Defect Generation in Generalized Quenching

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QAQC
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AD1

Adiabatic Dynamics: Quenching & Annealing.

Adiabatic Quantum Dynamics ➔ Slow Variation of quantum fluctuations

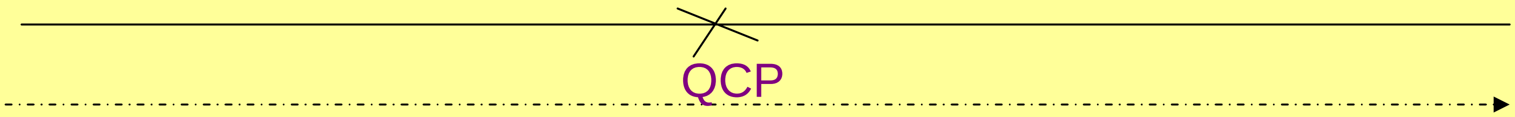
linear variation ➔ $(g - g_c) = t/\tau$

- System is prepared is initially in the ground state

- System should remain in its instantaneous ground state as long as protected by a gap (Δ) from excitations.

$\Delta^2 \tau >> 1$

What happens in passage through a quantum critical point ?



- Gap vanishes; diverging relaxation time

Diverging length scale : $\xi \propto (g - g_c)^{-\nu}$

Diverging time scale: $\xi_\tau \propto (g - g_c)^{-\nu z}$

Slide 2

AD1 Amit Dutta, 1/27/2009

ADIABATIC CONDITION IS NEVER SATISFIED IN THE VICINITY OF A QUANTUM PHASE TRANSITION.

Kibble-Zurek Scaling

Large τ Modes close to the critical modes contribute

The number of excitations (defects) in the final state decreases as a power-law of the tuning rate

Spins in *wrong* direction

$$n \propto \frac{1}{\tau^{\frac{\nu d}{\nu z+1}}}$$

ν and z are the critical exponents!! Universal behavior

- **Annealing:** Finite systems with random interactions; tune quantum fluctuations to zero—**Adiabatic theorem** → true ground state is expected

EXPERIMENTAL SITUATION

Classical Situation: Kibble-Zurek Scaling is verified

- liquid Crystals
- Superfluid Transition

Quantum Situation:

- Quantum Phase Transition from a superfluid to a Mott Insulator
  Greiner et al, Nature Physics (2002)

- Possibility: Ultra-cold atoms in optical lattices
  Wernsdorfer et al, Scince (1999)
  Duan, et al PRL (2003)

KIBBLE-ZUREK ARGUMENT:*

At what time non-adiabaticity dominates??

The tuning parameter varies linearly $\varepsilon = g - g_c = \frac{t}{\tau}$

Relaxation time ~ rate of change Hamiltonian at time $\hat{t}$

$$\hat{\xi}_\tau = \left(\frac{\hat{t}}{\tau}\right)^{-\nu z} = \frac{\varepsilon}{\dot{\varepsilon}} = \hat{t}$$

$$\hat{t} = \tau^{-\frac{\nu z}{\nu z+1}}$$ and $$\hat{\xi} = \tau^{-\frac{\nu}{\nu z+1}}$$

$$n = \frac{1}{\hat{\xi}^d} = \frac{1}{\tau^{\frac{\nu d}{\nu z+1}}}$$

For 1 d, $\nu = z = 1$, giving $\frac{1}{\hat{\xi}} \sim 1/\sqrt{\tau}$

Non-Linear Quenching: $\varepsilon = \left|\frac{t}{\tau}\right|^\alpha \operatorname{sgn}(t)$ → $n = \frac{1}{\tau^{\frac{\alpha \nu d}{\alpha \nu z + 1}}}$

Sen, Sengupta and Mondal, Phys. Rev. Lett. 101 016806 (2008)

Baraankov and Polkovnikon, Phys. Rev. Lett., 101, 076801 (2008)

* W. H. Zurek, U. Dorner, P. Zoller, Phys. Rev. Lett. **95**, 105701 (2005)

A. Polkovnikov, Phys. Rev. B. **72**, 161201(R), (2005)

B. Damski, Phys. Rev. Lett. Phys. Rev. Lett. 95, 105701 (2005)

J. Dziarmaga, Phys. Rev. Lett. 95, 035701 (2005)

R. W. Cherng and L. S. Levitov, Phys. Rev. A 73, 063405 (2006)

ADIABATIC PERTURBATION THEORY:

$$i\frac{\partial\psi}{\partial t} = H(\lambda)\psi;\psi = \sum_p a_p(t)\varphi_p(\lambda);\lambda = \frac{t}{\tau}$$

Critical point

$$\lambda = 0$$

Eigenfrequencies

$$\frac{d\tilde{a}_p}{dt} = -\sum_q \tilde{a}_p(\lambda)\langle p|\frac{d}{d\lambda}|q\rangle e^{i\tau\int d\lambda\omega_p(\lambda')-\omega_q(\lambda')d\lambda'};a_p(t) = \tilde{a}_p(\lambda)e^{-i\tau\int\omega_p(\lambda')d\lambda'}$$

- System starts from its ground State
- The system is translationally invariant

$$n_{ex} \sim \int\frac{d^dk}{(2\pi)^d}\left|\int_{-\infty}^{\infty} d\lambda\,\langle k|\frac{d}{d\lambda}|0\rangle e^{i\tau\int d\lambda\{\omega_p(\lambda')-\omega_o(\lambda')\}d\lambda'}\right|^2$$

Scaling forms:

$$\omega_k - \omega_o = \lambda^{z\nu} f\left(\frac{k}{\lambda^{\nu}}\right);\langle k|\frac{d}{d\lambda}|0\rangle = \frac{1}{\lambda} g\left(\frac{k}{\lambda^{\nu}}\right)$$

Scaling $\lambda = k^{1/\nu}\xi; k = \delta^{\nu/\nu z+1}$ →

$$n \propto \frac{1}{\tau^{\frac{\nu d}{\nu z+1}}}$$

A. Polkovnikov, Phys. Rev. B. 72, 161201(R), (2005)
Gritsev and Polkovnikov, C. De Grandi and Polkovnikov, Review Articles

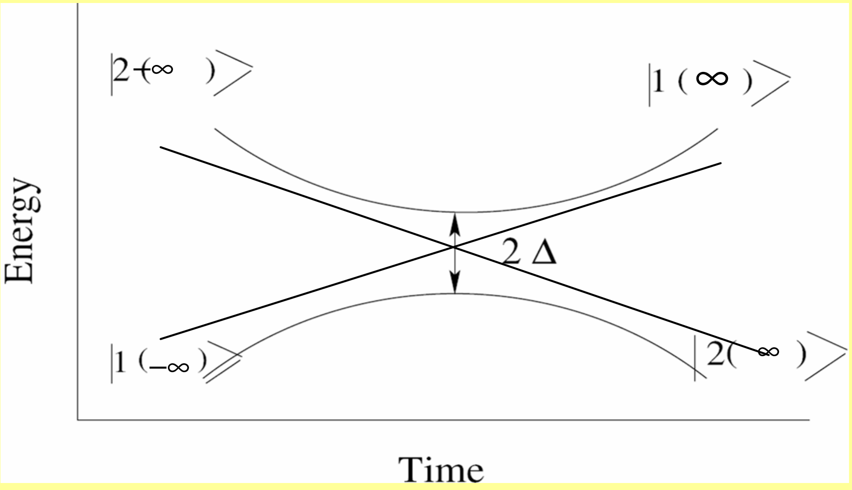
LANDAU-ZENER TRANSITION*

$$H = \varepsilon_1 |1\rangle\langle 1| + \varepsilon_2 |2\rangle\langle 2| + \Delta |1\rangle\langle 2| + \Delta |2\rangle\langle 1|$$

$$\varepsilon_1 - \varepsilon_2 = \frac{t}{\tau}; \quad i\frac{\partial \psi}{\partial t} = H\psi$$

$$\psi = C_1|1\rangle + C_2|2\rangle$$

$$i\frac{\partial}{\partial t}\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} \frac{t}{\tau} & \Delta \\ \Delta & -\frac{t}{\tau} \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$



