# Quantum steady states and phase transitions in the presence of non equilibrium noise

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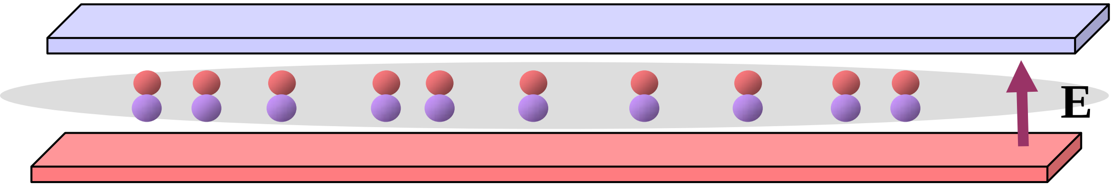
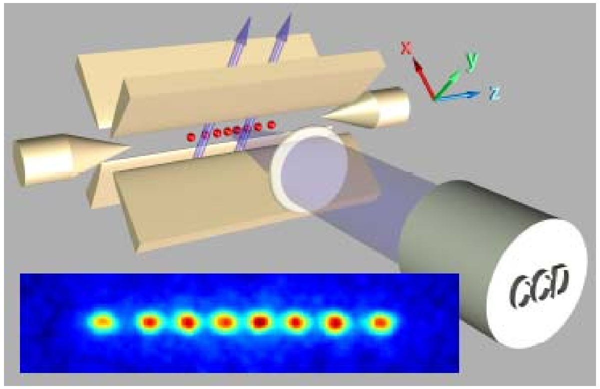
# Main point of this talk:

Low dimensional quantum systems subject to non equilibrium noise display novel quantum phase transitions & critical behavior

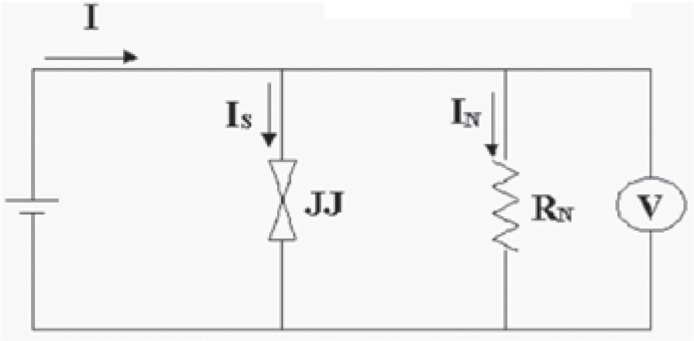
Ultracold polar molecules




Trapped ions




Shunted Josephson junction



# Critical correlations in 1d systems atomic systems

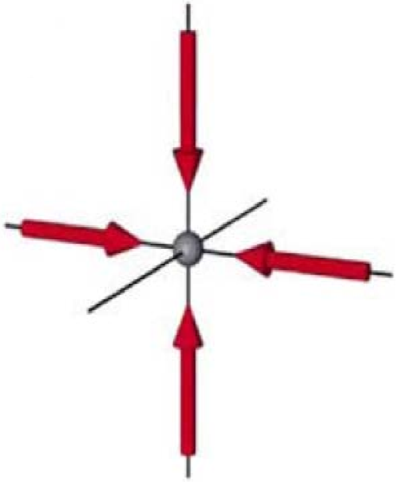
Long wavelength action of a 1d superfluid:

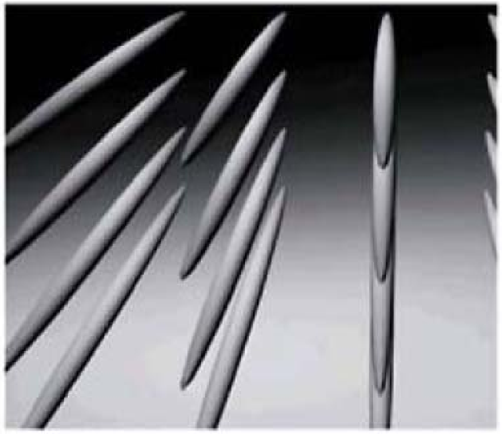
$$S = \frac{1}{2K}\int dx d\tau \left[(\partial_x \vartheta)^2 + (\partial_\tau \vartheta)^2\right] \Rightarrow \langle e^{i\vartheta(x)} e^{-i\vartheta(0)} \rangle \sim (1/x)^{1/2K}$$

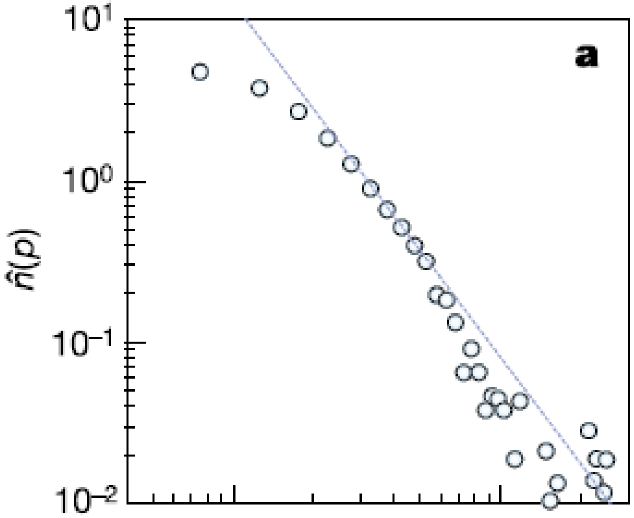
<u>Optical lattices</u>

Paredes et. al. (Bloch group), Nature 2004

Observed power-law peak in n(k):



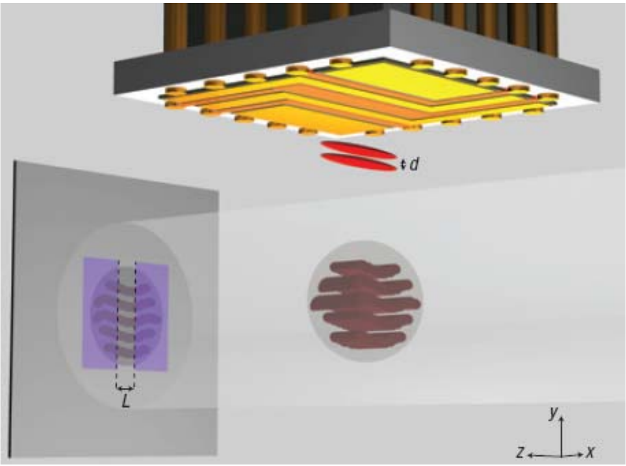




# Low dimensional ultracold atomic systems

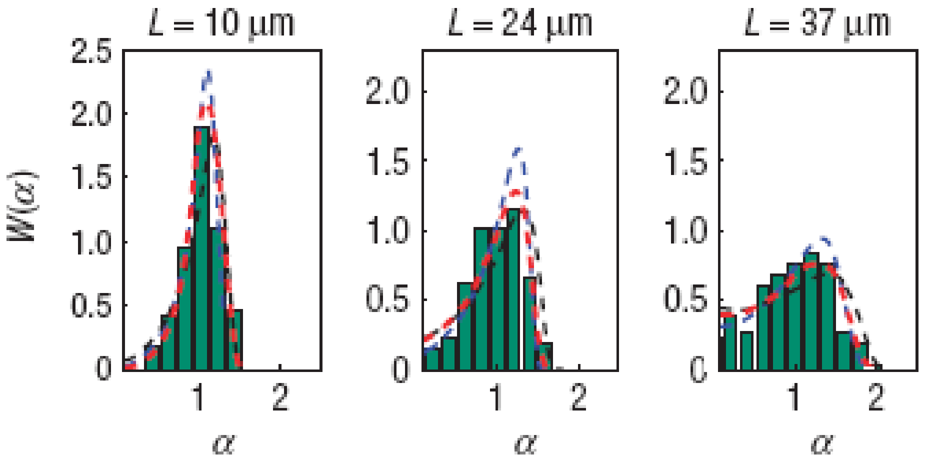
<u>Atom-chips</u>

Interference between independent condensates



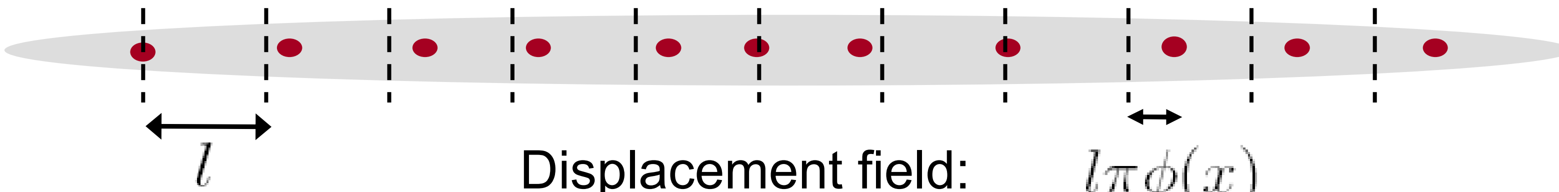

Correlations
encoded into fringe statistics

Hofferberth et. al. Nature Phys. 2008 (Vienna group)



# A brief review:
# Universal long-wavelength theory of 1D systems

Haldane (81)




Displacement field: $l\pi\phi(x)$

$$\rho(x) \approx \rho_0 - \frac{1}{\pi}\partial_x\phi + \rho_0\cos(2\pi\rho_0 x + 2\phi) + \dots$$

Long wavelength density fluctuations (phonons): $\delta\rho_0 = -\frac{1}{\pi}\partial_x\phi$

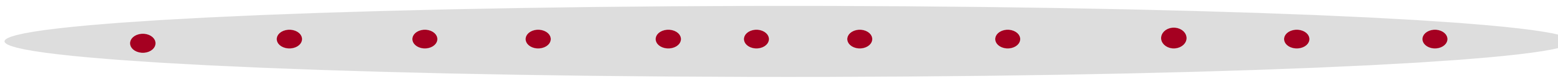
$$S_0 = \frac{K}{2}\int dx d\tau\left[(\partial_\tau\phi)^2 + (\partial_x\phi)^2\right]$$

Weak interactions: $K >> 1$

Hard core bosons: $K = 1$

Strong long range interactions: $K < 1$

# 1D review cont'd: Wigner crystal correlations



$$S_0 = \frac{1}{2K} \int dx\,dt \left[ (\partial_t \phi)^2 - (\partial_x \phi)^2 \right]$$

