Macroscopic charge quantization in single electron devices

I.S. Burmistrov & A.M.M.P

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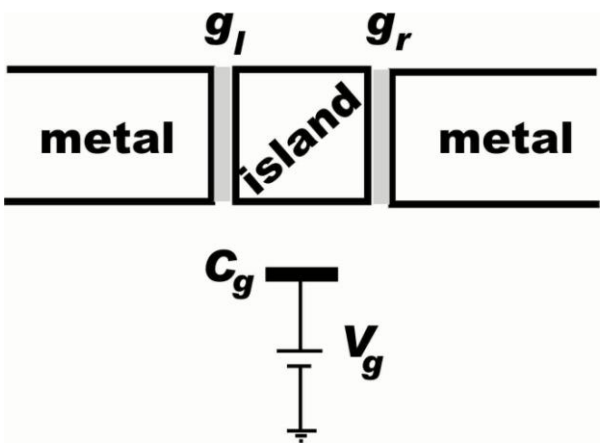
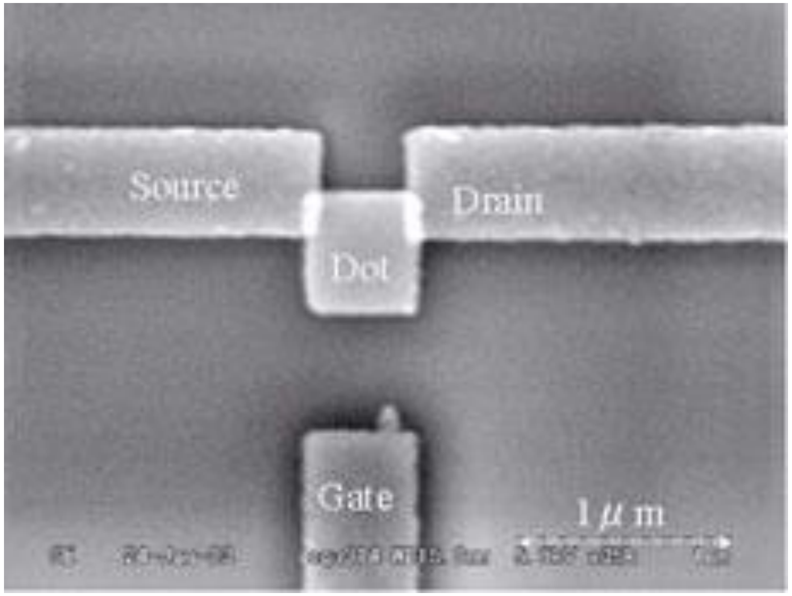
ICTS Mahabaleshwar 9-12-2009

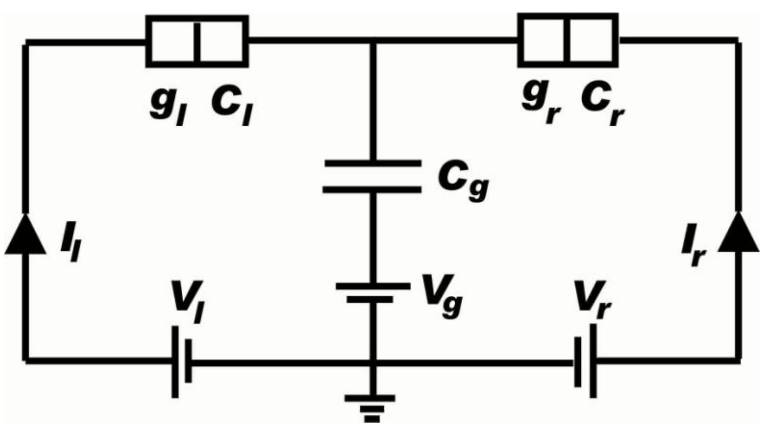
Introduction

- Single electron transistor (SET)





Equivalent circuit for SET



Introduction

- **The SET parameters**

Number of channels	$N \gg 1$
Tunneling matrix elements	$\|t\| \ll 1$
Charging energy	$E_c = \dfrac{e^2}{2C}$
Tunneling conductances	$g_{l,r}$
Level spacing on the island	$\delta \ll E_c$
Temperature	$\delta \ll T \lesssim E_c$
External charge	$q = C_g V_g / e$

Introduction

- How does discreteness of the electronic charge manifest itself at *T=0* ?

Semiclassical picture ($g_{l,r}=0$)

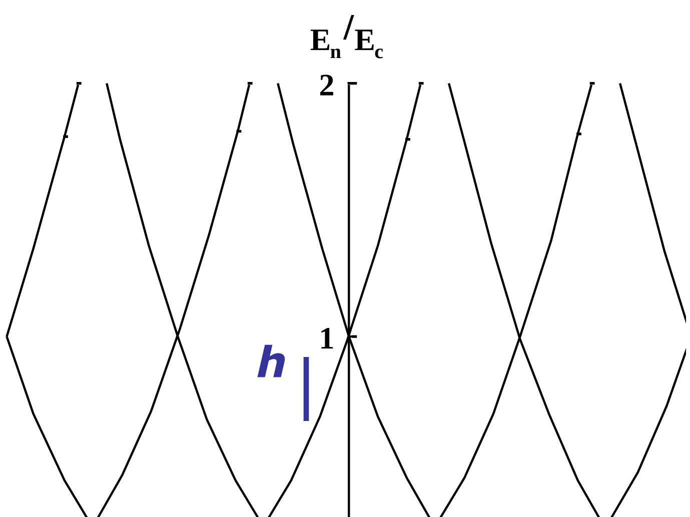
$$\hat{H}_c = E_c(\hat{n}-q)^2 \quad \Rightarrow \quad Z = \sum_{n=-\infty}^{\infty} \exp\left[-\beta E_c(n-q)^2\right]$$

$\hat{n}$ **= excess particle number on the island**

At low *T*

$$Q = \langle \hat{n} \rangle = \frac{1}{1+e^{\beta h}}, \quad 0 \leqslant q \leqslant 1$$

where $h = E_c(1-2q)$



Introduction

- How does discreteness of the electronic charge manifest itself at $T=0$?