Initial condition

$$C_1(-\infty) = 1; C_2(-\infty) = 0$$

Probability of non-adiabatic transition = $|C_1(\infty)|^2$

$$P_{\text{non-ad}} = \exp(-2\pi\gamma); \gamma = \frac{\Delta^2}{\frac{d}{dt}(\varepsilon_1 - \varepsilon_2)} = \Delta^2\tau$$

*, *Quantum Mechanics, Landau and Lifshitz; Zener, Proc. Of Royal Society A 1932*

S.Suzuki and M.Okada, in Quantum Annealing and Related Optimization Methods ,

edited by A. Das and B.K.Chakrabarti (Springer – Verlag, Berlin, 2005), p. 185

Diabatic Basis Vectors

$$|1\rangle, |2\rangle$$

Adiabatic Basis Vectors

Instantaneous Eigenstates

Diabatic Limit ← $\Delta^2\tau \ll 1; \Delta^2\tau \gg 1$ → Adiabatic Limit

$$i\frac{dC_1}{dt} = \frac{t}{\tau}C_1 + \Delta C_2; i\frac{dC_2}{dt} = -\frac{t}{\tau}C_2 + \Delta C_1$$

$$C_1 = \tilde{C}_1 \exp\left(i\int dt' \frac{t'}{\tau}\right); C_2 = \tilde{C}_2 \exp\left(-i\int dt' \frac{t'}{\tau}\right)$$

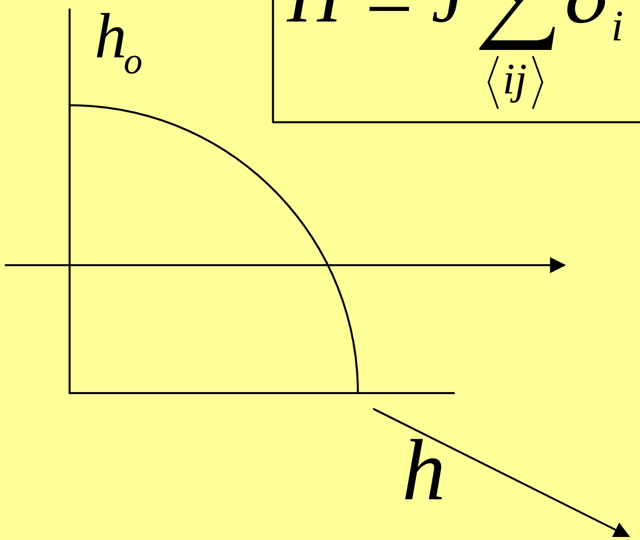
$$\left(\frac{d^2}{dt^2} - 2i\frac{t}{\tau}\frac{d}{dt} + \Delta^2\right)\tilde{C}_1(t) = 0$$

Scaling $t \rightarrow \frac{t}{\sqrt{\tau}}$ with initial condition $C_1(-\infty) = 1$

$$\left(\frac{d^2}{dt^2} - 2i\frac{d}{dt} + \Delta^2\tau\right)\tilde{C}_1(t) = 0$$

Probability of Excitations: $p = |C_1(\infty)|^2 = f(\Delta^2\tau)$

$$H = J\sum_{\langle ij\rangle}\sigma_i^x\sigma_j^x - h\sum_i \sigma_i^z - h_o\sum_i \sigma_i^x$$



QCP $\nu = z = 1$

Defect Density

$$n \propto \frac{1}{\sqrt{\tau}}$$

GENERALIZED QUENCHING SCHEMES

WE ADDRESS THE FOLLOWING QUESTIONS

- The system is repeatedly taken back and forth through the Quantum Critical point.
- Under a reversal of the magnetic field right at the quantum critical Point
- Effect of waiting at the quantum critical point for a time t_w

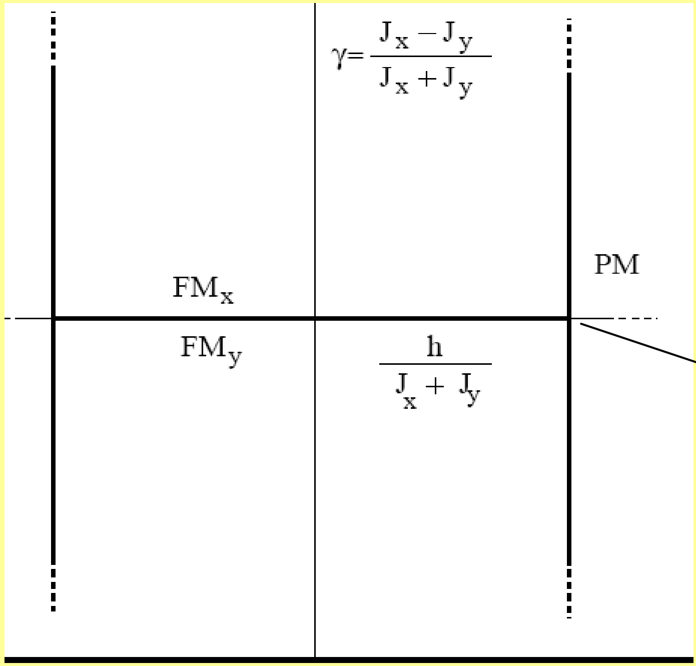
- *Mukherjee, Dutta and Sen, Phys. Rev. B 77, 214427 (2008)*
- *Divakaran, Dutta , Phys. Rev. B **79**, 22408 (2009)*
- *Divakaran, Dutta and Sen, arxiv:0910.5548*

TRANSVERSE XY MODEL: PHASE DIAGRAM

$$H = -\frac{1}{2}\left(J_x \sum_{\langle ij \rangle} S_i^x S_j^x + J_y \sum_{\langle ij \rangle} S_i^y S_j^y + h \sum_i S_i^z \right)$$

→ Ising Symmetry

$$\gamma = \frac{J_x - J_y}{J_x + J_y}$$



$J_x=J_y$, . Isotropic XY model ; Gapless for $h < J_x + J_y$

$J_y=0$; Transverse Ising Model; QPT: $h = J_x$

Ising Transition

$$h = \pm\left(J_x + J_y\right)$$

$$J_x = J_y$$

anisotropic transition

Damle and Sachdev, Phys Rev. Lett., 1996

)

Bunder and Mckenzie, Phys. Rev. B 60, 344 (1999)

How do we find it?

$$H = -\frac{1}{2}\left(J_x \sum_{\langle ij \rangle} S_i^x S_j^x + J_y \sum_{\langle ij \rangle} S_i^y S_j^y + h \sum_i S_i^z \right)$$

→ Spins coupled → Cannot treat a spin independently

↓

Way out → *Jordan Wigner transformation* → followed by *fourier transformation*

Jordan Wigner

↓

$$c_n = \left(\prod_{j=-\infty}^{n-1} \sigma_j^z \right) (-1)^n \sigma_n^-$$

$$c_n^+ = \left(\prod_{j=-\infty}^{n-1} \sigma_j^z \right) (-1)^n \sigma_n^+$$

→

$$H = -\sum_{k>0} \left\{ \left(J_x + J_y \right) \cos k + h \right\} \left(C_k^\dagger C_k + C_{-k}^\dagger C_{-k} \right)$$
$$-i \sum_{k>0} \left(J_x - J_y \right) \sin k \left(C_k^\dagger C_{-k}^\dagger + C_k C_{-k} \right)$$

↓

$$H_k = \begin{bmatrix} h + \left(J_x + J_y \right) \cos k & i \left(J_x - J_y \right) \sin k \\ -i \left(J_x - J_y \right) \sin k & -\left\{ h + \left(J_x + J_y \right) \cos k \right\} \end{bmatrix}$$

$|0\rangle, |k, -k\rangle$ *space*

Transverse quenching

$$h = \frac{t}{\tau}$$

$$i\frac{\partial}{\partial t}|\psi\rangle = \begin{bmatrix} \frac{t}{\tau} + (J_x + J_y)\cos k & i(J_x - J_y)\sin k \\ -i(J_x - J_y)\sin k & -\left\{\frac{t}{\tau} + (J_x + J_y)\cos k\right\} \end{bmatrix} |\psi\rangle$$

