Emergent Radiation in An Atom-Photon System

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Outline

- ***Atom-Photon Problem***
  - *Two-level atom + quantized single-mode radiation*
- ***Unified Boson***
  - *Pauli $\otimes$ Bose = Bose (new representation)*
    - *Or equivalently, Fermi $\otimes$ Bose = Bose*
  - *Emergent radiation at twice-resonance (non-interacting case)*

Outline

- ***Emergent Radiation***
  - *Interacting atom-photon model*
  - *Physical effects (one-photon correlation; population inversion)*
    --- half-harmonic generation
    --- atomic coherent state
  - *Atom-photon **entanglement***
    'Asymptotic' disentanglement
- ***Summary***

References

1. *B. Kumar, J. Phys. A: Math. Theor. 42, 245307 (2009)*
2. *B. Kumar, Phys. Rev. B 77, 205115 (2008) (Appendix B)*

Atom-Photon Problem

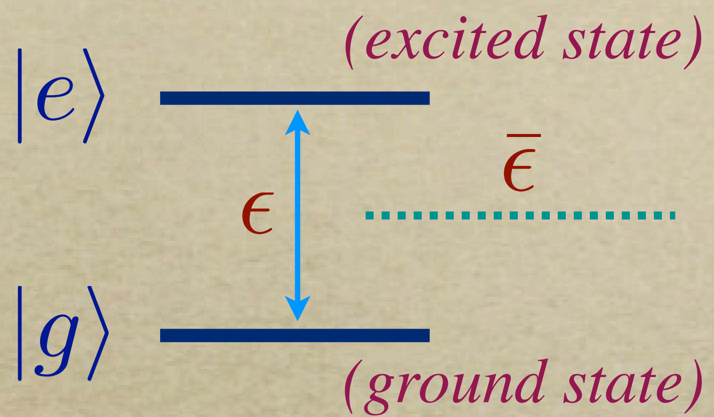
A quick introduction.

Comprehensive Refs. *Mandel and Wolf (1995)*
Scully and Zubairy (1997)

Atom-Field Problem

Two-Level Atom

$$H_{atom} = \frac{\epsilon}{2}\sigma^z + \bar{\epsilon}\,\mathbb{I}$$



Pauli Operators

$$\sigma^z = |e\rangle\langle e| - |g\rangle\langle g|$$
$$\sigma^+ = |e\rangle\langle g|$$
$$\sigma^- = |g\rangle\langle e|$$

$$\mathbb{I} = |e\rangle\langle e| + |g\rangle\langle g|$$

(Identity)

Atom-Field Problem

Quantized Electromagnetic Radiation

Electromagnetic Field Hamiltonian

$$H_{Field} = \sum_{\mathbf{k},\lambda} \omega_{\mathbf{k}\lambda} \left(\hat{b}^{\dagger}_{\mathbf{k}\lambda} \hat{b}_{\mathbf{k}\lambda} + \frac{1}{2} \right)$$

Single-Mode Radiation

$$H_{Field} = \omega \left(\hat{b}^{\dagger} \hat{b} + \frac{1}{2} \right)$$

Basis: $\{|m\rangle, \forall m = 0, 1, \cdots, \infty\}$ *(Fock states)*

Atom-Field Problem

Atom-Photon Interaction

In the long-wavelength approximation

(Rabi, 1937)

$$V_{dipole} = g\left(\hat{b}^\dagger + \hat{b}\right)\left(\sigma^+ + \sigma^-\right)$$

$$= g\left(\hat{b}^\dagger\sigma^- + \hat{b}\sigma^+\right) + g\left(\hat{b}^\dagger\sigma^+ + \hat{b}\sigma^-\right)$$

Slow processes *Fast processes*

Close to resonance, ignore fast processes

$$V_{JC} = g\left(\hat{b}^\dagger\sigma^- + \hat{b}\sigma^+\right)$$

Rotating Wave Approximation
(Jaynes and Cummings, 1969)

Interaction in the present study is
similar to V_{dipole}, but not same.

Emergent Radiation

Unified boson representation.
Emergent radiation.

