Writing and reading spin information on mobile electronic qubits: A new path to quantum computing?

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ICMP09, Mahabaleshwar, Dec. **2009**

Writing and reading spin information on mobile electronic qubits

Outline

Spintronics, quantum computing

Spin-Orbit interaction, Aharonov-Bohm effect

Spin field effect transistor

Spin filter: writing information on electron spinor

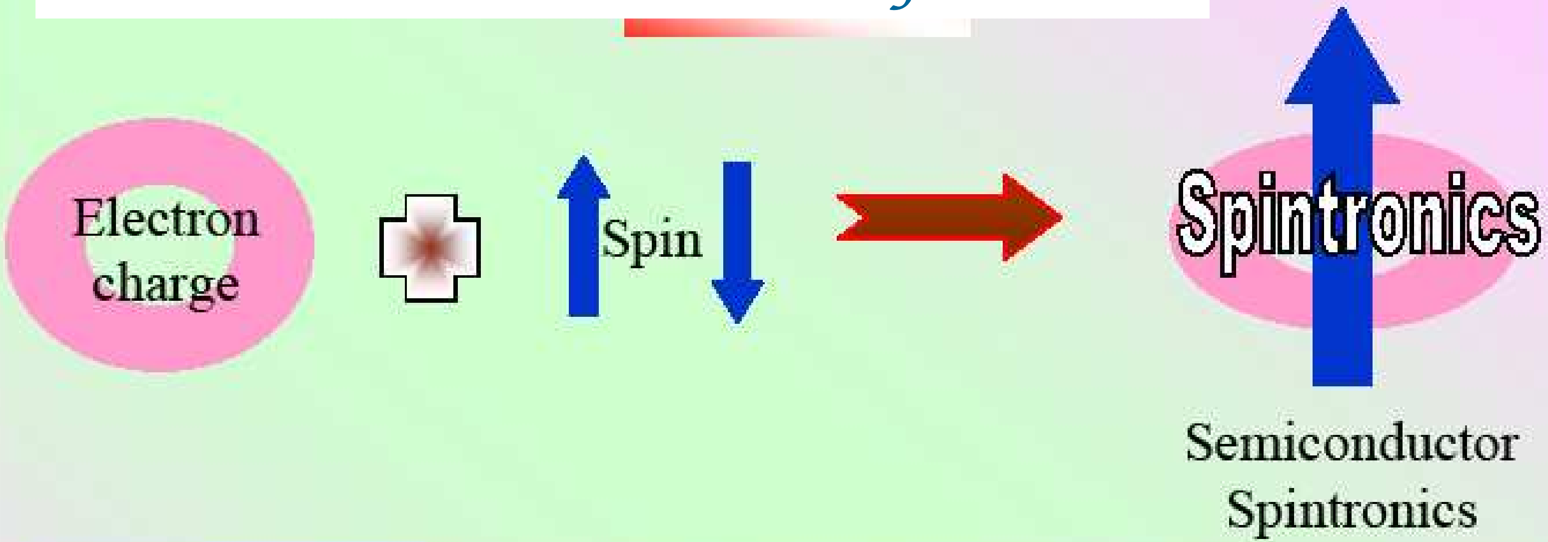
Quantum networks

Our spin filters: simple exercises in quantum mechanics

Our spin 'reader': measuring spinor via conductance

Conclusions

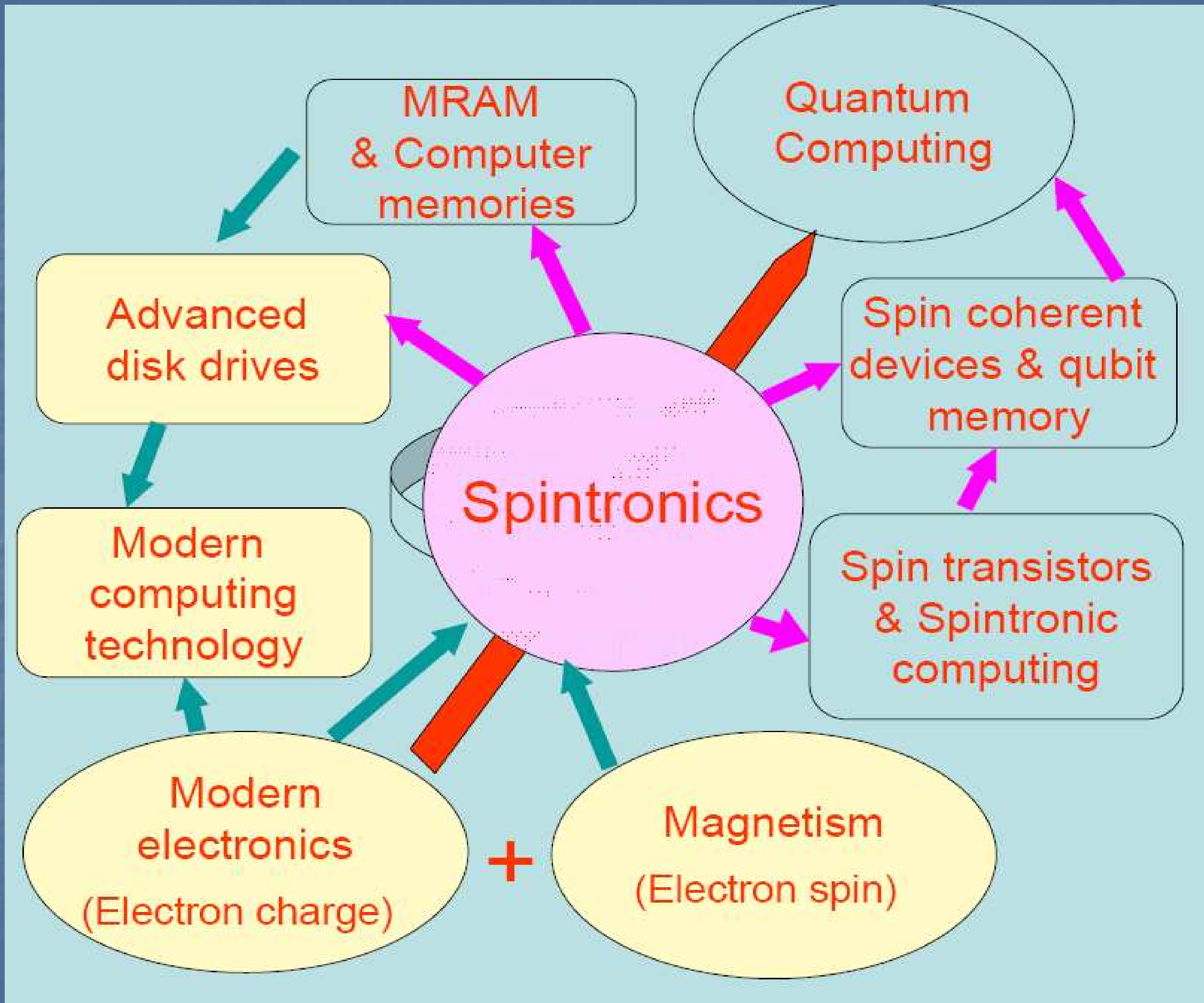
Alternative to electronics: spintronics



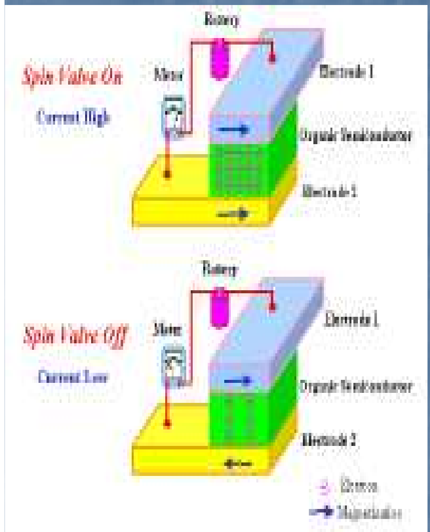
Electronic charge and spin can be merged together to form a new class of electronics called **Spintronics**

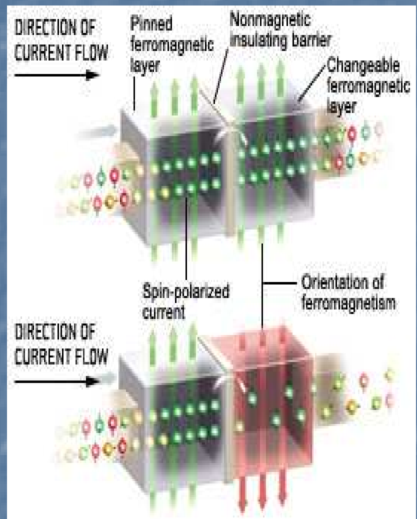
Data storage and information processing can be combined in a single device by exploiting spin and charge degrees of freedom of electrons simultaneously.

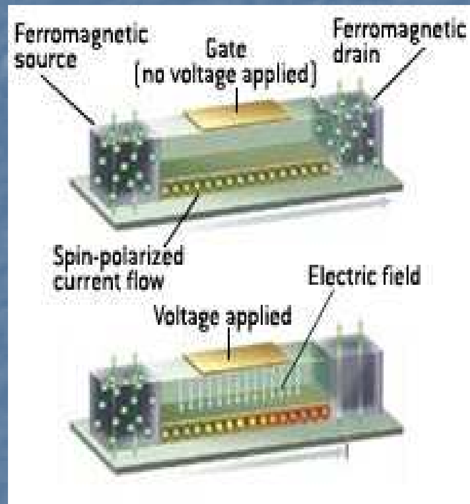
Spintronic devices will be smaller, more versatile and more robust than those currently making up silicon chips and circuit elements.

