Lecture 2.
Granular metals

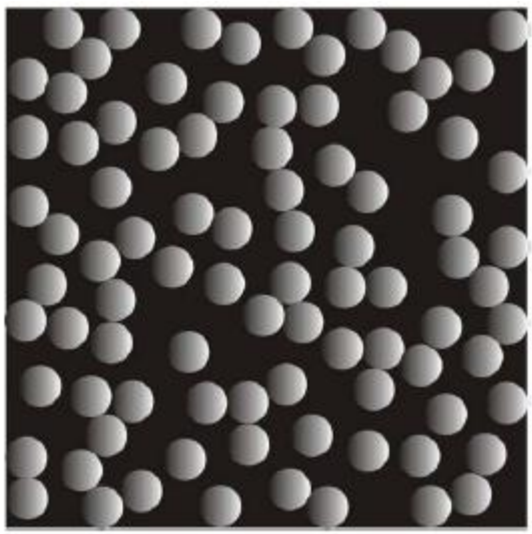
Plan of the Lecture

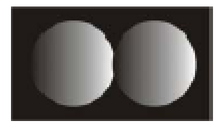
1) Basic energy scales
2) Basic experimental data
3) Metallic behaviour: logarithmic R(T)
4) Insulating behaviour: co-tunneling and ES law
5) Co-tunneling and conductance of diffusive wire

Review: I.Beloborodov *et al*, Rev. Mod.Phys.**79**, 469 (2007)

Metal grains in an insulating matrix

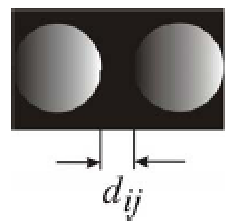
Grain radius $a >> \lambda_F$





Direct contact of metal grains.

Dimensionless conductance $g \approx \text{const}$



Intergrain tunneling through an insulating gap.

--thickness of insulating layer

Dimensionless conductance:

$$g_{ij} \propto \exp\{-2k_0 d_{ij}\}$$

Strong inhomogeneous fluctuations of g_{ij} due to exponential dependence on d_{ij}

Examples:

Conduction in granular aluminum near the metal-insulator transition

T. Chui, G. Deutscher,* P. Lindenfeld, and W. L. McLean

Phys Rev B 1981

Transport measurements in granular niobium nitride cermet films

R. W. Simon,* B. J. Dalrymple,* D. Van Vechten, W. W. Fuller, and S. A. Wolf

Phys Rev B 1987

Basic energy scales

Grain radius a is large on atomic scale: $k_F\, a >> 1$

Coulomb energy $E_c = e^2/a$

Level spacing inside grain $\delta = 1/(4a^3 \nu)$

Intra-grain Thouless energy $E_{th} = \hbar D_o/4a^2$

$$\delta \ << \ E_{th} \ << E_c$$

$g_o = E_{th}/\ \delta = 2500$

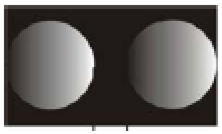
<u>Example</u>: Al grains with $a{=}20$ nm

$\delta = 0.02$ K $\qquad E_{th} = 50$ K $\qquad E_c = 1000$ K

Typical temperature range 4 K < T < 300 K

$\Gamma \sim 1$K for $g_T = 50$

Intergrain coupling



Intergrain tunneling through an insulating gap

Low transmission, but large number of transmission modes

$$\sigma_T = (4\pi e^2/\hbar)\,\nu^2 <|t_{pk}|^2>$$

$g_T = \sigma_T\, h/e^2$ - dimensionless inter-grain conductance

A) Granular metal $g_T \geq 1$

Effective diffusion constant

$D_{eff} = \Gamma a^2/\hbar << D_o$

Narrow coherent band $\Gamma = g_T \delta << E_{th}$

$\Gamma \sim 1K$ for $g_T = 50$

B) Granular insulator $g_T \leq 1$

Nearest-neighbors coupling only !

Experimental data: granular metals

Transport measurements in granular niobium nitride cermet films

R. W. Simon,* B. J. Dalrymple,* D. Van Vechten, W. W. Fuller, and S. A. Wolf

Phys Rev B 36 1964 (1987)

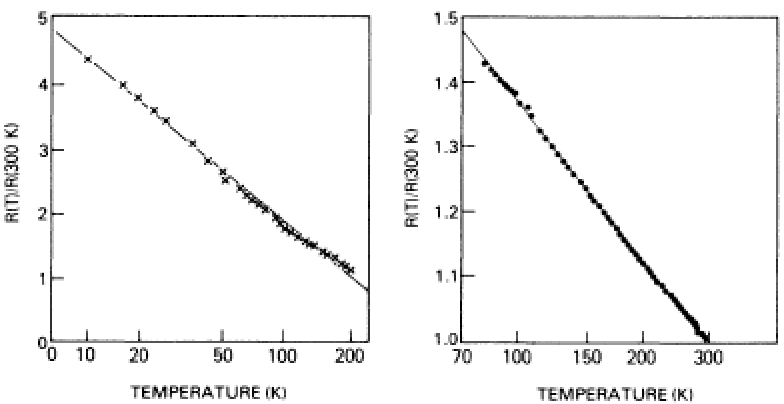


FIG. 5. The resistance normalized to the room-temperature value vs $\ln T$ for two different superconducting samples. The graph on the left is for a sample with $R_{\square}(300\ \mathrm{K})=2000\ \Omega/\square$, while that on the right is for $R_{\square}(300\ \mathrm{K})=100\ \Omega/\square$. The lines are guides to the eye.