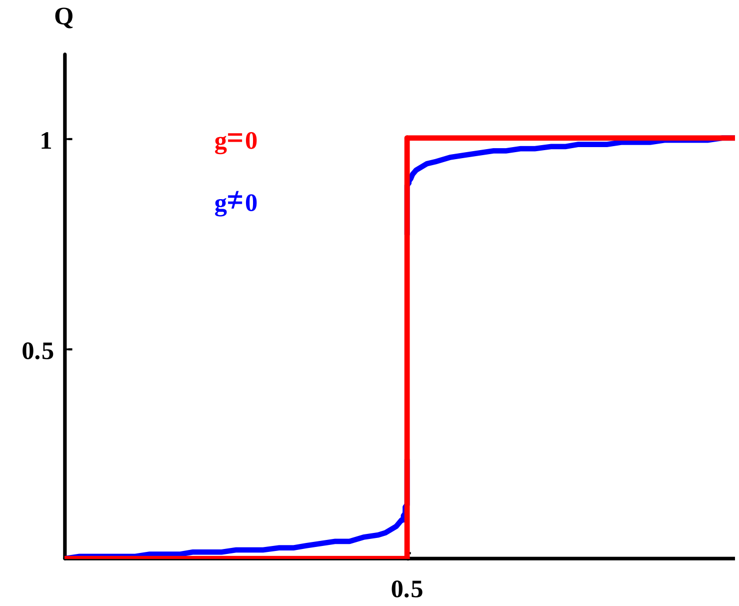
Island plus reservoir ($g_{l,r} \neq 0$, $g_{l,r} \ll 1$)

At low *T*

$$Q = \frac{1}{2}\frac{\frac{g}{2\pi^2}\ln\frac{1}{|1-2q|}}{1+\frac{g}{2\pi^2}\ln\frac{1}{|1-2q|}}, \qquad \frac{1}{2}-q \ll 1$$

Matveev, 1991
Schoeller, Schön, 1994

where $g = g_l + g_r$



Q is not quantized at $T=0$ for any $g \neq 0$!

What quantity is quantized at $T=0$?

Results

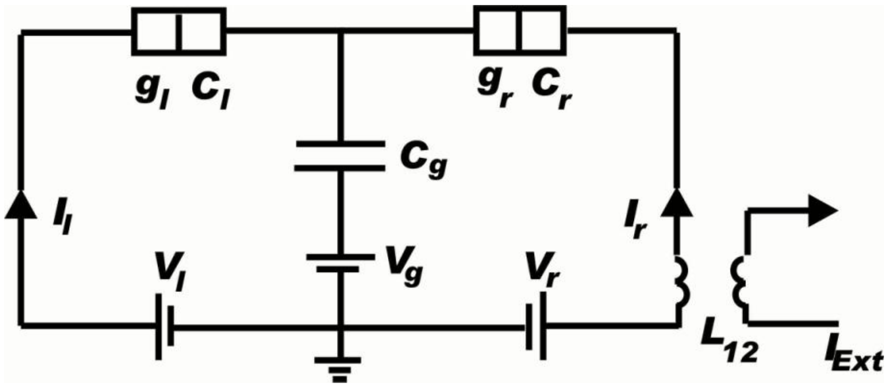
- **Novel quantity** q'

$$q' = Q - i\frac{(g_l + g_r)^2}{g_l g_r}\frac{\partial}{\partial V}\int_{-\infty}^{0} dt\langle[\hat{I}(0),\hat{I}(t)]\rangle\Big|_{V=0}$$

***I=I**$_l$**=-I**$_r$* **current into the SET**

Inductive coupling $\hat{H}_\Phi = \hat{I}\Phi$

$$q' = Q + \frac{(g_l + g_r)^2}{4\pi g_l g_r}\frac{\partial G(\Phi)}{\partial\Phi}\Big|_{\Phi=0}$$