Wigner crystal order parameter:

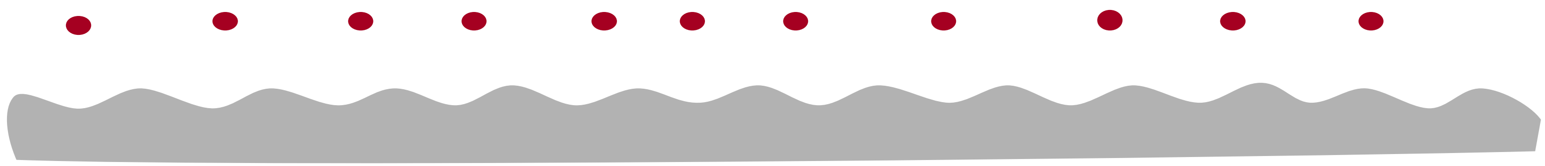
$$\langle \delta\rho_{2\pi/l} \rangle = \langle \cos\left(2\phi(x)\right) \rangle = 0$$

No crystalline order !

$$\langle \cos\left(2\phi(x)\right) \cos\left(2\phi(0)\right) \rangle \sim \frac{1}{x^{2K}}$$

Scale invariant critical state (Luttinger liquid)

# Review: 1D Mott transition (weak comensuarate lattice potential)



$$S = \frac{1}{2K}\int dx d\tau \left[(\partial_\tau \phi)^2 + (\partial_x \phi)^2\right] - V \int dx d\tau \cos(2\phi)$$

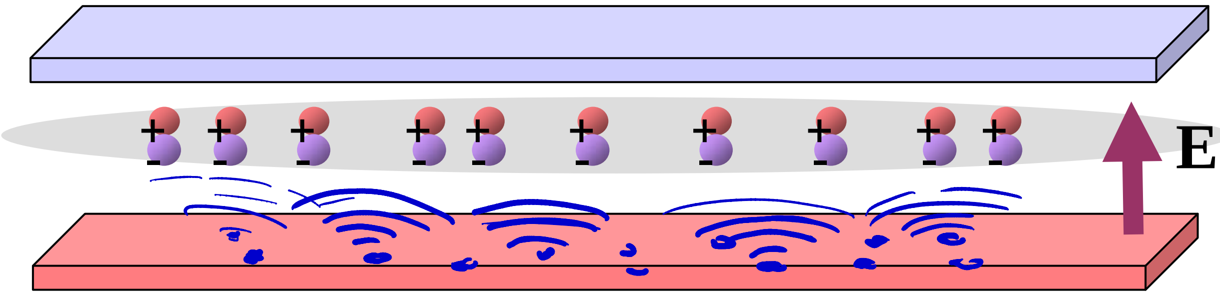
Scaling dimension of the lattice potential:

$$\langle \cos\left(2\phi(x)\right) \cos\left(2\phi(0)\right)\rangle \sim \frac{1}{x^{2K}} \quad \Rightarrow \quad [dx d\tau \cos(2\phi)] \sim [x]^{2-K}$$

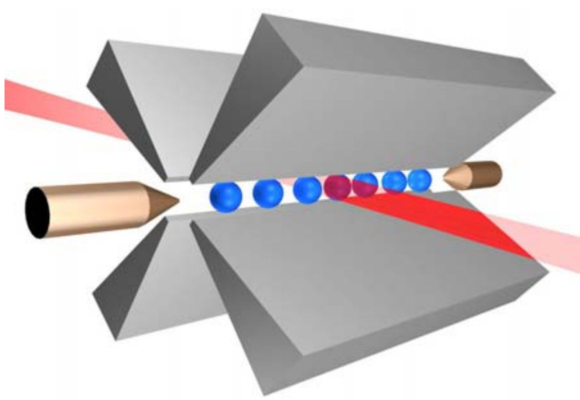
Quantum phase transition: K<2 – Pinning by the lattice (“Mott insulator”)
K>2 – Critical phase (1d Superfluid)

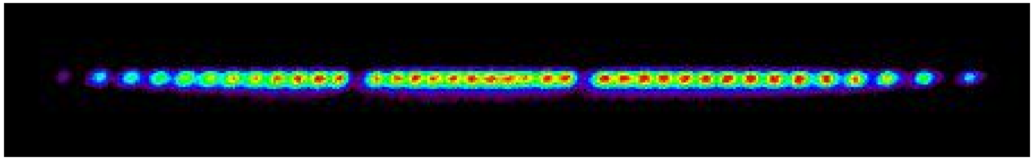
# New atomic systems: prone to external noise (non equilibrium)

Cold polar molecules



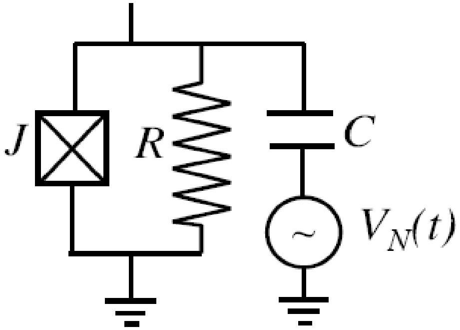

Trapped ions





(from NIST group )

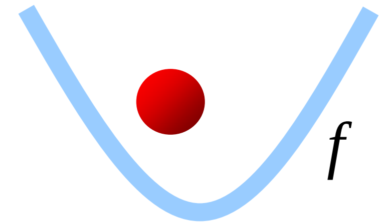
So are Josephson junctions:

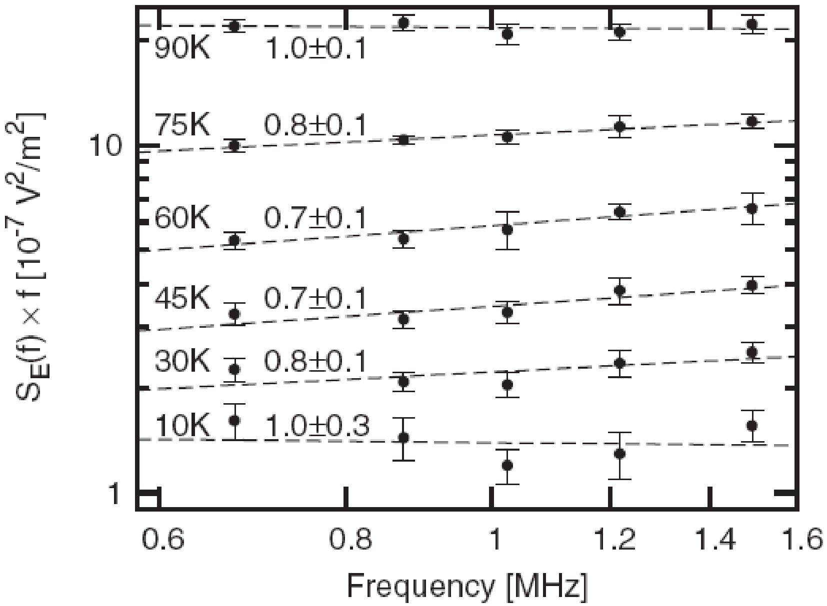


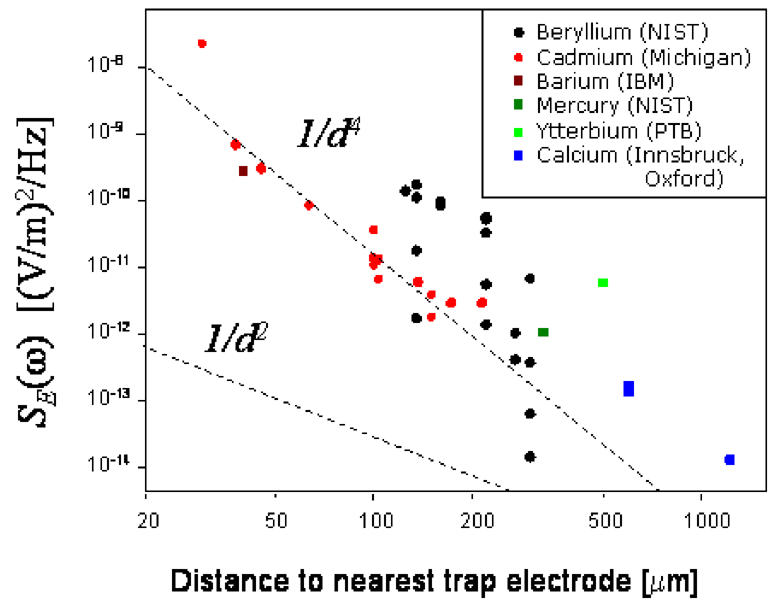
# Measured noise spectrum in ion trap

From dependence of heating rate on trap frequency.

Monroe group, PRL  (06), Chuang group, PRL (08)



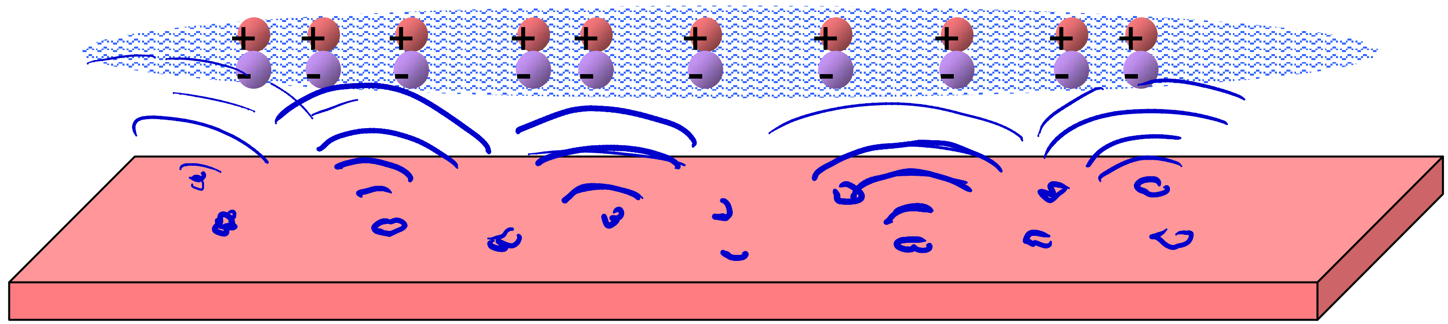

- Direct evidence that noise spectrum is *1/f*

- Short range spatial correlations (~ distance from electrodes)

$$\langle \delta E^{\star}_{q\omega} \delta E_{q\omega} \rangle \approx F_0/\omega$$

# General question:

What is the fate of quantum critical correlations and quantum phase transitions in presence of non equilibrium drive (e.g. external noise) ?



