Lecture 3.
Granular superconductors and Josephson Junction arrays

Plan of the Lecture

1). Superconductivity in a single grain

2) Granular superconductors: experiments

3) Theories of SIT. Which parameter drives the S-I transition ?

4) BKT transitions in 2D JJ arrays

5) Quantum transitions in 2D JJ arrays with magnetic field: intermediate “Bose metal” state

Reviews: I.Beloborodov *et al*, Rev. Mod.Phys.**79**, 469 (2007)
R.Fazio and H. van der Zant, Phys. Rep. **355,** 235 (2001)
V.Gantmakher and V.Dolgopolov, Russian Physics-USPEKHI (2009)

1) Superconductivity in a single grain

- What is the critical size of the grain a_c?
- What happens if $a < a_c$?
- Assuming $\xi_o >> a >> a_c$, what is the critical magnetic field ?

Critical grain size

Mean-field theory gap equation:

$$\Delta = (g/2)\ \Sigma_i\ \Delta/[\varepsilon_i^2 + \Delta^2]^{1/2}$$

$$\delta \sum_{\xi_k} \rightarrow \int_{-\infty}^{\infty} d\xi,$$

Level spacing $\delta << \Delta$ allows to replace sum by the integral and get back usual BCS equation

Grain radius $a >> a_c = (1/\ \Delta\ v\ )^{1/3}$

Ultra-small grains $a \ll a_c$

- No off-diagonal correlations
- Parity effect

------- E_F

--↑↓--

--↑↓--

K. Matveev and A. Larkin PRL **78**, 3749 (1997)

Perturbation theory w.r.t. Cooper attraction:

$$\Delta_P = \frac{1}{2}\lambda\delta$$

Take into account higher-order terms (virtual transitions to higher levels):

$$\lambda_R = \lambda/(1-\lambda\log(\epsilon_0/\delta)). \quad \Rightarrow \quad \Delta_P = \frac{\delta}{2\ln\frac{\delta}{\Delta}}$$

Critical magnetic field for small grain

$a_c << R << \xi_o$

A.Larkin 1965

Orbital critical field for the grain $$H_c^{\rm gr} = \frac{c}{e}\sqrt{\frac{5\Delta_0}{2R^2 D}},$$

Local transition temperature T_c is determined by equation:

$$\ln(T_c/T_{c0}) = \psi(1/2) - \psi(1/2 + \alpha/2\pi T_c).$$

Which follows from $$\Delta(\mathbf{r}) = \lambda_0 \pi T \sum_\omega f_\omega(\mathbf{r}), \qquad \Big(|\omega| + D(-i\nabla - 2e\mathbf{A}/c)^2/2\Big) f_\omega(\mathbf{r}) = \Delta(\mathbf{r}),$$

Deparing parameter (orbital) $$\alpha = R^2 D (eH/c)^2/5.$$

Zeeman term alone leads to $$H_c^z = \Delta_0/\sqrt{2}\mu_B$$

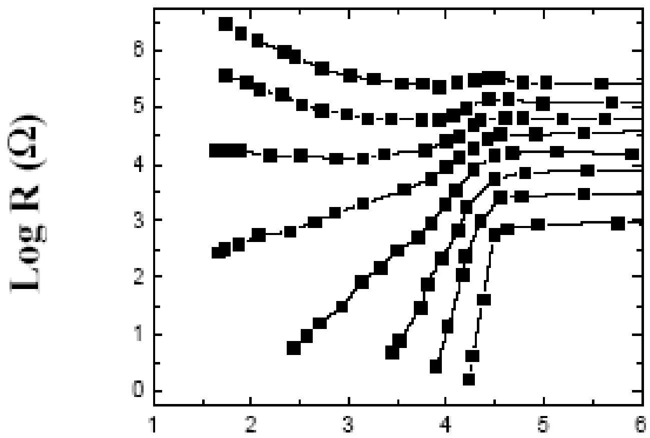
Orbital deparing prevails at $R > R_{o\text{-}z} \sim \sqrt{\frac{5}{D\Delta_0 m^2}} \sim \frac{1}{p_0}\sqrt{\frac{E_{Th}}{\Delta_0}}. >> a_c$

2) Granular superconductors: experiments

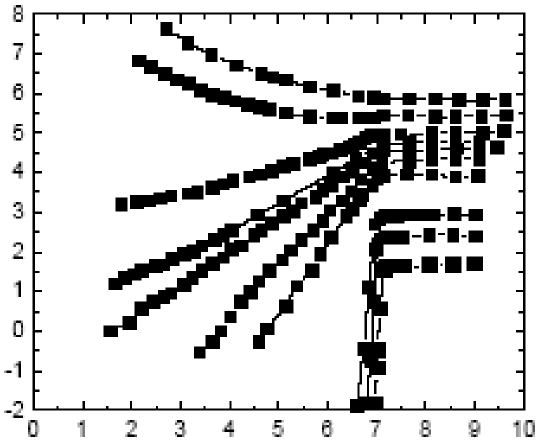
- Very thin granular films
- 3D granular materials
- E-beam - produced regular JJ arrays