Insulator-Superconductor Transition in 3D Granular Al-Ge Films

A. Gerber, A. Milner, G. Deutscher, M. Karpovsky, and A. Gladkikh

Phys Rev Lett.78, 4277 (1997)

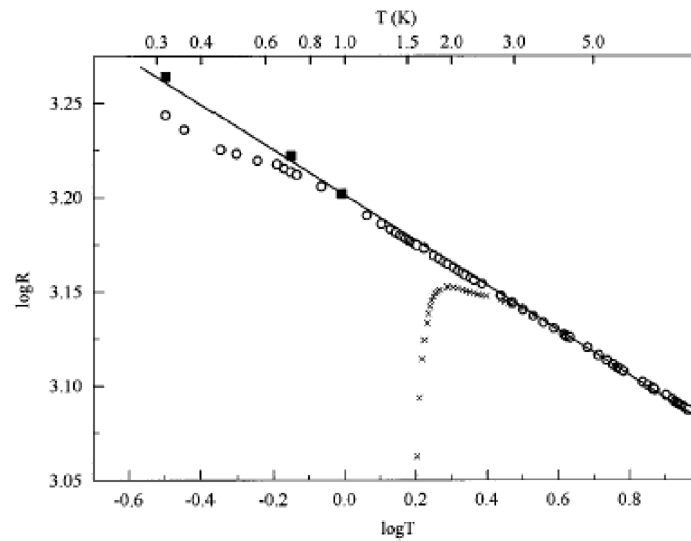


FIG. 3. Resistance of sample 3 as a function of temperature on a log-log scale, as measured at (zero) (×) and 100 kOe field (open circles). Open circles indicate resistance measured with a constant dc current $I = 10^{-5}$ A. Solid squares are zero bias resistances approximated from I-V measurements. Sample 3 room temperature resistance is 500 Ω.

$$R(T) = R_o \ln(T_o/T)$$

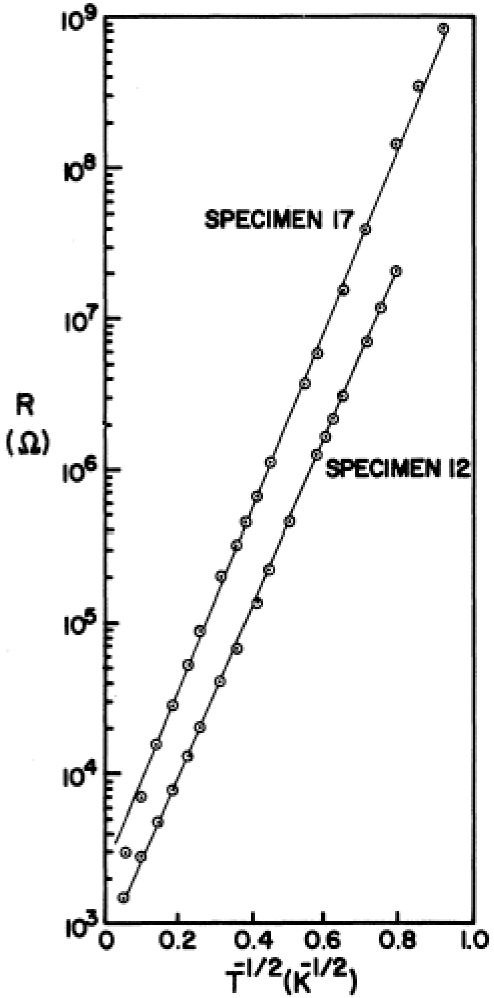
Coefficient in front of the Log is too large for interference corrections (Lecture 1)

Experimental data: granular insulators

Conduction in granular aluminum

T. Chui, G. Deutscher,* P. Lindenfeld, and W. L. McLean

Phys Rev B 23 6172 (1981)



Transport measurements in granular niobium nitride cermet films

R. W. Simon,* B. J. Dalrymple,* D. Van Vechten, W. W. Fuller, and S. A. Wolf

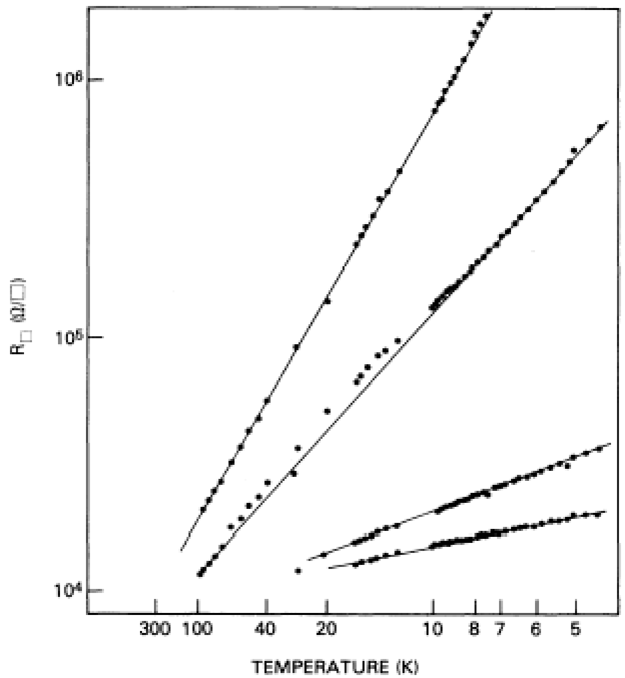


FIG. 4. The logarithm of the sheet resistance as a function of $1/\sqrt{T}$ for a variety of the nonsuperconducting samples. The lines are guides to the eye.

$$R = R_0 \exp(T_0/T)^{1/2}$$

Efros-Shklovsky

Variable-range hopping in presense of Coulomb gap

The origin of VRH ???

