

Quantum phase transitions of insulators, superconductors and metals in two dimensions

Talk online: sachdev.physics.harvard.edu

PHYSICS



HARVARD

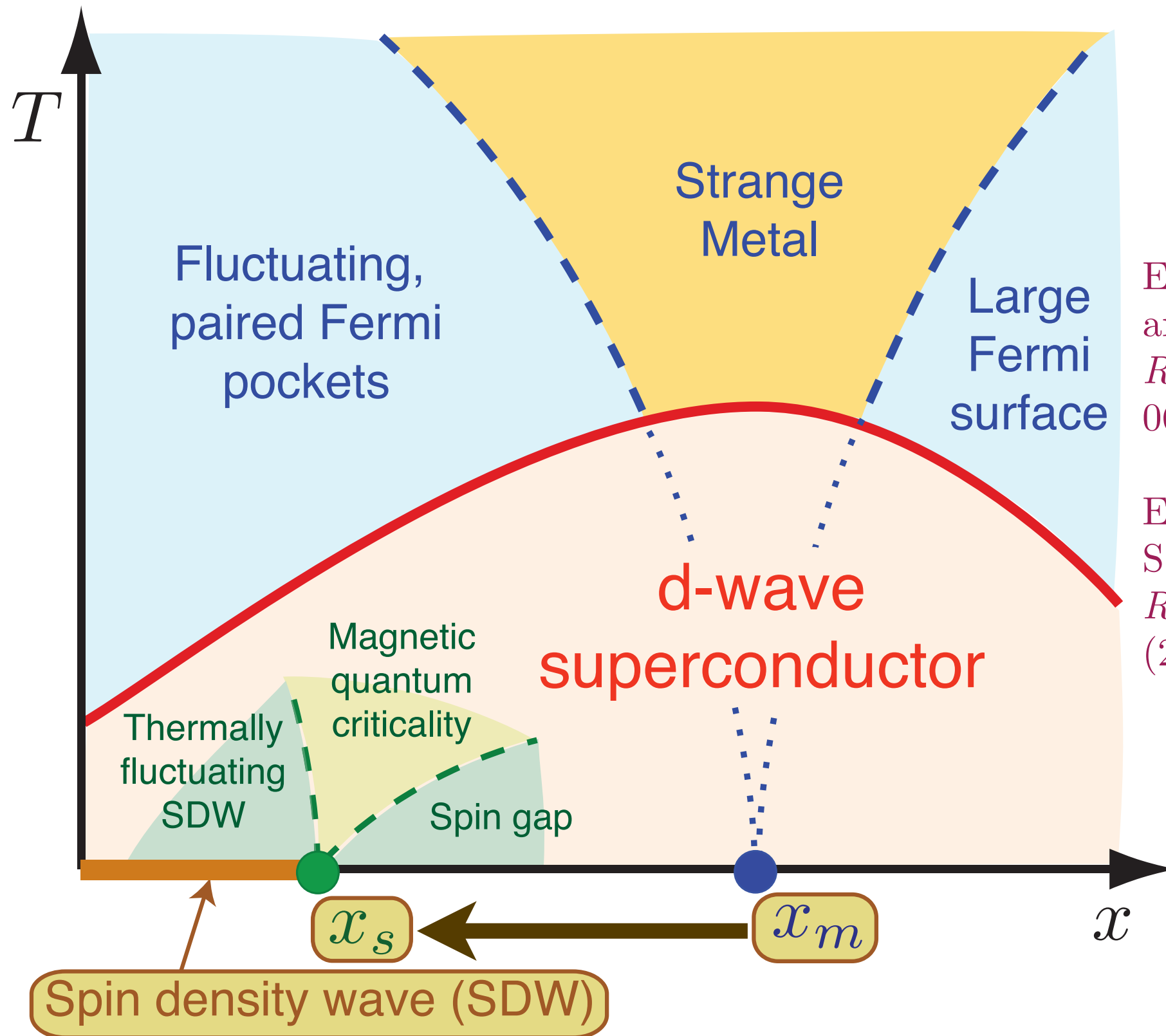
Outline

1. Phenomenology of the cuprate superconductors
(and other compounds)
2. QPT of antiferromagnetic insulators
(and bosons at rational filling)
3. QPT of d-wave superconductors:
Fermi points of massless Dirac fermions
4. QPT of Fermi surfaces:
 - A. Finite wavevector ordering (SDW/CDW):
“Hot spots” on Fermi surfaces
 - B. Zero wavevector ordering (Nematic):
“Hot Fermi surfaces”

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Theory of quantum criticality in the cuprates

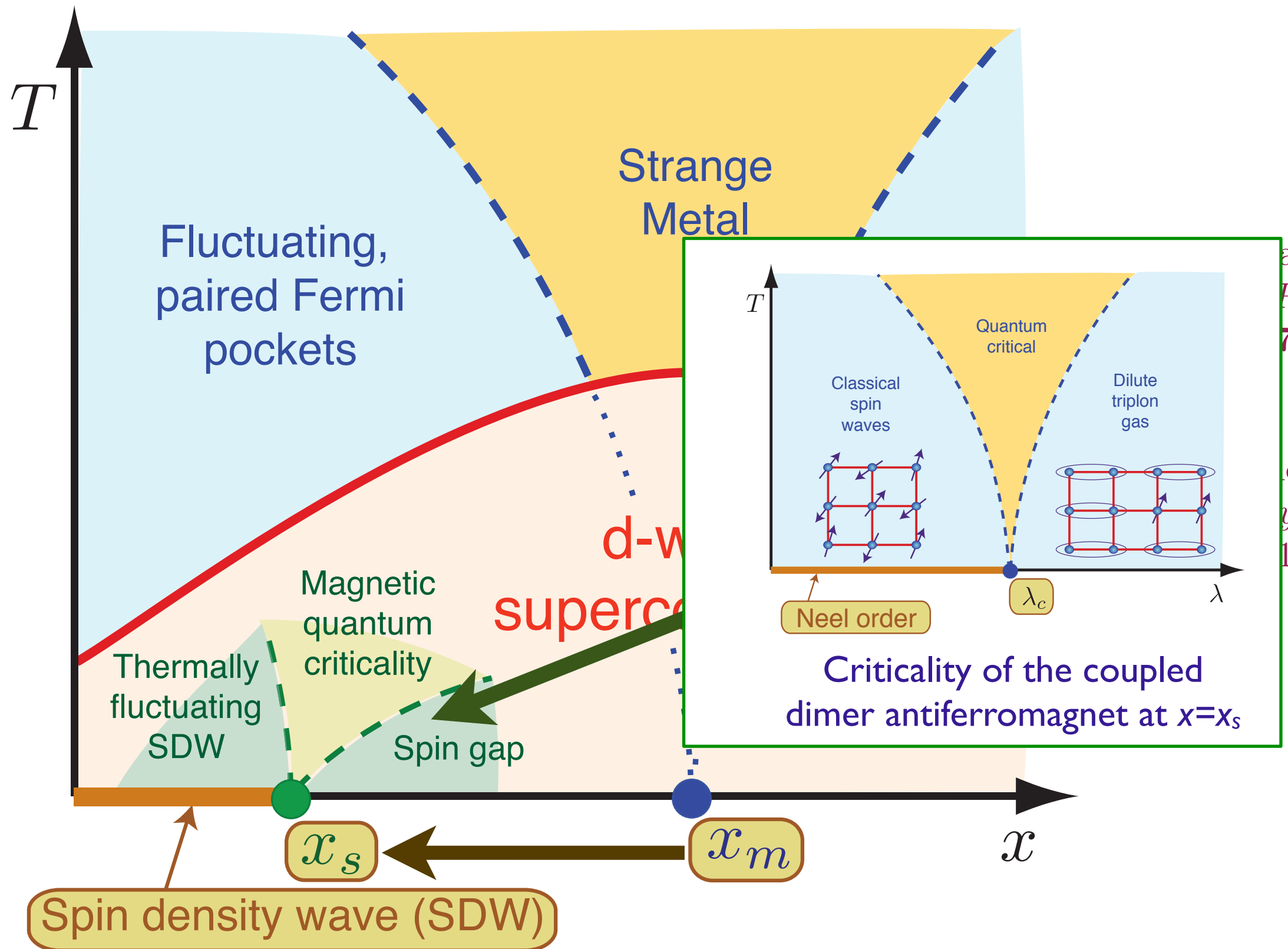


E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

E. G. Moon and S. Sachdev, *Phys. Rev. B* **80**, 035117 (2009)

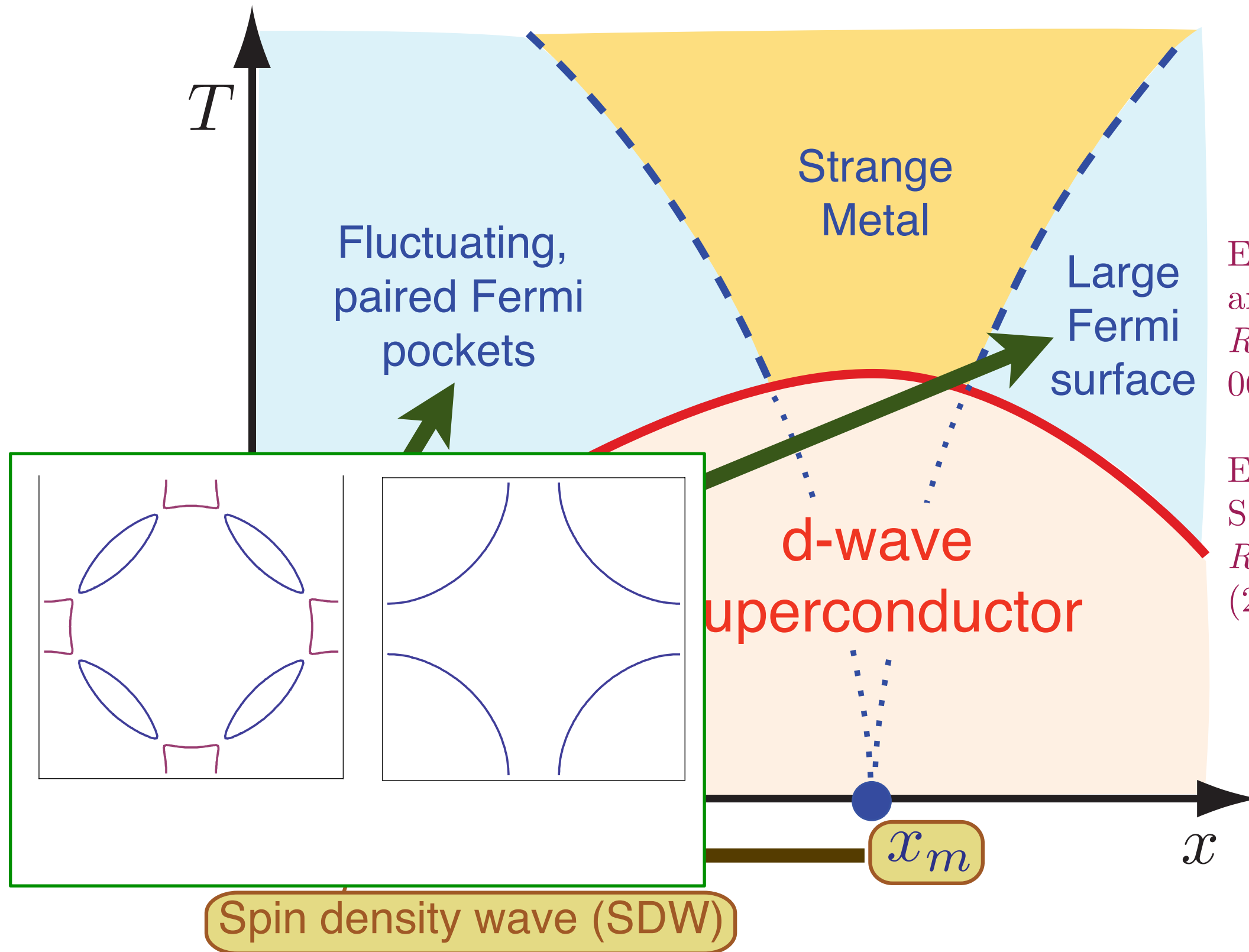
Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

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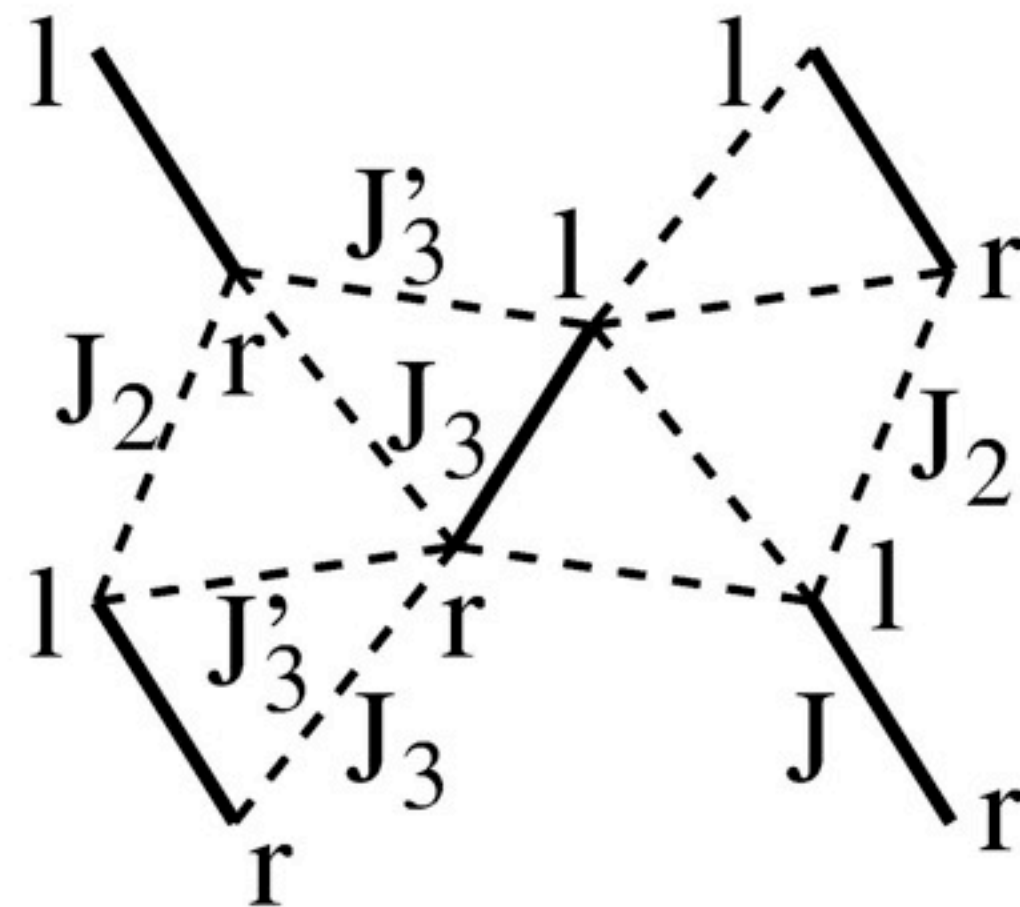
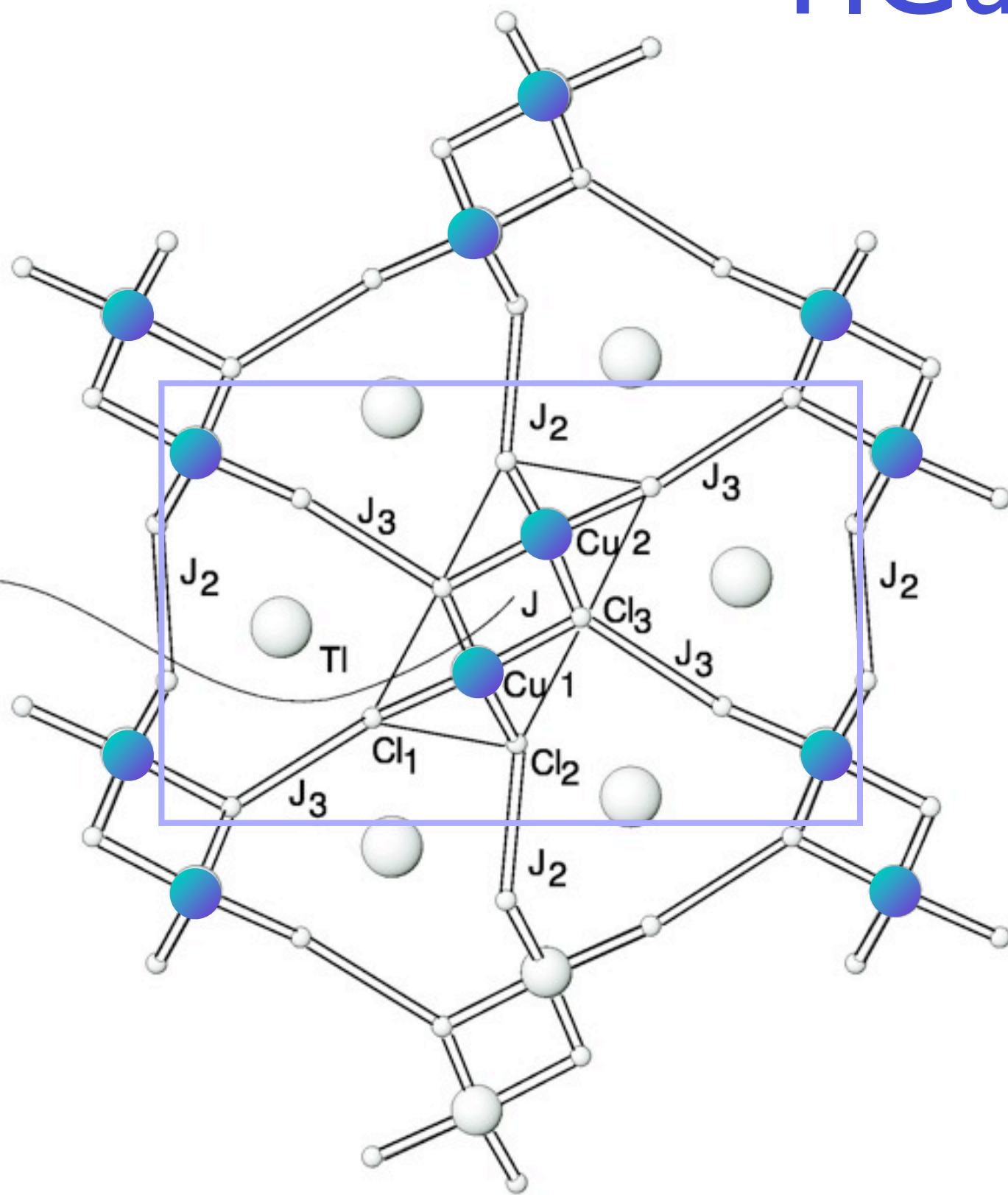