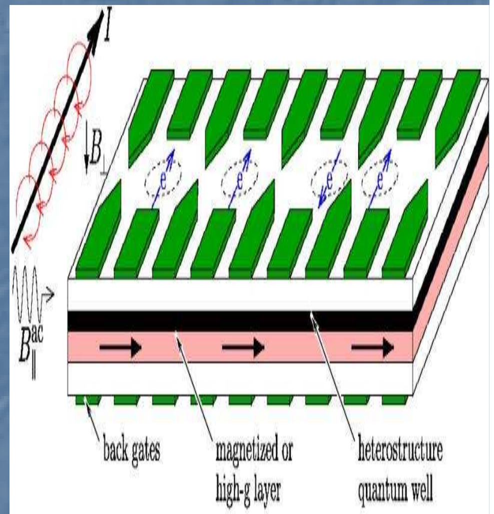

MRAM
& Computer
memories
Quantum
Computing
Advanced
disk drives
Spin coherent
devices & qubit
memory
Spintronics
Modern
computing
technology
Spin transistors
& Spintronic
computing
Modern
electronics
(Electron charge)
+
Magnetism
(Electron spin)

Spintronics: Exploiting the electron spin for information processing









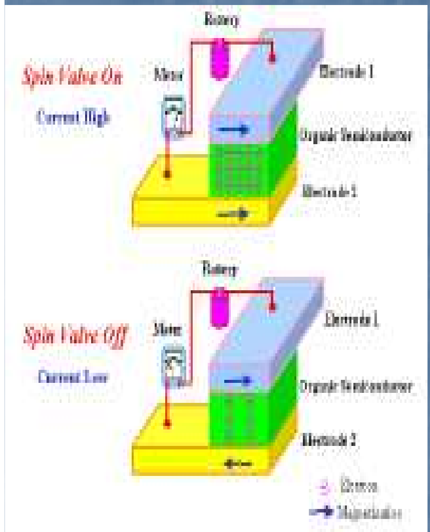
GMR valves (metals)

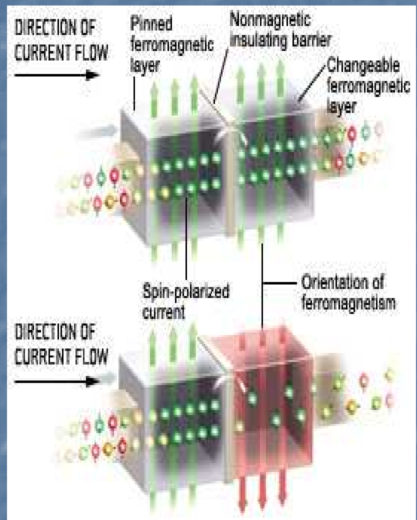
Magnetic tunnel junctions

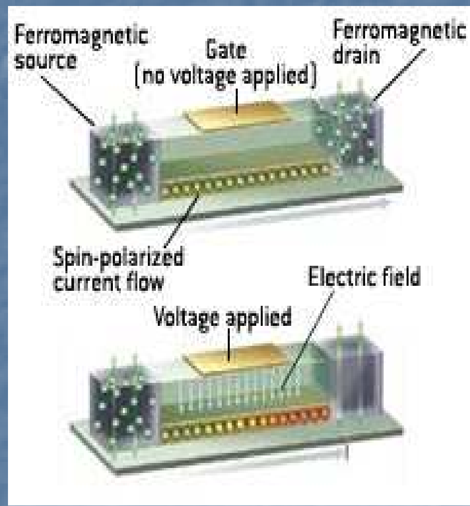
Semiconductor spintronic devices

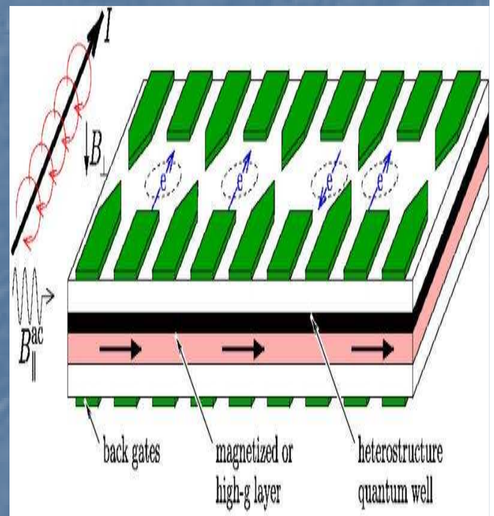
Static Spin qubits

Spintronics: Exploiting the electron spin for information processing









GMR valves (metals)

Magnetic tunnel junctions

Semiconductor spintronic devices

Spin qubits

Quantum mechanics:

Particle-wave duality

Schrödinger's wave equation

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},\,t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},\,t) + V(\mathbf{r})\Psi(\mathbf{r},\,t)$$

Dirac's equation: spin and spinor

$$\left(i\hbar\gamma^\mu\partial_\mu - mc\right)\psi = 0$$

Quantum mechanics:

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$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},\,t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},\,t) + V(\mathbf{r})\Psi(\mathbf{r},\,t) + \frac{\hbar}{(2M_0c)^2}\nabla V(\mathbf{r})(\hat{\boldsymbol{\sigma}}\times\hat{\mathbf{p}})\Psi(\mathbf{r},\,t)$$

Quantum computers

Conventional computers: information in **bits**,

0 or 1, +1 or -1, ⬆ or ⬇

Quantum computers: information in **Qubits**,

Electron described by **spinor**:

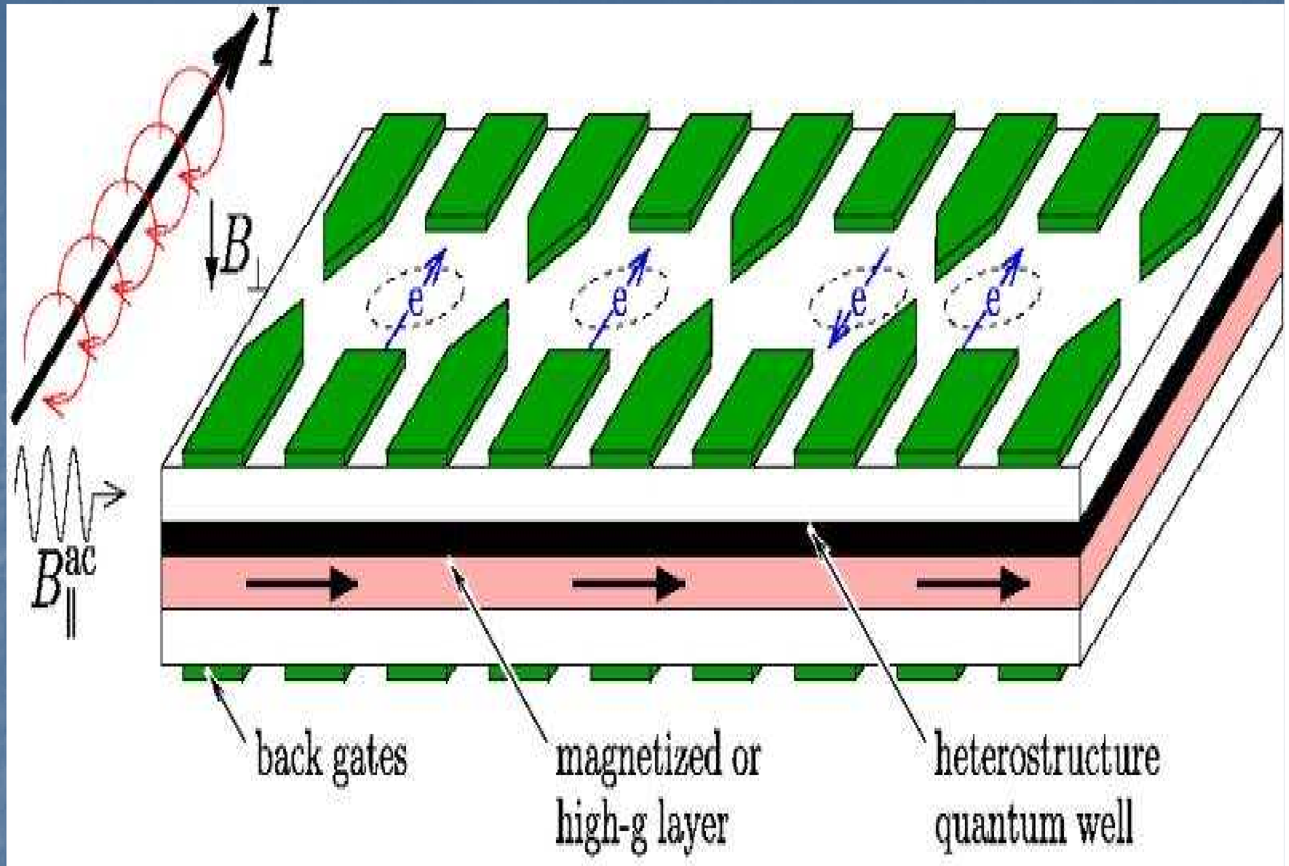
$$\psi = \alpha \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Complex numbers

Spinor is an eigenvector of $\mathbf{n} \cdot \boldsymbol{\sigma}$, the spin component along $\mathbf{n}$

$\alpha, \beta \Leftrightarrow \mathbf{n}$

Static qubits:



Here we discuss **mobile** qubits, in **mesoscopic semiconductor** devices

The Aharonov-Bohm (AB) Effect

- Classical Physics, e.g. Lorentz force

$$m\left(\frac{d^2\vec{r}}{dt^2}\right) = -e\left[\vec{E} + \frac{\vec{v}}{c}\times\vec{B}\right]$$