3) Metallic behaviour: theory of logarithmic R(T) dependence

Granular structure leads to a new (compared to usual metals) energy window $\Gamma \ll T \ll E_C$ where

- Coulomb energy is very important
- Coherent nature of transport is irrelevant

Useful formulation of the theory is in terms of phase variables

$\varphi_i(t) = (e/\hbar)\int^t V_i(t')dt'$ conjugated to grain charges Q_i

The reason: at $g_T >> 1$ phases are weakly fluctuating

Ambegaokar-Eckern-Schön functional

Phys Rev B **30**, 6419 (1984)

$$Z = \int \exp\left(-S\right) D\varphi, \qquad S = S_c + S_t$$

Action functional in imaginary (Matsubara) time

S_c describes the charging energy

$$S_c = \frac{1}{2e^2} \sum_{ij} \int_0^\beta d\tau C_{ij} \frac{d\varphi_i\left(\tau\right)}{d\tau} \frac{d\varphi_j\left(\tau\right)}{d\tau};$$

S_t stands for tunneling between the grains

$$S_t = (1/4)\, g \sum_{\langle ij \rangle} \int_0^\beta d\tau d\tau' \alpha\left(\tau - \tau'\right) \sin^2\left[\frac{\varphi_{ij}(\tau) - \varphi_{ij}(\tau')}{2}\right]$$

$$\alpha\left(\tau - \tau'\right) = T^2 \mathrm{Re} \sin^{-2}\left[\pi T(\tau - \tau' + i\eta)\right]$$

Perturbation theory

Quadratic approximation for the action: $$S_0 = T \sum_{\mathbf{q},n} \varphi_{\mathbf{q},n} G^{-1}_{\mathbf{q},n} \varphi_{-\mathbf{q},-n},$$

$$G^{-1} = (\omega_n^2/2e^2)\, C(\mathbf{q}) + (1/\pi)\, g_T |\omega_n|\, \beta(\mathbf{q})$$

$\omega_n = 2\pi n T$

$\beta(\mathbf{q}) = \Sigma_a (1 - \cos \mathbf{qa})$

$$\langle(\varphi_{ij}(0) - \varphi_{ij}(\tau))^2\rangle \sim (1/g_T)\, \Sigma_n (1-\cos \omega_n \tau)/[\, |\omega_n| + \omega_n^2/E_C g_T\,]$$

High-energy cutoff $E_C g_T = 1/(RC)$

Low-energy cutoff $1/\tau$

$$\langle(\varphi_{ij}(0) - \varphi_{ij}(\tau))^2\rangle \sim (1/g_T) \ln (E_C g_T \tau)$$

Intergain conductivity

Efetov, K. B., and A. Tschersich, 2003, Phys. Rev. B **67** 174205.

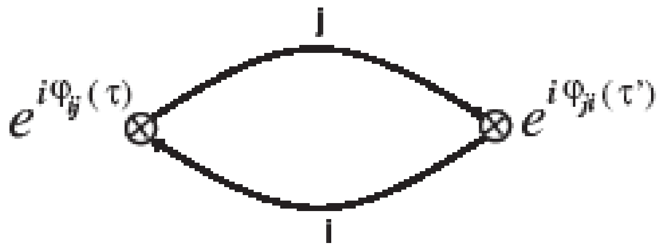


FIG. 9 This diagram represents the conductivity in the leading order in $1/g$. The crossed circles represent the tunnelling matrix elements $t_{kk'}^{ij}$, $e^{i\varphi_{ij}(\tau)}$ where phase factors appear from the gauge transformation.

$$\sigma(\omega) = \frac{2\pi e^2 g T^2 i a^{2-d}}{\omega} \int_0^\beta d\tau \, \frac{1 - e^{i\Omega\tau}}{\sin^2(\pi T \tau)} \exp\left(-\tilde{G}_{\mathbf{a}}(\tau)\right)$$

where the analytic continuation to the real frequency is assumed as $\Omega \to -i\omega$ and the function $\tilde{G}_{\mathbf{a}}(\tau)$ is

$$\tilde{G}_{\mathbf{a}}(\tau) = 4Ta^d \sum_{\omega_n > 0} \int \frac{d^d \mathbf{q}}{(2\pi)^d} G_{\mathbf{q}n} \sin^2 \frac{\mathbf{qa}}{2} \sin^2 \frac{\omega_n \tau}{2}.$$

1^{st} logarithmic correction:

$$g_T(T) = g_T - (4/z) \ln (g_T E_C/T)$$

here z is the lattice coordination number

The origin of this correction: discreteness of charge transfer. Formally, it is represented as non-linearity of the AES action in phase representation. Phase (voltage) fluctuations across each tunnel junction destroy coherence of electron states in neighboring grains, and suppress inter-grain conductivity.

