

ICOSAHEDRAL ORDER: *The Link between Quasicrystals and Metallic Glasses*

S. RANGANATHAN

**Department of Materials Engineering
Indian Institute of Science
Bangalore, India**

**School on Glass Formers and Glasses
JNCASR, Bangalore
January 19, 2010**

OUTLINE

- ❖ THE SIZE FACTOR
- ❖ PETTIFOR STRUCTURE MAPS
- ❖ QUASICRYSTALS
- ❖ THE SUPERTETRAHEDRON
- ❖ EFFICIENT CLUSTER PACKING
- ❖ BULK METALLIC GLASSES

INTERMETALLICS

- Zintl Phases
- Hume-Rothery Phases
- Frank-Kasper Phases
- Chemical Compounds
- Electron Compounds
- Size factor Compounds
- Where do Quasicrystals & Bulk Metallic Glasses fit in?

Atomic Parameters in Phase Stability

W Hume-Rothery, 1926

1 SIZE

2 ELECTRONEGATIVITY

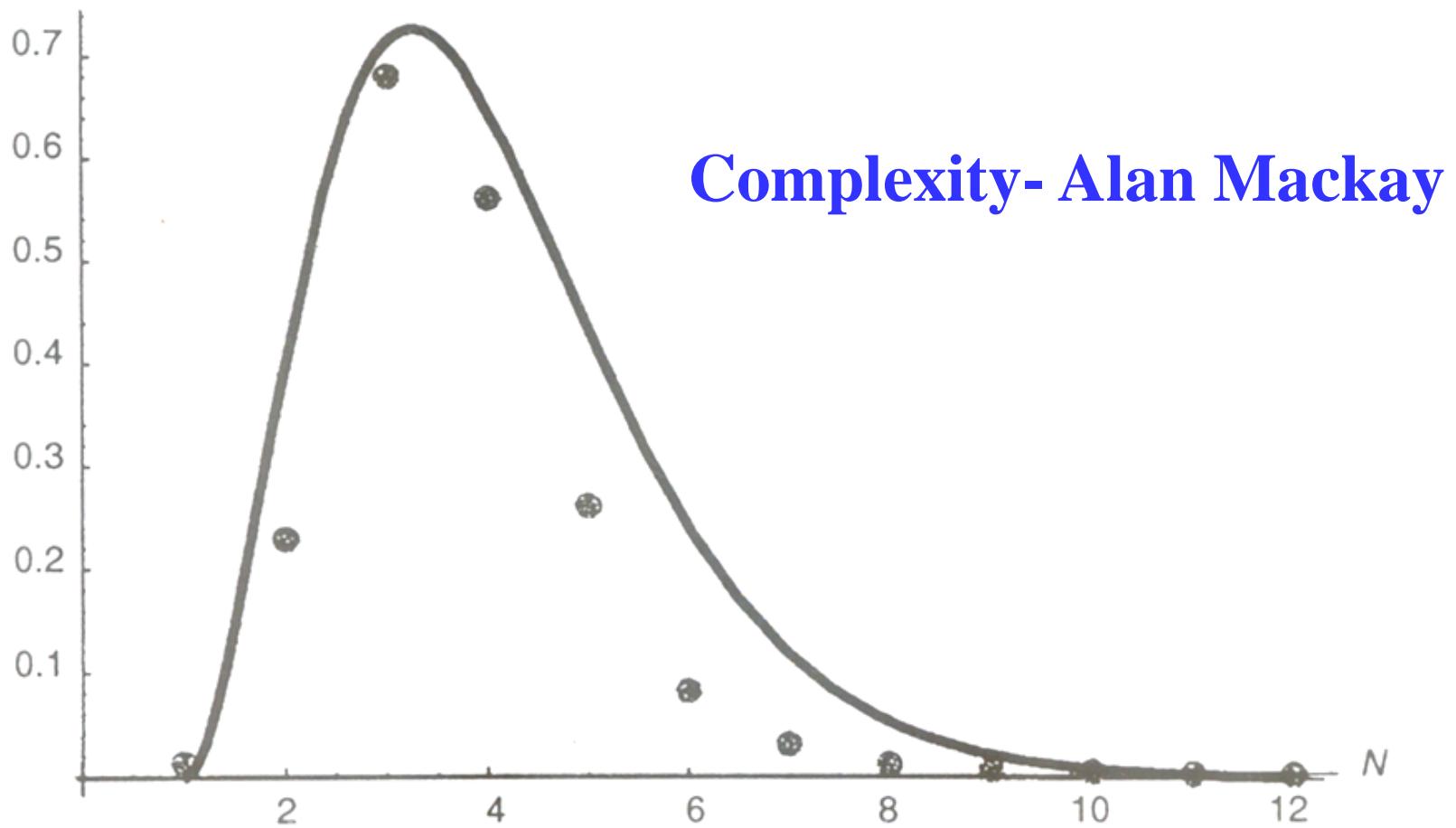
3 VALENCE ELECTRON CONCENTRATION

D Pettifor, 1984

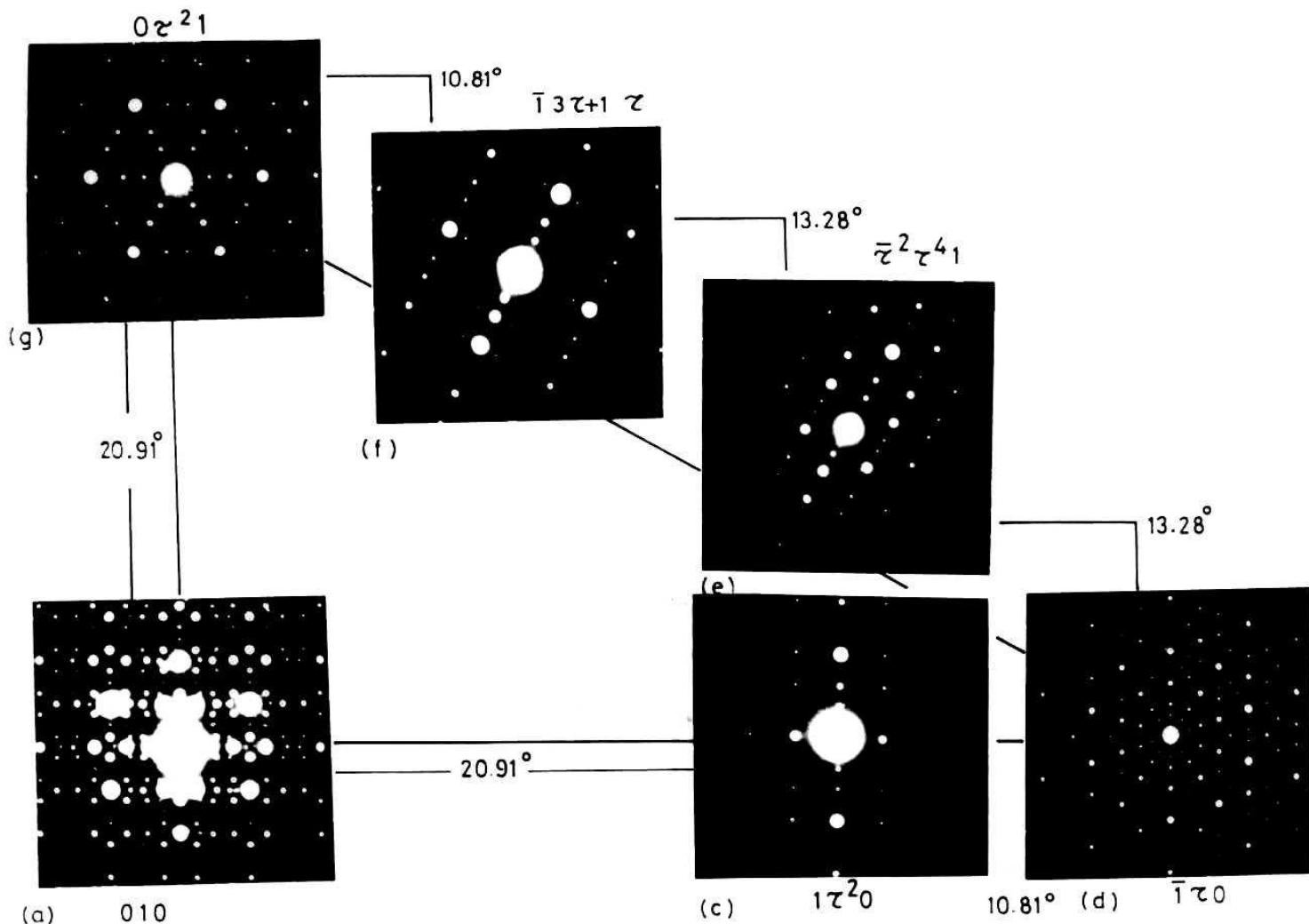
4 BOND ORBITALS (s, p, d, f- orbitals)

Distribution of Chemical Systems

Systems	Experimentally Known	Percent Known	Maximum Number
Unaries	100	100%	100
Binaries	4,000	81%	4,950
Ternaries	8,000	5%	161,700
Quaternaries	1,000	<1%	3,921,225



Frequency distribution of inorganic crystal structures having N different elements



Electron diffraction patterns from a rapidly solidified
Al-Mn alloy revealing Icosahedral symmetry

THE SIZE FACTOR

“My own view is that simple geometry...
... atomic sizes. . . will prove to be the main criterion
that in various subtle ways incorporates the others.”

A. I. Greer & R.W. Cahn, 1991

- Close packing of spheres of the same size
- Kepler’s Conjecture in 1609
- David Hilbert highlighted efficient packing
in a list of problems to guide mathematics
in the 20th century
- Hales solution in 1997

Spheres of Different Sizes

A major challenge

Structure of

Topologically close packed intermetallics

Quasicrystalline intermetallics

Metallic glasses

Bulk metallic glasses

Eutectic liquid compositions

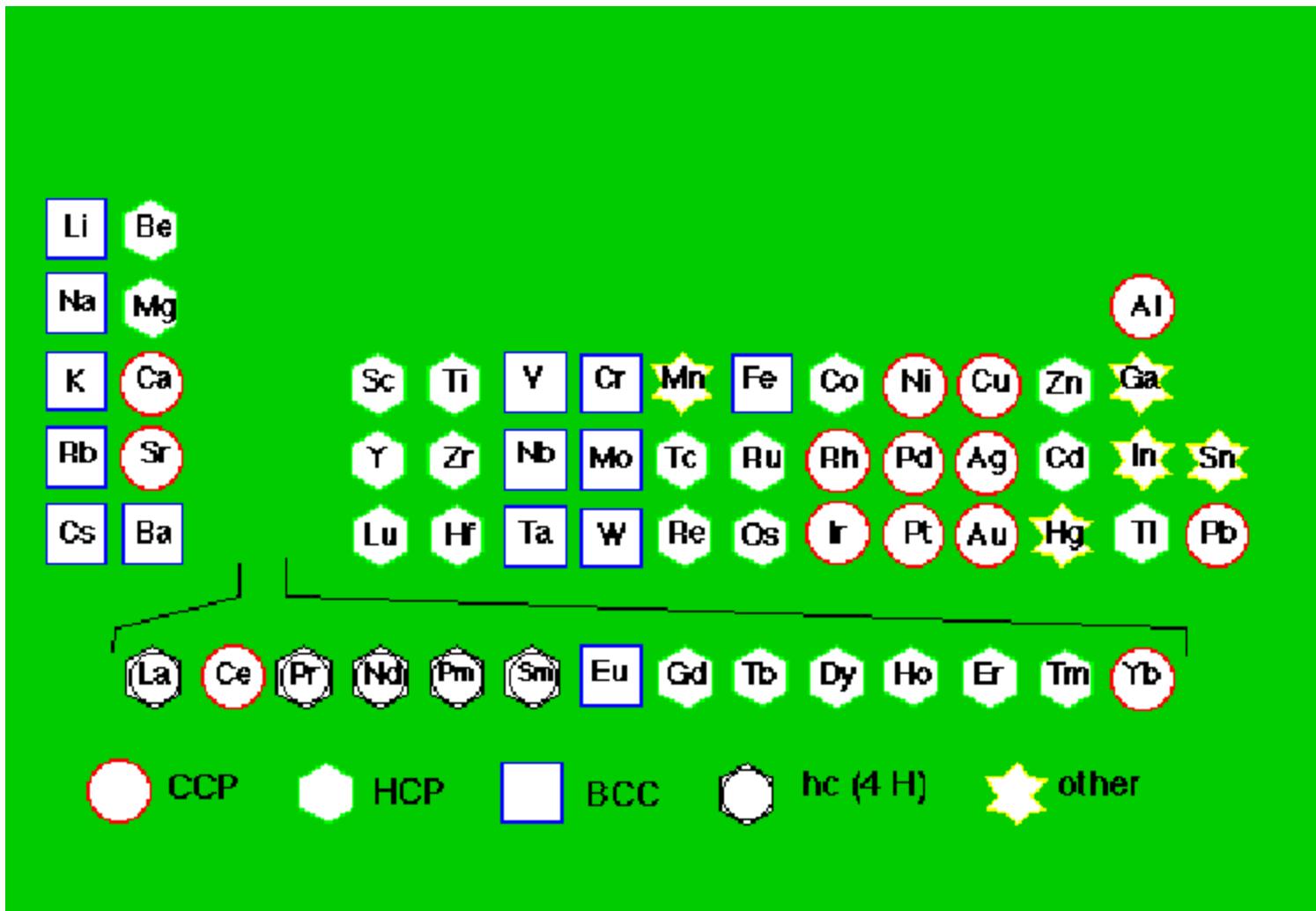
Clusters

Colloidal Crystals

Nanosuperlattices

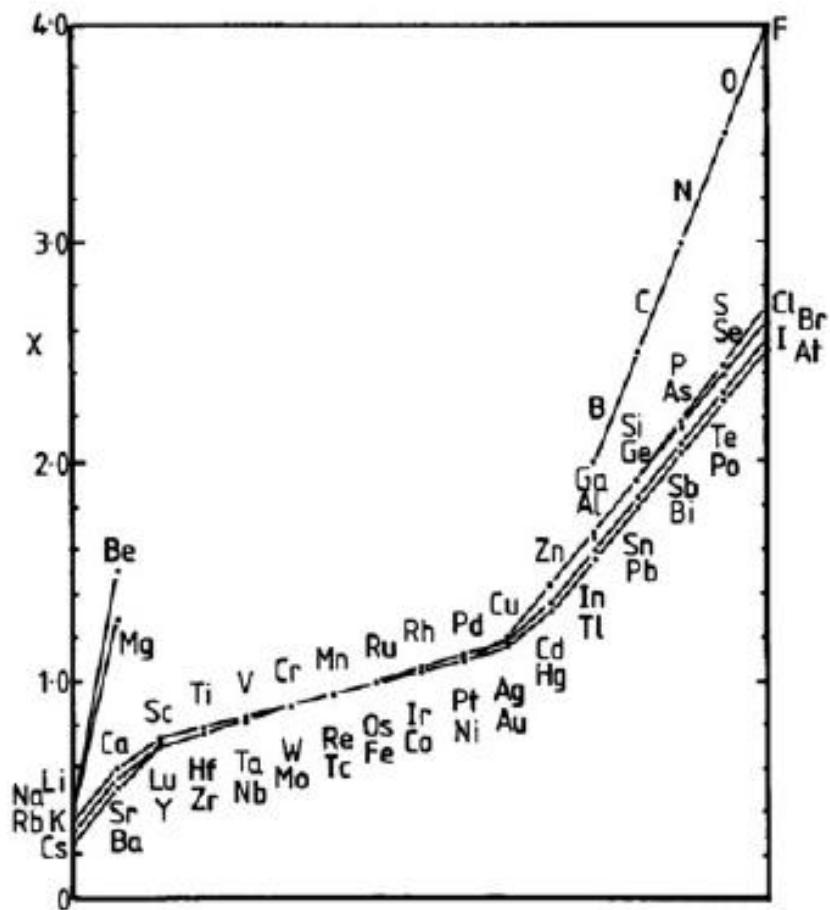
Frank 1952, Bernal 1959, Kasper 1959, Mackay 1977,
Gaskell 1978, Egami 1984, Miracle 2003, Ma 2006

