IMPORTANCE OF RARE FLUCTUATIONS IN QUANTUM CONDENSED MATTER SYSTEMS

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- ❖ INTRODUCTION: RARE EVENTS AND RARE FLUCTUATION PHENOMENA, AND "BLOCHITIS" IN CONDENSED MATTER PHYSICS
- ❖ ONE DIMENSIONAL EXAMPLE: DISORDERED QUANTUM SPIN CHAINS AND LOW TEMPERATURE MAGNETIC RESPONSE
- ❖ TWO DIMENSIONS: HALL CONDUCTANCE AT A QUANTUM HALL TRANSITION
- ❖ THREE DIMENSIONS: DIELECTRIC BEHAVIOR OF THE DISORDERED INSULATOR – LOW FREQUENCY RESPONSE NEAR THE INSULATOR-METAL TRANSITION
- ❖ THE TRANSVERSE FIELD ISING SPIN GLASS IN 2 AND 3 DIMENSIONS
- ❖ CONCLUDING REMARKS

INTRODUCTION TO RARE EVENTS AND FLUCTUATIONS

- Rare event/fluctuation – low probability, so why bother?

- CONSEQUENCES MAY BE EXTREME, e.g. :

- Traffic jams on single lane roads (Princeton versus Delhi)
- Stock Market Crashes
- Tsunamis
- Dopants in semiconductors (1 part in 10^7)

A SIMPLE EXAMPLE :

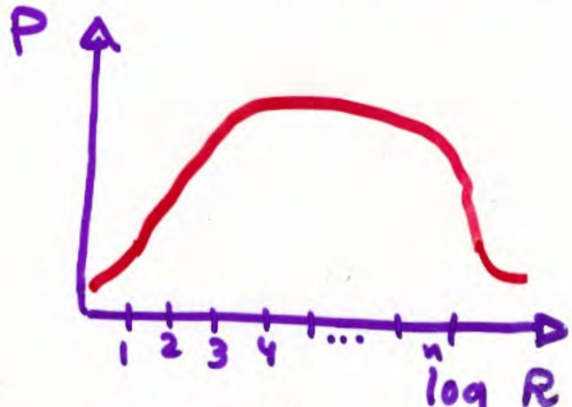
A ONE-DIMENSIONAL RESISTOR CHAIN

WEAK LINK (HIGH R)
DOMINATES THE CHAIN RESISTANCE

PROBLEM:

In Higher Dimensions, weak link rendered unimportant by parallel paths for conduction. (Traffic jams in one lane roads versus multi-lane roads)

Even in case of extremely broad distributions net resistance given by TYPICAL resistance (resistance at percolation threshold)



e.g. Ambegokar-Halperin-Langer formulation of Variable Range Hopping.

Net effect: “BLOCHITIS” in Condensed Matter Physics

- Understand system without disorder first, then perturb around it =>

- Effective Medium Theory, Average T-matrix approximation, Coherent Potential approximation, Mean field approach, …

- But MANY phenomena not captured, e.g. Percolation, Localization, Rare Fluctuation effects (Griffiths Singularities), Non-gaussian behavior in thermodynamic limit, etc.

GRIFFITH'S ARGUMENT (1969)

DILUTED FERROMAGNET

$$\mathcal{H} = \sum_{ij} J_{ij} \, S_i S_j$$

ABOVE PERCOLATION

FOR $T_c^0 > T > T_c$

T_c^0 ↙ PURE UNDILUTED SYSTEM

T_c ↘ ACTUAL SYSTEM

LOCAL REGIONS "ORDER" ⟹ LEAD TO ESSENTIAL SINGULARITIES IN M(H). (SUSCEPTIBILITIES $\chi^n \equiv d^n M / dH^n$ FINITE).

ORDERED CLUSTERS LEAD TO LONG TIME TAILS IN DYNAMICS (DHAR).

EXTENSION TO SPIN GLASSES (RANDOM J_{ij}) BY RANDERIA, SETHNA, PALMER. MC SIMULATIONS BY OGIELSKI - (?)

ANTIFERROMAGNETS WITH LARGE DISORDER

↑ QUENCHED, RANDOM

$$\mathcal{H} = \sum_{i,j} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

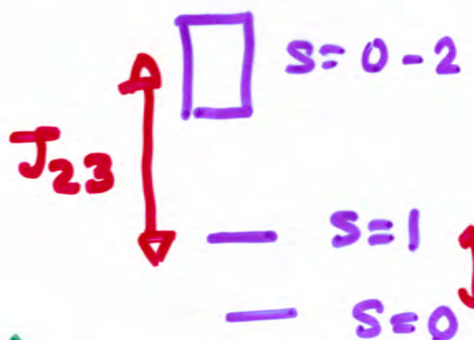
EXPTAL SYSTEMS: DOPED SEMICONDUCTORS, 1D ORGANIC CHAIN COMPOUNDS w/ DISORDER

NO QUENCHING OF SPINS DOWN TO mK, $\chi \sim T^{-\alpha}, \alpha \sim 0.7-0.9$

SPIN $\frac{1}{2}$ CHAIN

EXPLANATION:

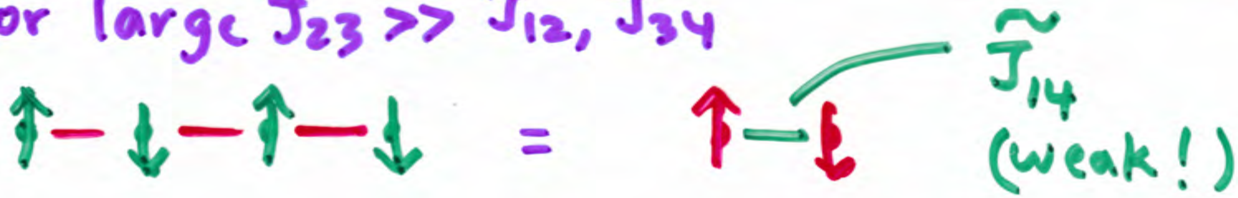
Spins with strong bonds form singlets, leaving weak bonds between remaining spins



$$\tilde{J}_{14} = \frac{J_{12} J_{34}}{2 J_{23}}$$

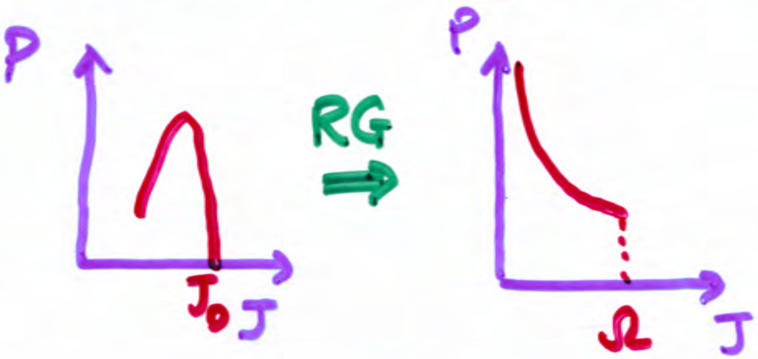
For $J_{23} = \infty$ two free spins

For large $J_{23} \gg J_{12}, J_{34}$



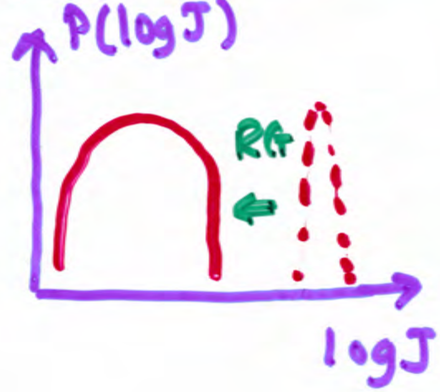
$$\ln \tilde{J}_{14} = \ln J_{12} + \ln J_{34} - \ln J_{23} - \ln 2$$

R.G. procedure generates very weak bonds, $P(J)$ becomes broader (on log scale) and eqn. becomes asymptotically exact



$$P(J,\Omega) \sim J^{-1+\lambda}$$

$$\lambda \sim \frac{1}{|\ln \Omega|} \to 0$$



Dasgupta + Ma, Bhatt + Lee, D. S. Fisher

Random Singlet or Valence-Bond Glass Phase

(Singlet pairing on large length, low energy, scales in a DISTRIBUTION SPECIFIC way)

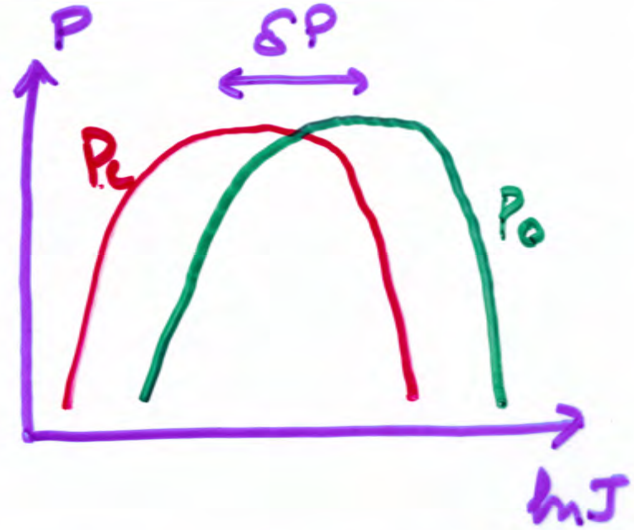
$$\chi \sim \frac{1}{T|\ln^2 T|} \qquad C \sim \frac{1}{|\ln^3 T|} \qquad \langle S_i \cdot S_j \rangle \sim \frac{1}{|i-j|^2}$$

(Fisher)

(dominated by few cases) mostly zero

EFFECT OF DIMERIZATION :

Hyman, Yang, Bhatt, Girvin (96)



$$\frac{d\,\delta P}{d \ln \Omega} = + \,\delta P$$

RELEVANT !

While decimating, more likely to generate weak even bond while removing one large odd bond, and two typical even bonds.

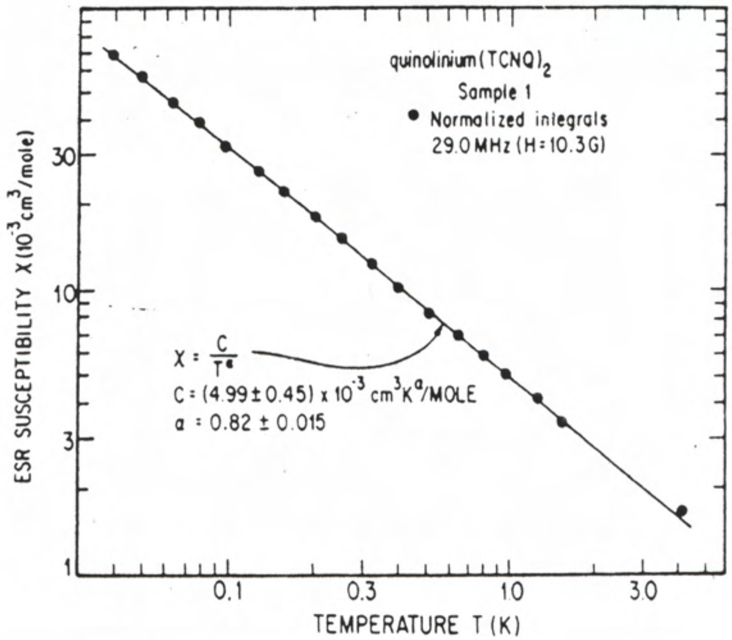
$\Rightarrow$ Develop low energy tail in P_e, P_e + P_o start separating.