# Outline

- 0D system: single JJ with 1/f charging noise.
  - simplest paradigm for a non-equilibrium critical point

- 1D systems: cold ions/polar molecules
  - Quantum critical steady states out of equilibrium
  - Correlations vs. linear response
  - Noise tuned Mott transition

- 2D/3D systems.
  - Phase diagram of coupled chains
  - noise stabilized critical phase (sliding LL)

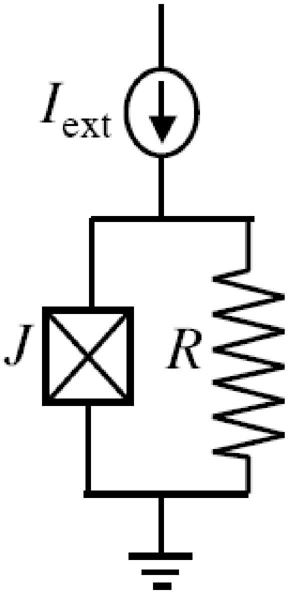
# Review: dissipative quantum phase transition in a single Josephson junction

Schmiedt PRL (83); Chakravarty PRL (83)

$$S = \frac{T}{V} \sum_{\omega} \vartheta_{-\omega} \left( R^{-1}|\omega| + \frac{1}{2} C\omega^2 \right) \vartheta_{\omega} - J \int d\tau \cos\vartheta(\tau)$$

Irrelevant

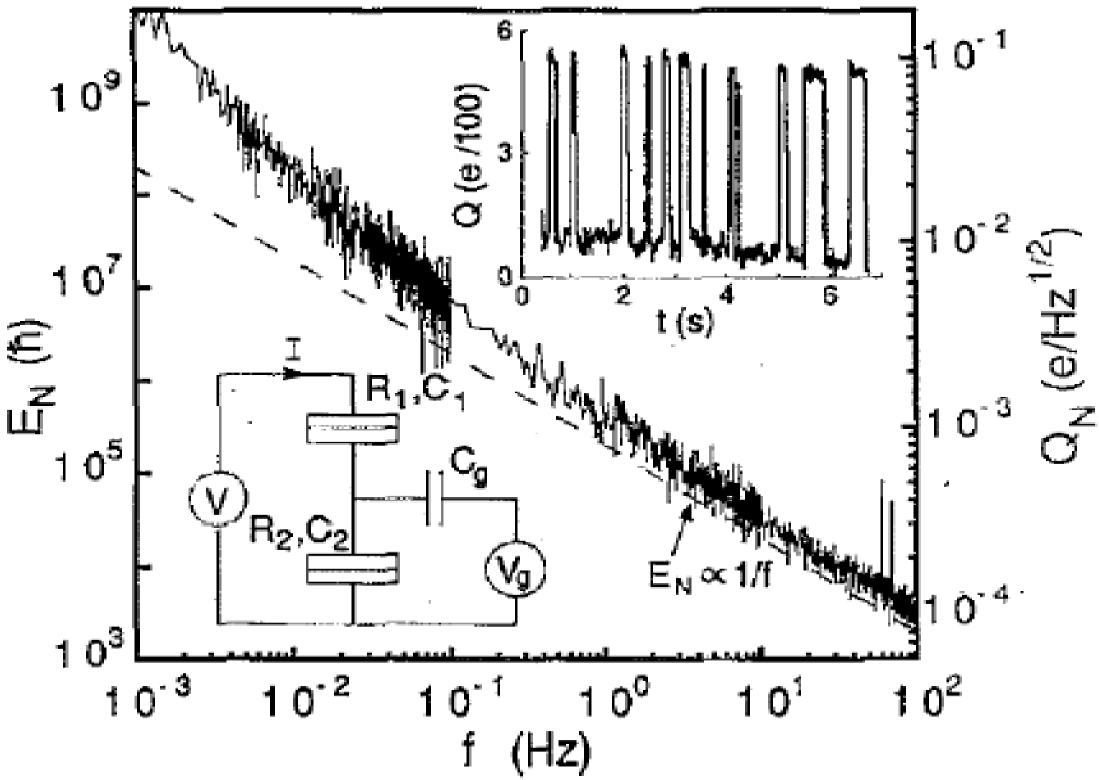
$$[d\tau \cos\vartheta] = [x]^{1-R}$$

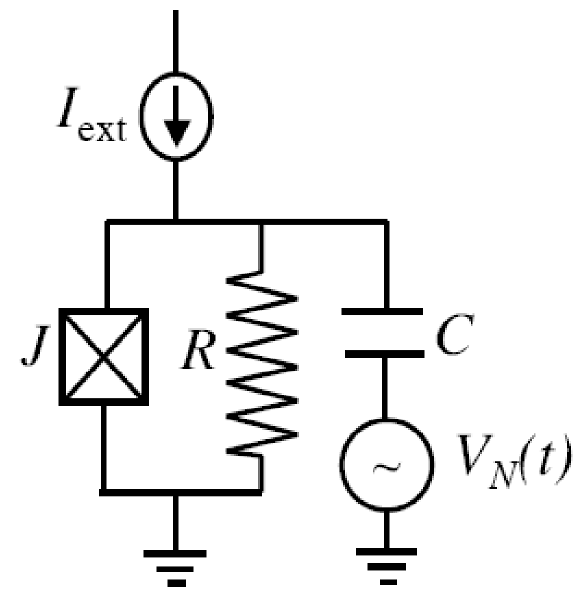



$R/R_Q < 1$ : Superconducting junction

$R/R_Q > 1$ : Normal junction (J irrelevant)

# Quantum Josephson junction with charging noise



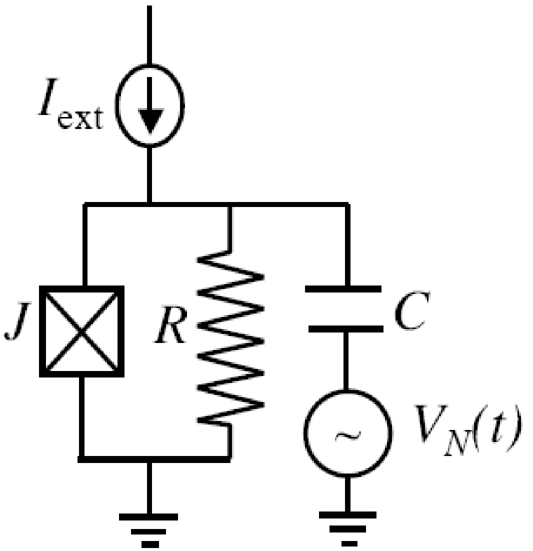




Small Josephson junctions exhibit charging noise with spectrum ~1/f

# Quantum Josephson junction with charging noise

Classical eq. of motion (Kirchoff's law):

$$I_{\text{ext}} = J \sin \vartheta + \frac{\hbar}{2eR}\dot{\vartheta} + \frac{C\hbar}{2e}\ddot{\vartheta} - C\dot{V}_N(t)$$




Quantum langevin dynamics (J=0, T=0)

$$\frac{1}{2}c\ddot{\theta} + \eta\dot{\theta} = \zeta(t) + \frac{1}{2}\dot{N}_0(t) \qquad \eta = \frac{R_Q}{2\pi R}$$

$$\langle \zeta^\star_\omega \zeta_\omega \rangle = \eta\omega \coth\left(\frac{\omega}{2T}\right)$$ Quantum / thermal noise from resistor

$$\langle \tilde{N}^*_{0\omega} \tilde{N}_{0\omega} \rangle = F_0 / |\omega|$$ External classical noise

# Equivalent Keldysh description

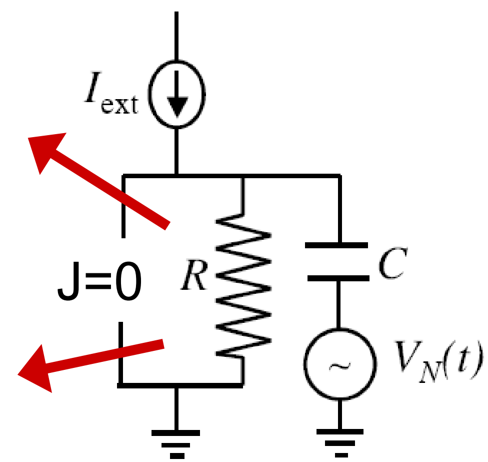
$$\langle \hat{O}(t) \rangle = \int \mathcal{D}f\, P[f(x,t')]\, \langle \psi_0 \,|\, U(-t)\hat{O}U(t) \,|\, \psi_0 \rangle = \int \mathcal{D}\phi_{cl}\mathcal{D}\hat{\phi}\; O(\phi_{cl}) e^{-S[\phi_{cl},\hat{\phi}]}$$

Classical noise — Includes bath variables

Take T=0 :

$$S_0 = \int d\omega \begin{pmatrix} \vartheta^*_{cl,\omega} & \hat{\vartheta}^*_\omega \end{pmatrix} \begin{pmatrix} 0 & \frac{1}{2}c\omega^2 - i\eta\omega \\ \frac{1}{2}c\omega^2 + i\eta\omega & -i\eta|\omega| - i\frac{\pi}{4}F_0|\omega| \end{pmatrix} \begin{pmatrix} \vartheta_{cl,\omega} \\ \hat{\vartheta}_\omega \end{pmatrix}$$

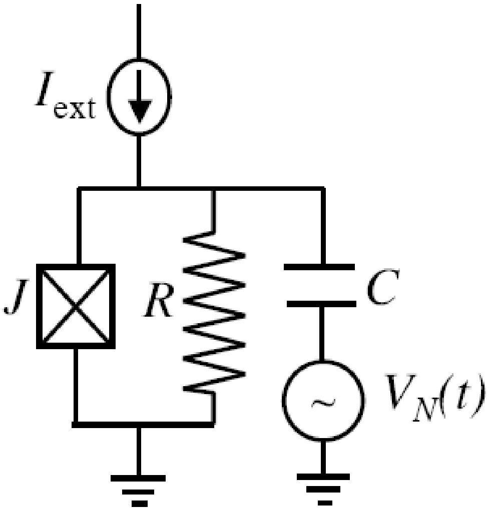
$$\langle \cos\left[\theta_{cl}(t) - \theta_{cl}(0)\right] \rangle \sim t^{-(1+\pi F_0/4\eta)/\pi\eta}$$




- External noise is a marginal perturbation
- Scale invariant (critical) steady state.