Thin quenched-condensed films

A.Frydman, O Naaman, R.Dynes 2002

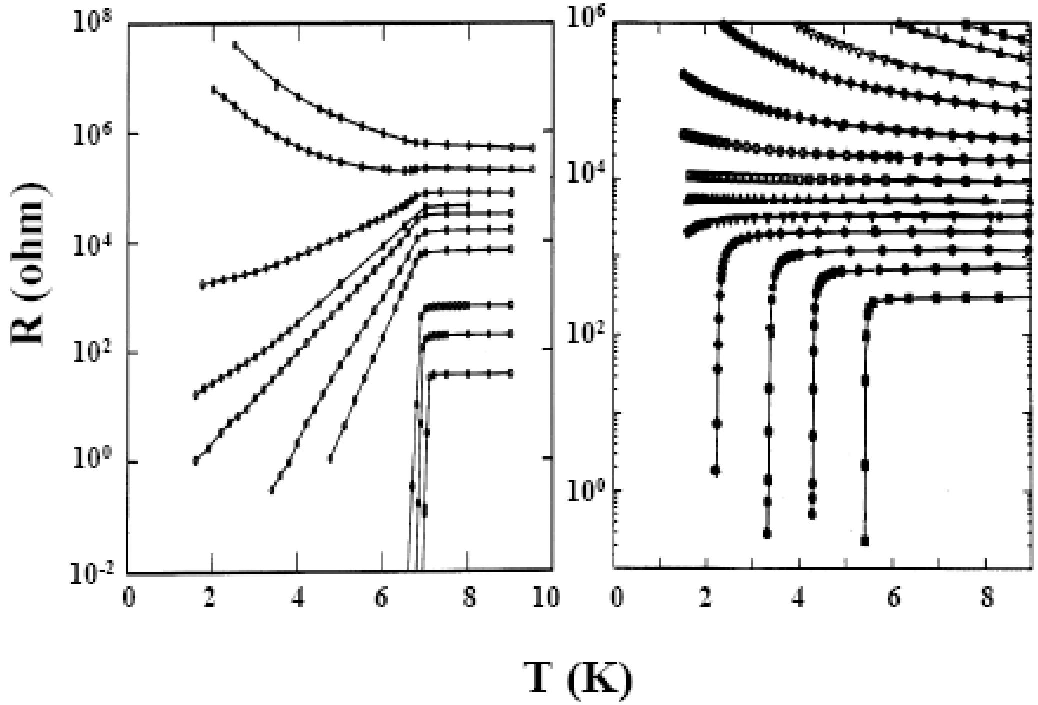


Sn grains



Pb grains

Granular v/s Amorphous films



A.Frydman
Physica C
391, 189 (2003)

FIG. 1. Resistance versus temperature for sequential layers of quench-condensed granular Pb (left) and uniform Pb evaporated on a thin Ge layer (right). Different curves correspond to different nominal thickness.

Onset of superconductivity in ultrathin granular metal films Phys Rev B **40** 182 (1989)

H. M. Jaeger,* D. B. Haviland, B. G. Orr,† and A. M. Goldman

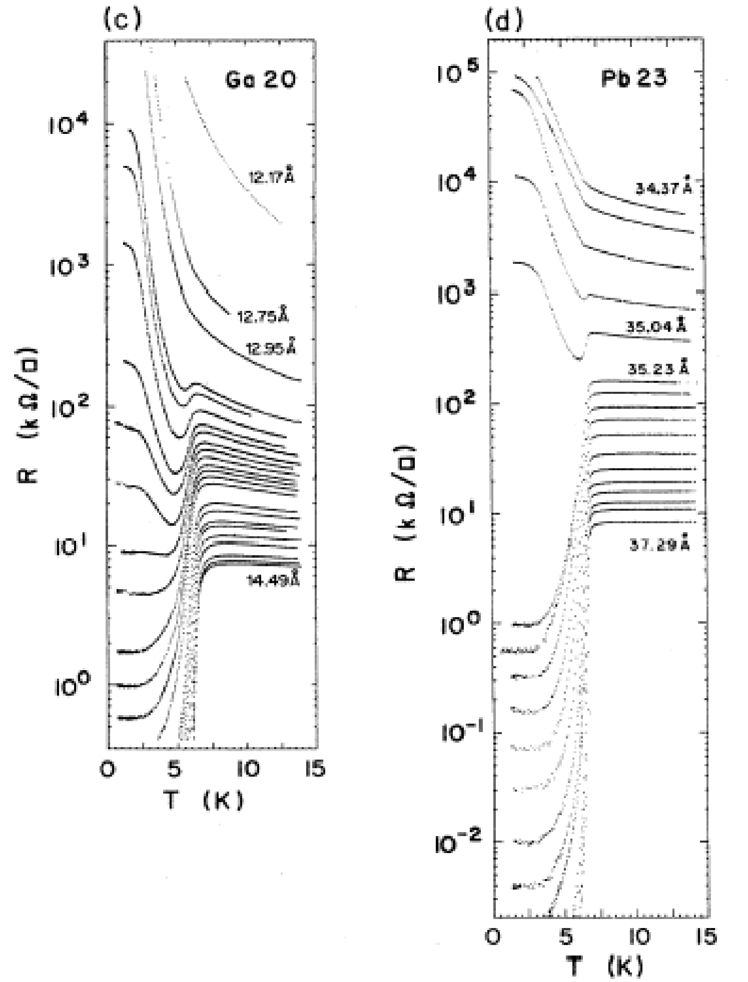


FIG. 2. The evolution of $R(T)$ curves for (a) Al, (b) In, (c) Ga, and (d) Pb, obtained *in situ* after successive increments of film thickness. Note that in all cases the entire evolution from insulating to globally superconducting spans an interval of nominal thickness of less than one monolayer.

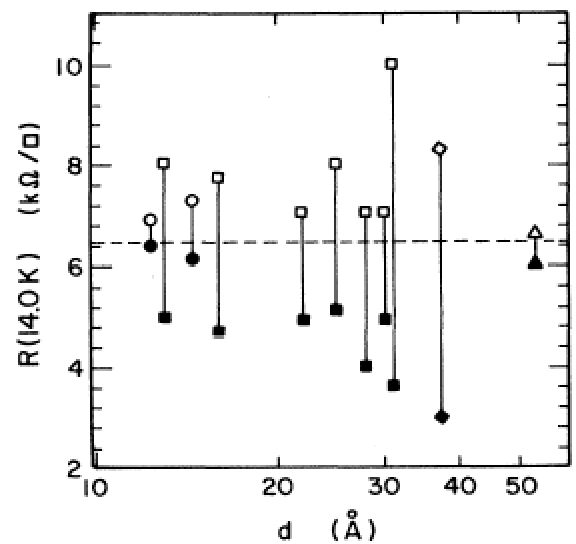


FIG. 6. The pairs of connected points represent the normal-state resistance of successive depositions in sequences of Ga (○), Sn (□), Pb (◇), and Al (△) films. The nominal thickness

Conclusion in this paper: control parameter is the normal resistance R. Its critical value is $R_Q = h/4e^2 = 6.5$ kOhm