Contrary to usual logarithmic corrections in 2D metals, this effect does not depend on dimensionality

Renormalization group

$$S_t = (1/4)\, g \sum_{\langle ij \rangle} \int_0^\beta d\tau d\tau' \alpha\left(\tau - \tau'\right) \sin^2\left[\frac{\varphi_{ij}(\tau) - \varphi_{ij}(\tau')}{2}\right]$$

Split phase variables into slow and fast parts

$$\phi_{ij\omega} = \phi^{(0)}_{ij\omega} + \overline{\phi}_{ij\omega}$$

where the function $\phi^{(0)}_{ij\omega}$ is the slow variable and it is not equal to zero in an interval of the frequencies $0 < \omega < \lambda\omega_c$, while the function $\overline{\phi}_{ij\omega}$ is finite in the interval $\lambda\omega_c < \omega < \omega_c$, where λ is in the interval $0 < \lambda < 1$.

Renormalization group -2

Substituting Eq. (2.66) into Eq. (2.65) we expand the action S_t up to terms quadratic in $\overline{\phi}_{ij\omega}$. Integrating in the expression for the partition function

$$Z = \int \exp\left(-S_t\left[\phi\right]\right) D\phi$$

over the fast variable $\overline{\phi}_{ij\omega}$ with the logarithmic accuracy we come to a new renormalized effective action $\tilde{S}_t$

$$\tilde{S}_t = 2\pi g \sum_{\langle i,j \rangle} \int_0^\beta \int_0^\beta d\tau d\tau' \alpha\left(\tau - \tau'\right) \times \sin^2\left[\frac{\phi_{ij}\left(\tau\right) - \phi_{ij}\left(\tau'\right)}{2}\right]\left(1 - \frac{\xi}{2\pi g d}\right)$$

where $\xi = -\ln\lambda$.

It follows from Eq. (2.67) that the form of the action is reproduced for any dimensionality d of the lattice of the grains. This allows us to write immediately the following renormalization group equation

$$\frac{\partial g\left(\xi\right)}{\partial \xi} = -4/z$$

Valid as long as $g \geq 1$

Conclusion from RG analysis:

The solution

$$g_T(T) = g_T - (4/z) \ln (g_T E_C/T) \qquad (1)$$

is valid down to $T^* \approx g_T E_C \exp[-(z/4) g_T]$

where $g_T(T^*) \sim 1$

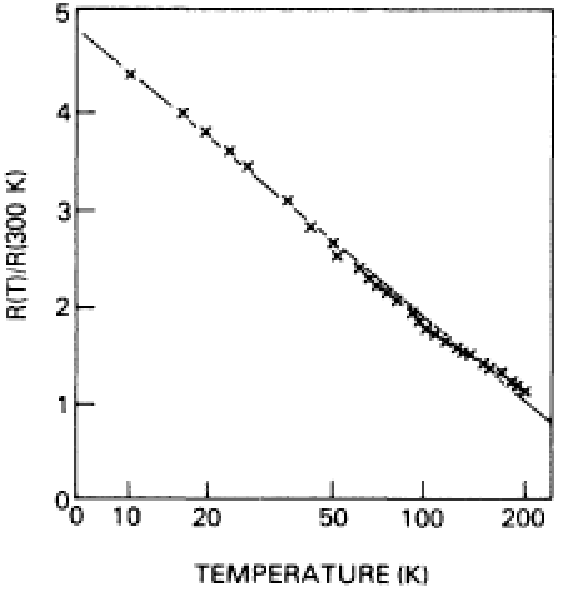
Solution (1) <u>does not</u> coinside with experimental result

$$R(T) = R_o \ln(T_o/T)$$

What is the reason for disagreement ?

1. R(T) approaching $R_K = h/e^2$

$R_{\square}(300\ \mathrm{K}) = 2000\ \Omega/\square$,



Ratio thickness/2a ~ 5

g_T (300K) ~ 2 g_T (10K) ~ 0.5

Condition $g_T >> 1$ is not fulfilled

2. Distribution of local g_{ij} is broad due to fluctuations of d_{ij}

$$g_{ij} \propto \exp\{-2k_0 d_{ij}\}$$

RG for disordered granular array

M.Feigelman, A.Ioselevich, M.Skvortsov, Phys.Rev.Lett. **93**, 136403 (2004)

Random initial values of tunneling conductances g_{ij}

Generalized RG equation

$$\frac{dg_{ij}}{dt} = -2g_{ij}R_{ij};$$

instead of

$$g(T) = g_0 - \frac{4}{z}\ln\frac{g_0 E_C}{T},$$

where $t(E) = \ln(\overline{g}_0 E_C/E)$ is the auxiliary RG "time"

R_{ij} is the actual resistance between grains i and j

RG for disordered array: solution

1) the case of relatively narrow original distribution $P_0(g)$: perturbation theory

for the square lattice ($z = 4$):

$$\frac{\sigma(E)}{\overline{g}(E)} = \frac{\sigma_0}{\overline{g}_0} \frac{\overline{g}_0/\overline{g}}{\sqrt{2\ln(\overline{g}_0/\overline{g})\ln\ln(\overline{g}_0/\overline{g})}}$$

where $\overline{g} \equiv \overline{g}(E) = \overline{g}_0 - \ln(\overline{g}_0 E_C/E)$

