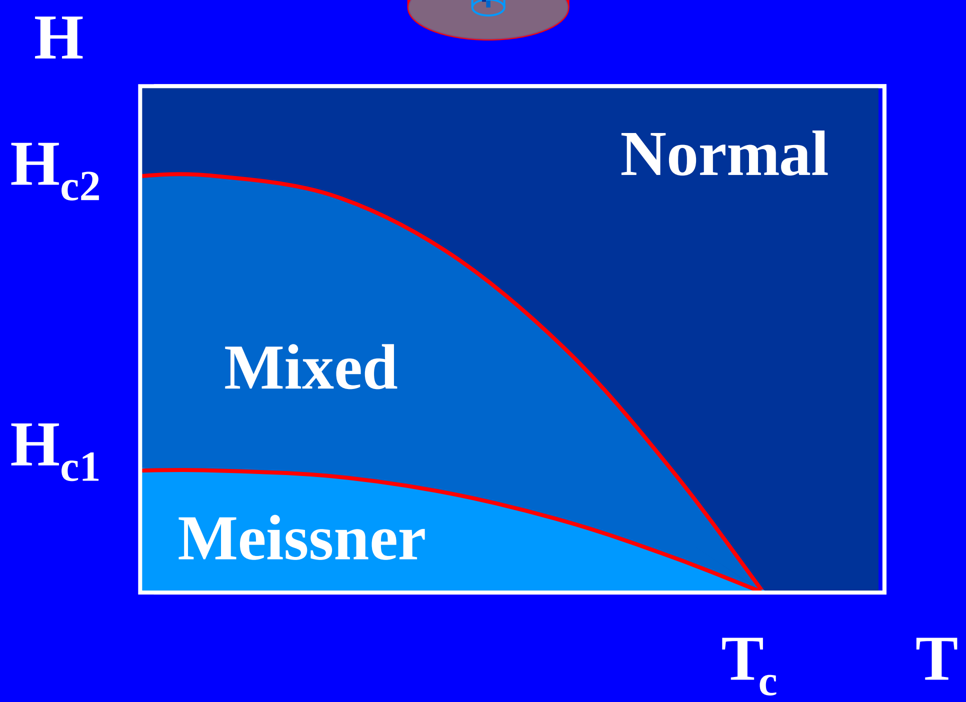


FIG. 2. 16-times phase-amplified interference micrograph of a single fluxon (film thickness =0.2 μm and sample temperature =4.5 K).

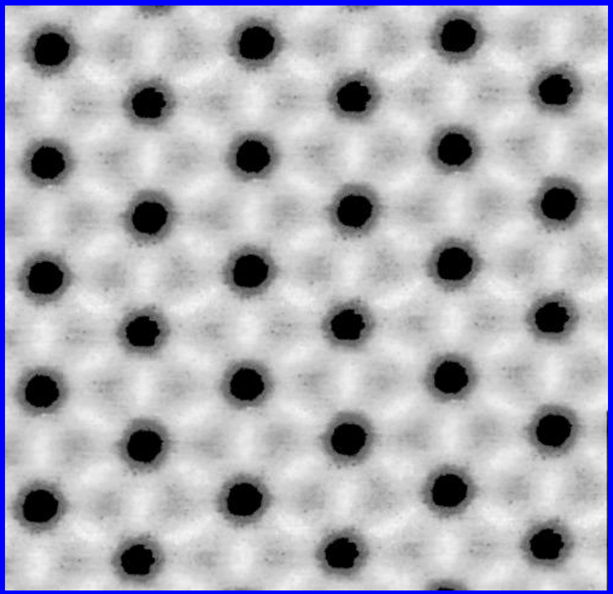


Electron tomography

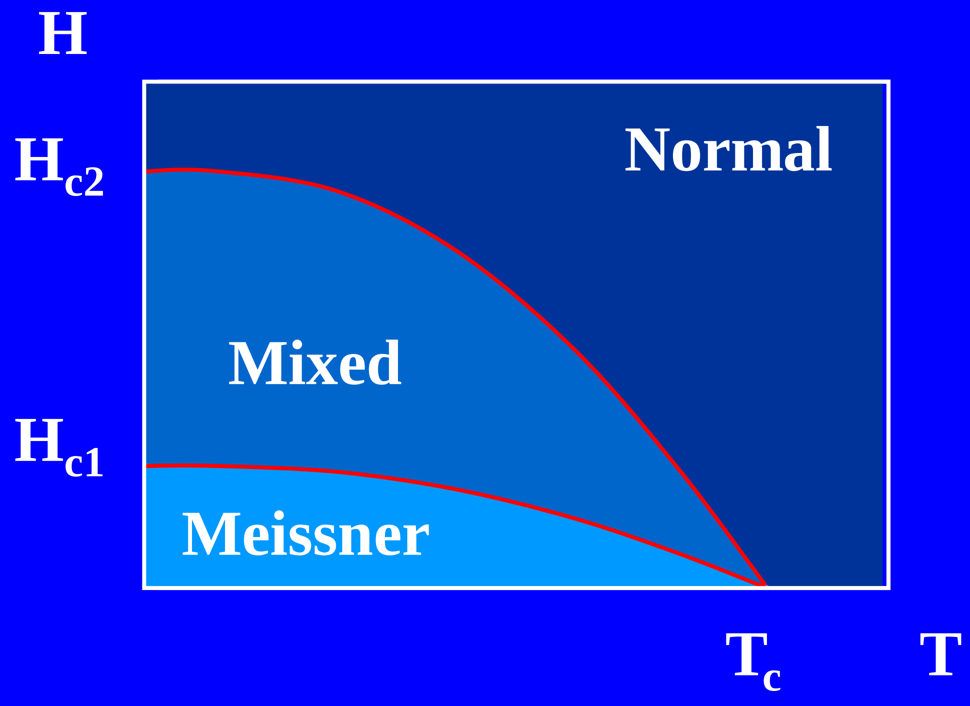
Tonomura et al, PRL66, 2519 (1993)

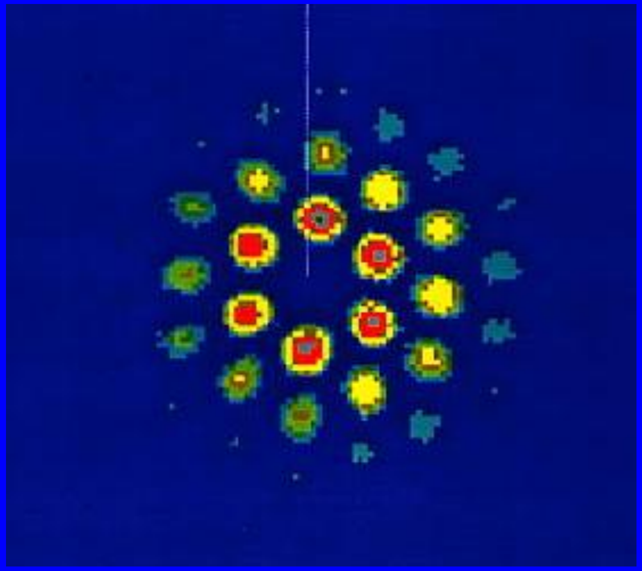
The second defining property, perfect diamagnetism, is lost

Vortex line repel each other forming highly ordered structures like flux line lattice (as seen by STM and neutron scattering)



Pan et al (2002)