Bulk granular superconductors

PHYSICAL REVIEW B VOLUME 27, NUMBER 7 1 APRIL 1983

Semiconductor-superconductor transition in granular Al-Ge

Y. Shapira and G. Deutscher

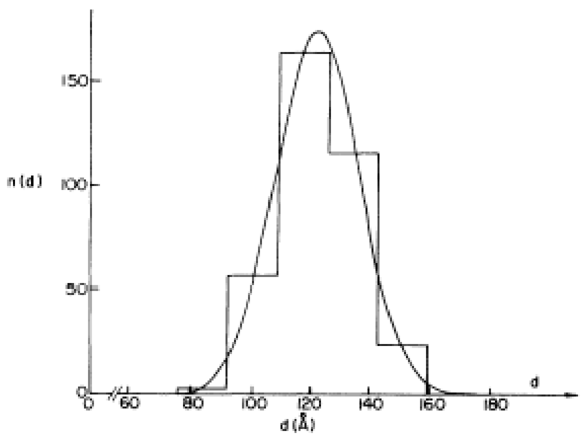


FIG. 2. Histogram of a distribution of grain size including a best fit to a normal distribution of the histogram. The data were taken from a micrograph of sample *B* (see Table I).

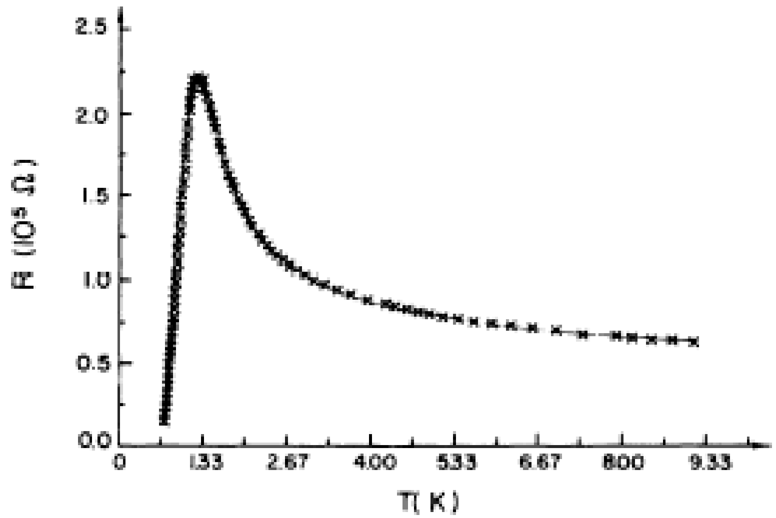


FIG. 4. Resistance *R* as a function of temperature for sample *E*.

Sample thickness 200 nm

Bulk granular superconductors

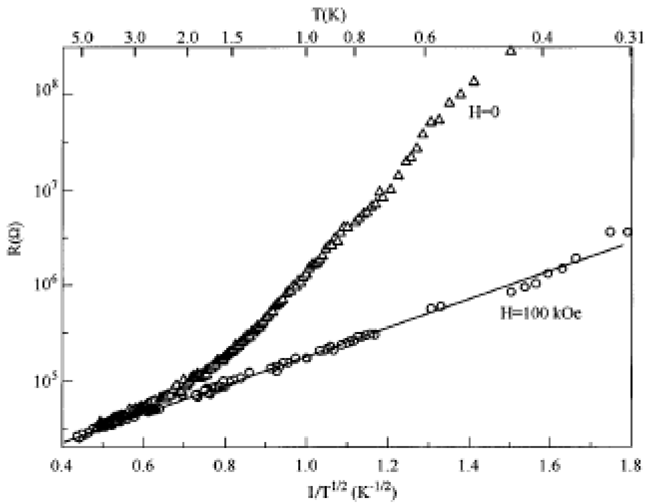


FIG. 1. Resistance of sample 1 measured at zero (triangles) and 100 kOe field (circles) as a function of the inverse square root of the temperature. Sample 1 room temperature resistance is $2 \times 10^3\ \Omega$.

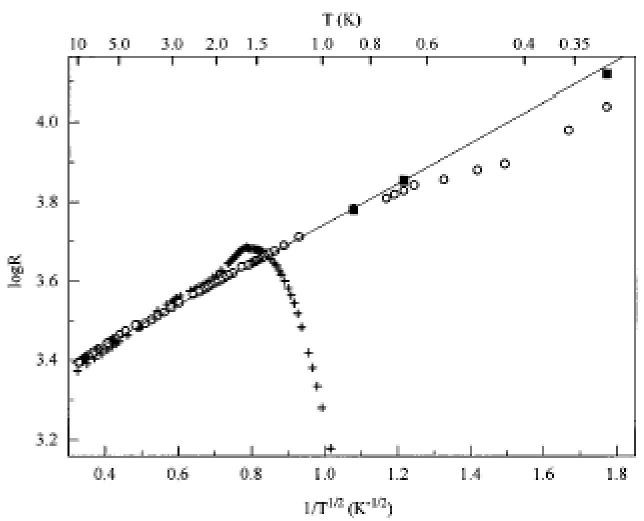


FIG. 2. Resistance of sample 2 measured at zero (crosses) and 100 kOe field (open circles) as a function of the inverse square root of the temperature. Open circles indicate resistance