Unified Boson

New Representation

$$\hat{a}^\dagger = \sqrt{2}\left[\sqrt{\hat{b}^\dagger\hat{b} + \frac{1}{2}}\sigma^+ + \hat{b}^\dagger\sigma^-\right]$$

$\hat{a}^\dagger, \hat{a}$: *new Bose operators*

$\vec{\sigma}$: *atomic operators*

$\hat{b}^\dagger, \hat{b}$: *radiation operators*

Hilbert Space Mapping

$$|n = 2m\rangle \quad := \quad |m\rangle \otimes |g\rangle$$

$$|n = 2m+1\rangle \quad := \quad |m\rangle \otimes |e\rangle$$

unified boson Fock states

Canonical & Invertible; Non-linear

Unified Boson

Inverse Representation

Pauli Operators

$$\sigma^z = -(-1)^{\hat{N}} := -\hat{\chi}$$

$$\sigma^+ = \frac{1-\hat{\chi}}{2} \frac{1}{\sqrt{\hat{N}}} \hat{a}^\dagger$$

first such 'unconstrained' bosonic representation for spin-1/2

Original Bose operators

$$\hat{b}^\dagger = \frac{\hat{a}^\dagger \hat{a}^\dagger}{\sqrt{2}} \left(\frac{1-\hat{\chi}}{2} \frac{1}{\sqrt{\hat{N}+2}} + \frac{1+\hat{\chi}}{2} \frac{1}{\sqrt{\hat{N}+1}} \right)$$

Unified Boson

Generalizations

Multi-level atoms

$$\hat{a}^\dagger = S^z S^+ \sqrt{\frac{3}{2}\hat{b}^\dagger \hat{b} + 1} \; - \; S^+ S^z \sqrt{\frac{3}{2}\hat{b}^\dagger \hat{b} + \frac{1}{2}} \; + \; \frac{\sqrt{3}}{2}\hat{b}^\dagger S^- S^-$$

(for a 3-level atom; S^z, $S^\pm$ are spin-1 operators)

Many 2-level atoms

Any number of Fermions can be generated out of a single bosonic mode.

$$\hat{f}_l^\dagger \sim \left(\frac{1}{\sqrt{\hat{N}}} \hat{a}^\dagger \right)^l$$

Unified Boson

Typical Problem in Terms of the Unified Boson

(for example, the dipole interaction model)

Non-interacting part

$$\omega\left(\hat{b}^\dagger\hat{b}+\frac{1}{2}\right)+\frac{\epsilon}{2}\sigma^z=\frac{\omega}{2}\left(\hat{a}^\dagger\hat{a}+\frac{1}{2}\right)+\frac{\omega-2\epsilon}{4}\hat{\chi}$$

Interaction

$$\hat{b}^\dagger\sigma^-+\hat{b}\sigma^+=\frac{1}{\sqrt{2}}\left(\hat{a}^\dagger\frac{1-\hat{\chi}}{2}+\frac{1-\hat{\chi}}{2}\hat{a}\right)$$

Slow processes

$$\hat{b}^\dagger\sigma^++\hat{b}\sigma^-=\frac{\hat{a}^\dagger\hat{a}^\dagger\hat{a}^\dagger}{\sqrt{2}}\frac{1}{\sqrt{(\hat{N}+3)(\hat{N}+1)}}\frac{1+\hat{\chi}}{2}+h.c.$$

Fast processes

Highly non-linear !