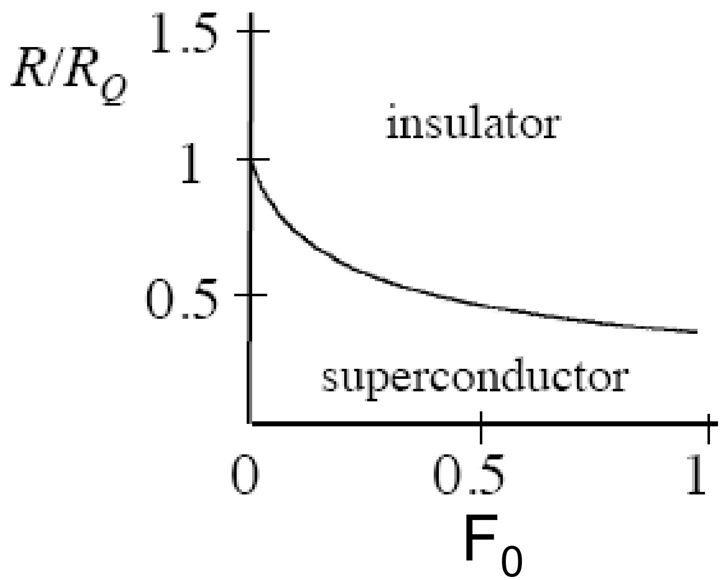
# Weak Josephson coupling ($E_J \ll E_C$)

$$S = S_0 - J \int dt \left[\cos(\vartheta_+) - \cos(\vartheta_-)\right]$$




Scaling of Josephson coupling in the critical state:

$$[dt \cos\vartheta] \sim [t]^{1-(1+2\pi F_0/\eta)/2\pi\eta}$$




Phase transition at a critical resistance tuned by noise:

$$\frac{1}{2\pi\eta^*} = \frac{R^*}{R_Q} = \frac{\sqrt{2\pi^2 F_0 + 1} - 1}{\pi^2 F_0}$$

Insulator pushes in to $R<R_Q$

# Strong Josephson coupling ($E_J$>>$E_C$)

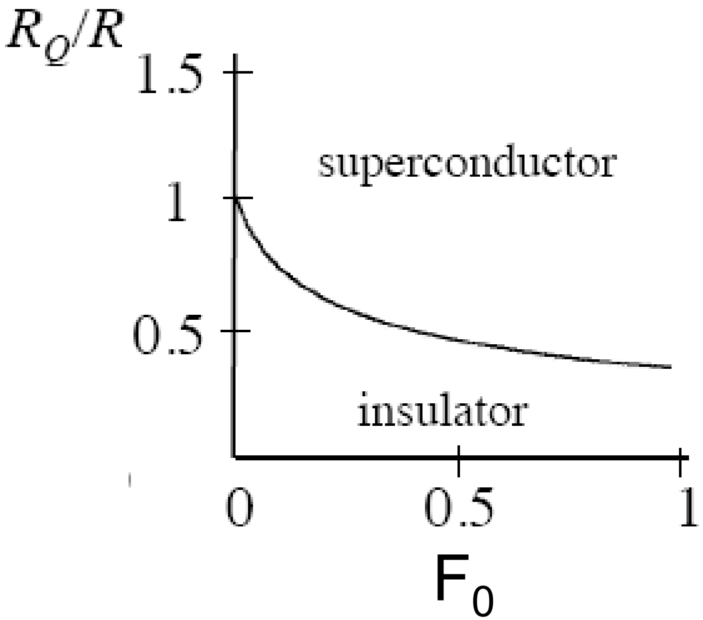
Employ duality: Cooper pair Phase slip

$$e^{i\theta} \longrightarrow e^{i\phi}$$

$$2\pi\eta \rightarrow 1/2\pi\eta$$

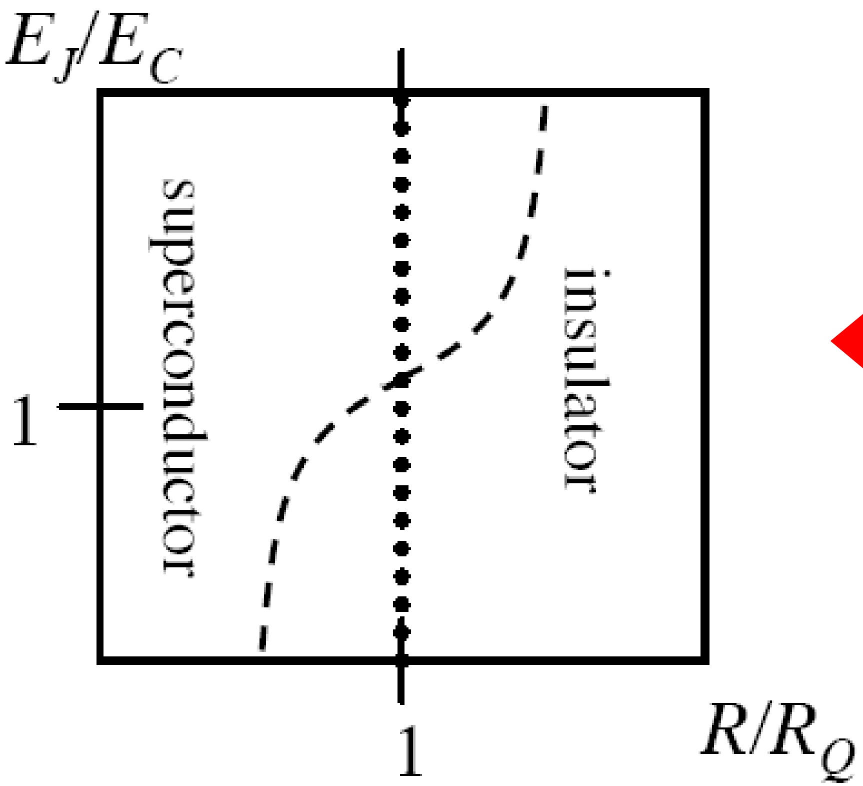
Phase slip action: $S_g = g\cos(\phi)$

$$R^*/R_Q = \pi^2 F_0/(\sqrt{2\pi^2 F_0 + 1} - 1)$$




Superconductor pushes to R>$R_Q$

# Phase diagram

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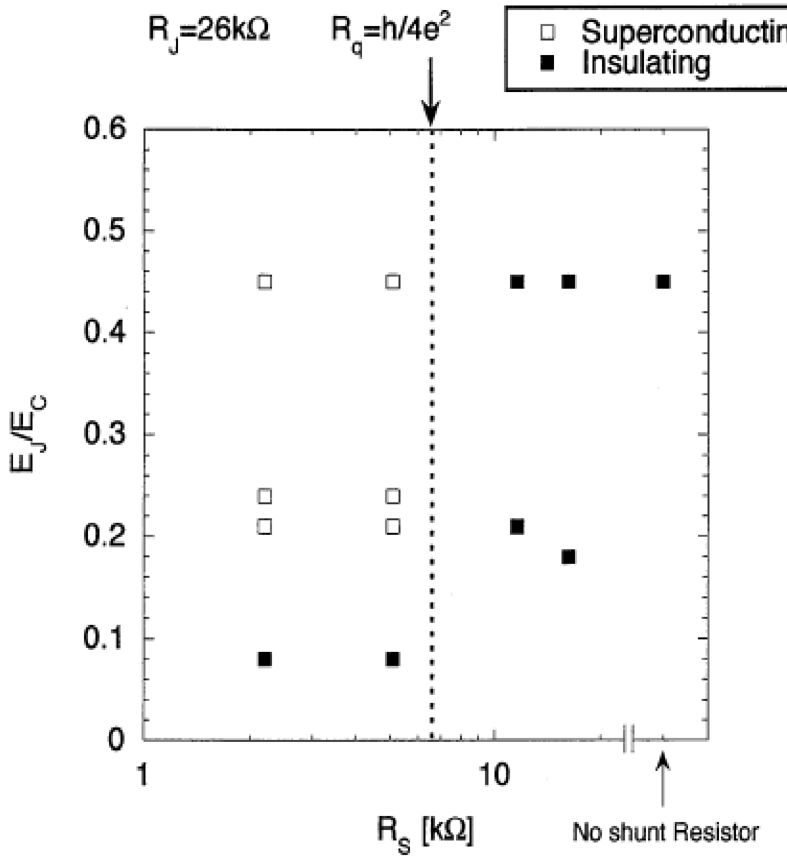
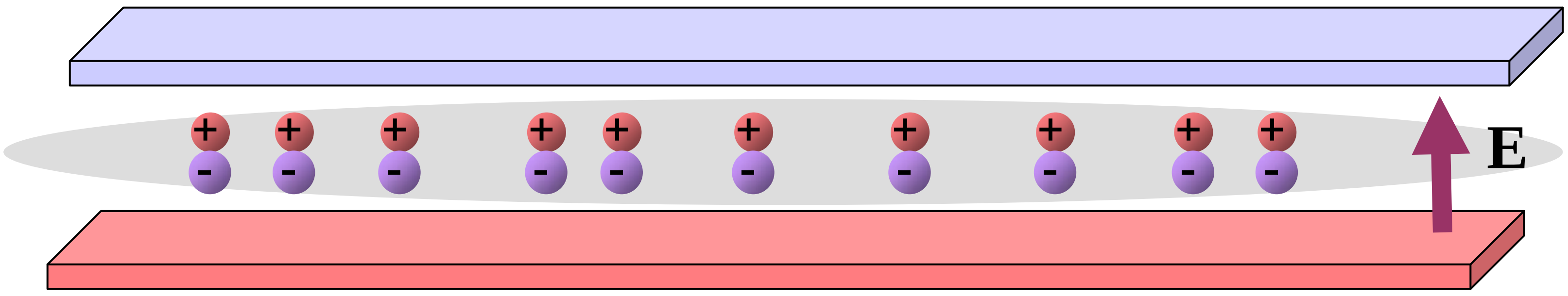