Relative width of the P(g) grows fastly under RG

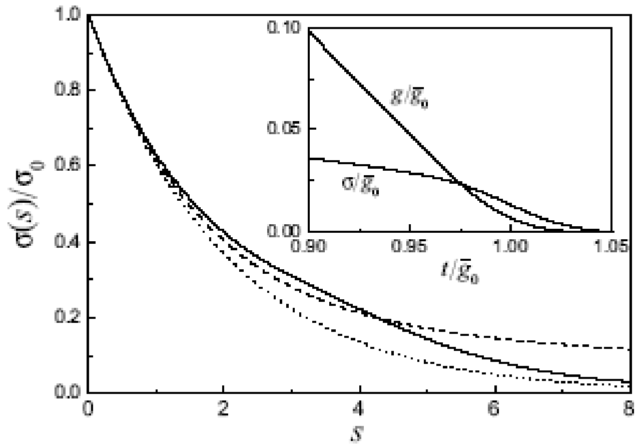


FIG. 2: Evolution of $\sigma(s)/\sigma_0$, Eq. (11), (dashed line), the result of numerical simulation of Eq. (2) with $\sigma_0/\overline{g}_0 = 0.1$ (solid line), and the EMA prediction $e^{-s/2}$ (dotted line). Inset: $\overline{g}(t)$ and $\sigma(t)$ near the MIT.

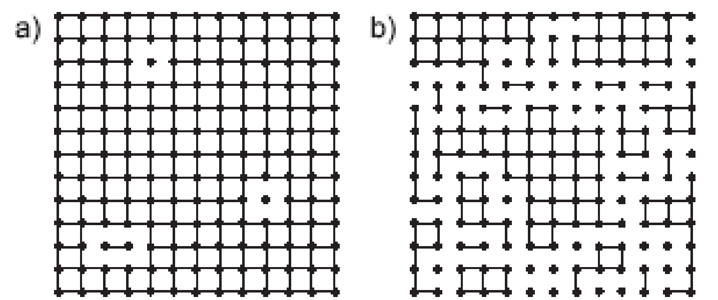


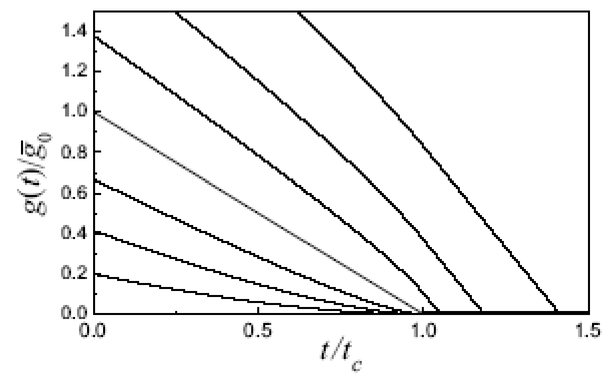
FIG. 1: Conducting bonds (with $g_{ij} > 0$) at different values of t. Results of numerical simulation of Eq. (2) on the lattice 20×20 with $P_0(g) = (2g_0\theta(g)/\pi)/(g^2+g_0^2)$. a) $t = 0.64g_0$ with the fraction of conducting bonds $N_{\rm cond} = 0.95$; b) $t = 1.08g_0$ with $N_{\rm cond} = 0.55$.

RG for disordered array: solution-2

2) Strong disorder: effective-medium approximation

$$R_{ij} = \left[g_{ij} + \left(\frac{z}{2} - 1 \right) g_{\text{eff}} \right]^{-1} \qquad \langle R_{ij} \left(g_{ij} - g_{\text{eff}} \right) \rangle_{g_{ij}} = 0.$$

$$\int_0^\infty P_0(g_0)\, dg_0 \frac{g[g_0, \{g_{\text{eff}}\}|t] - g_{\text{eff}}(t)}{g[g_0, \{g_{\text{eff}}\}|t] + \left(\frac{z}{2} - 1\right) g_{\text{eff}}(t)} = 0$$



Explicit solution for symmetric distributions of $\ln g \sim h_{ij} = \kappa(d_{ij} - \bar{d})$

for $t < t_c$,

$$g_{ij}(t) = \left[\frac{g_{ij}(0) - \bar{g}_0}{2\sqrt{g_{ij}(0)}} + \sqrt{\frac{[g_{ij}(0) + \bar{g}_0]^2}{4 g_{ij}(0)} - t} \right]^2$$

while for $t > t_c$

$$g_{ij}(t) = [g_{ij}(t_c) - 2(t - t_c)]\, \theta[g_{ij}(t_c) - 2(t - t_c)].$$

$$g_{\text{eff}}(t) = \bar{g}_0 - t \text{ for } t < t_c = \bar{g}_0$$

RG for disordered array: solution-3

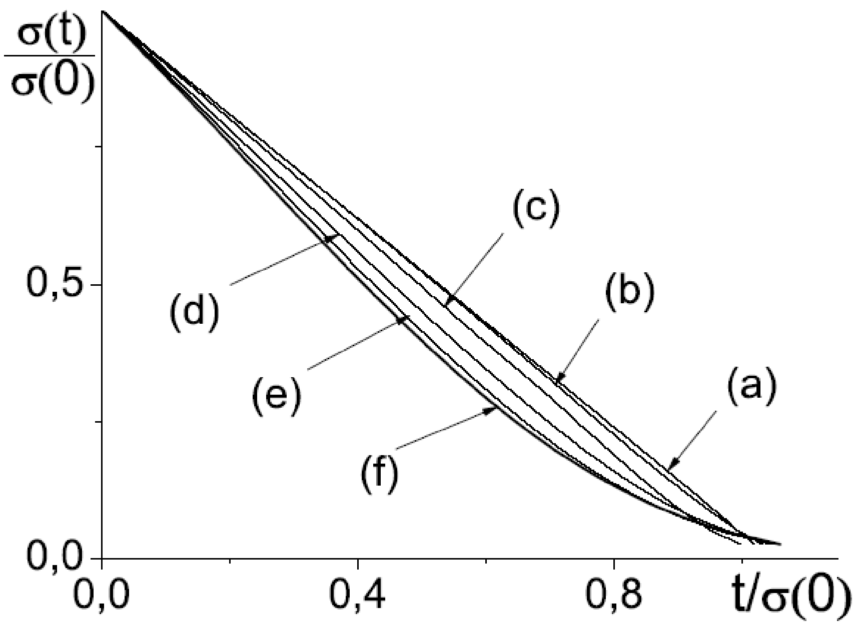
"Universal temperature dependence of the conductivity of a strongly disordered granular metal"