- Dynamics of each mode is an independent Landau-Zener problem
- For slow-driving only modes close to the critical point contribute

Focussing on the critcal point at h=J

$$i\frac{\partial}{\partial t}|\psi\rangle = \begin{bmatrix} \frac{t}{\tau} + k^2 & i\gamma k \\ -i\gamma k & -\left(\frac{t}{\tau} + k^2\right) \end{bmatrix} |\psi\rangle$$

Probability of excitations

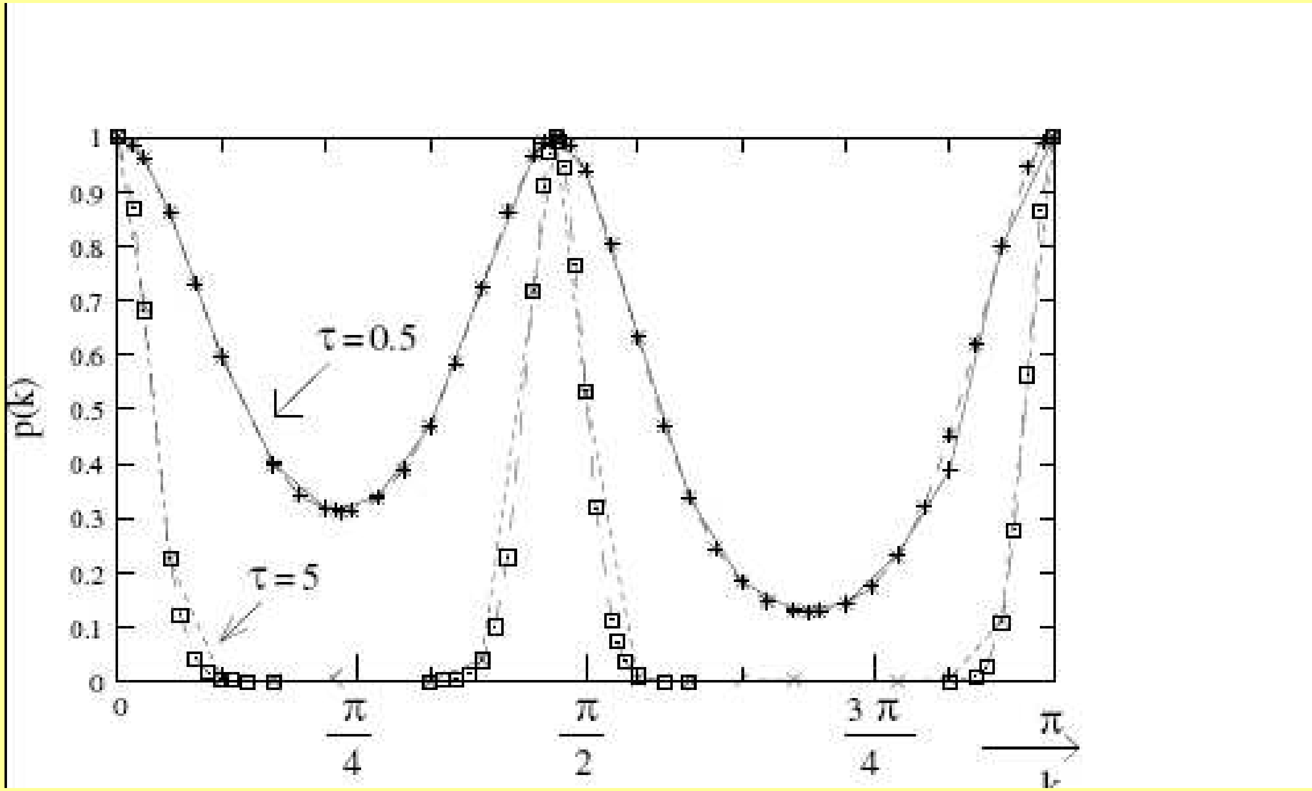
$$p_k = \exp(-\pi\gamma^2 k^2 \tau)$$

→ Effective rate

Defect Density

$$n = \int p_k dk \propto \frac{1}{\sqrt{\tau}}$$

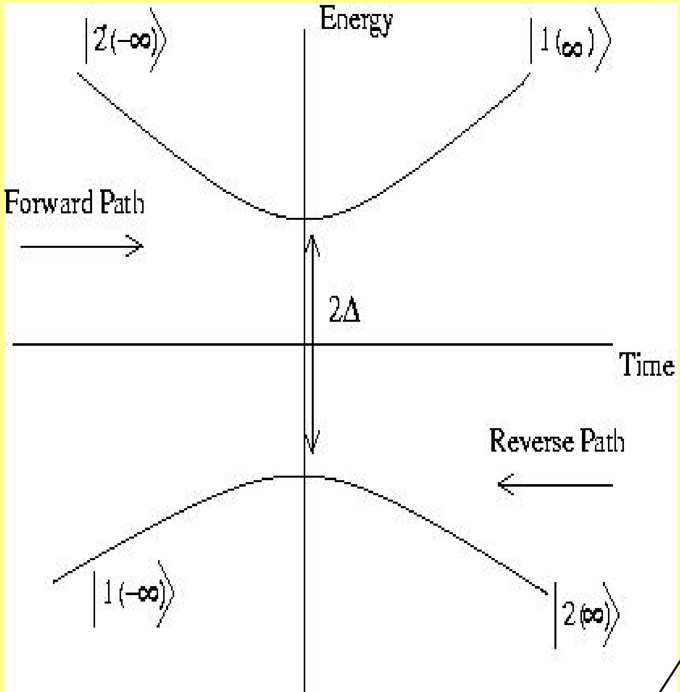
R. W. Cherng and L. S. Levitov, Phys. Rev. A 73, 063405 (2006)



1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
p(k)
τ = 0.5
τ = 5
0
π/4
π/2
3 π/4
π

Repeated Quenching of the transverse field through a quantum critical point

The system is quenched back and forth across the quantum critical point



$$p_k = 2e^{-\pi\Delta^2\tau}(1-e^{-\pi\Delta^2\tau})$$

Mukherjee, Dutta and Sen, Phys. Rev. B 77, 214427 (2008)

Generalized Landau-Zener form

Random initial phases

$$|C_1(2)|^2 = e^{-\pi\Delta^2\tau}|C_1(1)|^2 + \left(1-e^{-\pi\Delta^2\tau}\right)|C_2(1)|^2$$

$$|C_2(2)|^2 = e^{-\pi\Delta^2\tau}|C_2(1)|^2 + \left(1-e^{-\pi\Delta^2\tau}\right)|C_1(1)|^2$$

Recursive Relations after l passages

$$\left|C_1(-(-1)^{l+1}\infty)\right|^2 = e^{-2\pi\gamma}\left|C_1(-(-1)^l\infty)\right|^2 + (1-e^{-2\pi\gamma})\left|C_2(-(-1)^l\infty)\right|^2$$

$$\left|C_2(-(-1)^{l+1}\infty)\right|^2 = e^{-2\pi\gamma}\left|C_2(-(-1)^l\infty)\right|^2 + (1-e^{-2\pi\gamma})\left|C_1(-(-1)^l\infty)\right|^2$$

*E **Shimshoni**, Y Gefen - Ann. Phys, 1991*

The recursion relation do not contain any cross terms:

$$C_1(\infty)C_2(\infty)^* \text{ or } C_2(\infty)C_1(\infty)^*$$

In the limit of $t \longrightarrow \pm\infty$ $\quad C_1(t) \approx \exp(\pm i/\hbar \int dt' E(t'))$

The Two-Cross terms oscillate rapidly with the initial time; Different k-modes are uncorrelated. Vanishes over integration over momentum

After every passage through quantum Critical point, we are assuming a diagonal density matrix . A mixed state, Decoherence