Insulator-Superconductor Transition in 3D Granular Al-Ge Films

A. Gerber, A. Milner, G. Deutscher, M. Karpovsky, and A. Gladkikh

PHYSICAL REVIEW LETTERS VOLUME 78, NUMBER 22

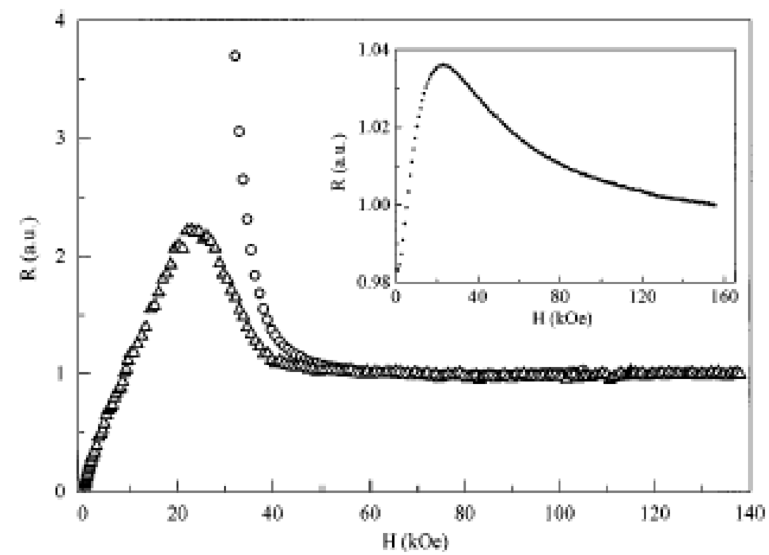


FIG. 4. Normalized resistances of sample 1 (open circles) and sample 2 (triangles) as a function of applied magnetic field measured at $T = 0.3$ K. Inset: magnetoresistance of sample 3 at $T = 2$ K normalized at $H = 160$ kOe.

Artificial regular JJ arrays

PHYSICAL REVIEW B VOLUME 50, NUMBER 6 1 AUGUST 1994-II

Charge solitons and quantum fluctuations in two-dimensional arrays of small Josephson junctions

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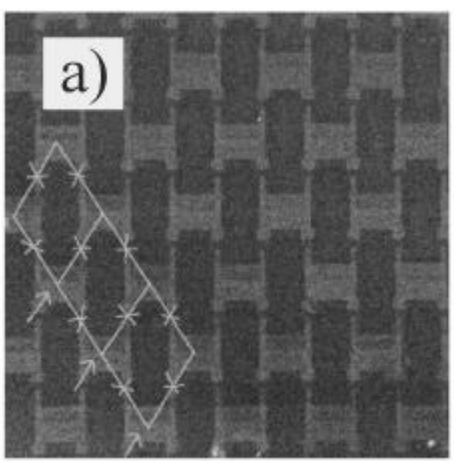
Department of Physics, Chalmers University of Technology and University of Göteborg, S-412 96 Göteborg, Sweden

PHYSICAL REVIEW B VOLUME 54, NUMBER 14 1 OCTOBER 1996-II

Quantum phase transitions in two dimensions: Experiments in Josephson-junction arrays

H. S. J. van der Zant, W. J. Elion, L. J. Geerligs, and J. E. Mooij
Department of Applied Physics and Delft Institute of Microelectronics and Submicron-technology (DIMES),



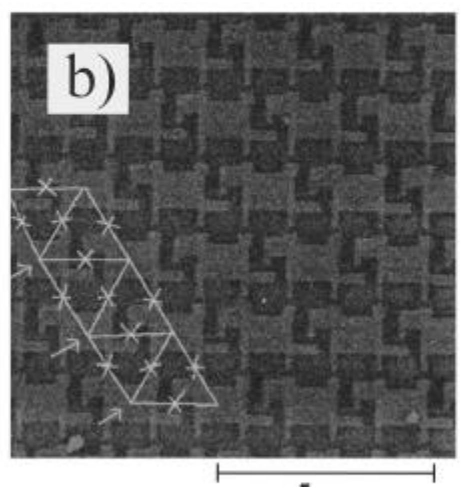


TABLE I. Sample parameters for our square (S) and triangular (T) arrays.

Sample	R_n (kΩ)	C (fF)	$\beta_c(T=0)$	E_J/k_B (K)	E_C/E_J	$\tau_V(f=0)$
S1	36.0	1.1	96	0.21	4.55	
S2	15.3	1.1	17.4	0.50	1.82	
S3	14.5	1.1	15.6	0.53	1.67	(0.28)
S4	11.5	1.1	12.4	0.66	1.25	0.4
S5	10.5	1.1	11.3	0.73	1.11	0.7
S6	5.0	1.1	5.4	1.5	0.56	0.83
S7	8.0	2.0	15.6	0.96	0.48	0.85
S8	6.8	1.7	11.3	1.1	0.45	0.88
S9	2.5	1.1	2.7	3.1	0.27	0.90
S10	3.3	3.5	11.3	2.3	0.14	
S11	1.14	1.1	1.2	6.7	0.13	0.95
T1	25.7	1.2	29	0.30	2.6	1.15
T2	23.8	1.7	39	0.32	1.7	1.6
T3	8.3	1.1	8.7	0.92	0.9	1.51
T4	4.7	1.1	7.2	1.6	0.35	1.85

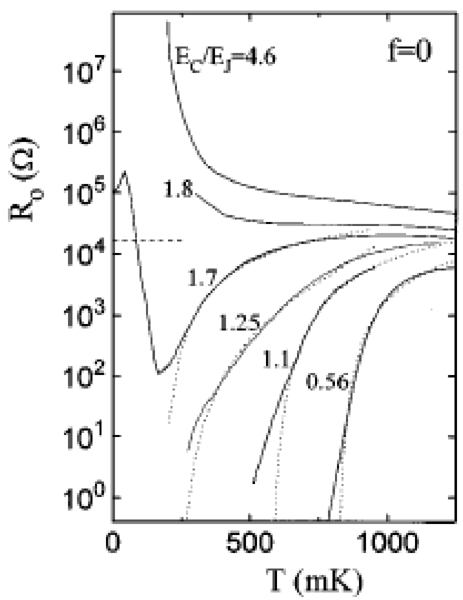


FIG. 5. The zero-field linear resistance per junction measured as a function of temperature for six different arrays. Dotted lines are fits to the vortex-KTB square-root cusp formula. The dashed horizontal line shows the zero-temperature universal resistance ($8R_q/\pi$=16.4 kΩ) of the S–I transition at f=0.