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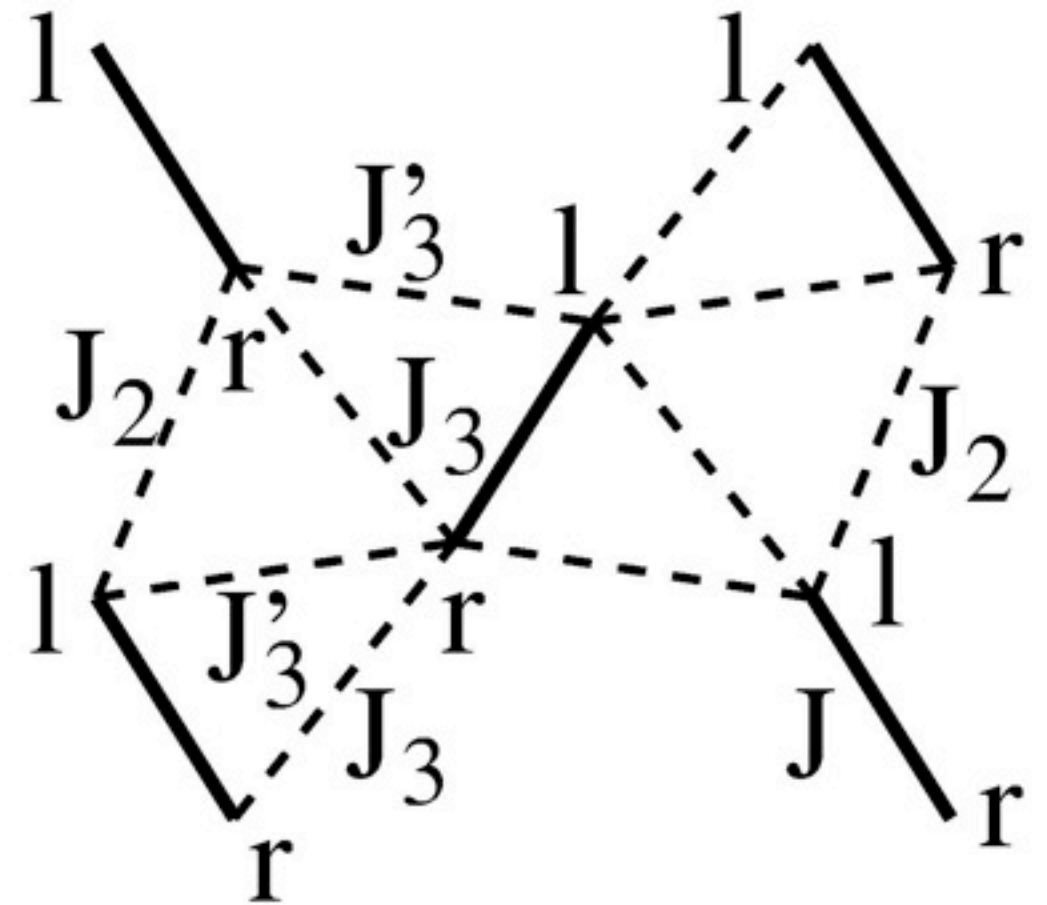
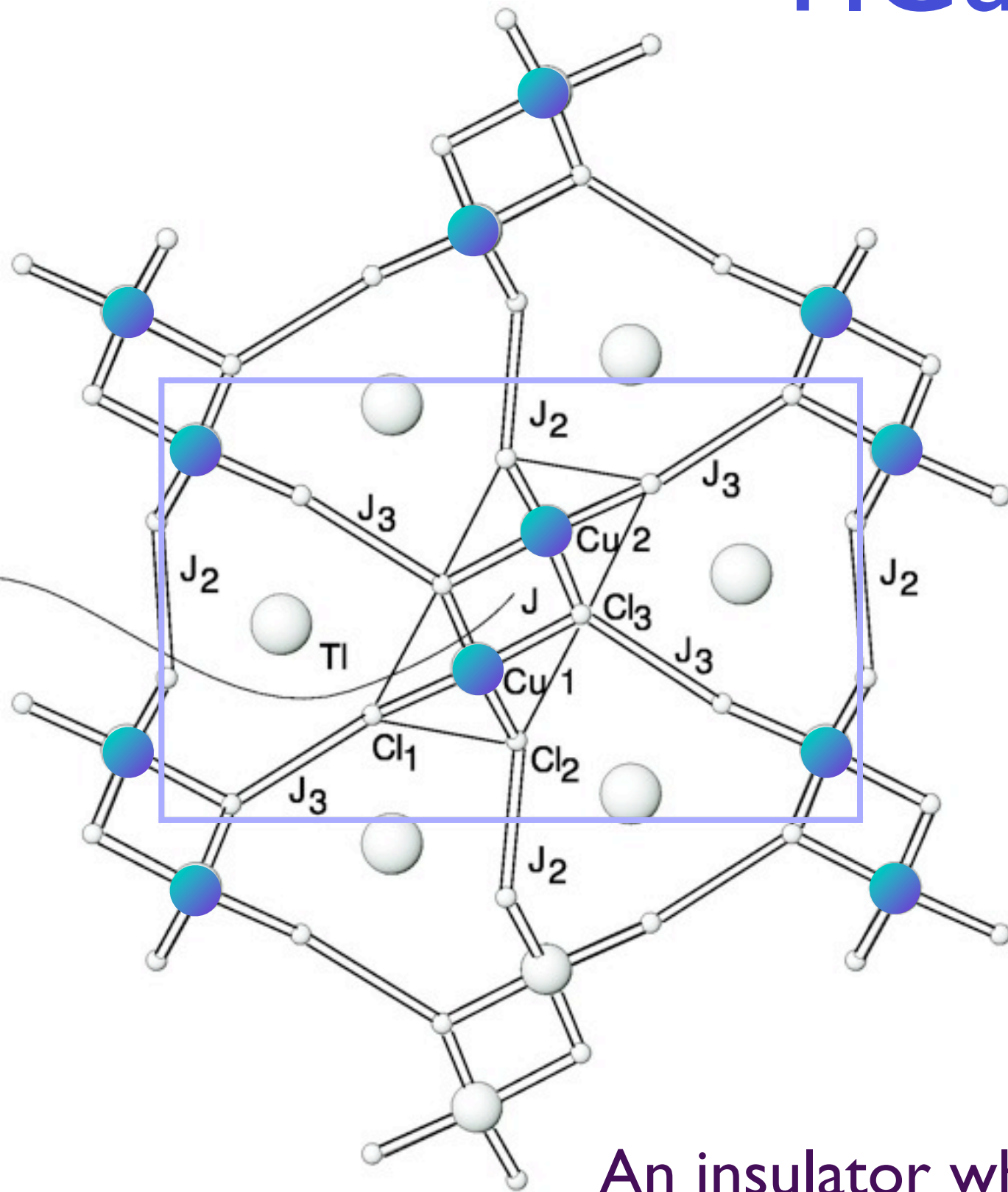
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Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

TlCuCl₃



TlCuCl_3



An insulator whose spin susceptibility vanishes exponentially as the temperature T tends to zero.

TlCuCl₃ at ambient pressure

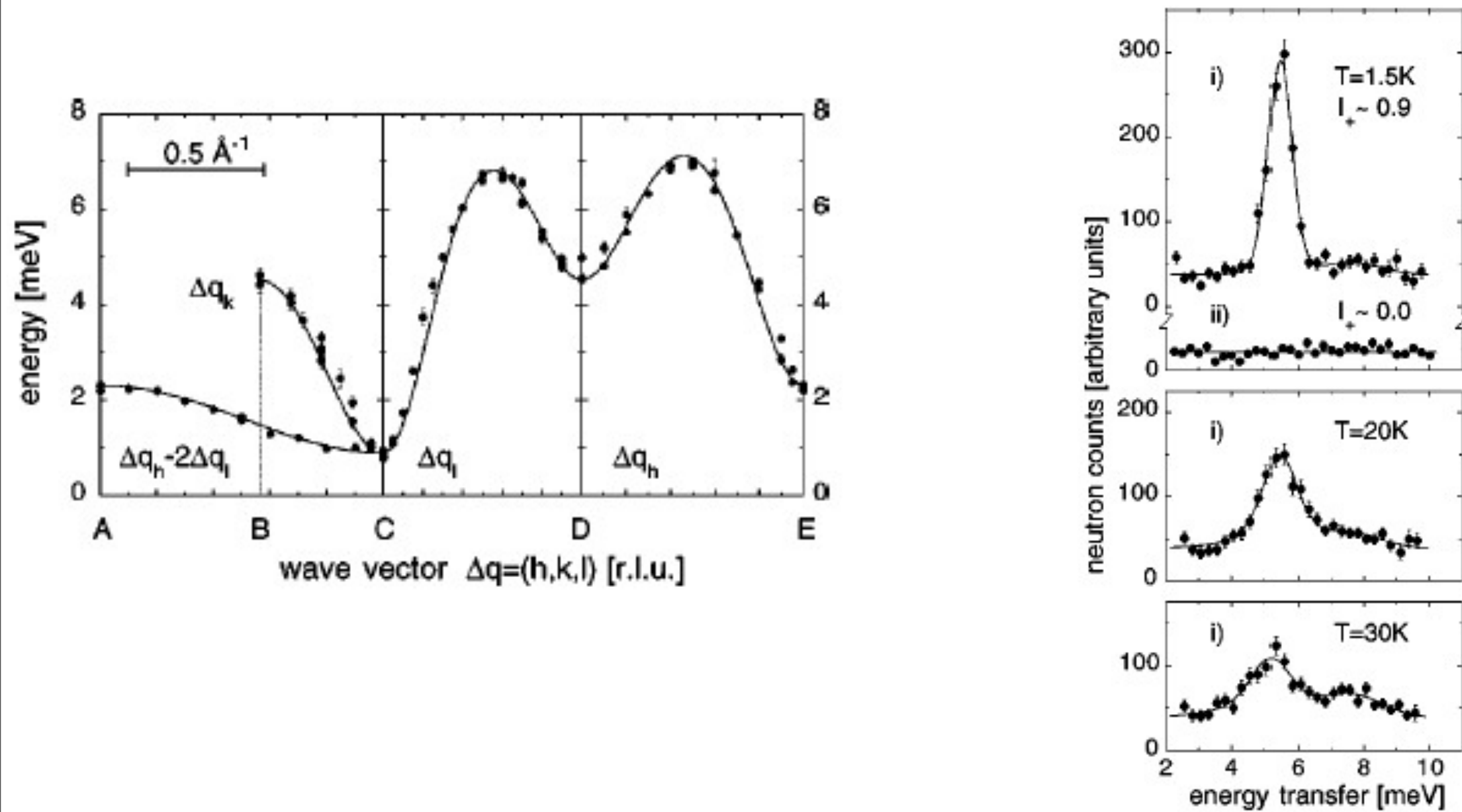
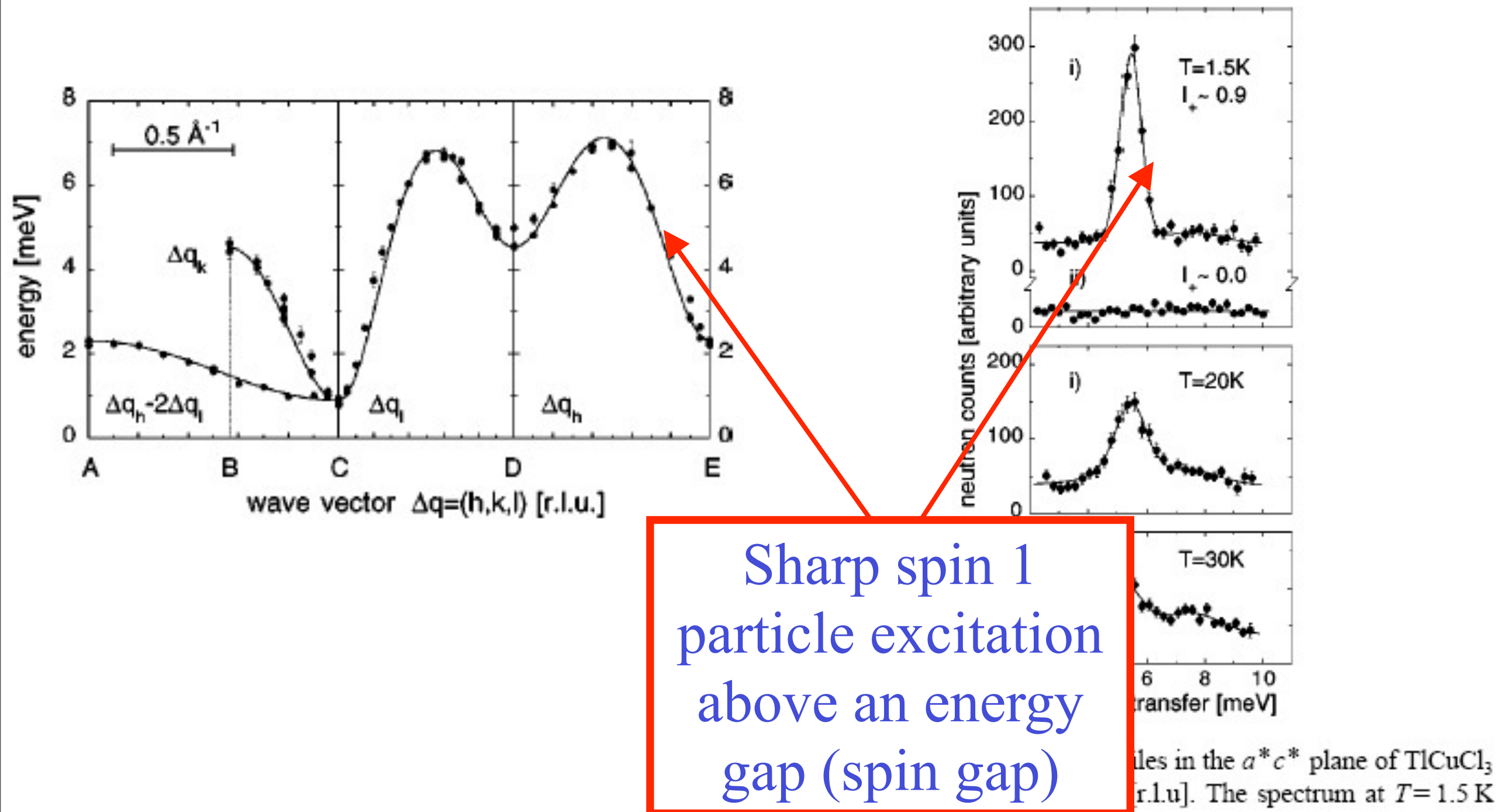


FIG. 1. Measured neutron profiles in the a^*c^* plane of TlCuCl₃ for $i=(1.35,0,0)$, $ii=(0,0,3.15)$ [r.l.u.]. The spectrum at $T=1.5\text{ K}$

N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer and H. Mutka, *Phys. Rev. B* 63 172414 (2001).

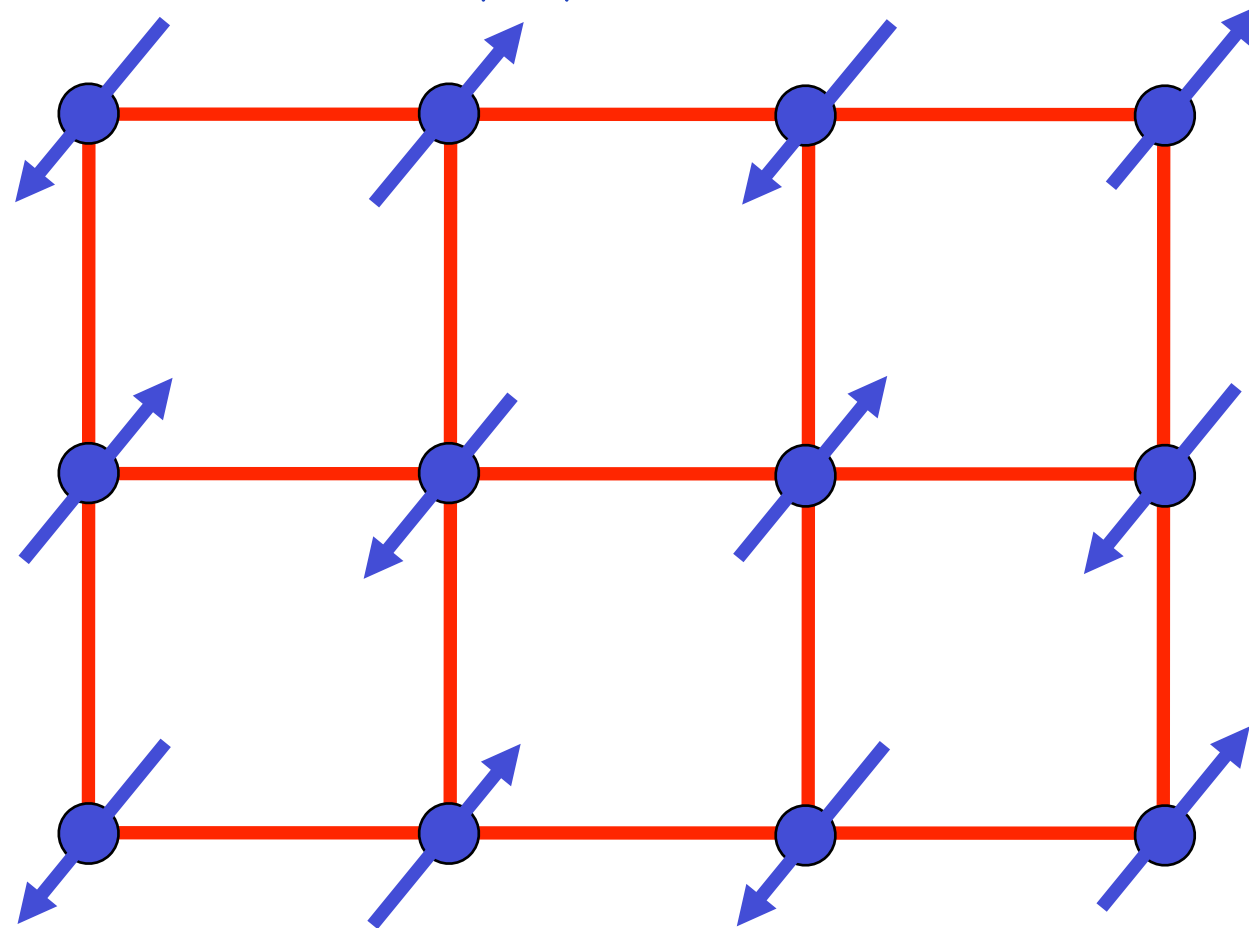
TlCuCl₃ at ambient pressure



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Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state has long-range Néel order

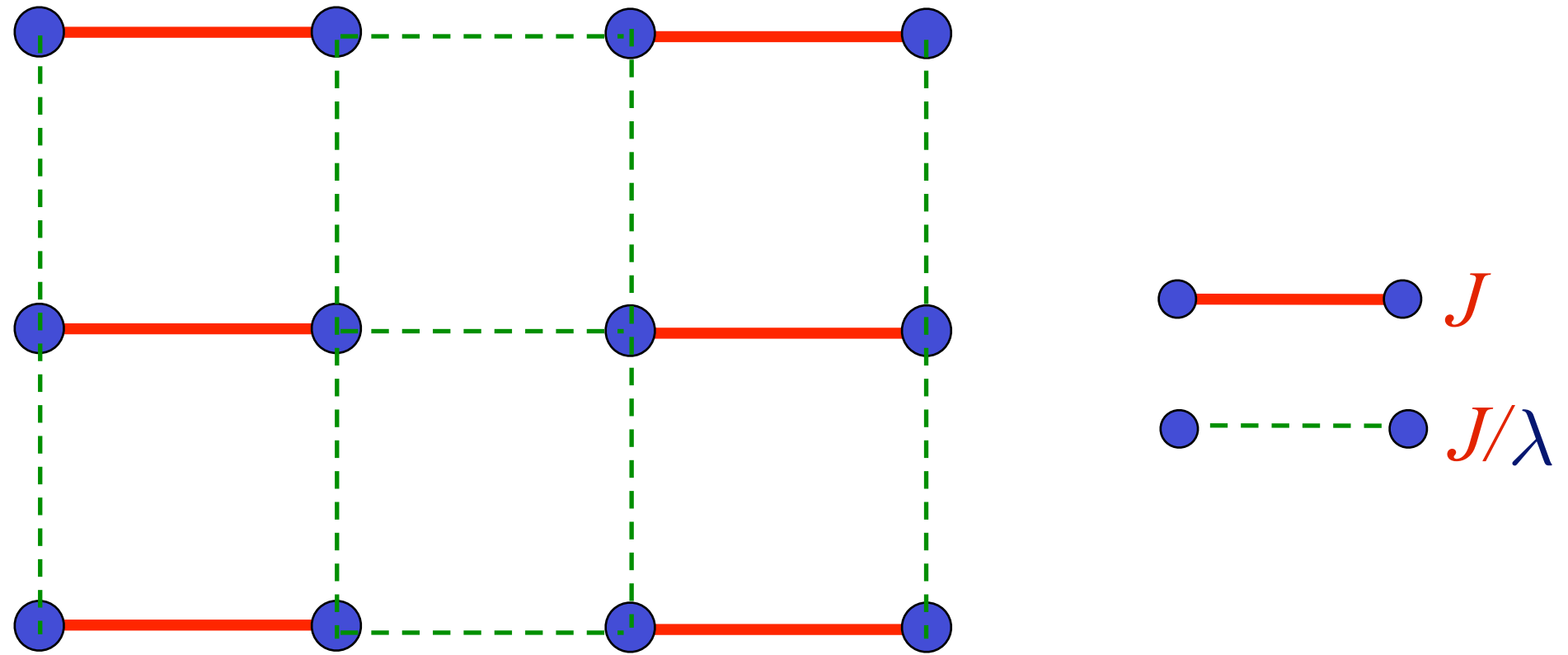
Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

Square lattice antiferromagnet

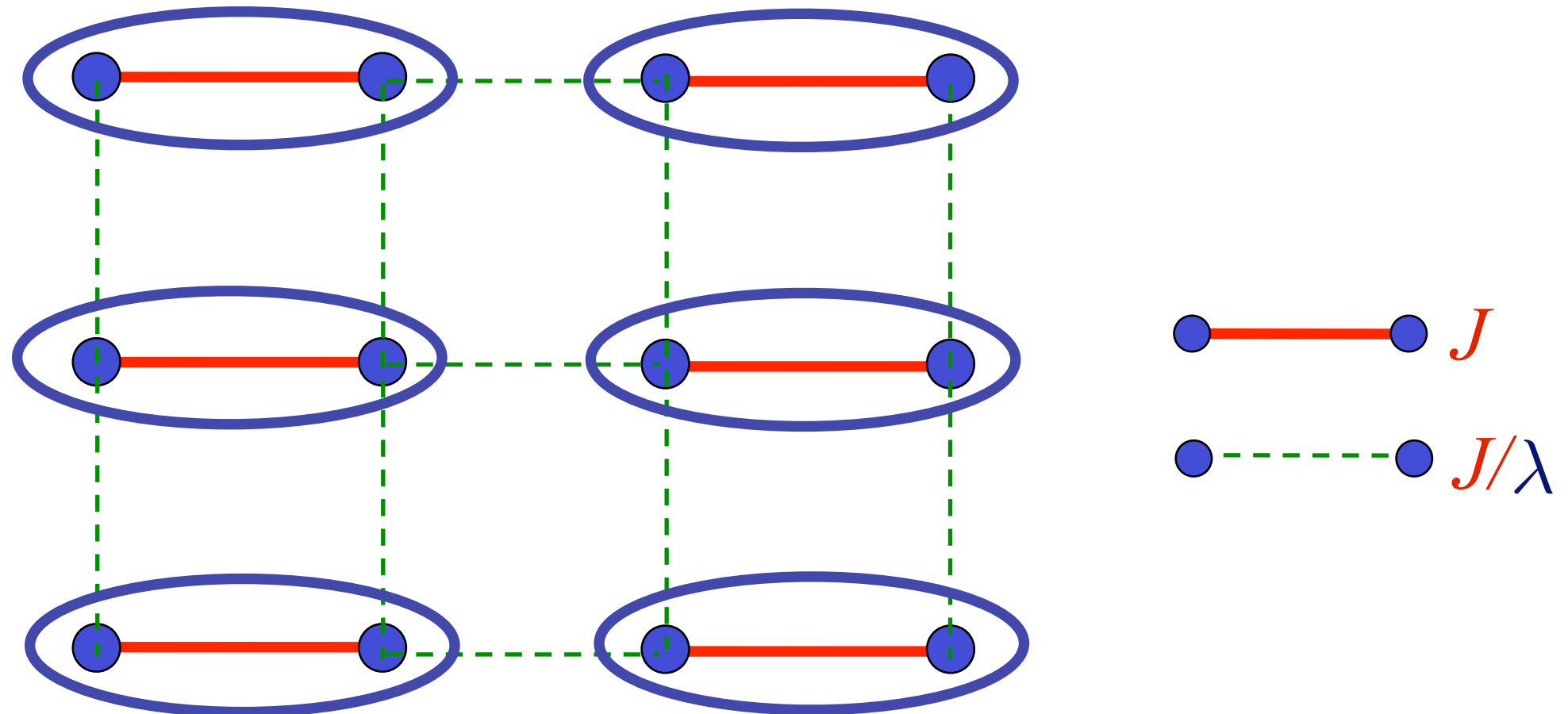
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Weaken some bonds to induce spin entanglement in a new quantum phase