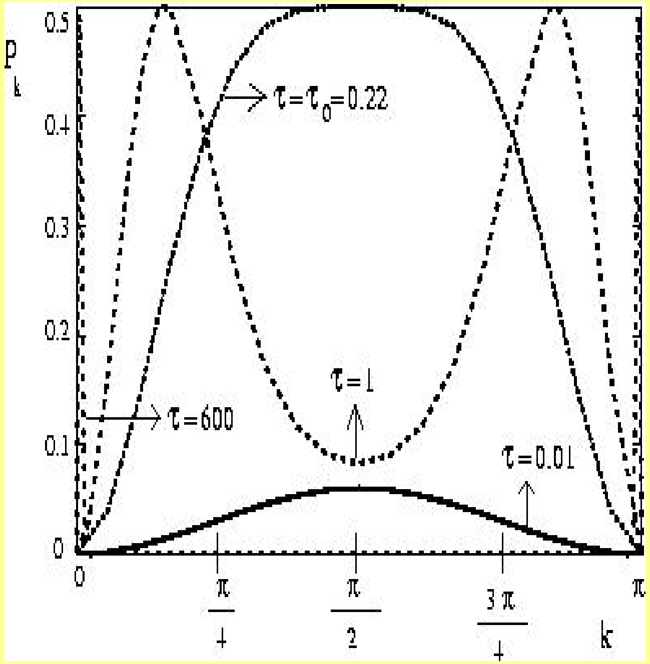
E **Shimshoni**, Y Gefen - Ann. Phys, 1991 $\quad 2P_{LZ}(1-P_{LZ})(1-\cos\psi)$

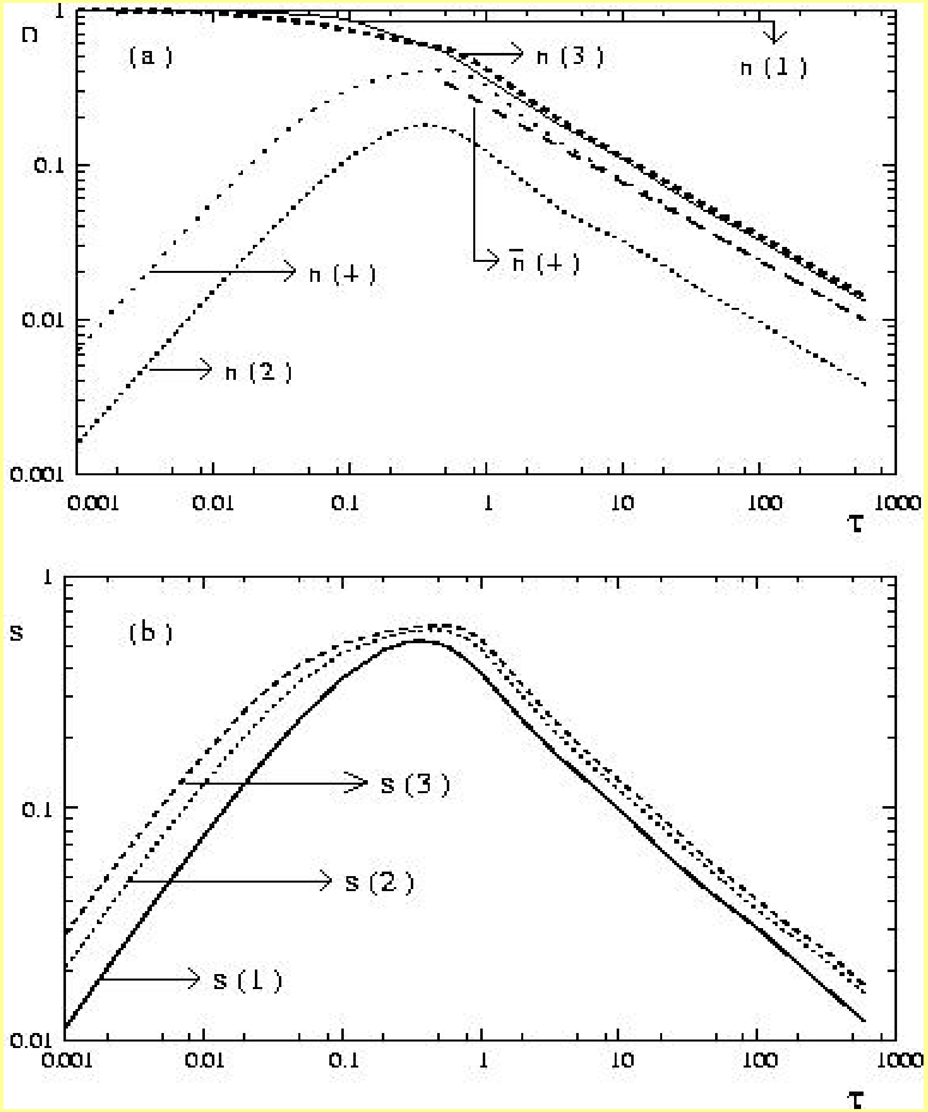
$$S = -\int dk\, p_k \ln(p_k) + (1-p_k)\ln(1-p_k)$$

→ Diagonal Entropy

A. Polkovnikov, arxiv0806.2861

Situation for two successive half-periods l=2





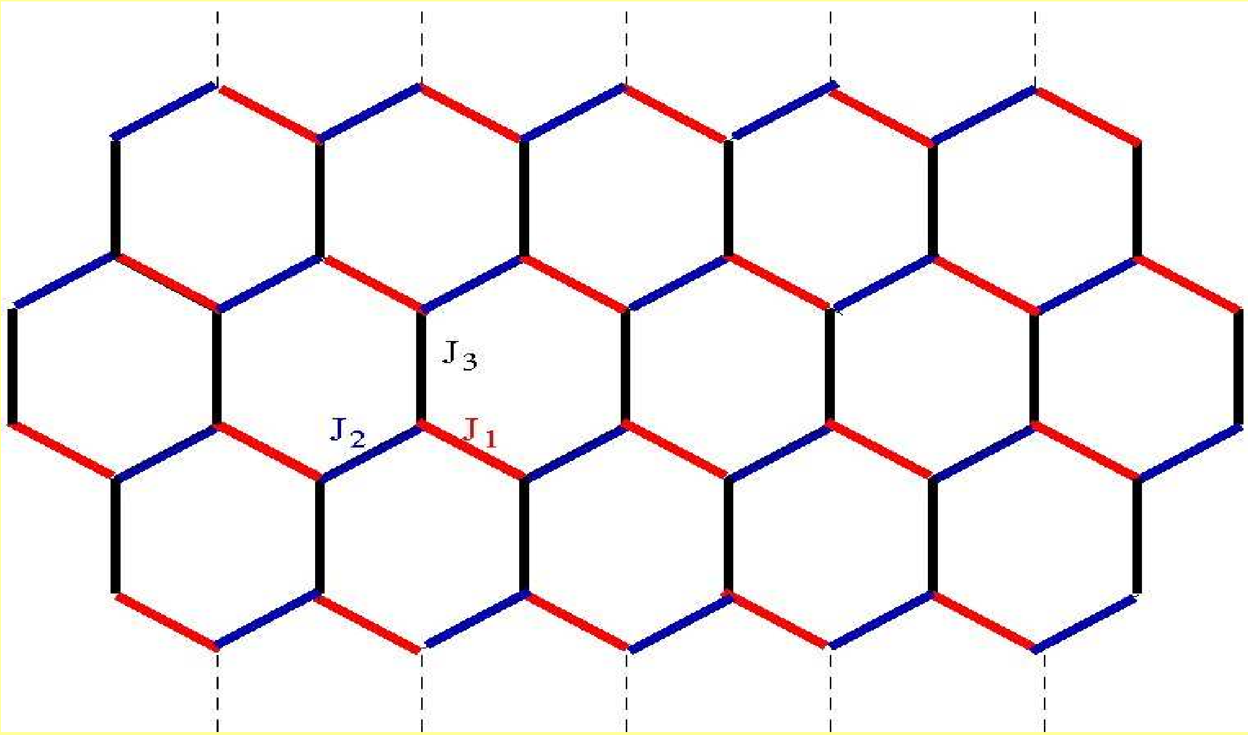
Non-adiabatic transition Probability

$$p_k = 2e^{-\pi k^2 \tau}(1 - e^{-\pi k^2 \tau})$$

Reversal of the parameter right at the Quantum Critical point:

2 dimensional Kitaev Model

$$H_{2d} = \sum_{n+l=even} J_1 \sigma^x_{n,l} \sigma^x_{n+1,l} + J_2 \sigma^y_{n-1,l} \sigma^y_{n,l} + J_3 \sigma^z_{n,l} \sigma^z_{n,l+1}$$