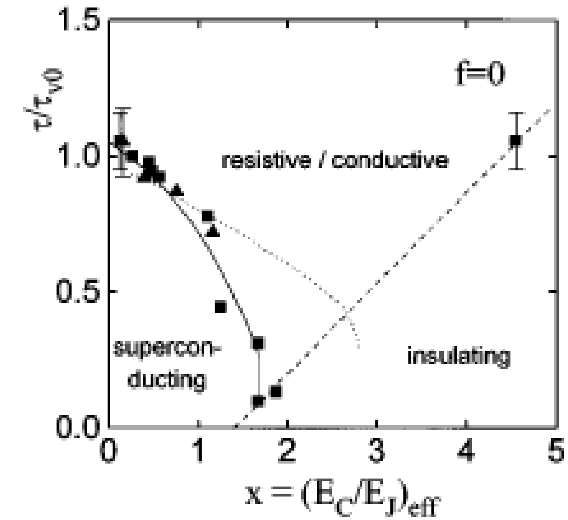


FIG. 6. Measured phase diagram of our square arrays (solid squares) and triangular (solid triangles) in zero magnetic field, showing the superconductor-to-insulator transition at $(E_C/E_J)_{eff}\approx1.7$. The solid line is a guide to the eye connecting the data points and the dotted line at the superconducting side is the result of a recent calculation (Ref. 39).

What is the parameter that controls SIT in granular superconductors ?

- Ratio E_C/E_J ?

- Dimensionless conductance
  $g = (h/4e^2)\, R^{-1}$? (for 2D case)

Note that in Lec.2 we used another definition: $g_T = (h/e^2)\, R$

3) Theoretical approaches to SIT

- K.Efetov ZhETF **78**, 2017 (1980) [Sov.Phys.-JETP **52**, 568 (1980)]

  Hamiltonian for charge-phase variables

- M.P.A.Fisher, Phys.Rev.Lett. **65**, 923 (1990)

  General “duality” *Cooper pairs – Vortices* in 2D

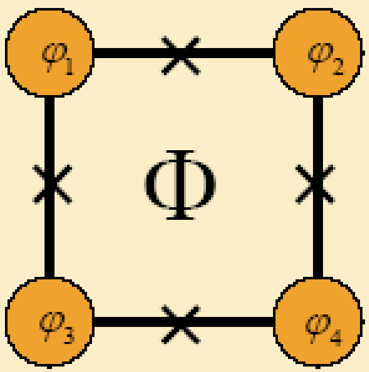
- R.Fazio and G.Schön, Phys. Rev. B**43,** 5307 (1991)

  Effective action for 2D arrays

K.Efetov's microscopic Hamiltonian

JOSEPHSON ARRAYS

Elementary building block



Ideal Hamiltonian:

$$H = \frac{1}{2}\sum_{i,j} C_{ij}^{-1} q_i q_j + E_J \cos(\varphi_i - \varphi_j - 2\pi \frac{\Phi_{ij}}{\Phi_0}) \quad q_i = 2e\, i \frac{d}{d\varphi_i}$$

C_{ij} - capacitance matrix E_J - Josephson energy

Control parameter

$x = E_C / E_J$

$E_c = e^2/2C$

Artificial arrays: major term in capacitance matrix is n-n capacitance C

q_i and φ_i are canonically conjugated

Logarithmic Coulomb interaction

Artificial arrays with dominating capacitance of junctions: $C/C_0 > 100$

DELSING, CHEN, HAVILAND, HARADA, AND CLAESON

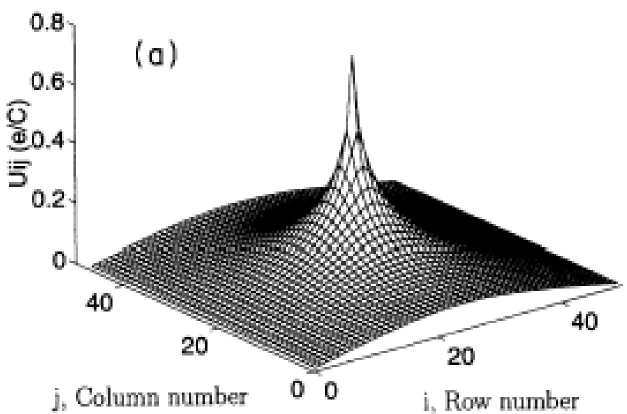


FIG. 1. Exact solution for the potential distribution $U(i,j)$ in a 49×47 junction array, where the ratio between the junction capacitance and the self-capacitance of each electrode is 400. The current leads are connected a $i=1$ and 49. (a) A single electron in the center of the array giving rise to a single-electron

Coulomb interaction of elementary charges

$$U(R) = 2E_C\left(\frac{1}{2\pi}\ln R_{ij} + \frac{1}{4}\right)$$

$$E_C = \frac{e^2}{2C}$$

For Cooper pairs, x by factor 4

M.P.A.Fisher's duality arguments

- Competition between Coulomb repulsion and Cooper pair hopping:

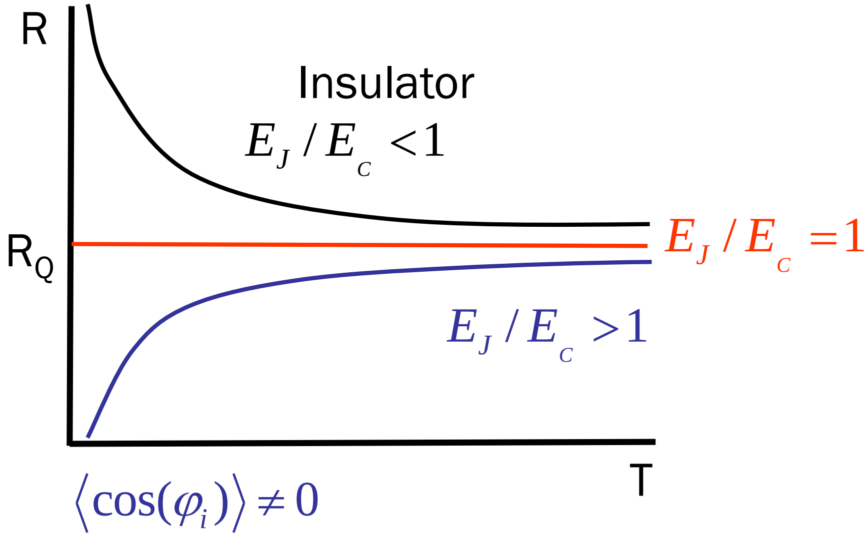
  Duality charge-vortex: both charge-charge and vortex-vortex interaction are Log(R) in 2D.