A. R. Akhmerov, A. S. Ioselevich JETP Lett. **83**(5), 211-216 (2006); cond-mat/0602088

$$g_{ij} = \overline{g}_0 e^{h_{ij}}, \qquad h_{ij} = -2\kappa(d_{ij} - \langle d \rangle), \qquad \Delta \equiv \sqrt{\langle h^2 \rangle} \gg 1$$

Universal solution:

$$\frac{G_\square(T)}{G_\square(T_R)} = F^{(2)}\left[\frac{\ln(T_R/T)}{G_\square(T_R)}\right],$$



Conclusion:

strong disorder in h_{ij} might be a reason for

$$R(T) = R_o \ln(T_o/T)$$

4) Insulating behaviour: co-tunneling and Efros-Shklovsky law

Usual hopping insulator (doped semiconductor): Mott law

$t_{ij} \sim \exp(- r_{ij}/a)$ a is the localization length of Hydrogen-like orbital

$P_{ij} \sim \exp(- 2 r_{ij}/a) \exp(- \varepsilon_{ij}/T)$ ε_{ij} is the energy difference between states i and j

$\varepsilon_{ij} \sim (r_{ij})^{-d} \nu^{-1}$ Optimize over r_{ij}: $\min_R (2R/a + 1/R^d T)$

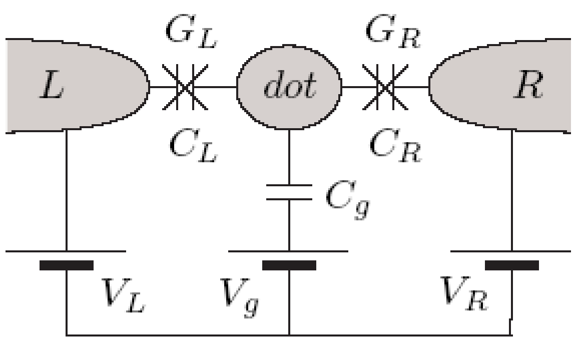
$$P_{opt} \sim \exp[-(T_M/T)^a] \qquad a = 1/(d+1)$$

Doped semiconductor with a Coulomb gap: ES law

$\nu(\varepsilon) \sim \varepsilon^{d-1}$ ⇒ $\varepsilon(R) \sim 1/R$ Now R_{opt} is found from $\min_R (2R/a + \mathrm{const}/RT)$

⇒ $P_{opt} \sim \exp[-(T_o/T)^{1/2}]$ $T_o = C e^2/\kappa a$

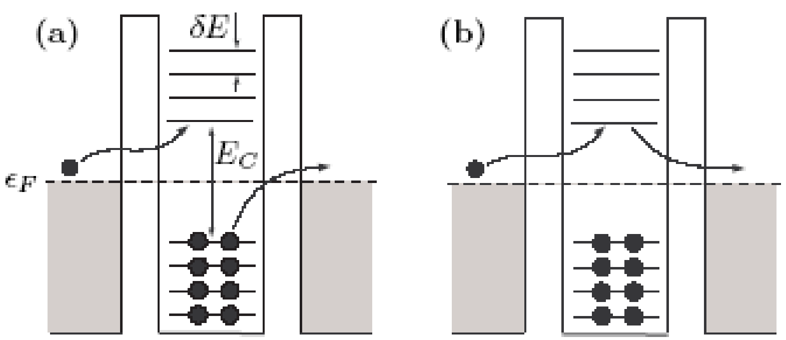
Co-tunneling: correlated tunnling



$G_{orto} \sim \min(g_L, g_R) \exp(-E_C/T)$

Consider now 2-nd order amplitude with charged grain in a virtual state
(Averin & Nazarov PRL 65, 2446, 1990)

Review: Leonid I. Glazman[1] and Michael Pustilnik[2] cond-mat/0501007



(a) inelastic co-tunneling (b) elastic co-tunneling

Level widths:

$$\Gamma_{\alpha n} = \pi\nu \left| t_{\alpha n}^2 \right|$$

$$G_\alpha = \frac{4e^2}{\hbar}\frac{\Gamma_\alpha}{\delta E}$$

Co-tunneling: inelastic and elastic

Inelastic:

$$A_{n,m} = \frac{t^*_{Ln} t_{Rm}}{E_C}.$$

$$G_{in} \sim \frac{e^2}{h} \sum_{n,m} \nu^2 \left| A^2_{n,m} \right| \sim \frac{e^2}{h} \frac{1}{E_C^2} \sum_{n,m} \Gamma_{Ln} \Gamma_{Rm},$$