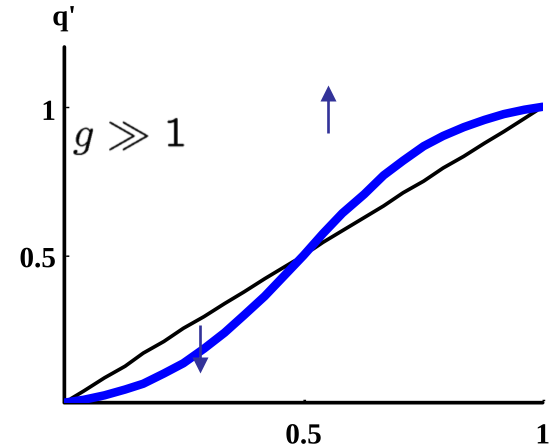
$$\Phi = L_{12} I_{Ext}$$

Results

- **Temperature dependence of q'**

Weak coupling $g >> 1$

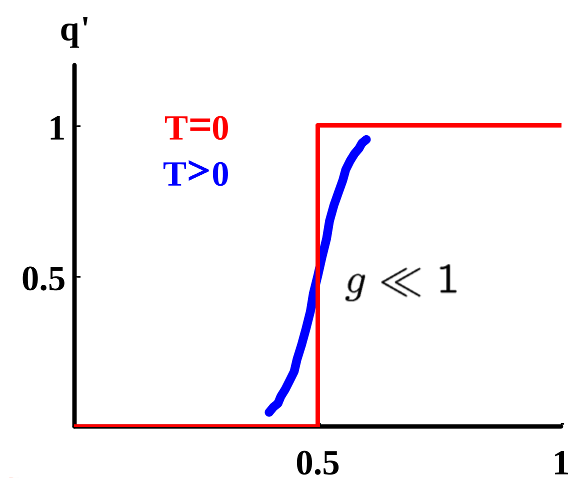
$$q'(T) = q - \frac{g^3 E_c}{24\pi T} e^{-g/2} \sin 2\pi q, \qquad T \gg E_c g^3 e^{-g/2}$$



Strong coupling $g << 1$ and $\frac{1}{2} - q \ll 1$

$$q'(T) = \frac{1}{1 + e^{\beta h'}}, \qquad T \ll E_c$$

$$h' = \frac{h}{1 + \frac{g}{2\pi^2} \ln \frac{E_c}{\max\{T, h\}}}$$

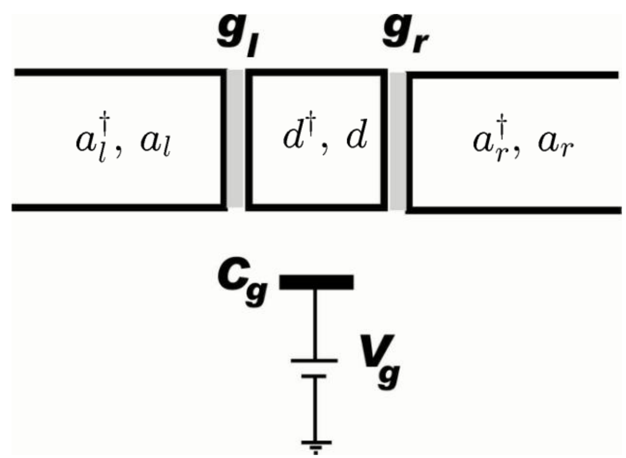


q' is robustly quantized independent of g

AES model

Microscopic Hamiltonian

$$
\begin{aligned}
\hat{H} - \mu\hat{N} &= \hat{H}_0 - \mu\hat{N} + \hat{H}_c + \sum_{\alpha=l,r} \hat{H}_T^{(\alpha)} \\
\hat{H}_0 - \mu\hat{N} &= \sum_{k,\alpha=l,r} (\epsilon_{k\alpha} - \mu_\alpha) a^\dagger_{k\alpha} a_{k\alpha} + \sum_p (\epsilon_p - \mu) d^\dagger_p d_p \\
\hat{H}_c &= E_c \left(\sum_p d^\dagger_p d_p - q \right)^2 \\
\hat{H}_T^{(\alpha)} &= \sum_{k,p} t^{(\alpha)}_{kp} a^\dagger_{k\alpha} d_p + \text{h.c.}
\end{aligned}
$$



i) Decoupling H_c by Hubbard-Stratonovich field $\phi(\tau)$

ii) Integration over fermions to lowest order in H_T

AES model

Effective action

$$\mathcal{S}_{AES}[\phi] = \frac{g}{4}\int_0^\beta d\tau_1 d\tau_2 \alpha(\tau_{12}) e^{-i[\phi(\tau_1)-\phi(\tau_2)]} - iq\int_0^\beta d\tau\dot{\phi} + \frac{1}{4E_c}\int_0^\beta d\tau\,\dot{\phi}^2$$

$$\phi(\beta) = \phi(0) + 2\pi W$$

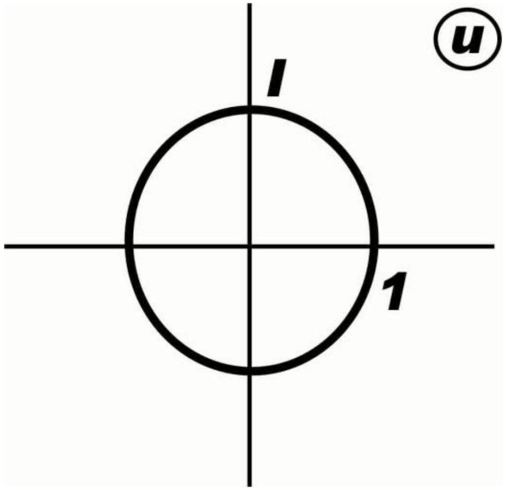
integer

V. Ambegaokar, U. Eckern, G. Schön, 1982

where $\tau_{12}=\tau_1-\tau_2$ and $$\alpha(\tau) = -\frac{T^2}{\sin^2 \pi T\tau} = \frac{T}{\pi}\sum_n |\omega_n| e^{i\omega_n\tau}$$