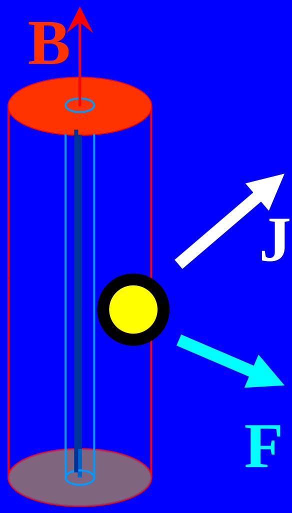


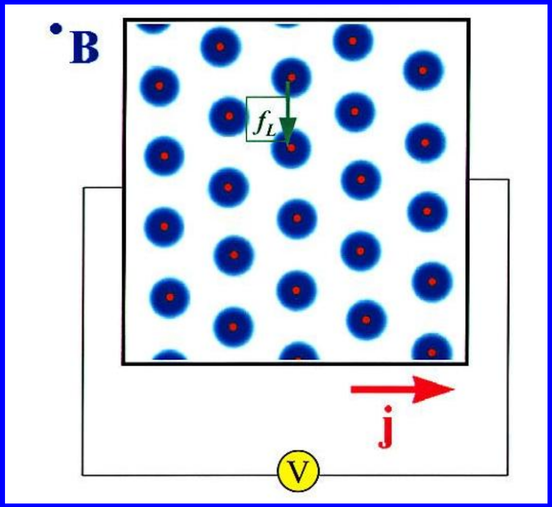


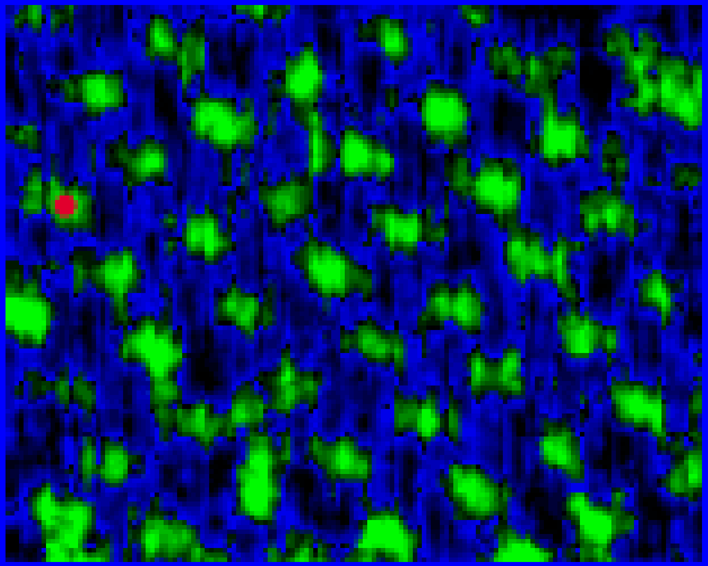
Park et al (2000)

Flux flow

Fluxons are light and move. The motion is generally a friction dominated one with energy dissipated in the vortex cores. Electric current "induces" the flux flow, causing voltage via phase slips.







The first defining property, zero resistivity, is also lost in magnetic field above Hc1

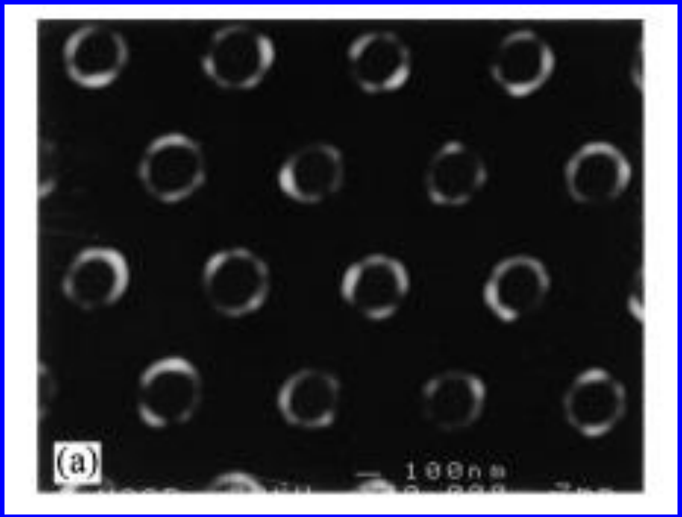
Field driven flux motion probed by STM on NbSe2

Troyanovsky et al (04)

This however is not the end of the story

Pinning of vortices

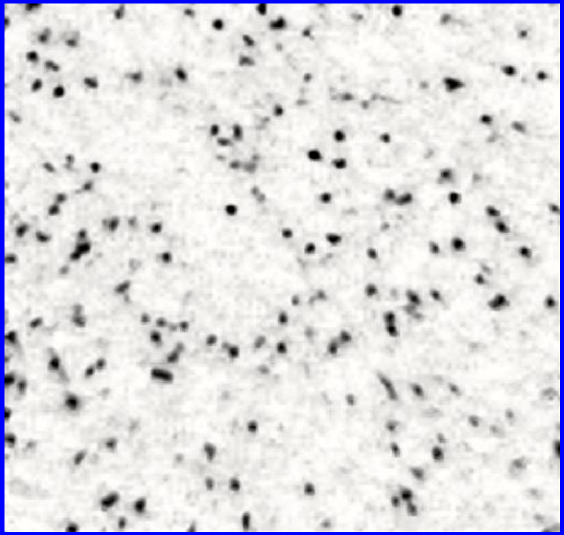
Artificial



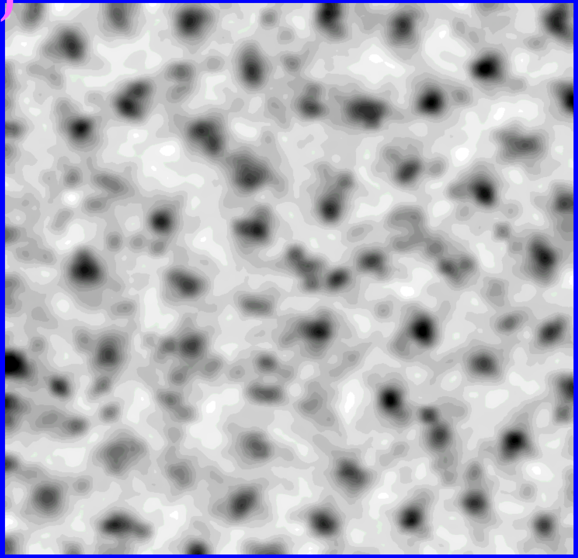
Schuler et al, PRL79, 1930 (1996)

Intrinsic random disorder

STM of both the pinning centers (top) and the vortices (bottom)



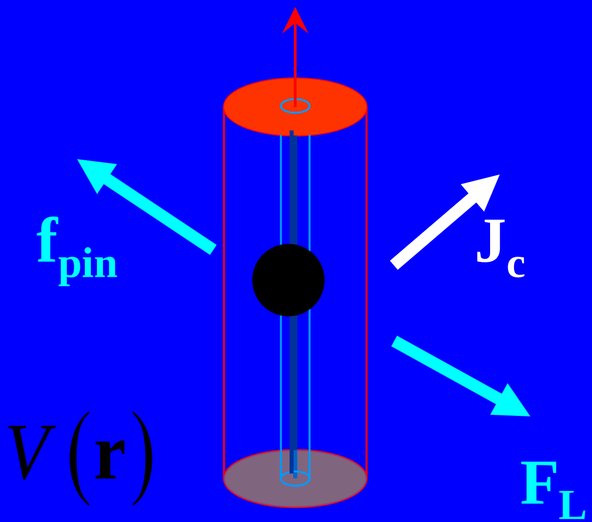
Pan et al
PRL 85, 1536 (2000)



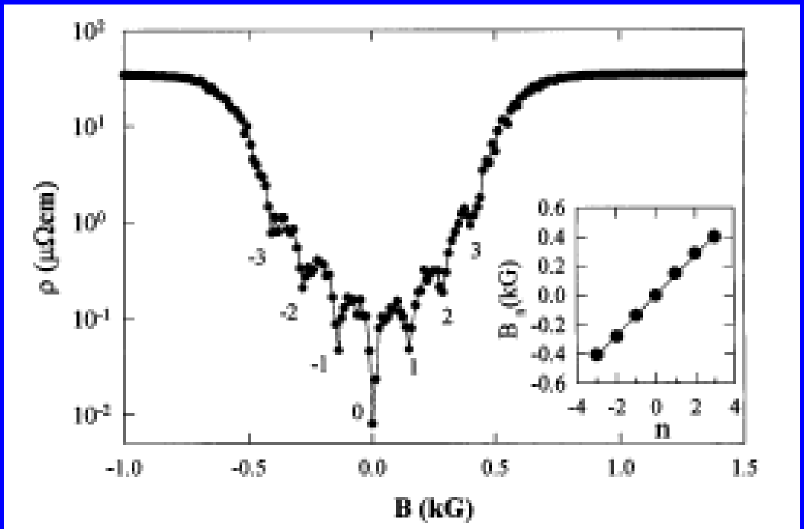
Recently techniques were developed to effectively pin the vortices on the scale of coherence length. The most effective pinning is achieved at the matching field – one vortex per pin. Fortunately this case is also the simplest to treat theoretically.