STOP PROCEDURE (RG) WHEN DISTRIBUTIONS SEPARATE

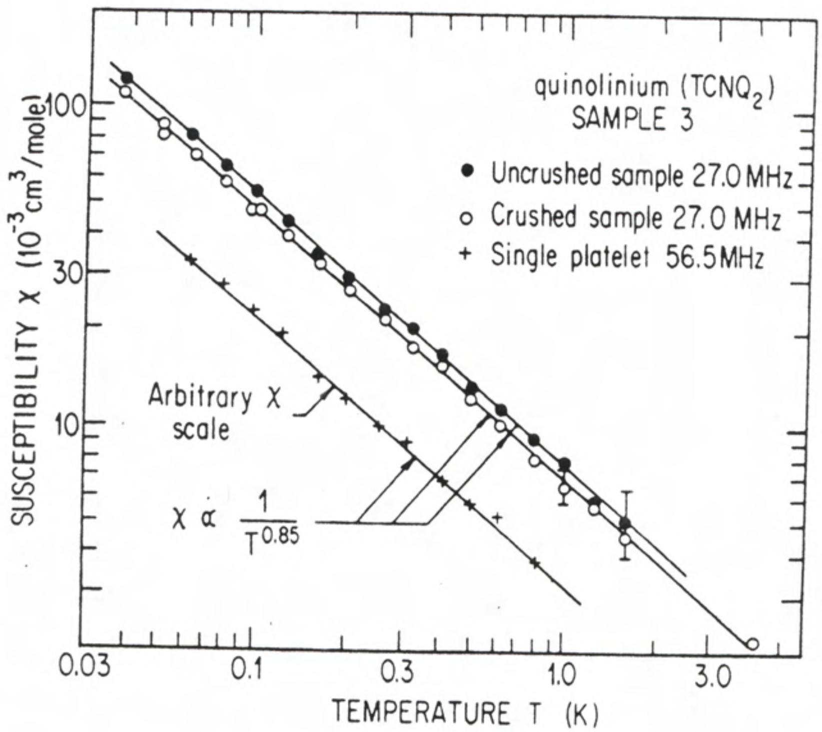
UNCOUPLED DIMERS WITH $P(J) \sim J^{-1+\lambda}$

$$\Rightarrow \chi \sim T^{-1+\lambda}, \quad C \sim T^{\lambda}, \quad \langle S_i \cdot S_j \rangle \sim e^{-|i-j|/\xi}$$

(GRIFFITHS PHASE)

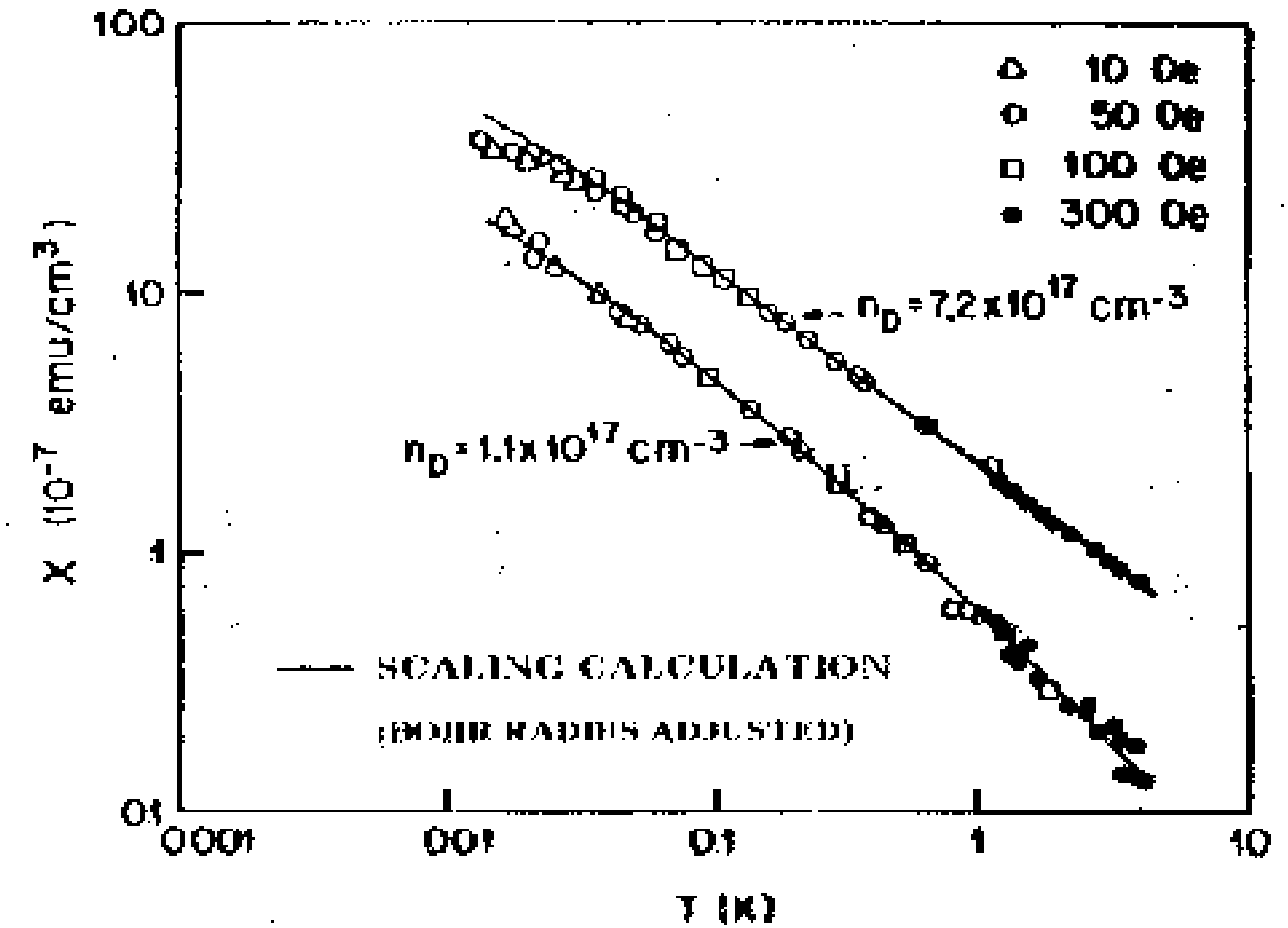


quinolinium (TCNQ)$_2$
Sample 1
Normalized integrals
29.0 MHz (H = 10.3 G)
$\chi = \frac{C}{T^{\alpha}}$
C = (4.99 ± 0.45) x 10^{-3} cm^3K^{α}/MOLE
α = 0.82 ± 0.015
ESR SUSCEPTIBILITY χ ($10^{-3}cm^3$/mole)
TEMPERATURE T (K)



quinolinium (TCNQ$_2$)
SAMPLE 3
Uncrushed sample 27.0 MHz
Crushed sample 27.0 MHz
Single platelet 56.5 MHz
Arbitrary χ scale
$\chi \propto \frac{1}{T^{0.85}}$
SUSCEPTIBILITY χ ($10^{-3}cm^3$/mole)
TEMPERATURE T (K)

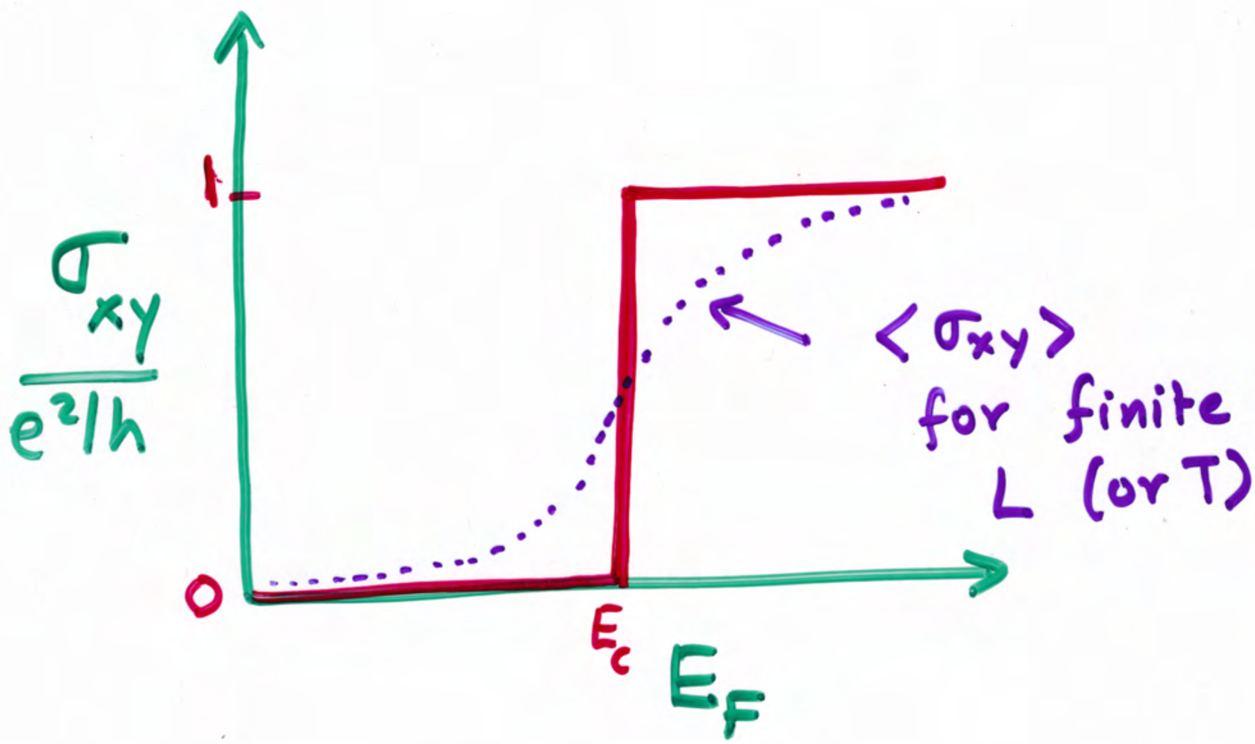
Magnetic Susceptibility of Si:P in Insulating Phase



Data: K. Andres *et al*, Phys. Rev. B **24**, 244 (1981)

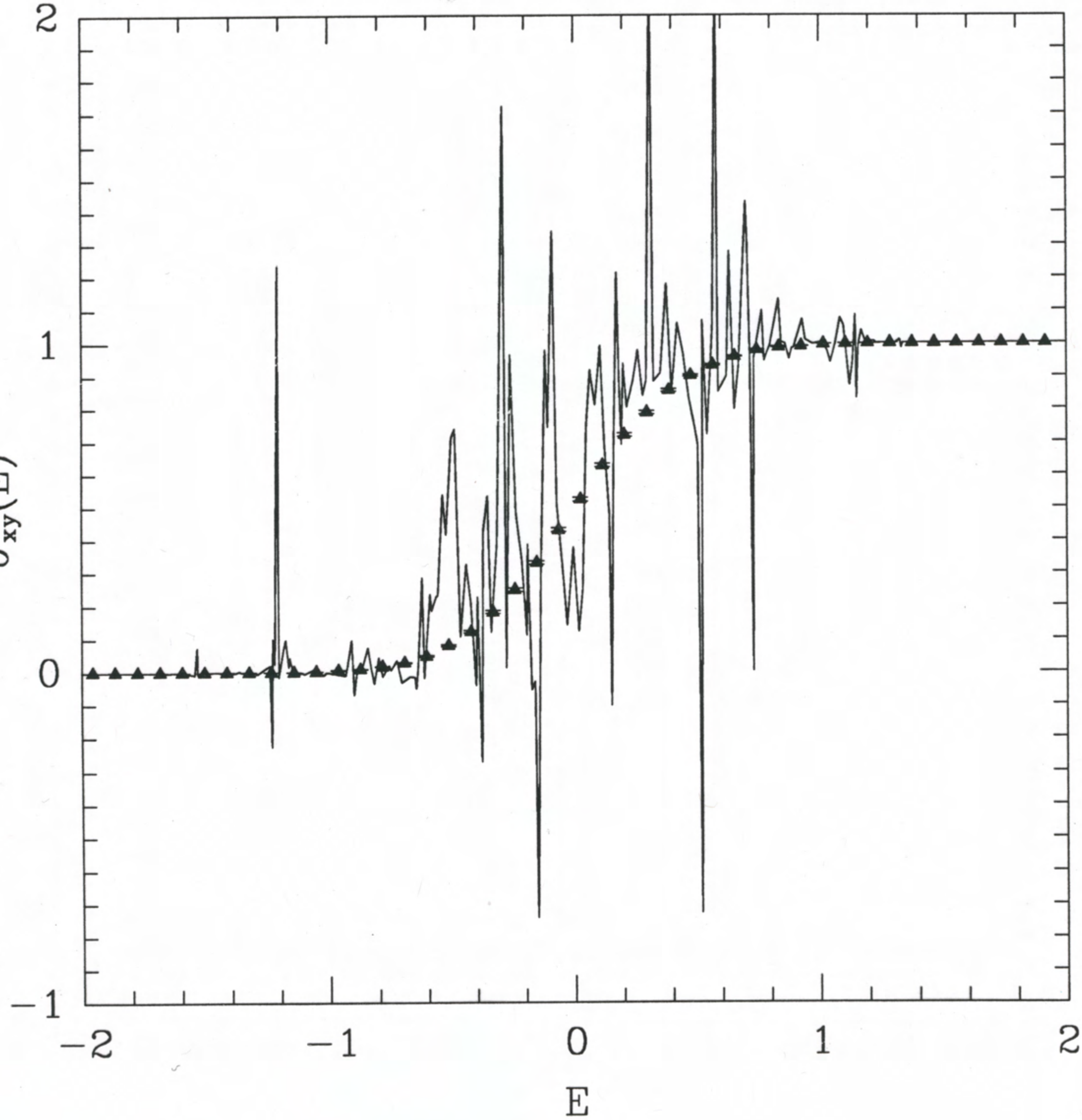
Theory: Bhatt & Lee, Phys. Rev. Lett. **48**, 344 (1982)