and

$$g = g_l + g_r, \qquad g_{l,r} = 8\pi^2 N|t_{l,r}|^2 \nu_{l,r}\nu_d$$



Convenient variables

$$u = e^{i2\pi T\tau}, \qquad f(u) = e^{i\phi(\tau)}$$

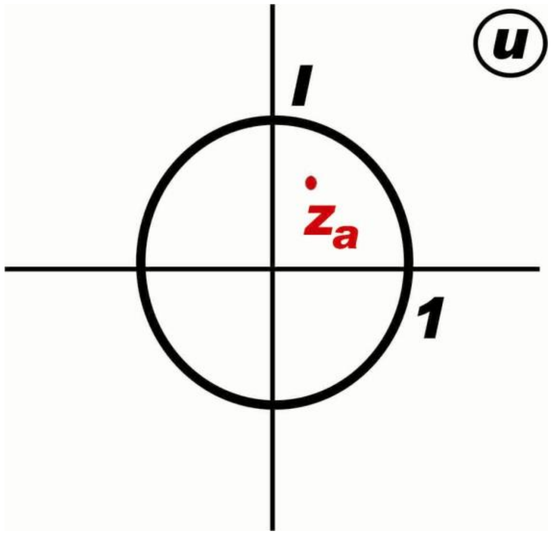
$$S_{AES} = \frac{g}{2}\sum_n |n||f_n|^2 + \frac{\pi^2 T}{E_c}\sum_n n^2|\phi_n|^2 - iq\int_0^\beta d\tau\dot{\phi}$$

AES model

Instantons

Topological charge (winding number)

$$\mathcal{C}[\phi] = \frac{1}{2\pi}\int_0^\beta d\tau\dot{\phi} = W$$



Instanton solution

$$f_{inst} = \prod_{a=1}^{|W|} \frac{u - z_a}{1 - u\bar{z}_a}$$

Instanton *W>0* $|z_a|<1$

Anti-instanton *W<0* $|z_a|>1$

Korshunov, 1987
Nazarov, 1999

***2|W|* zero modes !**

Classical action

$$\mathcal{S}_{AES}^{inst} = \frac{g}{2}|W| - i2\pi q W + \frac{\pi^2 T}{E_c}\prod_{a,b=1}^{|W|}\frac{1 + z_a\bar{z}_b}{1 - z_a\bar{z}_b}$$

Response to change in the boundary condition

Formal response parameters

Expansion in topological sectors

$$Z = \sum_W Z[W], \qquad Z[W] = \int_{\phi(\beta)=\phi(0)+2\pi W} \mathcal{D}[\phi] e^{-S_{AES}[\phi]}$$

Let us *formally* define

$$\begin{aligned} \frac{g'}{4\pi} &= \mathrm{Im}\, \frac{1}{2\pi i Z} \sum_W \frac{\partial Z[x]}{\partial x}\Big|_{x=W} \\ q' &= \mathrm{Re}\, \frac{1}{2\pi i Z} \sum_W \frac{\partial Z[x]}{\partial x}\Big|_{x=W} \end{aligned}$$

Note that $\overline{Z[W,q]} = Z[W,-q] = Z[-W,q]$ **then**

$$\begin{aligned} \frac{g'(T)}{4\pi} &= \sum_{W=0}^{\infty} A_W(T) \cos 2\pi W q \\ q'(T) &= \sum_{W=1}^{\infty} B_W(T) \sin 2\pi W q \end{aligned}$$

$$\Longrightarrow$$

$$\begin{aligned} q = k \quad &\rightarrow \quad q'(T) = k \\ q = k + \frac{1}{2} \quad &\rightarrow \quad q'(T) = k + \frac{1}{2} \end{aligned}$$

Response to change in the boundary condition

- **Response parameters from correlation functions**

From *W*-*n* to *W*

$$\phi(\tau) \to \phi(\tau) - \omega_n \tau \quad \Longrightarrow \quad Z[W-n] = Z[W]\left\langle e^{-\beta\widehat{K}(i\omega_n) - i\omega_n\beta\widehat{Q} - \omega_n{}^2\beta/(4E_c)}\right\rangle_W$$

where

$$\begin{aligned}\widehat{Q} &= q + \frac{iT}{2E_c}\int_0^\beta d\tau\dot{\phi} \\ \widehat{K}(i\omega_n) &= \frac{g}{4\beta}\int_0^\beta d\tau_1 d\tau_2\left[e^{i\omega_n\tau_{12}} - 1\right]\alpha(\tau_{12})e^{i[\phi(\tau_2)-\phi(\tau_1)]}\end{aligned}$$

Hence

$$\frac{1}{2\pi i Z}\sum_W \frac{\partial Z[x]}{\partial x}\bigg|_{x=W} = Q + \frac{K^R(\omega)}{\omega}, \quad \omega \to 0$$

where $Q = \langle\widehat{Q}\rangle, \quad K^R(\omega) = \langle\widehat{K}(i\omega_n \to \omega + i0)\rangle$

$$\frac{g'}{4\pi} = \mathrm{Im}\,\frac{K^R(\omega)}{\omega}, \quad q' = Q + \mathrm{Re}\,\frac{K^R(\omega)}{\omega}, \quad \omega \to 0$$

Response to change in boundary condition

- **Analytic continuation** $K(i\omega_n)$

Correlation function

$$D(i\omega_n) = \int_0^\beta \frac{d\tau_1 d\tau_2}{\beta} e^{-i\omega_n \tau_{12}} \left\langle e^{i\phi(\tau_1) - i\phi(\tau_2)} \right\rangle$$

then

$$K(i\omega_n) = \frac{g}{4\beta} \int_0^\beta d\tau_1 d\tau_2 \left[e^{i\omega_n \tau_{12}} - 1 \right] \alpha(\tau_{12}) D(\tau_{21})$$

$$\Downarrow$$

$$K^R(\omega) = \frac{g}{4\pi^2} \int dE dE' \,\mathrm{Im}\, D^R(E) \frac{E'}{E' - E + \omega + i0} [n_B(E) - n_B(E')]$$

$$\mathrm{Im}\, K^R(\omega) = -\frac{g}{4\pi} \int dE \,\mathrm{Im}\, D^R(E) (E - \omega) [n_B(E) - n_B(E - \omega)]$$