- Quantum Physics, Schrödinger equation

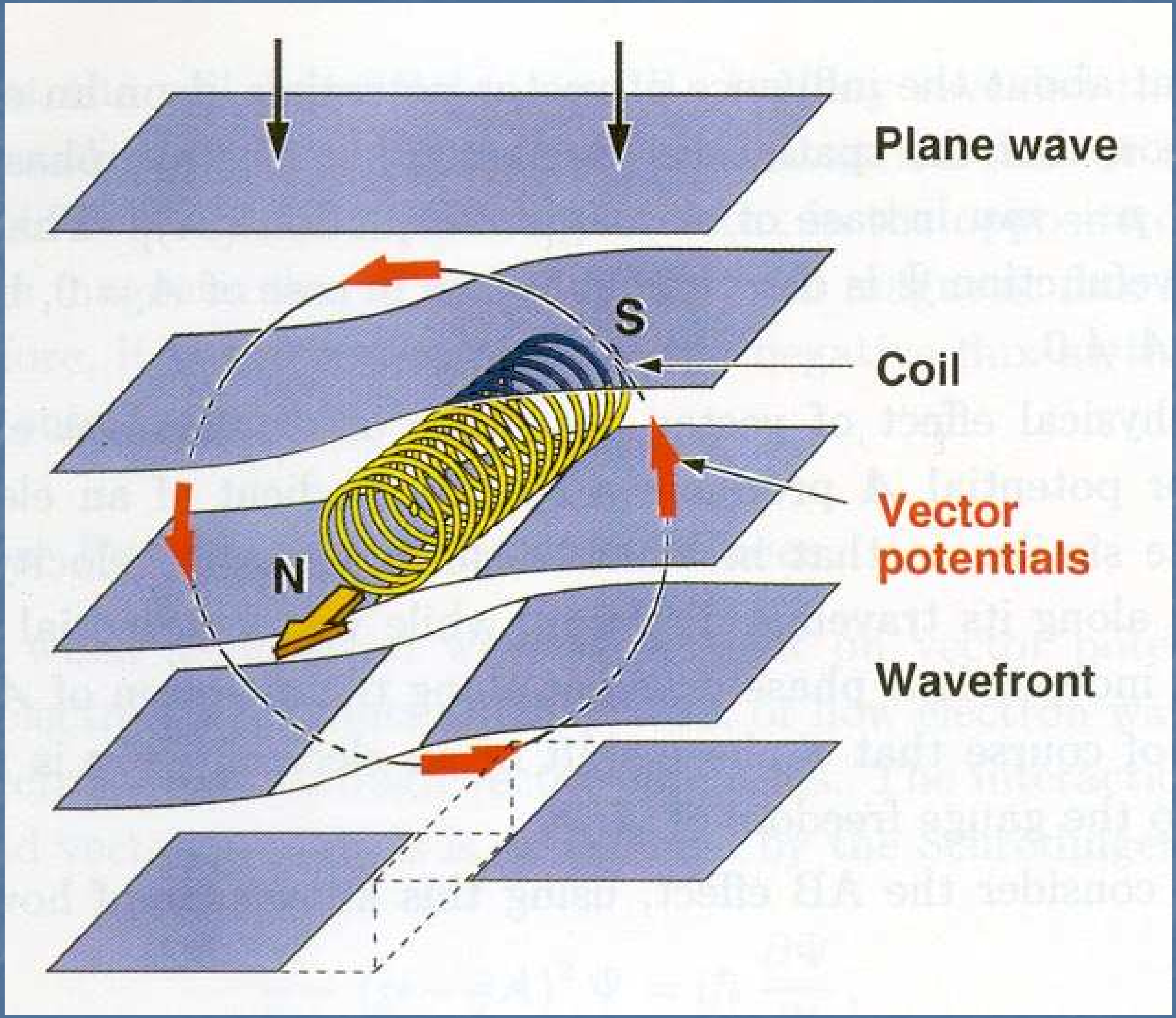
$$(H_o + V)\Psi = \frac{i\hbar\partial\Psi}{\partial t}$$

  - $V = -eED$ and $H_o = \frac{p^2}{2m}$, where $\vec{p} = m\vec{v} + \frac{e\vec{A}}{c}$
  - with E electric field, D electrode's separation

- Aharonov and Bohm (AB), *Phys.Rev.* **115**, 485 (1959)

- Phase Shift $\Delta\phi = \frac{1}{\hbar}\int L dt = \frac{1}{\hbar}\int\left(m\vec{v} + \frac{e\vec{A}}{c}\right).d\vec{s} - \frac{1}{\hbar}\int eEDdt$

A. Tonomura



Spin-orbit interactions

Dirac:

$$\hat{H}_{SO} = \frac{\hbar}{(2M_0c)^2} \nabla V(\mathbf{r})(\hat{\boldsymbol{\sigma}} \times \hat{\mathbf{p}}).$$

Entin-Wohlman, Gefen, Meir, Oreg (1989, 1992)

$$\frac{1}{2m}(\mathbf{p} + e\,\mathbf{A}_{\text{s.o.}}/c)^2, \quad \mathbf{A}_{\text{s.o.}} = (\hbar/4mc)\boldsymbol{\sigma} \times \mathbf{E}$$

$$\psi(x+L) = \exp[i\lambda \mathbf{d}\cdot\boldsymbol{\sigma}/2 + 2\pi\Phi/\Phi_0]\psi(x)$$

Aharonov-Casher

Spin-orbit interactions

Dirac:

$$\hat{H}_{SO} = \frac{\hbar}{(2M_0c)^2}\nabla V(\mathbf{r})(\hat{\boldsymbol{\sigma}}\times\hat{\mathbf{p}}).$$

Rashba: 2DEG, confined to a plane by an asymmetric potential along z:

$$\mathcal{H}_{\rm SO}^{\rm R} = \alpha_R(p_y\sigma_x - p_x\sigma_y),$$

Strength of Rashba term can be tuned by gate voltage!

A spinor ψ entering from the left
and travelling a distance *L* along the *x*-axis
will be multiplied by the 2x2 unitary matrix

$$e^{-i\alpha\sigma_y}$$

$$\alpha = k_{SO}L$$

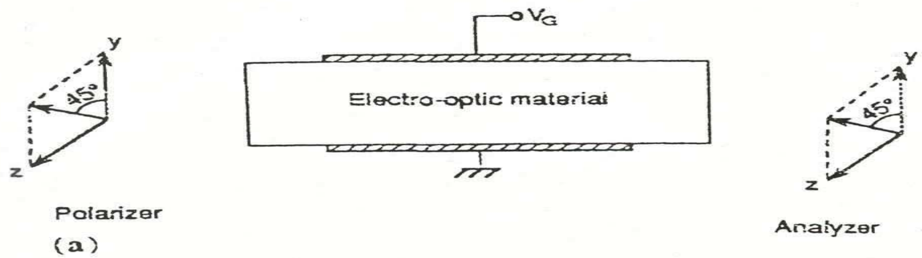
Rotation of spin direction around *y*-axis

Spin field effect transistor

Electronic analog of the electro-optic modulator

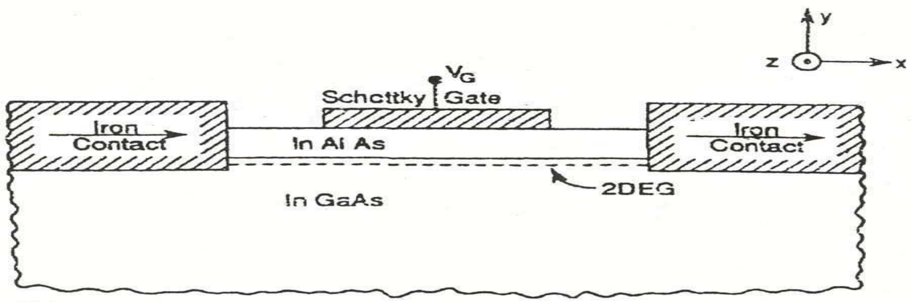
Supriyo Datta and Biswajit Das
School of Electrical Engineering, Purdue University, West Lafayette, Indiana 47907

Appl. Phys. Lett. 56 (7), 665   12 February 1990



(a)

$$\underset{(45^\circ\ \text{pol.})}{\begin{pmatrix}1\\1\end{pmatrix}} = \underset{(z\ \text{pol.})}{\begin{pmatrix}1\\0\end{pmatrix}} + \underset{(y\ \text{pol.})}{\begin{pmatrix}0\\1\end{pmatrix}} . \qquad P_0 \propto \left| (1\ \ 1)\begin{pmatrix}e^{ik_1L}\\e^{ik_2L}\end{pmatrix}\right|^2 = 4\cos^2\frac{(k_1-k_2)L}{2} .$$



(b)

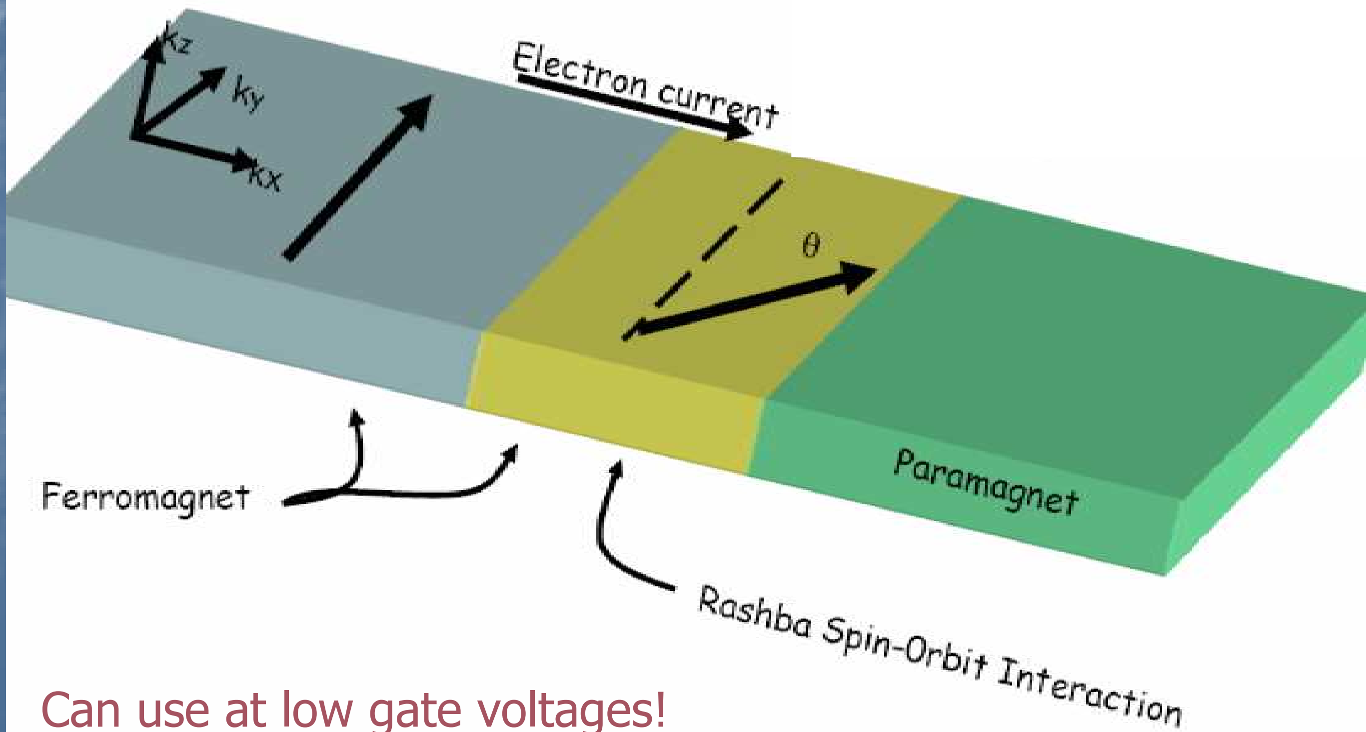
FIG. 1. (a) Electro-optic modulator; (b) proposed electron wave analog of the electro-optic modulator.