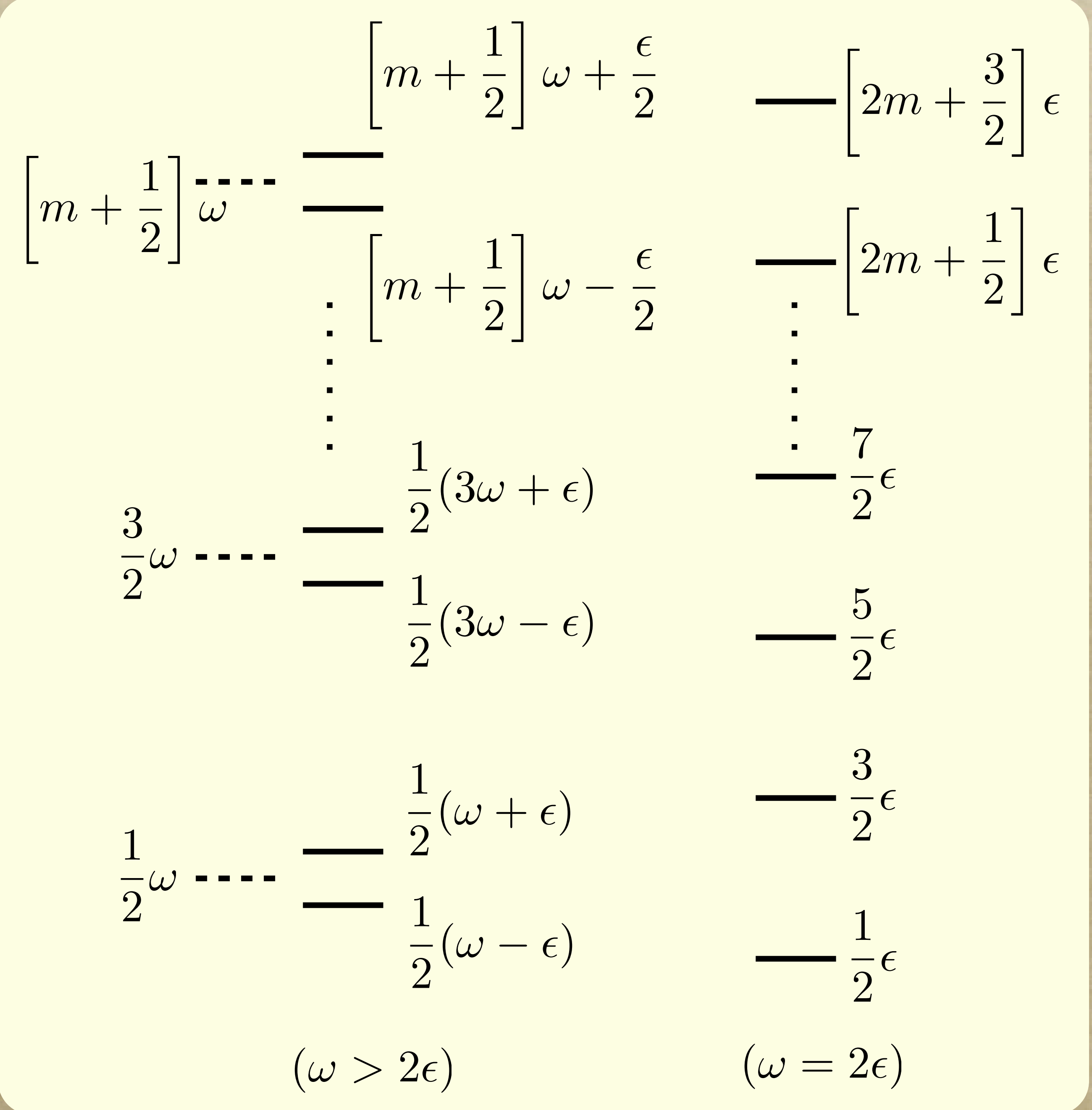
Emergent Radiation

Non-Interacting Case

$$2\epsilon\left(\hat{b}^\dagger\hat{b}+\frac{1}{2}\right)+\frac{\epsilon}{2}\sigma^z$$

$$=\epsilon\left(\hat{a}^\dagger\hat{a}+\frac{1}{2}\right)$$

twice-resonant
non-interacting system
is same as
'free' unified radiation



Emergent Radiation

Interacting Atom-Photon Model

$$H_0 = 2\epsilon H_b + \frac{\epsilon}{2}\sigma^z + \xi\sqrt{2}\left[\sqrt{H_b}\sigma^x + \left(\hat{b}^\dagger\sigma^- + \hat{b}\sigma^+\right)\right]$$

Non-interacting

New Interaction.
Scalar type "Dipole-radiation" interaction !