CONDUCTANCE DISTRIBUTION AT A QUANTUM HALL STEP

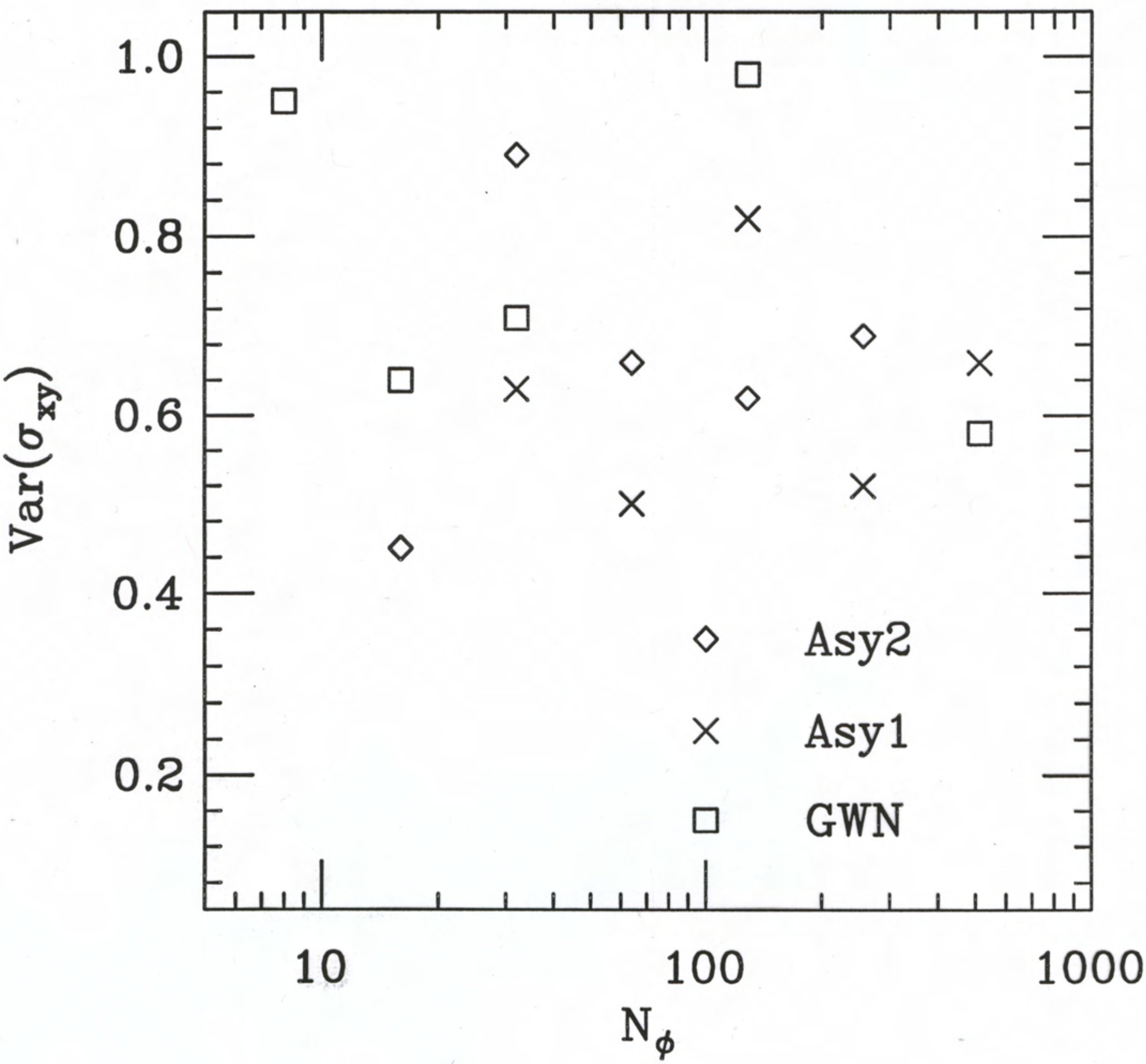


For $E \neq E_c$ $P(\sigma_{xy})$ is Gaussian around $\langle \sigma_{xy} \rangle$ for sufficiently large L ($\langle \sigma_{xy} \rangle \simeq 0$ or 1)

What is $P(\sigma_{xy})$ at E_c or for $\langle \sigma_{xy} \rangle \neq 0, 1$?



2
1
0
−1
$\sigma_{xy}(E)$
−2
−1
0
1
2
E



1.0
0.8
0.6
0.4
0.2
10
100
1000
Var(σ_{xy})
N_ϕ
Asy2
Asy1
GWN

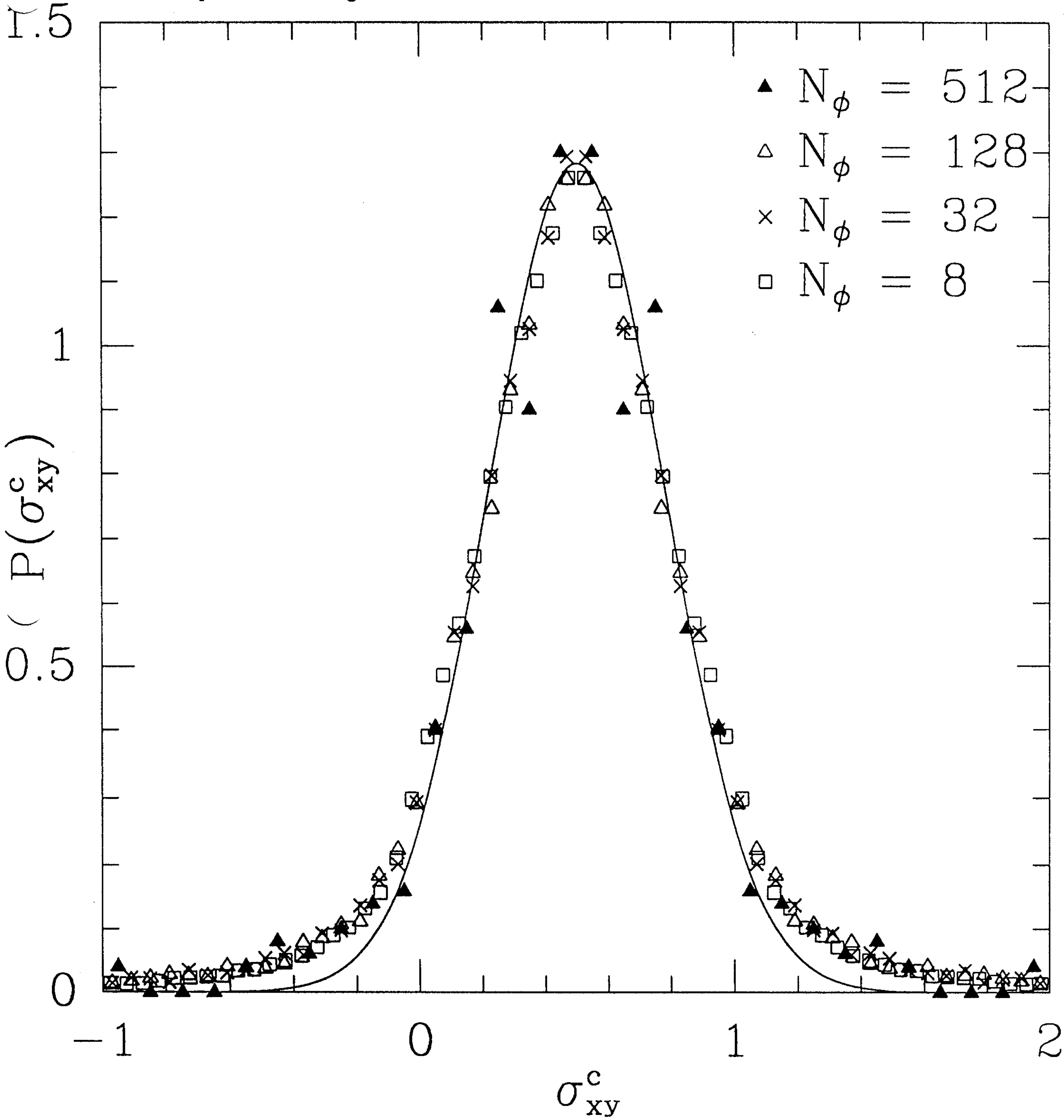
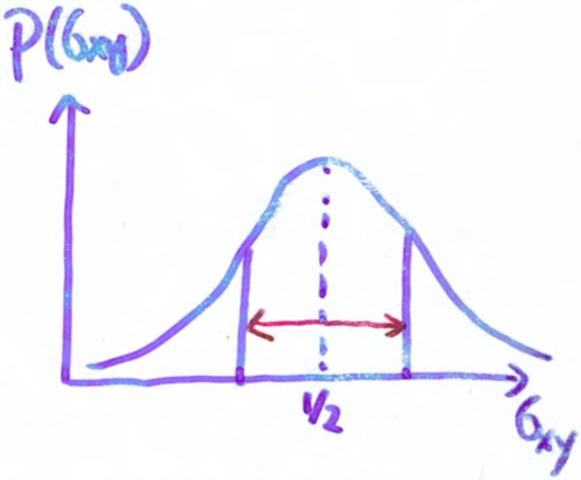


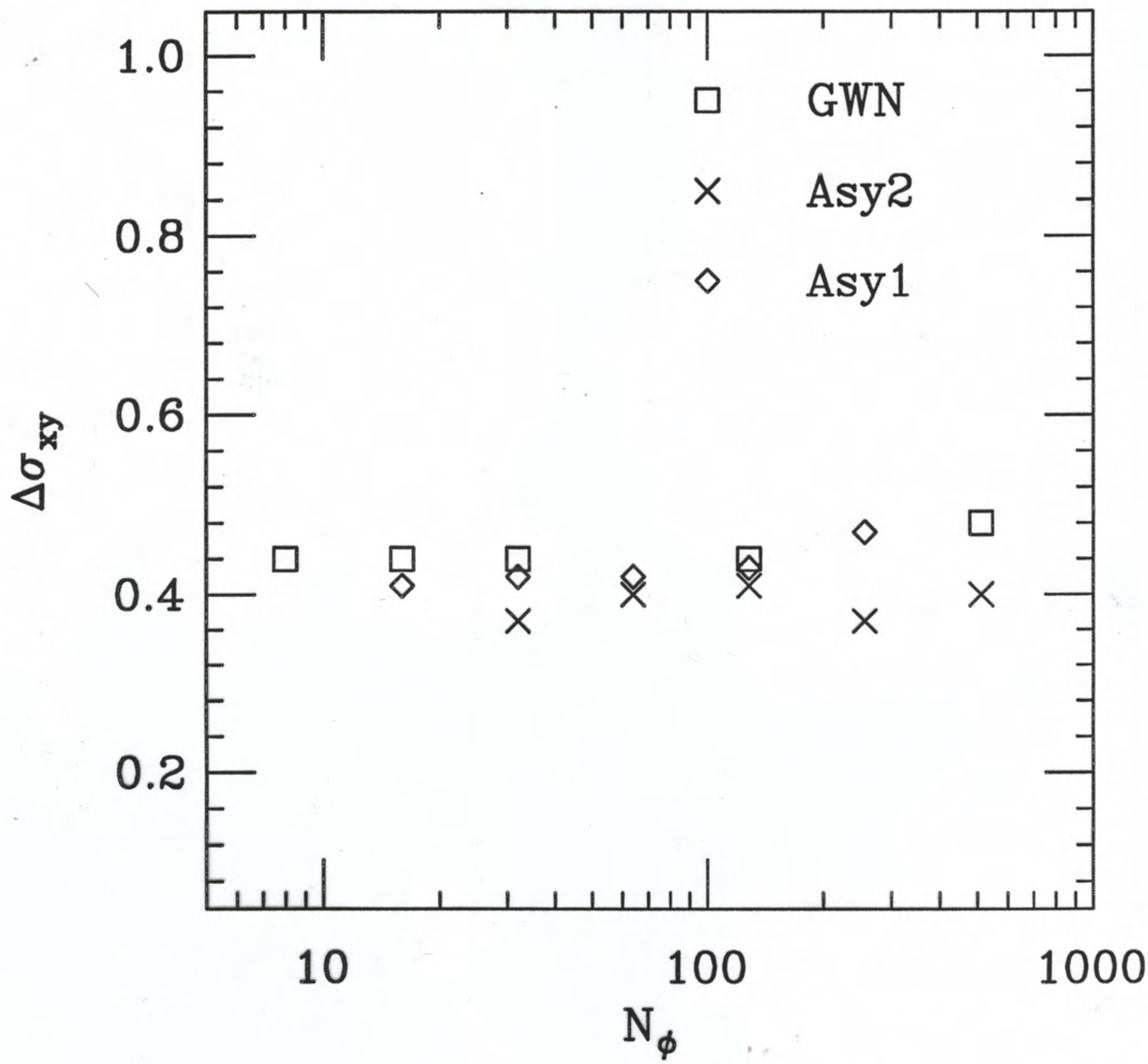
Figure 5.7: Probability distribution functions for Hall conductivity at the critical energy for Gaussian white noise potential

$$\int_{-\infty}^{\sigma^1_{xy}} P(\sigma_{xy})\, d\sigma_{xy} = \frac{1}{4}$$

$$\int_{-\infty}^{\sigma^2_{xy}} P(\sigma_{xy})\, d\sigma_{xy} = \frac{3}{4}$$

$$\Delta\sigma_{xy} = \sigma^2_{xy} - \sigma^1_{xy}$$





POWER LAW TAILS OF $P(\sigma^c_{xy})$

$$\sigma_{xy} \sim \sum_{i,j} \frac{|M_{ij}|^2}{(E_i - E_j)^2}$$

Large value of σ_{xy} due to near degeneracy of pairs of eigenstates

If $M \sim$ constant $\quad E = E_i - E_j$

$$P(\sigma)\, d\sigma = \phi(E)\, dE$$

For GUE $\phi \sim E^2$ for small E, $\sigma \sim \frac{1}{E^2}$

$$\therefore \; P(\sigma) = \phi(E) \frac{dE}{d\sigma} \sim E^2 \cdot E^3 \sim E^5$$

$$\sim \frac{1}{\sigma^{2.5}}$$

Need to check what M does.