Square lattice antiferromagnet

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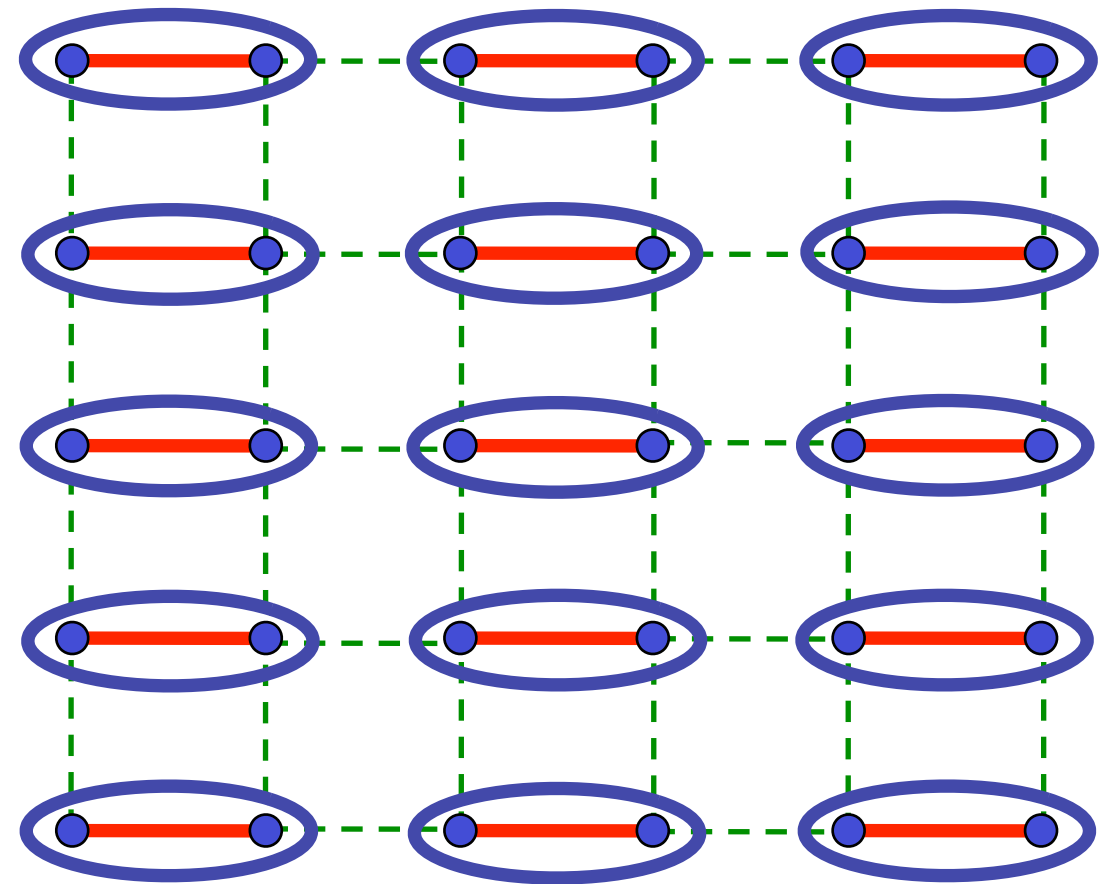
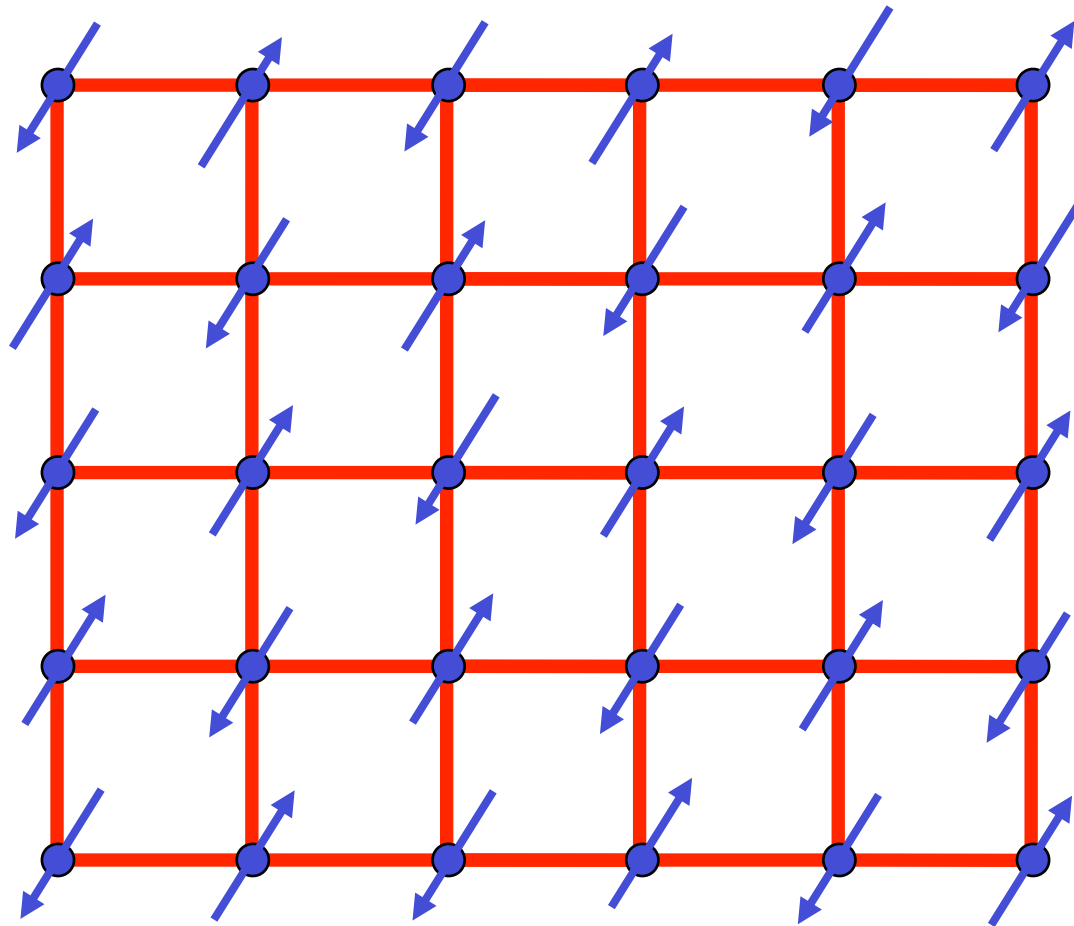


Ground state is a “quantum paramagnet”
with spins locked in valence bond singlets

$$\text{Valence bond singlet} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$



$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

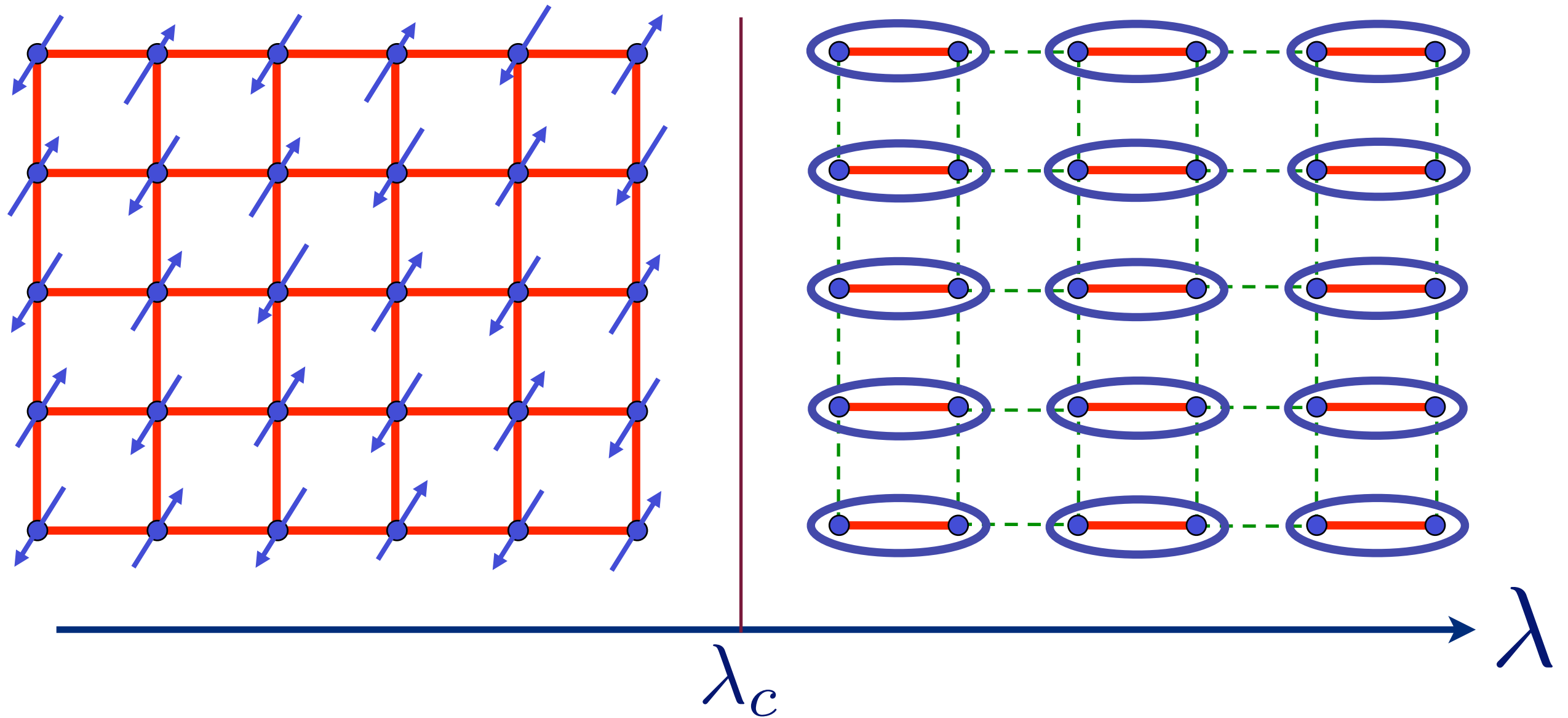


λ_c

λ

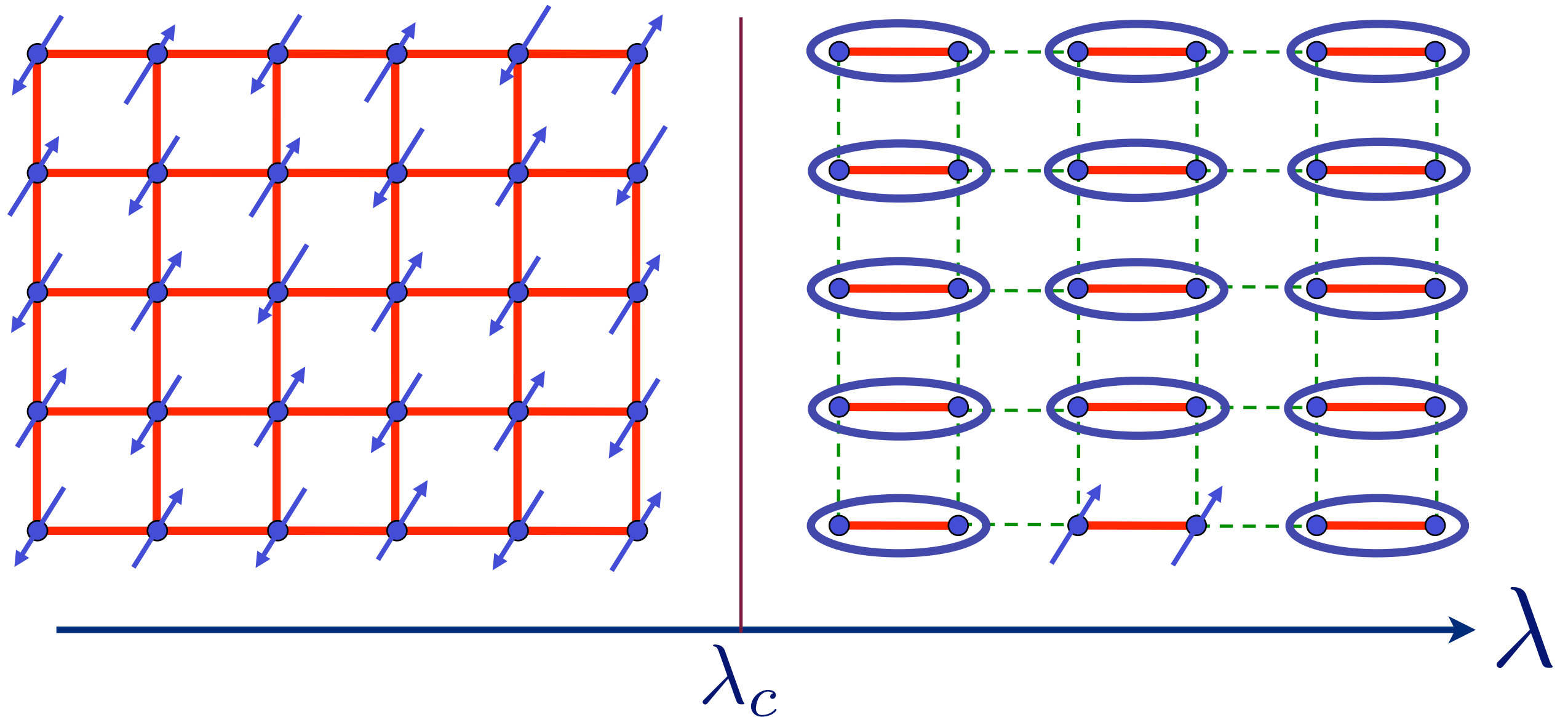
Quantum critical point with non-local entanglement in spin wavefunction

M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev. B* **65**, 014407 (2002).

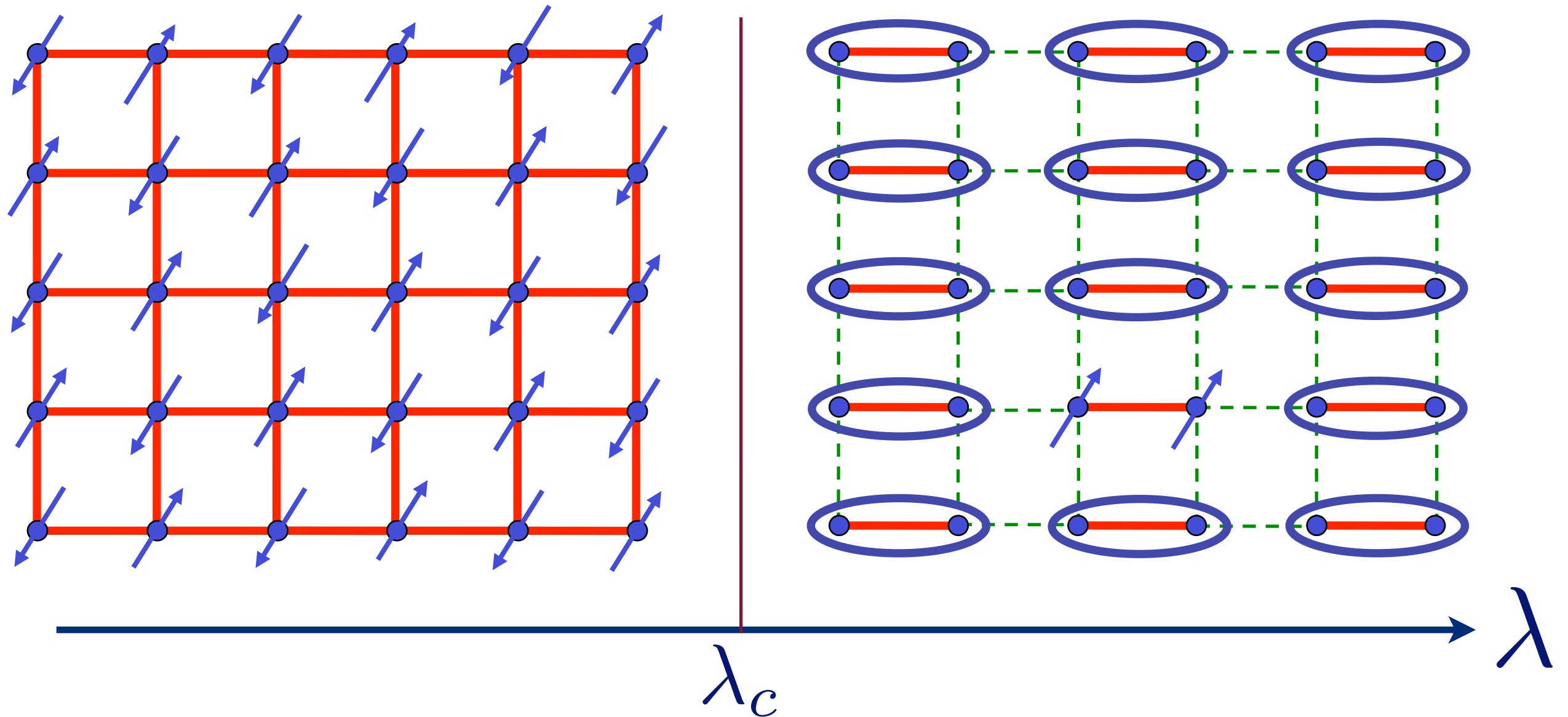


← Pressure in TlCuCl_3

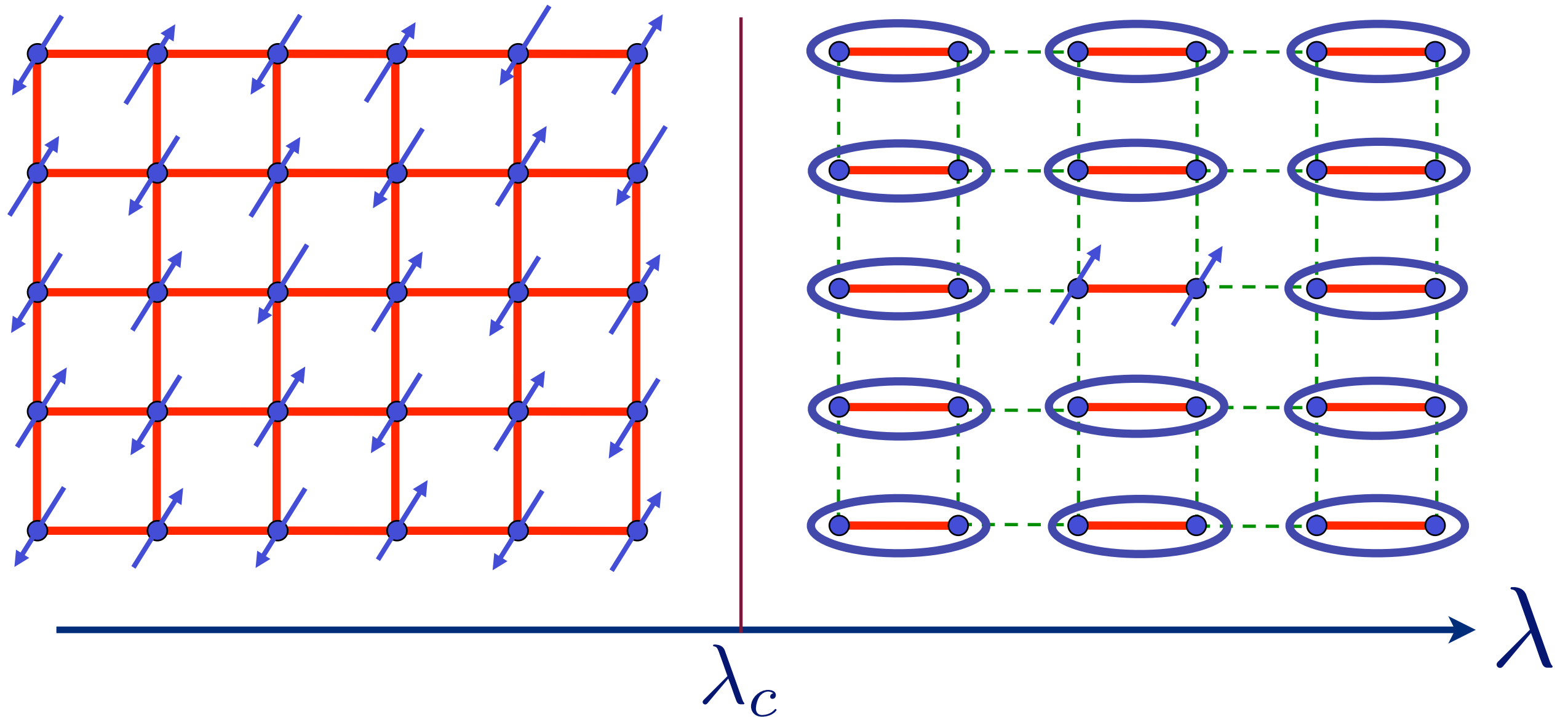
Excitation spectrum in the paramagnetic phase