A single vortex description of the pinned state

A pinning center acts as an attractive force on the vortex since the energy loss due to the necessity to create a vortex core is reduced.



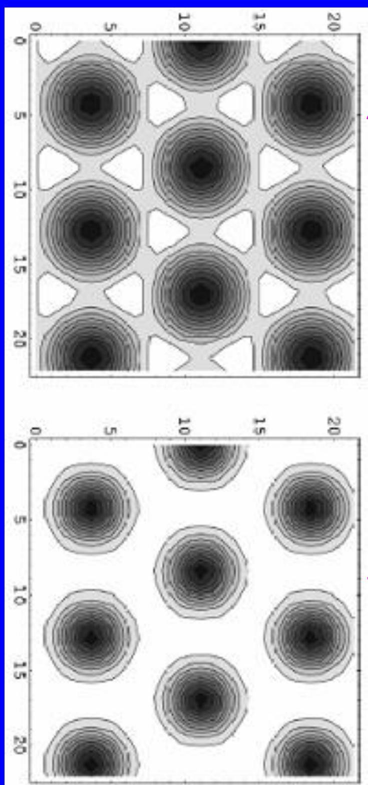
When the pinning force is able to oppose the Lorentz force, the flux motion stops, electric field cannot penetrate the material and the superconductivity is restored.

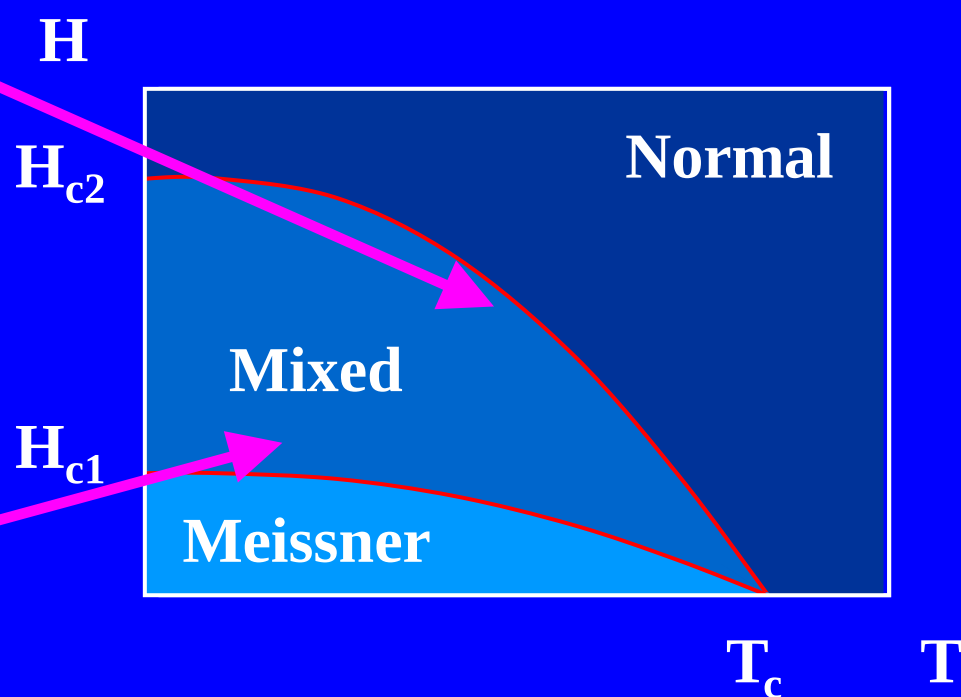


Schuler, PRL79, 1930 (1996)

Two complementary theoretical approaches to the mixed state

For $H \gg H_{c1}$ vortex cores almost overlap. Instead of lines one just sees array of superconducting "islands"





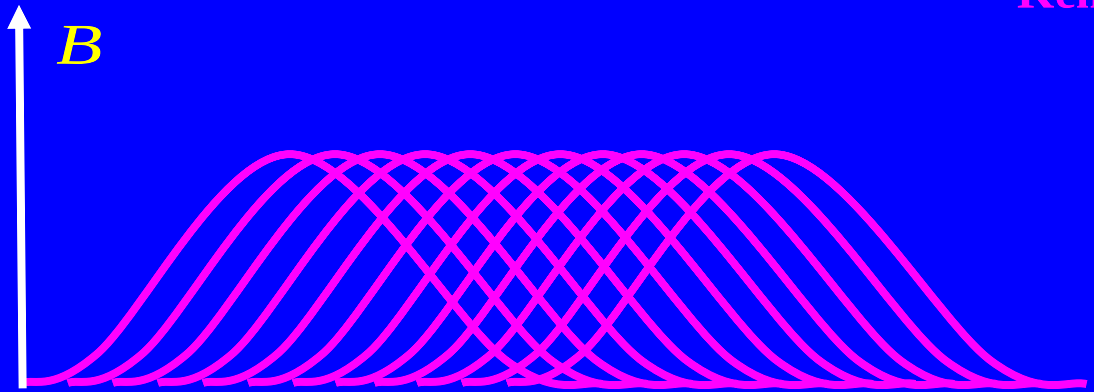
The Landau level description for constant **B**

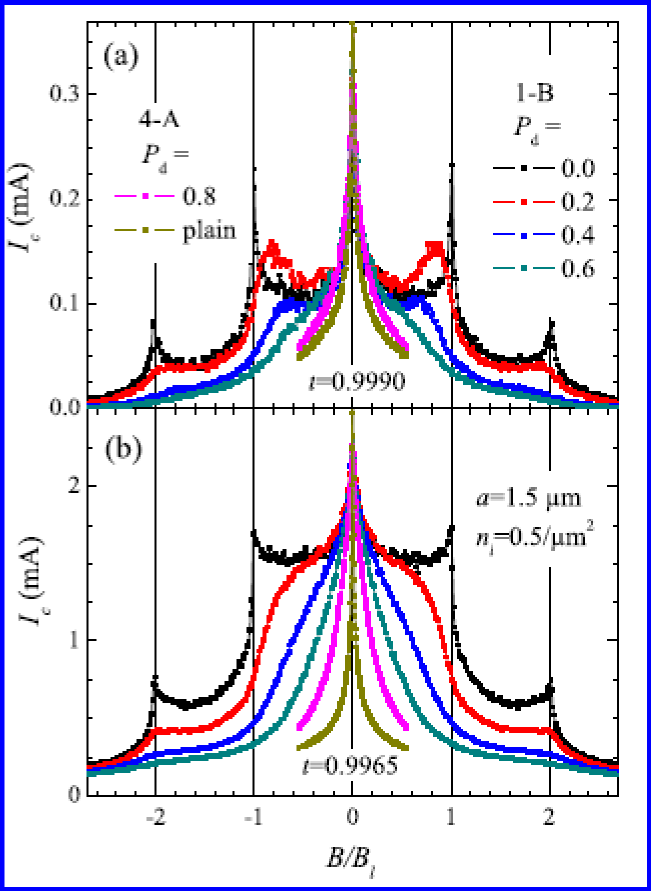
London appr. for infinitely thin lines

For $H \ll H_{c2}$ vortices are well separated and have very thin cores

Homogeneity of magnetic field

Kemmler et al, PRB79, 184509 (2009)





Homogeneity of magnetic induction B for $a < \lambda$ is a result of overlap of magnetic fields of roughly $\kappa^2 B / H_{c2}$ magnetic fields of individual vortices Magnetization (although inhomogeneous) is small ($\kappa^{-2} H_{c2}$) and one replaces B(r)=H

$$b = B/H_{c2}; \quad t = T/T_c \simeq 1; \; \lambda = \lambda_0 \left(1 - t^2\right)^{-1/2}; \; \xi = \xi_0 \left(1 - t^2\right)^{-1/2}; \kappa = \lambda / \xi >> 1$$

Outline

1. The Ginzburg – Landau description of the charged BEC in magnetic field.
2. Basic mathematical tool – perturbation theory around a bifurcation point. Abrikosov lattice as an example.
3. How current carrying states look like in the Landau level basis?
4. Current carrying pinned states, Labousch parameter and the critical (depining) current.
5. Solutions of the time dependent GL equations. Electric field and dissipation in the moving vortex matter.
6. Some generalizations and open questions.

