Many body quantum interference:
Seeing strongly correlated states of ultracold Atoms

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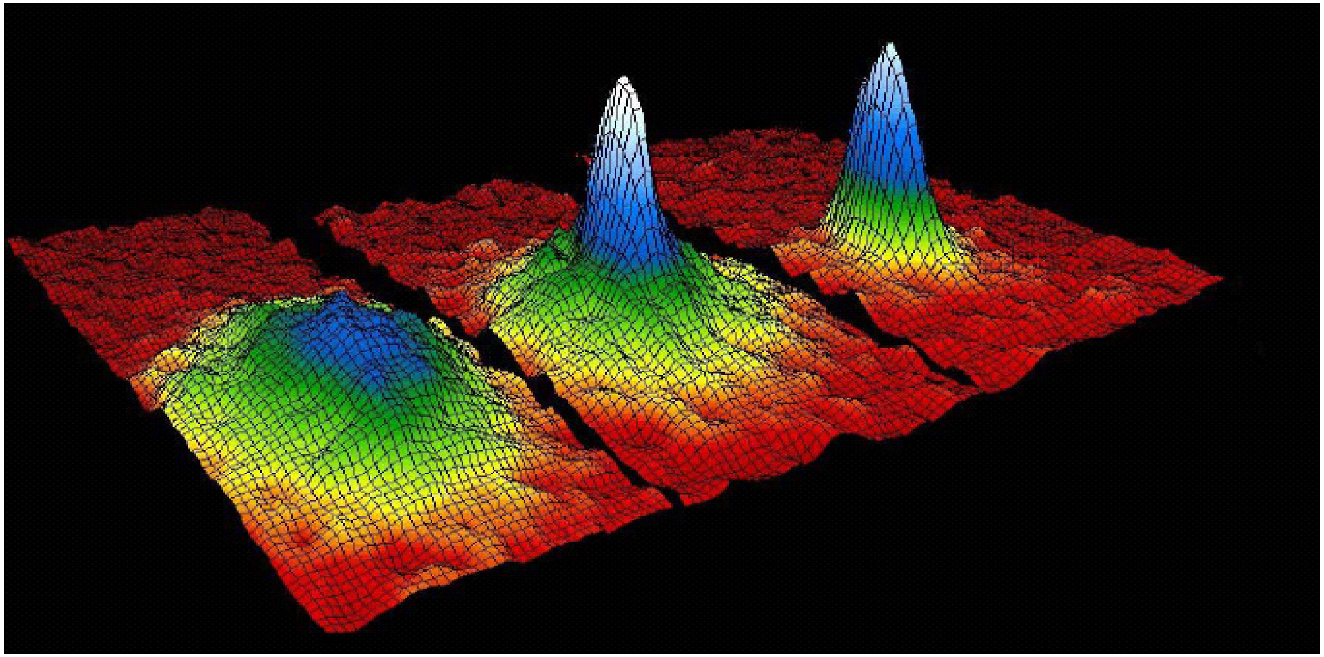
Thanks:
E. Demler, V. Gritsev, M. Lukin,
L. Mathey, A. Polkovnikov,
A. Vishwanath

Bose-Einstein condensation in the 90's



$n \sim 10^{14}\,\mathrm{cm}^{-3}$ $\qquad$ $T_{\mathrm{BEC}} \sim 1\mu\mathrm{K}$

Extremely dilute. Scattering length is much smaller than characteristic inter-particle distances. Interactions are weak