  Vortex motion generates voltage: $V=\varphi_o j_V$

  Charge motion generates current: $I=2e\, j_c$

  At the self-dual point the currents are equal → $R_Q=V/I=h/(2e)^2=6.5k\Omega$.

Insulator is a superfluid of vortices



In favor of this idea:
usually SIT in films occurs at R near R_Q

Problems: i) how to derive that duality ?
ii) What about capacitance matrix in granular films ?
iii) Critical R(T) is not flat usually

Can we reconcile Efetov's theory and result of "duality approach" ?

We need to account for capacitance renormalization to due to virtual electron tunneling via AES [PRB **30**, 6419 (1984)] action functional

$$S[\varphi] = S_c[\varphi] + S_{ts}[\varphi] + S_J[\varphi]$$

Virtual tunneling

$$\alpha_s(\tau) = \frac{1}{\pi^2}\Delta^2 K_1^2(\Delta|\tau|),$$

Charging

$$S_c = \frac{1}{2e^2}\sum_{ij}\int_0^\beta d\tau C_{ij}\frac{d\varphi_i(\tau)}{d\tau}\frac{d\varphi_j(\tau)}{d\tau},$$

$$S_{ts} = g\sum_{\langle ij\rangle}\int_0^\beta d\tau d\tau' \alpha(\tau-\tau')\sin^2\left[\frac{\varphi_{ij}(\tau)-\varphi_{ij}(\tau')}{2}\right]$$

↓

$$S_{ts} = (3g/32\,\Delta)\sum_{<ij>}\int_0^\beta \dot{\varphi}_{ij}^2(\tau)d\tau,$$

Josephson (if dφ/dt << Δ)

$$S_J = -\frac{1}{2}\sum_{\langle ij\rangle}\int_0^\beta J_{ij}\cos\left(2\left[\varphi_i(\tau)-\varphi_j(\tau)\right]\right)d\tau$$

$$C_{ind} = (3/16)\, ge^2/\Delta$$

Mean-field estimate with renormalized action

$$\sum_{\langle i,j\rangle} J_{ij} \cos[2(\varphi_i - \varphi_j)] \to J z \langle \cos 2\varphi \rangle_{MF} \sum_j \cos 2\varphi_j .$$

SC transition at $1 = \frac{zJ}{2} \int_0^\beta \Pi_s(\tau)\, d\tau, \qquad \Pi_s(\tau) = \langle \exp(2i[\varphi(\tau) - \varphi(0)]) \rangle ,$

T=0: $zJ = 8E_0(g) .$ $J = g\Delta/2$

Strong renormalization of C: $E_0(g) = c\Delta/g,$

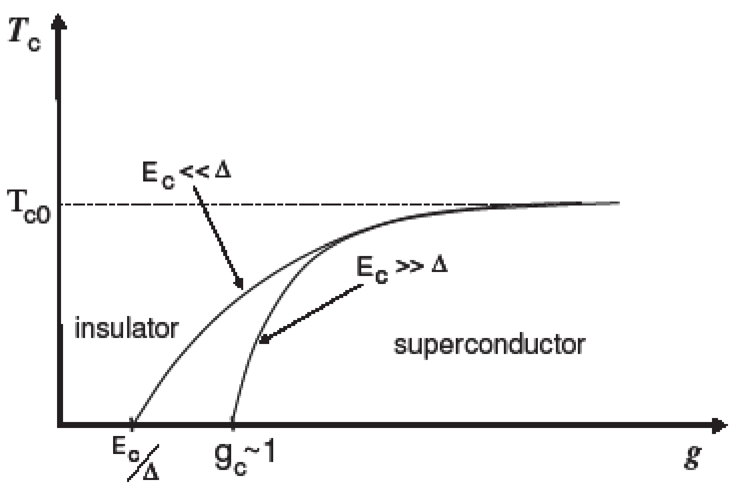


FIG. 17 The sketch of the phase diagram of a granular superconductor in coordinates critical temperature vs. tunnelling conductance for two cases $E_c \ll \Delta_0$ and $E_c \gg \Delta_0$. At large tunnelling conductances $g \gg 1$ the Coulomb interaction is screened due to electron intergrain tunnelling and the transition temperature is approximately given by the single grain BCS value. In the opposite case $g \ll 1$ the boundary between the insulating and superconducting states is obtained comparing the Josephson $E_J = g\Delta$ and Coulomb E_c energies.

Can one disentangle "g" and "E_C/E_J" effects ?

Superconductor-insulator duality for the array of Josephson wires

I. V. Protopopov and M. V. Feigel'man   JETP Lett. **85**(10), 513 (2007)

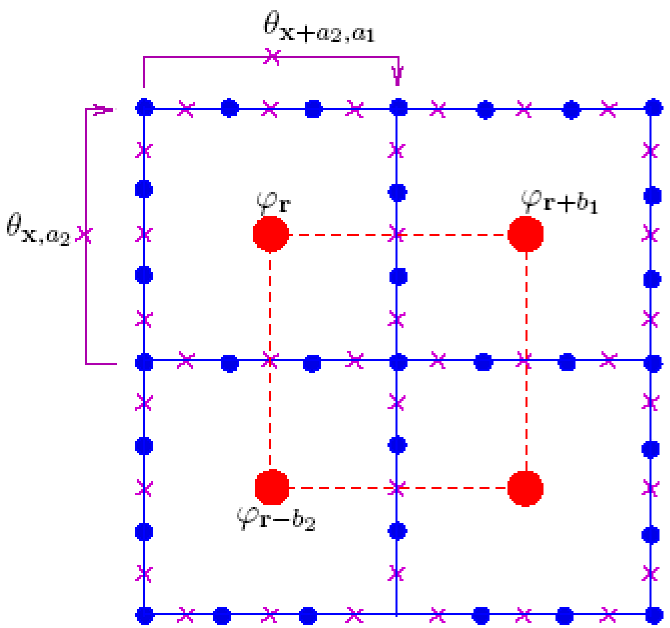


Figure 1: Fig. 1. The array of Josephson wires. Small circles represent the superconducting islands connected by Josephson junctions (crosses). The phase differences $\theta_{\mathbf{x},a_\mu}$ are defined on the bonds of the array. The large circles denote the vertices of the dual lattice.