A. Kiatev: Ann. Phys. 321,2 (2006)

Sengupta, Sen and Mondal, Phys. Rev. Lett. 2008

ONE DIMENSIONAL VERSION

$$J_3 = 0$$

$$H_{1d} = \sum_{n=1}^{N} J_1 \sigma_{2n}^x \sigma_{2n+1}^x + J_2 \sigma_{2n-1,l}^y \sigma_{2n}^y$$

Jordan-Wigner transformation:

$$a_n = \left(\prod_{j=-\infty}^{2n-1} \sigma_j^z \right) \sigma_{2n}^y ; b_n = \left(\prod_{j=-\infty}^{2n} \sigma_j^z \right) \sigma_{2n+1}^x$$

Majorana Fermions

$$a_n^\dagger = a_n ; b_n^\dagger = b_n$$

$$H = \sum_{k=0}^{\pi} \psi_k^\dagger H_k \psi_k$$

$$H_k = 2i \begin{pmatrix} 0 & -J_1 - J_2 e^{-ik} \\ J_1 + J_2 e^{+ik} & 0 \end{pmatrix}$$

$$H_k = 2 \begin{pmatrix} J_- \sin k & J_+ \cos k \\ J_+ \cos k & J_- \sin k \end{pmatrix}$$

$$\varepsilon_k = \pm 2 \sqrt{(J_- \sin k)^2 + (J_+ \cos k)^2}$$

QCP: $J_- = 0, k = \dfrac{\pi}{2}$

QUENCHING SCHEME WITH REVERSAL:

$$J_- = \frac{t}{\tau} \quad \text{for } -\infty < t < 0$$
$$= -\frac{t}{\tau} \quad \text{for } 0 < t < \infty$$

→ Discontinuous change at the QCP

LZ problem for each mode with reversal at the minimum Gap::

$$|\psi(t)\rangle = C_1|1\rangle + C_2|2\rangle$$

$$\psi(t=0) = \alpha|1\rangle + \beta|2\rangle$$

$$i\frac{\partial}{\partial t}|\psi\rangle = \begin{pmatrix} \frac{t}{\tau} & \Delta \\ \Delta & -\frac{t}{\tau} \end{pmatrix}|\psi\rangle \quad --(1)$$

$$\psi(t<0)|_{t=0} = \psi(t>0)|_{t=0}$$

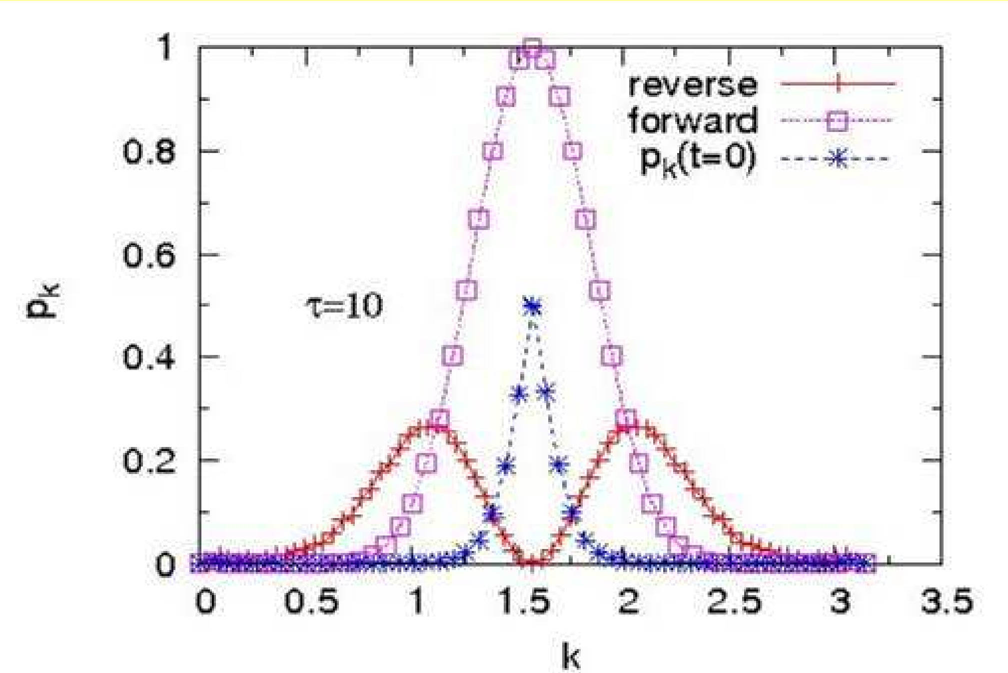
$$p = |C_2(t\to\infty)|^2 = \frac{1}{2}(1-e^{-2\pi\Delta^2\tau})\left|\frac{\Gamma(1-i\Delta^2\tau/2)}{\Gamma(1+i\Delta^2\tau/2)} + i\frac{\Gamma(1/2-i\Delta^2\tau/2)}{\Gamma(1/2+i\Delta^2\tau/2)}\right|^2$$

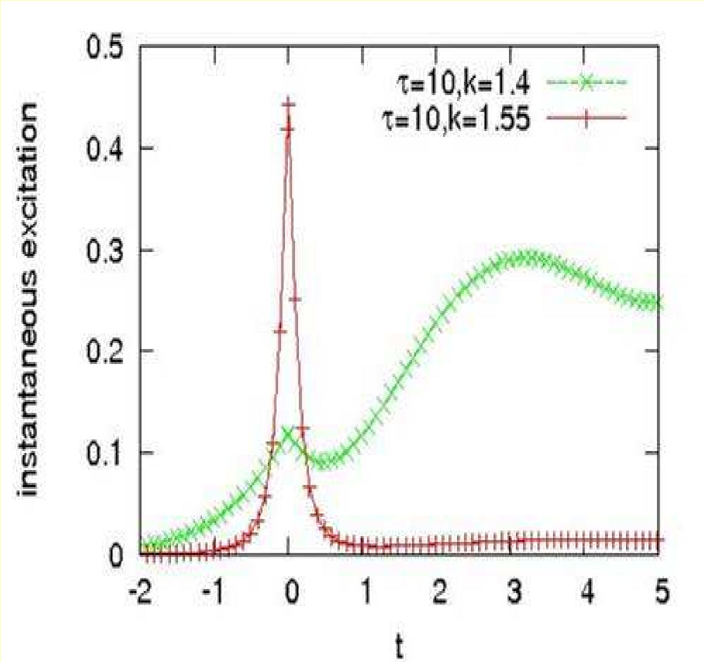
*Garanin and Schilling, Phys Rev B 66, 174438 (2002)