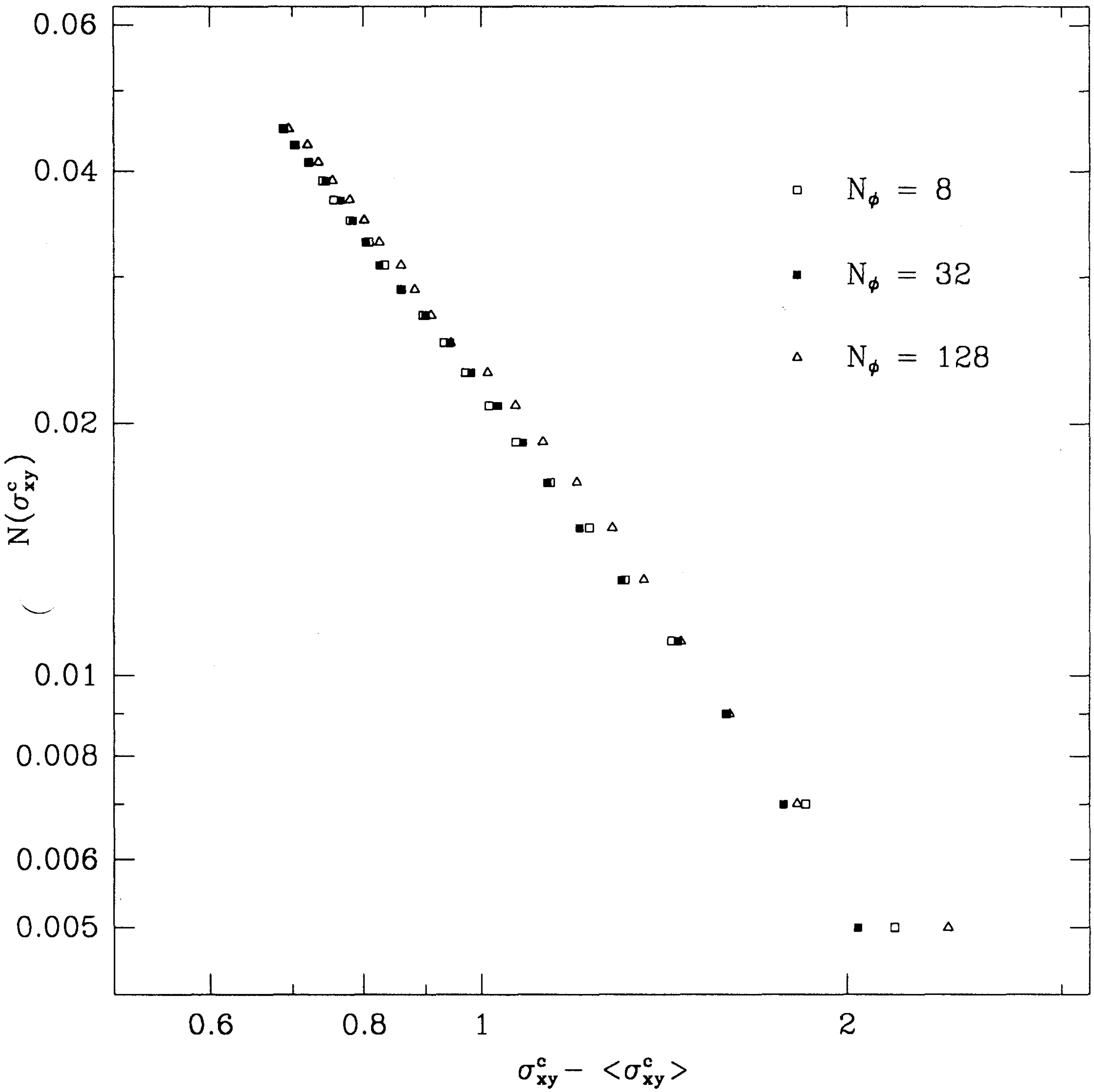
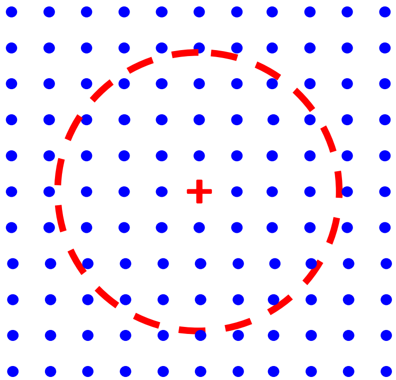


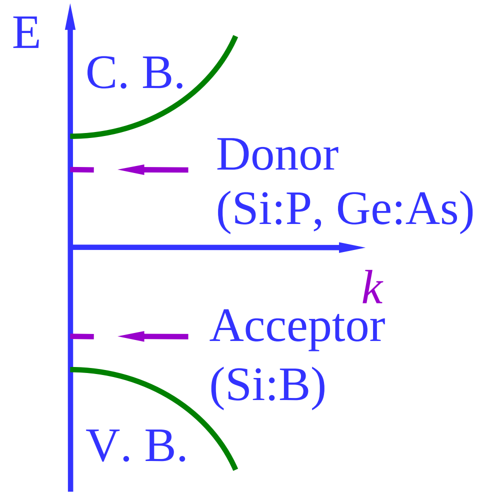
Figure 5.10: Integrated density of states in the tail of distribution of σ_{xy} for Gaussian white noise potential on a double logarithmic plot.

Impurity States in Semiconductors



$$H = H_{cryst} - \frac{e^2}{\varepsilon r}$$

impurity potential

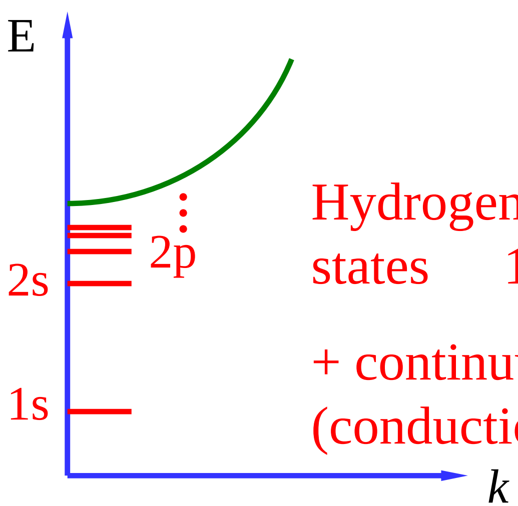


Perturbation theory gives

$$\Psi(\vec{r}) = \phi_{k_0}(\vec{r})\chi(\vec{r})$$

Bloch wave   Hydrogenic envelope →

$$\left[-\frac{1}{2m^*}\nabla^2 - \frac{e^2}{\varepsilon r}\right]\chi = E\chi$$



Hydrogenic bound states   1s, 2s, 2p, ...

+ continuum (conduction band)

"Rydberg" $\frac{m^* e^4}{2\varepsilon^2\hbar^2} \sim$ 10 meV (Ge), 30 meV (Si)

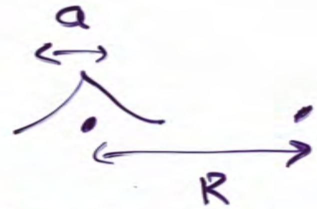
"Bohr radius" $\frac{\varepsilon\hbar^2}{m^* e^2} \sim$ 50 Å (Ge), 20 Å (Si)

AC CONDUCTIVITY IN DISORDERED INSULATORS
(low freq.)

$$\sigma(\omega) \sim N^2(E_F)\ \omega^2$$

$N^2(E_F)$: Density of initial & final states (NO GAP)

ω^2: one for energy transfer, one for exclusion principle



$$\omega = E_0 e^{-R/a}$$

$$\Rightarrow R = a \ln \frac{E_0}{\omega}$$

DOMINATED BY RESONANT PAIRS

$$\sigma(\omega) \sim N^2 \omega^2 (eR)^2 R^2 \Delta R \sim \omega^2 \ln^4 (E_0/\omega)$$

LOGARITHMIC ENHANCEMENT (MOTT)

ADD COULOMB (e^2/r) INTERACTION

$\Rightarrow$ Replace ω by Δ (Coulomb Gap in Single particle DOS)

EXPT $\sigma(\omega) \sim \omega^{0.8}$ $\leftrightarrow$ $\sigma(\omega) \sim \omega \ln^3 (1/\omega)$ (EFROS-SHKLOVSKII) 1975-81

$$\Rightarrow \epsilon(0) \sim \int \frac{\sigma(\omega)}{\omega^2} d\omega \sim \text{logarithmically divergent!}$$

WHAT HAPPENS WHEN INSULATOR $\rightarrow$ METAL ?

As you approach I-M Transition, "insulating" physics restricted to $r > \xi$

localization length $\to \infty$

Expect resonant pairs not to matter (phase space $\to 0$)

NOT SO! ES analysis, with modifications, valid for $r > \xi$, and yields

$$\sigma(\omega) \sim \epsilon \frac{\omega}{\ln\left(\frac{1}{\omega}\right)} \qquad \epsilon(\omega) = \epsilon + \epsilon \ln|\ln \omega| + \ldots$$

Bhatt + Ramakrishnan (1984)

The dipole moment of relevant dipoles grows and net effect is low frequency condutivity grows! (becomes part of scaling description, and cannot be neglected)

EXPERIMENT BY Paalanen et al.