PERIODIC TABLE OF METAL STRUCTURE



Issue :Combination of topological and chemical identities₁₀

THE CHEMICAL SCALE



From David Pettifor (1984)
Note anomaly in places for Be & Mg

PETTIFOR MAP

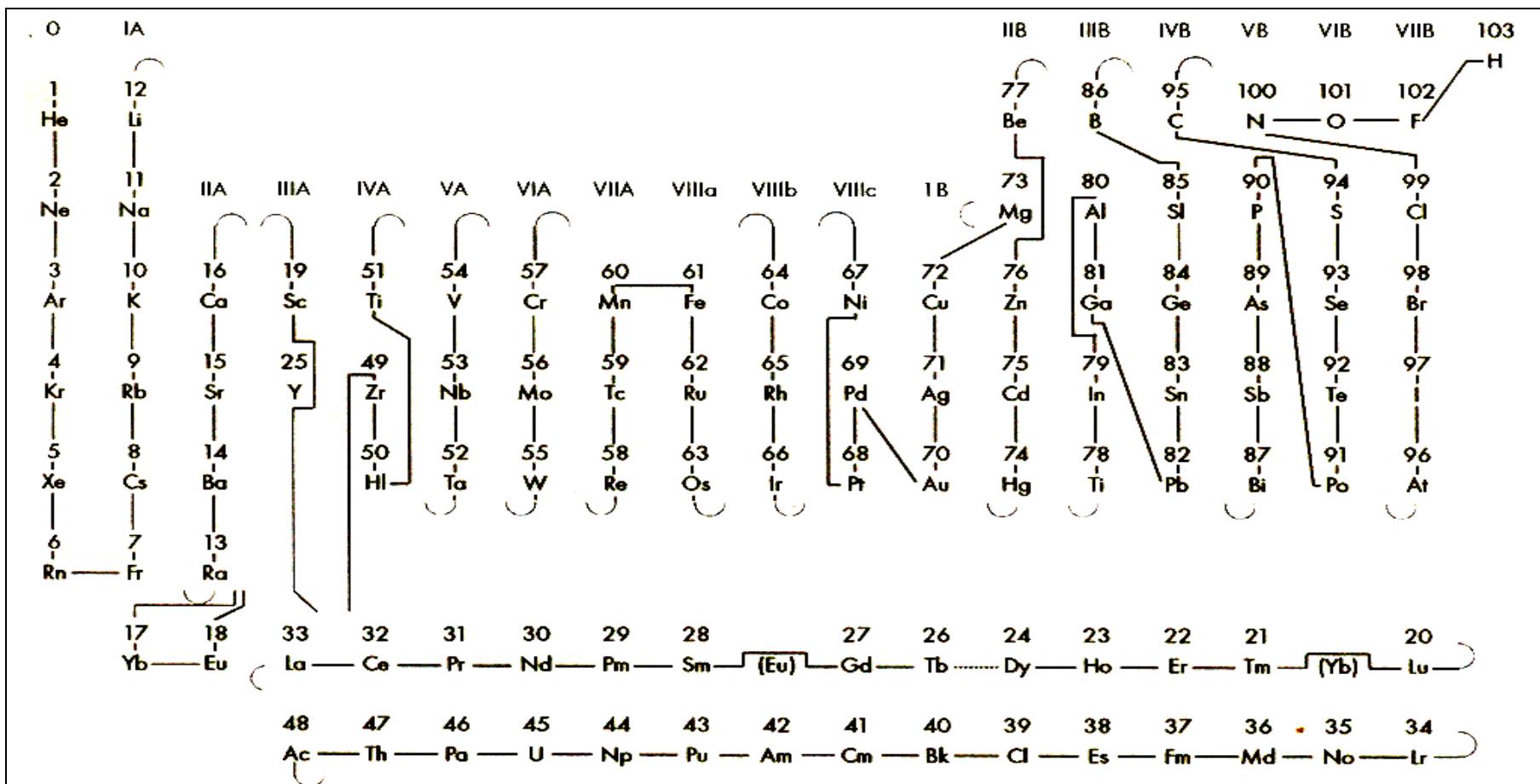
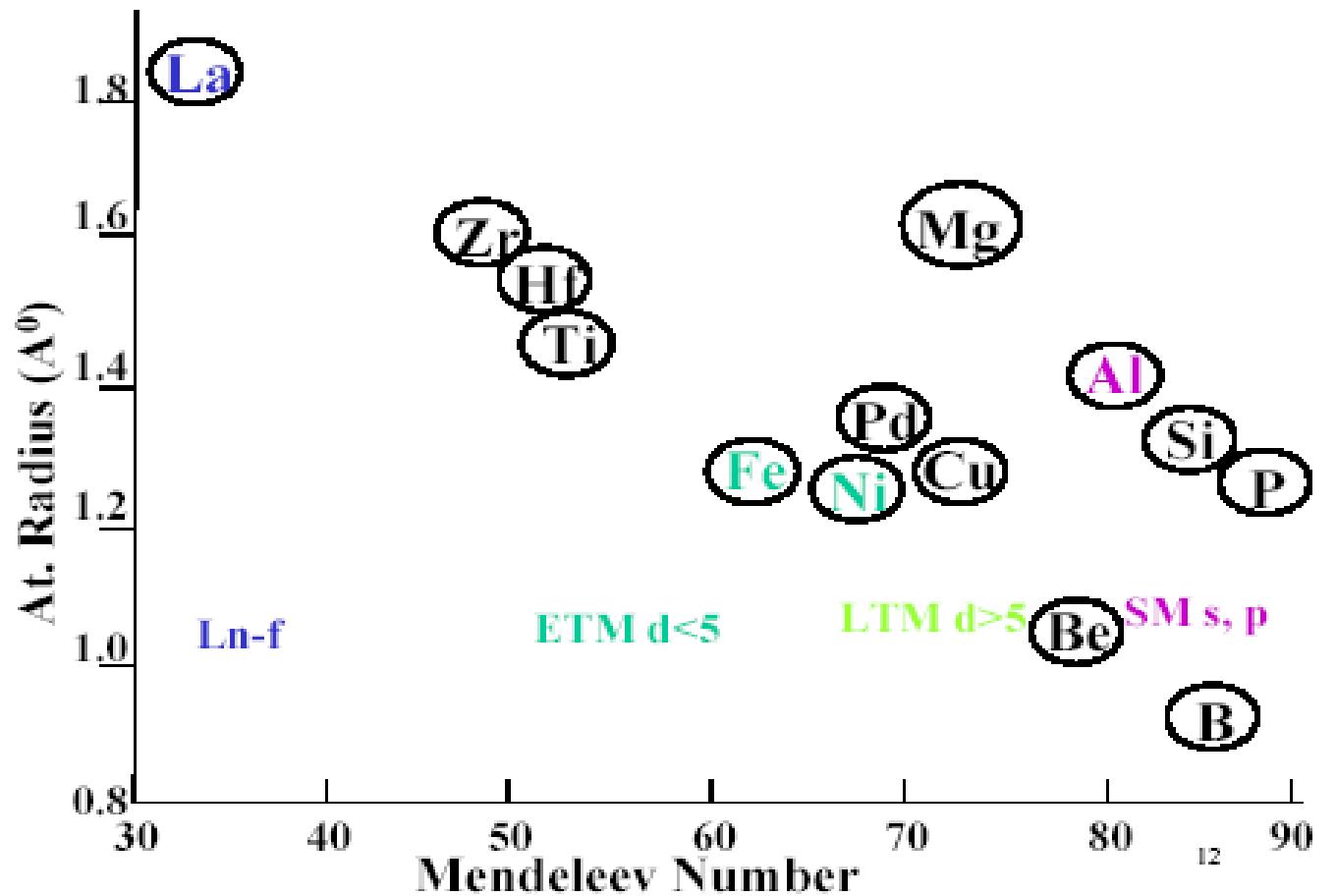
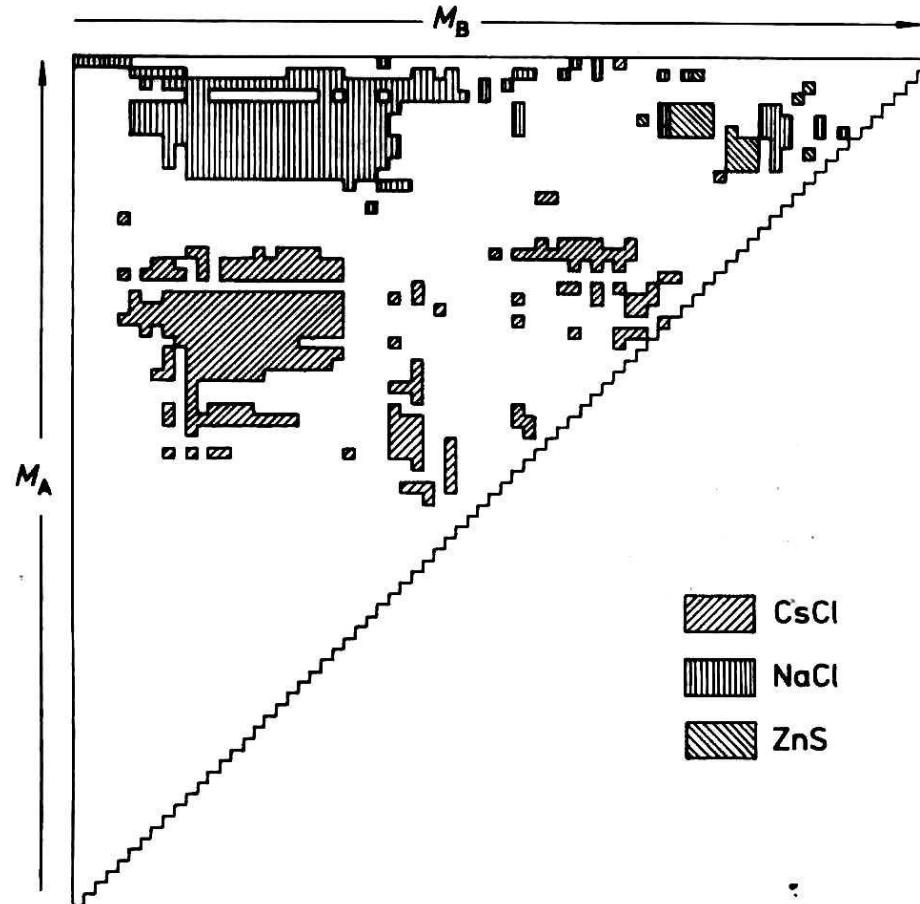


Figure 5: The string running through this modified periodic table puts all the elements in sequential order, thereby defining their Mendeleev number

Pettifor assignment of Mendeleev numbers to elements
by using a string through the modified Periodic Table

Size versus Mendeleev Number





Pettifor map with Mendeleev number as the discriminator for AB type binary compounds

AB STRUCTURE MAP

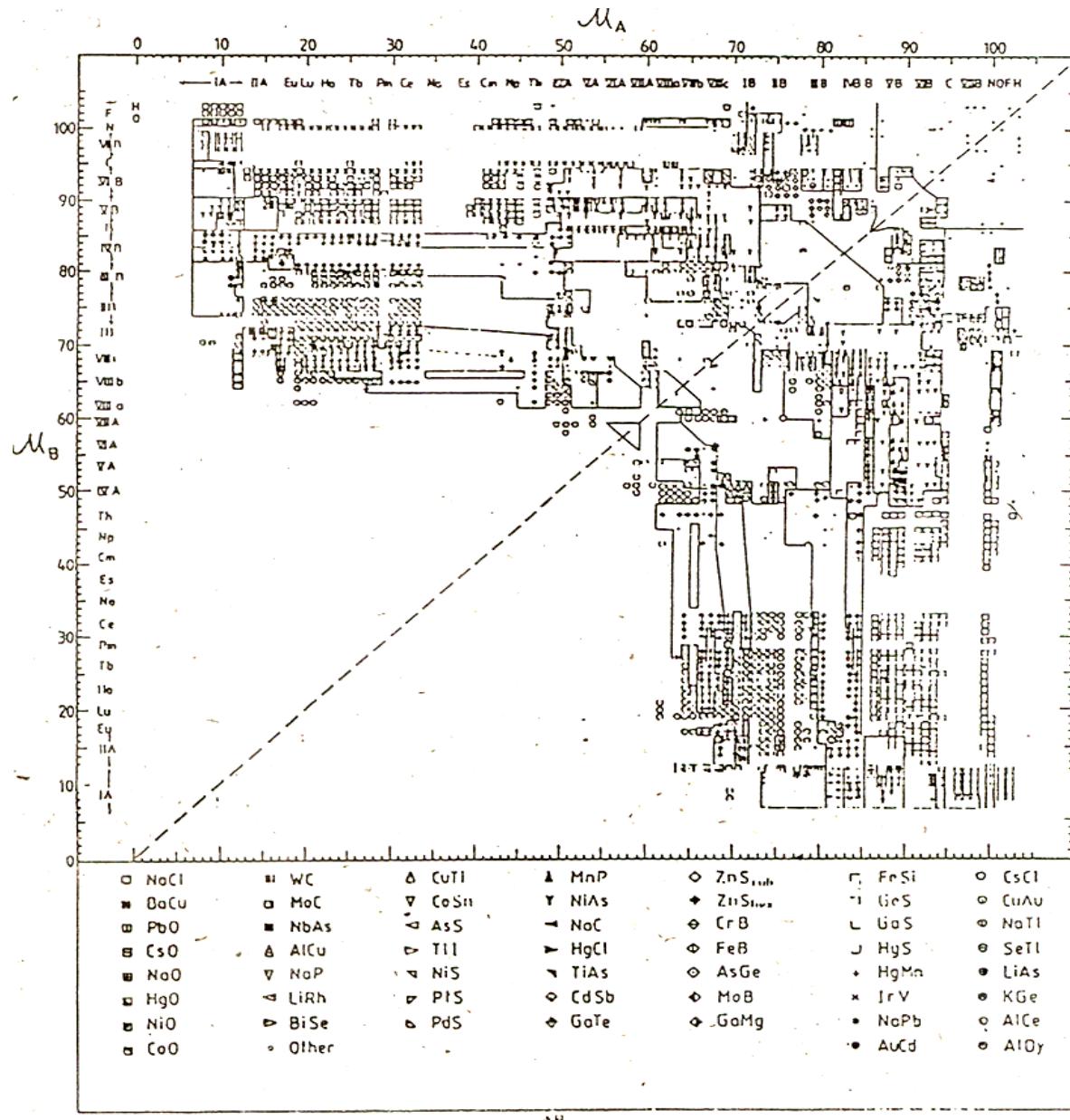


Figure 2-2. The AB structure map (Pettifor, 1988a).

Pseudo-Binary Intermetallics

If $A_x B_y$ is considered as the binary alloy, a quaternary alloy with constituent atoms as A, B, C and D may be treated as a pseudo-binary

$$(A_x C_{1-x})(B_y D_{1-y}).$$

The average Mendeleev numbers M^*_A and M^*_B are given by

$$M^*_A = x M_A + (1-x) M_C$$

$$M^*_B = y M_B + (1-y) M_D.$$

Successfully applied so far only to AB and AB_3 compounds.
Our analysis extend to
to $A_2 B_3$, $A_5 B_2$ AB_2 and AB_6 stoichiometry and related quasicrystals and Laves Phases

CLASSIFICATION OF QUASICRYSTALS :

Majority Component versus Large Atom

Alloy	Discovery	Year	Majority Component	Large Atom
Al-Mn 14	Shechtman	1984 (1982)	Aluminium	Aluminium
Mg 36-Zn38-Al25	Ramachandra Rao	1985	Zinc	Magnesium
Mg38-Zn15-Cu5-Al42	Ranganathan	1985	Aluminium	Magnesium
Ti-V-Ni	Kuo	1985	Titanium	Titanium
Ga-Mn	Tartas	1985	Gallium	Gallium
Al60-Li30-Cu10	Audier	1986	Aluminium	Lithium
Ti40-Zr40-Ni20	Molokanov	1990	Zirconium	Zirconium
Zn60-Mg30-RE10	Luo	1993	Zinc	Rare Earth
Zr-Ni-Cu-Al	Koester	1996	Zirconium	Zirconium
Hf-Ni-Cu-Al	Inoue	2000	Hafnium	Hafnium
Cd65-Mg20-RE15	Tsai	2000	Cadmium	Rare Earth
Cd-Yb	Tsai	2000	Cadmium	Ytterbium
Zn80-Mg5-Sc15	Ishimasa	2001	Zinc	Scandium
Cu48-Ga34-Mg3-Sc15	Ishimasa	2001	Copper	Scandium
Ag38-In38-Mg8-Ca16	Tsai	2001	Silver-Indium	Calcium