1. Phenomenological GL description of charged BEC

The GL energy for inhomogeneos superconductor

$$F=\int_r \frac{\hbar^2}{2m^*}\left|\mathbf{D}\Psi\right|^2+\alpha\left[T-T_c\left(\mathbf{r}\right)\right]\left|\Psi\right|^2+\frac{\beta}{2}\left|\Psi\right|^4$$

The friction dominated dynamics is described by the TDGL

$$\frac{\hbar^2\gamma}{2m^*}\frac{\partial}{\partial t}\psi\left(x,t\right)=-\frac{\delta}{\delta\psi^*\left(x,t\right)}F\left[\psi,\psi^*\right]$$

γ **the normal state inverse diffusion constant**

The electromagnetic field is minimally coupled to the order parameter:

$$\mathbf{D}=\nabla-\frac{ie^*}{c\hbar}\mathbf{A}\left(\mathbf{r},t\right);\; D_t=\frac{\partial}{\partial t}-\frac{ie^*}{\hbar}\Phi\left(\mathbf{r},t\right)$$

The current density is

$$\mathbf{J}=\frac{i\hbar e^*}{2m^*}\left[\Psi^*\mathbf{D}\Psi-\Psi\left(\mathbf{D}\Psi\right)^*\right]+\sigma_n\mathbf{E}$$

The vortex dynamics then can be simulated

Disintegration of the magnetic flux at the normal line into vortices at type II SC

2. A systematic expansion for the unpinned Abrikosov lattice solution

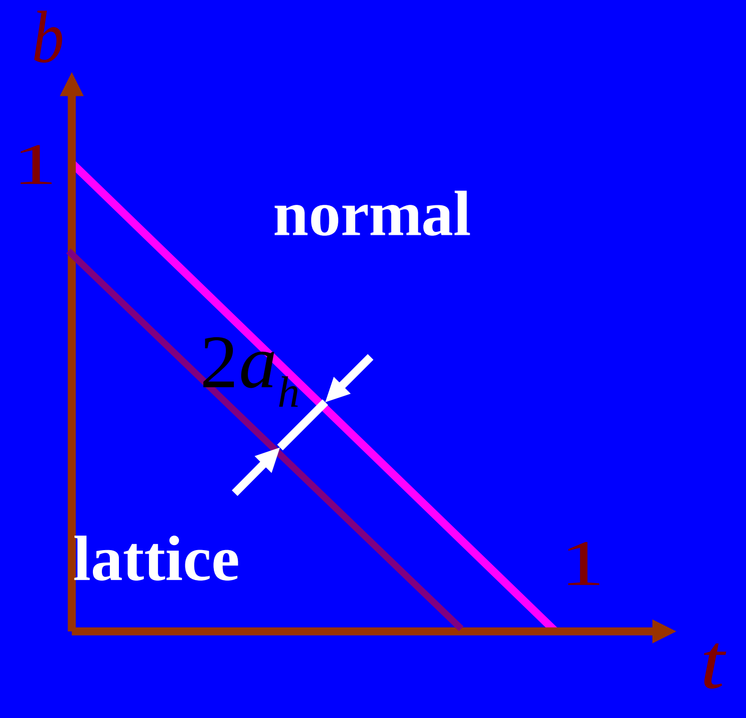
GL equations (using ξ as a unit of length, $\psi^2 = \Psi^2 / \left(2\Psi_0^2\right)$ and neglecting pinning are

$$\frac{\delta F}{\delta \psi^*} = H\psi - a_h \psi + \psi \left|\psi\right|^2 = 0$$

$$H = -\frac{1}{2} D^2 - \frac{b}{2} \qquad H\varphi_{kN} = Nb\varphi_{kN}$$

$$a_h = \frac{1}{2}\left(1 - t - b\right);$$

$$t = T / T_c, \quad b = B / H_{c2}$$



Is the distance from the bifurcation point in which the nonzero solution disappears

Perturbation theory in a_h

$$\psi = a_h^{1/2}\left(\psi_0 + a_h\psi_1 + a_h^2\psi_2 + ..\right)$$

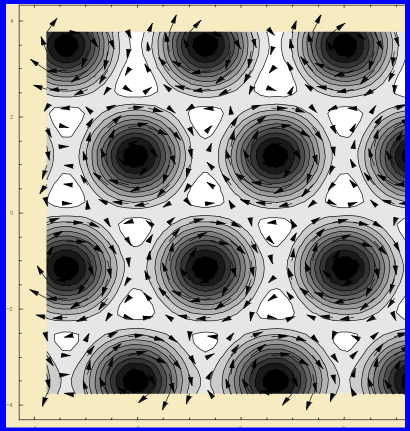
Lascher. PRA140, 523 (65)

The leading ($a_h^{1/2}$) order equation gives the lowest LLL restriction:

$$H\psi_0 = 0 \Rightarrow \psi_0 = C_0\varphi_{k0}$$

$$\varphi_{kN} = T_k\varphi_N;\ \varphi_N = \sum_{l=-\infty}^{\infty} H_N\left(b^{1/2}y + \frac{2\pi l}{b^{1/2}}\right)\exp\left[2i\pi l\left(\frac{l}{2} - x\right) - \frac{b}{2}\left(y + \frac{2\pi l}{b}\right)^2\right]$$

The next to leading order, $a_h^{3/2}$, the equation is:



$$H\psi_1 - C_0\varphi + C_0^2C_0^*\varphi|\varphi|^2 = 0$$

Making a scalar product with φ one obtains

$$\left\langle \varphi \middle| H\psi_1 - C_0\varphi + C_0^2C_0^*\varphi|\varphi|^2 \right\rangle$$

$$-1 + |C_0|^2\left\langle|\varphi|^4\right\rangle = 0 \Rightarrow C_0 = \beta_A^{-1/2}$$

Corrections

Higher order correction will include the higher Landau level contributions

$$\psi_1 = \sum_{N=0}^{\infty} C_1^N \varphi_N$$

Scalar product with $\varphi_N, N > 0$ **gives**

$$\left\langle \varphi_N \middle| H\psi_1 \right\rangle = N\, b\, C_1^N \Rightarrow C_1^N = \frac{C_0^{3/2}}{N\, b} \left\langle \varphi_N{}^* \varphi \left|\varphi\right|^2 \right\rangle$$

$$C_1^{(6)} = -\frac{0.279}{6\,b}; \quad C_1^{(12)} = \frac{0.025}{12\,b}$$

The LLL component is found from the order $a_h{}^{5/2}$ **etc.**

$$f = -\frac{a_h^2}{2\beta_A} - \frac{0.044}{6^2}\frac{a_h^3}{b} + \frac{0.056}{6^3}\frac{a_h^4}{b^2}$$