Physical observables

- **Tunneling conductance *G* of the SET**

Standard trick (α=*l*,*r*)

$$\begin{aligned}\widehat{H}_T^{(\alpha)} &= X_\alpha e^{iV_\alpha t} + X_\alpha^\dagger e^{-iV_\alpha t}\\ \widehat{I}_\alpha &= iX_\alpha e^{iV_\alpha t} - iX_\alpha^\dagger e^{-iV_\alpha t},\end{aligned} \qquad \widehat{X}_\alpha = \sum_{k,p} t_{kp}^{(\alpha)} a_{k\alpha}^\dagger d_p$$

Average current

$$I_\alpha = -i\int_{-\infty}^{t} dt' \langle [\widehat{I}_\alpha(t), \widehat{H}_T^{(\alpha)}(t')]\rangle$$

Retarded correlation function

$$K_\alpha^R(\omega) = i\int_{-\infty}^{\infty} dt\, e^{i\omega t}\theta(t)\langle [X_\alpha(t), X_\alpha^\dagger(0)]\rangle$$

$$\Rightarrow \quad I_\alpha = 2\,\mathrm{Im}\, K_\alpha^R(\omega = V_\alpha)$$

Matsubara correlation function

$$\begin{aligned}K_\alpha(i\omega_n) &= \frac{1}{2}\int_{-\beta}^{\beta} d\tau e^{i\omega_n\tau}\langle T_\tau X_\alpha(\tau) X_\alpha^\dagger(0)\rangle\\ &= \frac{g_\alpha}{4\beta}\int_0^\beta d\tau_1 d\tau_2 e^{i\omega_n\tau_{12}}\alpha(\tau_{12})D(\tau_{21}) \equiv \frac{g_\alpha}{g}K(i\omega_n)\end{aligned}$$

$$\Rightarrow \quad K_\alpha^R(\omega) = \frac{g_\alpha}{g}K^R(\omega)$$

Physical observables

Tunneling conductance *G* of the SET

Average current

$$I_\alpha = \frac{1}{2\pi}\frac{g_\alpha g'}{g}V_\alpha, \qquad g' = 4\pi\,\mathrm{Im}\frac{K^R(\omega)}{\omega}, \quad \omega \to 0$$

By definition

$$I \equiv I_l = -I_r = \frac{G}{2\pi}(V_r - V_l) \equiv \frac{G}{2\pi}V$$

$$\Downarrow$$

$$G = \frac{g_l g_r}{(g_l + g_r)^2}g'$$

Ben-Jacob, Mottola, Schön, 1983

Physical observables

- **Novel quantity q'**

Real part of retarded function

$$\mathrm{Re}\, K_\alpha^R(\omega = V_\alpha) = -\frac{i}{2}\int_{-\infty}^{t} dt' \langle [\hat{I}_\alpha(t), \hat{I}_\alpha(t')] \rangle$$

hence

$$\lim_{\omega\to 0} \mathrm{Re}\, \frac{K^R(\omega)}{\omega} = -\frac{i}{2}\frac{(g_l+g_r)^2}{g_l g_r}\frac{\partial}{\partial V}\int_{-\infty}^{t} dt' \langle [\hat{I}(t), \hat{I}(t')] \rangle \Big|_{V=0}$$

$$\Downarrow$$

$$\begin{aligned} q' &= Q - i\frac{(g_l+g_r)^2}{2 g_l g_r}\frac{\partial}{\partial V}\int_{-\infty}^{0} dt \langle [\hat{I}(0), \hat{I}(t)] \rangle \Big|_{V=0} \\ &\equiv Q + \frac{(g_l+g_r)^2}{4\pi g_l g_r}\frac{\partial G(\Phi)}{\partial \Phi}\Big|_{\Phi=0} \end{aligned}$$

where *G(Φ)* is conductance in the presence of $\hat{H}_\Phi = \hat{I}\Phi$

Notice

$$Q = q + \frac{T}{2E_c}\frac{\partial \ln Z}{\partial q} = q + \frac{iT}{2E_c}\int_0^\beta d\tau \langle \dot{\phi} \rangle$$

Weak coupling – g>> 1

- **Perturbation theory in *1/g* (*W=0*)**

Quadratic part of the action

$$S_{AES}^{(2)} = g \sum_{n>0} \left[n + \frac{2\pi^2 T}{gE_c} n^2 \right] |\phi_n|^2 \quad \Longrightarrow \quad \langle \phi_n \phi_{-n} \rangle = \frac{1}{g} \frac{1}{|n| + 2\pi^2 T n^2/(gE_c)}$$

Correlation function

$$D(\tau_{12}) \approx 1 - \frac{1}{2} \langle [\phi(\tau_1) - \phi(\tau_2)]^2 \rangle$$

$$D(i\omega_n) = \beta \delta_{n,0} \left[1 - \frac{2}{g} \sum_{m>0} \frac{1}{m + 2\pi^2 T m^2/(gE_c)} \right] + \frac{\beta}{g} \frac{1}{|n| + 2\pi^2 T n^2/(gE_c)}$$

$$\text{Im}\, D^R(E) = \pi \left[1 - \frac{2}{g} \ln \frac{gE_c}{T} \right] \lim_{\eta \to 0} \beta\eta\, \delta(E - \eta)$$

$$\Downarrow$$

Guinea, Schön, 1986

$$g'(T) = g - 2 \ln \frac{gE_c}{T}, \qquad q'(T) = q$$

Weak coupling – g>> 1

Instanton contribution (W= 1)