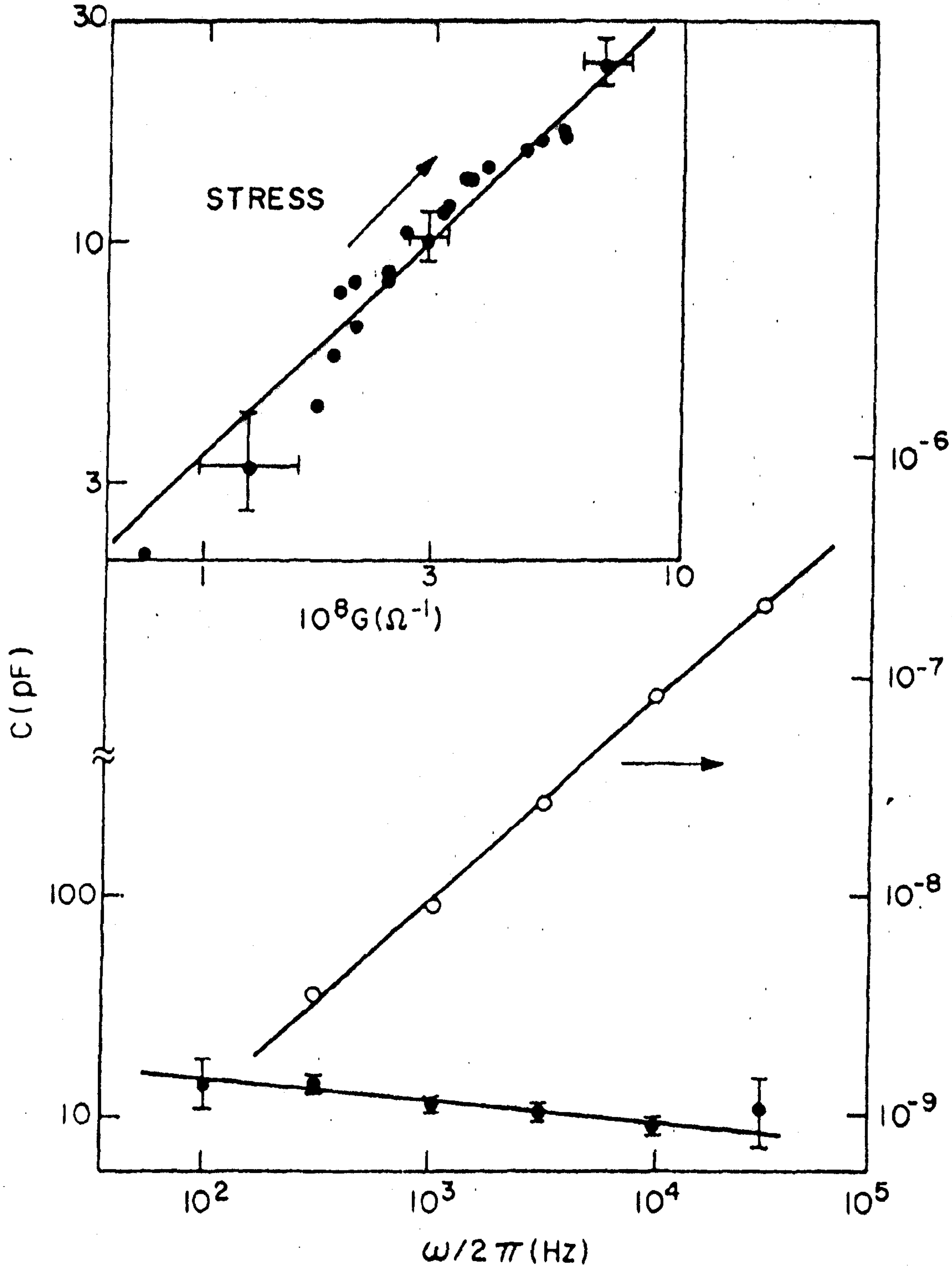
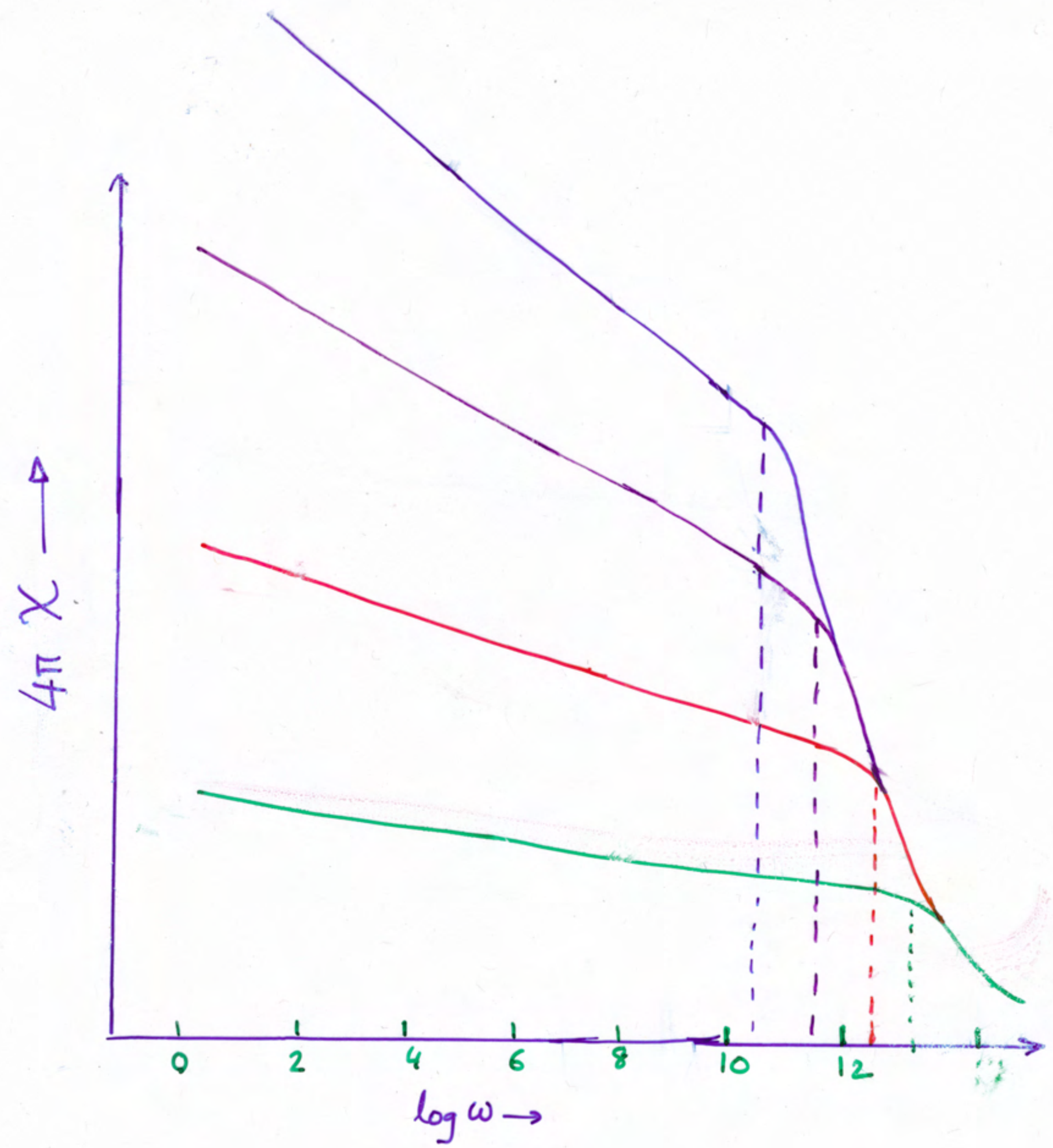


FIG. 2. Variation of conductance G (open circles) and donor capacitance C (solid circles) with frequency $\omega/2\pi$ at $T=13$ mK at a typical stress [$S=1.72$ kbar, $4\pi\chi(0)\simeq 180$]. Solid lines are fits by the forms ω^s and ω^{s-1} respectively, with $s=0.9\pm0.1$ (data also consistent with other forms, see text). Inset shows proportionality between real and imaginary parts of the conductance (extrapolated to $T=0$ K) at 31 kHz as $n\rightarrow n_c^-$.



4π χ →
0
2
4
6
8
10
12
log ω →

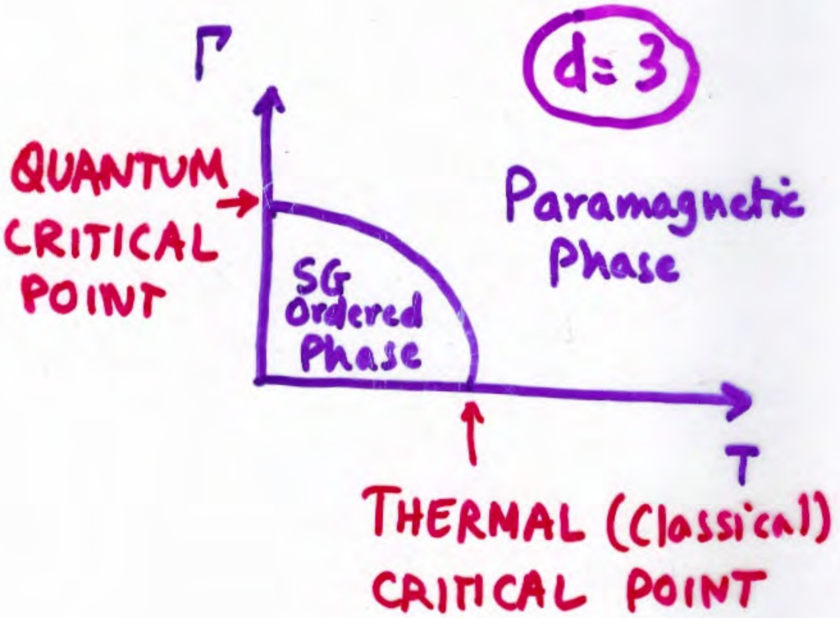
THE TRANSVERSE-FIELD ISING SPIN GLASS

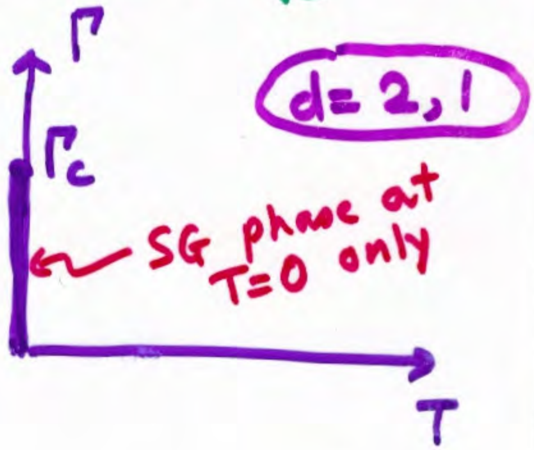
$$\mathcal{H} = \sum_{\langle ij \rangle} J_{ij} S_i^z S_j^z - \Gamma \sum S_i^x$$

$\sum_{\langle ij \rangle} J_{ij} S_i^z S_j^z$ → ISING SPIN GLASS ORDERING IN S^z: $|\uparrow\rangle$

$\Gamma \sum S_i^x$ → TRANSVERSE FIELD ALIGNS S^x

$$|\rightarrow\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right)$$





ISSUES:

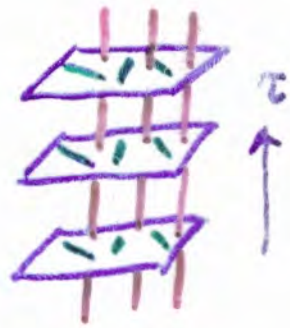
- UNIVERSALITY CLASSES FOR Q.P.T.
- UNIVERSAL QUANTITIES AT Γ_c (EXPONENTS AMPLITUDES)
- UNUSUAL FEATURES OF Q.P.T.

Path Integral Representation

d- QM system $\Leftrightarrow$ $(d+1)$- Classical system

Trotter-Suzuki mapping:

$$H = -\sum_{<i,j>} J_{ij} S_i^z S_j^z - \Gamma \sum_i S_i^x$$



$$\Updownarrow$$

$$H_{eff}^{L_\tau} = -\Delta\tau \sum_{\tau=1}^{L_\tau} \sum_{<i,j>} J_{ij} S_i(\tau) S_j(\tau) - J_F \sum_{\tau=1}^{L_\tau} \sum_i^{L^d} S_i(\tau) S_i(\tau+1)$$

(Ising)

FM n.n. coupling J_F along τ

SG coupling J_{ij} in each "spatial" hyperplane

Size of mapped system $L^d \times L_\tau$, has **nontrivial anisotropy**

$$\xi_\tau \sim \xi^z (T \sim T_c)$$

← conventional dynamic scaling

(z: dynamic exponent)

The mapping preserves **critical properties** and

$$\Gamma \Leftrightarrow T_{cl}$$

$$L_\tau \Leftrightarrow \beta_{qm} = \frac{1}{T_{qm}}$$

$$L_\tau \rightarrow \infty \Leftrightarrow T_{qm} \rightarrow 0$$

- Monte Carlo simulation on classical model

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to have activated rather than conventional dynamic scaling, one would expect the peaks in g to grow broader with increasing L, when plotted on a logarithmic scale as in

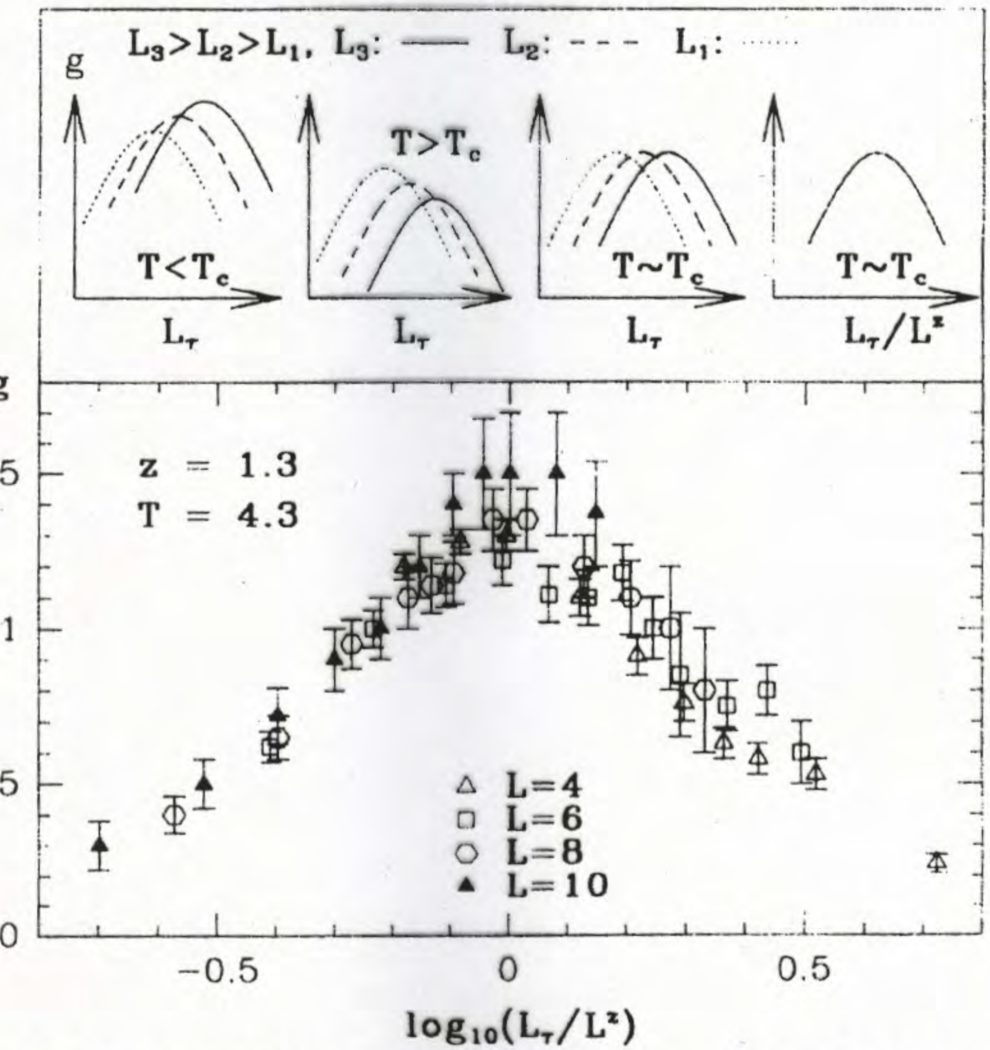


FIG. 1. Scaling of the coupling constant g [Eq. (4)] as sample size and shape are varied. Top plots schematically show the behavior below, at, and above T_c. The main graph shows the actual g, computed as a function of the scaled sample shape at T_c. The dynamic exponent $z \cong 1.3$ and $T_c \simeq 4.3$ are chosen for the best collapse of the data for different sizes onto one curve in this plot.