Number of terms ~ T/δE

$$\overline{G_{in}} \sim \left(\frac{T}{\delta E}\right)^2 \frac{e^2}{h} \frac{\Gamma_L \Gamma_R}{E_C^2} \sim \frac{G_L G_R}{e^2/h} \left(\frac{T}{E_C}\right)^2$$

Elastic:

$$A_{el} = \sum_n t^*_{Ln} t_{Rn} \frac{\text{sign}(\epsilon_n)}{E_C + |\epsilon_n|}$$

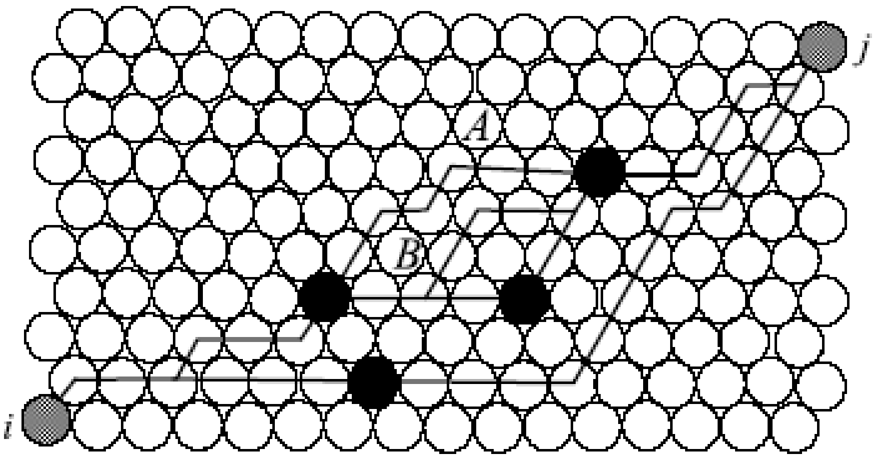
$$\overline{|A^2_{el}|} = \frac{\Gamma_L \Gamma_R}{(\pi\nu)^2} \sum_n (E_C + |\epsilon_n|)^{-2}.$$

$$G_{el} = \frac{4\pi e^2 \nu^2}{\hbar} \left| A^2_{el} \right| \sim \frac{G_L G_R}{e^2/h} \frac{\delta E}{E_C}$$

Variable range cotunneling and conductivity of a granular metal

M. V. Feigel'man[1], *A. S. Ioselevich*[1]

Pis'ma v ZhETF, vol. 81, iss. 6, pp. 341 – 347



Four different strings contributing to the $i \to j$ transition in a particular realization of array. For a shown partition ("inelastic grains" $\{l\}$ are depicted as filled circles, elastic grains $\{m\}$ – as open ones) only two strings (A and B) contribute to interference effects

$$g_{ij} \propto e^{-\frac{\varepsilon_{ij}}{T}} \left(\frac{t}{E_C}\right)^{2N} \times$$
$$\times \sum_{L=0}^{N} \frac{\left(\frac{2|\Delta_{ij}|^2}{\delta^2}\right)^L \left(\frac{E_C}{\delta}\right)^{N-L}}{(2L+1)!} F_{NL}.$$

N is the total grain number in the path

L is the number of "inelastic" grains

$$\Delta_{ij} = \varepsilon_j - \varepsilon_i - E_c^{(ij)}$$

General expression can be analyzed for local Coulomb only (screening by gate)

Variable range cotunneling - limiting cases

$$g_{ij} \propto \exp\left\{-\frac{\varepsilon_{ij}}{T}\right\} \begin{cases} \left(\frac{\tilde{A}_1 g\delta}{8\pi^2 E_C}\right)^{N_{ij}}, & \text{elastic,} \\ \left(\frac{e^2 \tilde{A}_2 g |\Delta_{ij}|^2}{16\pi^2 N_{ij}^2 E_C^2}\right)^{N_{ij}}, & \text{inelastic.} \end{cases}$$

$g \equiv Gh/e^2 = 8\pi^2 (t/\delta)^2 \ll 1$ is the average conductance between neigbouring grains

Usual optimization over N and ε leads to

$\sigma \sim e^{-\sqrt{T_{ES}/T}}$ with $T_{ES} = \mathcal{L}(T) E_C$ and

$$\mathcal{L}(T) = \begin{cases} c_1 \ln\left(\frac{8\pi^2 E_C}{\tilde{A}_1 g\delta}\right), & T \ll T_c, \\ c_1 \ln\left(\frac{16\pi^2 E_C^2}{e^2 \tilde{A}_2 g T^2 \mathcal{L}^2}\right), & T \gg T_c, \end{cases}$$

$T_{ES} \propto \bar{a}^{-1}$

$T_c \propto a^{-2}.$

crossover temperature $T_c \sim \sqrt{E_C \delta}/\mathcal{L}$

Variable-range co-tunneling: estimates

Example: Al grains with a=20 nm

$\delta = 0.02$ K $\quad E_{th} = 50$ K $\quad E_c = 1000$ K

With $g \sim 0.3$ one finds $\mathcal{L} \approx 10$

Thus $T_{ES} \approx 10^4$ K and $T_c \approx 0.5$ K

Conclusions: inelastic co-tunneling dominates,
magneto-resistance is very weak

5) Co-tunneling and conductance of weakly coupled quasi-1D wire

M.V.Feigel'man and A.S.Ioselevich, JETP Lett. **88**, 767-771 (2008); arXiv:0809.1325