$$\underset{(+x\ \text{pol.})}{\begin{pmatrix}1\\1\end{pmatrix}} = \underset{(+z\ \text{pol.})}{\begin{pmatrix}1\\0\end{pmatrix}} + \underset{(-z\ \text{pol.})}{\begin{pmatrix}0\\1\end{pmatrix}} .$$

Das and Datta (1990): The **Spin field effect transistor**

Tunable with gate voltage

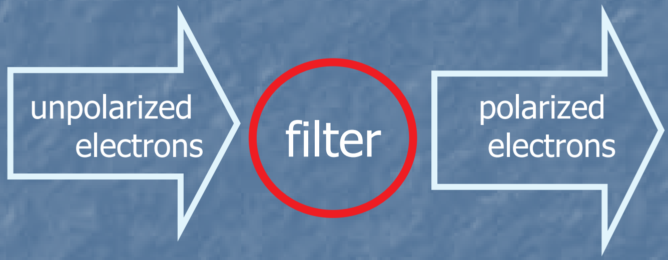
$$H_{SO} = \lambda(\vec{p} \times \hat{z}) \cdot \vec{s}$$



Can use at low gate voltages!

‘Writing’ on spinor: **Spin filtering**

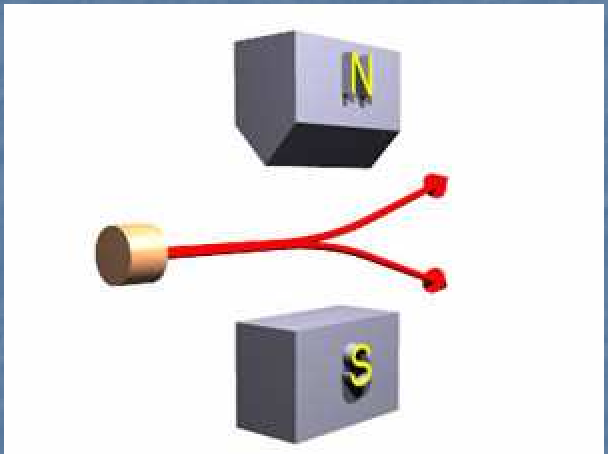
Work with **mobile** electrons,
Generate **spin-polarized** current out of an unpolarized source



‘Writing’ on spinor: **Spin filtering:**

Work with mobile electrons,
Generate spin-polarized current out of an unpolarized source

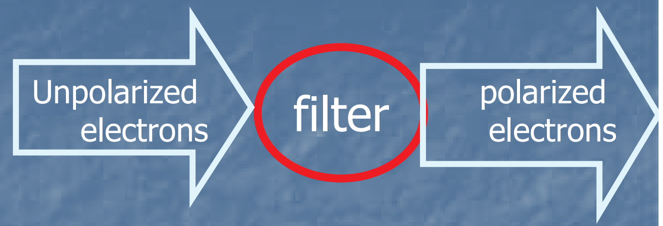
Textbook method: Stern-Gerlach splitting



Based on **Zeeman** splitting,
Requires large fields, separation of beams not easy due to uncertainty

Spin filtering:

Generate **spin-polarized** current out of an **unpolarized** source



Earlier work: usually calculate **spin-dependent conductance**, and generate **partial** polarization, which varies with parameters.

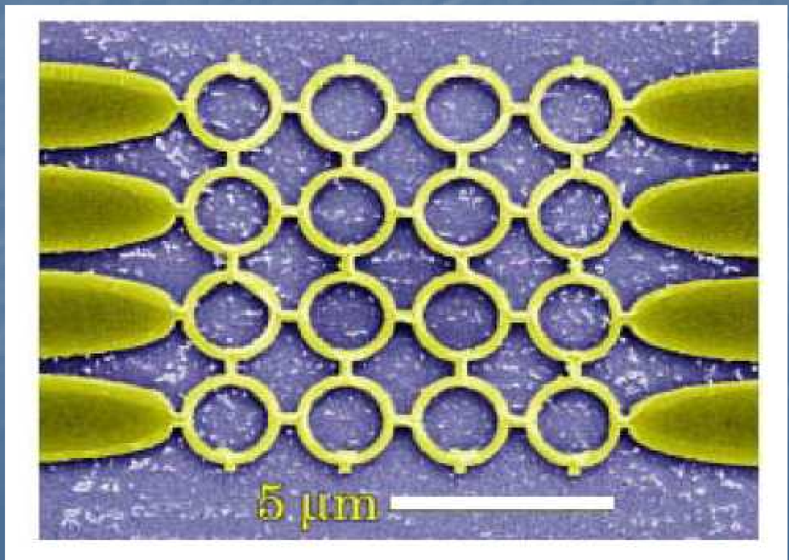
Our aim: obtain **full** polarization, in a **tunable** direction → **quantum networks**

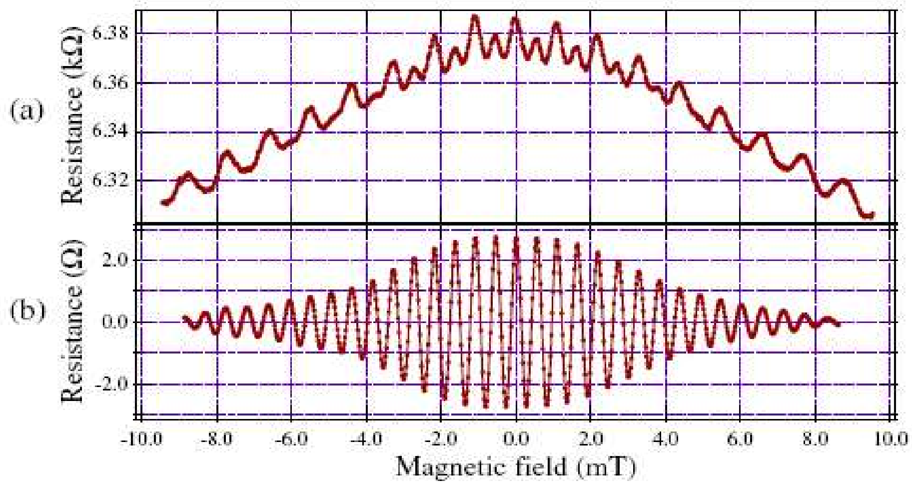
Quantum networks

PRL **97**, 196803 (2006) PHYSICAL REVIEW LETTERS week ending 10 NOVEMBER 2006

Experimental Demonstration of the Time Reversal Aharonov-Casher Effect

Tobias Bergsten,[1,3] Toshiyuki Kobayashi,[1] Yoshiaki Sekine,[1] and Junsaku Nitta[1,2,3]
[1]*NTT Basic Research Labs, 3-1 Morinosato-Wakamiya, Atsugi-shi, Kanagawa 243-0198, Japan*
[2]*Graduate School of Engineering, Tohoku University, 6-6-02 Aramaki-Aza Aoba, Aoba-ku, Sendai 980-8579, Japan*
[3]*CREST-Japan Science and Technology Agency, Kawaguchi Center Building, 4-1-8, Hon-cho, Kawaguchi-shi, Saitama 332-0012, Japan*