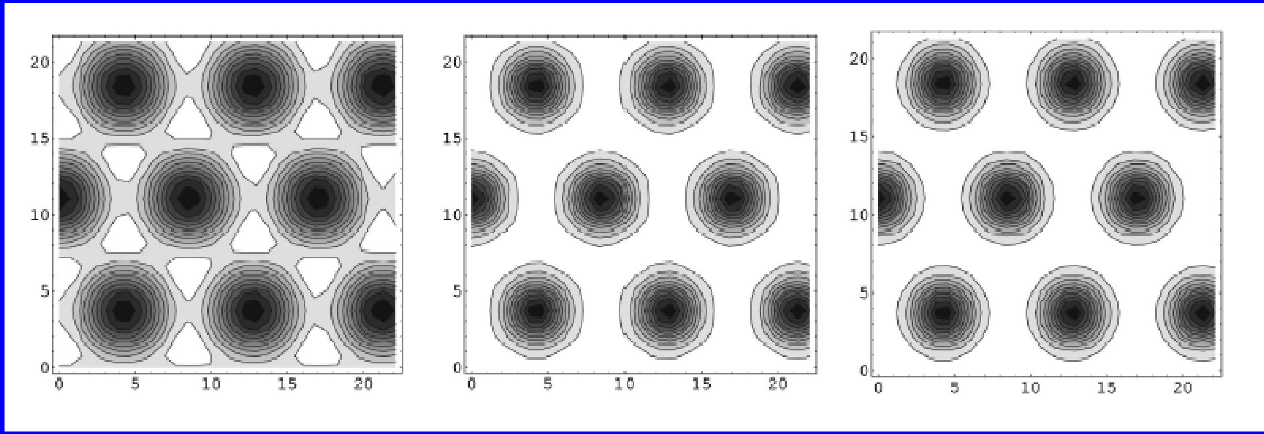
Li, B.R. PRB60, 9704 (99)

Therefore the perturbation theory in a_h is useful up to surprisingly low fields and temperatures, roughly above the line $b = \frac{1}{13}(1-t)$

$T/T_c = .5$

$B/H_{c2} = .1$

$\Rightarrow a_h = .2$



LLL is by far the leading contribution above this line. Of course currents in this state are purely diamagnetic. Generally in the absence of electric fields

$$j(r) = j_{transport} + j_{magn}(r)$$

the overall current of the equilibrium state is zero in accordance with a generalization of the Bloch theorem.

Bloch theorem for GL

Assume that a configuration ψ has minimal energy and carries a transport current

$$j=\frac{1}{vol}\int_r \frac{i}{2}\left[\psi^*\mathbf{D}\psi-\psi\left(\mathbf{D}\psi\right)^*\right]\neq 0$$

Then the confuguration $\psi'=e^{ikr}\psi$ Has lower energy for $k=-j$

$$\frac{1}{vol}F\left[\psi'\right]=\int_r \frac{1}{2}\left|\mathbf{D}\psi\right|^2+\left[a+V\left(\mathbf{r}\right)\right]\left|\psi\right|^2+\frac{1}{2}\left|\psi\right|^4$$

Bohm, PR (59)

$$=\frac{1}{vol}F\left[\psi'\right]+kJ+k^2/2<\frac{1}{vol}F\left[\psi'\right]$$

So even when pinning is present the system simply finds a state in which the current vanishes, while the supercurrent carrying states are excitations, which have exremely large relaxation times in a superconductor.

3. Persistent current in magnetic field

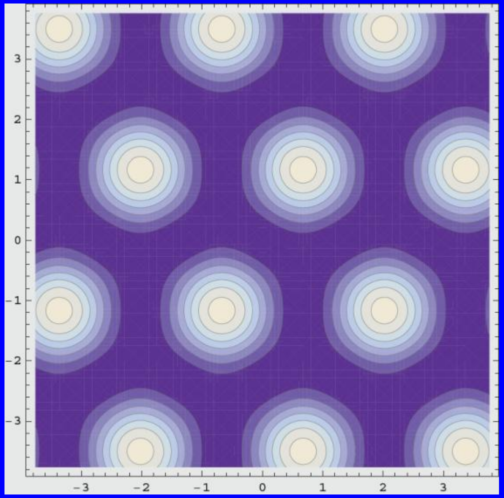
LLL is not enough

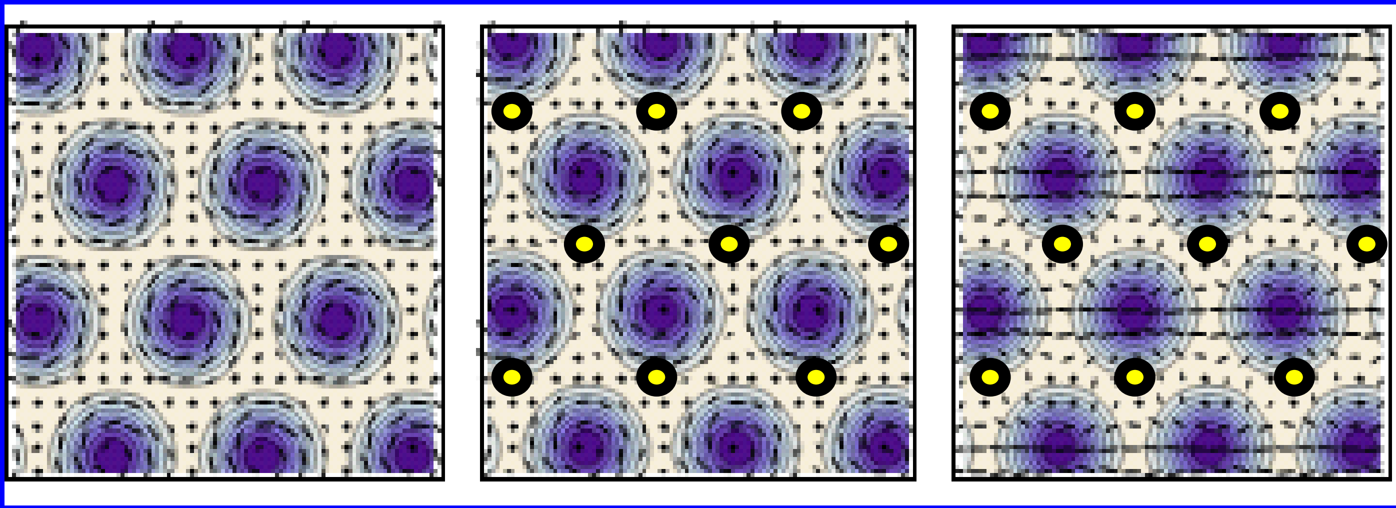
When all the vortices are pinned there is current without dissipation.

The order parameter configuration cannot belong to LLL, since for a general LLL configuration

$$J_i = \propto \varepsilon_{ij} \partial_j \left(|\Psi|^2 \right)$$

Affleck, Brezin. NPB257, 451 (85)

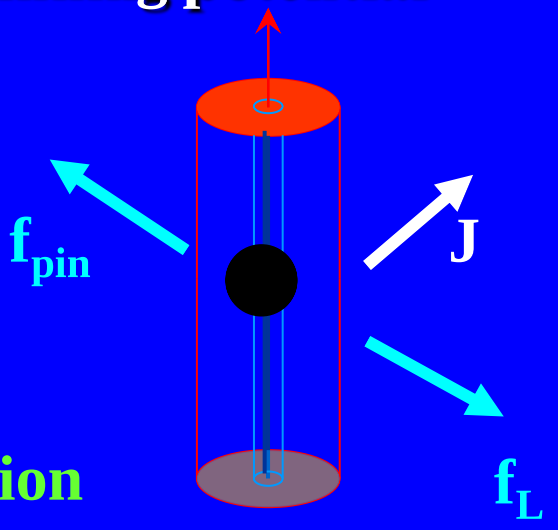




A small 1LL correction like $\psi = \varphi_0 + 0.02\varphi_1$

Produces an appreciable net current of one percent (the unit of current is the depairing current of superconductor)

The force balance equation for a periodic pinning potential



The force balance equation

$$-f_L = \frac{B}{c}\int \hat{z}\times J = -\int \nabla V\left|\Psi\right|^2 = f_{pin}$$

Multiplying a covariant derivative of GL equation

$$-\frac{1}{2}D^2\psi - (1-t-V)\psi + \psi\rho = 0$$

By ψ^* one obtains

$$\psi^*\left(-\frac{1}{2}D_iD^2 + \partial_i\rho + \partial_iV\right)\psi + \left(-\frac{1}{2} + \rho + V\right)\psi^* D_i\psi = 0$$

$$\frac{1}{2}D^2\psi^* D_i\psi$$

$$\psi^*\left(\frac{1}{2}\overleftarrow{D}^2 D_i - \frac{1}{2}D_iD^2\right)\psi + \rho\partial_i\rho + \rho\partial_iV = 0$$