This model allows exact duality transformation

Control parameter $q = \tilde{E}_J/\tilde{E}_C = 4N^2 v/\pi^2 E_J.$

$$v = \frac{2^{11/4}}{\sqrt{\pi}} (E_J^3 E_C)^{1/4} \exp\left[-2\sqrt{\frac{2E_J}{E_C}}\right]$$

Experimentally, it allows study of SIT in a broad range of g and/or E_J/E_C

4) Charge BKT transition in 2D JJ arrays

Logarithmic interaction of Cooper pairs 2e

$$U(R) = 8 E_C \left(\frac{1}{2\pi} \ln R_{ij} + \frac{1}{4} \right)$$

R.Fazio and G.Schön,
Phys. Rev. B**43**, 5307 (1991)

Temperature of BKT transition is $T_2 = E_C/\pi$ Not observed !

The reason: usually T_2 is above parity temperature $T^* \approx \Delta / \ln \mathcal{M} \ll \Delta$

$$\mathcal{M} = V\nu(0)\sqrt{8\pi T \Delta} \sim 10^4 - 10^5$$

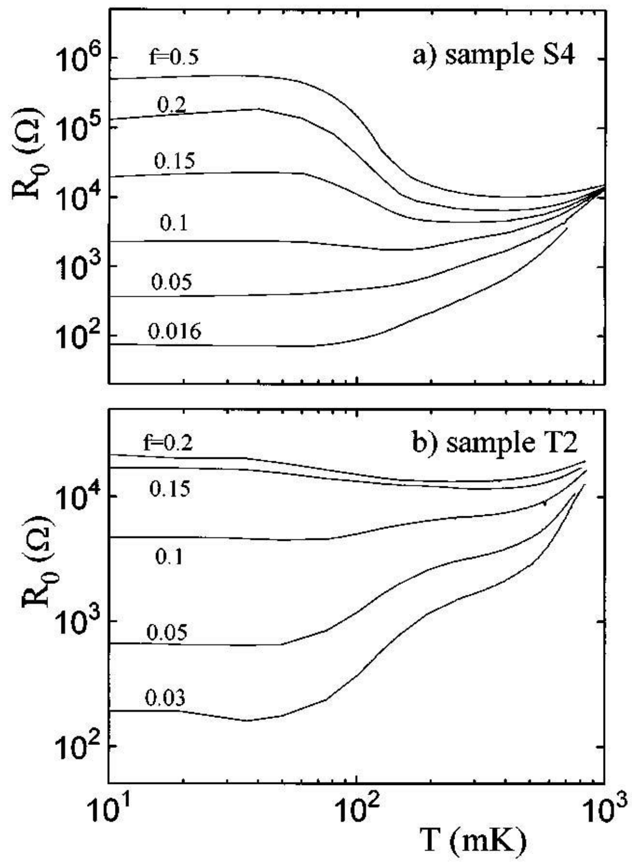
Interaction of pairs is screened by quasiparticles

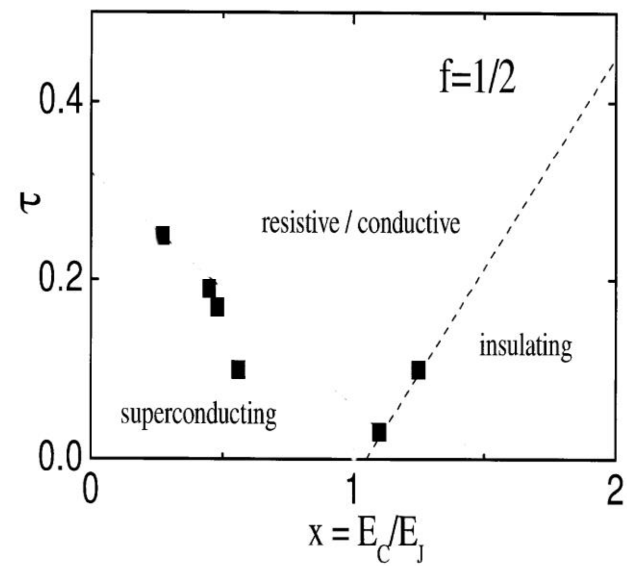
Charge BKT is at $T1 = E_C/4\pi$ (unless T^* is above T_2)

M. V. Feigel'man, S. E. Korshunov and A. V. Pugachev,
Pis'ma ZhETF **65**, 541 (1997)

5) "Bose metal" in JJ array ?

van der ZANT, ELION, GEERLIGS, AND MOOIJ





At non-zero field simple Josephson arrays show temperature-independent resistance with values that change by orders of magnitude.

Dice array (E.Serret and B.Pannetier 2002; E.Serret thesis, CNRS-Grenoble)