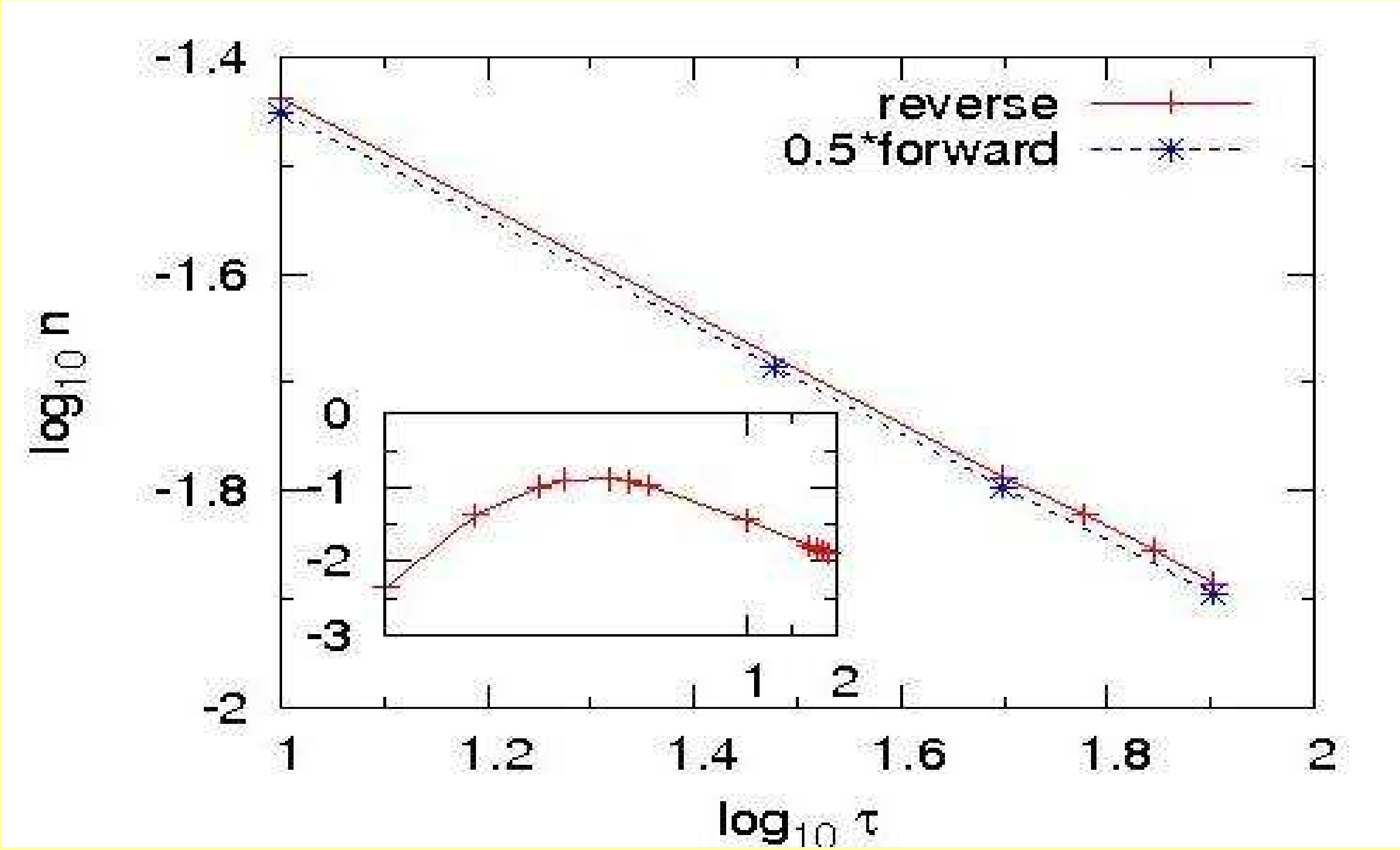
For Kitaev Model:

$$p_k = |C_{2k}(t \to \infty)|^2 = \frac{1}{2}(1 - e^{-2\pi\alpha}) \left| \frac{\Gamma(1 - i\alpha/2)}{\Gamma(1 + i\alpha/2)} + i \frac{\Gamma(1/2 - i\alpha/2)}{\Gamma(1/2 + i\alpha/2)} \right|^2$$

$$\alpha = \frac{\tau \cos^2 k}{\sin k}$$ → Effective Rate







Defect Density as a function of the quenching rate

LANDAU- ZENER PROBLEM WITH WAITING AT THE MINIMUM GAP:

Recall the LZ Hamiltonian
Minimum Gap at t=0

$$\psi = C_1 |1\rangle + C_2 |2\rangle$$

$$i\frac{\partial}{\partial t}|\psi\rangle = \begin{pmatrix} \frac{t}{\tau} & \Delta \\ \Delta & -\frac{t}{\tau} \end{pmatrix} |\psi\rangle - -(1)$$

- The dynamics is initiated at $t \rightarrow -\infty$ and system is brought to $t = 0$
- The system is allowed to relax at the minimum gap for a time t_w
- Linear driving is again resumed again from $t = t_w$ to $t = +\infty$

- **How does the waiting influence the dynamics?**
- **How does t_w alter the probability of excitations? Can one estimate it exactly?**
- **We need to visualize LZ dynamics in two parts.**

Vitanov and Garraway, Phys. Rev. A, 53, 4288 (1996)

Landau-Zener revisited with a new technique:

Initial condition: $C_1(-\infty)=1, C_2(-\infty)=0$

The wave function at the minimum gap (Exactly known)*

$$\psi(0)=\alpha|1\rangle+\beta|2\rangle;|\alpha|^2+|\beta|^2=1;|\alpha|^2-|\beta|^2=\exp\left(-\pi\Delta^2\tau/2\right)$$

Idea of time reversal

$$\psi(-\infty)=\begin{pmatrix}1\\0\end{pmatrix}\rightarrow\psi(0)=\begin{pmatrix}\alpha\\\beta\end{pmatrix}$$

Orthogonality: $\psi(-\infty)=\begin{pmatrix}0\\1\end{pmatrix}\rightarrow\psi(0)=\begin{pmatrix}\beta^*\\-\alpha^*\end{pmatrix}$ Up to a phase

Question? **What are the wave functions at** $t=0$ **those will evolve to**

$\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ **at** $t=+\infty$??

* *Vitanov, Phys. Rev. A 59, 988 (1999); C. De Grandi and A. Polkovnikov, 0910.2236*

Idea of time reversal

$$-i\frac{\partial}{\partial t}\sigma^z|\psi\rangle=\begin{pmatrix}-\frac{t}{\tau} & \Delta\\ \Delta & \frac{t}{\tau}\end{pmatrix}\sigma^z|\psi\rangle--(2)$$

Dynamics occurring in (1) for $t=0$ to $-\infty$ is same as in (2) with

$$t'\rightarrow -t;\psi'\rightarrow\sigma^z\psi$$

t' Going from 0 to ∞

$$\psi(0)=\sigma^z\begin{pmatrix}\alpha\\ \beta\end{pmatrix}=\begin{pmatrix}\alpha\\ -\beta\end{pmatrix}\rightarrow\psi(\infty)=\begin{pmatrix}1\\ 0\end{pmatrix}$$

$$\psi(0)=\sigma^z\begin{pmatrix}\beta^*\\ -\alpha\end{pmatrix}=\begin{pmatrix}\beta^*\\ \alpha^*\end{pmatrix}\rightarrow\psi(\infty)=\begin{pmatrix}0\\ 1\end{pmatrix}$$

$$\psi(0)=\begin{pmatrix}\alpha\\ \beta\end{pmatrix}=\left[\begin{pmatrix}\alpha\\ -\beta\end{pmatrix}\begin{pmatrix}\alpha^* & -\beta^*\end{pmatrix}+\begin{pmatrix}\beta^*\\ \alpha^*\end{pmatrix}\begin{pmatrix}\beta & \alpha\end{pmatrix}\right]\begin{pmatrix}\alpha\\ \beta\end{pmatrix}$$

$$=\left(|\alpha|^2-|\beta|^2\right)\begin{pmatrix}\alpha\\ -\beta\end{pmatrix}+2\alpha\beta\begin{pmatrix}\beta^*\\ \alpha^*\end{pmatrix}$$

What is $\psi(+\infty)$??

$$\psi(\infty)=\left(|\alpha|^2-|\beta|^2\right)\begin{pmatrix}1\\ 0\end{pmatrix}+2\alpha\beta\begin{pmatrix}0\\ 1\end{pmatrix}$$

Probability of excitation:

$$P=\left(|\alpha|^2-|\beta|^2\right)^2=\exp(-\pi\Delta^2\tau)$$

Exact Result

LZ dynamics with waiting at the mimimum gap

For $t=0$ to $t=t_w$

$$i\frac{\partial}{\partial t}|\psi\rangle = \begin{pmatrix} 0 & \Delta \\ \Delta & 0 \end{pmatrix}|\psi\rangle$$

Oscillation between two levels

$$\psi(t_w) = \frac{\alpha+\beta}{2}e^{-i\Delta t_w}\begin{pmatrix}1\\1\end{pmatrix} + \frac{\alpha-\beta}{2}e^{+i\Delta t_w}\begin{pmatrix}1\\-1\end{pmatrix}$$

$$\psi(t_w) = \left[\begin{pmatrix}\alpha\\-\beta\end{pmatrix}\begin{pmatrix}\alpha^* & -\beta^*\end{pmatrix} + \begin{pmatrix}\beta^*\\\alpha^*\end{pmatrix}\begin{pmatrix}\beta & \alpha\end{pmatrix}\right]\psi(t_w) = \begin{pmatrix}\alpha^* & -\beta^*\end{pmatrix}\psi(t_w)\begin{pmatrix}\alpha\\\beta\end{pmatrix} + \begin{pmatrix}\beta & \alpha\end{pmatrix}\psi(t_w)\begin{pmatrix}\beta^*\\\alpha^*\end{pmatrix}$$