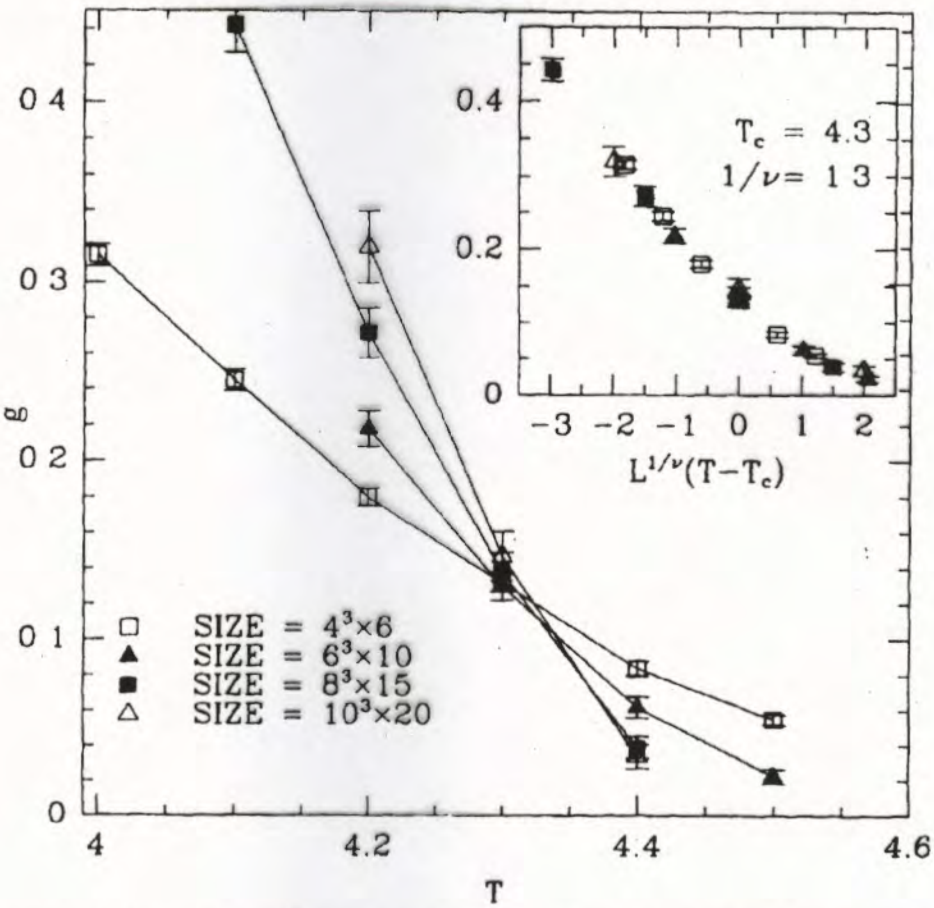


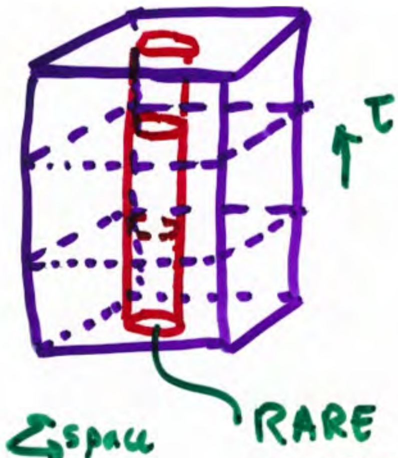
FIG. 2. The coupling constant, g, vs temperature for the scaled sample shape determined by the maximum of g in Fig. 1. The crossing indicates T_c. The inset shows the best collapse of these data onto one scaling curve.

Fig. 1. There is no sign of such a broadening, so we conclude that these data are more consistent with conventional, rather than activated, dynamic scaling.

Having determined z, we can fix the scaled shape and study the dependence of g on the scaled size L/ξ. We fix the scaled shape to be near the maximum of g vs L_τ in order to be insensitive to slight errors in our estimate of z, or to the rounding error because of the requirement that

GRIFFITHS SINGULARITIES IN PARAMAGNETIC PHASE

(Guo, Bhatt, Huse, PRB 54, 3336 (96))



RARE REGION OF SIZE L WITH STRONG BONDS / LESS FRUSTRATION (LOCALLY ORDERED)

$$P(L) \sim \exp(-c_1 L^d)$$

Like 1D Ising Ferro. with coupling $\propto L^d$

$$\therefore \; \xi \sim \exp(c_2 L^d)$$

$$(c_2 \sim 1/T)$$

⇓

LOCAL AUTOCORREL.

$$C(\tau) \sim \int_0^\infty dL \, P(L) \, e^{-\tau/\xi(L)}$$

$$\sim \tau^{-\lambda} \quad \text{power law}$$

Local (on-site) susceptibilities

$$m_i = \sum_{\tau=1}^{L_\tau} S_i(\tau)$$

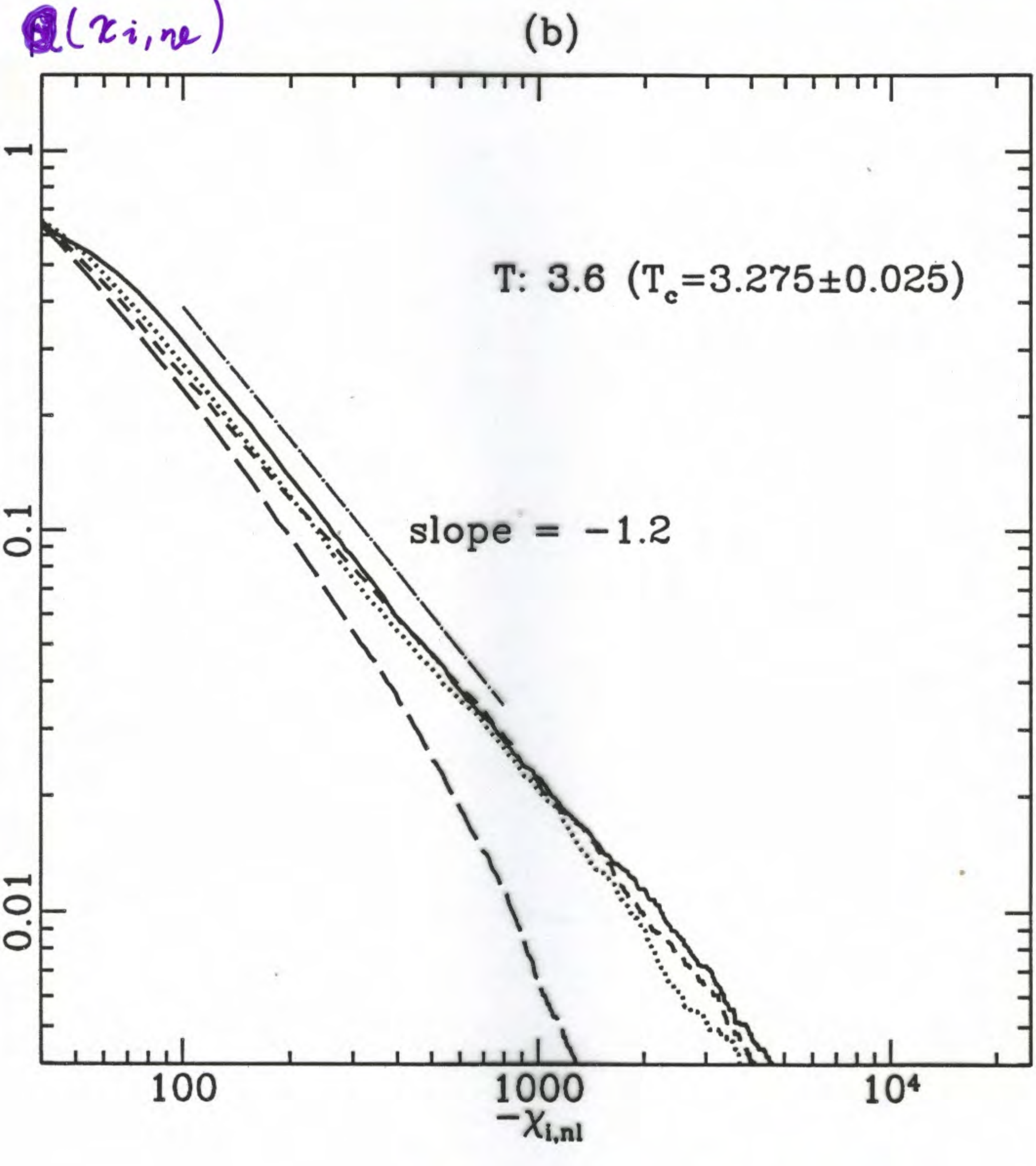
$$\chi_{i,l} = [\langle m_i \rangle^2] / L_\tau$$

$$\chi_{i,nl} = [\langle m_i^4 \rangle - 3\langle m_i^2 \rangle^2] / L_\tau$$

$$\bar{\chi}_{l/nl} = \int \chi_{i,l/nl} \, P(\chi_{i,l/nl}) \, d\chi$$

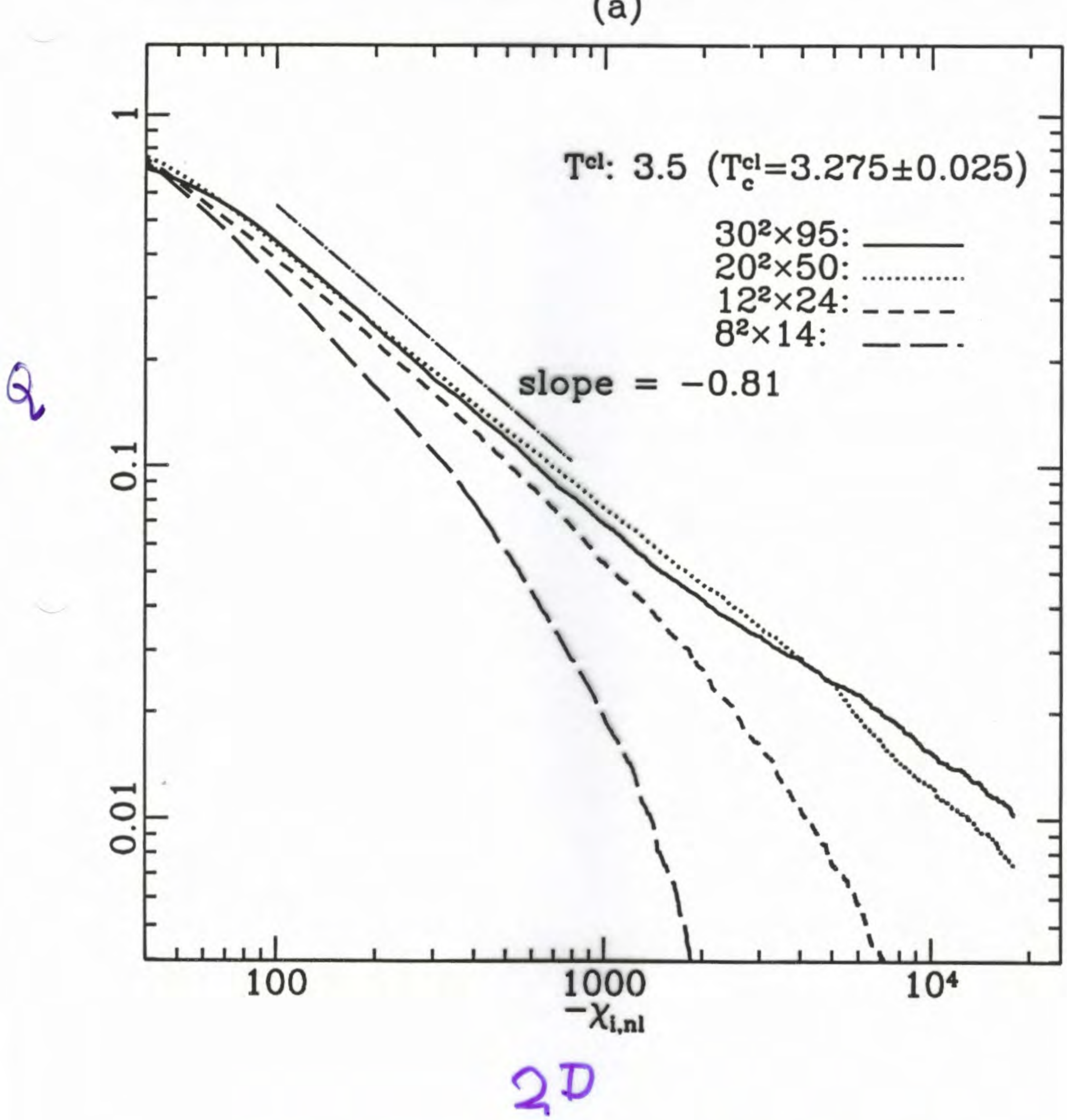
diverges if $P(\chi_i) \sim \chi_i^{-(s+1)}$ at large χ_i with $s < 1$