Jaynes-Cummings interaction

$$H_b = \hat{b}^\dagger\hat{b} + \frac{1}{2}$$

Surprisingly, we could show that it is a ***unitary cousin*** *of the* ***fast terms*** *in V_{dipole}.*

Emergent Radiation

Emergent Radiation in the Interacting Model

$$H_0 \;=\; 2\epsilon H_b + \frac{\epsilon}{2}\sigma^z + \xi\sqrt{2}\left[\sqrt{H_b}\sigma^x + \left(\hat{b}^\dagger\sigma^- + \hat{b}\sigma^+\right)\right]$$

$$= \; \epsilon\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) - \xi\left(\hat{a}^\dagger + \hat{a}\right)$$

$$\xrightarrow{\mathcal{D}} \epsilon\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) - \frac{\xi^2}{\epsilon}$$

$$\mathcal{D}(x) = e^{-x\left(\hat{a}^\dagger - \hat{a}\right)}$$

where $x = \frac{\xi}{\epsilon}$

$$|\psi_n(x)\rangle = \mathcal{D}(x)|n\rangle$$

Eigen-spectrum $$E_n = \epsilon\left(n + \frac{1}{2}\right) - \frac{\xi^2}{\epsilon}$$

'free' unified radiation

Emergent Radiation

Interacting Atom-Photon Model

Consider the fast terms in V_{dipole}.

$$\hat{b}^\dagger \sigma^+ + \hat{b}\sigma^-$$

$$= \sqrt{\hat{M}} e^{-i\hat{\Phi}} \sigma^+ + \sigma^- e^{i\hat{\Phi}} \sqrt{\hat{M}}$$

For the unitary operator

$$\mathcal{U}_{\hat{\Phi}} = e^{-\frac{i}{2}\sigma^z \hat{\Phi}}$$

$$\mathcal{U}^\dagger_{\hat{\Phi}} \left(\hat{b}^\dagger \sigma^+ + \hat{b}\sigma^- \right) \mathcal{U}_{\hat{\Phi}} = \sqrt{H_b} \sigma^x$$

Number-Phase representation

$$\hat{b}^\dagger = \sqrt{\hat{M}} e^{-i\hat{\Phi}}$$

$$\hat{M} = \hat{b}^\dagger \hat{b}$$

$$\left[\hat{M}, \hat{\Phi} \right] = i$$

$\hat{M}$ & $\hat{\Phi}$ are Hermitian.

The unfamiliar term in our interaction.

Physical Effects

Half-harmonic generation.
Atomic coherent states.

Half-Harmonics

One-Photon Correlation

Original radiation electric field

$$E_b(t) \propto \hat{b}^\dagger(t) + \hat{b}(t)$$

One photon correlation function

$$\mathcal{G}(t) = \left\langle \left[\hat{b}^\dagger(t) + \hat{b}(t)\right] \left[\hat{b}^\dagger(0) + \hat{b}(0)\right] \right\rangle$$

Heisenberg time dependence

$$\hat{b}(t) = e^{iH_0 t}\hat{b}e^{-iH_0 t}$$

$$\langle \hat{O} \rangle = tr\{\hat{\rho}\hat{O}\}$$

Expectation

Half-Harmonics

One-Photon Correlation

Weak-coupling limit: $\xi \ll \epsilon$

$$\mathcal{G}(t) \approx \langle \hat{b}^\dagger \hat{b} \rangle e^{i2\epsilon t} + \langle \hat{b} \hat{b}^\dagger \rangle e^{-i2\epsilon t} + 2\frac{\xi^2}{\epsilon^2} \{ e^{i\epsilon t} [\langle \sigma^+ \sigma^- \rangle + B_1 \langle \sigma^- \sigma^+ \rangle] + ((e^{-i\epsilon t})) + ((e^{\pm i\epsilon t})) \} + \mathcal{O}(x^3)$$

Strong-coupling limit: $\xi \gg \epsilon$

Half-harmonic generation.

$$\mathcal{G}(t) \approx c_0 + \frac{1}{2} \left[\langle \hat{a}^\dagger \hat{a} \rangle e^{i\epsilon t} + \langle \hat{a} \hat{a}^\dagger \rangle e^{-i\epsilon t} \right] + \frac{9}{32x^2} \left[\langle \hat{a}\hat{a}\hat{a}^\dagger \hat{a}^\dagger \rangle e^{-i2\epsilon t} + \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a}\hat{a} \rangle e^{i2\epsilon t} \right]$$

original mode is sub-dominant in the strong coupling limit.

obvious similarity, but clear differences, with the parametric down-conversion.