Classical action

$$S_{AES}^{inst} = \frac{g}{2} \pm i2\pi q$$

Gaussian fluctuations about instanton

$$S_{AES}^{inst} = \frac{g(\lambda)}{2} \pm i2\pi q$$

where $\lambda = \beta(1-|z|^2)$ **and** $g(\lambda) = g - 2\ln gE_c\lambda$

Panyukov, Zaikin, 1991
Wang, Grabert, 1996

Correlation function

$$D_{inst}(i\omega_n) = \int_0^\beta \frac{d\lambda}{\lambda^2} g\, e^{-g(\lambda)/2} \begin{cases} e^{i2\pi q}\lambda^2(1-\lambda T)^{|n|-1}, & n<0 \\ 2\beta^2(1-\lambda T)\cos 2\pi q, & n=0 \\ e^{-i2\pi q}\lambda^2(1-\lambda T)^{n-1}, & n>0 \end{cases}$$

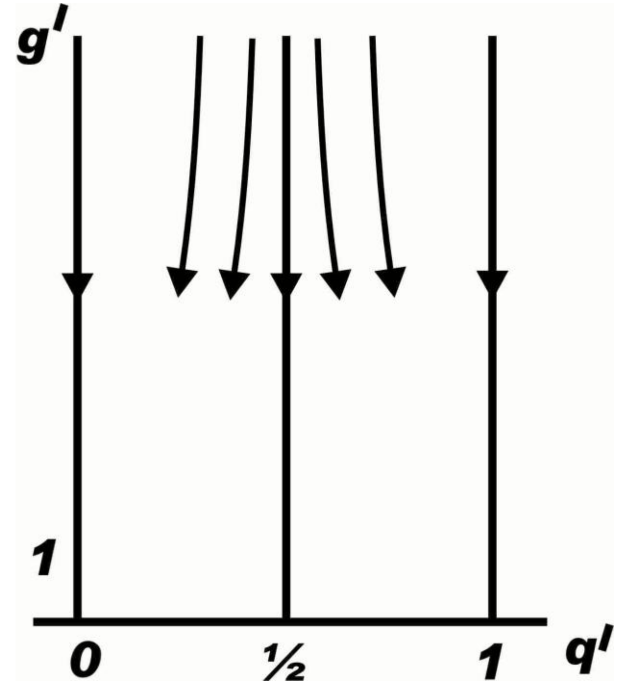
Analytic continuation

$$D_{inst}^R(\omega) = \int_0^\beta \frac{d\lambda}{1-\lambda T} g e^{-g(\lambda)/2}\left[e^{-i2\pi q} e^{-i\omega\beta/(2\pi)\ln(1-\lambda T)} + \left(1 - 2\frac{(1-\lambda T)^2}{\lambda^2 T^2}\right)\lim_{\eta\to 0}\frac{\eta}{\omega-\eta+i0}\cos 2\pi q\right]$$

Weak coupling – g>> 1

- **Instanton contribution (*W*= *1*)**

At $T \gg E_c g^3 e^{-g/2}$



$$g'(T) \;=\; g - 2\ln\frac{gE_c}{T} - \frac{g^3 E_c}{6T} e^{-g/2}\cos 2\pi q$$

$$\boxed{q'(T) \;=\; q - \frac{g^3 E_c}{24\pi T} e^{-g/2}\sin 2\pi q}$$

Altland, Glazman, Kamenev,Meyer, 2006

Notice

$$Q(T) = q - \frac{g^2 E_c}{\pi T} e^{-g/2} \ln\frac{E_c}{T}\sin 2\pi q \quad \Longrightarrow \quad \frac{|q'-q|}{|Q(T)-q|} \sim \frac{g}{\ln \beta E_c} \gg 1$$

Strong coupling – g<< 1

Effective action for |q-1/2|≈ 1 and T<< E_c

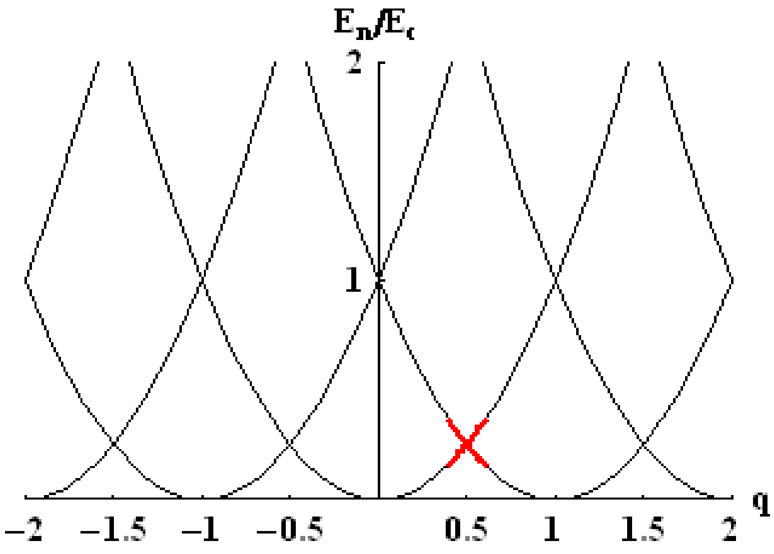
Isolated island (*g=0*)

$$Z = \sum_{n=0}^{\infty} e^{-\beta E_c (n-q)^2} \approx e^{-\beta E_c q^2}(1 + e^{-\beta h})$$

where *h=E_c(1-2q)*

Two-level Hamiltonian

$$\hat{H}_0 = \frac{h}{2}\hat{\sigma}_z \longrightarrow Z = 2\cosh\frac{\beta h}{2}$$