Yagi et. al. JPSJ (1997)

# Ultra cold polar molecules



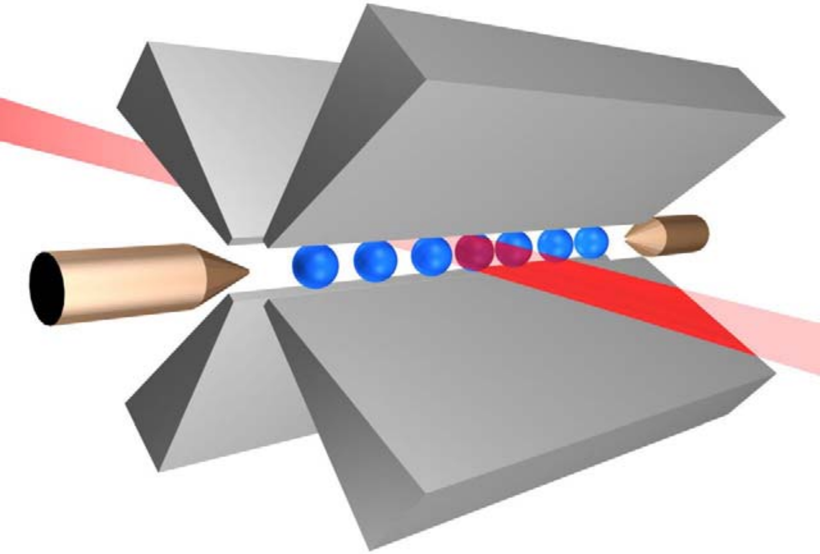

Polarizing electric field: $\mathbf{E(x,t)} = \mathbf{E_0} + \delta\mathbf{E(x,t)}$

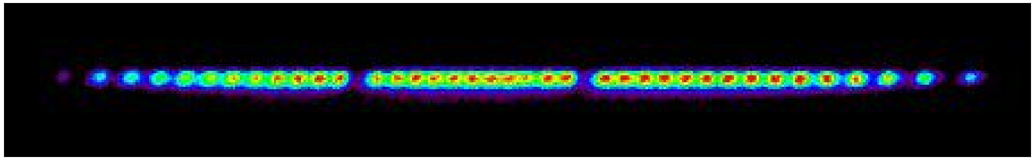
$$H = H_0 - \alpha_m \int dx \; \delta E(x,t)\hat{\rho}(x,t)$$

Molecule polarizability

System is subject to electric field noise from the electrodes !

# Linear ion trap



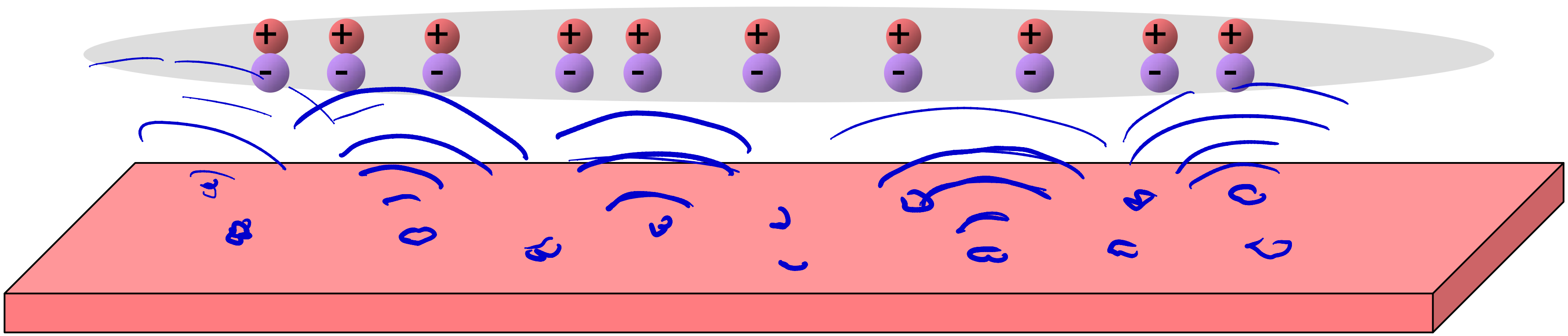


Again expect linear coupling to the noise:

$$H = H_{Coul} - Q \int dx \; \delta V(x,t) \hat{\rho}(x,t)$$

( Another complication: long range interaction )

# Coupling to external noise in long wavelength theory

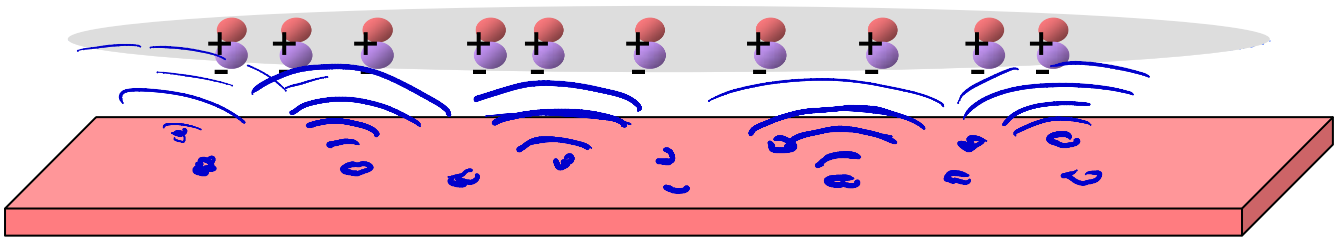



$$g \int dx\, \delta E(x,t)\hat{\rho}(x,t) \rightarrow \int dx\, f(x,t)\partial_x \phi + \int dx\, \zeta(x,t) \cos[2\phi(x)]$$

Long wavelength component of noise >> Component of noise at wavelengths near the inter-particle spacing

The "backscattering" $\zeta$ can be neglected if the distance to the noisy electrode is much larger than the inter-particle spacing.

# Effective harmonic theory of the noisy system




(Quantum) Langevin dynamics:

$$K^{-1}\left(\partial_t^2\phi - \partial_x^2\phi\right) + \eta\partial_t\phi = \xi(x,t) + \partial_x f(x,t)$$

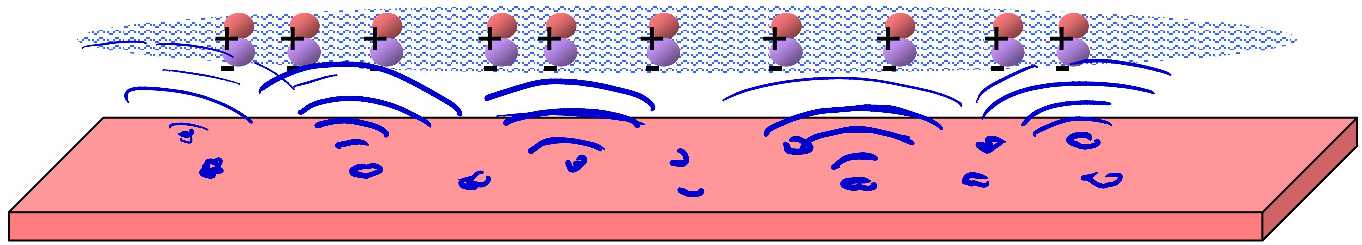
$$\langle \xi_{q\omega}^{*}\xi_{q\omega}\rangle = \eta\omega\coth\left(\frac{\omega}{2T}\right)$$ Thermal bath

$$\langle f_{q,\omega}^{\star} f_{q,\omega}\rangle = \frac{2\pi F_0}{|\omega|}$$ External noise

Dissipative coupling to bath needed to ensure steady state
(removes the energy pumped in by the external noise)

Implementation of bath: immersion in condensate or continuous cooling.
In ion traps laser cooling provides the required dissipative force.

# Equivalent Keldysh description (at T=0)



$$S_0 = \sum_{\omega,q} \begin{pmatrix} \phi_{cl}^* & \hat{\phi}^* \end{pmatrix} \begin{pmatrix} 0 & \frac{1}{\pi K}(\omega^2 - q^2) - i\eta\omega \\ \frac{1}{\pi K}(\omega^2 - q^2) + i\eta\omega & -i\eta|\omega| - i\frac{q^2 F_0}{2\pi|\omega|} \end{pmatrix} \begin{pmatrix} \phi_{cl} \\ \hat{\phi} \end{pmatrix}$$

Crystalline (CDW) correlations ($\eta \to 0$):

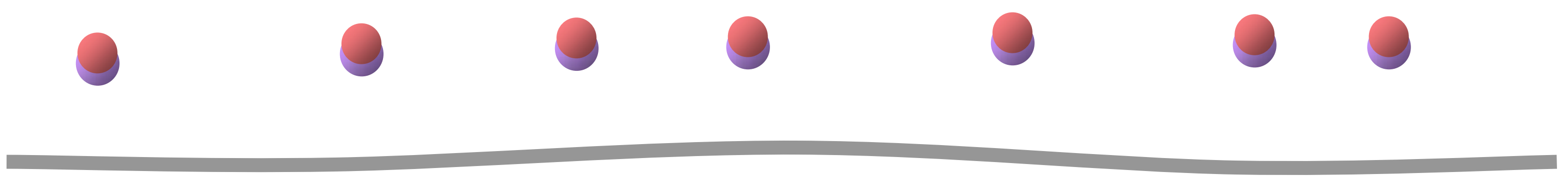
$$\langle \cos(2\phi_{cl}(x)) \cos(2\phi_{cl}(0)) \rangle \sim x^{-2K(1+F_0/2\pi\eta)}$$

- Noise is a marginal perturbation → critical steady state
- $1/\eta$ is an IR cutoff (correlations decay exponentially at longer scales)