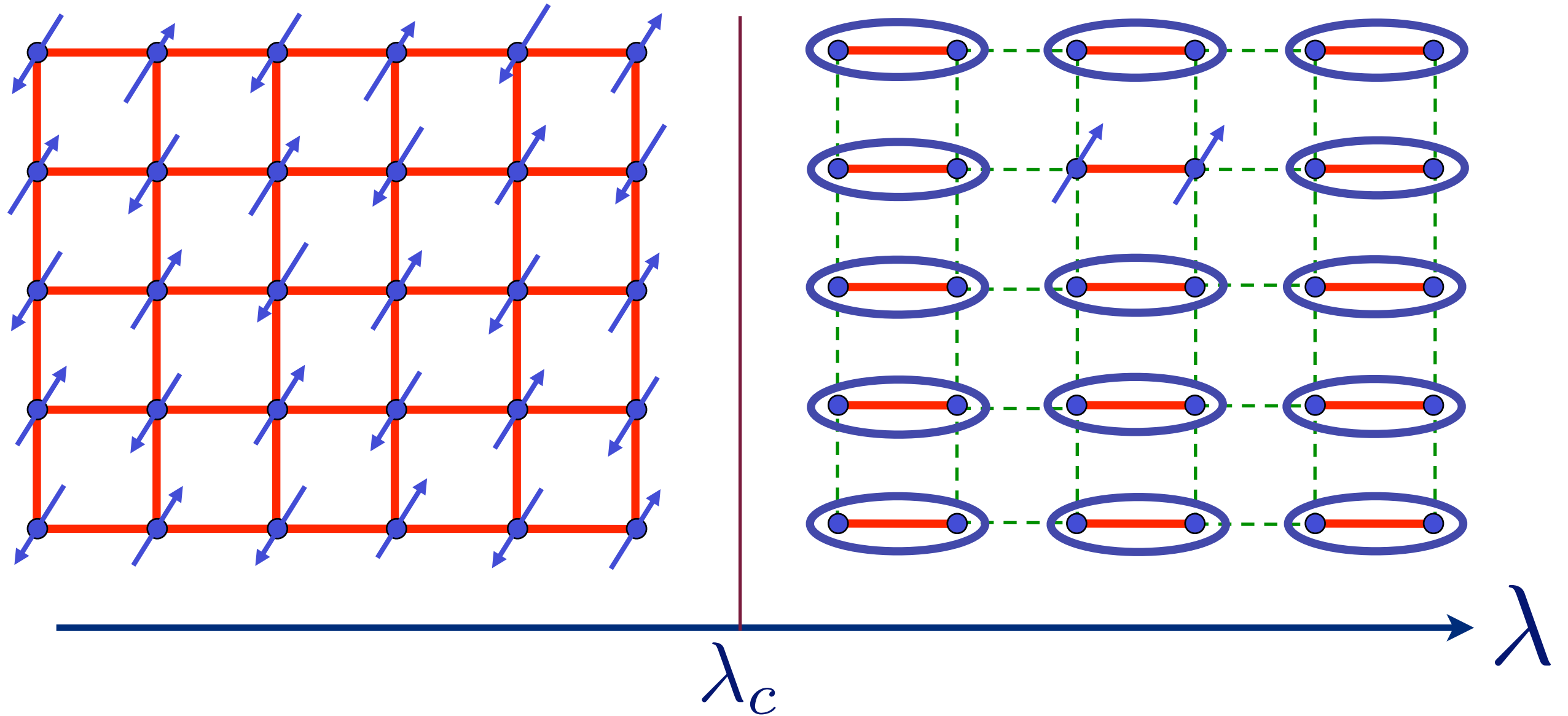
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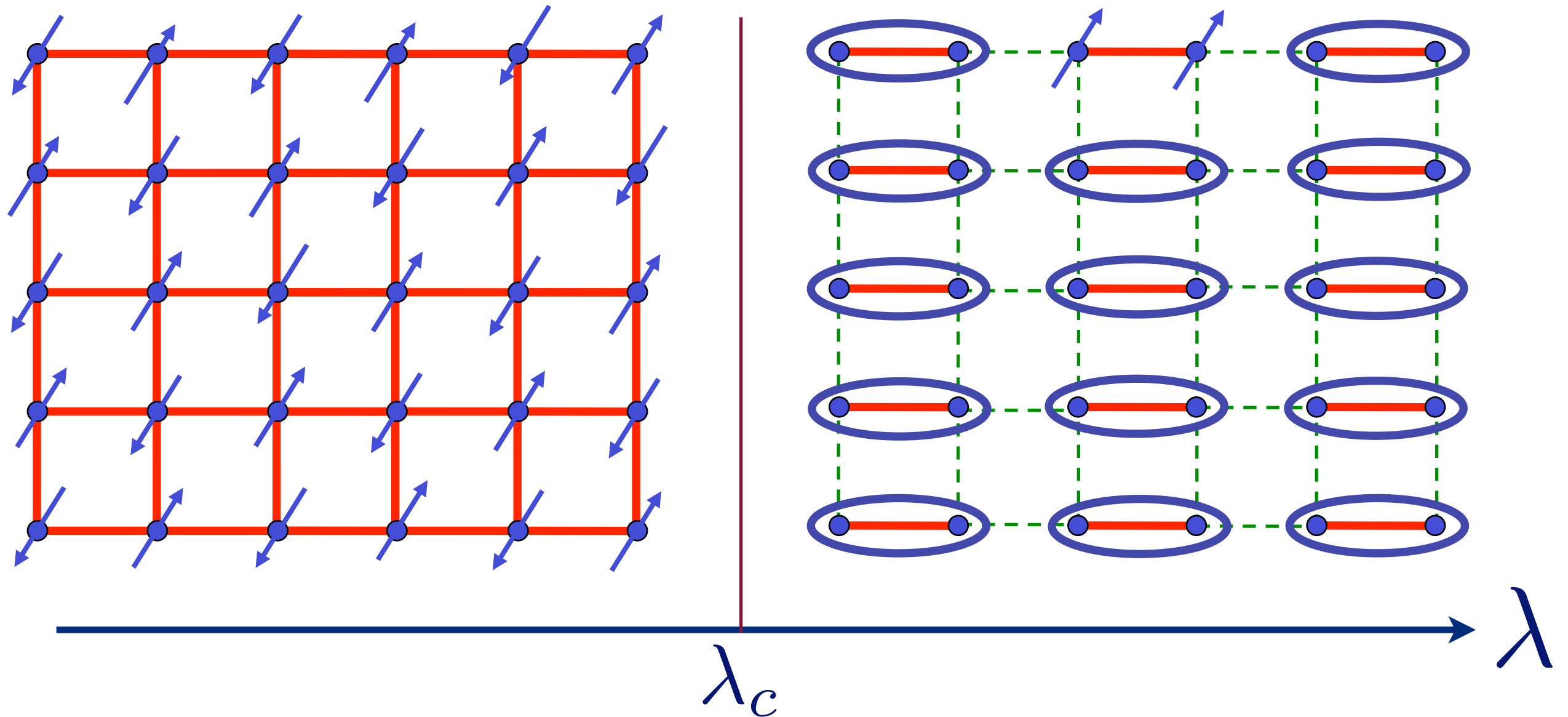
Excitation spectrum in the paramagnetic phase



Excitation spectrum in the paramagnetic phase

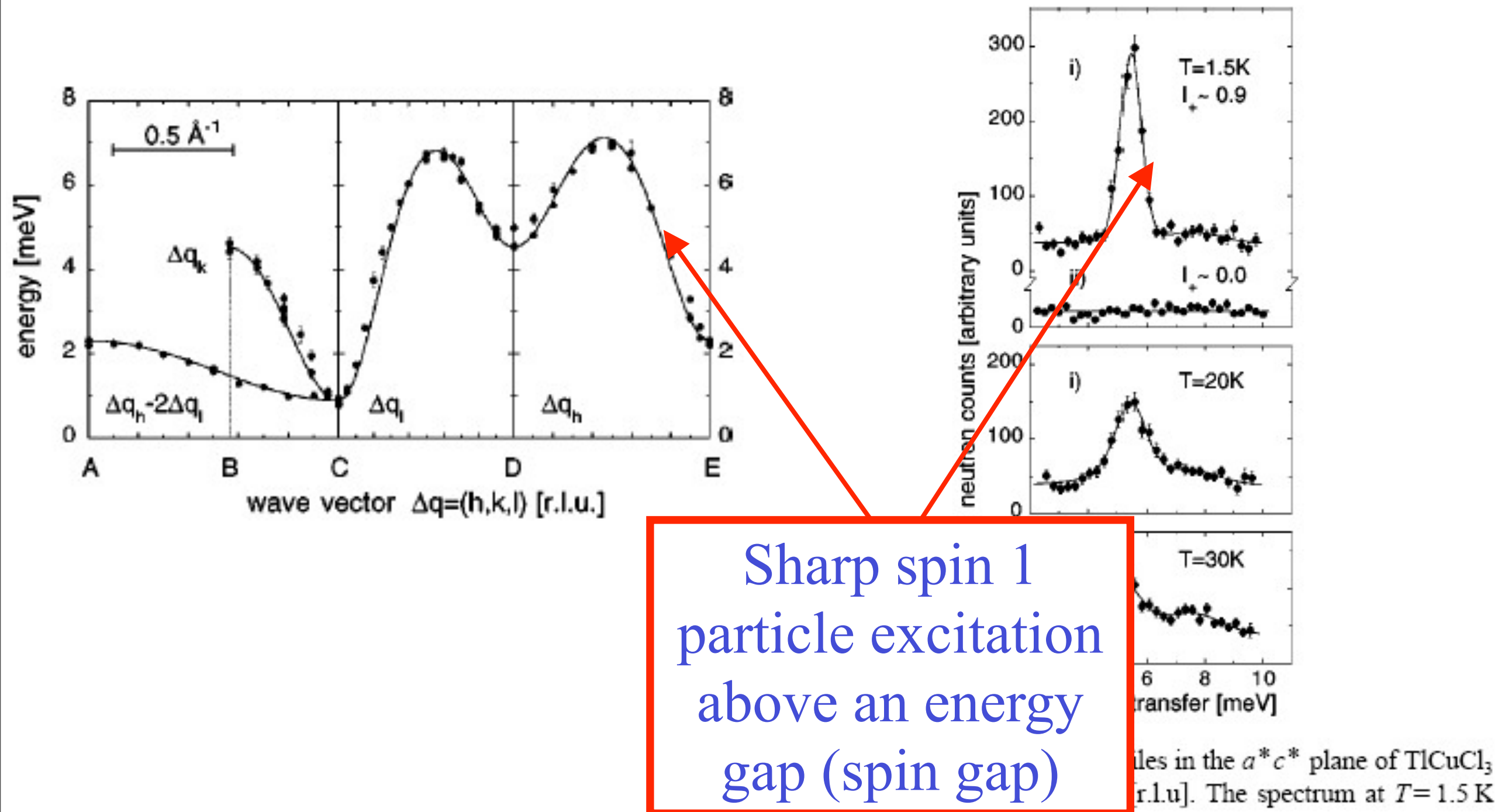


Excitation spectrum in the paramagnetic phase



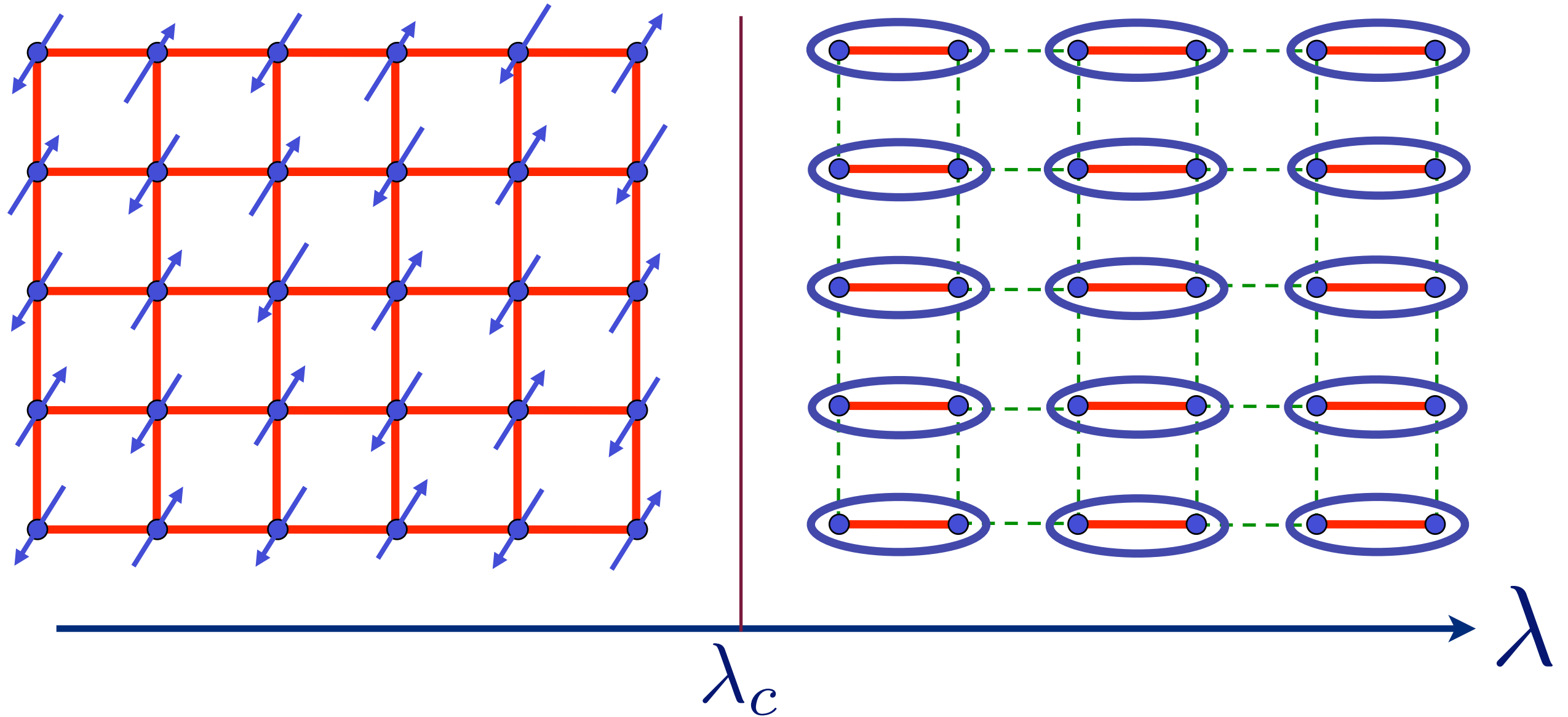
Sharp spin 1
particle excitation
above an energy
gap (spin gap)

TlCuCl₃ at ambient pressure

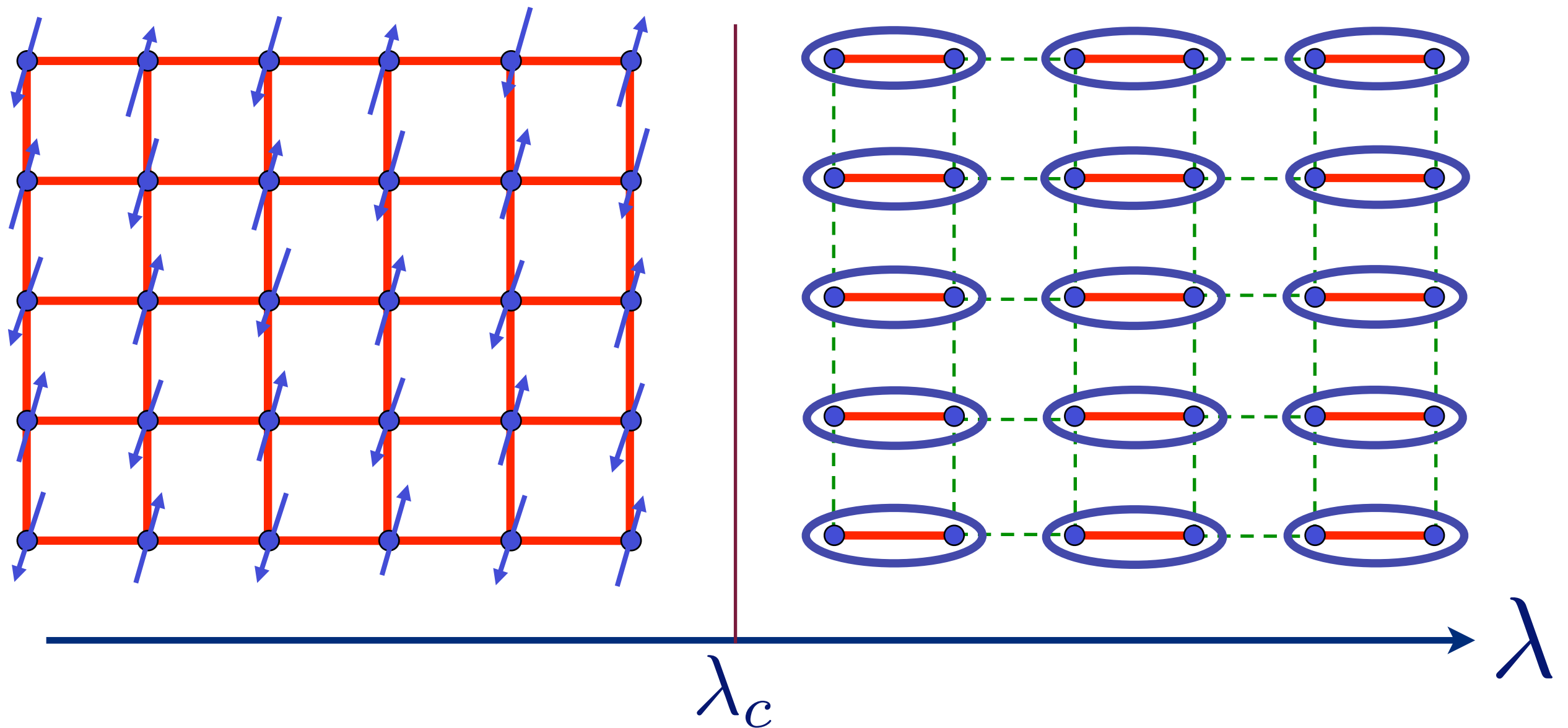


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Excitation spectrum in the Néel phase