$$Q(\chi_{i,nl}) = \int P(\chi_{i,nl})\, d\chi_{i,nl} \sim \chi^{-s}$$



2D QUANTUM MODEL

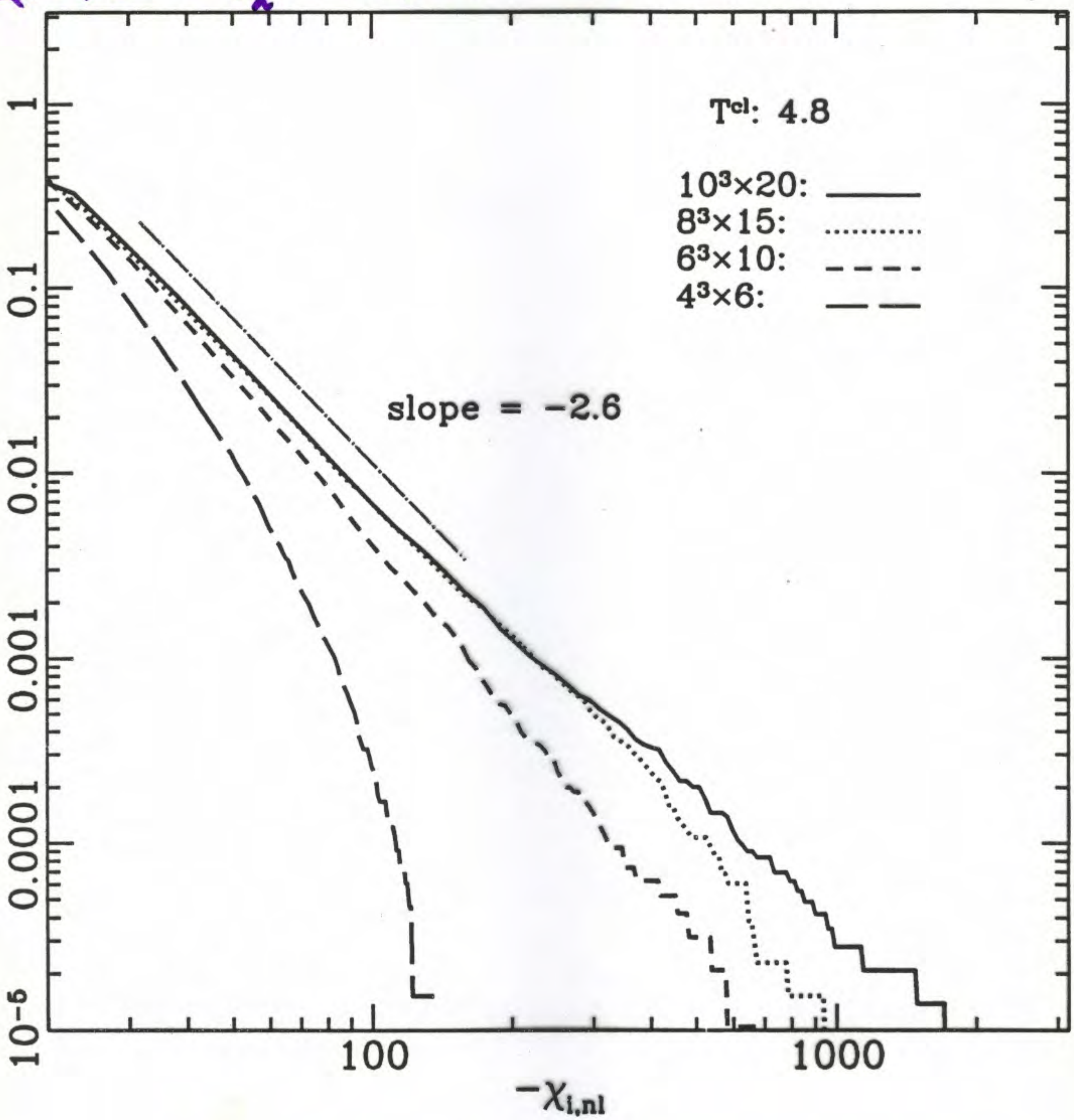
→ 3D Classical Model



s < 1 IMPLIES DIVERGENCE IN

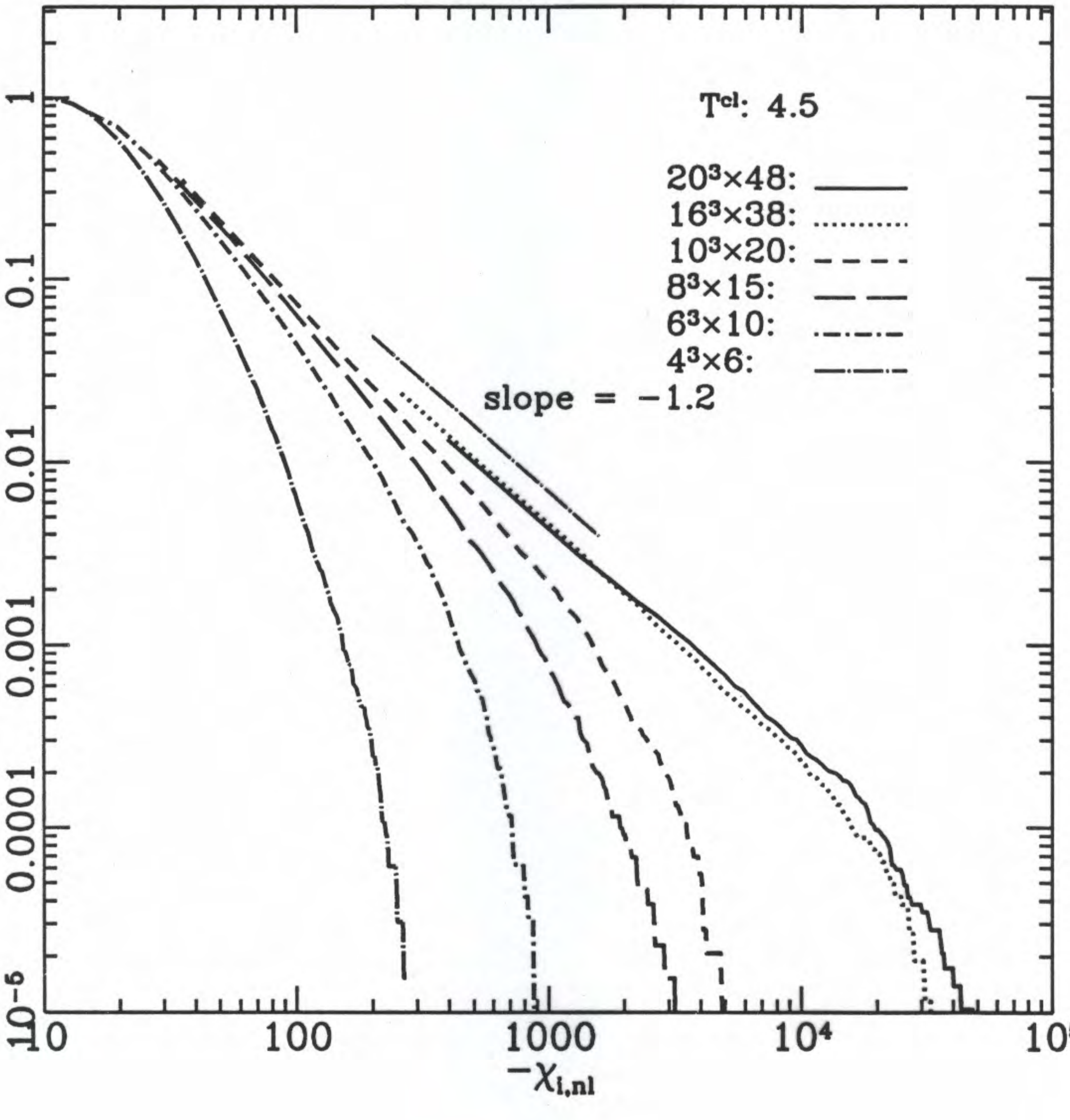
$\overline{\chi_{i,nl}} = \int \chi_{i,nl}\, P(\chi_{i,nl})\, d\chi$ ABOVE T_c

$$Q(\chi_{i,nl}) = \int_{\chi}^{\infty} P(\chi_{i,nl})\, d\chi_{i,nl} \sim \chi^{-s}$$