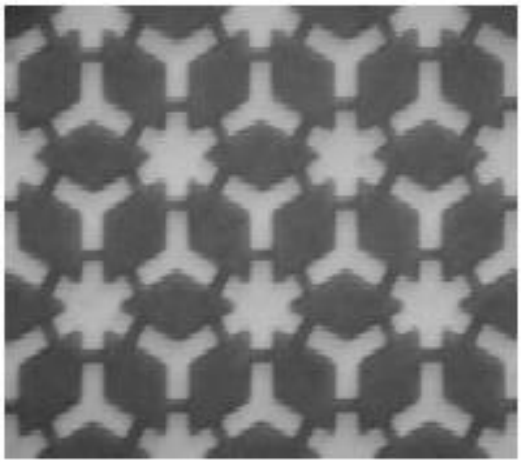
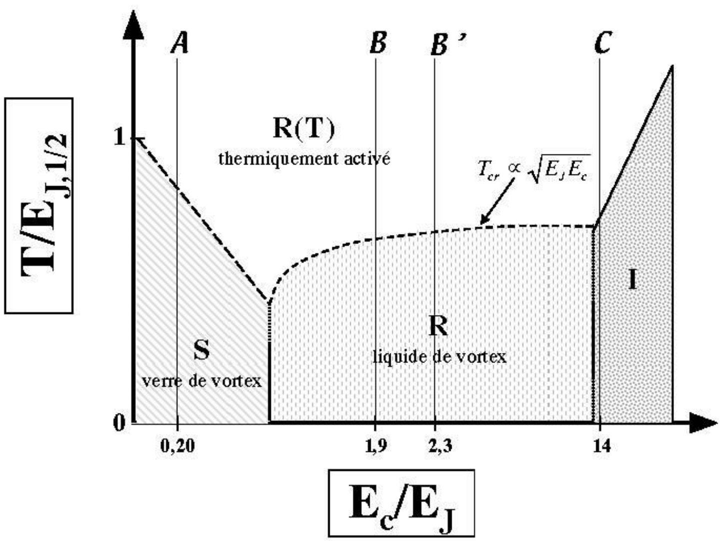


Foto from arxiv:0811.4675

In-Cheol Baek, Young-Je Yun, and Mu-Yong Choi



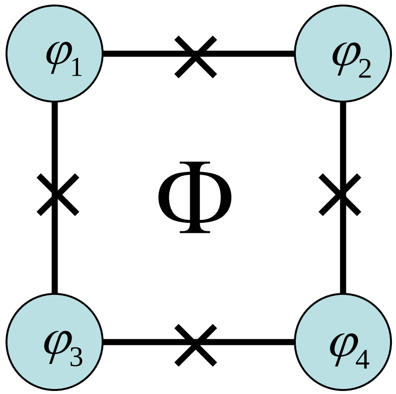
At non-zero field Josephson arrays of more complex (dice) geometry show temperature independent resistance in a wide range of E_J/E_c

The origin of “Bose metal” is unknown

Hypothesis:
it might be related to charge offset noise

JOSEPHSON ARRAYS

Elementary building block

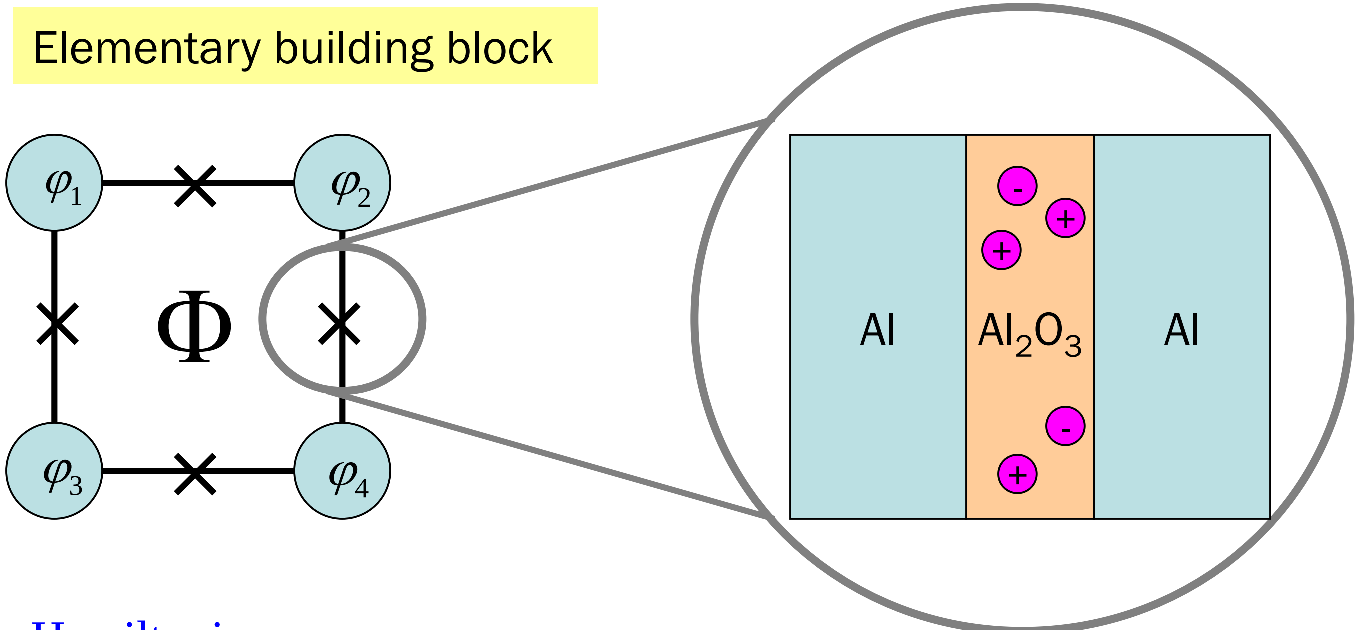


Ideal Hamiltonian:

$$H = \frac{1}{2}\sum_{i,j} C_{ij}^{-1} q_i q_j + E_J \cos(\varphi_i - \varphi_j - 2\pi \frac{\Phi_{ij}}{\Phi_0}) \quad q_i = 2e\, i \frac{d}{d\varphi_i}$$

C_{ij} - capacitance matrix E_J - Josephson energy

JOSEPHSON ARRAYS



More realistic Hamiltonian:

$$H = \frac{1}{2}\sum_{i,j} C_{ij}^{-1}(q_i + Q_i)(q_j + Q_j) + (E_J + \delta E_J)\cos(\varphi_i - \varphi_j - 2\pi\frac{\Phi_{ij} + \delta\Phi}{\Phi_0}) \quad q_i = 2e\, i\frac{d}{d\varphi_i}$$

C_{ij} - capacitance matrix E_J - Josephson energy

$Q_i = Q_i^0 + Q_i(t)$ - induced charge (static and fluctuating)

$\delta\Phi = \delta\Phi^0 + \delta\Phi(t)$ - static flux due to area scatter and flux noise

$\delta E_J = \delta E_J{}^0 + \delta E_J(t)$ - static scatter of Josephson energies and their time dependent fluctuations.