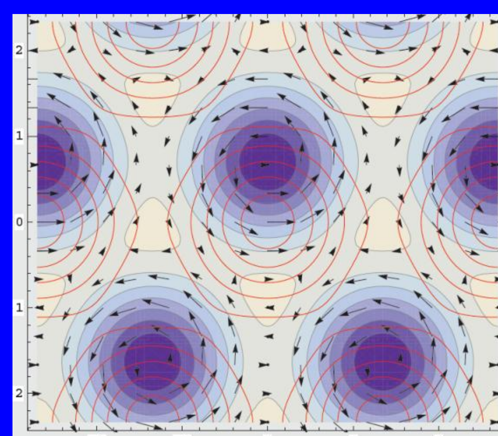
Taking the sample integral

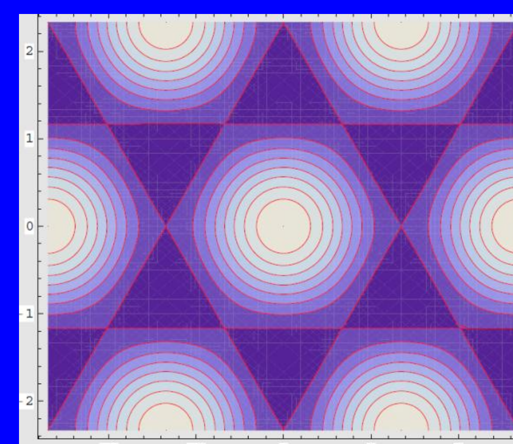
$$\left\langle \frac{1}{2}\psi^{*}\left|\overleftarrow{D}^{2} D_{i} - D_{i} D^{2}\right|\psi \right\rangle + \langle \rho\partial_{i}\rho + \rho\partial_{i}V \rangle = 0$$

and using the commutator $\left[D^{2}, D_{i}\right] = i\varepsilon_{ij} b D_{j}$ **and , dropping full derivatives due to periodicity, one obtains**

$$\left\langle \frac{1}{2}\psi^{*} i\varepsilon_{ij} b D_{j}\psi \right\rangle = -\langle \rho\partial_{i}V \rangle \Rightarrow -\varepsilon_{ij} b j_{j} = -\langle \rho\partial_{i}V \rangle$$

This relation is making a calculation of the persistent supercurrent in a lattice pinned by an arbitrary periodic potential at matching field very simple.

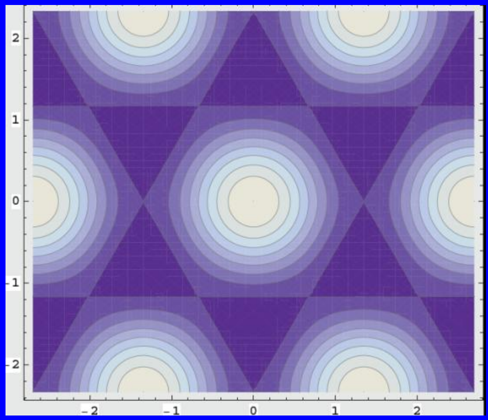




4. Current carrying pinned states, Labousch parameter and the critical (depining) current.

Solution of GL with a periodic pinning potential

We consider a periodic potential

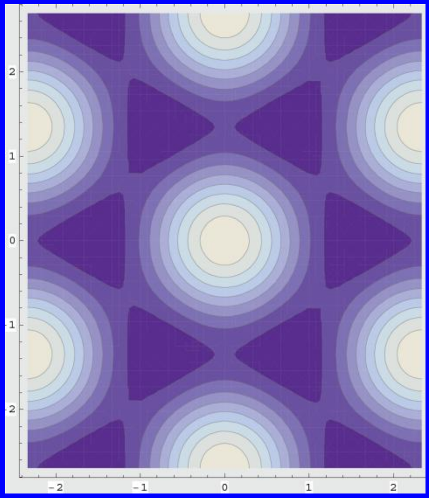


$$V(r) = \sum_Q V_Q e^{ik\cdot r},$$

$$V_{11} = V_{01} = V_{10} = v$$

In this case a conflict between interactions of vortices and pinning potential is avoided and quasimomentum k is conserved

$$H = -\frac{1}{2}D^2 - \frac{b}{2} - \varepsilon_k \qquad a_h = \frac{1-t-b}{2} - \varepsilon_h$$



One can systematically expand solutions of GL eqs. around the “new” bifurcation point for the inhomogeneous case to first order in pinning potential $\varepsilon_h = \langle \varphi_k | V(r) | \varphi_k \rangle$

Current vs displacement of vortices

To first order in potential and a_h the order parameter is

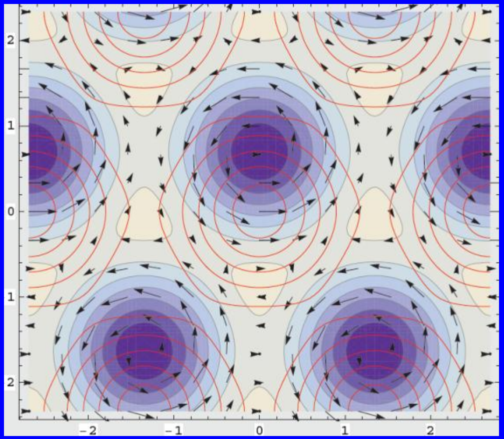
$$\psi = \left(a_h / \beta_A\right)^{1/2} \psi_0 + O\left(a_h^{\ 3/2}\right) \qquad \psi_0 = \varphi_{k0} + \sum_{N=1} c_{N0}^{*} \varphi_{kN}$$

$$c_{N0} = \frac{\left\langle \varphi_{kN} \left| V \right| \varphi_{k0} \right\rangle}{-Nb} = \frac{(-)^{N+1}}{Nb\sqrt{N!\left(2b\right)^{N}}} \sum_{Q} Q^{N} V_{Q}\, g\left(Q,k\right) \qquad Q = Q_x + iQ_y$$

and it carries a current

$$j_i = b^{-1} \varepsilon_{ij} \left\langle \rho \partial_i V \right\rangle$$

$$j = j_x + i j_y = -i \frac{a_h}{b\beta_A} c_{01}$$



This is not rotation invariant with the maximal value achieved when the quasimomentum is along the x axis. It is important to note that in addition to the displacement, the shape of vortices changes in the current carrying states

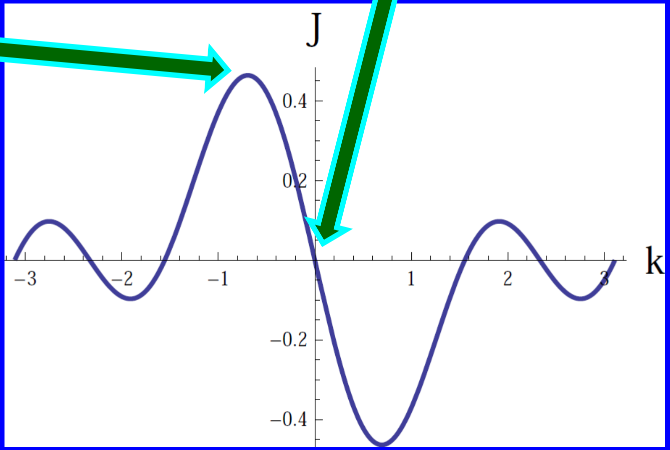
Critical (depinning) current

It is interesting that the transition to the flux flow state as current is increased passed j_c is always at same quasimomentum (same place within the unit cell)