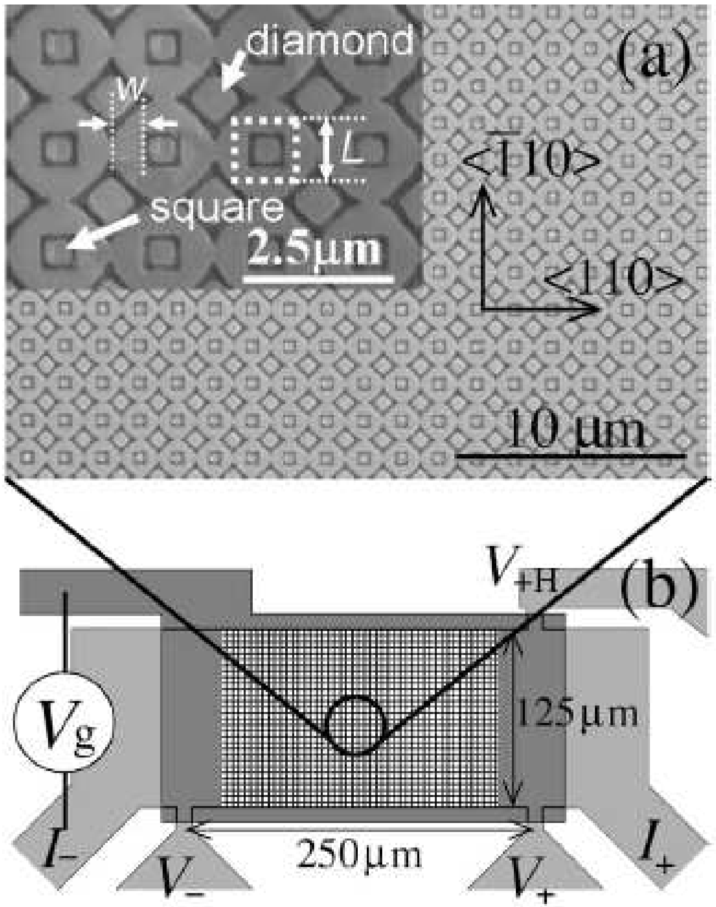


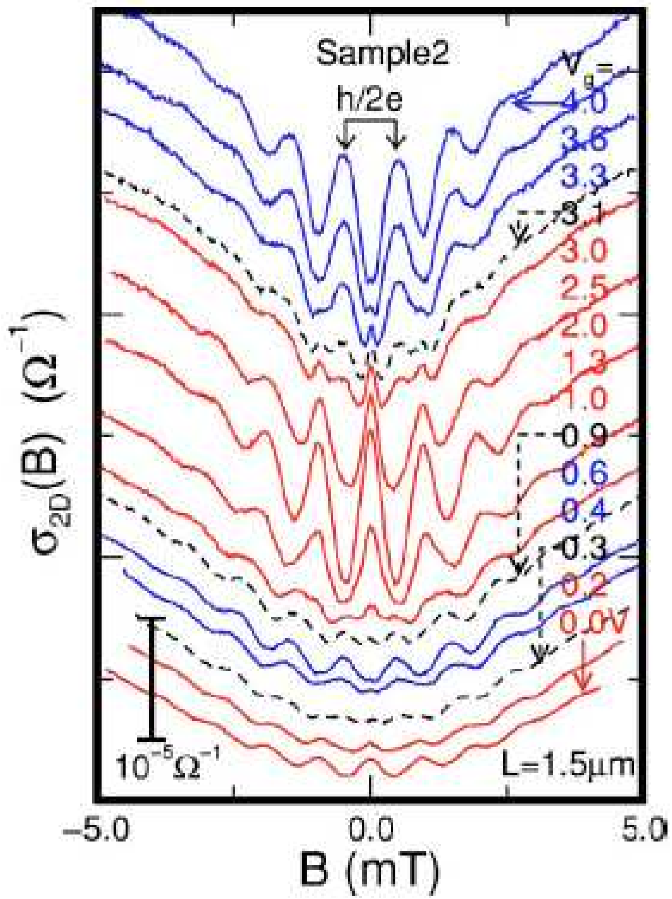


PHYSICAL REVIEW B **74**, 041302(R) (2006)

Experimental realization of a ballistic spin interferometer based on the Rashba effect using a nanolithographically defined square loop array

Takaaki Koga,[1,2,3,*] Yoshiaki Sekine,[2] and Junsaku Nitta[2,3,†]
[1]*PRESTO, Japan Science and Technology Agency, 4-1-8, Honchou, Kawaguchi, Saitama 332-0012, Japan*
[2]*NTT Basic Research Laboratories, NTT Corporation, 3-1, Morinosato-Wakamiya, Atsugi, Kanagawa 243-0198, Japan*
[3]*CREST, Japan Science and Technology Agency, 4-1-8, Honchou, Kawaguchi, Saitama 332-0012, Japan*

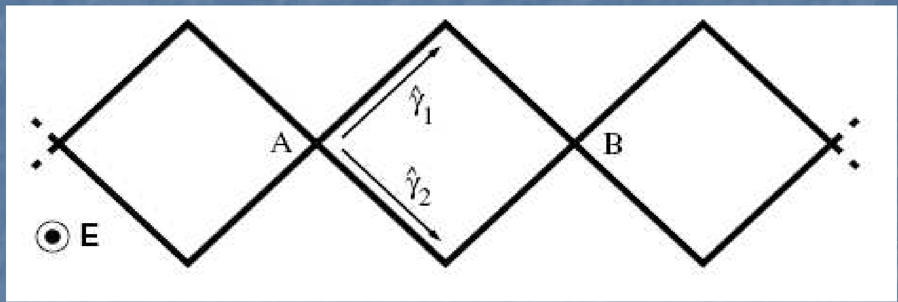




VOLUME 93, NUMBER 5 PHYSICAL REVIEW LETTERS week ending 30 JULY 2004

Rashba-Effect-Induced Localization in Quantum Networks

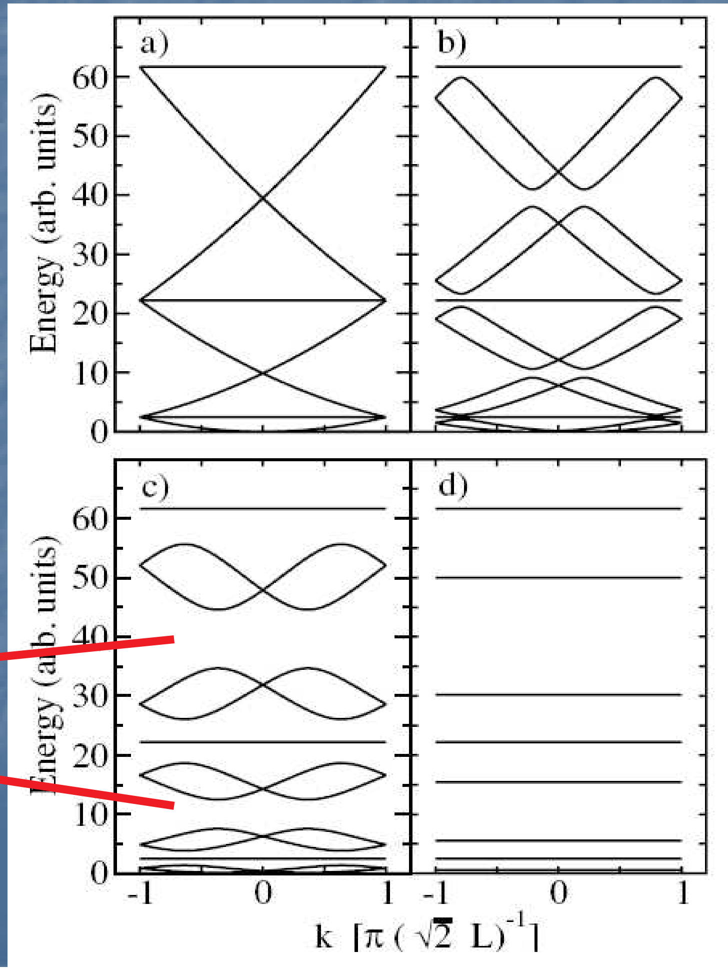
Dario Bercioux,[1] Michele Governale,[2] Vittorio Cataudella,[1] and Vincenzo Marigliano Ramaglia[1]
[1]Coherentia-INFM and Dipartimento di Scienze Fisiche, Università degli studi "Federico II," I-80126 Napoli, Italy
[2]NEST-INFM and Scuola Normale Superiore, Piazza dei Cavalieri 7, I-56126 Pisa, Italy



k →

k= momentum of electrons along the chain

Gaps: SO cage
Evanescent modes



PHYSICAL REVIEW B **78**, 125328 (2008)

Spin filtering by a periodic spintronic device

Amnon Aharony,[1,*] Ora Entin-Wohlman,[1,*] Yasuhiro Tokura,[2] and Shingo Katsumoto[3]
[1]*Department of Physics and the Ilse Katz Center for Meso- and Nano-Scale Science and Technology, Ben Gurion University, Beer Sheva 84105, Israel*
[2]*NTT Basic Research Laboratories, NTT Corporation, Atsugi-shi, Kanagawa 243-0198, Japan*
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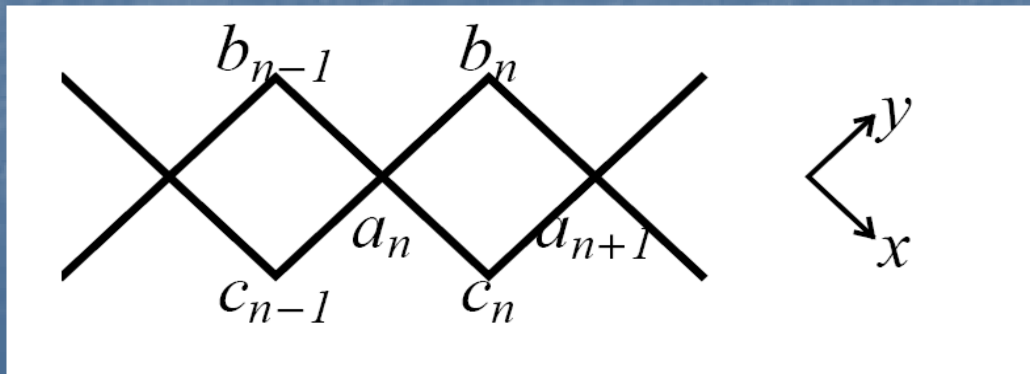
(Received 27 May 2008; revised manuscript received 3 September 2008; published 30 September 2008)

For a linear chain of diamondlike elements, we show that the Rashba spin-orbit interaction (which can be tuned by a perpendicular gate voltage) and the Aharonov-Bohm flux (due to a perpendicular magnetic field) can combine to select only one propagating ballistic mode, for which the electronic spins are fully polarized along a direction that can be controlled by the electric and magnetic fields and by the electron energy. All the other modes are evanescent. For a wide range of parameters, this chain can serve as a spin filter.