Rational Approximants to Quasicrystals

The Colouring Problem

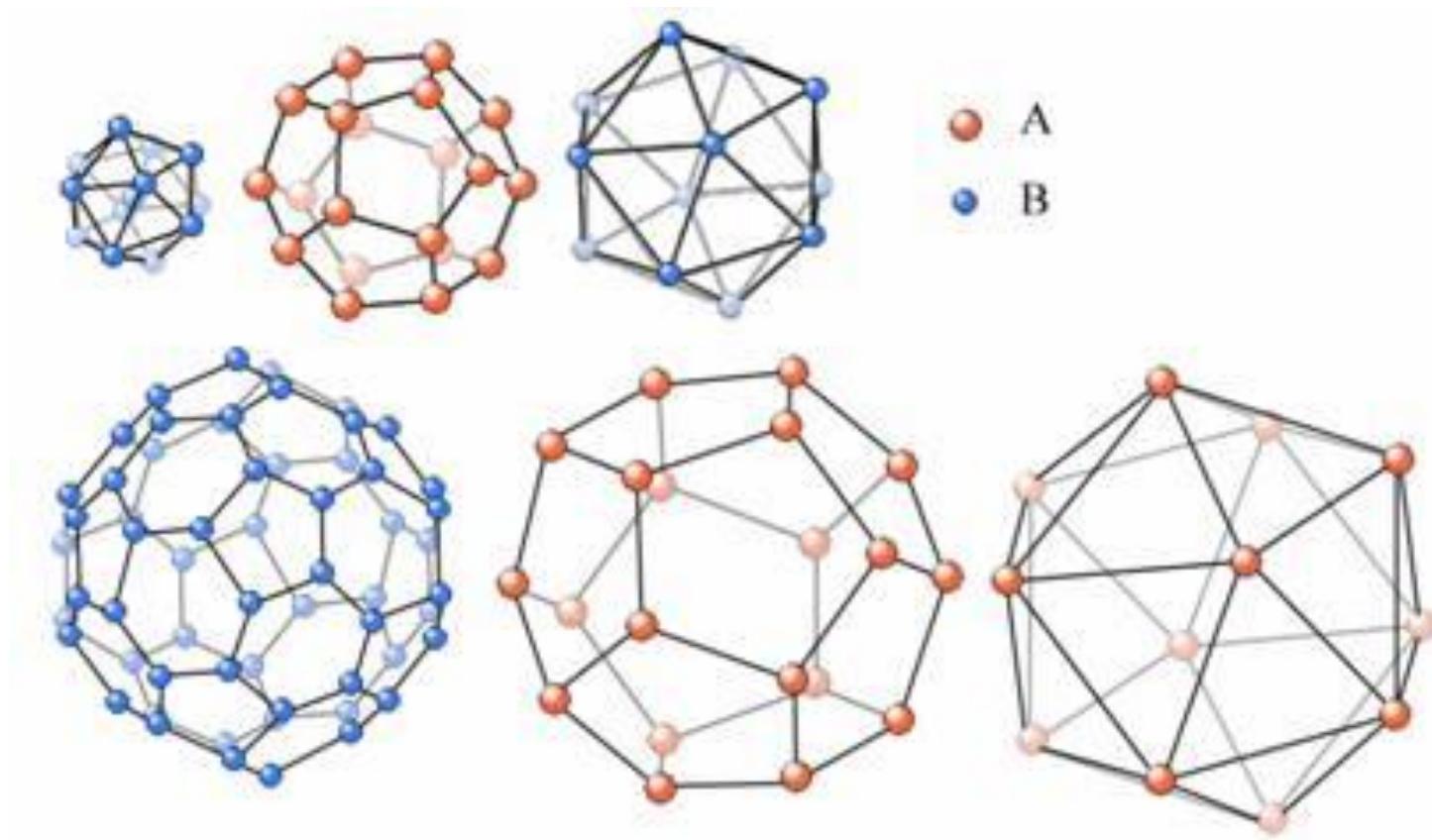
1. Bergman Approximant - Li, Mg

2 Mackay Approximant - Al, Ga

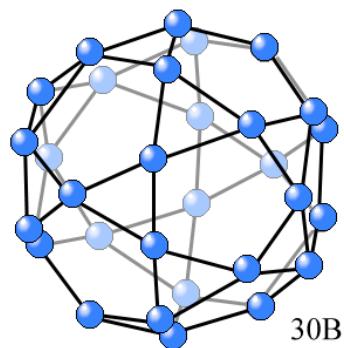
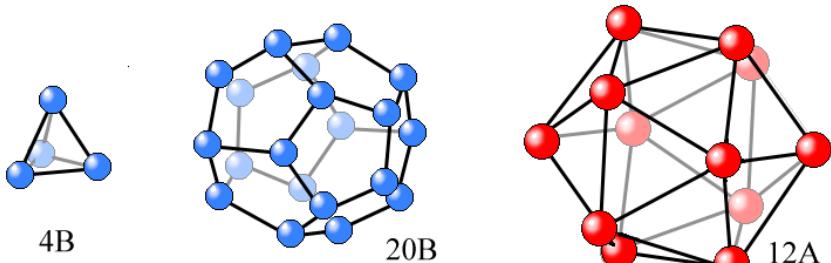
3 Kuo Approximant- Ti, Zr, Hf

4 Tsai Approximant- RE, Ca, Sc,

Bergman cluster

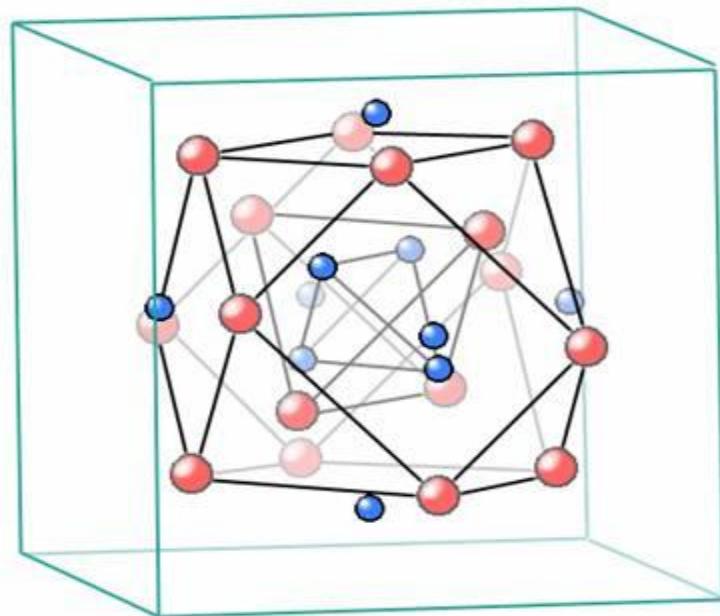


The Bergman Cluster with seven shells (117 atoms):
a central atom, surrounded by an icosahedron (12 atoms),
a dodecahedron (20 atoms), a second icosahedron (12 atoms),
a truncated icosahedron (60 atoms),
a dodecahedron (20 atoms) and an icosahedron (12 atoms).

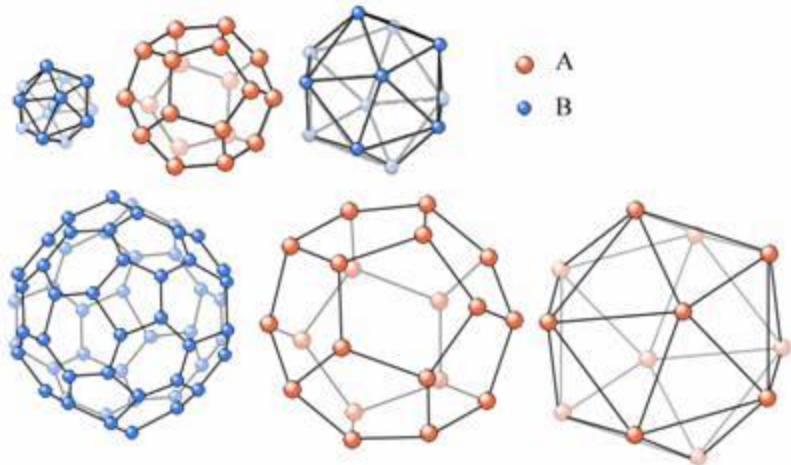
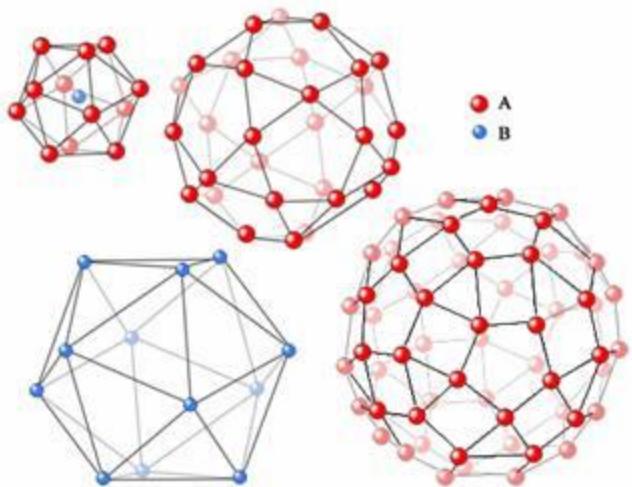


- A Large atoms
(Yb, Ca, Sc, RE)
- B Small atoms
(Zn, Cd)

Tsai



Kuo



Mackay

Acta Materialia, 2006

Bergman

QUASICRYSTALS AS PSEUDO-BINARY INTERMETALLICS

	Bergman Class	Mackay Class	Kuo Class	Tsai Class
Binary	---	Al-Mn	Ti-Ni, Zr-Pd	Cd-Yb, Cd-Ca
Ternary	Mg-(Zn, Al) Mg(Cu, Al) Mg(Zn, Ga) Li-(Cu,Al)	Al-(Cu,Fe) Al(Pd, Mn)	Ti-Zr-Ni Ti-Hf-Ni	Cd-Mg-RE Zn-Mg-RE Zn-Mg-Sc
Quaternary	Mg-(Cu,Zn, Al) Mg-(Zn, Al, Ga) (Li,Mg)- (Zn,Al)	(Al,Ga)-(Pd, Mn) Al-(Pd, V,Co)	Ti-Zr-Ni-Cu	Cu-Ga-Mg-Sc Ag-In-Mg-Cd (Zn, Mg)- (RE1,RE2)

1: 2 Stoichiometry & R ` 1.225

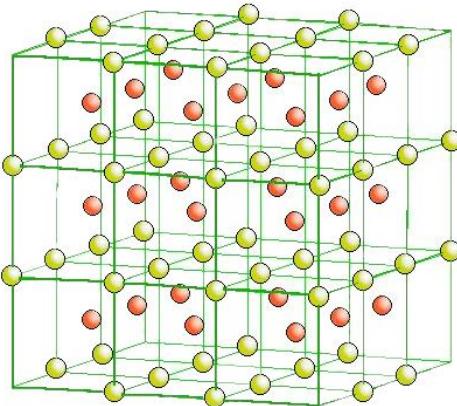
Laves Phase A B₂

Anti-Laves Phase A₂B (Giessen glasses)

**Atomic environment Laves phase CN 16 and CN12
anti-Laves phase CN 15 and 10**

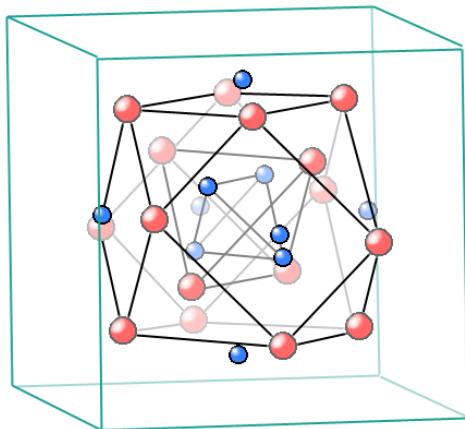
Average coordination number of both is 13.33!

**The supertetrahedron as a common building block
Dong -2004**

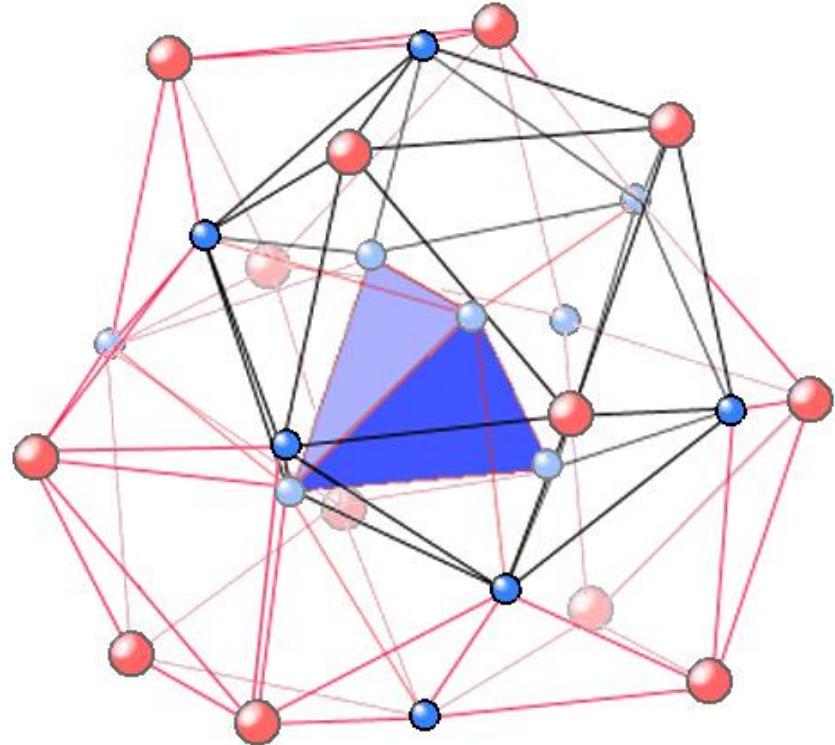


Gamma brass

1928 Bradley & Thewlis
Modified B2



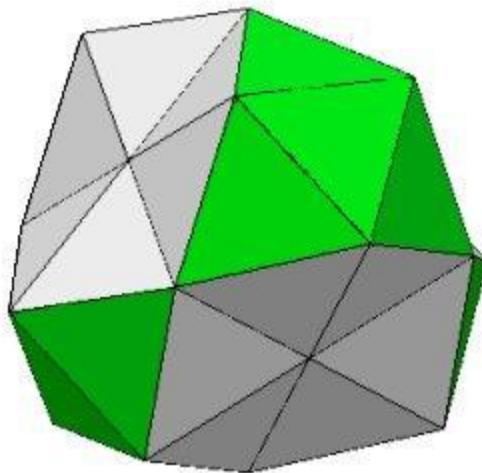
1933 Bradley & Jones
Nested polyhedra



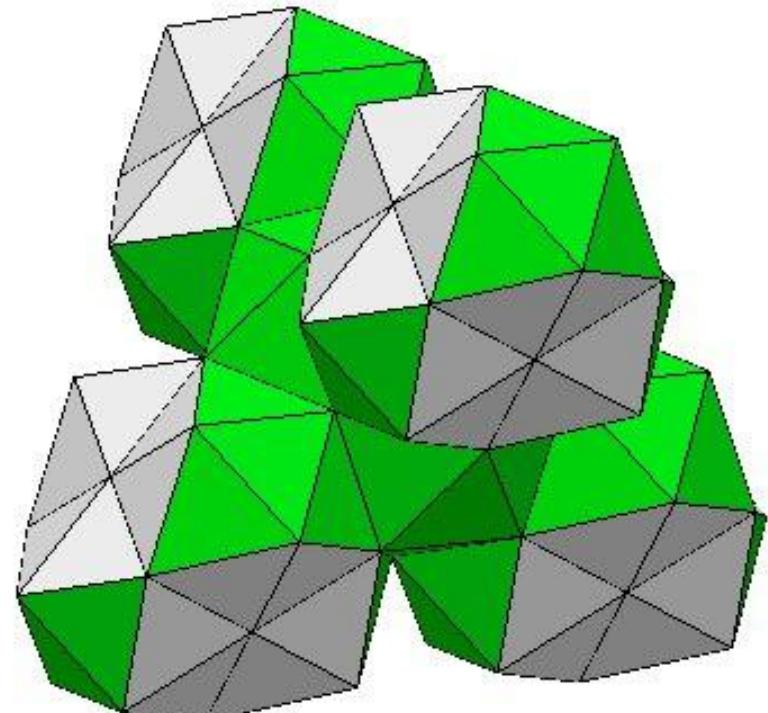
Supertetrahedron
Four interpenetrating icosahedra

Anti-Laves phase

A_2B (Ti_2Ni , Zr_2Ni etc)



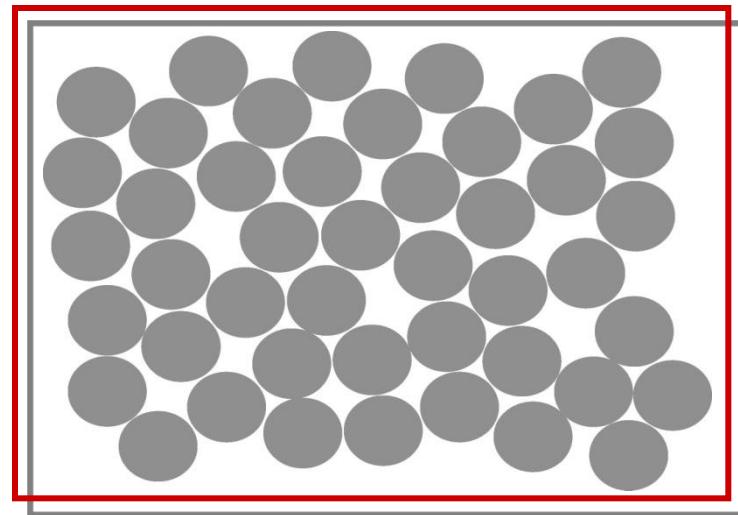
Supertetrahedra sharing
atoms to form a
'diamond-type' network