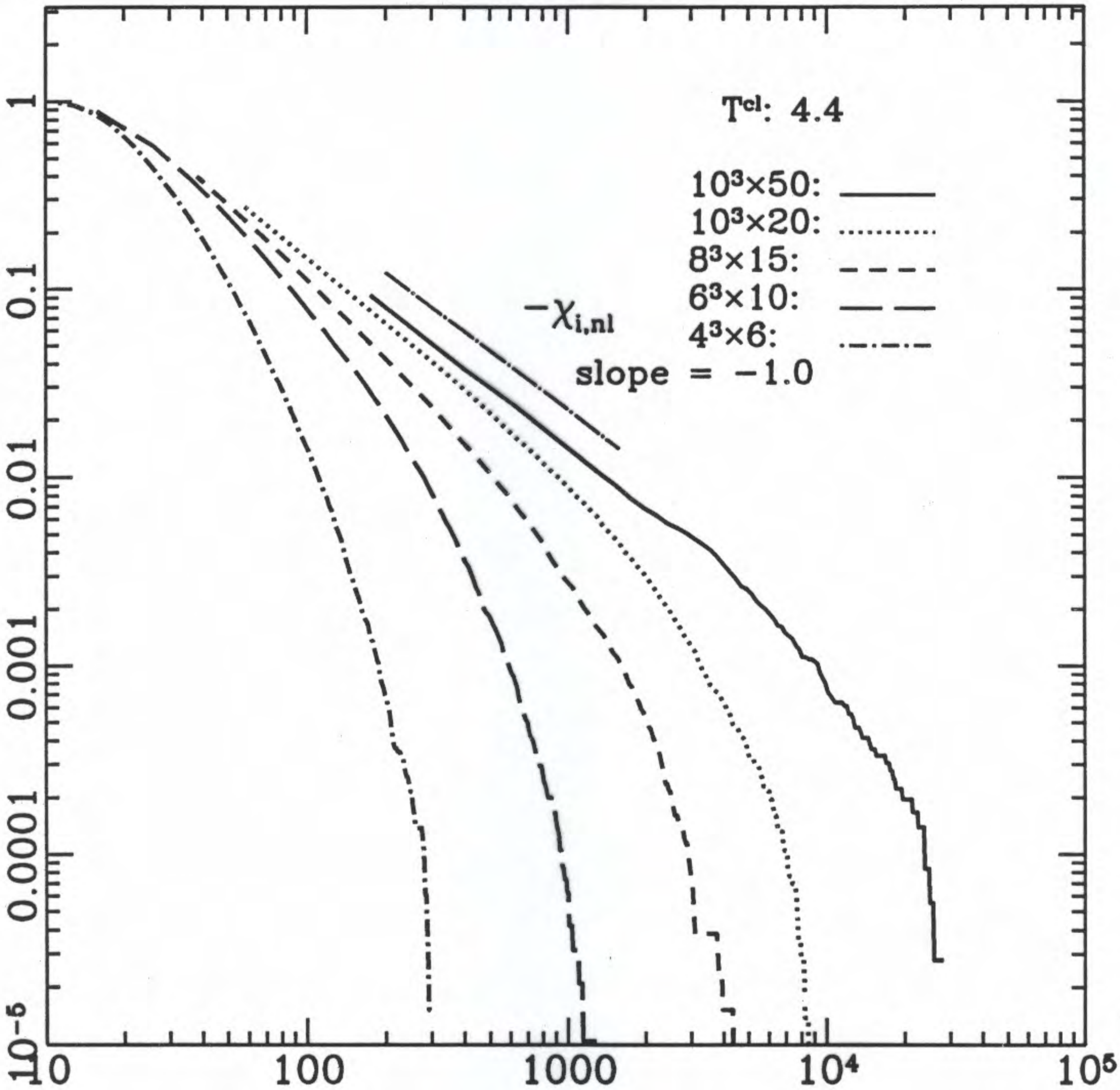


Previous work: $T_c \approx 4.32 \pm 0.03$

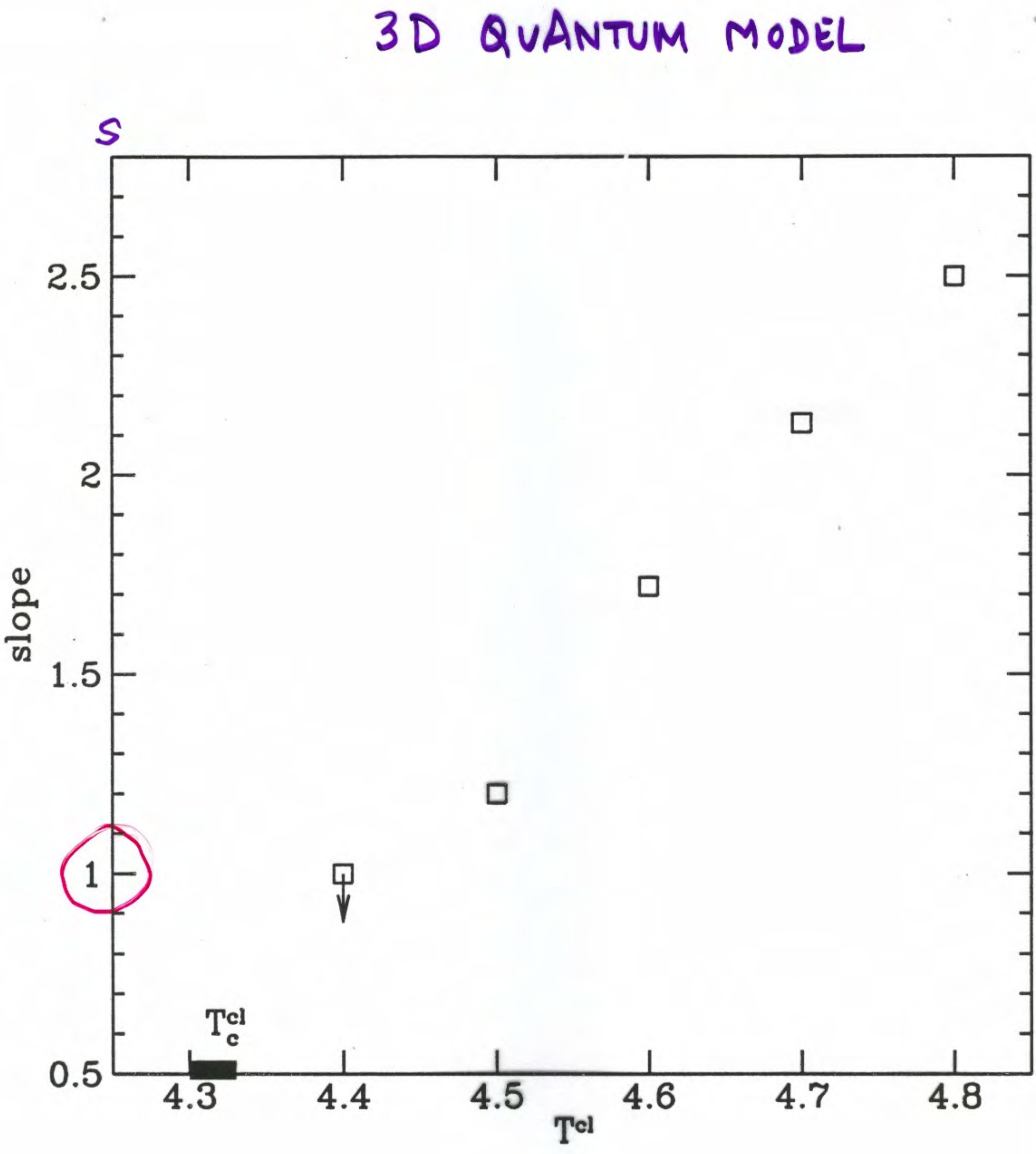
- Massively parallel machines
- Large systems



T^{cl}: 4.5
$20^3\times48$:
$16^3\times38$:
$10^3\times20$:
$8^3\times15$:
$6^3\times10$:
$4^3\times6$:
slope = −1.2
1
0.1
0.01
0.001
0.0001
10^{-5}
10
100
1000
10^4
10^5
$-\chi_{i,nl}$



T^{cl}: 4.4
$10^3\times50$:
$10^3\times20$:
$8^3\times15$:
$6^3\times10$:
$4^3\times6$:
$-\chi_{i,nl}$
slope = −1.0
1
0.1
0.01
0.001
0.0001
10^{-5}
10
100
1000
10^4
10^5


3D QUANTUM MODEL
S
slope
2.5
2
1.5
1
0.5
4.3
4.4
4.5
4.6
4.7
4.8
T^{cl}
T_c^{cl}

RESULT :

Griffiths type rare regions lead to divergent non linear susceptibilities in the paramagnetic phase in both 2+1 & 3+1 d. (Not a 1d phenomenon)

Region smaller as d is increased

~ 9% of T_c for 2+1

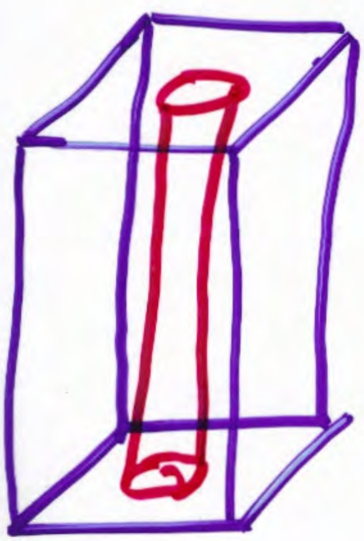
~ 2% of T_c for 3+1

Would muddy the experimental determination of γ, but probably not enough to account for discrepancy.

⇒ DIVERGENCE DUE TO GRIFFITHS EFFECTS — Non universal.

WHY MORE IMPORTANT IN QUANTUM SYSTEMS ?

Statics/Thermodynamics & Dynamics tied.



A rare compact cluster in d-dimensional quantum model appears like a LINE defect in (d+1) dim classical model.

Stronger effects than for point-like defects in purely classical model.

CONCLUDING REMARKS

- **Rare Fluctuations DO matter in (Quantum) Condensed Matter systems with disorder, for a variety of properties – dielectric, transport, magnetic**
- **In 1D quantum spin chains, dominate low-T thermodynamics: (i) divergent magnetic response, (ii) varying exponents, and (iii) separation of phase transition from thermodynamic singularities.**
- **Similar behavior for (ii) and (iii) seen in 2D and 3D for transverse field Ising spin glass, but effect decreases with increasing D.**
- **In 2D quantum Hall transitions, non-Gaussian statistics with power law tails of Hall conductivity**
- **For MI transition in disordered 3D systems, resonant pairs lead to infra-red divergent dielectric response which rides on top of and is part of scaling behavior of the conductivity**
- **Other examples - magnetic response in disordered insulator as well as disordered metal, long time tails in dynamics, disordered superconductors?**

CONCLUDING REMARKS

I. Rare Fluctuation and Large Disorder Effects are consequential, and occur in a variety of aspects of quantum systems – dielectric properties, magnetic behavior, (Hall) conductivity fluctuations, and a host of other phenomena.

II. In dielectric response, “resonant states” dominate low frequency ac conductivity and appear to be part of the scaling description of the Insulator-Metal transition.

III. Magnetic behavior of highly disordered quantum antiferromagnets: “vise-like grip” of the strongest bonds enslaves all spins and sets the hierarchy of ordering in a manner not seen in (classical) Ising spin systems with same bond distribution.

In 1D, system flows to infinite disorder fixed point characterized by large difference between average and typical spin-spin correlations; with dimerization, appearance of Griffiths phase – continuously variable exponents, short range spin correlation.

In higher dimensions for appropriate (dilute) systems, similar behavior persists for several orders of energy – slow meandering in Hamiltonian space – irrelevance of fixed point?

IV The distribution of Hall conductance at a QH step is characterized by long, power-law tails, so the second moment does not exist. This is due to rare instances of eigenvalues being unusually close.

V. The list goes on – this talk has gone over only a few examples.

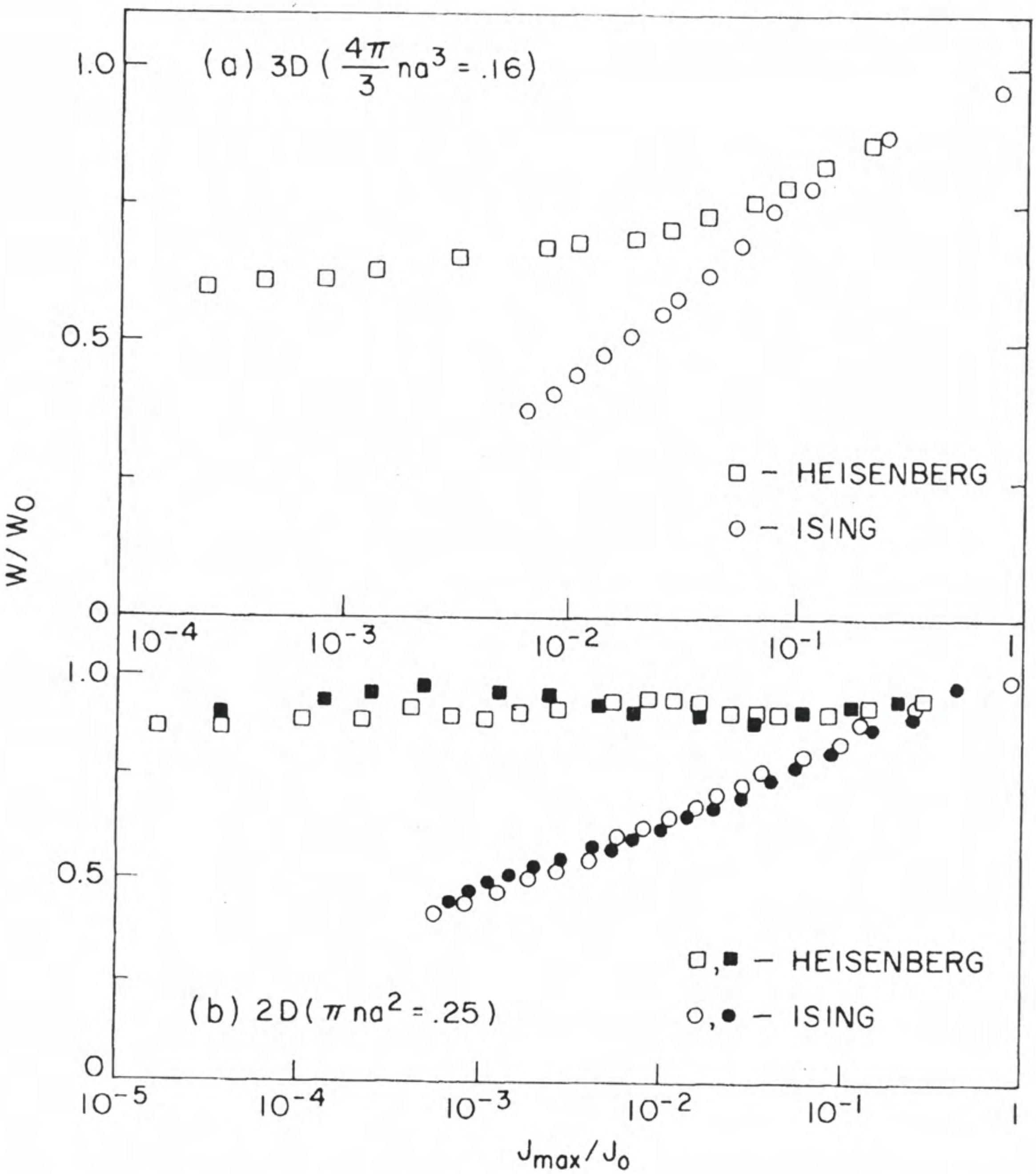


FIG. 2. Width of the distribution of the *logarithm* of the nearest-neighbor coupling for the spatially random Heisenberg and Ising systems as a function of maximum coupling (temperature), normalized to the bare (initial) value in (a) 3D, (b) 2D.