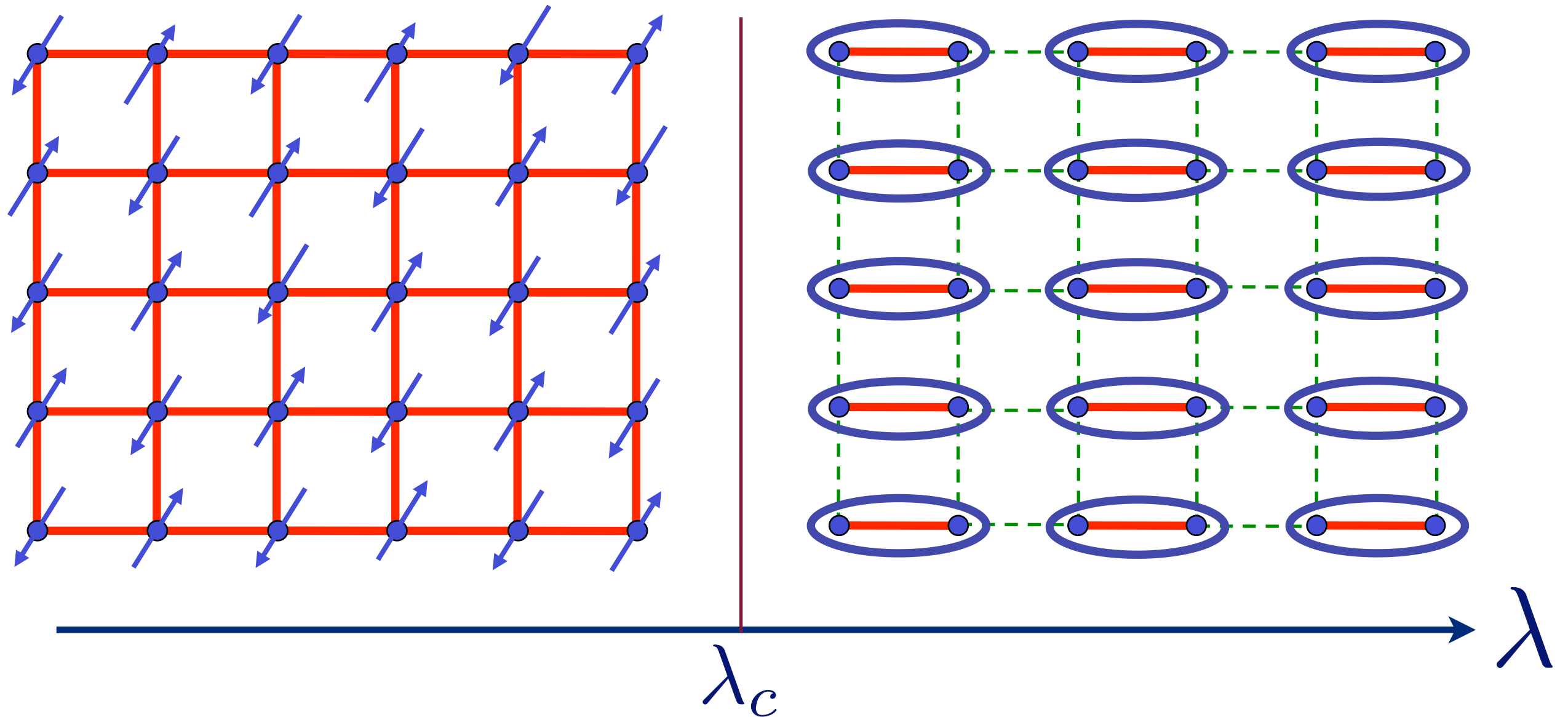


Excitation spectrum in the Néel phase



Spin waves

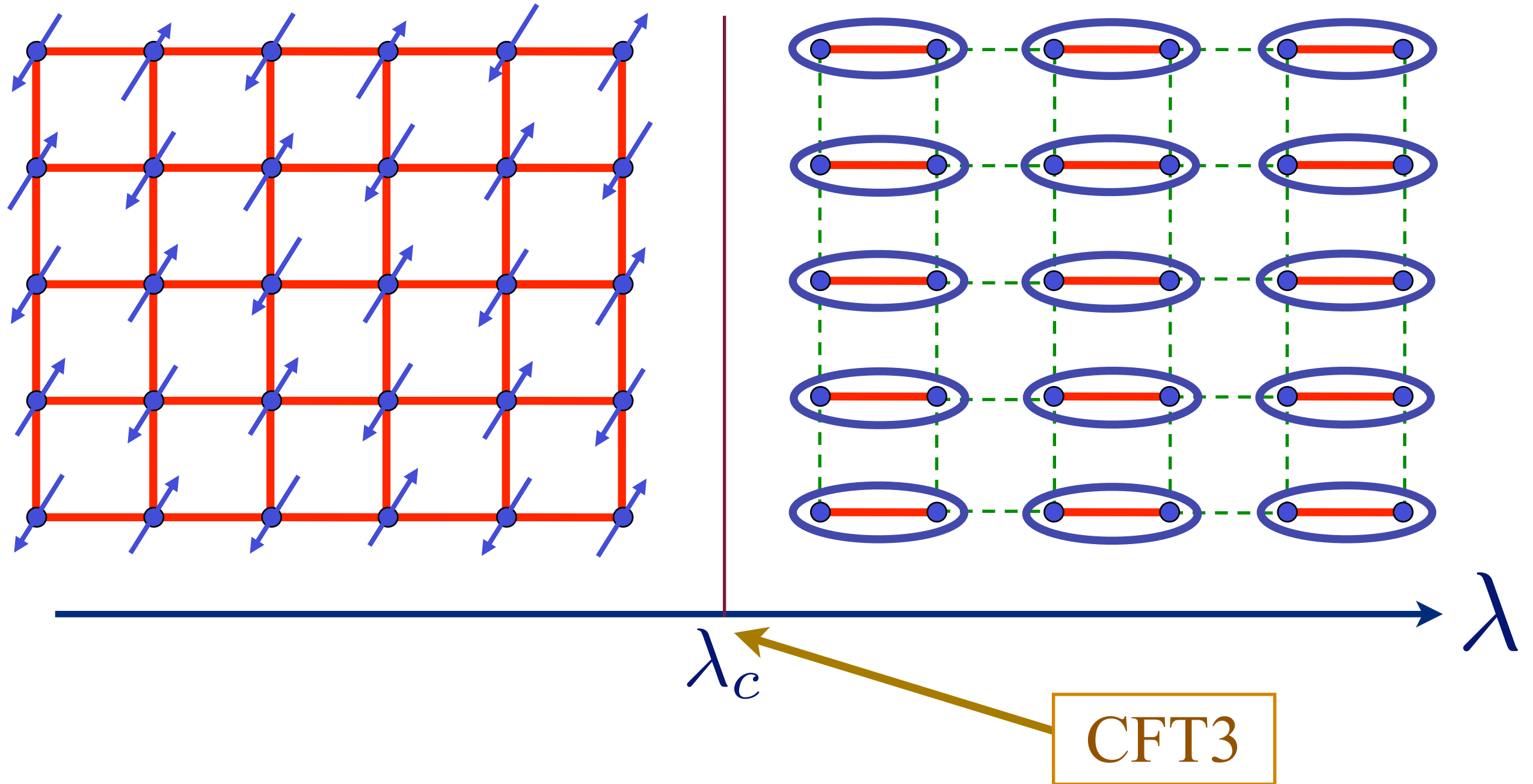
Excitation spectrum in the Néel phase



Spin waves

Derivation of field theory of critical point

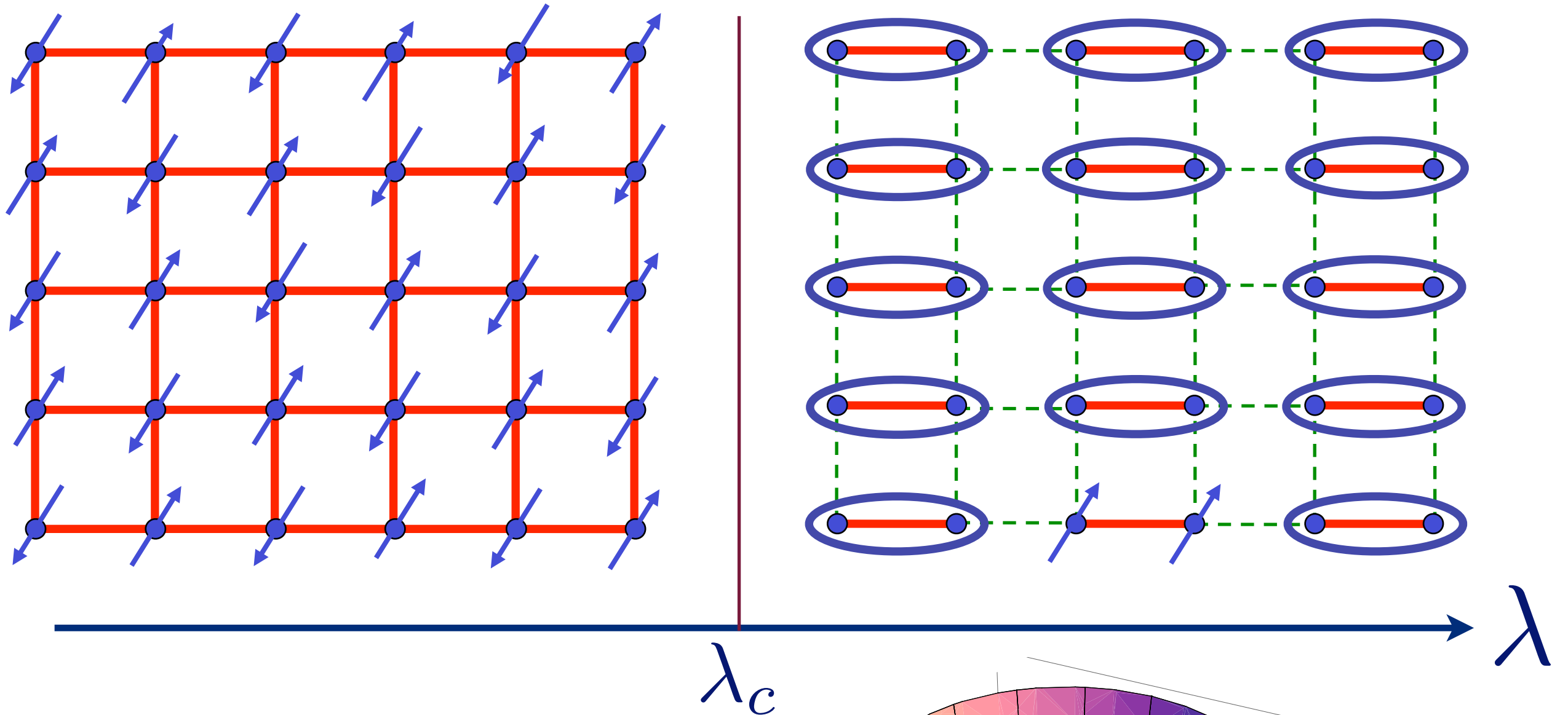
Description using Landau-Ginzburg field theory



$O(3)$ order parameter $\vec{\varphi}$

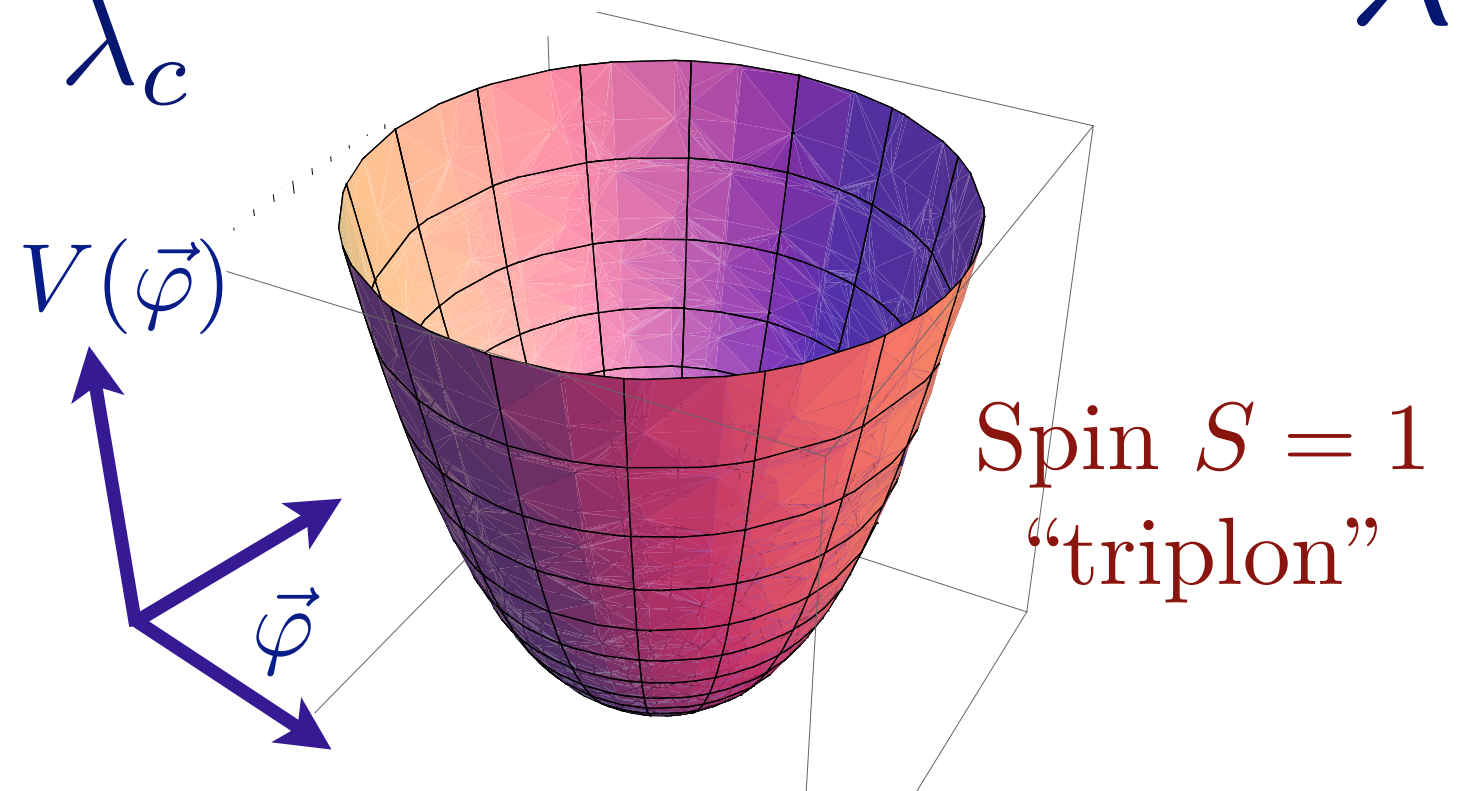
$$\mathcal{S} = \int d^2 r d\tau \left[(\partial_\tau \varphi)^2 + c^2 (\nabla_r \vec{\varphi})^2 + (\lambda - \lambda_c) \vec{\varphi}^2 + u (\vec{\varphi}^2)^2 \right]$$

Excitation spectrum in the paramagnetic phase

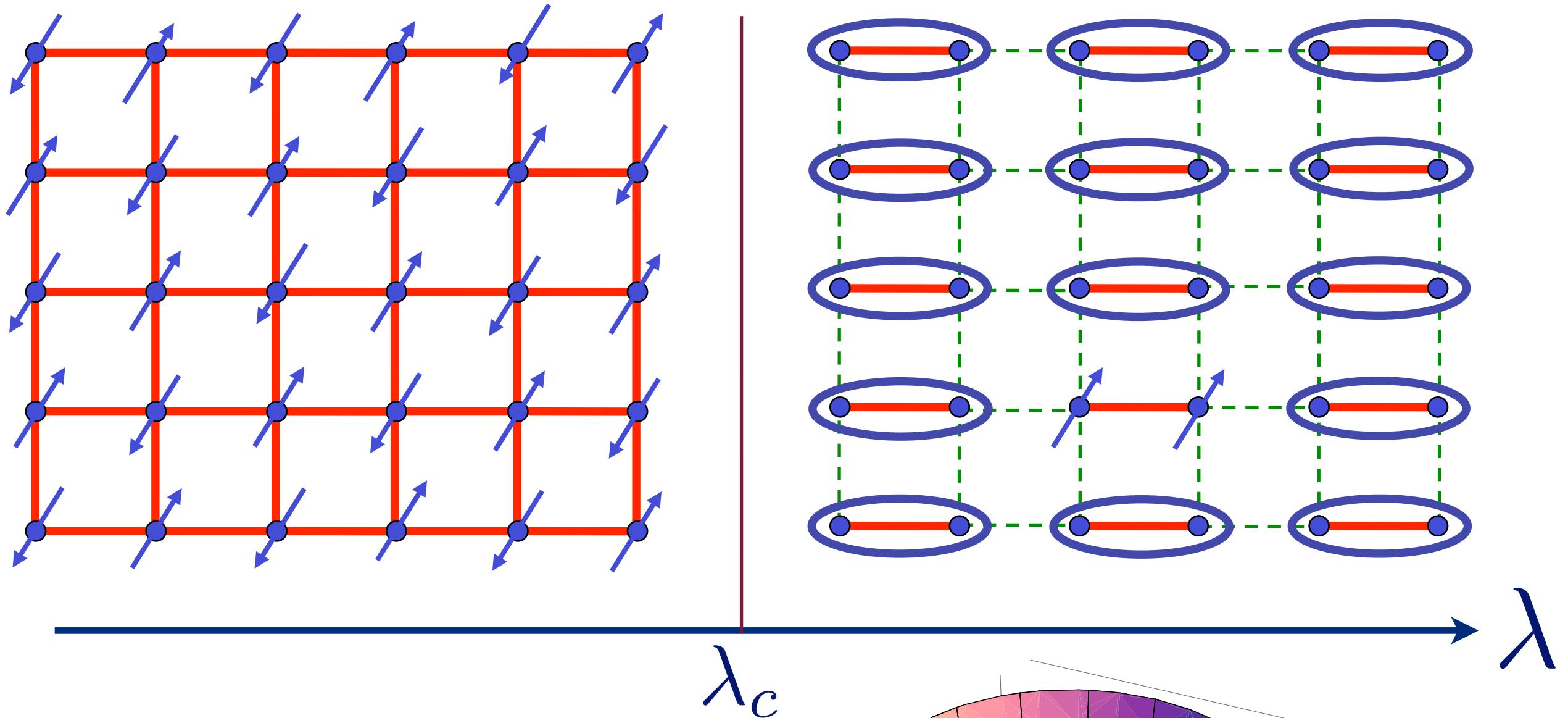


$$V(\vec{\varphi}) = (\lambda - \lambda_c) \vec{\varphi}^2 + u (\vec{\varphi}^2)^2$$

$$\lambda > \lambda_c$$

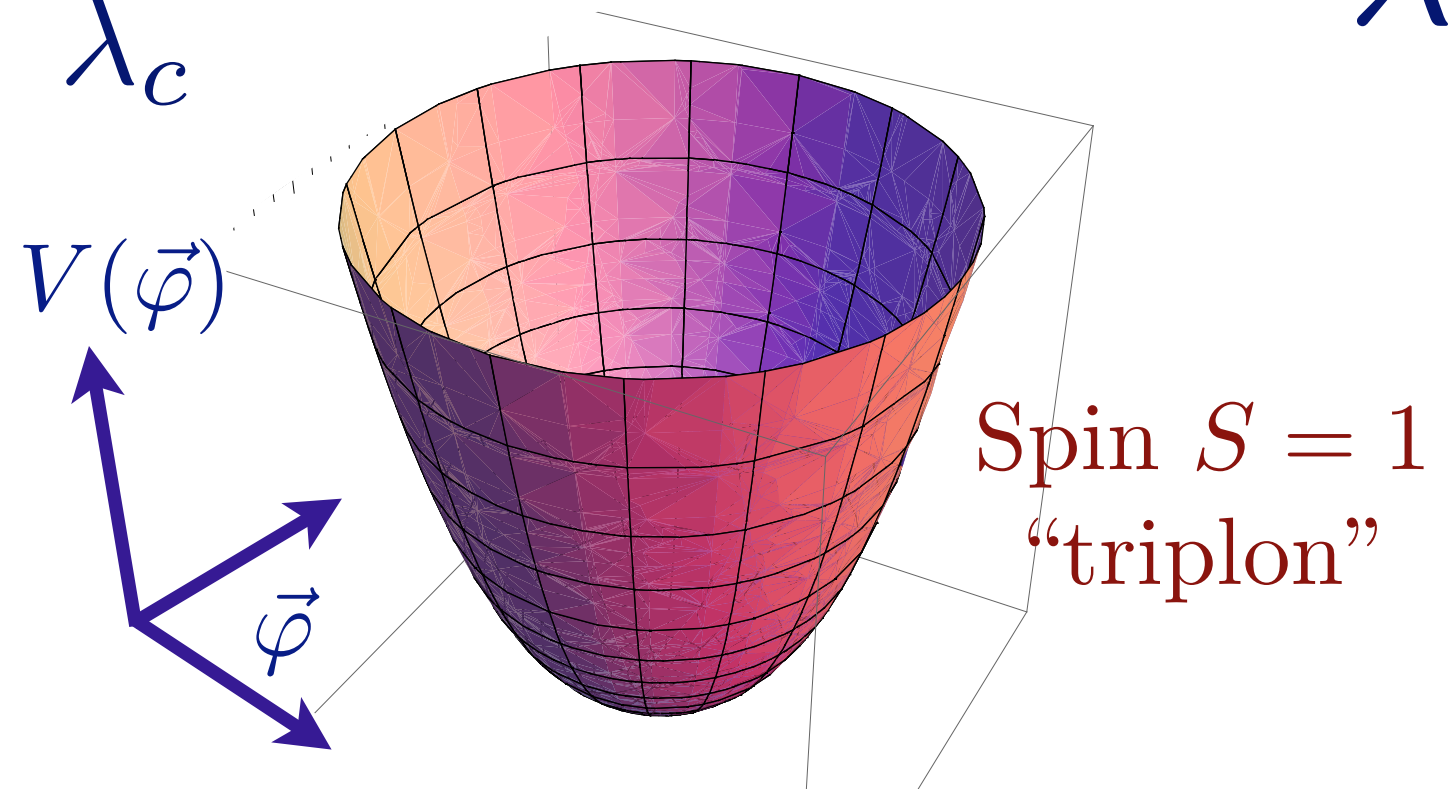


Excitation spectrum in the paramagnetic phase

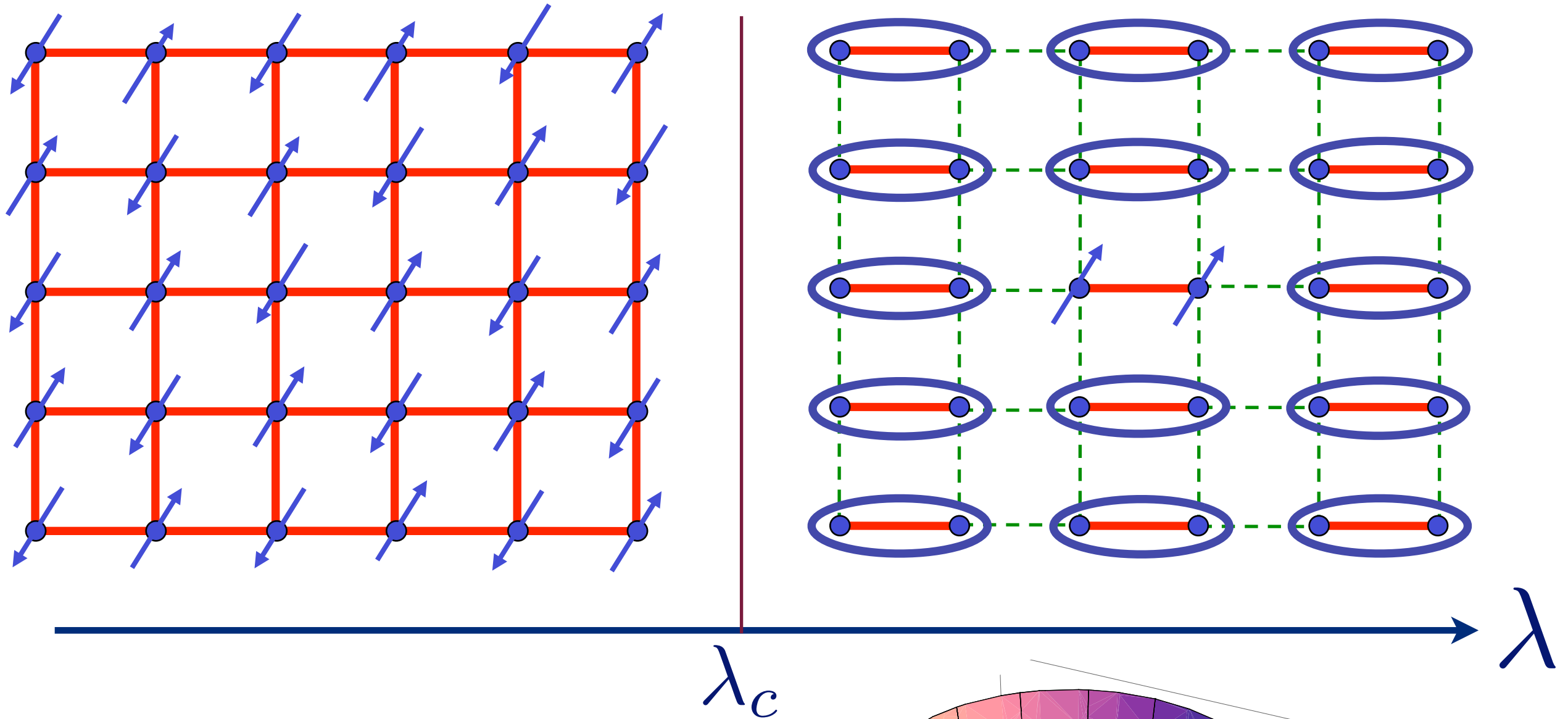


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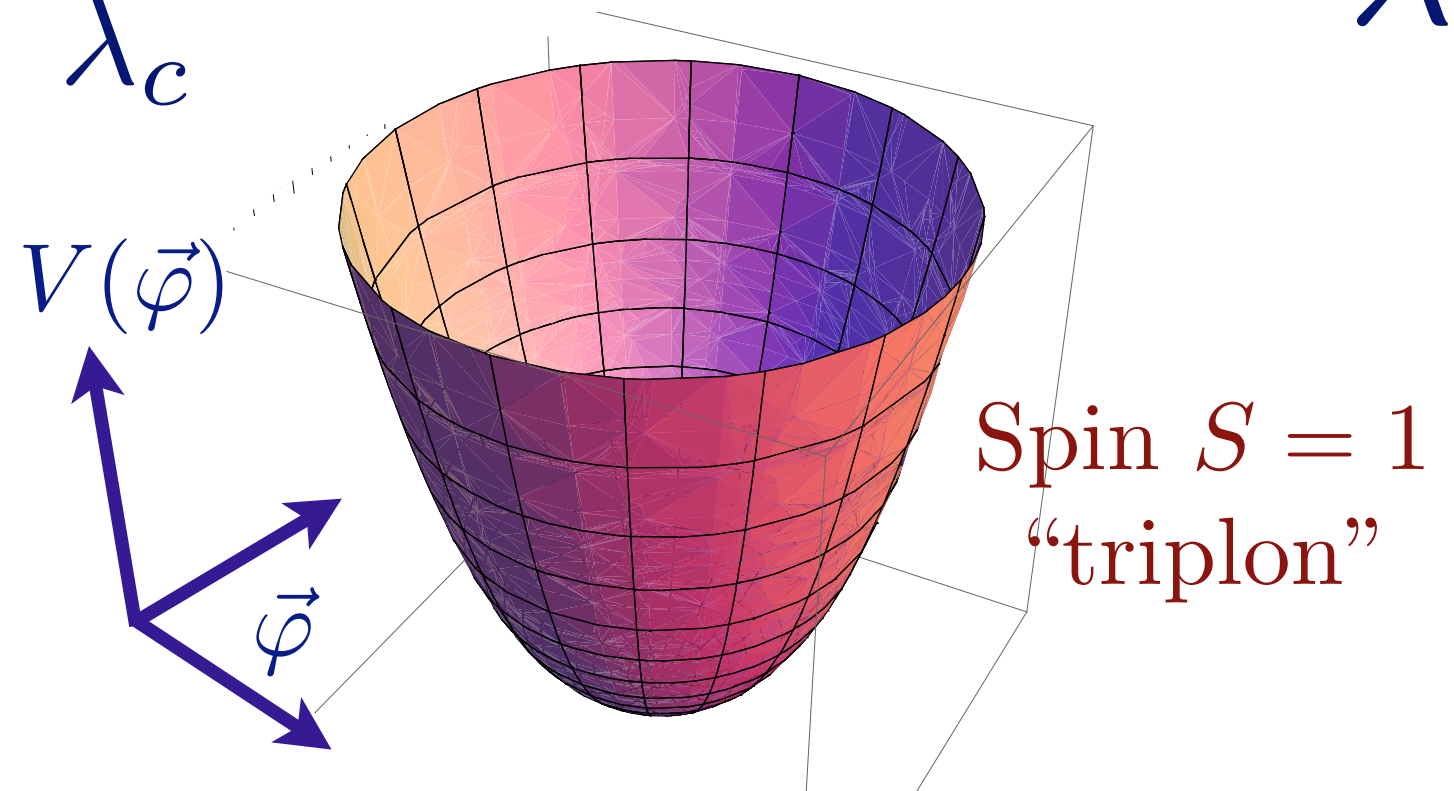