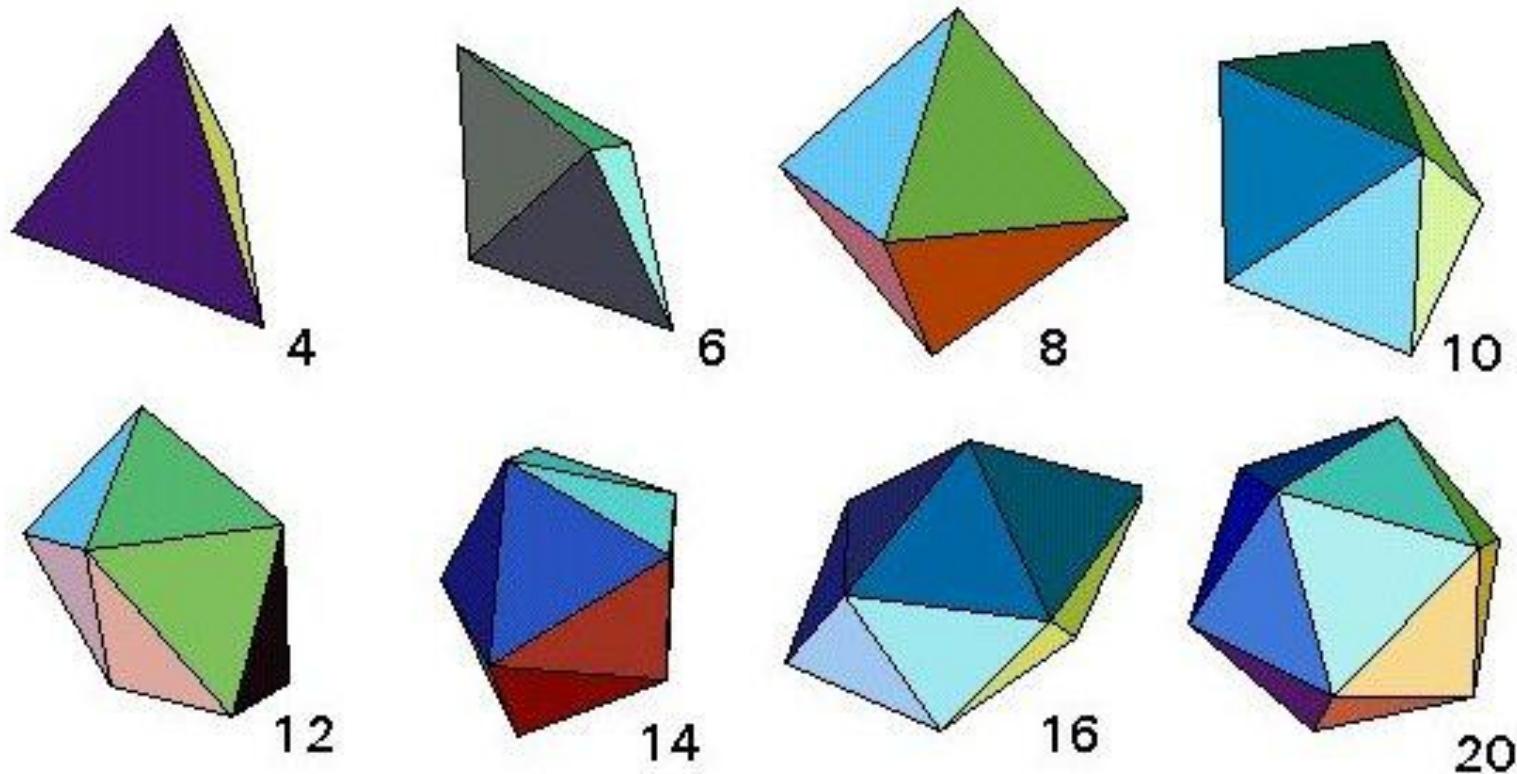
DENSE RANDOM PACKING

A Statistical Model

- A dense random packed structure of equal-sized spheres is characterized by:
 - a packing fraction of 0.6366
 - frequently observed specific local atomic clusters
 - tetrahedra, half-octahedra, trigonal prisms, Archimedian antiprisms, tetragonal dodecahedron
 - the absence of medium-and long-range order



Miracle 2004

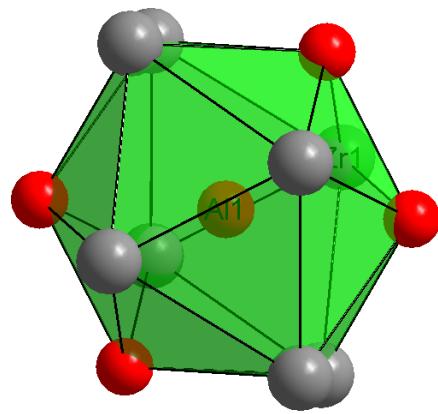


The eight Bernal convex deltahedra

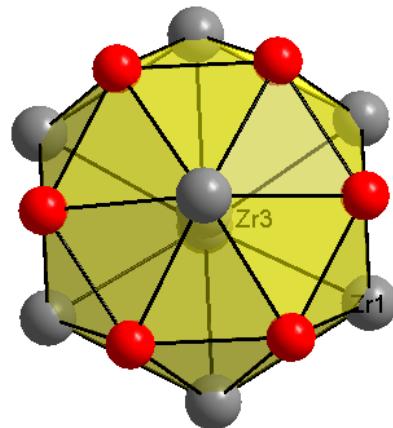
Bernal; *Nature*, 185, (1959)

FRANK-KASPER PHASES

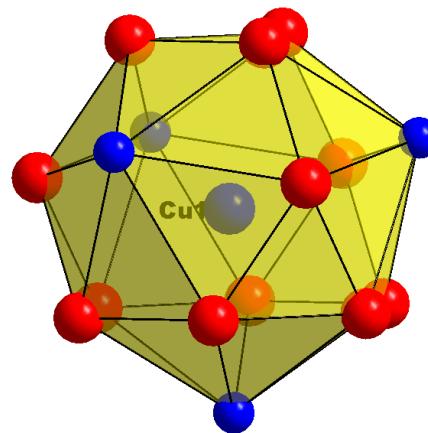
**CN
12**



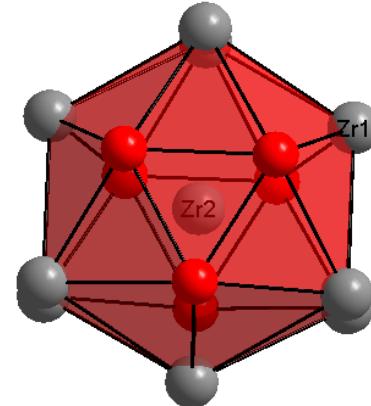
**CN
14**

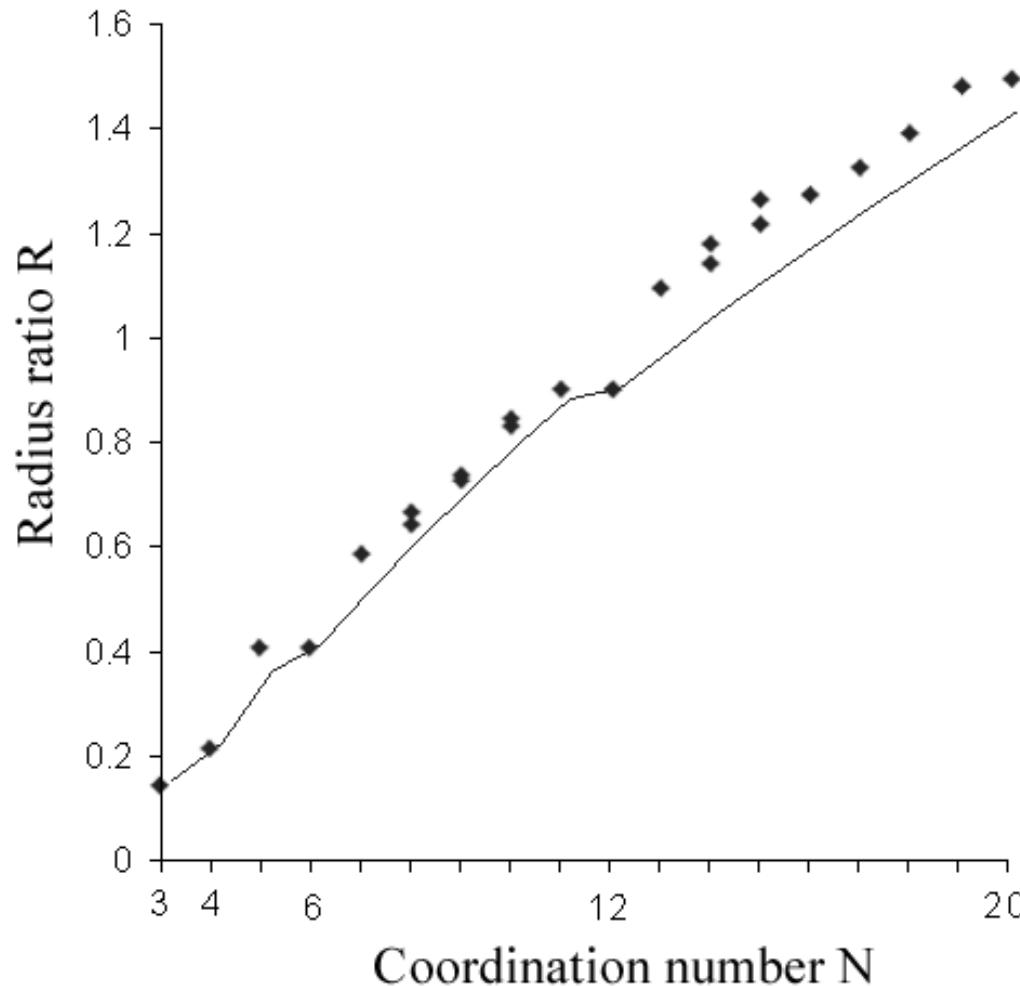


CN 16



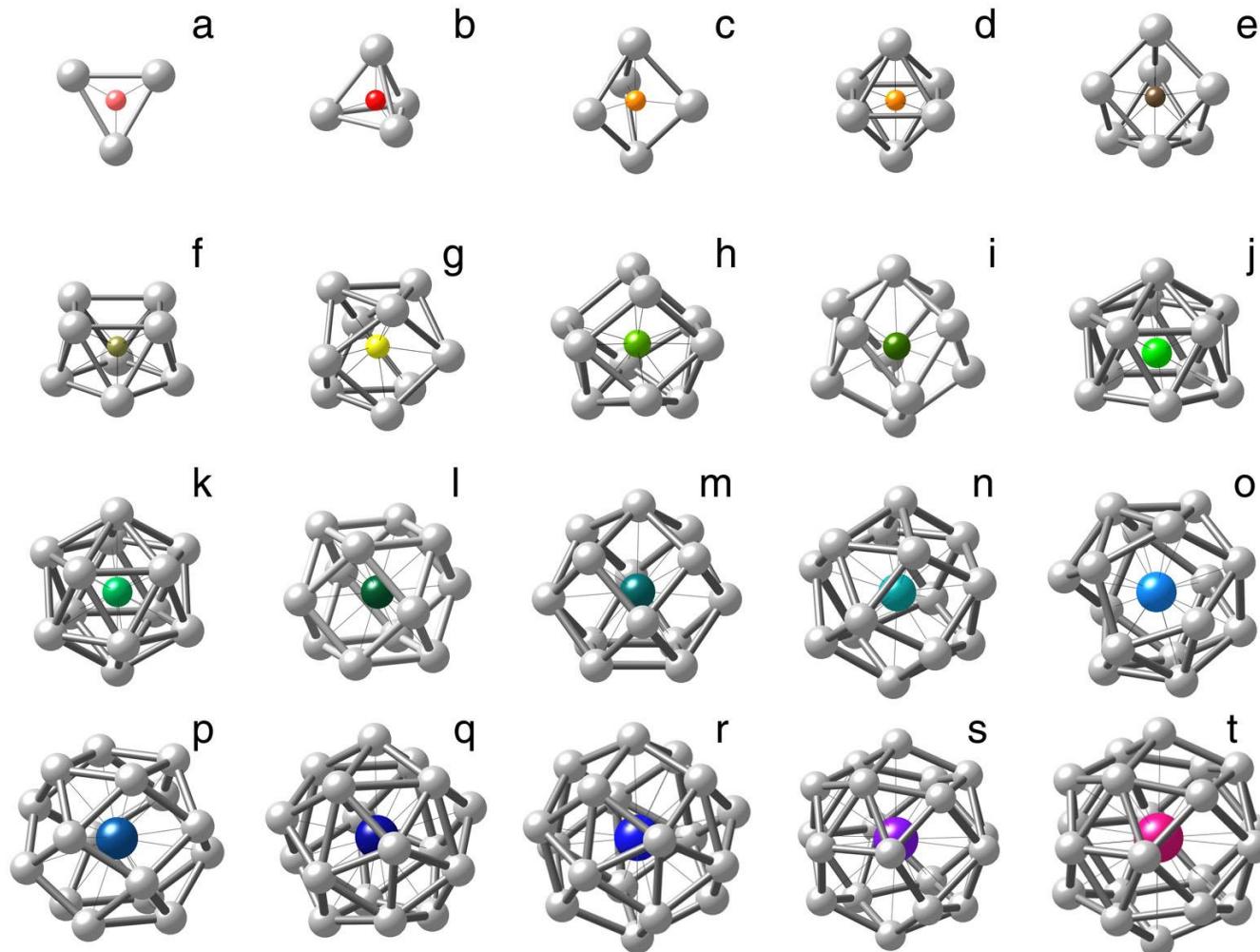
**CN
15**





Radius ratios of hard sphere clusters with $3 \leq N \leq 20$
The solid line is an idealized relation from Miracle et al (2003)
shown for comparison

CANDIDATE EFFICIENTLY PACKED ATOMIC CLUSTERS



Miracle, Lord and Ranganathan; “Candidate Atomic Cluster Configurations in Metallic Glass Structures”

STRUCTURAL MODELS

□ β and γ solutes form efficiently packed clusters with Ω atoms only in the 1st coordination shell

- these clusters overlap with α clusters in the 1st coordination shell
- β and γ clusters form regular arrays within the DCP structure

Miracle, D. B. *Nature Mater.* **3**, 697–702 (2004).

<12-10-9> Glass



<17-12-10> Glass



Analysis of Composition Dependence of Formation of Ternary Bulk Metallic Glasses from Crystallographic Data on Ternary Compounds

A. Takeuchi^a, S. Ranganathan^b, B.S. Murty^c and A. Inoue^a

^a Institute for Materials Research, Tohoku University, Sendai 980-8577, Japan

^b Department of Metallurgy, Indian Institute of Science, Bangalore 560 012, India

^c Dept. Metal. Mater. Eng., Indian Institute of Technology, Madras, Chennai 600 036, India

Audience Hall, Bulk Metallic Glasses (1), August 28, 2006, 10:15~10:30

Outline

1. Introduction

1-1. Early studies on stabilization of glassy phase, bulk metallic glasses (BMGs)

1-2. Purpose

2. Methods

2-1. Seven classes of BMGs (C-1~C-7): chemical species

2-2. Three types of BMGs (L-, M-, S-type): relative atomic size

of the main constituent (Large, interMediate, Small) in ternary alloys

2-3. **L-M-S composition diagram**

2-4. Data source

3. Results and Discussion

3-1. L-M-S composition diagram for ternary compounds

3-2. Representative compounds

3-3. L-M-S composition diagram for BMGs and representative compounds

4. Summary

(2) Stabilization of glassy phase ← stoichiometry of compounds

- W. Hume-Rothery and E. Anderson, Phil. Mag., 5 (1960), 383-405.

$A:B = 1:6, 1:3, 1:2, 2:3$ ← eutectic compositions ← binary phase diagram by Hansen (1958))

$A_{12}B$ ← Frank icosahedral unit

- R.St. Amand and B.C. Giessen, Scripta Metall., 12 (1978) 1021-1026.