Like the conventional case

$$\psi(+\infty) = \begin{pmatrix}\alpha^* & -\beta^*\end{pmatrix}\psi(t_w)\begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}\beta & \alpha\end{pmatrix}\psi(t_w)\begin{pmatrix}0\\1\end{pmatrix}$$

Probability of excitations:

$$p_{t_w} = \left[\left(|\alpha|^2 - |\beta|^2\right)^2 \cos(\Delta t_w) - i(\alpha^*\beta - \alpha\beta^*)\sin(\Delta t_w)\right]^2$$

Closed form expressions:

$$\Delta^2\tau \to 0;\ p_{t_w} \approx e^{-\pi\Delta^2\tau/2}\cos^2(\Delta(t_w + \sqrt{\pi\tau});\ \Delta^2\tau \to \infty;\ p_{t_w} \approx \frac{1}{16\Delta^2\tau}\sin^2(\Delta(t_w)$$

Application to Quenching

One-dim Kitaev Model

$$H_k = 2\begin{pmatrix} J_- \sin k & J_+ \cos k \\ J_+ \cos k & J_- \sin k \end{pmatrix}$$

Quenching Scheme

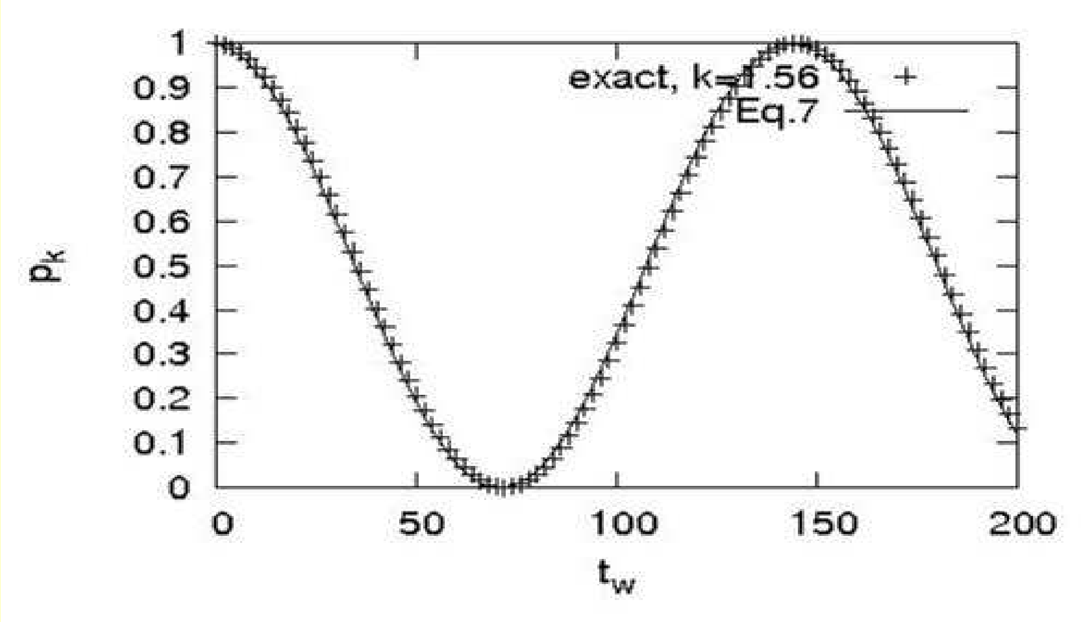
$$J_-(t) = \frac{t}{\tau} \quad \text{for } -\infty < t \le 0$$
$$= 0 \text{ for } 0 < t \le t_w$$
$$= \frac{t - t_w}{\tau} \quad \text{for } t_w \le t < \infty$$

$$\Delta^2 \tau = \frac{\pi \tau \cos^2 k}{\sin k}; \; J_+ = 1$$

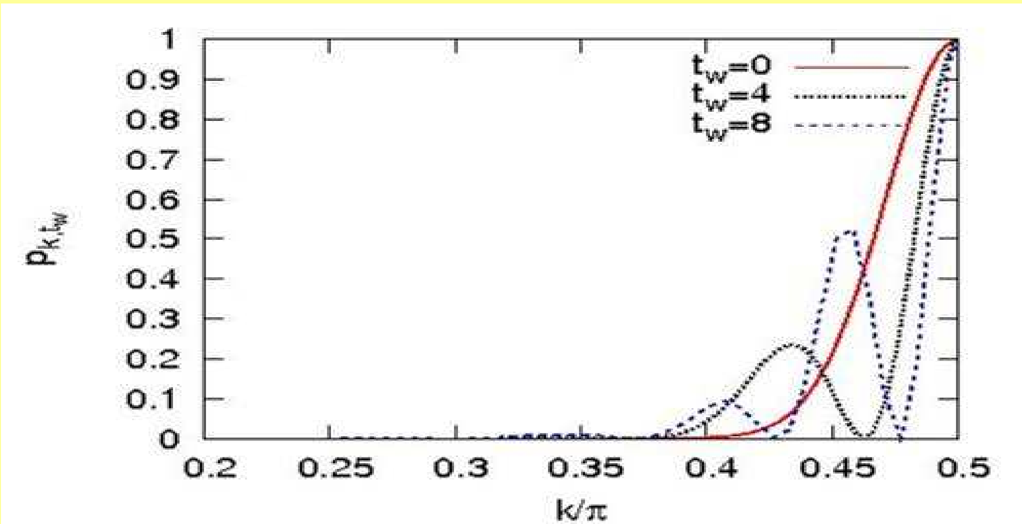
For critical modes: Use the diabatic approximation

$$p_{k,t_w} \approx e^{-\pi\tau\cos^2 k/\sin k} \cos^2\left[2\cos k\left(t_w + \frac{1}{2}\sqrt{\frac{\pi\tau}{\sin k}}\right)\right]$$

Comparison: Numerical and Analytical Results:



Excitation probability as a function of waiting time



Secondary Maxima

$$\cos k = \frac{m\pi}{2\left(t_w + \frac{\sqrt{\pi}}{2}\right)}$$

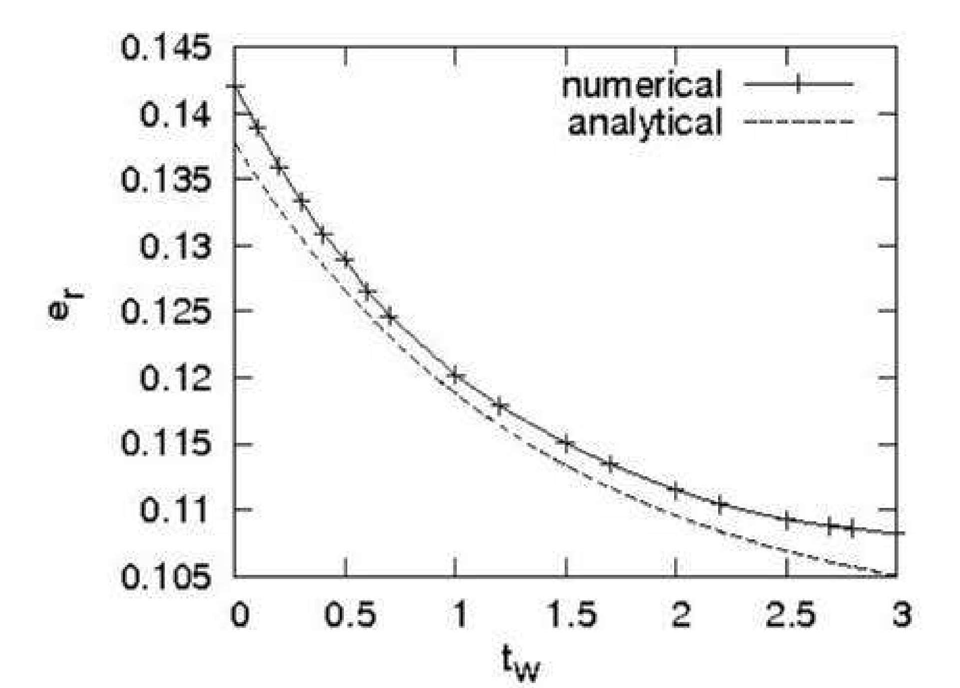
Quenching with Waiting

Take Operator
$$O=\frac{1}{N}\sum_m\left(\sigma_{2m}^x\sigma_{2m+1}^x-\sigma_{2m-1}^y\sigma_{2m}^y\right)$$