Excitation spectrum in the paramagnetic phase

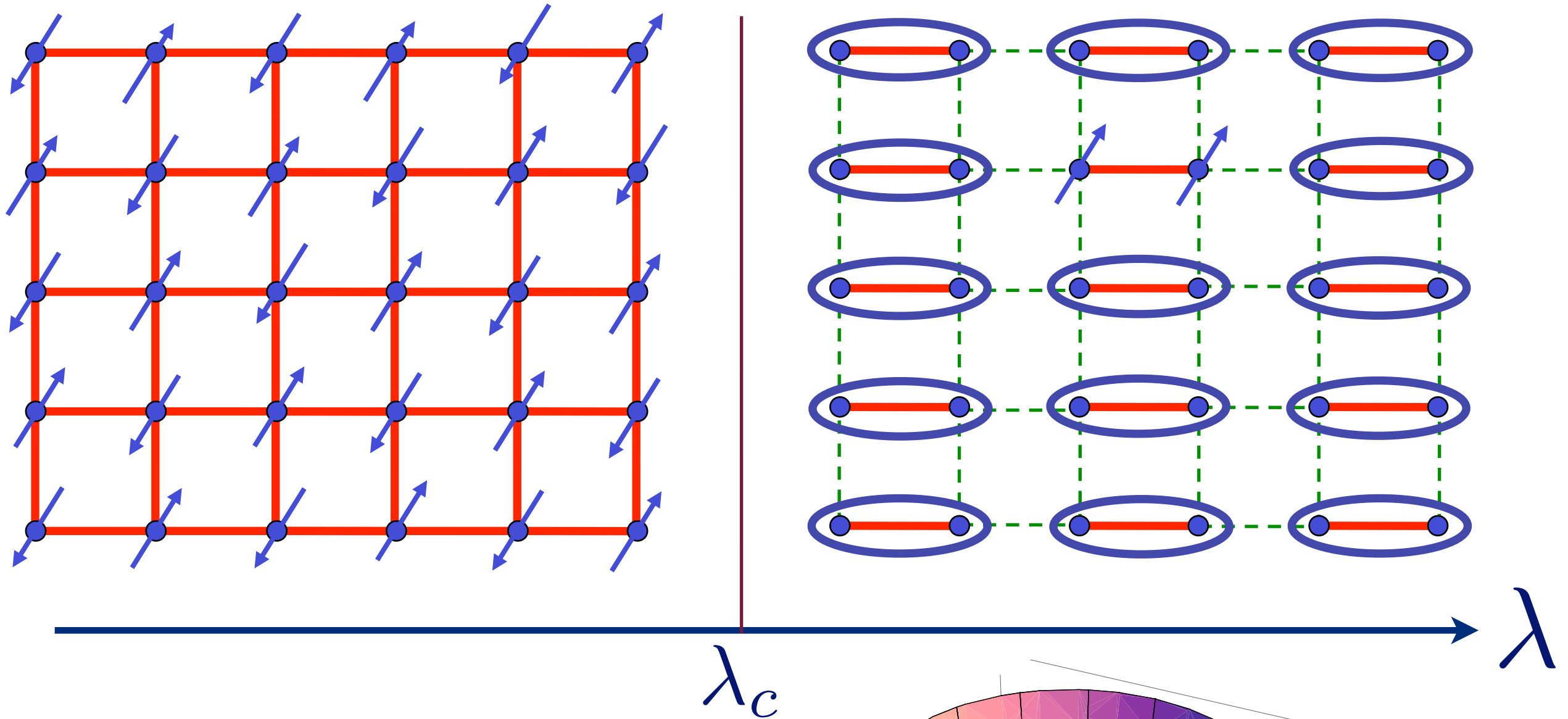


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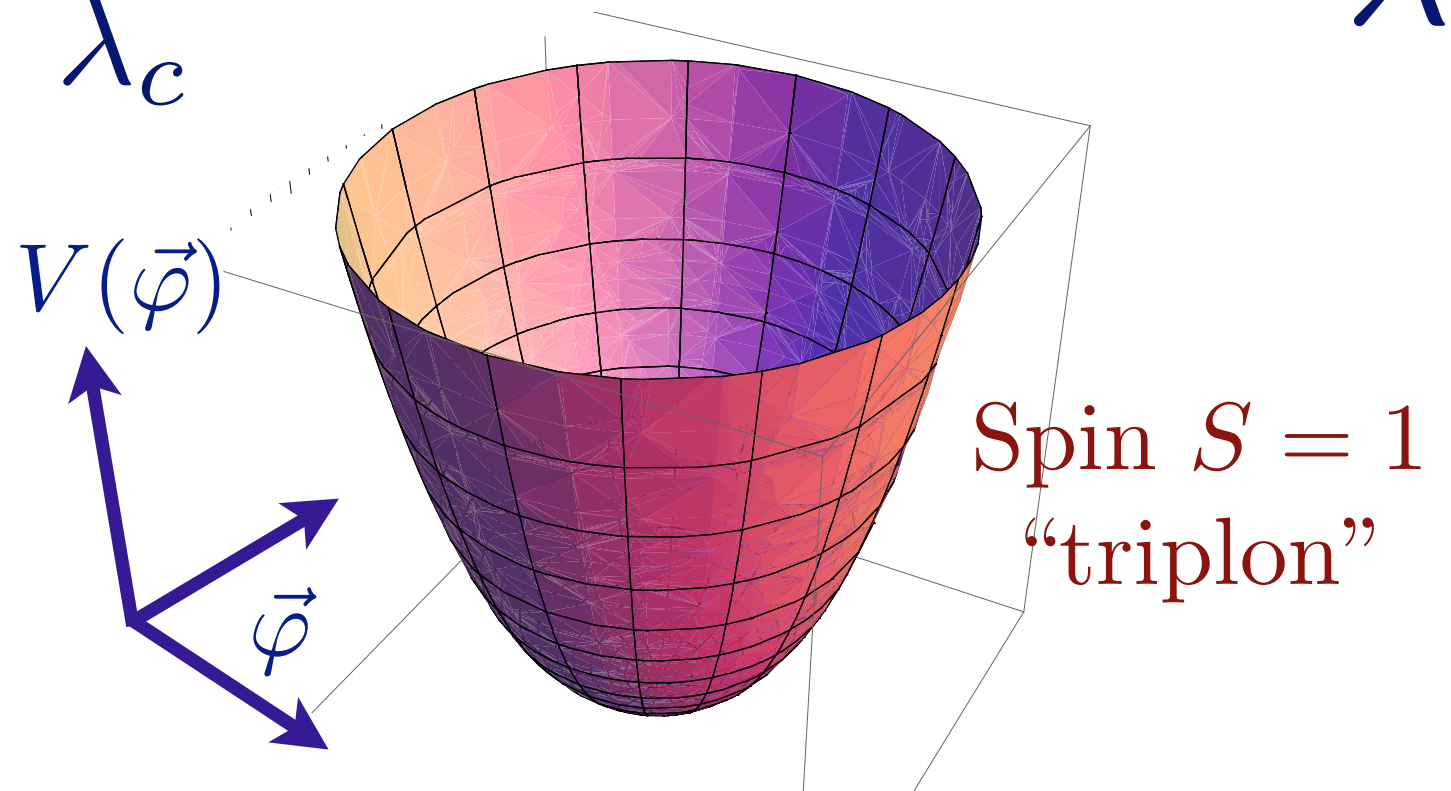


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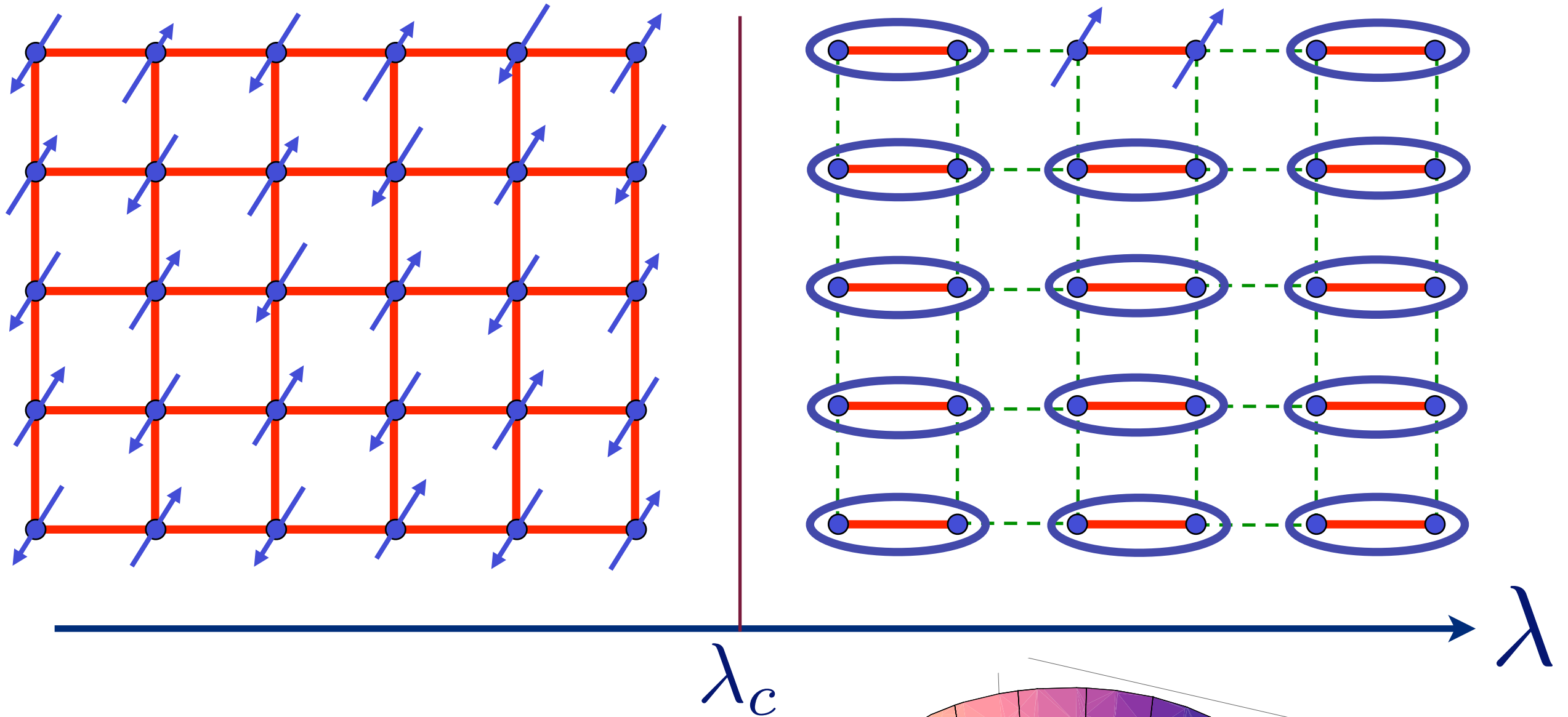


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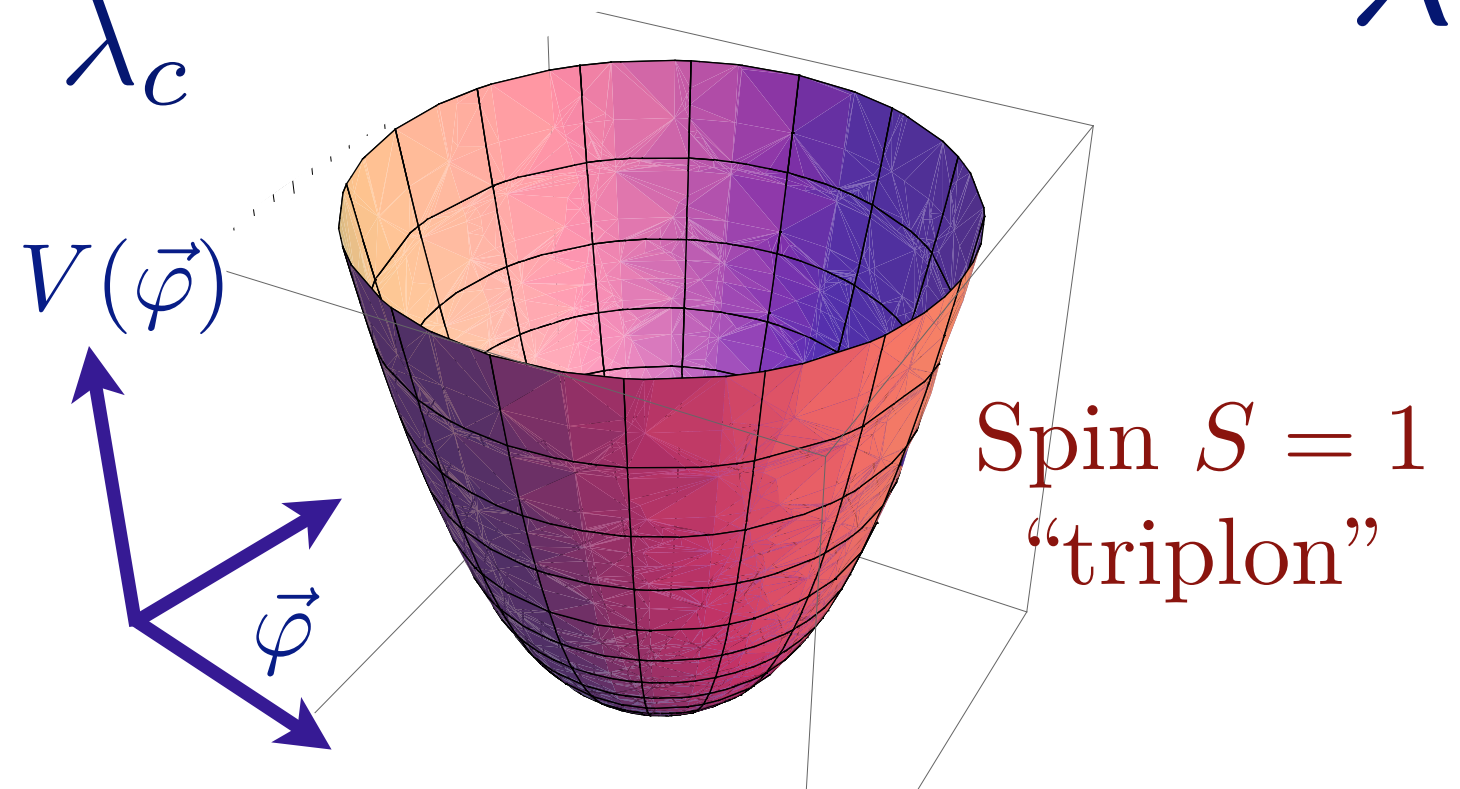