Ca-based metallic glasses: ← anti-Laves composition ← A_2B (Laves phase)

Absence of systematic researches on the formation of
ternary BMGs based on crystallographic data

Ternary phase
diagram

Crystallographic data on ternary compounds

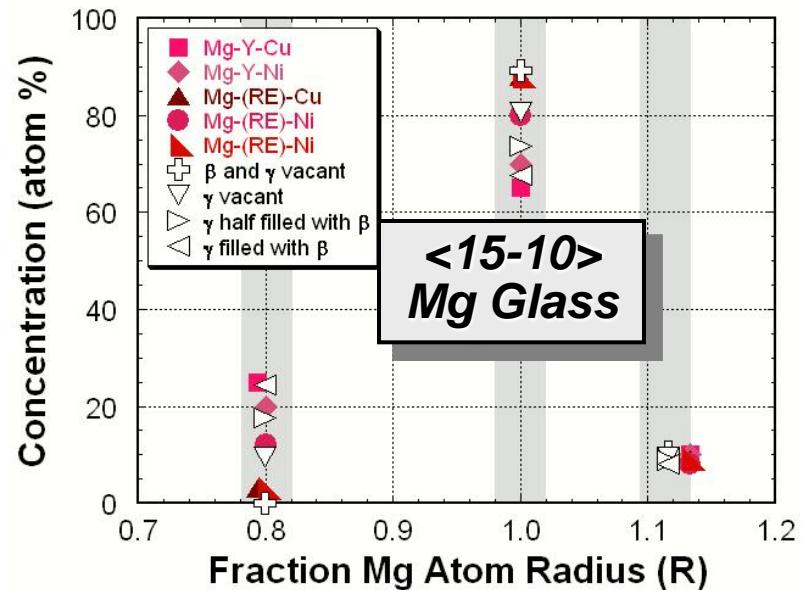
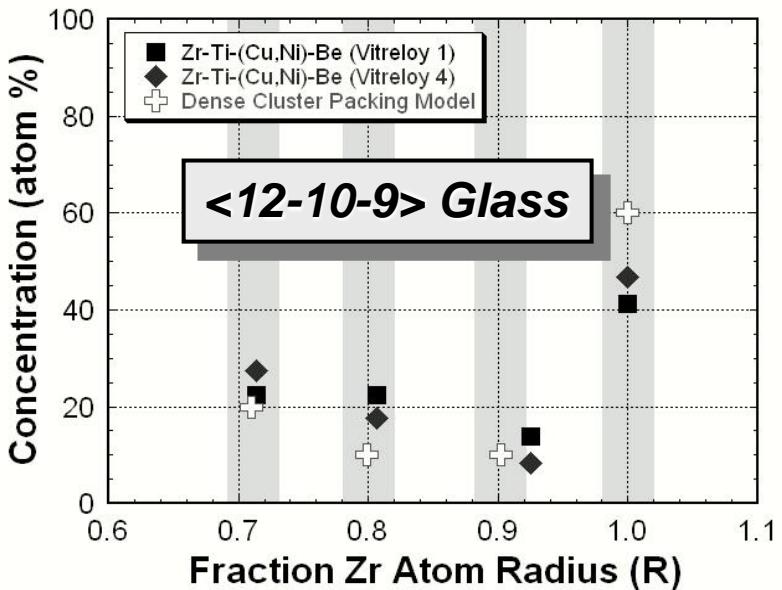
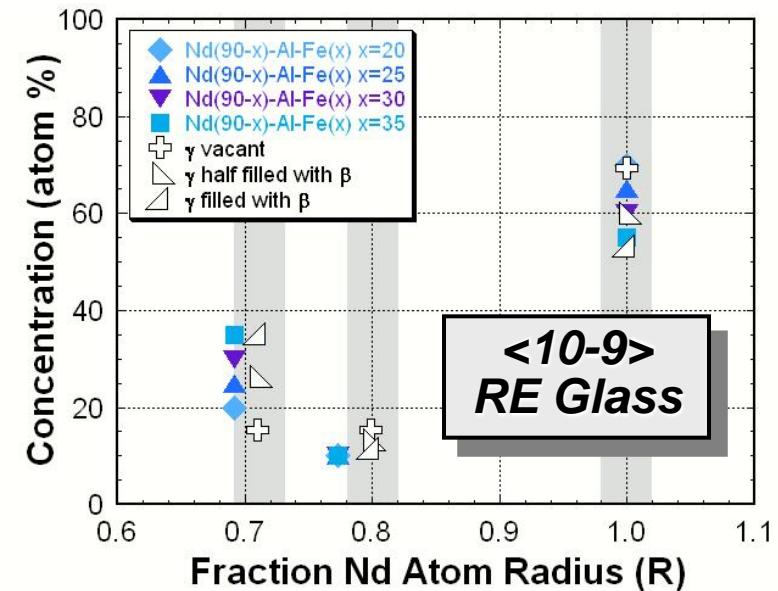
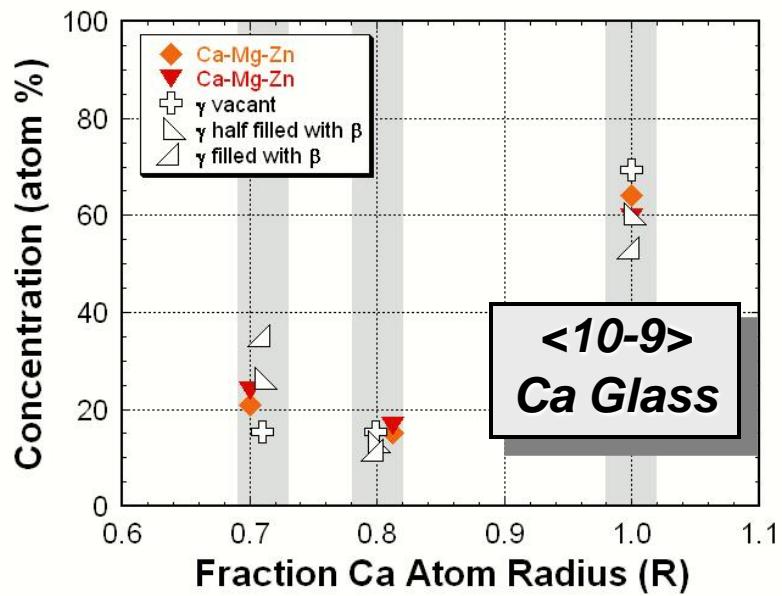
- crystalline structure (local atomic arrangements of BMGs)
- stoichiometry of compounds

Classification of BMGs: 2-1, 2-2

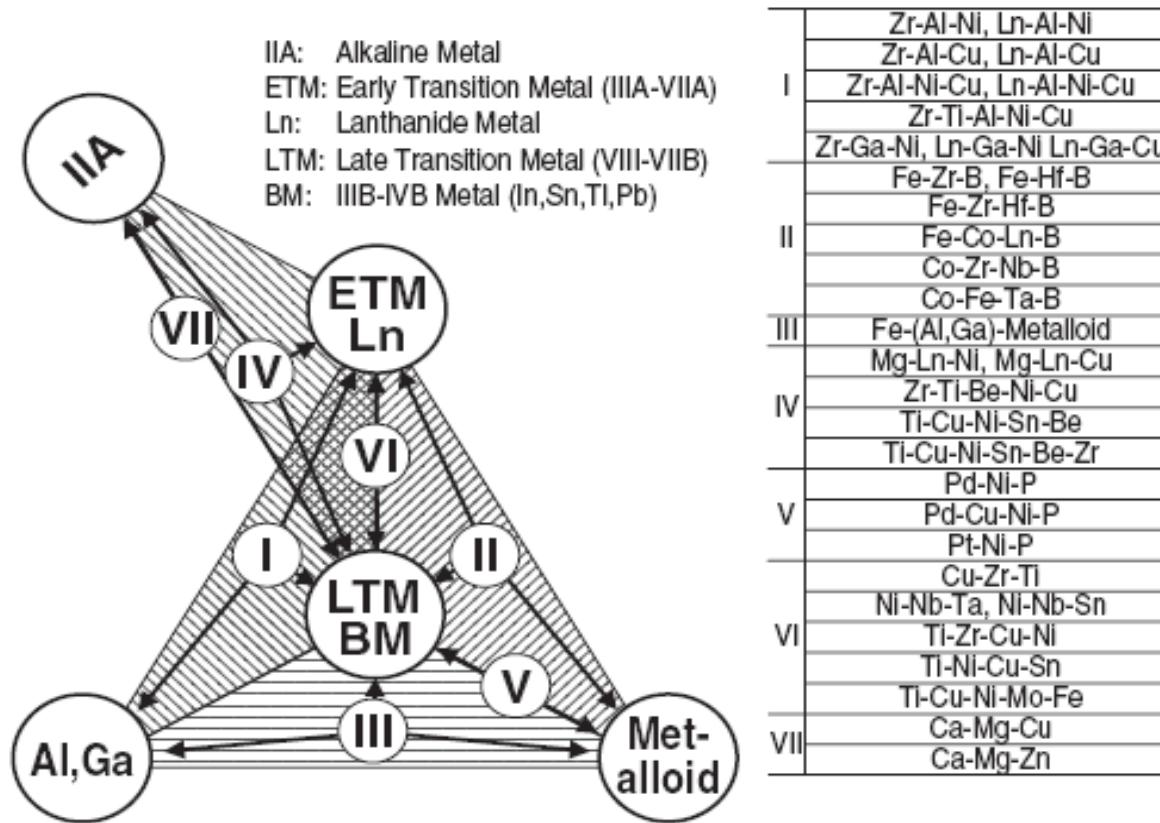
To identify general trends of composition
dependence of formation of ternary BMGs

ALLOY COMPOSITIONS

Ca, RE, Zr, Mg Glasses



Classification of Bulk Metallic Glasses



A Inoue Acta Mater 2001

Five chemical types, Five ternary groups

A Takeuchi & A Inoue Mater Trans 2005

Five slightly different chemical types, Seven groups

2 . Methods

2-1. Seven classes of BMGs (C-1~C-7)* : combinations of class of constituents

Class of BMGs

*A. Takeuchi and A. Inoue: Mater. Trans., 46 (2006), 2817-2829.

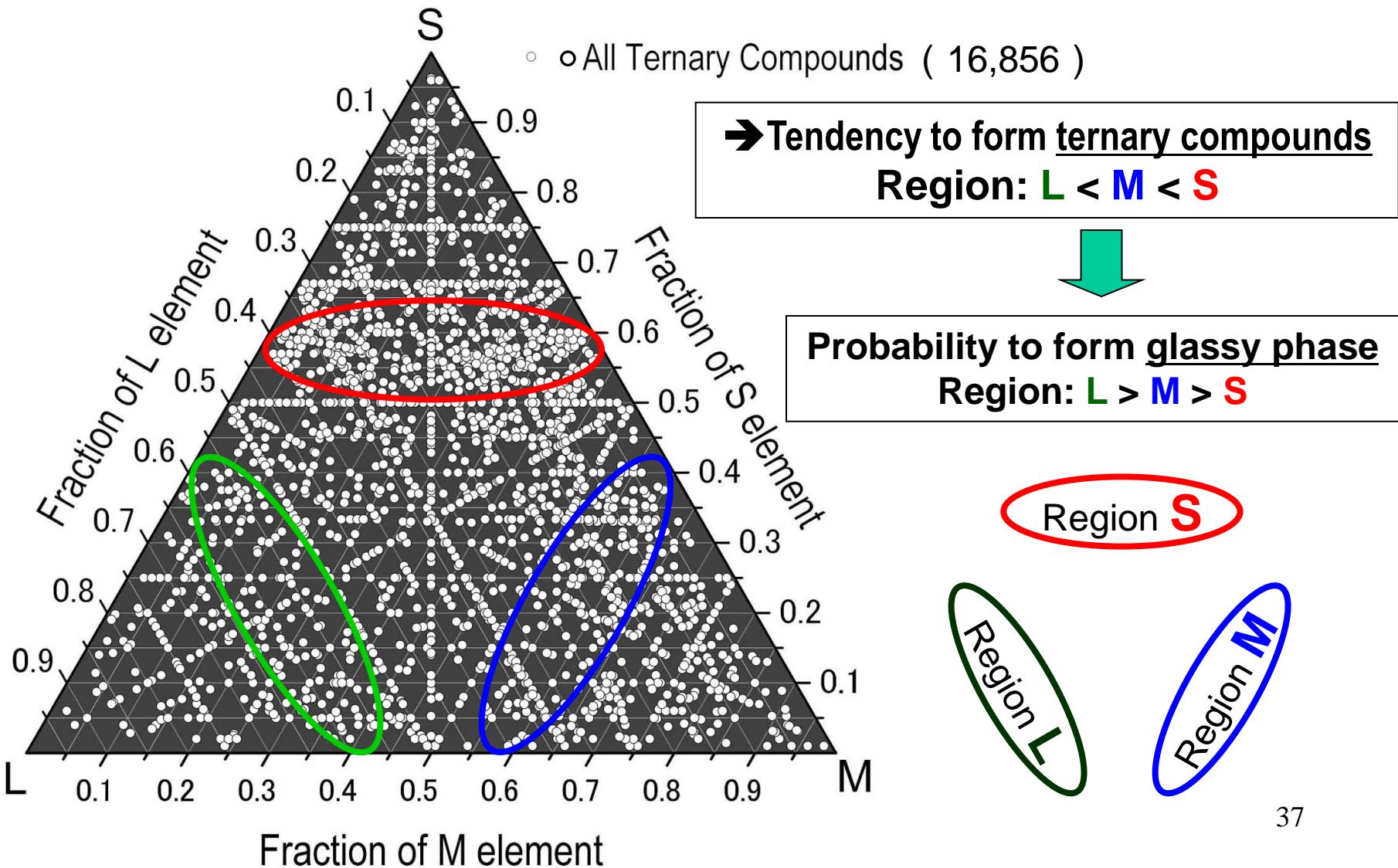
Class	Representative alloy system	Combinations of class of constituents
C-1	La-Al-Ni, Zr-Al-Ni	(ETM,Ln)-(Al,Ga)-(LTM,BM)
C-2	Fe-Zr-B	(LTM,BM)-(ETM,Ln)-(Metalloid)
C-3	Fe-(Al,Ga)-Metalloid	(LTM,BM)-(Al,Ga)-(Metalloid)
C-4	Mg-Cu-Y	(IIA)-(LTM,BM)-(ETM,Ln)
C-5	Pd-Ni-P	(LTM,BM)-(Metalloid)
C-6	Cu-Zr-Ti	(LTM,BM)-(ETM,Ln)
C-7	Ca-Mg-Cu	(IIA)-(LTM,BM)

Class of constituents

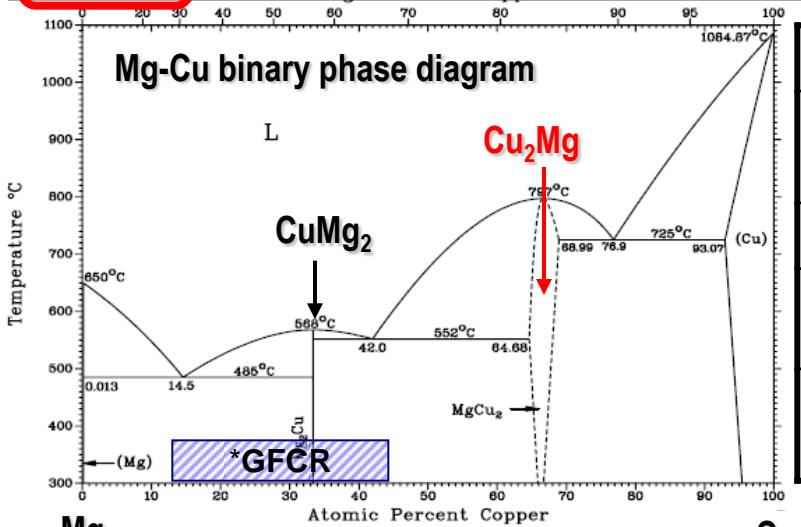
IIA:	Alkaline Earth Metal	(Be,Mg,Ca)
ETM:	Early Transition Metal_(IIIA-VIIA)	(Zr,Ti,Nb,...)
Ln:	Rare Earth Metal	(La,Nd,Gd,...)
LTM:	Late Transition Metal_(VIII-VIIB)	(Fe,Co,Ni,...)
BM:	B-group Metal(IIIB-IVB)	(In,Sn,Pb,...)
Metalloid:		(P,C,B,Si,Ge)
Al,Ga:		36

3 . Results and discussion

3-1. L-M-S composition diagram for ternary compounds



Cu_2Mg : Laves /Anti-Laves relationship



*GFCR from "Guide for amorphous alloy formation",
eds. U. Mizutani, Agne Tech. Center (1987).