Earlier work concentrated on spin-dependent conductance, averaged over electron energies, did not concentrate on spin filtering

Our aim: use simplest quasi-1D model to generate spin filtering

Our main conclusion: can achieve **full filtering** provided we use **both** spin-orbit and Aharonov-Bohm

We use tight-binding quantum networks,

$$\epsilon \Psi_u = -J \sum_v U_{uv} \Psi_v$$

2-component spinor at node *u*

2x2 unitary matrix, representing hopping from *v* to *u*

Continuum versus tight-binding networks:
AA + Ora Entin-Wohlman, J. Phys. Chem. **113**, 3676 (2009); ArXiv: 0807.4088

General solution: $$\psi_a(n) = \sum_{i=1}^{4} A_i e^{iq_i \bar{L} n} \chi_a(q,\mu)$$

4 solutions, which appear in pairs, $\pm q_i$

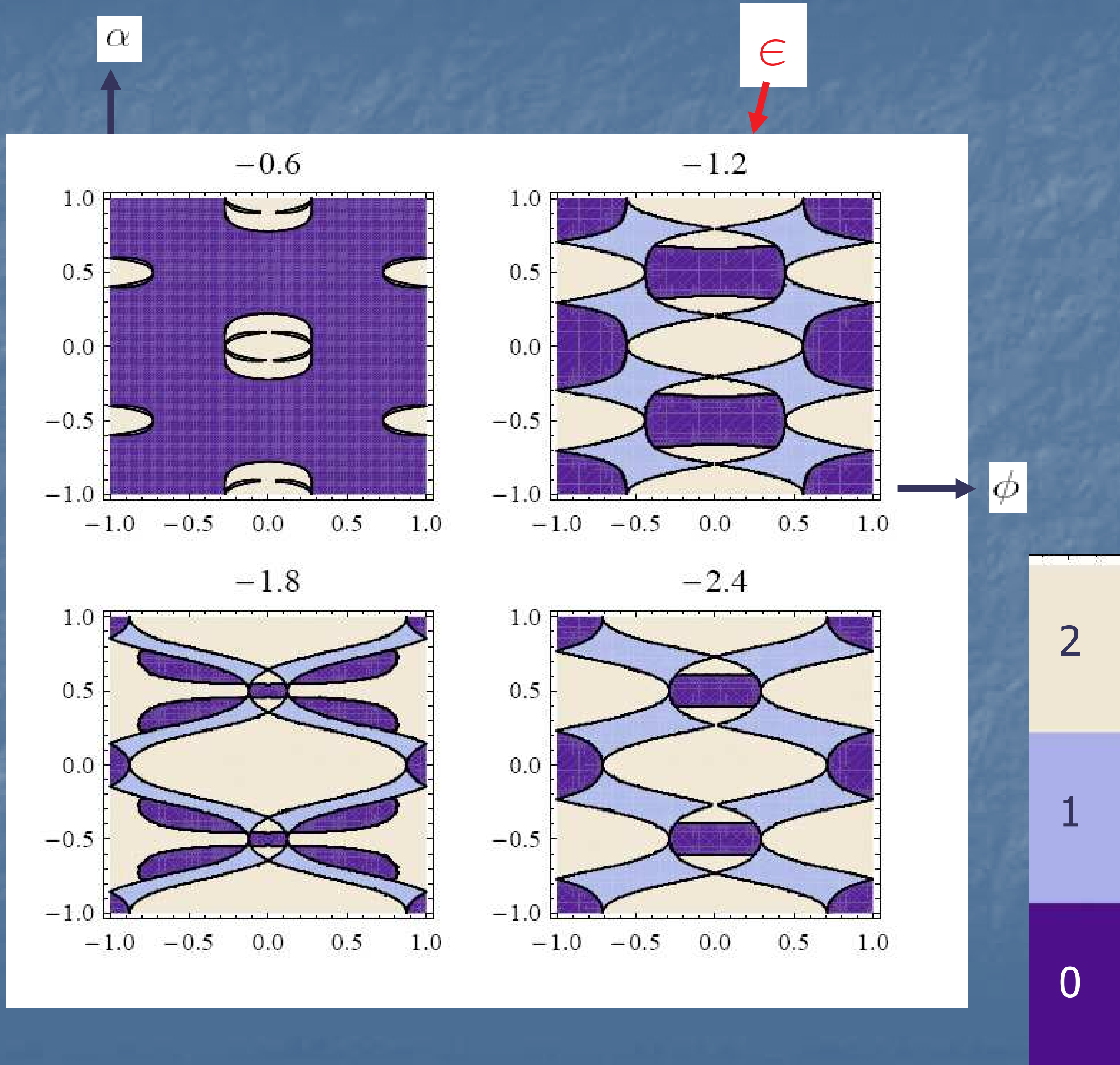
Real *q*: 'running' solution.
Complex *q*: evanescent solution.

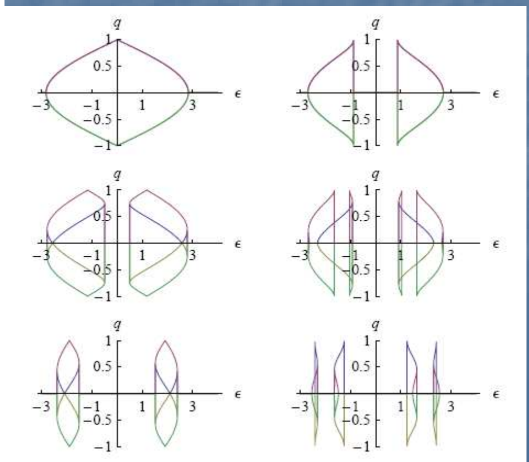
Ballistic conductance = $(e^2/h)g(E_F)$
g= number of solutions which run from left to right: *g*= 0, 1 or 2

→ For a broad range of parameters, there is only **one** running solution, and then the electrons are fully polarized!

Ballistic conductance *g*

$(e^2/h)\,g(E_F)$





Problems:

How to realize long chain?

How to read information from spinor?

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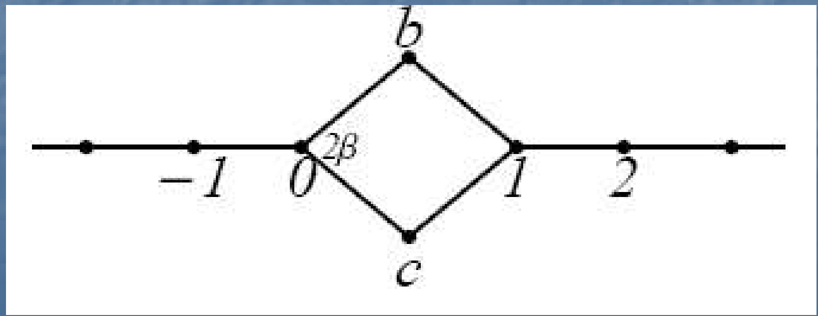
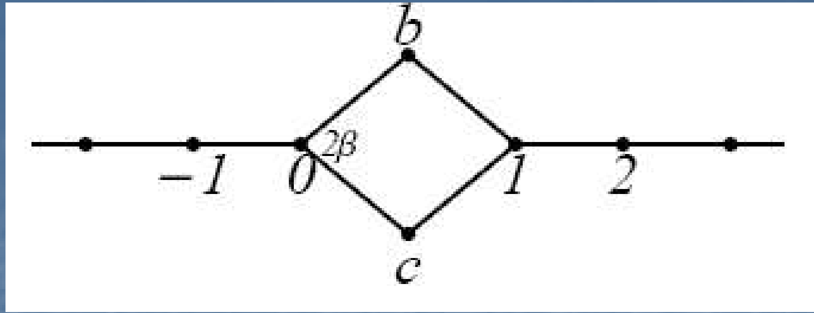
How to read information from spinor?

Single diamond



Single diamond



$$(\epsilon-\epsilon_u)\psi(u) = -\sum_v J_{uv}U_{uv}\psi(v)$$

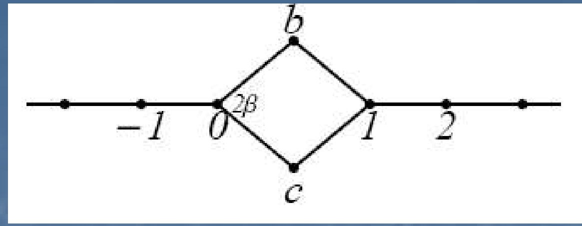
$$U_{uv} = \exp\left[i\frac{\pi BL}{\Phi_0}\hat{\gamma}_{uv}\times\hat{z}\cdot\mathbf{r}_u + ik_{SO}L\hat{\gamma}_{uv}\times\hat{z}\cdot\boldsymbol{\sigma}\right]$$

$$U_{0b} = \exp(i\alpha\sigma_1), \quad U_{b1} = \exp(-i\phi/2 - i\alpha\sigma_2)$$
$$U_{0c} = \exp(-i\alpha\sigma_2), \quad U_{c1} = \exp(i\phi/2 + i\alpha\sigma_1)$$

$$\sigma_1 = \sin\beta\sigma_x - \cos\beta\sigma_y$$
$$\sigma_2 = \sin\beta\sigma_x + \cos\beta\sigma_y$$

$$(\epsilon-\epsilon_b)\psi(b) = -J\big(U_{0b}^\dagger\psi(0) + U_{b1}\psi(1)\big),$$
$$(\epsilon-\epsilon_c)\psi(c) = -J\big(U_{0c}^\dagger\psi(0) + U_{c1}\psi(1)\big),$$
$$\epsilon\psi(0) = -J\big(U_{0b}\psi(b) + U_{0c}\psi(c)\big) - j\psi(-1),$$
$$\epsilon\psi(1) = -J\big(U_{b1}^\dagger\psi(b) + U_{c1}^\dagger\psi(c)\big) - j\psi(2).$$

Eliminate *b, c:*



$$(\epsilon - \epsilon_0 - \gamma_b - \gamma_c)|\psi(0)\rangle = \mathbf{W}|\psi(1)\rangle - j|\psi(-1)\rangle,$$
$$(\epsilon - \epsilon_1 - \gamma_b - \gamma_c)|\psi(1)\rangle = \mathbf{W}^\dagger|\psi(0)\rangle - j|\psi(2)\rangle,$$

$$\mathbf{W} = \gamma_b U_{0b} U_{b1} + \gamma_c U_{0c} U_{c1} = d + b_y \sigma_y + b_z \sigma_z,$$
$$d = a_+[c^2 - s^2 \cos(2\beta)],$$
$$b_y = -2ia_+ cs \cos\beta, \quad b_z = ia_- s^2 \sin(2\beta),$$