Atomic-Coherent State

Population Inversion

$|n=0\rangle = |m=0\rangle \otimes |g\rangle$ ***Initial states*** $|n=1\rangle = |m=0\rangle \otimes |e\rangle$

$W_0(t) = \langle 0|\sigma^z(t)|0\rangle$ ***Population inversion*** $W_1(t) = \langle 1|\sigma^z(t)|1\rangle$

$$W_0(t) = -e^{-4x^2(1-\cos\epsilon t)} \qquad (x = \xi/\epsilon)$$

$$W_1(t) = \left[1 - 8x^2\left(1 - \cos\epsilon t\right)\right] e^{-4x^2(1-\cos\epsilon t)}$$

Rabi frequency is coupling independent !!

Strong coupling case (x>>1)

$$W_0(t) = -W_1(t) = -\delta_{t=t_l} \qquad (t_l = 2\pi l/\epsilon, \quad l = 0, 1, \cdots)$$

Atom lives in a coherent state!

Entanglement

Atom-photon entanglement in the emergent radiation state.

'Asymptotic' disentanglement & atomic coherent state.

Entanglement

Measure of Bipartite Entanglement in a Pure State

deviation from Idempotency

$$\mathcal{E} = 1 - tr\hat{\rho}_A^2 = 1 - tr\hat{\rho}_R^2$$

where

$$\hat{\rho}_A = tr_R\hat{\rho}$$ *reduced density matrix of the Atom*

$$\hat{\rho}_R = tr_A\hat{\rho}$$ *reduced density matrix of the Radiation*

We compute entanglement in the lowest emergent radiation state.