Compound	CuMg_2	Cu_2Mg
Crystal structure, Feature	Orthor ombic	A prototype of Cubic Laves-phase , High melting temp.
GFCR*	Inside	Outside
The No. comp. in ternary system	5	474
Description with atomic size**	SL_2	S_2L

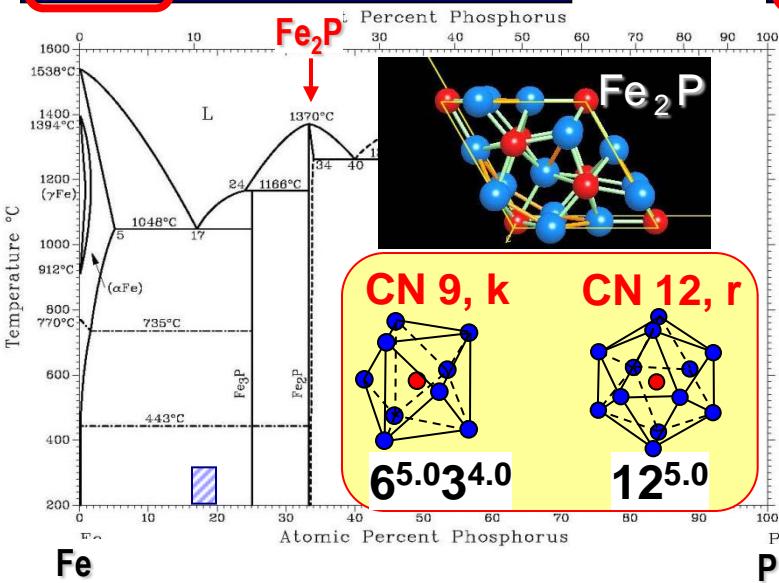
CN 12, r

125.0

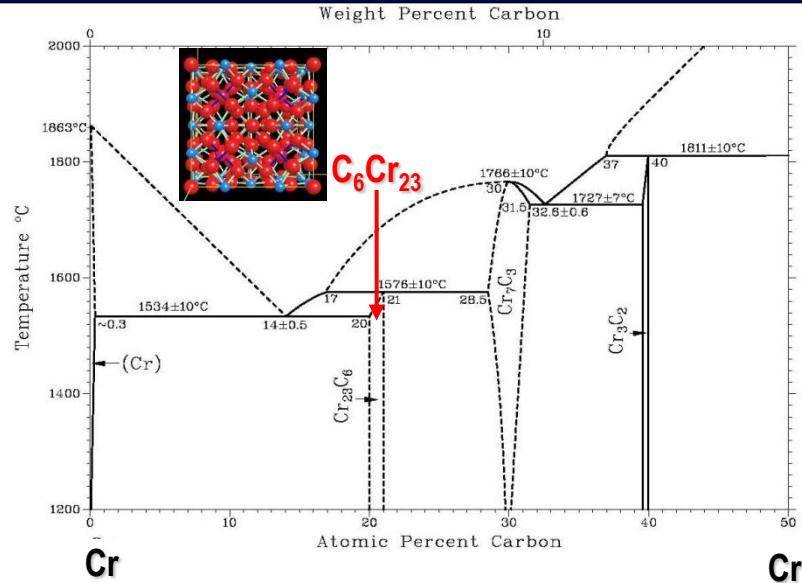
* GFCR: Glass-Forming Composition Region

**Atomic radius : Mg(0.160 nm), Cu(0.128 nm), "Metal Databook", Marzen (2004)

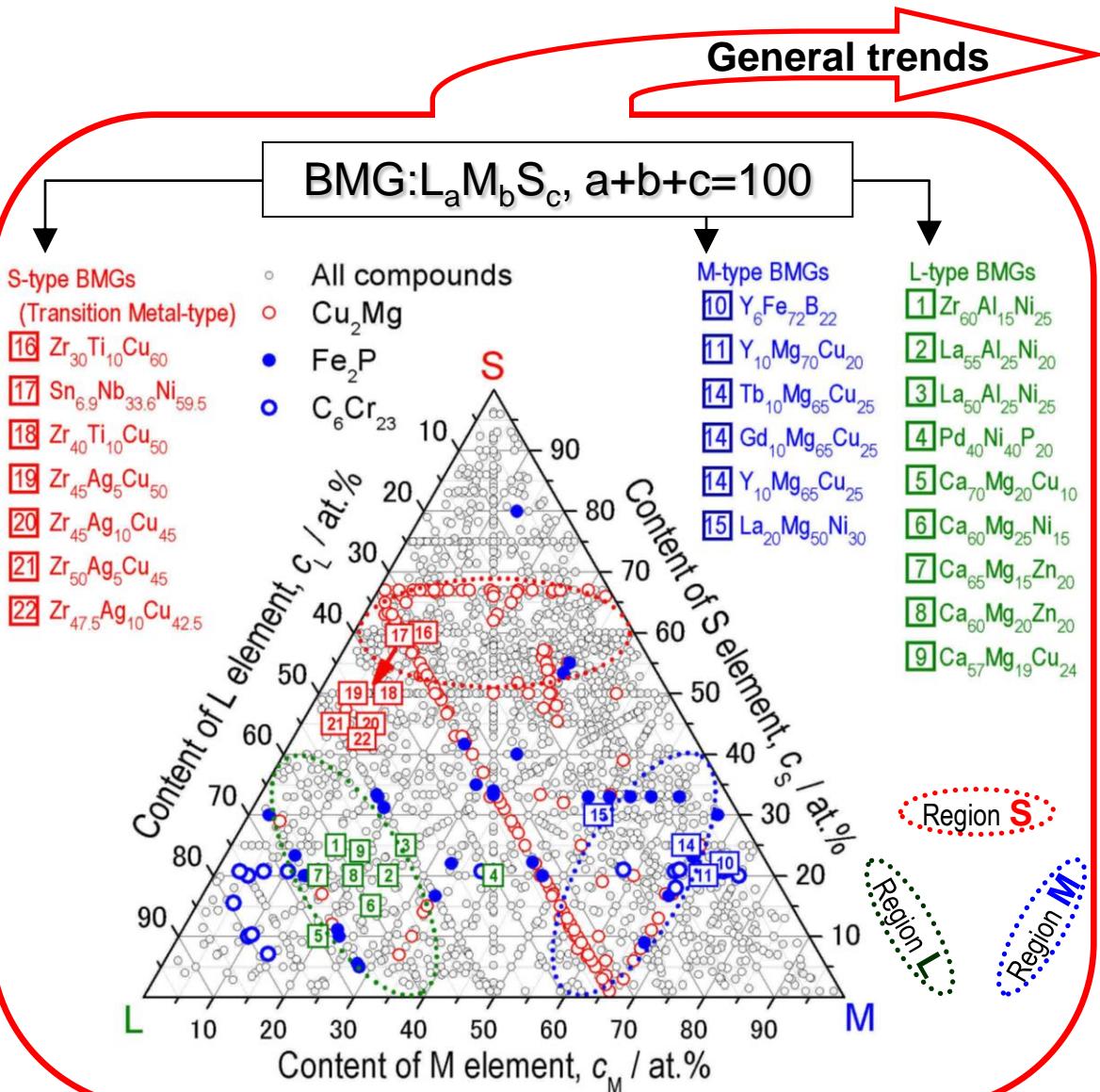
Fe_2P Metal-Metalloid



C_6Cr_{23} local atomic arrangements for Fe-based BMGs



3-3. L-M-S composition diagram for BMGs and representative compounds



(1) Ternary BMGs compositions

L;M-type BMGs (□○)

- Independent to Cu₂Mg (○)
- BMGs content: $c_S \sim 20$ at.%

L-type BMGs (□)

- Formation: Region L ~ L₆₀M₂₀S₂₀
- anti-Laves relation to Cu₂Mg

M-type BMGs (□)

- relate to Fe₂P,C₆Cr₂₃ (●○)
- Formation: edge of Region M, M-S side (~L₁₀M₇₀S₂₀)

S-type BMGs (□)

- [16,17] overlap with Cu₂Mg (○) in Region S
 - low glass-forming ability (GFA)
 - shift of main constituent (from S-[16,17] to L-[18-22]-rich side)*
 - increase in GFA
- Formation: Region S, $c_M \sim 10$ at. %

(2) Ternary compounds compositions

Fe₂P,C₆Cr₂₃ (●○) at around Region L

- Influence to L-type BMG formation

4. Summary

1. Introduction: Stabilization of glassy phase ← eutectic reaction
← local atomic arrangements, stoichiometry of compounds
2. Methods: Seven classes of BMGs (C-1~C-7) and three types of BMGs (L-,M-,S-type),
L-M-S composition diagram, Crystallographic data from ternary phase diagrams

3. Results and Discussion

3-1. L-M-S composition diagram for ternary compounds

→ ternary compounds tend to form in S-rich corner (Region S) ←

3-2. Representative compounds → (1) Cu_2Mg , (2) Fe_2P and (3) C_6Cr_{23}

3-3. L-M-S composition diagram for BMGs and representative compounds

S-type (transition metal) BMGs: → influenced by Cu_2Mg → low-glass forming ability ←

(ex: Cu-based BMGs) → shift of main constituent in atomic size from S to L

→ Formation: Region S, L-S side ($c_M \sim 10$ at. %)

M-type BMGs: → independent to Cu_2Mg , influenced by Fe_2P and C_6Cr_{23}

(ex: Fe- and Mg-based BMGs) → Formation: edge of Region M: M-S side ($\sim \text{L}_{10}\text{M}_{70}\text{S}_{20}$)

L-type BMGs: most stable BMGs → (1) anti-Laves relationship to Cu_2Mg

(ex: La-, Zr-, and Ca-based BMGs) → (2) ternary compounds in Region L: infrequent

→ somewhat affected by Fe_2P and C_6Cr_{23}

→ Formation: Region L: around $\text{L}_{60}\text{M}_{20}\text{S}_{20}$)



$c_S \sim 20$ at. %

Chemical identity on the basis of Mendeleev Numbers

6 types based on orbitals and MN

New types
f electrons

Metal (MN)
Ca (16), Sc(19)
Lu (20).. Dy(24), Y (25)
Gd(27), Ce(32), La (33)

Takeuchi & Inoue

d electrons (ETM)

Zr(49), Hf(50), Ti(51), Ta(52), Nb(53)

Ln

+
ETM

d electrons (LTM) **Fe(61), Co(64), Ni(67), Pt(68), Pd(69)** LTM + Sn
Au(70), Ag(71), Cu(72)

Mg (73), Zn(76), Be (77)

Be, Mg, Ca

sp electrons metallic

Al (80), Ga (81), Sn (83)

Al, Ga

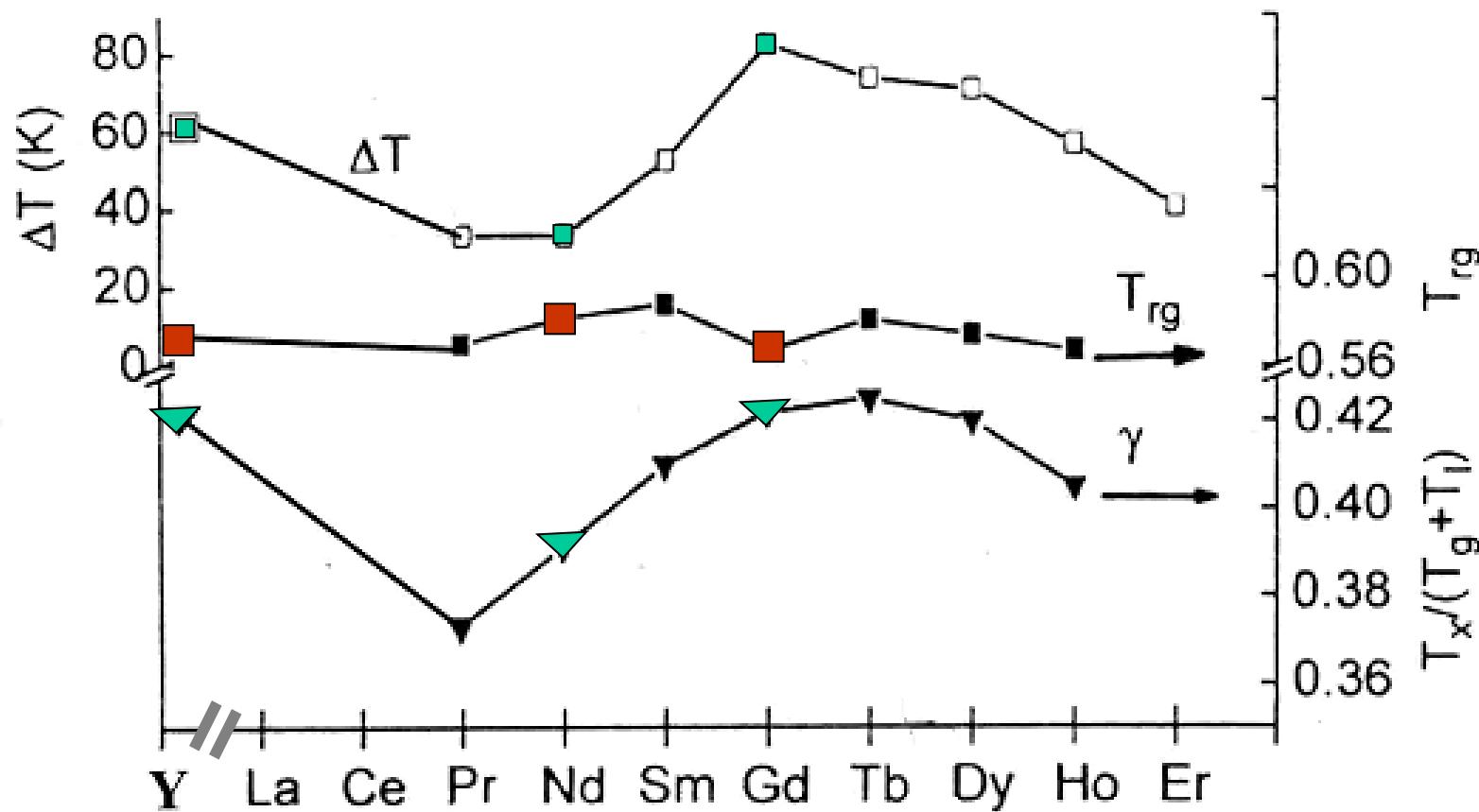
sp electrons metalloid

Ge (84), Si (85), B (86), P (90), C (95) Metalloid

Takeuchi & Inoue ANMM Sept 2005

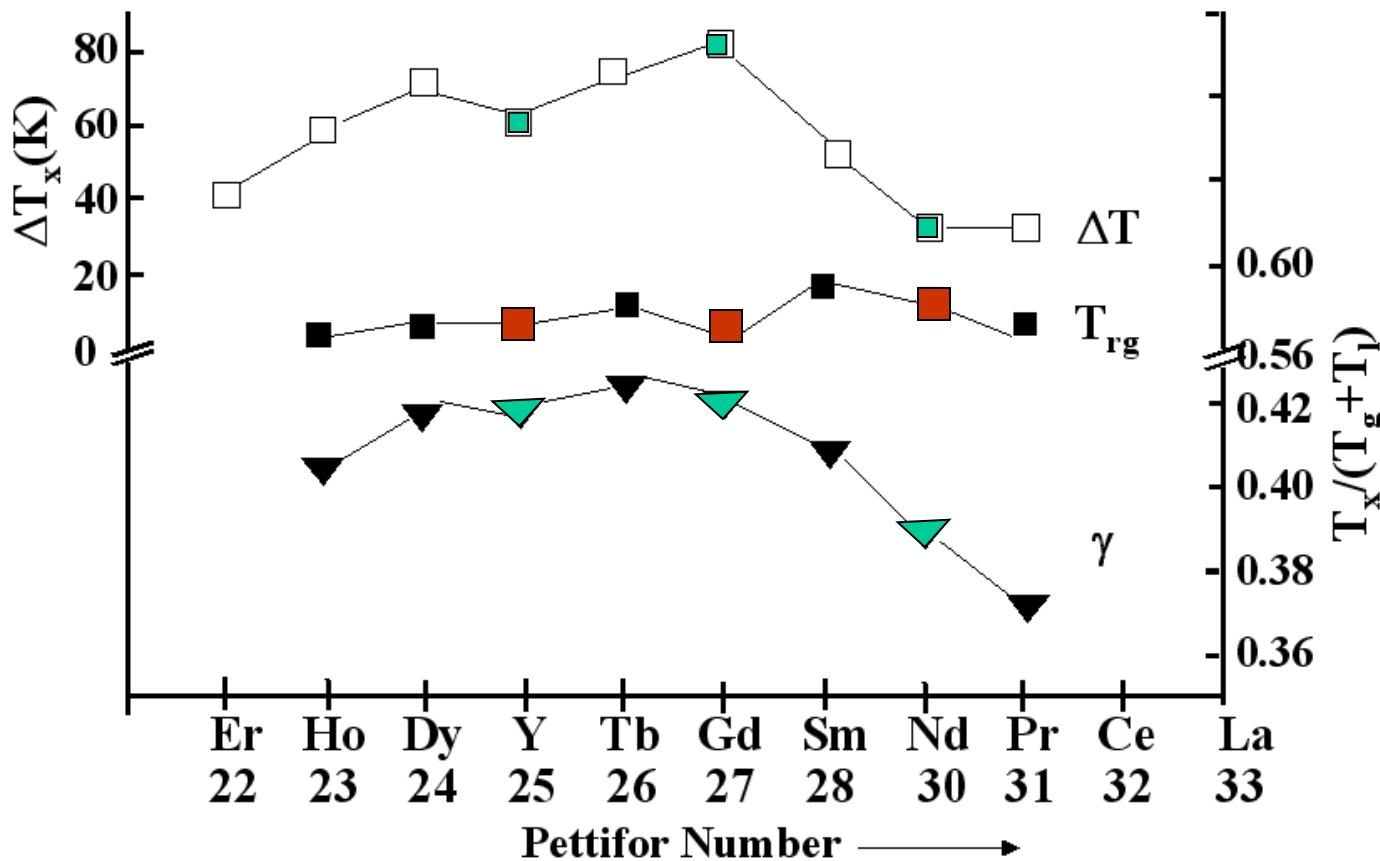
Ranganathan, Murty, Inoue & Takeuchi , 2006 in progress⁴¹