Non-unitary!

$$c = \cos\alpha,\ s = \sin\alpha$$

$$a_\pm = \gamma_b e^{-i\phi/2} \pm \gamma_c e^{i\phi/2}.$$

$$\gamma_j = J^2/(\epsilon - \epsilon_j),\ j = b, c$$

Electron from left:

$$\psi(n) = e^{ikna}\chi_{in} + re^{-ikna}\chi_r, \quad n \le 0,$$
$$\psi(n) = te^{ik(n-1)a}\chi_t, \quad n \ge 1,$$

$$t|\chi_t\rangle = \mathcal{T}|\chi_{in}\rangle$$

$$t|\chi_t\rangle = \mathcal{T}|\chi_{in}\rangle$$

$$\mathcal{T} = 2ij\sin(ka)\mathbf{W}^\dagger\left(Y - \mathbf{W}\mathbf{W}^\dagger\right)^{-1}$$

$$Y = (X + \epsilon_0)(X + \epsilon_1), \quad X = \gamma_b + \gamma_c + je^{-ika}.$$

$$\mathbf{W}\mathbf{W}^\dagger = A + \mathbf{B}\cdot\boldsymbol{\sigma}$$

$$\mathbf{W}\mathbf{W}^\dagger|\chi_\pm^{\mathbf{n}}\rangle = \lambda_\pm|\chi_\pm^{\mathbf{n}}\rangle$$

$$\mathbf{n}\cdot\boldsymbol{\sigma}|\chi_\pm^{\mathbf{n}}\rangle = \pm|\chi_\pm^{\mathbf{n}}\rangle$$

$$\mathbf{n} = \mathbf{B}/|\mathbf{B}| = \left(2cs\cos\beta,\ 0,\ c^2 - s^2\cos(2\beta)\right)/\sqrt{1 - s^4\sin^2(2\beta)}$$

Depends only on Rashba and on AB flux!

$$\lambda_\pm = A \pm |\mathbf{B}|$$

$$|\chi_{in}\rangle = c_+|\chi_+^{\mathbf{n}}\rangle + c_-|\chi_-^{\mathbf{n}}\rangle$$

$$t|\chi_t\rangle = c_+ t_+|\chi_+^{out}\rangle + c_- t_-|\chi_-^{out}\rangle$$

$$|\chi_\pm^{out}\rangle = \mathbf{W}^\dagger|\chi_\pm^{\mathbf{n}}\rangle/\sqrt{|\lambda_\pm|}$$

$$|\chi_\pm^{out}\rangle \equiv |\chi_\pm^{\mathbf{n}'}\rangle$$

$$\mathbf{n}' = (-n_x, 0, n_z)$$

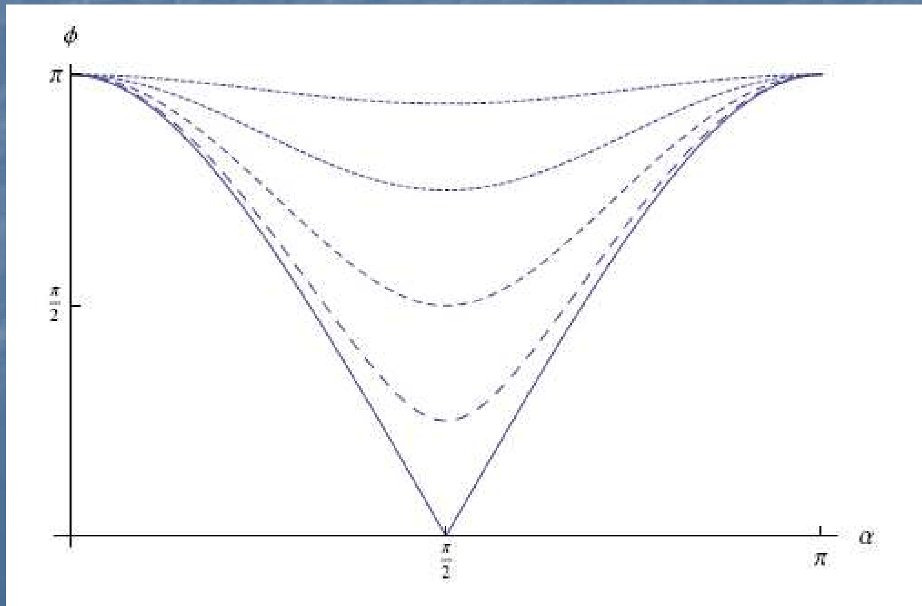
Choose parameters so that

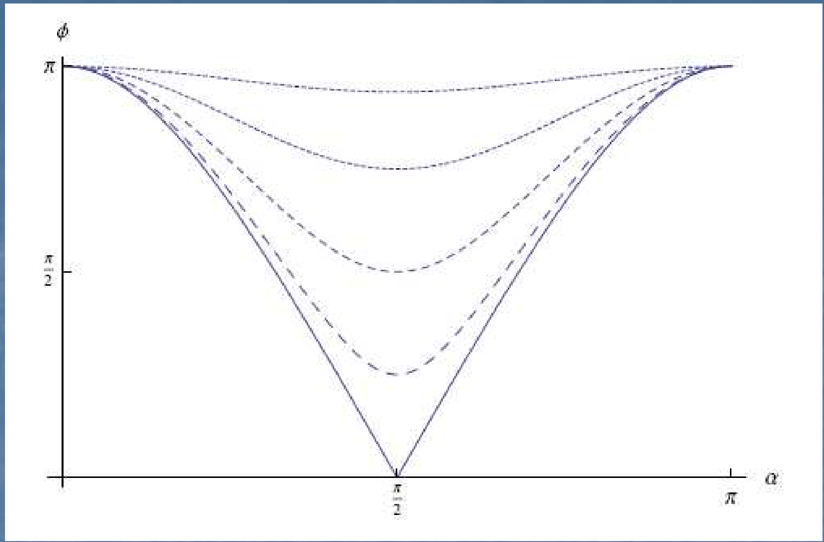
$$\mathbf{W}^{\dagger}|\chi_{-}^{\mathbf{n}}\rangle = 0$$

$$t|\chi_t\rangle = c_{+}t_{+}|\chi_{+}^{\mathbf{n}'}\rangle$$

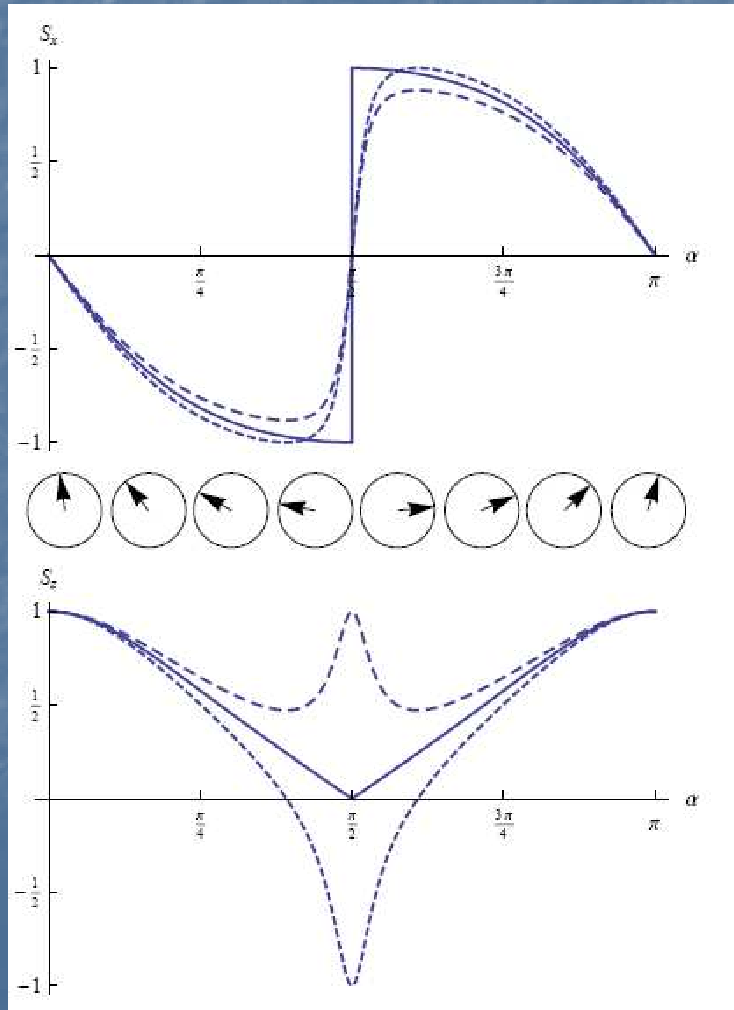
Full filtering!