$$j_x = -0.46\frac{a_h}{b\beta_A}k_x = -0.46\frac{a_h}{\beta_A}y$$

$$k_c = (0.695, 0)\, b^{1/2}$$

$$j_c = 1.5\frac{a_h}{\beta_A}v$$



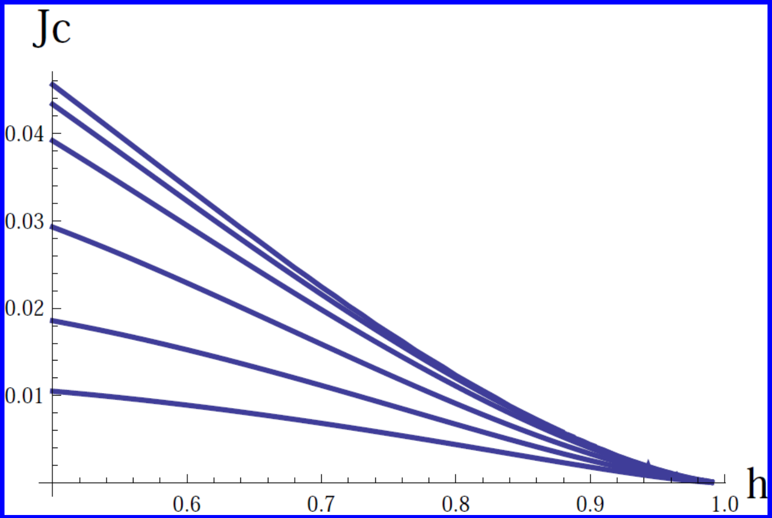
One calculates easily both higher orders in potential and in the expansion parameter a_h and finds that beyond certain pinning strength the perturbation theory breaks down. In this case a simple variational method is successful

Beyond perturbation theory in potential

It is quite enough to consider a variational method in which the configuration is restricted two the lowest two LL.

$$\psi = c_0 \varphi_{k0} + c_1 \varphi_{k1}$$

Beyond certain potential the critical current stops rising. Qualitatively the best pinning is achieved when the gradient of the pinning potential is proportional to the Abrikosov vortex superfluid density



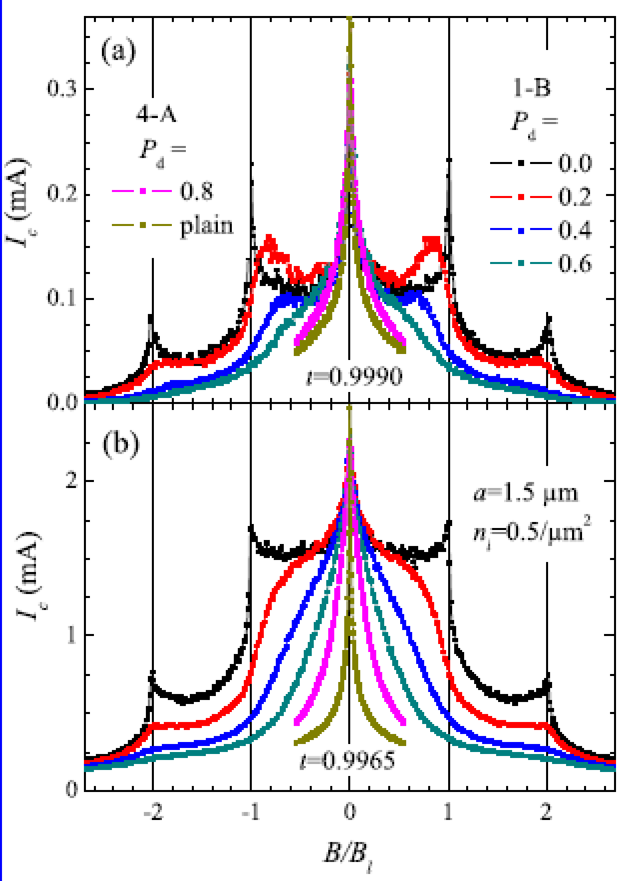
In press B.R., B. Shapiro, I. Shapiro, PRB (2010)

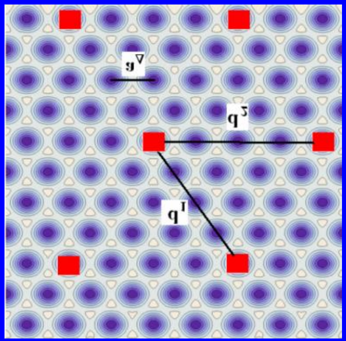
Beyond critical current Lorentz force becomes larger than the pinning force, vortices start moving and electric field enters the superconductor. Since electric field is inhomogeneous, Maxwell equations should be solved. Simplicity is lost.

Comparison with randomly distributed pinning centers

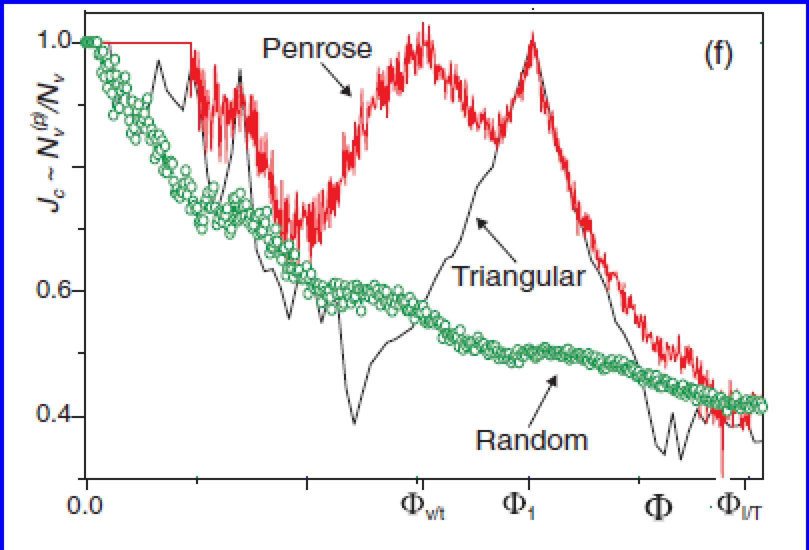
When the filling fraction is fractional, the critical current becomes much lower due to interstitial vortices, which are very weakly pinned by interactions with strongly pinned vortices that "neutralize" pinning centers.

Kemmler et al, PRB79, 184509 (2009)





Misko, Savel'ev, Nori, PRL95, 177007 (2005)



Random or incommensurate component beyond the periodic array actually lead to slightly better pinning

5. Flux flow

When current significantly exceeds critical, electric field is present and, due to superposition between vortices, is also homogeneous in sufficiently dense vortex matter

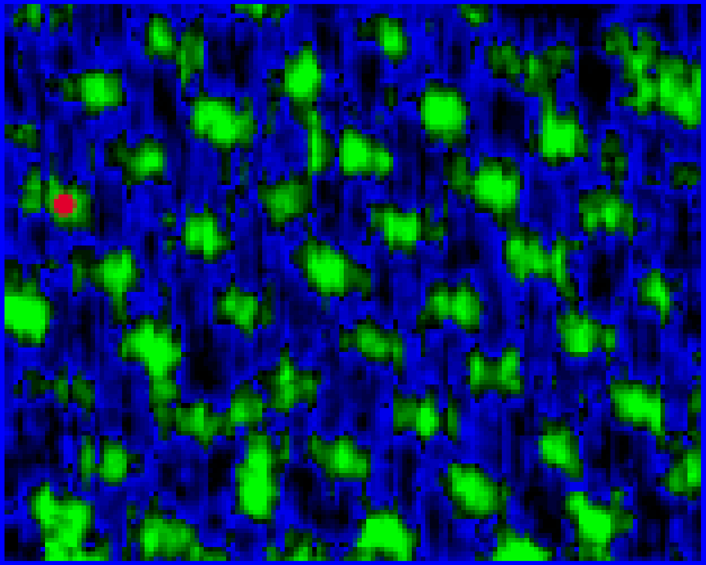
Hu, Thompson, PRL27, 1352 (75)

The friction dominated dynamics is described by the TDGL

$$\frac{\hbar^2\gamma}{2m^*} D_t \psi = -\frac{\delta}{\delta\psi^*} F\left[\psi,\psi^*\right]$$

$$D_t = \frac{\partial}{\partial t} - \frac{ie^*}{\hbar}\Phi(r); \quad \Phi(r) = Ey$$