$\text{Mg}_{65}\text{Cu}_{25}\text{RE}_{10}$

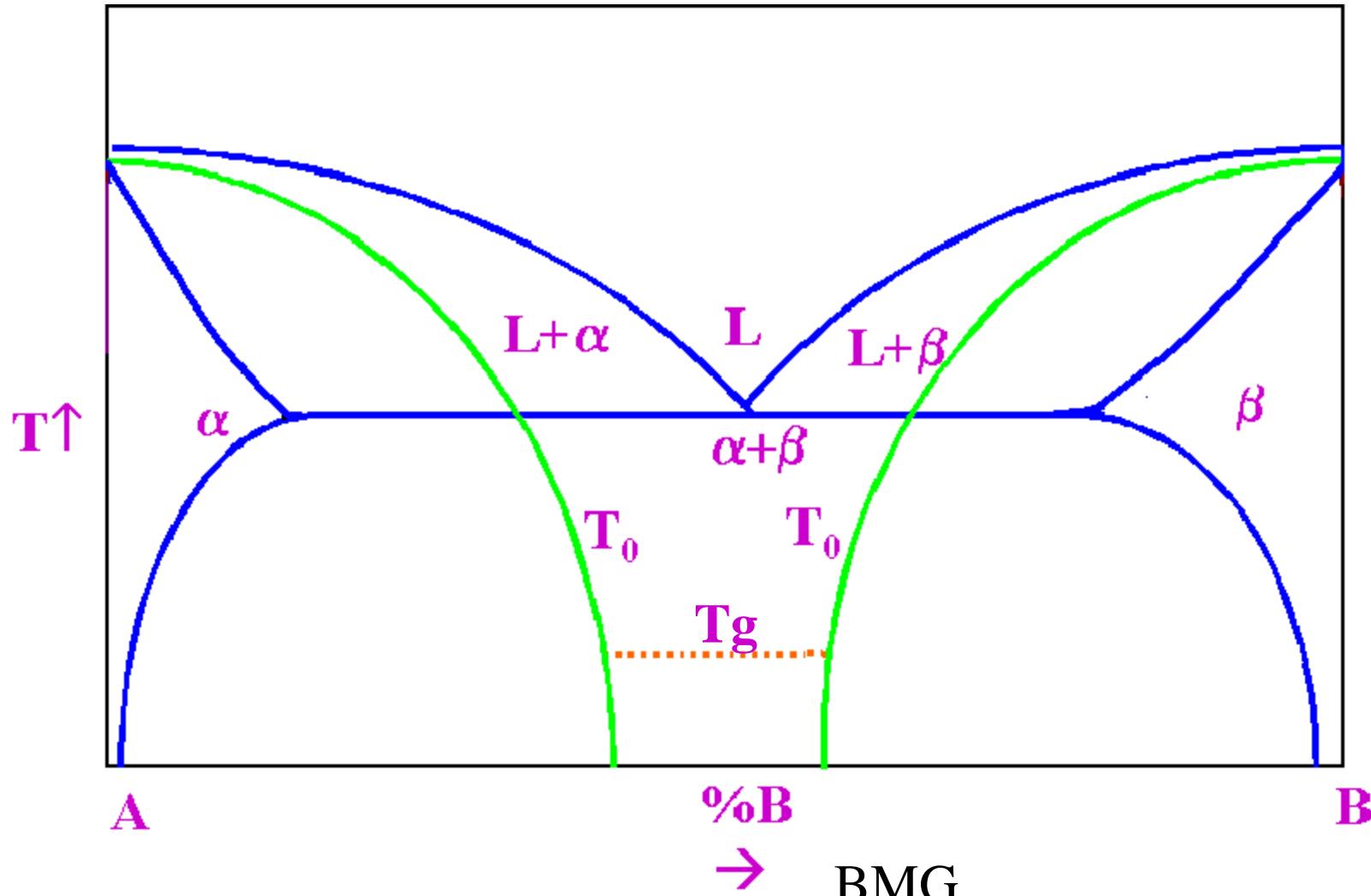


GFA parameters of the $\text{Mg}_{65}\text{Cu}_{25}\text{RE}_{10}$ alloys: super cooled liquid region ($\Delta T = T_x - T_g$), reduced glass transition temperature ($T_{rg} = T_g/T_l$) and γ ($T_x/(T_g + T_l)$)

$\text{Mg}_{65}\text{Cu}_{25}\text{RE}_{10}$

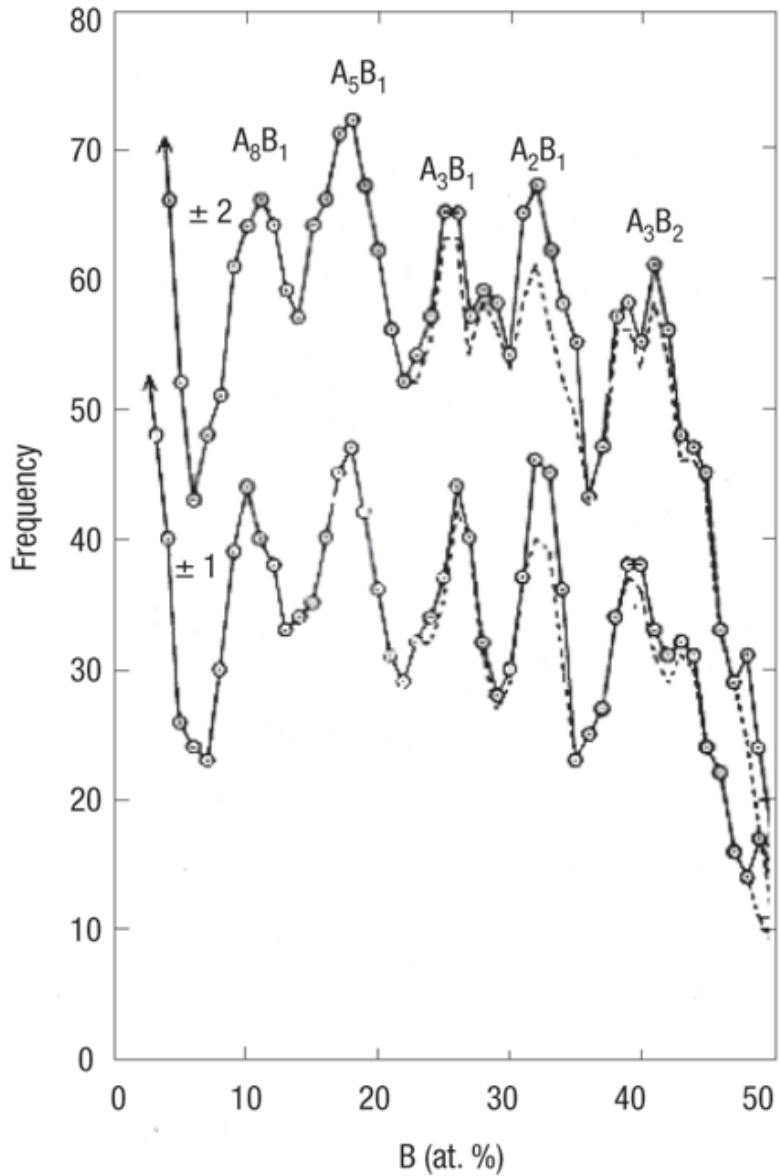


GFA parameters of the $\text{Mg}_{65}\text{Cu}_{25}\text{RE}_{10}$ alloys:
super cooled liquid region ($\Delta T = T_x - T_g$),
reduced glass transition temperature ($T_{rg} = T_g/T_l$)
and γ ($T_x/(T_g + T_l)$)



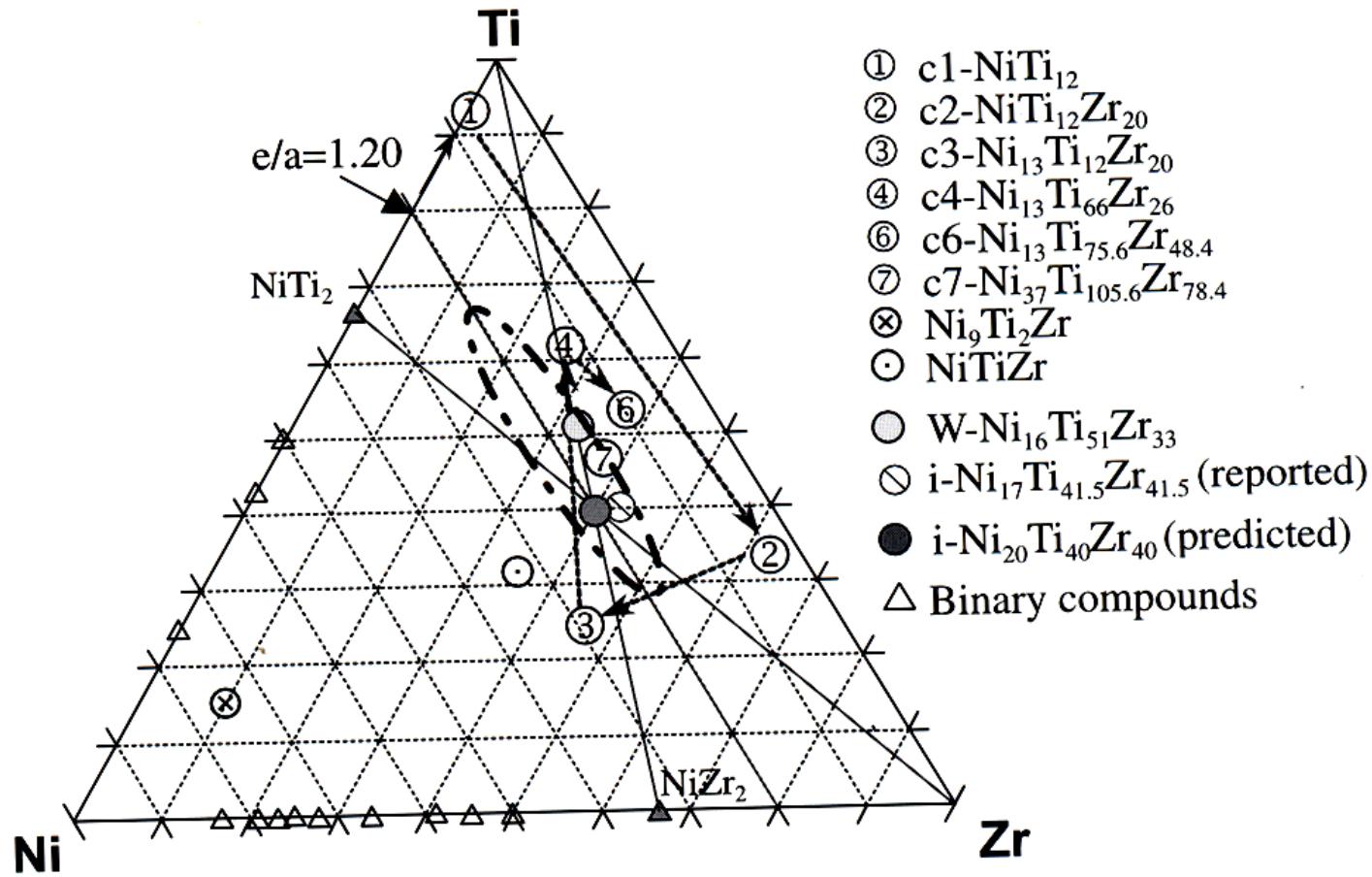
Strict stoichiometry ?

EQUILIBRIUM PHASE DIAGRAM & POLYMORPHOUS PHASE DIAGRAM



D. Stockdale,
Proc. Roy. Soc. A **152**, 81 (1935).
 W. Hume-Rothery & E. Anderson, *E. Phil. Mag.* **5**, 383–405 (1960).
 A.R. Yavari,
Nature Materials, **4**, 2,3 (2005)

1. Binary eutectics by telescoping multinary eutectics-
 Pseudobinary & ternary eutectics-
2. Hume-Rothery explanation of eutectic composition in terms of icosahedral clusters
 S Ranganathan



J. Phys. D: Appl. Phys. 40 (2007) R273–R291

TOPICAL REVIEW

From clusters to phase diagrams: composition rules of quasicrystals and bulk metallic glasses

C Dong¹, Q Wang, J B Qiang, Y M Wang, N Jiang, G Han, Y H Li, J Wu and J H Xia

State Key Laboratory for Materials Modification by Laser, Ion and Electron Beams, Dalian¹⁶

EARLY TRANSITION METAL- LATE TRANSITION METAL ALLOYS Glass & Quasicrystal forming ability