$$\cos(\phi/2) = s^2 \sin(2\beta)$$





Polarization of outgoing spins

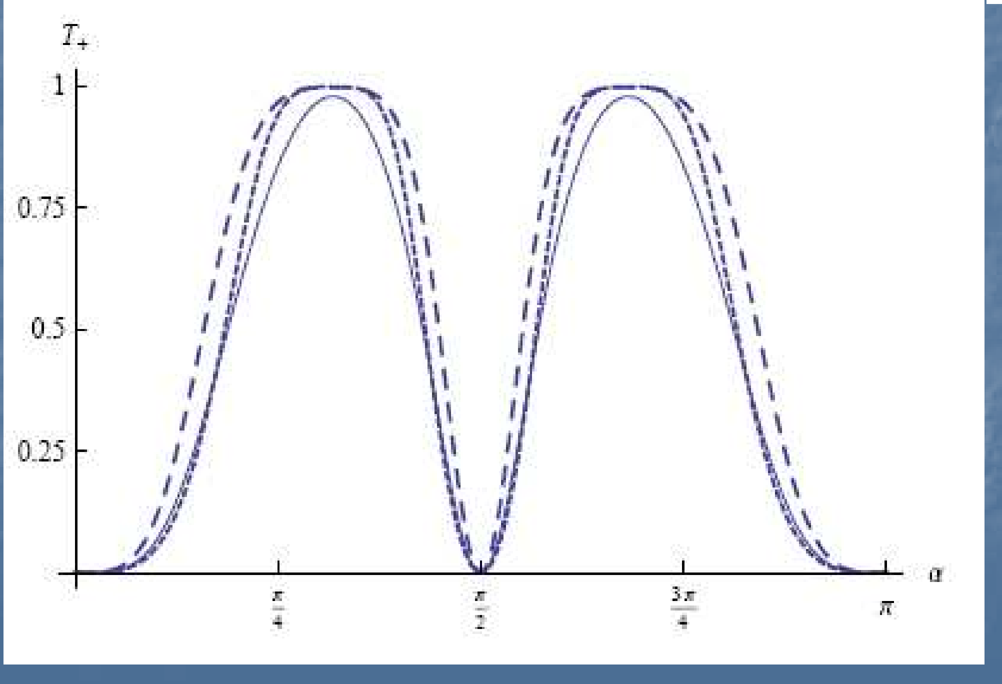


$$|\chi_{in}\rangle = c_+|\chi_+^{\mathbf{n}}\rangle + c_-|\chi_-^{\mathbf{n}}\rangle$$

$$t|\chi_t\rangle = c_+ t_+ |\chi_+^{\mathbf{n}'}\rangle$$

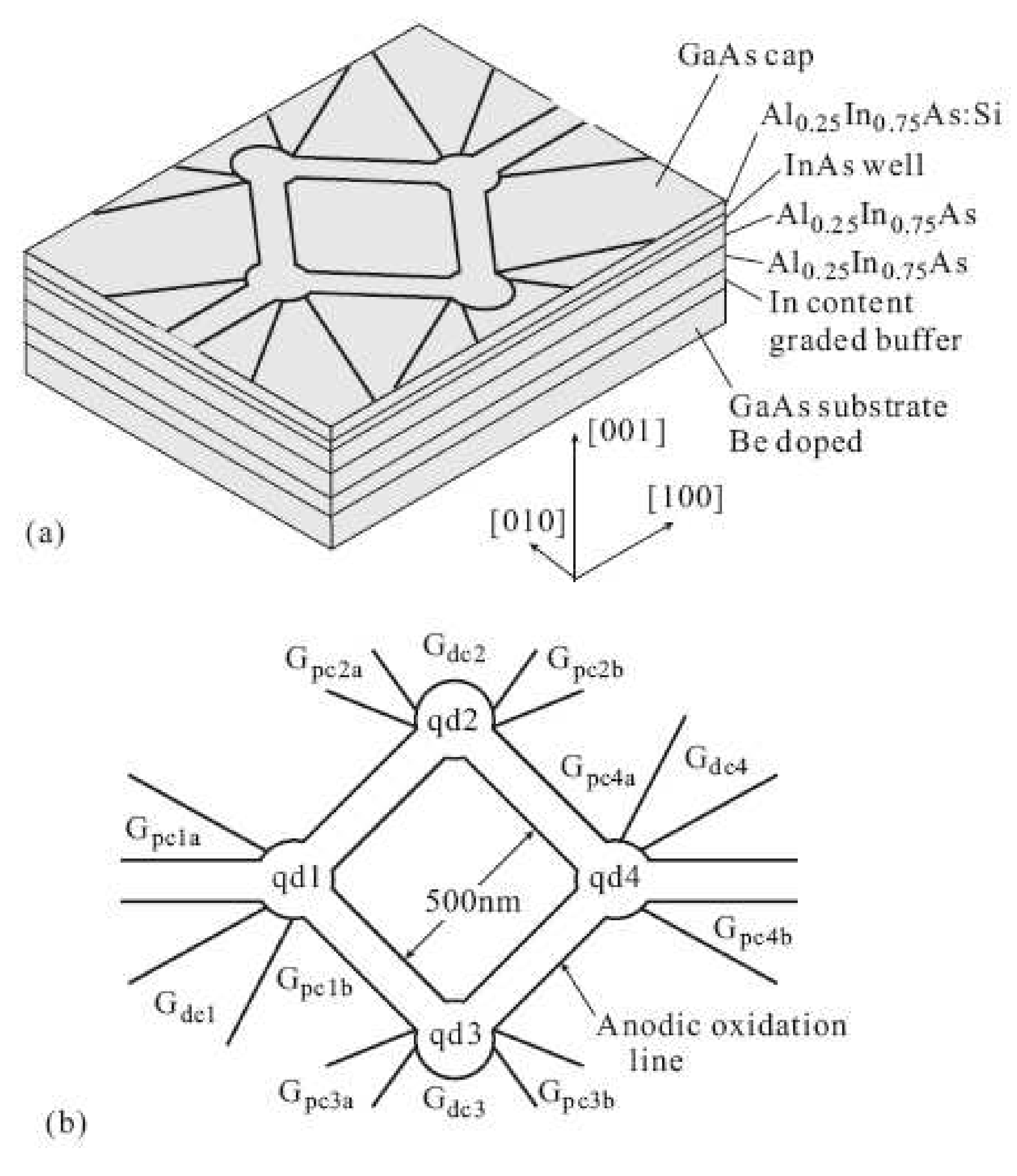
$$|c_+|^2 = |\langle|\chi_+^{\mathbf{n}}|\chi_{in}\rangle|^2$$

$$|c_+|^2 = \frac{1}{2}[1 + \mathbf{s}\cdot\mathbf{n}]$$



Transmitted currrent
Proportional to $|c_+|^2$:
Can measure incoming spin polarization
Via measurements of the transmission!
→ 'READING'.

Experimental realization



Conclusions:

Need **both** Aharonov-Bohm and spin-orbit to maintain a **single** outgoing solution, with **unique spin**.

Spin is **sensitive to parameters**: small changes in parameters switch the direction of the filtered spin.

Can work at **fixed small magnetic field**, with small changes in electric field or in electron energy.

More to do:

Add **Dresselhaus** spin-orbit interaction?

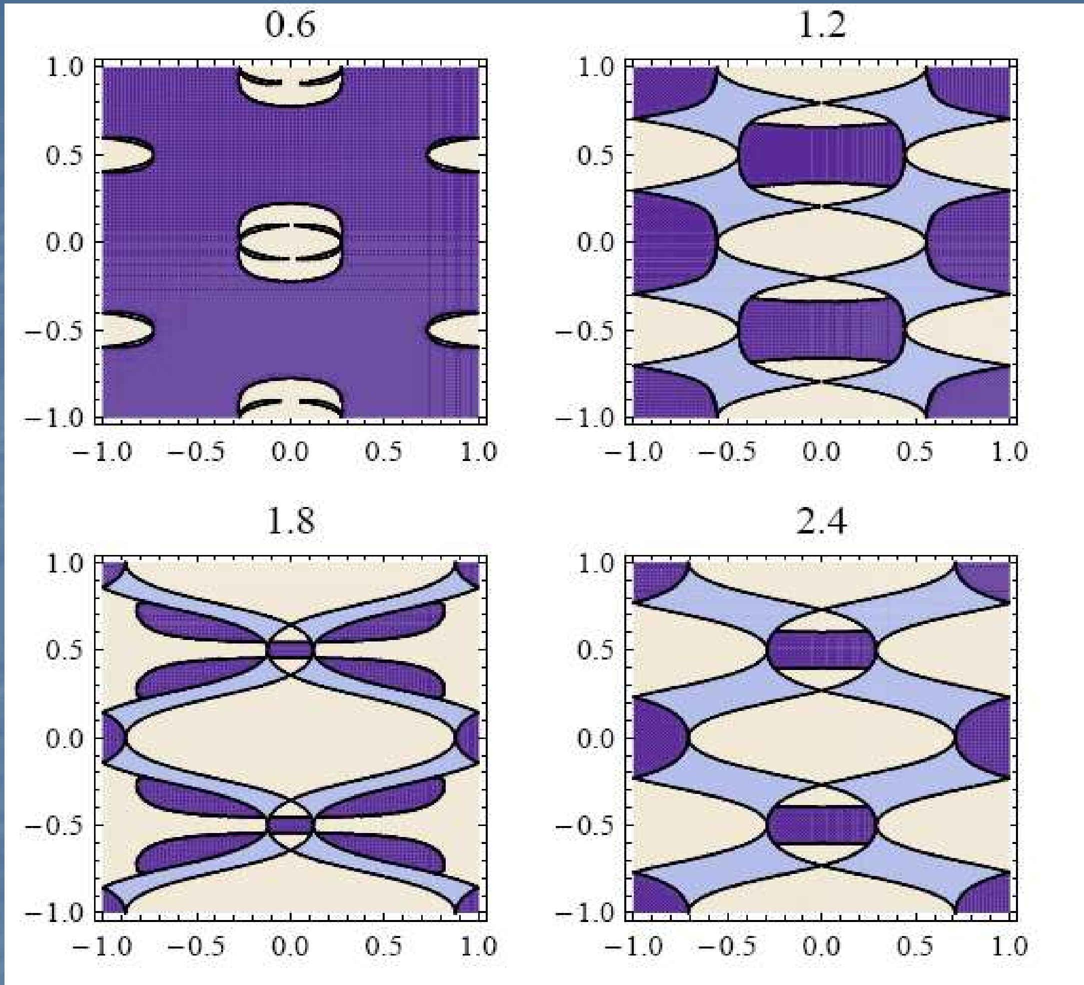
Add **Zeeman** field – Aharonov-Casher? Berry phase?

Dissipation: stochastic noise? phonons? Dephasing?

Add **e-e interactions?**

How can we **combine** beams to perform **computing?**

Postdoc positions



Thank you