# Dynamic response: Bragg spectroscopy

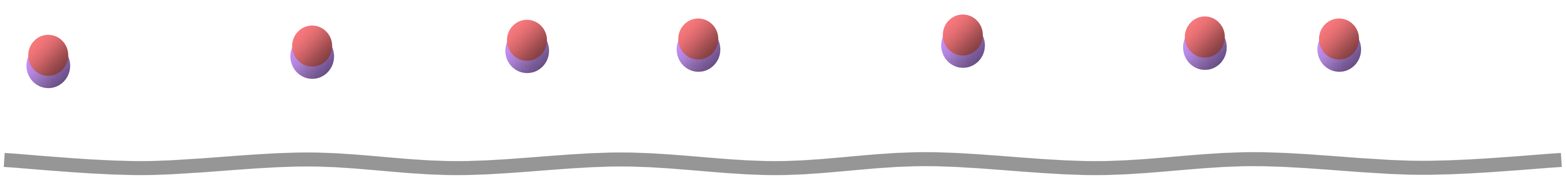
$$\chi(x,t) = i\langle \rho\left(\phi_f(x,t)\right)\left[\rho\left(\phi_f(0,0)\right) - \rho\left(\phi_b(0,0)\right)\right]\rangle$$

Long wavelength modulated lattice ($q \ll 2\pi\rho_0 = q_0$):



$$\Rightarrow \quad \hat{\rho}(x,t) \approx \partial_x \phi(x,t)$$

Modulated lattice at $q \sim 2\pi\rho_0 = q_0$ :

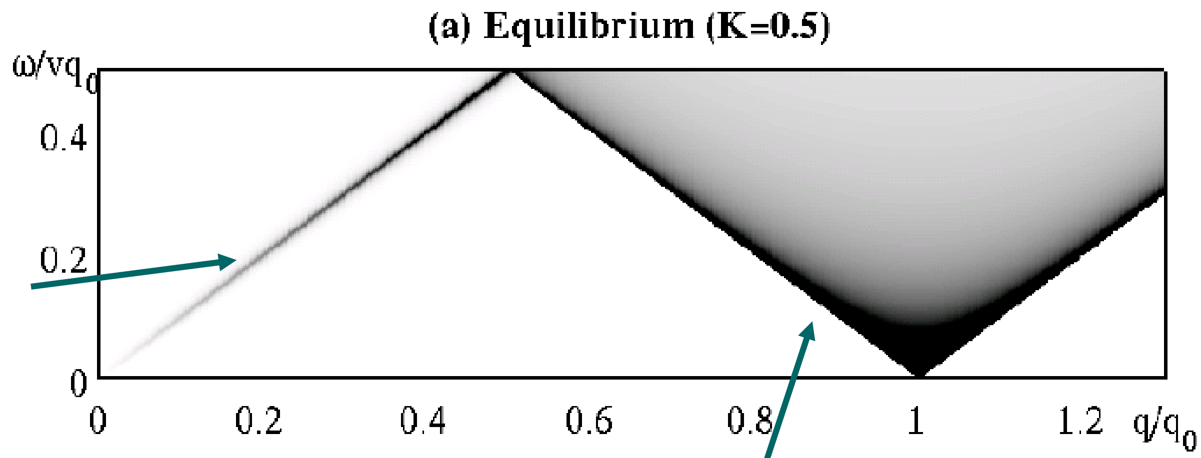


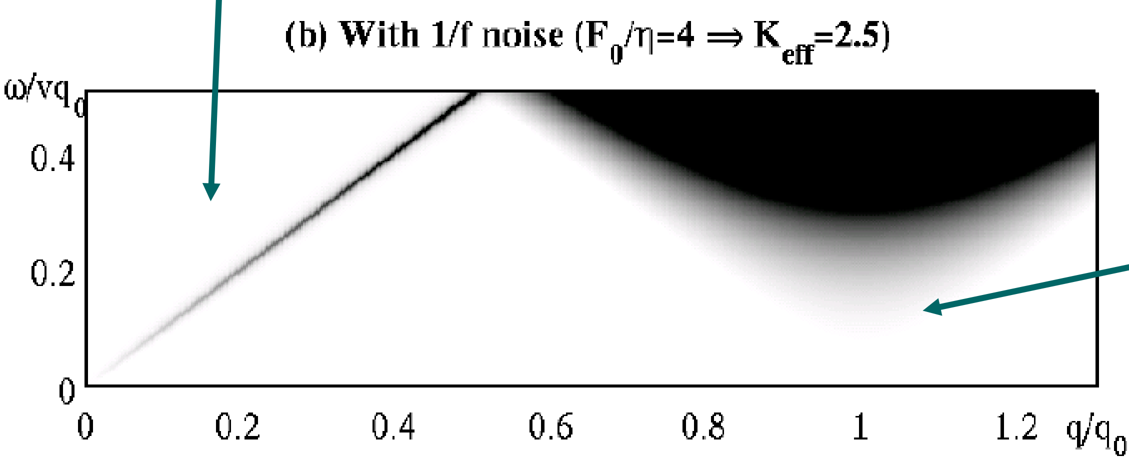
$$\Rightarrow \quad \hat{\rho} \approx \cos\left(2\phi(x,t)\right)$$

# Dynamic response: Bragg spectroscopy

Response at long wavelengths ($q \ll q_0$) :

$$\chi''(q,\omega) \sim K|q|\delta(\omega - v|q|)$$




Response at q near $q_0$ :

$$\chi''(q,\omega) \sim (\omega^2 - v^2\delta q^2)^{K_{eff}-1} \times \Theta(\omega^2 - v^2\delta q^2)$$

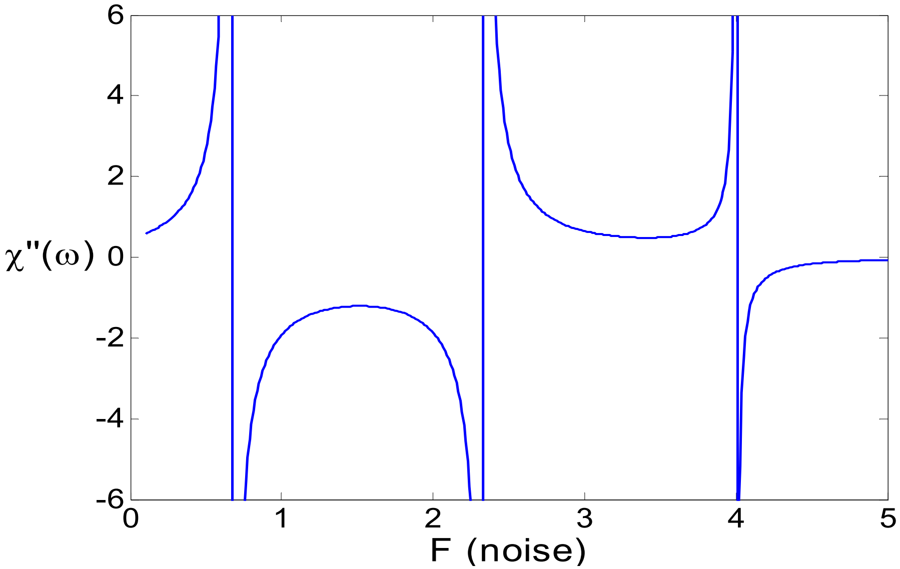
$$K_{eff} = K(1 + F_0/\eta)$$

- Response at long wavelength is unaffected by noise.
- Response near $q \sim 2\pi\rho_0$ is strongly affected

# Energy loss of probe field

$$\dot{E}_{probe} = \lambda^2 \omega \chi''(q,\omega)$$

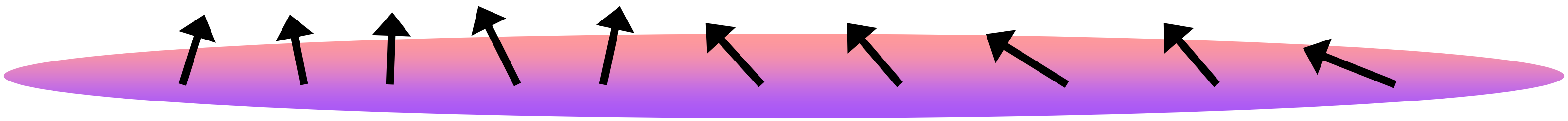
$$= \frac{\lambda^2}{4} \frac{|\omega|}{\Gamma^2(K_{eff})} \frac{\sin(\pi K)}{\sin(\pi K_{eff})} (\omega^2 - \delta q^2)^{K_{eff}-1} \Theta(\omega^2 - \delta q^2)$$



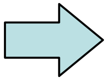

( K=0.6 )

- Non equilibrium: both absorption and stimulated emission possible
- Divergences for certain combinations of K and $F_0$ (?)

# Phase correlations (Off diagonal order)



Density is conjugate to phase (~Josephson relation): $\partial_x \phi = K\dot{\theta}$



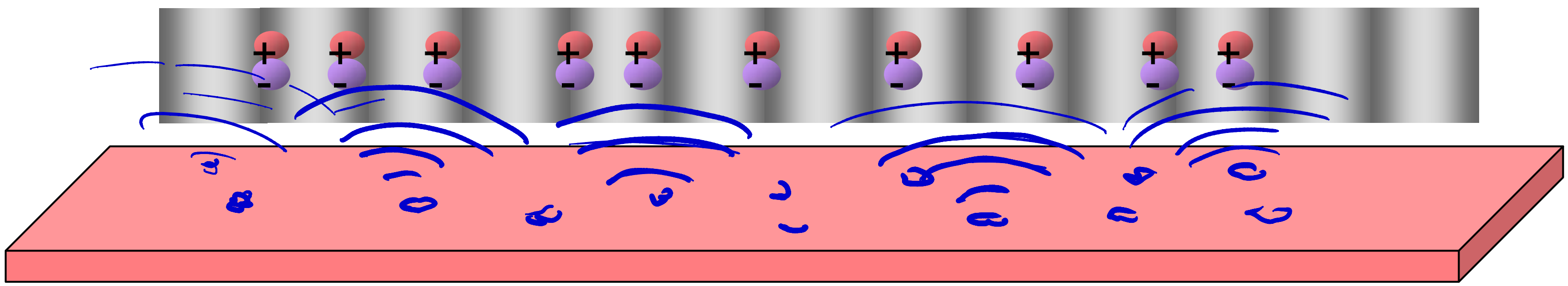
$$\langle \cos\left[\theta_{cl}(x) - \theta_{cl}(0)\right]\rangle \sim x^{-(1+F_0/\eta)/2K}$$

Noise harms both density and phase correlations!