Hf70-Cu30

Hf70-Cu20-Ag10

Hf70- Cu20-(Pt/Pd)10 a--qc

Hf73-Pd27 a-qc

Li, Ranganathan & Inoue, Acta mater 2001

Zr41.5-Ti41.5-Ni17 a-- qc

Zr41.5-Hf41.5-Ni17

Basu, Louzguine, Inoue ,JNCS 2004

Ti40-Zr20- Hf20-(LTM= Ni/Pd/Pt)20

Nano qc

Chen, Louzguine,Kubota, Ranganathan & Inoue Scripta mater. 2005

Zr-80Pt20

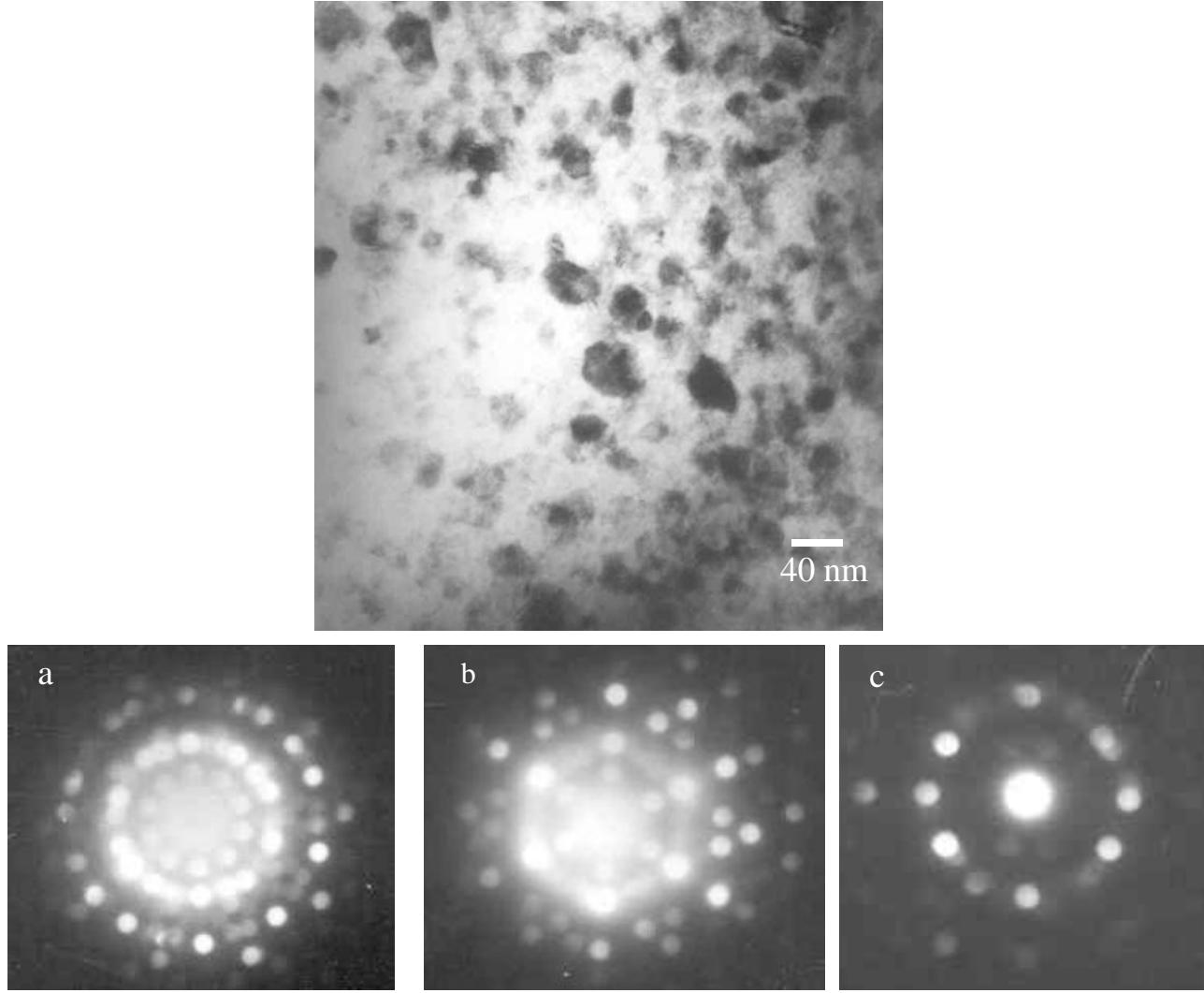
Zr70-(Cu/Ag/Au)10-Pt20

Zr70-Pd 30

Zr70-(Ag/Au)10-Pd20

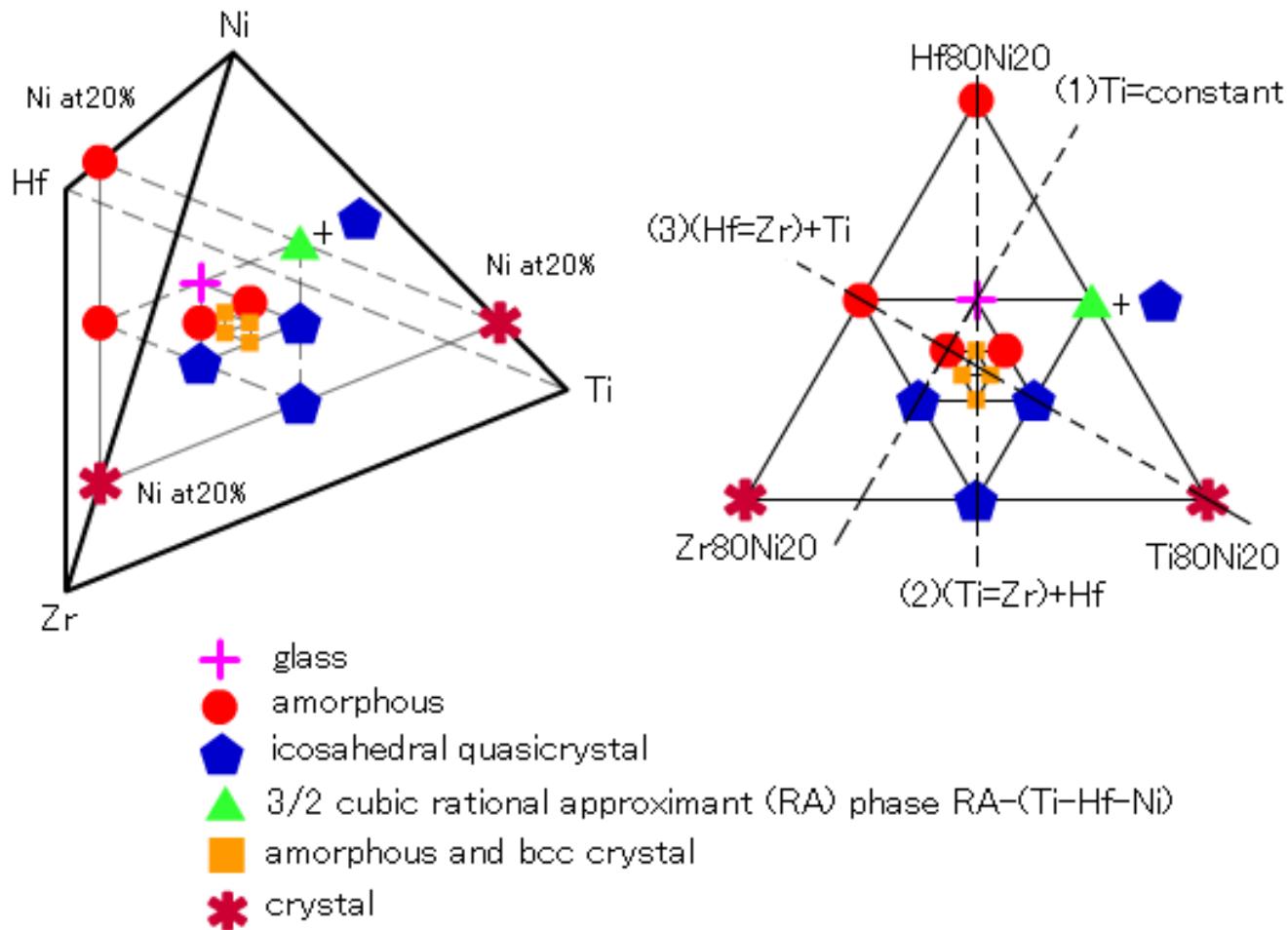
Saida, Ranganathan & Inoue

Multicomponent alloys viewed as pseudo lower order alloys



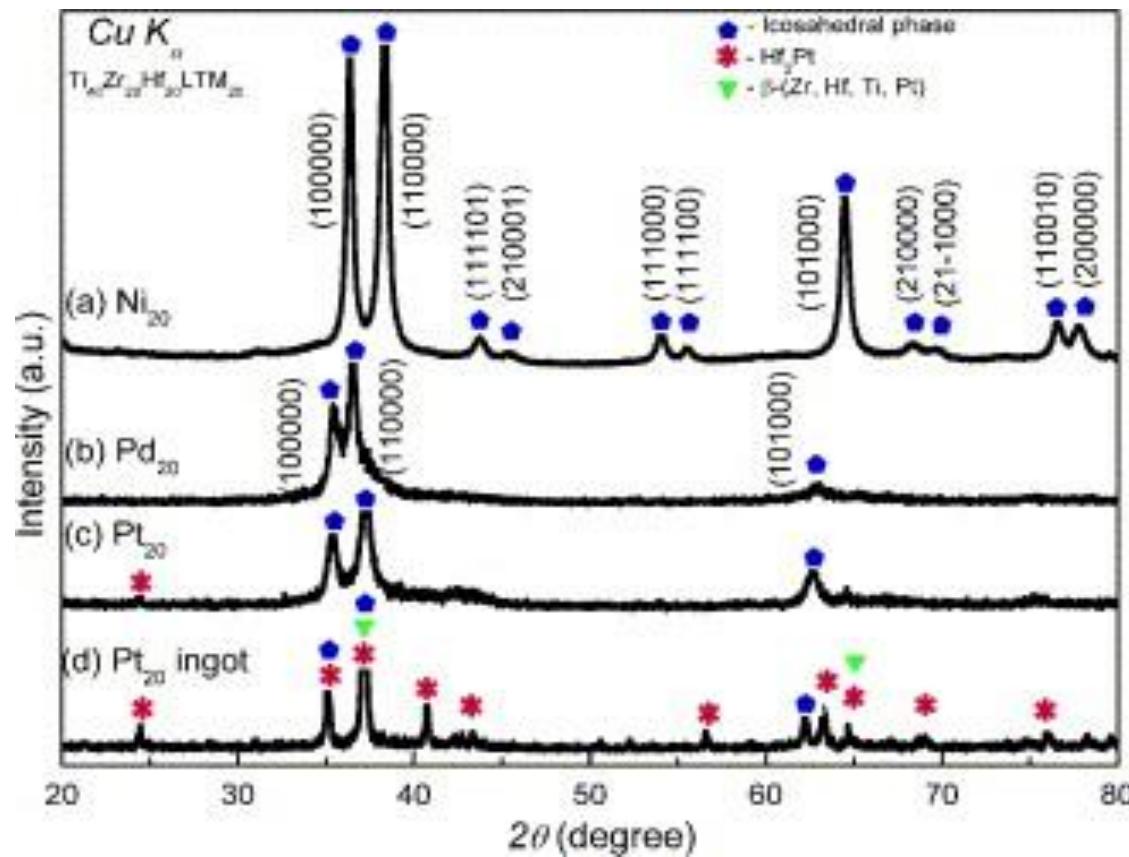
Bright field electron micrograph of a nanoquasicrystallised Zr-Ti-Ni alloy with nano beam electron diffraction pattern showing
a) 5-fold b) 3-fold c) 2-fold symmetry

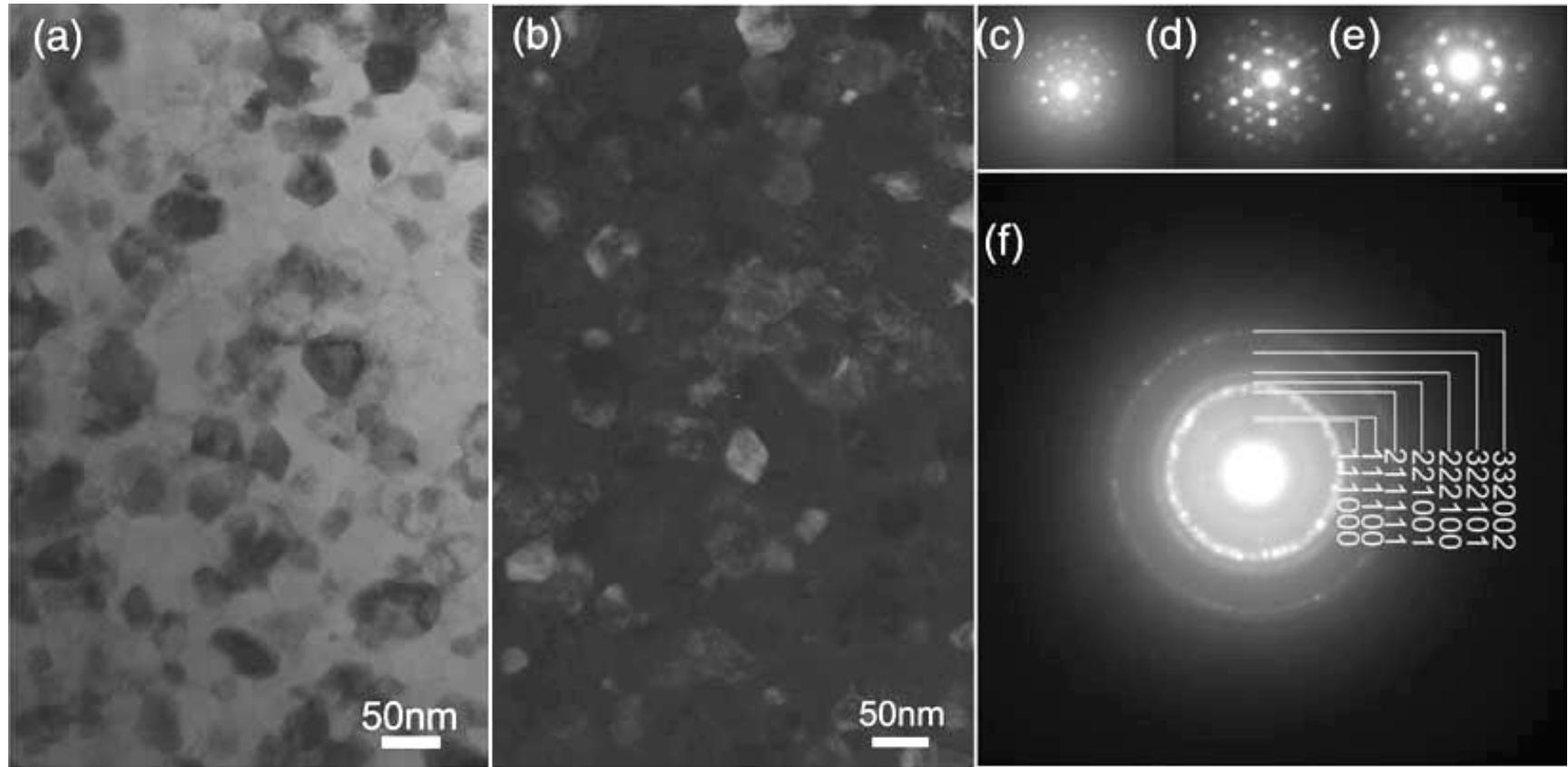
Phases in Melt-spun Alloys



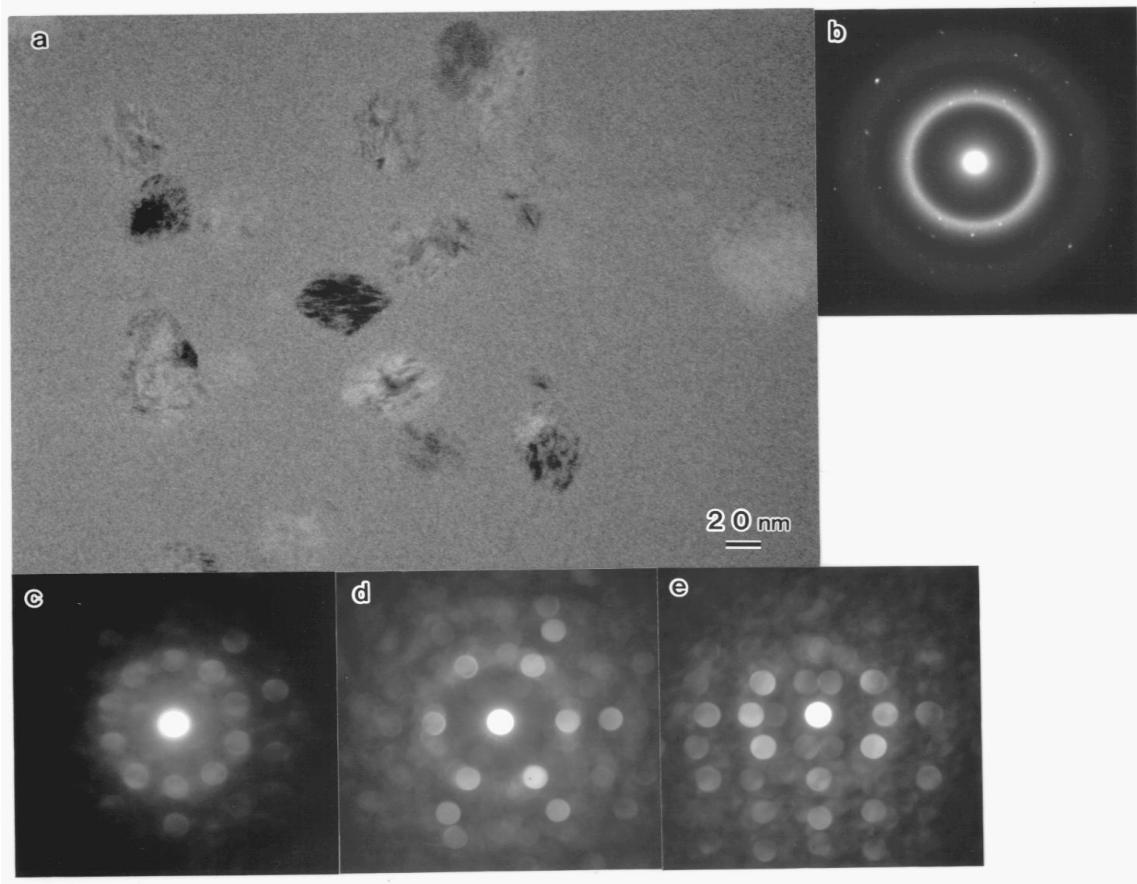
XRD patterns

- (a) Ti40-Zr20-Hf20- Ni20 (b) Ti40-Zr20-Hf20-Pd 20
© Ti40 -Zr20-Hf20- Pt20 alloys
(d) Ti40- Zr20- Hf20- Pt 20 ingot





TEM of the melt-spun $\text{Ti}_{40}\text{Zr}_{20}\text{Hf}_{20}\text{Ni}_{20}$ alloy. (a) Dark-field (b) bright field image (c-e) nano beam diffraction patterns of five-, three- and twofold symmetry respectively and (f) selected-area electron diffraction pattern



Bright field TEM image (a), selected area diffraction pattern (b) and nanobeam electron diffraction patterns (c)-(e) of the rapidly solidified $\text{Zr}_{70}\text{Au}_{10}\text{Pt}_{20}$ alloy. The beam diameter for electron diffraction is $1\mu\text{m}$ in (b) and is 2.4 nm in (c)-(e) (J. Saida, A. Inoue S. Ranganathan)⁵²

QUASICRYSTALS AND METALLIC GLASSES: A COMPARISON

Bond Orbital	Large Atom	Quasicrystal	Bulk Metallic Glass
s-electrons	Li, Mg	Li-Cu-Al Mg-Zn-Al	(Mg-Cu-Y)
p-electrons	Al, Ga	Al-Pd-Mn Ga-Cu-Co	Al-La-Ni (Al-rich marginal)
d- electrons (ETM)	Zr, Hf, Ti	Zr-Ti-Ni Zr-Ti-Ni-Cu Zr-Pd-Cu	Zr-Ni-Al Zr-Ni-Cu-Al Zr-Ti-Ni-Cu-Be
d- electrons (LTM)	Fe, Co, Ni Pd, Pt	-	Fe-Ni-P-B Pd-Ni-P-B
f-electrons	Ln	Ln-Zn-Mg Ln-Cd-Mg	La-Ni-Al (Mg-Cu-Y) (Al-Ni-La)

CONCLUSIONS- I

- The close packing of spheres of different sizes favours intrinsically polytetrahedral packings involving icosahedral order. This extends from atomic to micrometre dimensions .-e.g colloids
- Icosahedral order applies to crystalline and quasicrystalline intermetallics as well as bulk metallic glasses.
- The size ratio close to 1.225 is favoured for both large atom minority and majority compositions.
- The topological complexity in BMG s extends up to four components only (Miracle 2004).

CONCLUSIONS- II

- The chemical complexity for topologically close packed crystalline and quasicrystalline intermetallics extends to two
- The Pettifor structure mapping approach allows to consider both topology and chemistry together.
- For bulk metallic glasses the chemical complexity can be three or four. Binary BMGs are an exception .
- The composition of eutectics with strict stoichiometry mirrors that of resultant BMGs. There is a strong connection in the stoichiometry of higher order eutectics so that many appear to be psuedo binary or ternary.

ACKNOWLEDGEMENTS

Collaboration with

Prof A Inoue, Prof A Takeuchi, Prof H Kimura,
Prof J Saida and Prof D Louzguine

Discussions with Dr Eric Lord and Dr Daniel
Miracle, Prof B S Murty

Comments from
Prof David Pettifor and Prof Alan Mackay

Sponsors
DRDO, India; AOARD, Tokyo