Excitation spectrum in the paramagnetic phase

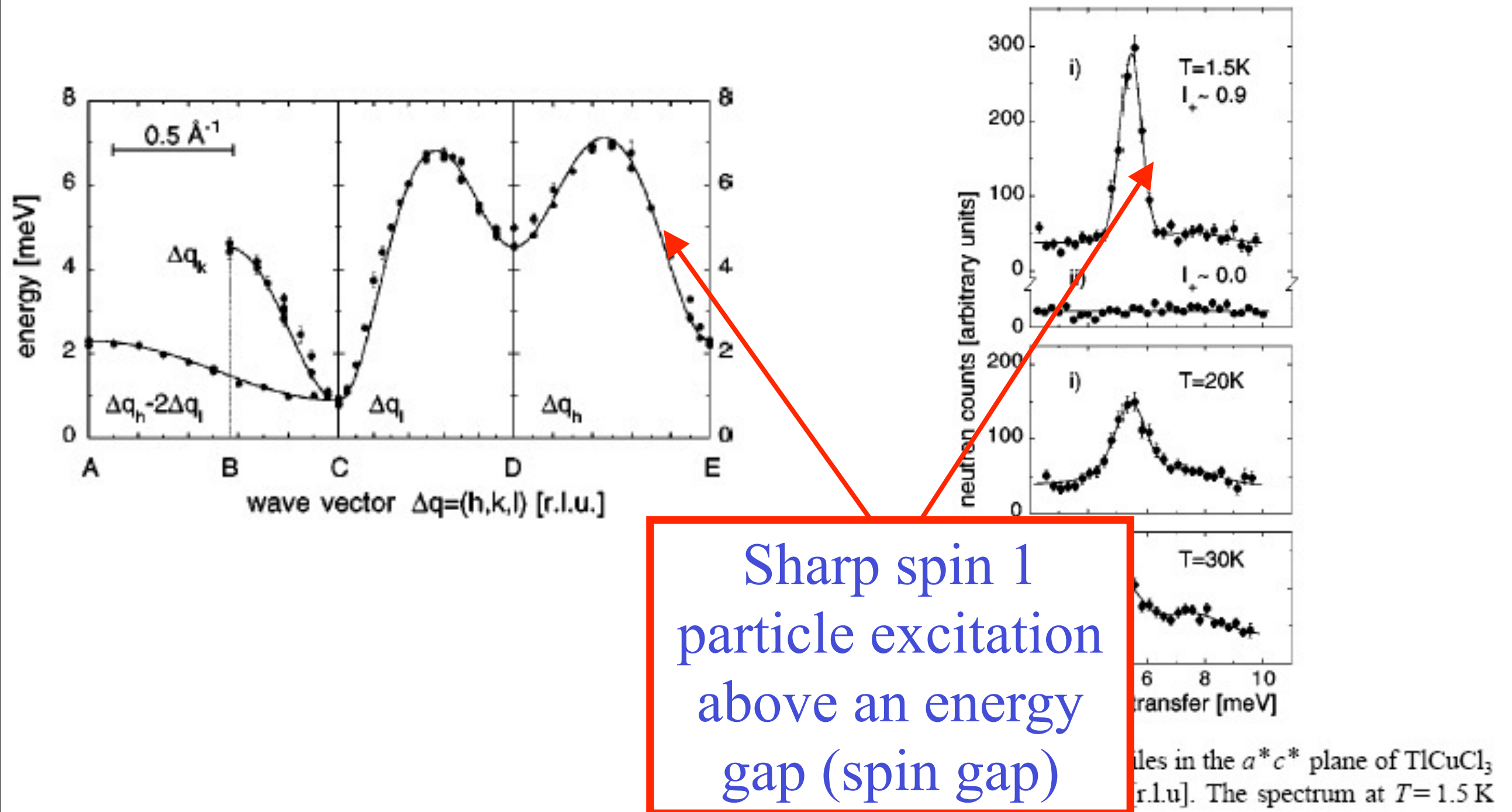


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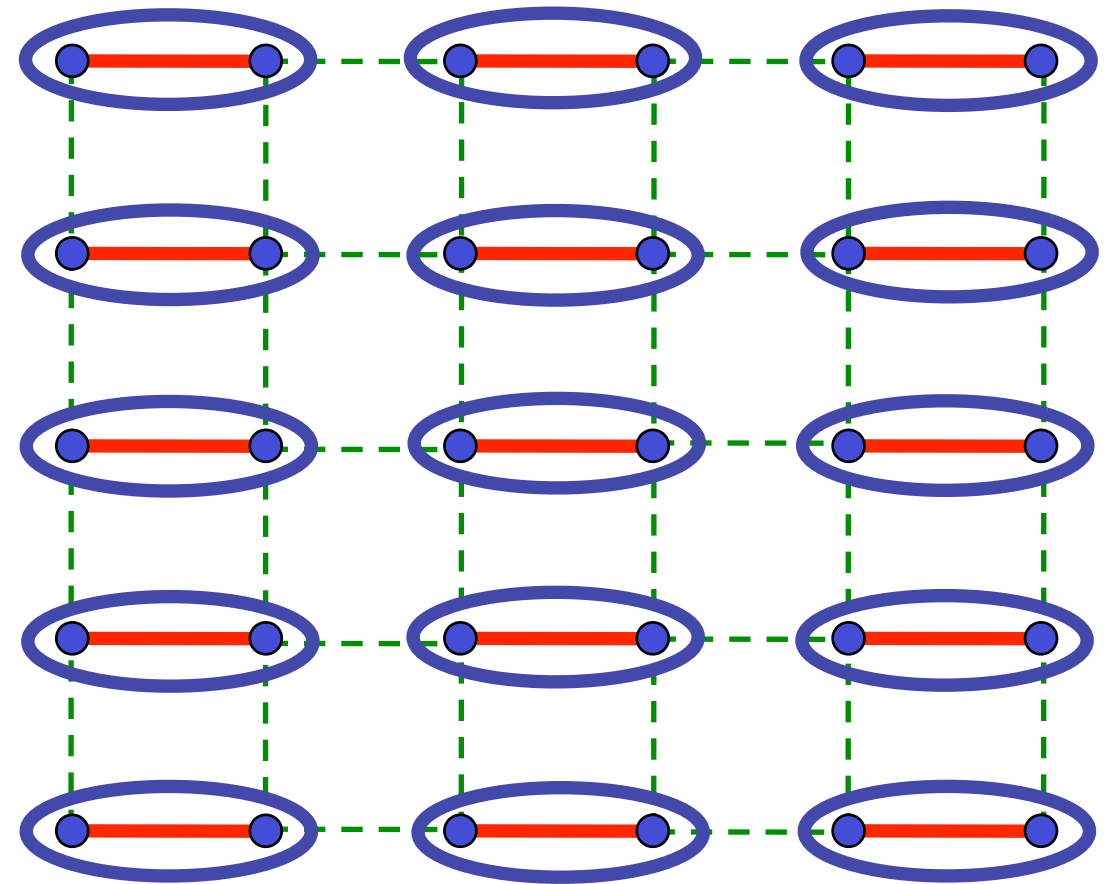
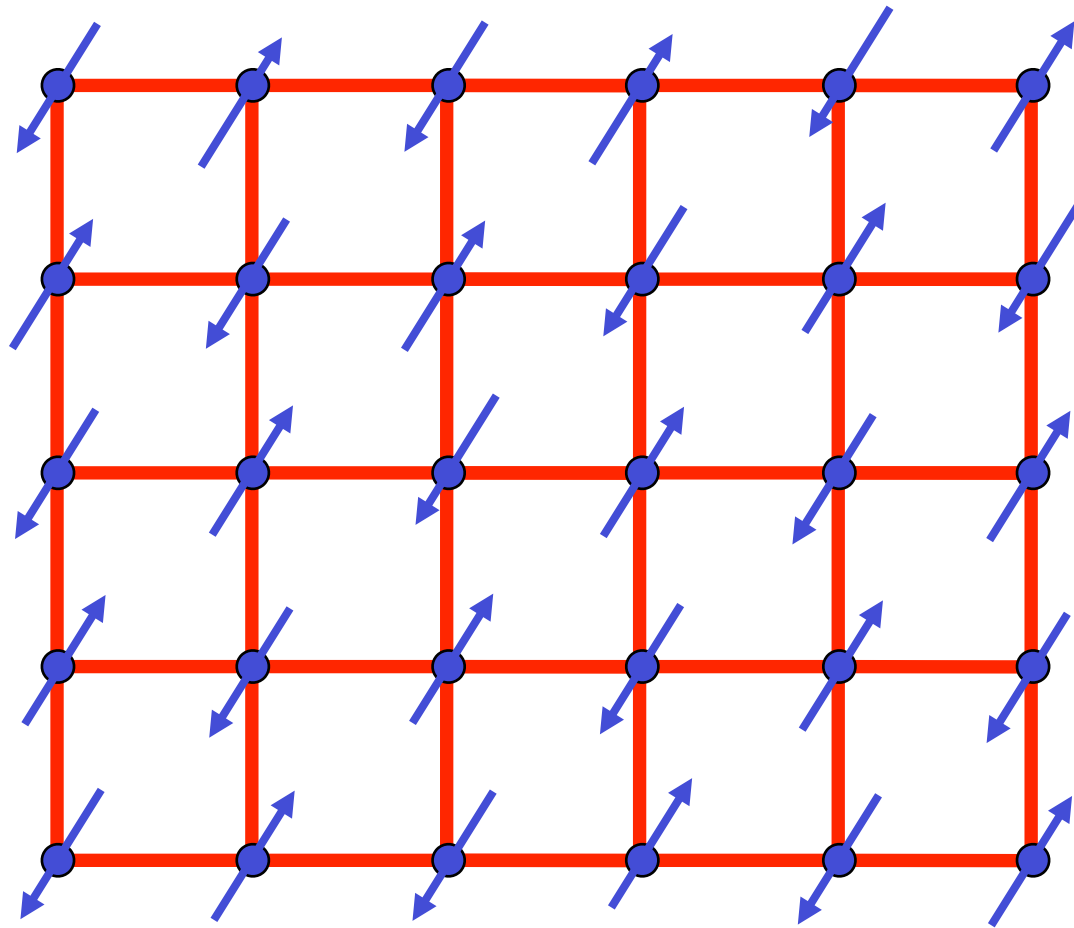


TlCuCl₃ at ambient pressure



N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer and H. Mutka, *Phys. Rev. B* 63 172414 (2001).

Excitation spectrum in the Néel phase

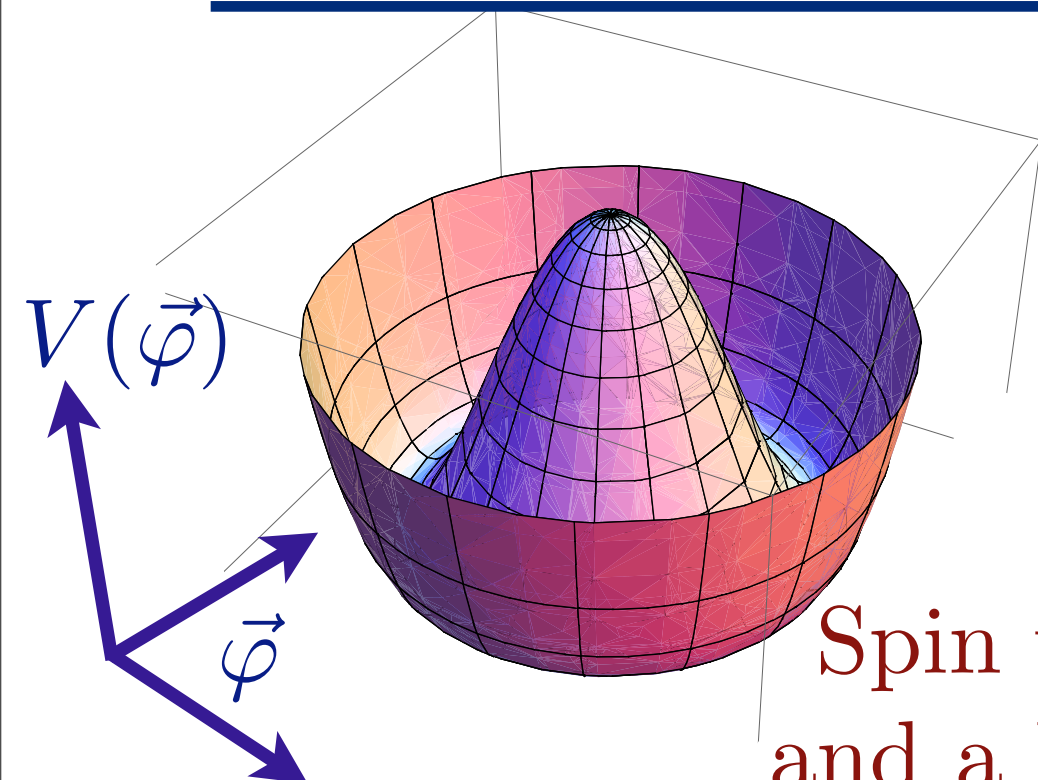


λ_c

λ

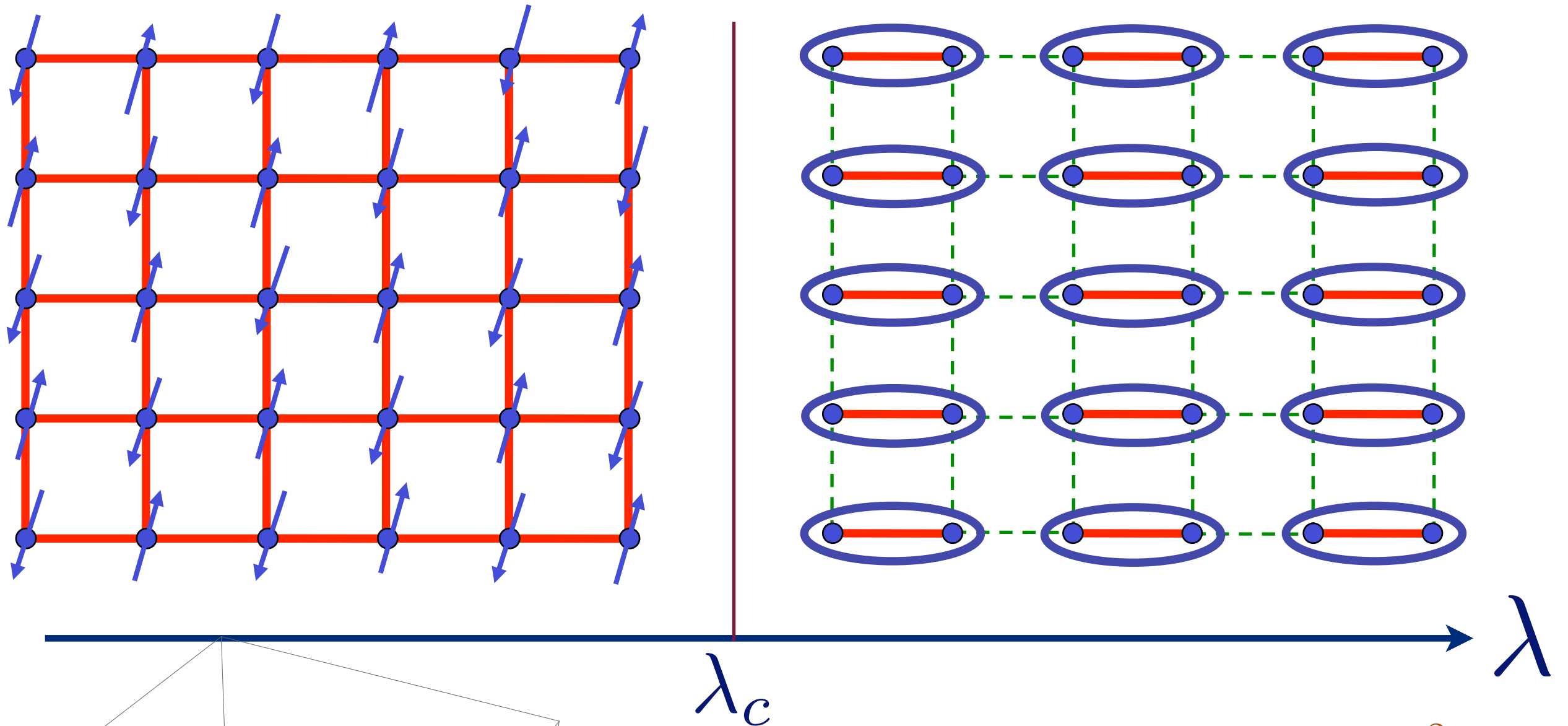
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$$\lambda < \lambda_c$$



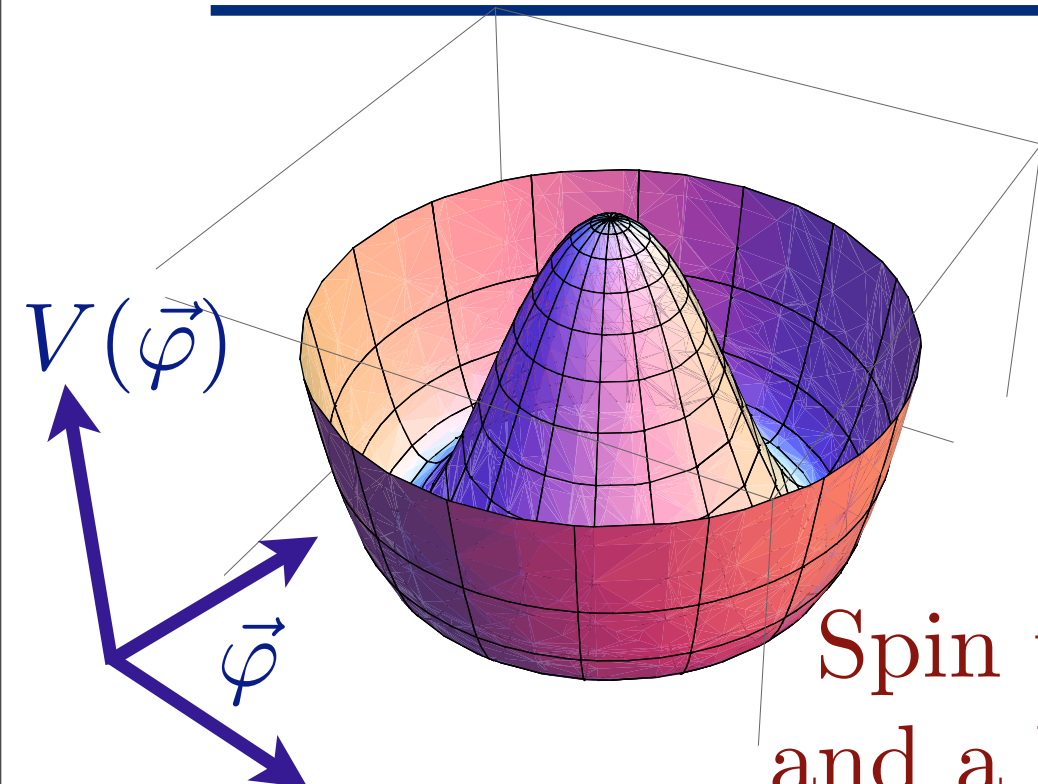
Spin waves (“Goldstone” modes)
and a longitudinal “Higgs” particle

Excitation spectrum in the Néel phase



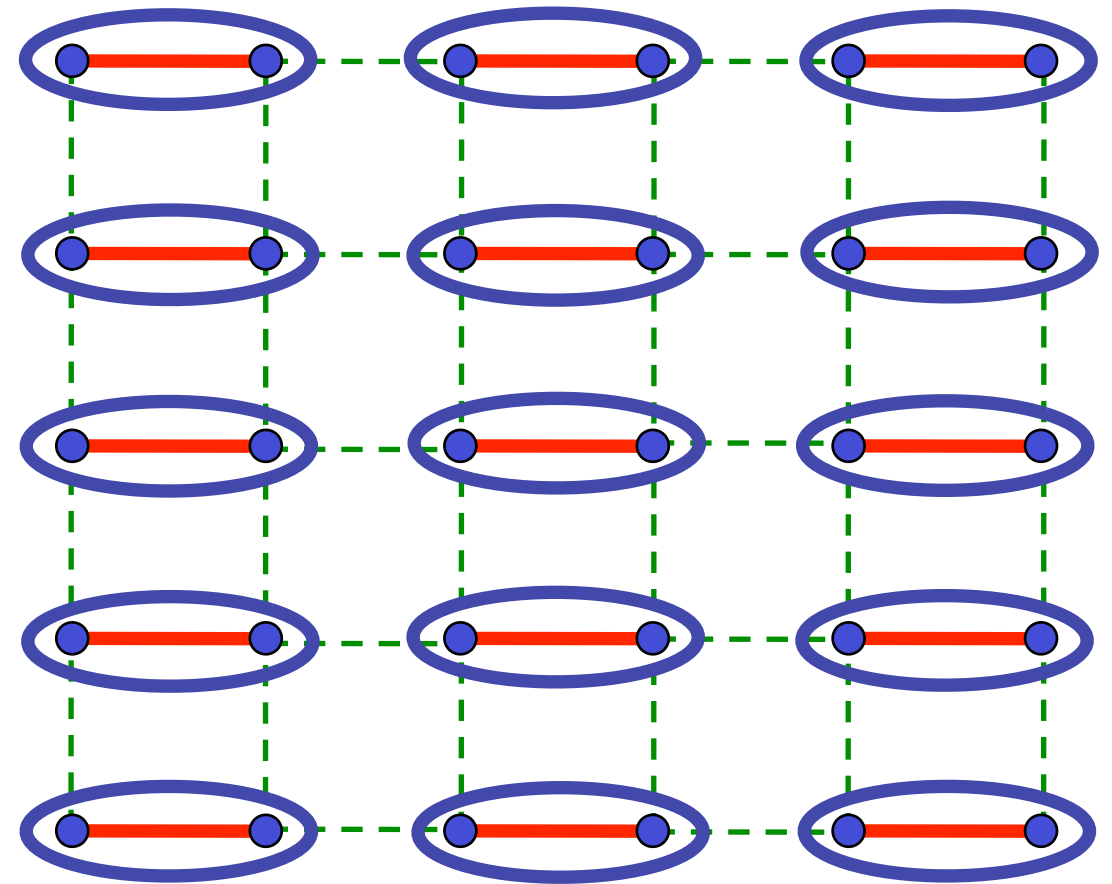
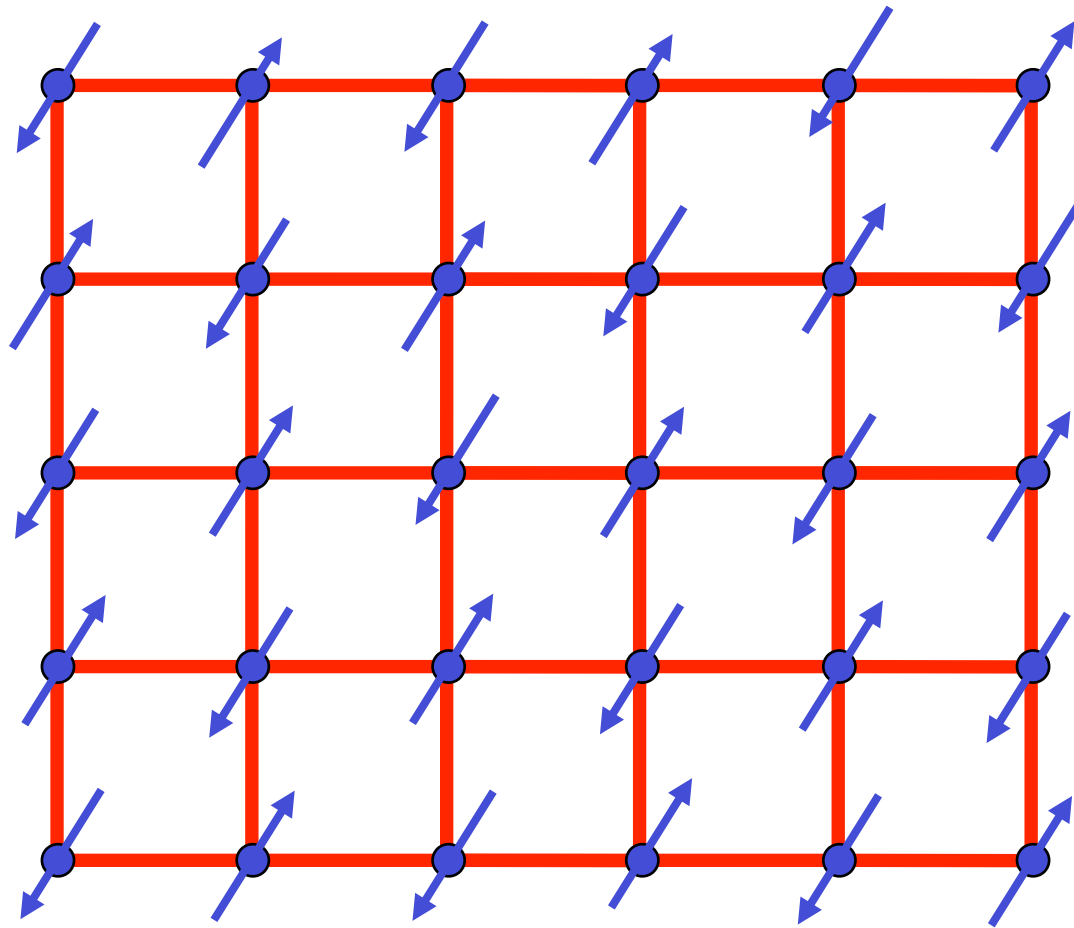
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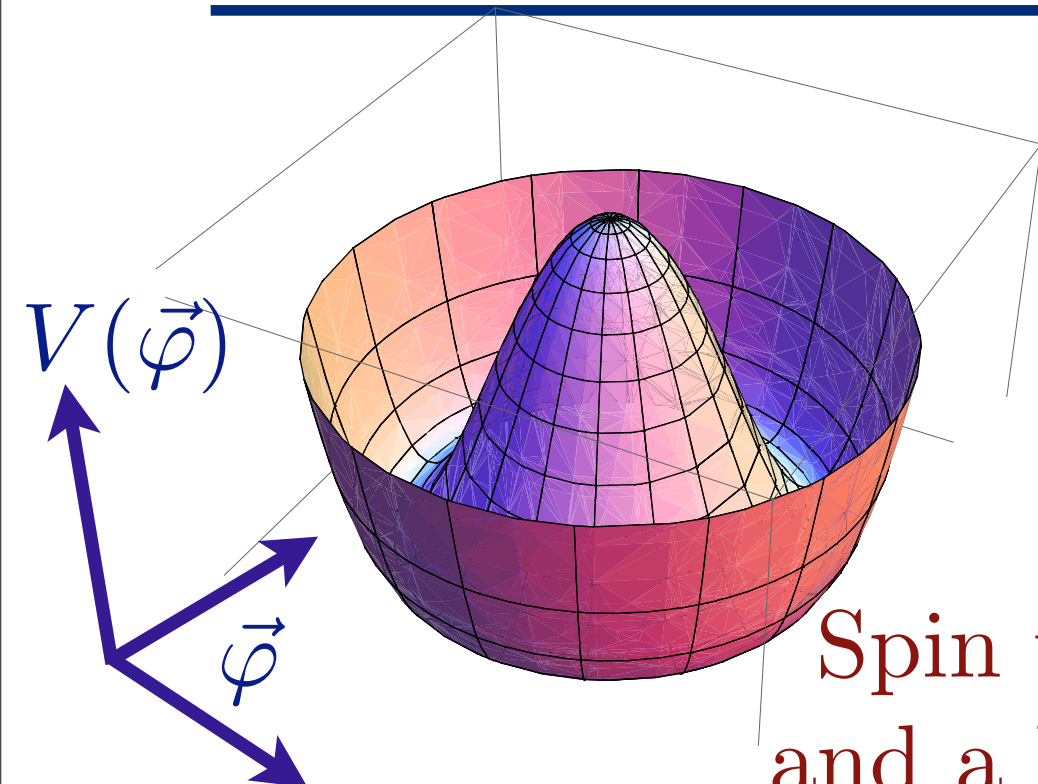