Average charge

$$Q = \frac{1}{2} - \frac{\partial}{\partial h} T \ln Z = \frac{1}{1 + e^{\beta h}}$$

Operators

$$e^{\pm i\phi} \rightarrow \hat{\sigma}_{\pm}$$

Strong coupling – g<< 1

- **Effective action for $|q-1/2| \approx 1$ and $T \ll E_c$**

Island plus tunneling ($g \neq 0$)

$$\mathcal{S}_{AES} \rightarrow \mathcal{S} = \int_0^\beta d\tau \bar{\psi}\left(\partial_\tau - \eta + \frac{h}{2}\sigma_z\right)\psi + \frac{g}{4}\int_0^\beta d\tau_1 d\tau_2 \alpha(\tau_{12})(\bar{\psi}\sigma_-\psi)(\tau_1)(\bar{\psi}\sigma_+\psi)(\tau_2)$$

Similar to
Larkin, Melnikov, 1971
Sachdev, Ye, 1993
$\vec{S}\cdot\vec{S}$

Pseudofermions $\bar{\psi}, \psi$ ($\eta \rightarrow -\infty$)

Grand canonical partition function

$$Z = \lim_{\eta\rightarrow-\infty} \frac{\partial}{\partial e^{\beta\eta}} Z_{pf}$$

$$\langle O \rangle = \lim_{\eta\rightarrow-\infty} \left[\frac{Z_{pf}}{Z}\frac{\partial}{\partial e^{\beta\eta}}\langle O\rangle_{pf} + \langle O\rangle_{pf}\right]$$

Strong coupling – g<< 1

Pseudofermions

Spin language

$$\hat{H}_0 = \frac{h}{2}\hat{\sigma}_z \longrightarrow Z = e^{\beta h/2} + e^{-\beta h/2}$$

Fermion language

$$\begin{array}{ll} h/2 \\ -h/2 \end{array} \qquad \begin{array}{lcc} 0\,0 & \rightarrow & \cancel{1} \\ 1\,0 & \rightarrow & e^{-\beta h/2} \\ 0\,1 & \rightarrow & e^{\beta h/2} \\ 1\,1 & \rightarrow & \cancel{1} \end{array} \quad \xrightarrow{N_{pf}=1} \quad Z = e^{\beta h/2} + e^{-\beta h/2}$$

$$Z_{pf} = \sum_{N_{pf}} e^{\beta\eta N_{pf}}\, \mathrm{Tr} e^{-\beta \hat{H}} \quad \rightarrow \quad Z = \lim_{\eta \to -\infty} \frac{\partial}{\partial e^{\beta\eta}} Z_{pf}$$

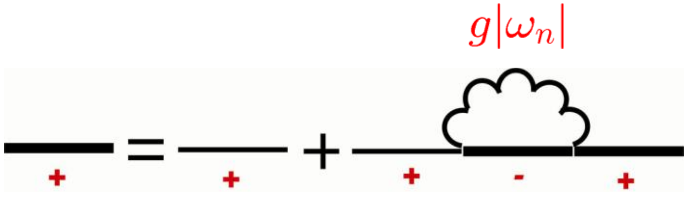
Strong coupling – g<< 1

- **Leading logarithmic approximation –** $g^n \ln^n$

Green function renormalization

$$\begin{aligned} G_{0\pm}^{-1}(i\epsilon_n) &= i\epsilon_n + \eta \mp h/2 \\ G_{\pm}^{-1}(i\epsilon_n) &= (i\epsilon_n + \eta)\gamma(i\epsilon_n) \mp \gamma_s(i\epsilon_n)h/2 \end{aligned}$$

$$\begin{aligned} \gamma(x) &= 1 + \frac{g}{4\pi^2}\int_0^x \frac{dy}{\gamma(y)} \\ \gamma_s(x) &= 1 - \frac{g}{4\pi^2}\int_0^x \frac{dy\,\gamma_s(y)}{\gamma^2(y)} \end{aligned}$$



$$x = \ln \frac{E_c}{\max\{h, |\epsilon_n|\}}$$

$$\gamma(x) = \gamma_s^{-1}(x) = \left(1 + \frac{g}{2\pi^2}\ln\frac{E_c}{\max\{h, |\epsilon_n|\}}\right)^{1/2}$$

Strong coupling – g<< 1

- **Leading logarithmic approximation:** $g^n \ln^n$

$$Z = \frac{2}{\gamma} \cosh \frac{\beta h'}{2}$$

$$h' = \frac{h}{1 + \frac{g}{2\pi^2} \ln \frac{E_c}{\max\{h,T\}}} \qquad \gamma^2 = 1 + \frac{g}{2\pi^2} \ln \frac{E_c}{\max\{h,T\}}$$