Destroys the duality between the two

# Instabilities of the steady state: Non-equilibrium phase transitions

# Effect of a weak commensurate lattice potential



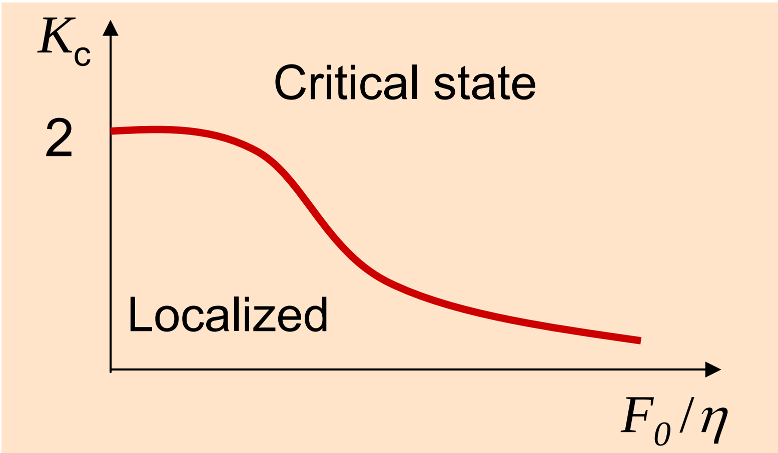
Without lattice: Scale invariant steady state.

How does the lattice change under a scale transformation?

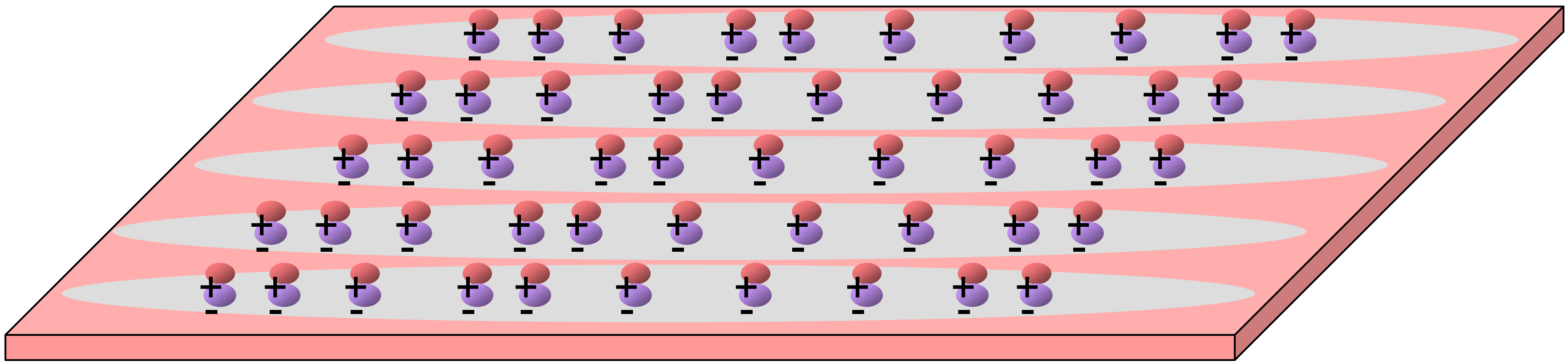
$$\langle \cos(2\phi(x)) \cos(2\phi(0)) \rangle \sim x^{-2K(1+F_0/\eta)} \Rightarrow [dx dt \cos(2\phi)] \sim [x]^{2-K(1+F_0/\eta)}$$

Phase transition tuned by noise power

(Supported also by a full RG analysis within the Keldysh formalism)



# 1D-2D transition of coupled tubes

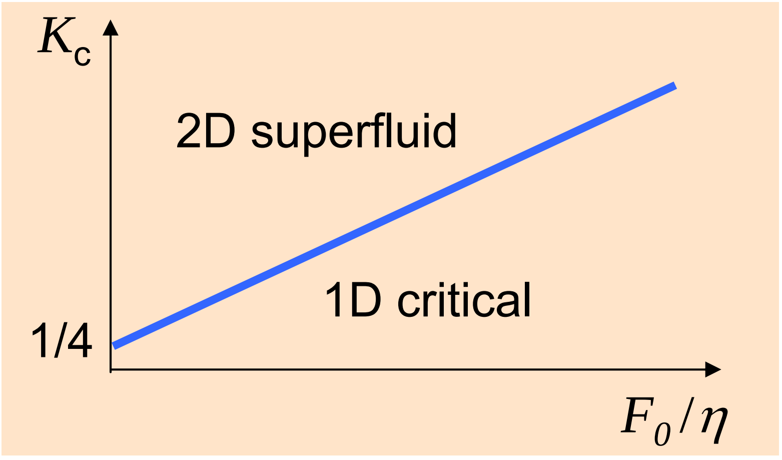


Scaling of the inter-tube hopping:

$$\langle \cos\left[\theta_{cl}(x) - \theta_{cl}(0)\right]\rangle \sim x^{-(1+F_0/\eta)/2K}$$

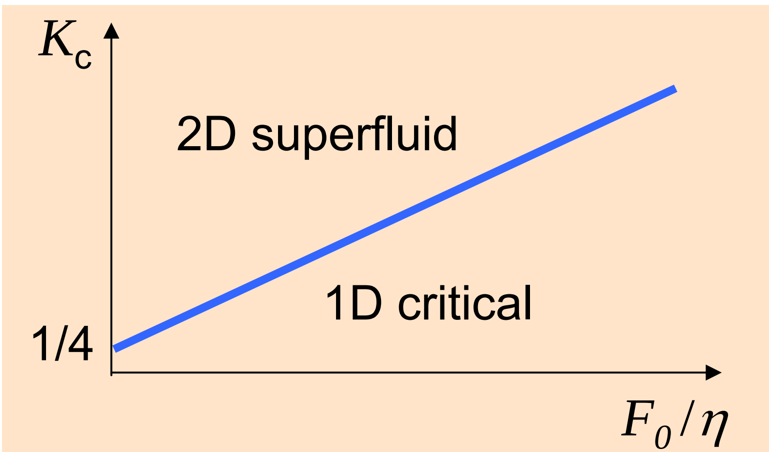
$$\Downarrow$$

$$[dx dt \cos\left(\theta_i(x) - \theta_j(x)\right)] = x^{2-(1+F_0/\eta)/2K}$$

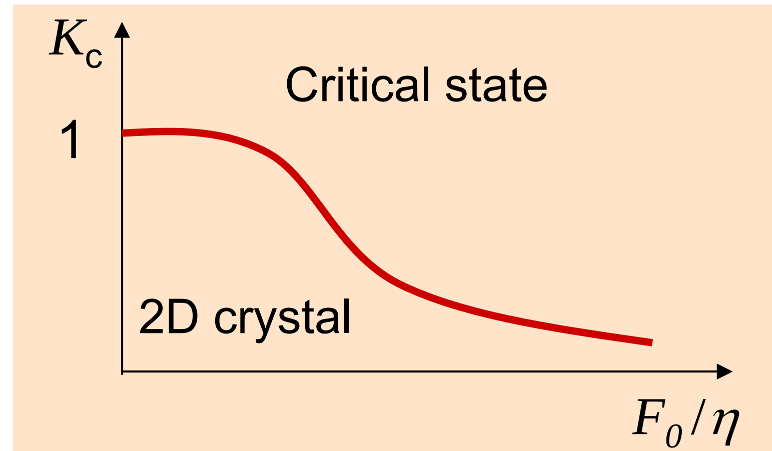


# Global phase diagram

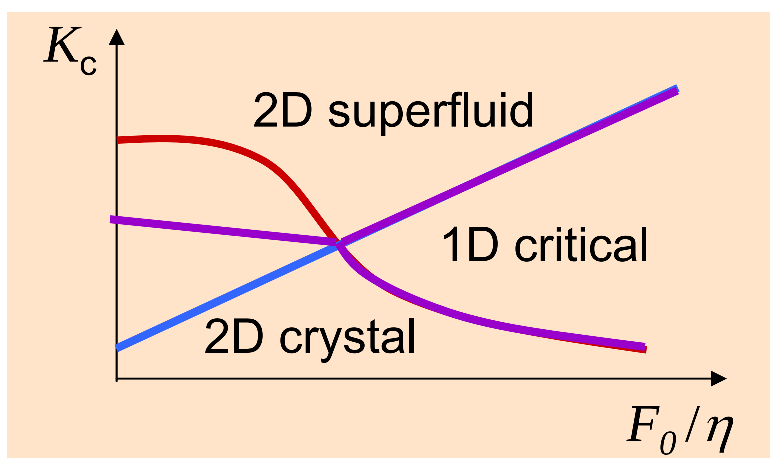
Inter-tube tunneling




Inter-tube interactions



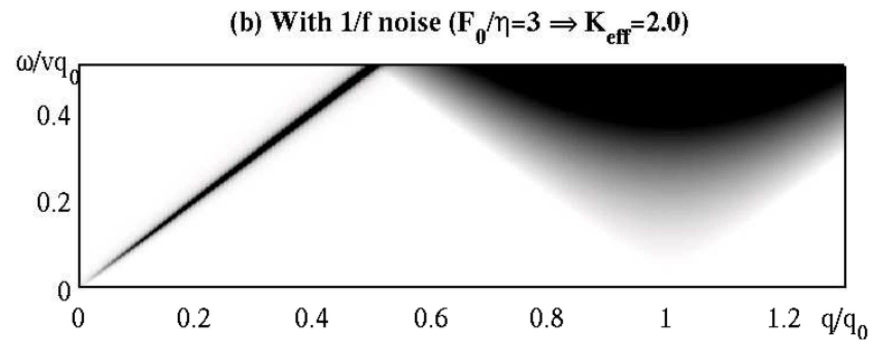

Both perturbations



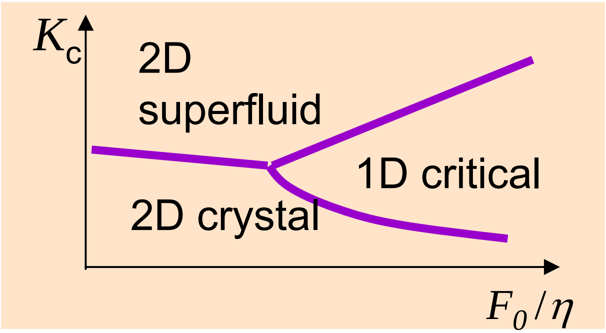

Sliding phases are stabilized by the external noise !