Here E is constant and this makes it possible to apply the bifurcation perturbation theory again



Troyanovsky et al (04)

Bifurcation perturbation theory in constant electric field

Using the natural units $\tau_{GL} = \gamma\xi^2$ $\quad E_{GL} = \dfrac{4\hbar}{e^* t_{GL}\xi} \quad \mathrm{E} = \dfrac{E}{E_{GL}}$

Time dependent GL equation can be written as

$$L\psi - a_h\psi + \psi|\psi|^2 = 0$$

$L = D_t + H + \dfrac{\mathrm{E}^2}{2b^2}$ **is not Hermitean.**

Electric field therefore is an additional pair breaker. The critical line beyond which just a trivial normal solution exists is

$$1 - t - b - \mathrm{E}^2/b^2 = 0 \Rightarrow \quad H_{c2}(T,E) = H_{c2}\left(1 - t - \mathrm{E}^2/b^2\right)$$

$$a_h = -\frac{1}{2}\left(1 - t - b - \mathrm{E}^2/b^2\right)$$

Hu, Thompson, PRL27, 1352 (75)

The adaptations to the method are the following. One first looks for eigenfunctions of the linear part of the equation

$$L\phi_{Np\omega} = \Theta_{Np\omega}\phi_{Np\omega} \qquad \Theta_{Np\omega} = Nb + i\left(\omega - vk\right)$$

The right eigenfunctions are:

$$v = \mathrm{E}b^{-3/2}$$

$$\phi_{Np\omega} = \mathrm{e}^{i(kx-\omega t)}\, H_N\left[b^{1/2}\left(y - k/b + iv\right)\right]\exp\left[-\frac{b}{2}\left(y - k/b + iv\right)^2\right]$$

Note the "wave" exponential despite absence of Galileo invariance (due to microscopic disorder tied to the rest frame)

Within the bifurcation method one uses scalar products. In the present case these should be formed with the left eigenfunctions:

$$\bar{\phi}_{Nk\omega} = \mathrm{e}^{-i(kx-\omega t)}\, H_N\left[b^{1/2}\left(y - k/b + iv\right)\right]\exp\left[-\frac{b}{2}\left(y - k/b + iv\right)^2\right] \neq \phi^*_{Nk\omega}$$

The orthonormality relations take a form:

$$\int_{x,y,t} \bar{\phi}_{Nk\omega}(x,y,t)\phi_{N'k'\omega'}(x,y,t) = \sqrt{\pi}\delta_{NN'}\delta(k-k')\delta(\omega-\omega')$$

Assuming, as in statics, an expansion $\psi = a_h^{1/2}\left(\psi_0 + a_h\psi_1 + ..\right)$ **,**

the leading, $a_h^{1/2}$**, order equation is the LLL constraint**

$L\psi_0 = 0$ **implying** $N = 0, \quad \omega = vk$

For each moving lattice symmetry one gets normalization from the $a_h^{3/2}$ **order:**

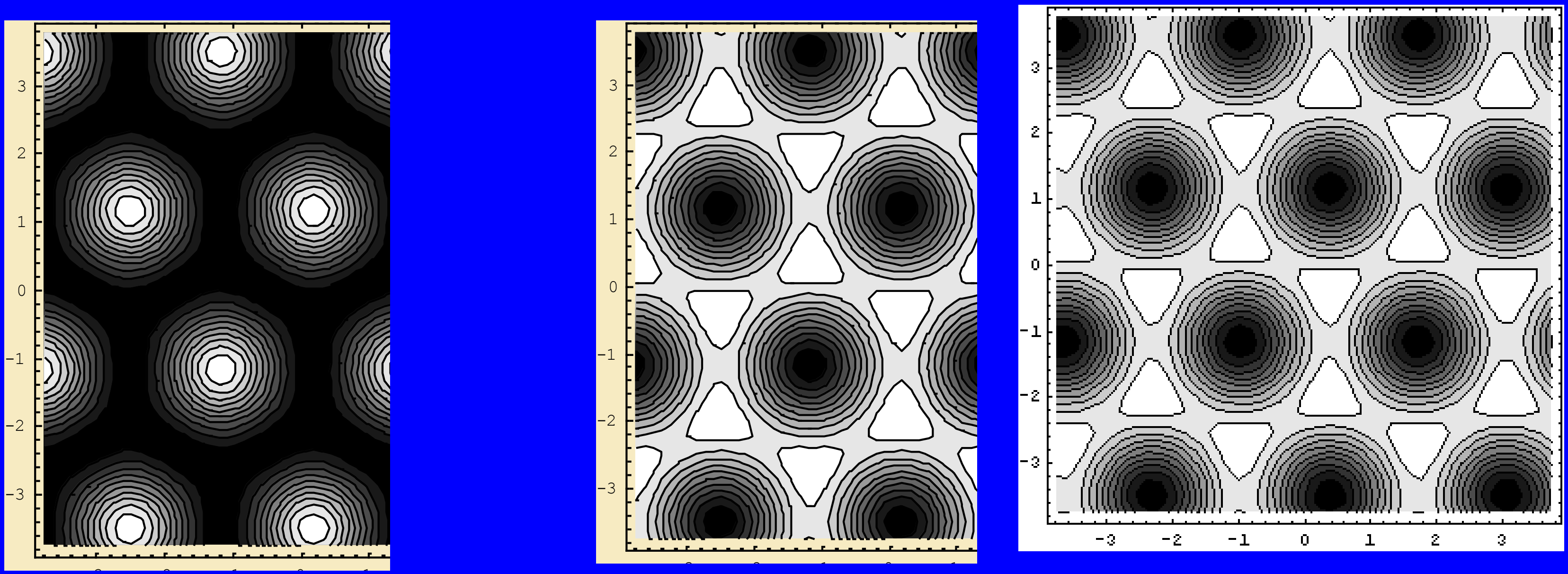
$$C_0 = \beta_A^{-1/2}(v); \quad \beta_A(v) = \left\langle \bar{\varphi}\varphi^*\varphi^2 \right\rangle$$

Li, Malkin, B.R., PRB70, 214529 (04)

LLL supercurrent density:

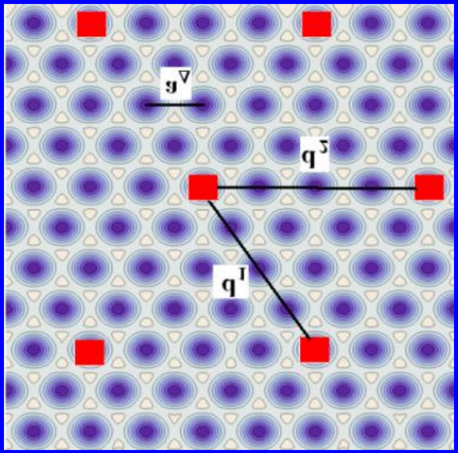
$$J_i = \frac{\hbar e^*}{m^*}\left[\frac{1}{2}\partial_j\left(|\Psi|^2\right) + v_i|\Psi|^2\right]$$

The moving lattice solution



dissipation

superfluid density $p = \langle E \cdot J \rangle = \frac{\hbar^2 \gamma}{2m^*} \langle |D_t \psi|^2 \rangle$



The lattice is no longer hexagonal, but is slightly deformed.

In the presence of periodic pinning the corrections and the AC conductivity can be obtained.

Maniv, B.R., Shapiro, PRB80, 134512 (09)

Conclusions

1. **Bifircation point perturbation theory is a convenient systematic universal method which can be applied to vortex matter in type II superconductors when electromagnetic field is essentially homogeneous.**

B.R., Li, Rev. Mod. Phys.82 , 109 (2010)

2. **It was applied to describe quantitatively nonequilibrium supercurrent carrying states supported by a periodic array of pins of arbitrary shape and the flux flow at sufficiently large flux velocities.**

3. **Pins on the scale of coherence length can manipulate the distribution of the order parameter. The critical current is maximized when gradient of potential is proportional to the Abrikosov lattice superfluid density.**