Residual Energy
$$e_r=\langle\psi_f(t\to\infty)|O|\psi_f(t\to\infty)\rangle-\langle\psi_{true}|O|\psi_{true}\rangle$$

$$e_r=\int_0^{\pi/2}\frac{dk}{2\pi}8\sin k$$

$$e_r \approx \frac{0.32}{\sqrt{\tau}}\left(1+0.37e^{-2.3t_w/\sqrt{\tau}}\right)$$



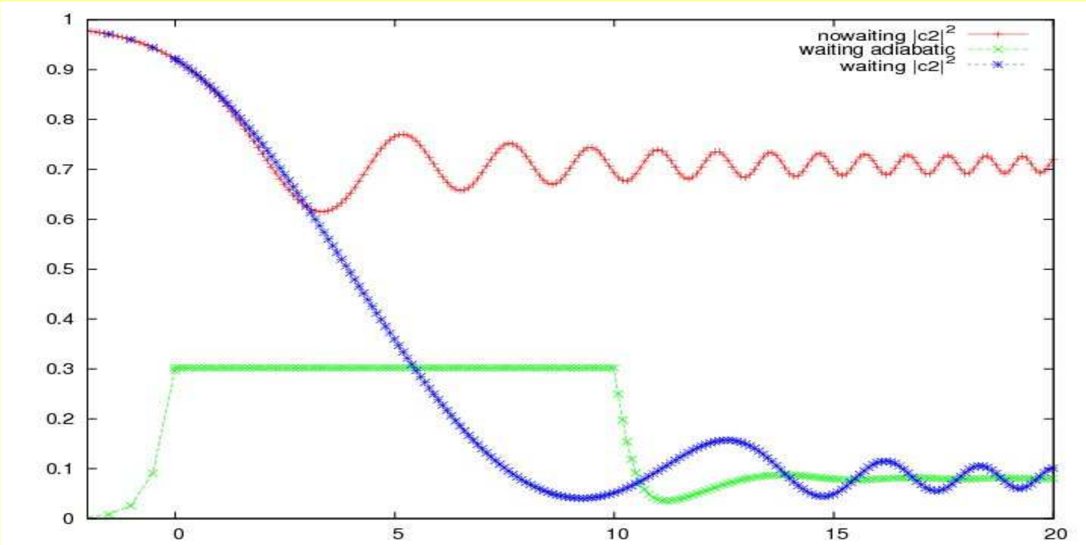
WHY THIS DECAY ?

During the Waiting:

$$H_k = 2\begin{pmatrix} 0 & \cos k \\ \cos k & 0 \end{pmatrix}$$

- Oscillation between two levels: Frequency proportional to $2\cos k$
- Very small for the modes close to the critical modes
- Waiting time << Time period
- For critical modes excitation probability gets smaller $|C_1(t_w)|^2 < |C_1(t=0)|^2$

- Non-critical modes do not contribute to the residual energy



Instantaneous excitations

WHY WE CHOOSE KITAEV MODEL?

$$H_k = 2\begin{pmatrix} J\_\sin k & \cos k \\ \cos k & J\_\sin k \end{pmatrix}$$

Minima for all the modes occur at the critical point $t=0$

Not true for transverse XY models

$$\begin{bmatrix} \frac{t}{\tau} + k^2 & i\gamma k \\ -i\gamma k & -\left(\frac{t}{\tau} + k^2\right) \end{bmatrix}$$

Consider the large τ limit

$$k \sim \frac{1}{\sqrt{\tau}}$$

→ Landau-Zener Formula

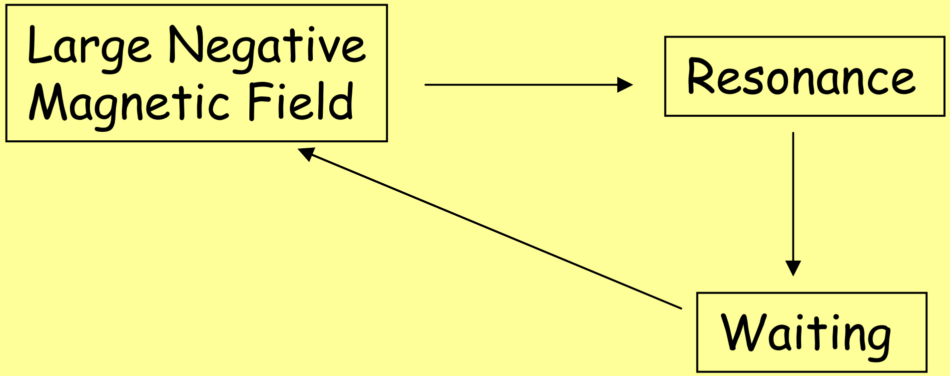
Scale $t' = \frac{t}{\sqrt{\tau}}$

$$t' + \frac{1}{2\gamma\sqrt{\tau}}$$

→ Vanishes for $\tau \to \infty$

EXPERIMENTS SINGLE MOLECULAR MAGNETS

$Mn_{12}Ac$ → Single Molecular Magnet with $S = 0$



Streched Exponenetical decay in number particles that has crossed the barrier

Waiting followed by reversal   Experiment for forward??

Caveats: **Non-Linear Term**
LZ for all rates?

Wernsdorfer, et al Phys. Rev. B 72, 214429 (2005)

** D. Pahlen and S Hill, AIP Conference Proceeding*

Experiments: Optical Lattices

Duan, Demler and Lukin, PRL 91,090402 (2003)

Kitaev Model in Optical Lattices: Defects and Correlations

Altman, Demler and Lukin, Phys. Rev. A 70,013603 (2004)

Evolution of defect Correlations can be detected by spatial noise correlation measurements

CONCLUSION

Passage through quantum critical point → *Defect Generations*

Integrable Spin Chains

Linear Drive Across QCP → *Decoupled Landau-Zener Problems*

Generalized Quenching Dynamics →

- *Repeated Dynamics,*
- *Reversal Dynamics*
- *Waiting at the minimum Gap*

→ *Interesting Variants of LZ*

→ *Interesting Defect and Entropy Generation*

We have Calculated defect density Solving the LZ pronlems

Experiments:

- *Optical Lattices*
- *Single Molecular Magents*

DECOHERENCE

$$\rho_k = \begin{pmatrix} p_k & -q_k^* \\ -q_k & 1-p_k \end{pmatrix}$$

$$G(x, x', \infty, T) = \int dk e^{-ik(x-x')} \begin{pmatrix} p_k & q_k \\ q_k^* & 1-p_k \end{pmatrix}$$

$$T \to \infty$$

$$\cos(T \cos k)$$

ONE-DIMENSIONAL KITAEV MODEL

Quantum Phase Transition: No change in Symmetry

$$\sigma_j^x = \tau_{j-1}^x \tau_j^x ; \sigma_j^y = \prod_{k=j}^{2N} \tau_k^y$$

$$H_{1d} = \sum_{j=1}^{N} J_1 \tau_{2j-2}^x \tau_{2j}^x + J_2 \tau_{2j}^y$$

$J_1 > J_2;$ ⟶ Long-range order in τ_x

Feng et al, Phys. Rev. Letts., PRL 98, 087204 (2007)

$$f(x) \propto x^{z} \quad \text{for} \quad x >> 1$$

$$f(x) \to \text{constant} \quad ;$$

$$f(x) \to x^{\alpha}$$

$$g(x) \propto x^{-1/\nu} \text{for} \quad x >> 1$$

$$g(x) \propto x^{\beta} \text{for} \quad x << 1$$