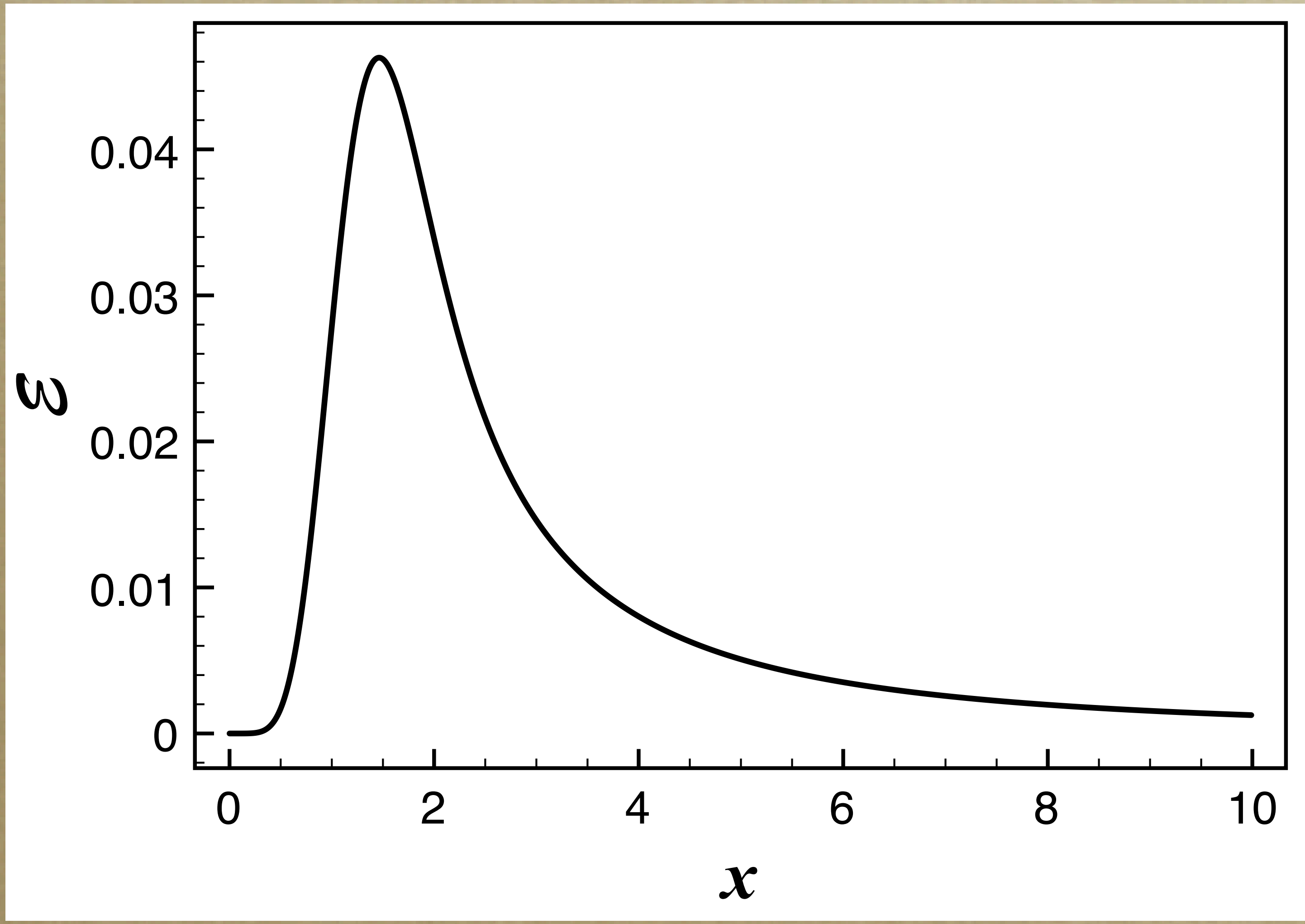
$$|\psi_0\rangle = e^{-x(\hat{a}^\dagger - \hat{a})}|0\rangle$$

$$= e^{-\frac{x^2}{2}} \sum_{m=0}^{\infty} \frac{x^{2m}}{\sqrt{(2m)!}} |m\rangle \otimes \left[|g\rangle - \frac{x}{\sqrt{2m+1}} |e\rangle \right]$$

Entanglement

<u>*Entanglement in |ψ₀⟩*</u>

$$\mathcal{E}(x) = \frac{1}{2}\left(1 - e^{-4x^2}\right) - 2\mathcal{A}^2(x)e^{-2x^2}$$



$$\mathcal{A}(x) = x\sum_{m=0}^{\infty}\frac{x^{4m}}{(2m)!}\frac{1}{\sqrt{2m+1}}$$

Asymptotic Disentanglement

$\mathcal{E} \to 0$ ***as*** $x \to \infty$

Atomic Coherent State $\frac{|g\rangle - |e\rangle}{\sqrt{2}}$

Entanglement

“ Quantum Information ”

Emergent radiation is an interesting ‘quantum informatic’ state, in which the information ‘qubit’ and the carrier ‘light’ are integrated into a whole which itself behaves as ‘light’.

Summary

Summary

- *Presented a canonical and invertible unified boson representation. Bose $\bigotimes$ Pauli = Bose*
- *Introduced and discussed the 'emergent' radiation in an atom-photon system at twice-resonance.*
- *The interaction, which supports the emergent radiation behavior, has surprising resemblance and partial connections with the V_{dipole}. It's different however.*
- *Physically, the emergent radiation implies a half-harmonic effect, especially in the strong coupling limit.*

Summary

- *The population inversion oscillates, but the frequency of its oscillation is coupling independent, in contrast to the usual Rabi oscillations.*
- *The atom-photon entanglement in the interacting case varies non-monotonically.*
- *Asymptotically, atom & radiation disentangle into an atomic coherent state and a half-harmonic state.*
- *The present idea is also applicable to spin-resonance or any spin-boson system.*