λ_c

λ

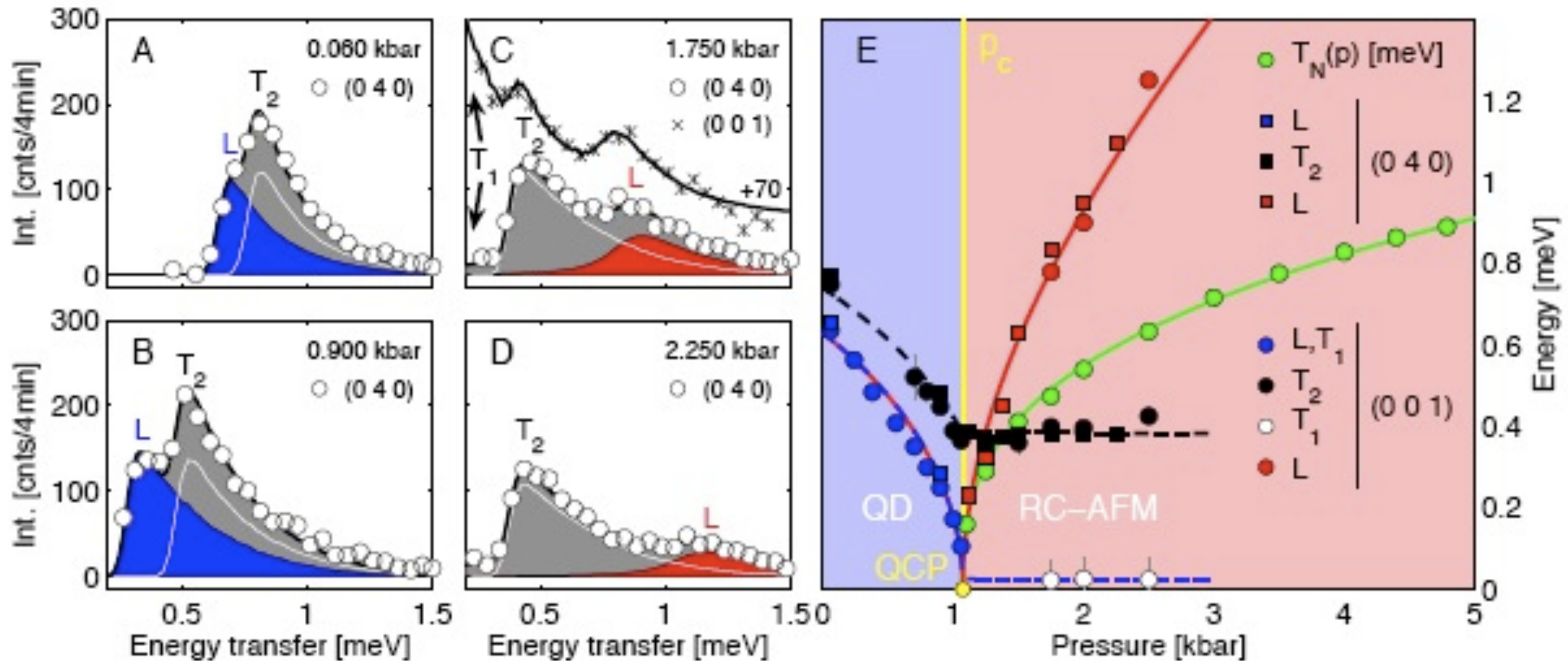
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Spin waves (“Goldstone” modes)
and a longitudinal “Higgs” particle

TlCuCl₃ with varying pressure



Observation of 3 → 2 low energy modes, emergence of new longitudinal mode (the “Higgs boson”) in Néel phase, and vanishing of Néel temperature at quantum critical point

Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

Prediction of quantum field theory

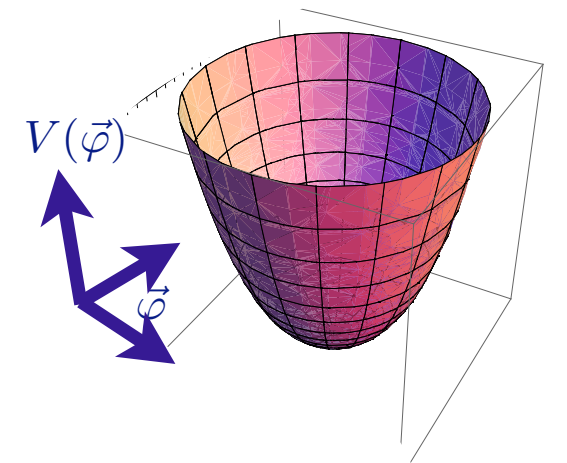
Potential for $\vec{\varphi}$ fluctuations: $V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u (\vec{\varphi}^2)^2$

Paramagnetic phase, $\lambda > \lambda_c$

Expand about $\vec{\varphi} = 0$:

$$V(\vec{\varphi}) \approx (\lambda - \lambda_c)\vec{\varphi}^2$$

Yields 3 particles with energy gap $\sim \sqrt{(\lambda - \lambda_c)}$



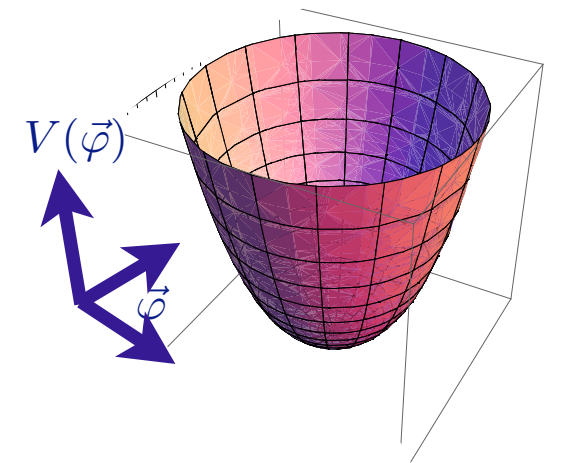
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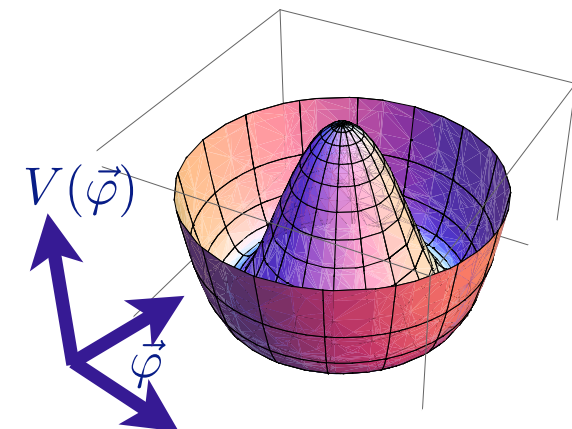


Yields 3 particles with energy gap $\sim \sqrt{(\lambda - \lambda_c)}$

Néel phase, $\lambda < \lambda_c$

Expand $\vec{\varphi} = (0, 0, \sqrt{(\lambda_c - \lambda)/(2u)}) + \vec{\varphi}_1$:

$$V(\vec{\varphi}) \approx 2(\lambda_c - \lambda)\varphi_{1z}^2$$

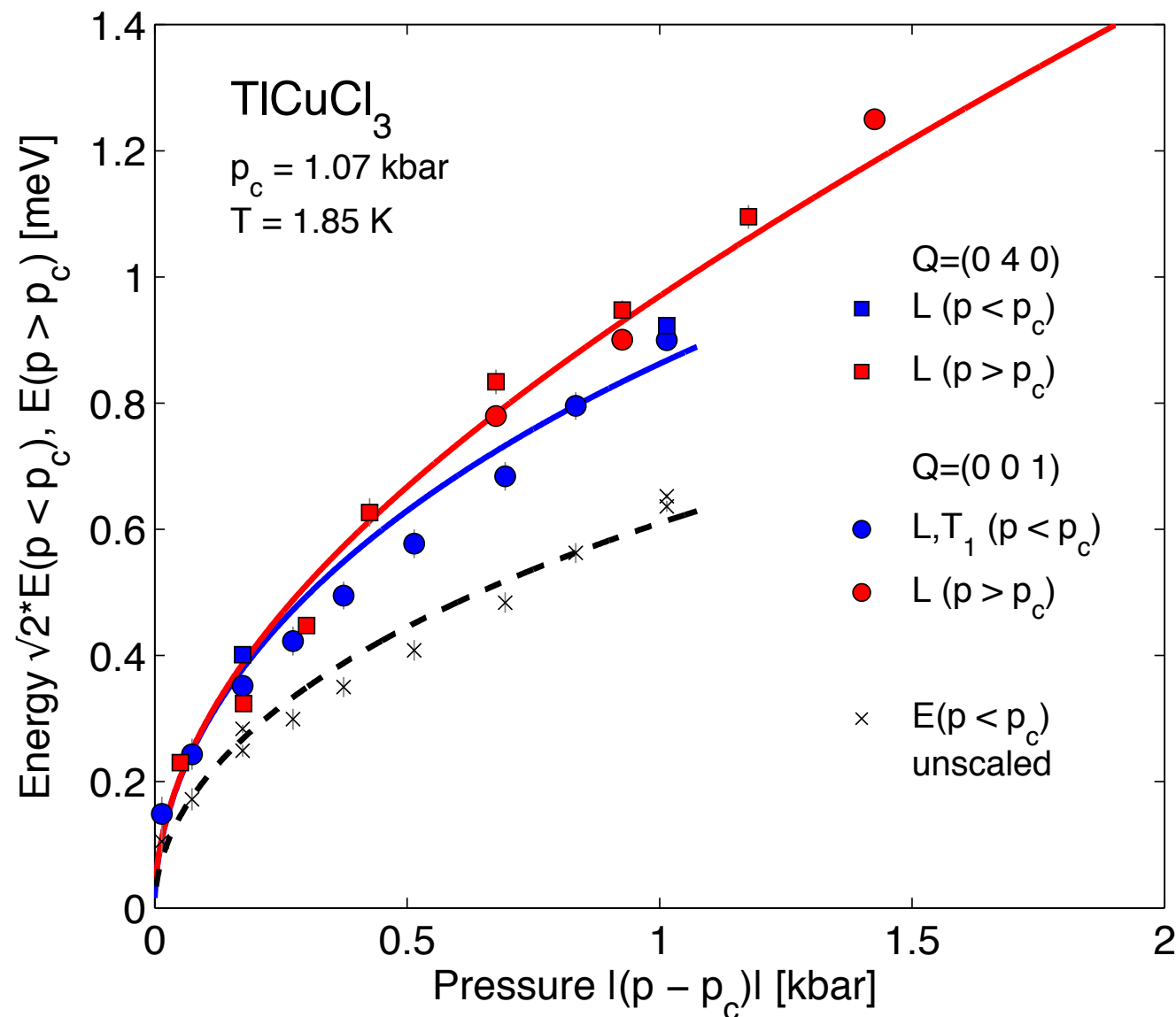


Yields 2 gapless spin waves and one Higgs particle with energy gap $\sim \sqrt{2(\lambda_c - \lambda)}$

Prediction of quantum field theory

$$\frac{\text{Energy of Higgs particle}}{\text{Energy of triplon}} = \sqrt{2}$$

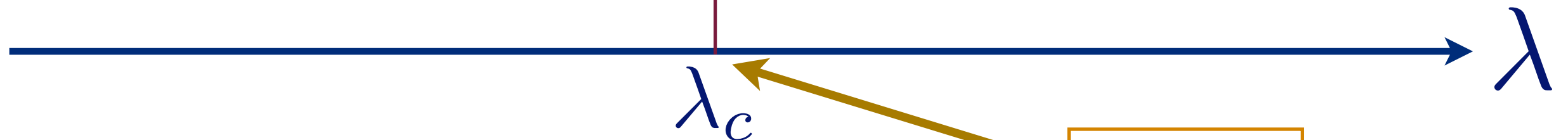
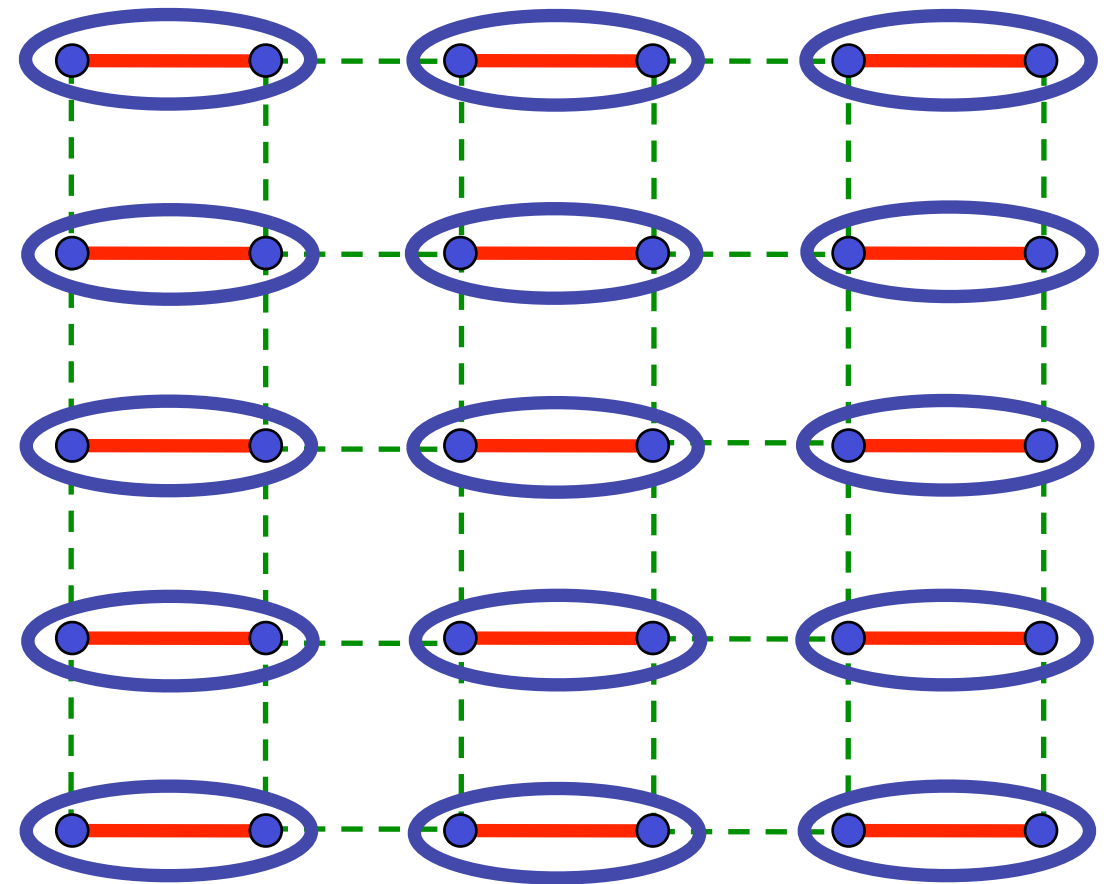
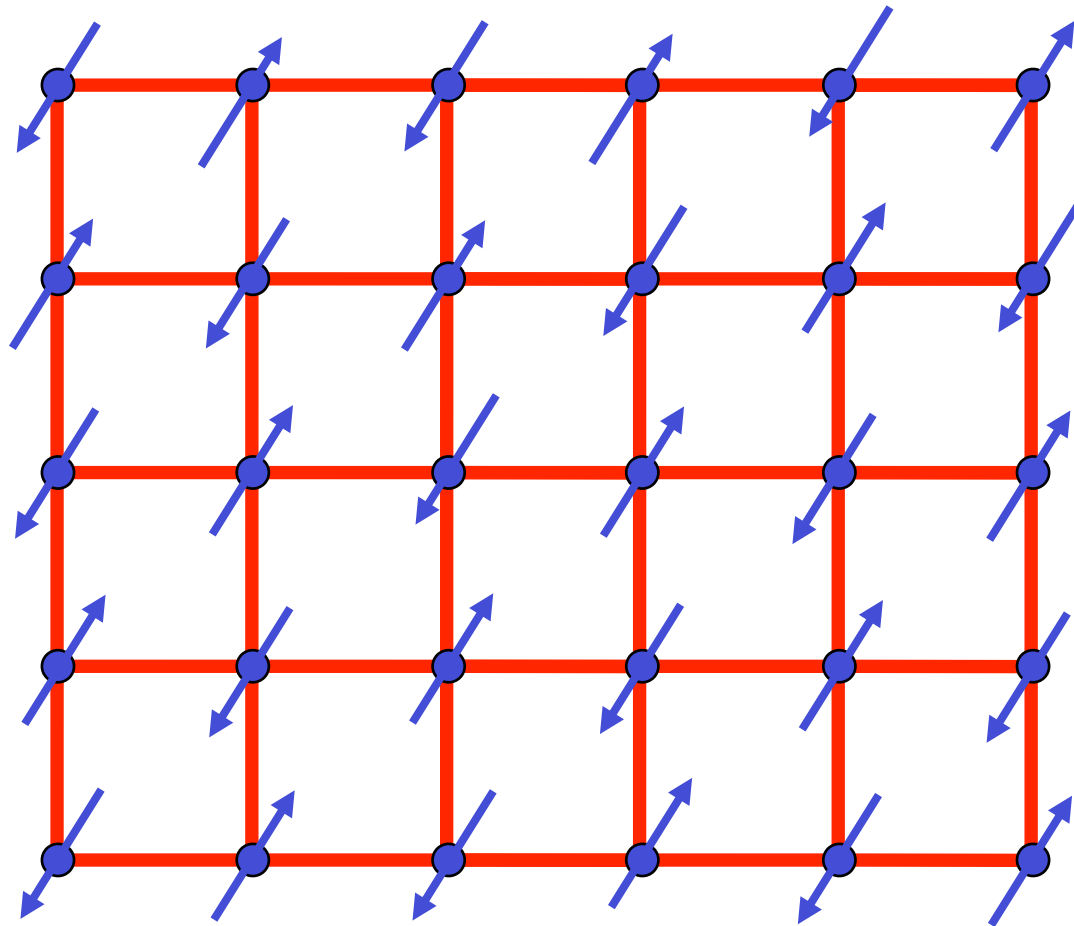
$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u(\vec{\varphi}^2)^2$$



S. Sachdev, arXiv:0901.4103



$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$



CFT3

$O(3)$ order parameter $\vec{\varphi}$

$$\mathcal{S} = \int d^2 r d\tau \left[(\partial_\tau \varphi)^2 + c^2 (\nabla_r \vec{\varphi})^2 + s \vec{\varphi}^2 + u (\vec{\varphi}^2)^2 \right]$$

Quantum Monte Carlo - critical exponents

Table IV: Fit results for the critical exponents ν , β/ν , and η . We summarize results including a variation of the critical point within its error bar. For the ladder model (top group of values) fit results and quality of fits are also given at the previous best estimate of α_c . The bottom group are results for the plaquette model. Numbers in [...] brackets denote the $\chi^2/\text{d.o.f.}$ For comparison relevant reference values for the 3D $O(3)$ universality class are given in the last line.

α_c	ν^a	β/ν^b	η^c
1.9096 $-\sigma$	0.712(4) [1.8]	0.516(2) [0.5]	0.026(2) [0.2]
1.9096	0.711(4) [1.8]	0.518(2) [1.1]	0.029(5) [0.8]
1.9096 $+\sigma$	0.710(4) [1.8]	0.519(3) [2.5]	0.032(7) [1.4]
1.9107 ^d	0.709(3) [1.7]	0.525(8) [15.3]	0.051(10) [12]
1.8230 $-\sigma$	0.708(4) [0.99]	0.515(2) [0.84]	0.025(4) [0.15]
1.8230	0.706(4) [1.04]	0.516(2) [0.40]	0.028(3) [0.31]
1.8230 $+\sigma$	0.706(4) [1.10]	0.517(2) [1.6]	0.031(5) [0.80]
Ref. 49	0.7112(5)	0.518(1)	0.0375(5)

^a $L > 12$.

^b $L > 16$.

^c $L > 20$.

^dPrevious best estimate of Ref. 19.

S. Wenzel and W. Janke, *Phys. Rev. B* **79**, 014410 (2009)

M. Troyer, M. Imada, and K. Ueda, *J. Phys. Soc. Japan* (1997)

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Field-theoretic
RG of CFT3
E.Vicari *et al.*

S.Wenzel and W.Janke, *Phys. Rev. B* **79**, 014410 (2009)

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