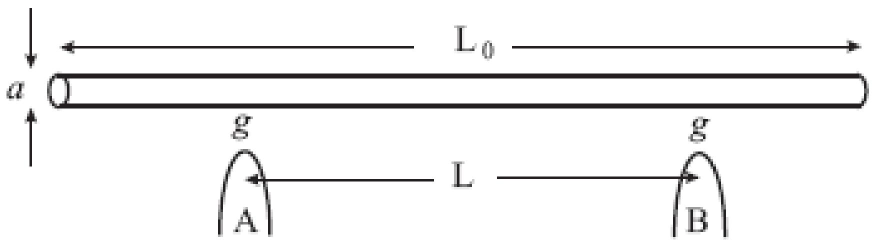


FIG. 1: Electrons tunnel between a wire of length L_0 and diameter a and two leads A and B, placed symmetrically with respect to the center of the wire, at distance L from each other.

Experimental results:

$$G \equiv dI/dV \propto V^{\alpha} \ \ (\text{low } T), \qquad G \propto T^{\alpha} \ \ (\text{high } T).$$

Usual view: "Luttinger Liquid"

However, such a scaling is observed for multi-channel diffusive conductors

Our explanation: inelastic co-tunneling + Coulomb anomaly

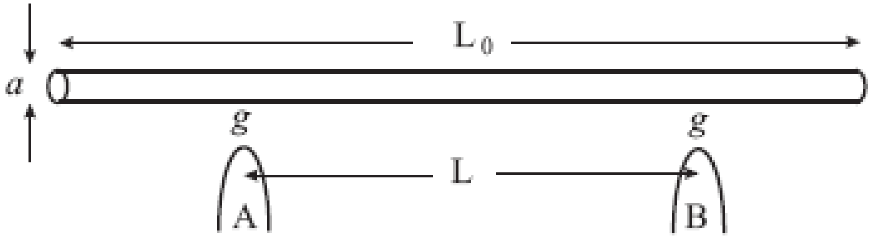


FIG. 1: Electrons tunnel between a wire of length L_0 and diameter a and two leads A and B, placed symmetrically with respect to the center of the wire, at distance L from each other.

weak tunnel contacts A and B with conductances $g \ll 1$

Total classical resistance

$$R(L)/(h/e^2) \equiv L/\xi \gtrsim 1$$

$\xi \sim N_{\rm ch} l$ is the localization length, l is the mean free path, and $N_{\rm ch} \gg 1$ is the number of channels.

We consider relatively high temperatures, i.e. no Anderson localization:

$$T \gg T_{\rm Loc} \sim D\xi^{-2} \sim v_F/N_{\rm ch}^2 l.$$

Our main result reads as follows:

$$G_{AB}^{(2)} \sim g^2 \left(\frac{\max(T, eV)}{E_C(L)} \right)^{\alpha^*} \quad \text{where} \quad \alpha^* = \frac{R(L)}{(h/2e^2)}$$

Exponentially suppressed in L/ξ - in spite of the absence of localization

Coulomb zero-bias anomaly

- A.Finkelstein ZhETF 1984
- Yu.Nazarov ZhETF 1989
- L.Levitov & A.Shytov cond-mat/9607136

$$g \longrightarrow g e^{-S(T)}$$

$S(T)$ is the Action of charge spreading process

$$S \sim \int_T \frac{d\omega}{2\pi\omega} \int_{1/\xi} \frac{d^d\mathbf{q}}{(2\pi)^d} \frac{U_q}{\omega + \sigma_q q^2 U_q},$$ $J(q,\omega)$

Potential energy

$U_{\mathbf{q}} = 2\pi e^2/q$

Source term: $\mathcal{J}_1 = [\delta(\tau+\tau_0) - \delta(\tau-\tau_0)]\delta(x+L/2)$

Usual result in quasi-1D:

$$S_1(T,V) \approx \begin{cases} 0.76\sqrt{\frac{E_C(\xi)}{T}}, & \text{for } eV \ll \sqrt{E_C(\xi)T}, \\ E_C(\xi)/(eV), & \text{for } eV \gg \sqrt{E_C(\xi)T}. \end{cases}$$

Coulomb anomaly + inelastic co-tunneling

$$g \longrightarrow g e^{-S(T)}$$

$$S \sim \int_T \frac{d\omega}{2\pi\omega} \int_{1/\xi} \frac{d^d\mathbf{q}}{(2\pi)^d} \frac{U_q}{\omega + \sigma_q q^2 U_q}, \quad J(q,\omega)$$

The crucial point of our approach is that we describe contunnelling through a diffusive wire by the same semi-classical equations (11,12), but with modified source

$$\mathcal{J}_2 = [\delta(\tau+\tau_0) - \delta(\tau-\tau_0)][\delta(x+L/2) - \delta(x-L/2)] \qquad \mathcal{J}_2(\omega, q) = -4i \sin(\omega\tau_0)\sin(qL/2)$$

which describes *simultaneous* tunnelling of an electron and a hole via both contacts.

In the case of high voltage

$$S_2(T, V \to 0) \approx \frac{2L}{\xi} \ln\left(\frac{\omega_{\max}}{T}\right) \qquad \tilde{S}_2(T, \tau_0) \approx \frac{2L}{\xi} \ln(\omega_{\max}\tau_0), \quad \tau_0^* = \frac{4L(eV)}{\xi},$$