# Summary: Non-equilibrium critical steady states and phase transitions of low dimensional systems subject to 1/f noise

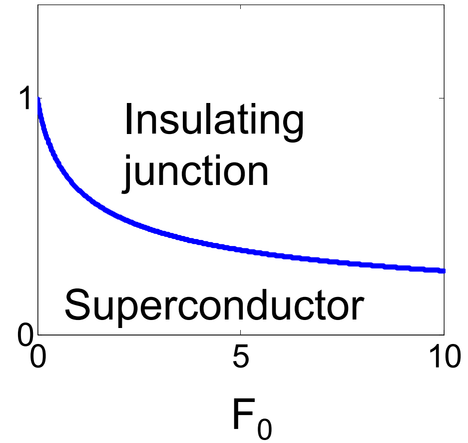
- Powerlaw correlations and response in the critical steady state




- Novel phase transitions tuned by a competition of noise and quantum fluctuations




- Dissipative transition of a shunted Josephson junction at a non universal shunt resistance



# Outlook

- Description of the “gapped” steady states?
  Variational approach

- Potentially interesting solid state applications:
  Superconducting nanowires, nanotubes, …
  Compute I-V curves – Variational approach

- Critical points and phase transitions in higher dimensional driven systems?

- Coupling to a finite temperature bath ?

# Variational approach to non-equilibrium states

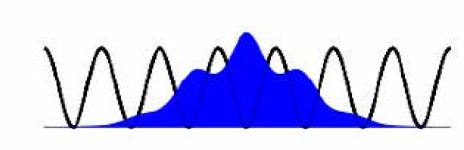
## Digression: Equilibrium variational approach

"Self consistent harmonic approximation"

Fisher, Zwerger (1985)

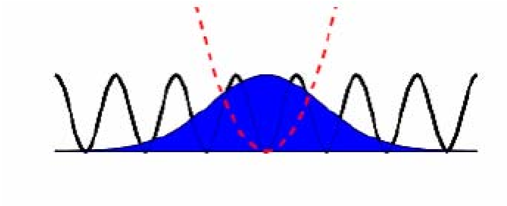
Exact Hamiltonian

$$H = \frac{\hat{Q}^2}{2C} - J\cos(\hat{\theta}) + H_{bath}(\hat{\theta})$$



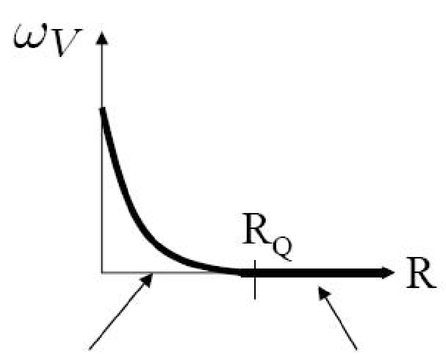
Variational Hamiltonian

$$H = \frac{\hat{Q}^2}{2C} - \frac{1}{2}\omega_V^2\hat{\theta}^2 + H_{bath}(\hat{\theta})$$



Minimize $\langle H \rangle_V$

$$\Rightarrow \omega_V^2 \sim \Delta\left(\frac{J}{\Delta}\right)^{\frac{1}{1-R/R_Q}}$$




Quantum phase transition between superconductor and insulator

# Time dependent variational approach

Consists in minimizing the "effective action"

$$\Gamma_V = \int dt \, \langle \, \psi_V(t) \, | \, i\partial_t \text{ - } H(t) \, | \, \psi_V(t) \, \rangle$$

This method has been used to describe single particle

quantum mechanics by guessing the appropriate $| \, \psi_V(t) \, \rangle$

| |
|---|
| Jackiw, Kerman (1979) |
| Kramer, Saraceno (1981) |

How to extend to many body problems?

# We extend the time dependent variational approach to many-body systems

(1) Use a variational Hamiltonian instead of a variational wavefunction

$$|\,\psi_V(t)\,\rangle \;= U_V(t)\,|\,\psi_0\,\rangle \;= T e^{i\int_0^t dt'\;\; H_V(t')}\,|\,\psi_0\,\rangle$$

$$\Longrightarrow \quad \Gamma_V = \int dt\,\langle\,\psi_0\,|\,U^\dagger(t)\,(H_V(t) - H(t))\,U(t)\,|\,\psi_0\,\rangle$$

(2) Express the effective action as a Keldysh expectation value

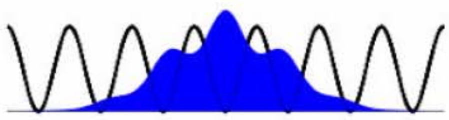
$$\Gamma_V = \int \mathcal{D}\phi \int_{-\infty}^{\infty} dt\,[H_V(\phi_F,t) - H(\phi_F,t)]\,e^{i\int_{-\infty}^{t} dt'\;\; L_V(\phi_F,\phi_B,t')}$$

Does the variational approach capture the non-equilibrium phase transition?

# Many body variational approach: (1) Steady State

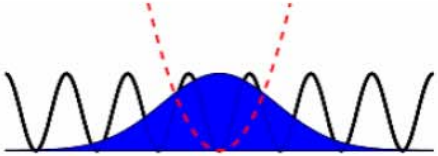
Exact Hamiltonian

$$H(t) = \frac{\hat{Q}^2}{2C} - J\cos(\hat{\theta}) + f(t)\hat{\theta} + H_{bath}(\hat{\theta})$$



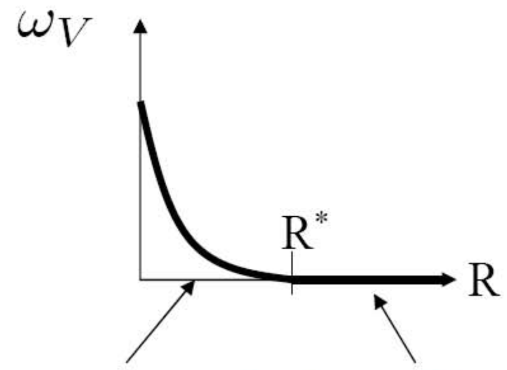
Variational Hamiltonian

$$H(t) = \frac{\hat{Q}^2}{2C} - \frac{1}{2}\omega_V^2\hat{\theta}^2 + f(t)\hat{\theta} + H_{bath}(\hat{\theta})$$



Minimize $\Gamma_V$ ⇨

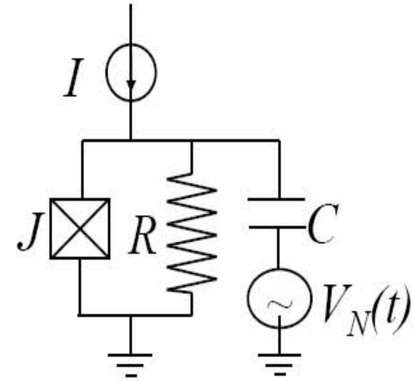
$$\frac{R^*}{R_Q} = \frac{\sqrt{2\pi^2 F_0 + 1} - 1}{\pi^2 F_0}$$




Non equilibrium phase transition between a superconductor and insulator

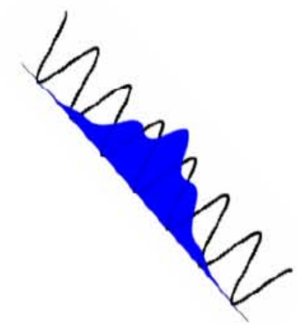
# Many-body variational approach: (2) Current bias

Goal: compute IV curve in the localized phase




Original Hamiltonian

$$H(t) = \frac{\hat{Q}^2}{2C} - J\cos(\hat{\theta}) + f(t)\hat{\theta} + H_{bath}(\hat{\theta})$$



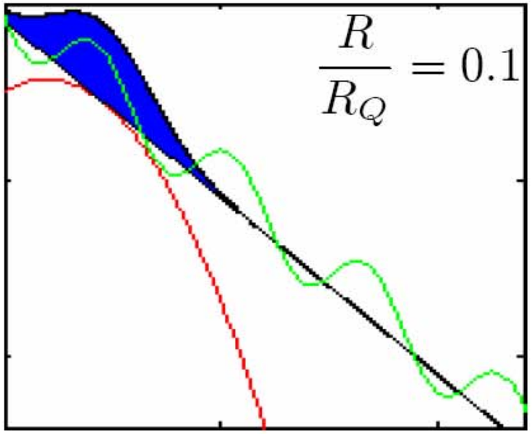
The voltage is given by $V = \partial_t \theta(t)$

Variational Hamiltonian

$$H(t) = \frac{\hat{Q}^2}{2C} - \frac{1}{2}\omega_V^2\left(\hat{\theta} - \alpha(t)\right)^2 + f(t)\hat{\theta} + H_{bath}(\hat{\theta})$$

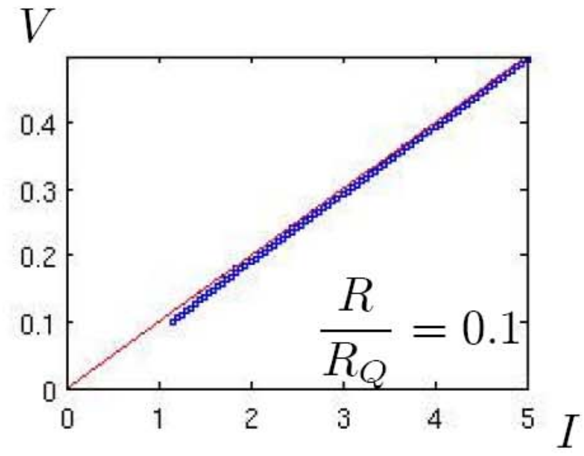
# Many-body variational approach: IV curve

Minimize $\Gamma_V$ $\Rightarrow$




The voltage is given by the average velocity

Repeat for different values of the current




IV curve in the localized phase

# IV curve: compare with universal results

The localized state of the shunted JJ

is dual to the critical state

→ universal behavior in the localized state

Ingold, Nazarov (1992)

Kane, Fisher (1992)

Many-body time dependent variational approach

Universal result

$$I - RV \sim V^{1-2R/R_Q}$$