Average charge

$$Q = \frac{1}{2}\left(1 - \frac{1}{\gamma^2} \tanh \frac{\beta h'}{2}\right)$$

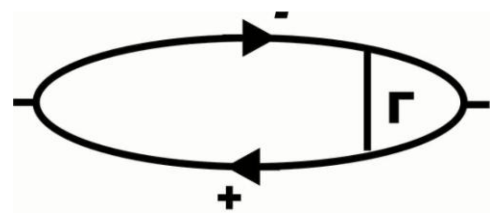
T=0 **- Matveev, 1991**
T>0 - Schoeller, Schön, 1994

Strong coupling – g<< 1

- **Correlation function $D(i\omega_n)$**

$$D(i\omega_n) = -\frac{1}{2\cosh\beta h'/2}\lim_{\eta\to-\infty}\frac{\partial}{\partial e^{\beta\eta}}T\sum_{\epsilon_m}\frac{1}{(i\epsilon_m+\eta)\gamma-\gamma_s h/2}\frac{1}{(i\epsilon_m+i\omega_n+\eta)\gamma-\gamma_s h/2}$$

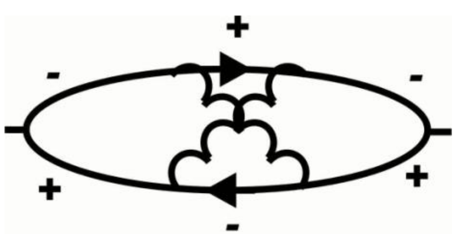
$$= \frac{\tanh\beta h'/2}{\gamma^2}\frac{1}{i\omega_n+h'}$$



$\Gamma = 1$

$$D^R(\omega) = \frac{\tanh\beta h'/2}{\gamma^2}\frac{1}{\omega+h'+i0}$$

No vertex renormalization !



$$\propto g^2 \ln\frac{E_c}{\max\{h,T\}}\frac{1}{i\omega_n+h}$$

Strong coupling – g<< 1

- **Physical observables q' and g'**

Correlation function $K(\omega)$ for $\omega \to 0$

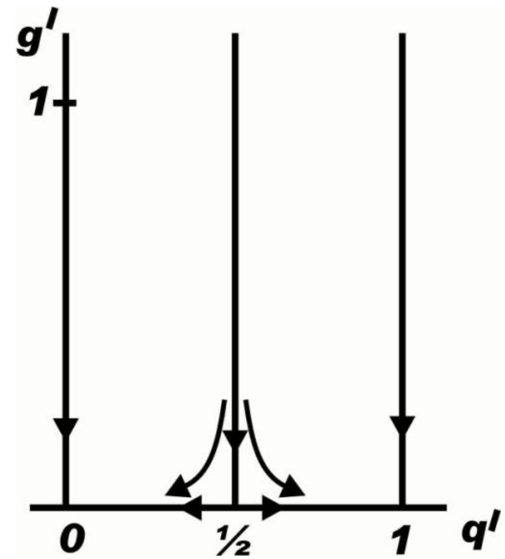
$$\mathrm{Im}\,\frac{K(\omega)}{\omega} = \frac{g}{8\pi\gamma^2}\frac{\beta h'}{\sinh \beta h'}$$

$$\mathrm{Re}\,\frac{K(\omega)}{\omega} = -\frac{g}{4\pi^2\gamma^2}\ln\frac{E_c}{\max\{T,h\}}\tanh\frac{\beta h'}{2}$$

$$\Downarrow$$

$$g' = \frac{g/2}{1+\frac{g}{2\pi^2}\ln\frac{E_c}{\max\{T,h\}}}\frac{\beta h'}{\sinh\beta h'}$$

Schoeller, Schön, 1994



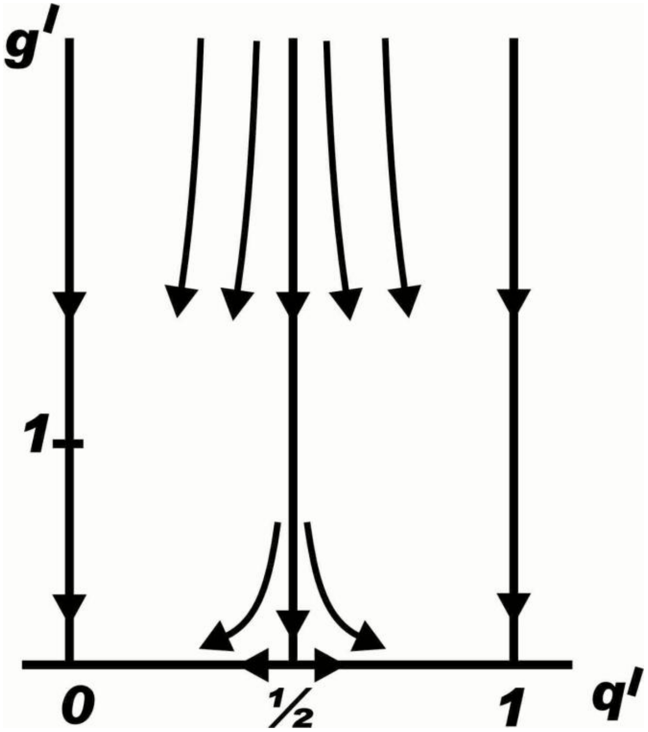
$$\boxed{q' = \left(1+e^{\beta h'}\right)^{-1}}$$

with $$h' = \frac{h}{1+\frac{g}{2\pi^2}\ln\frac{E_c}{\max\{h,T\}}}$$

q' is quantized independent of g !

Physical observables

- *T* - dependence



Conclusions

- **Novel quantity q' - completely analogous to σ_{xy} in QHE**

- **Unlike the average charge Q on the SET island, q' is robustly quantized as $T \rightarrow 0$**