Gross-Pitaevskii description works well

$$\langle b(x) \rangle = \psi(x) = \sqrt{\rho(x)}\, e^{i\varphi(x)}$$

$$v_s(x) = \frac{\hbar}{m} \nabla \varphi(x)$$

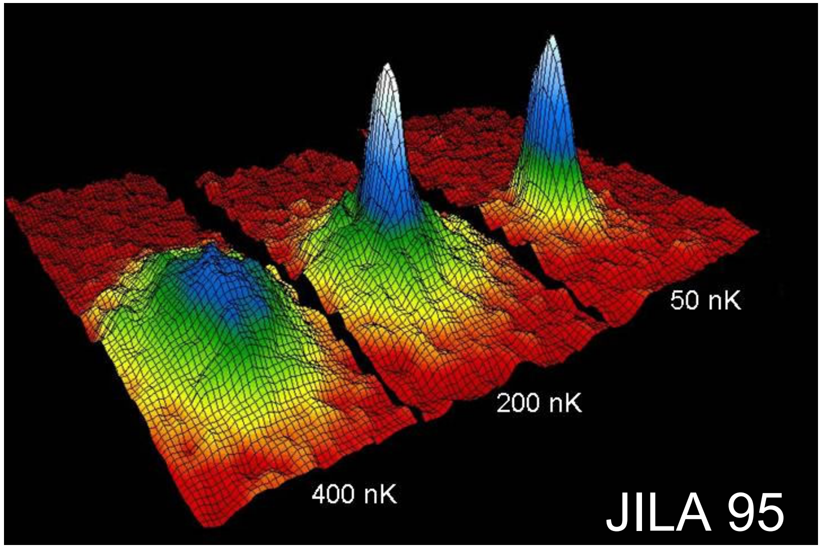
Broken U(1) symmetry

Classical field equation for the condensate wave function

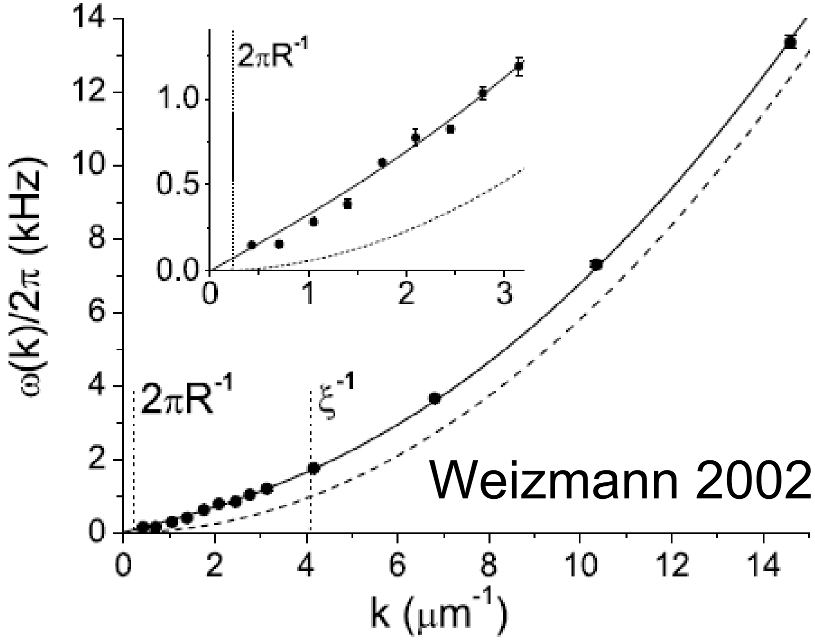
$$-i \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + \left[V(x) + u|\psi|^2 \right] \psi$$

Enormous success in describing ultra cold dilute atomic gasses !

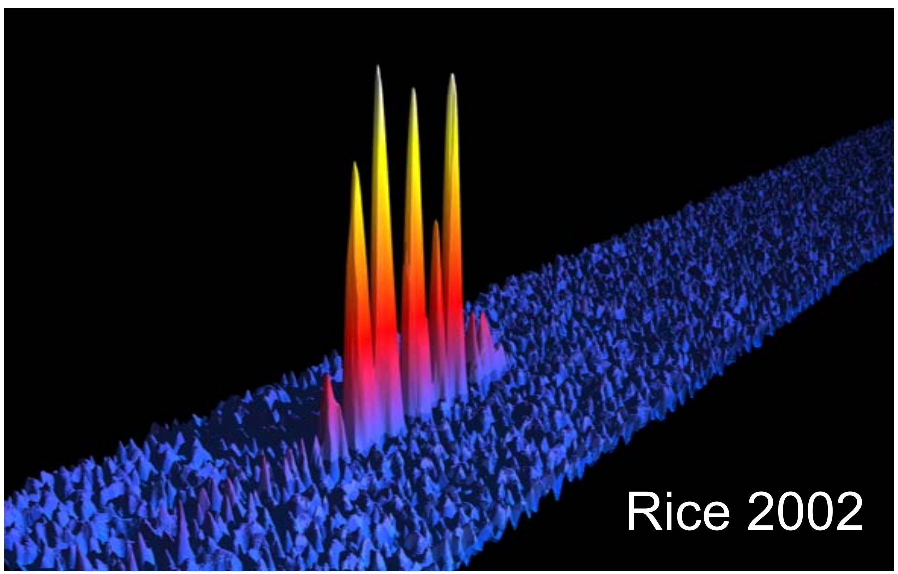
BEC



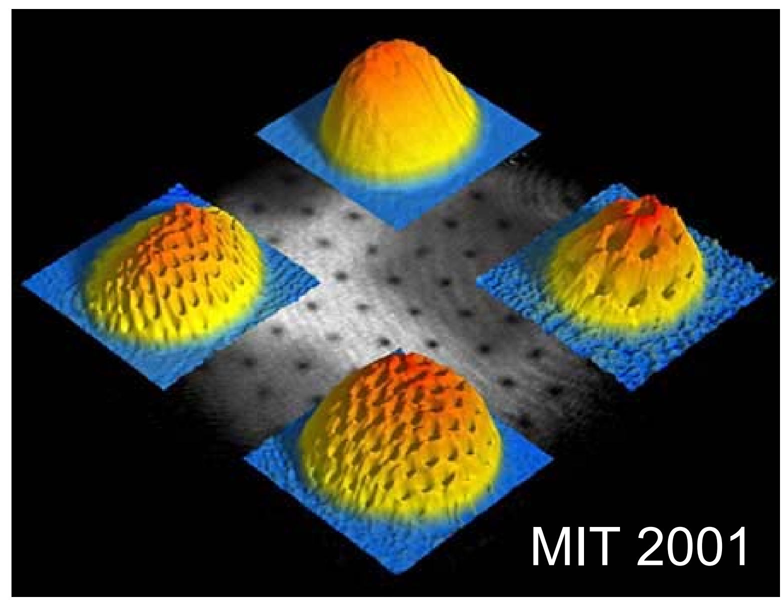
Excitation spectrum



Bright solitons

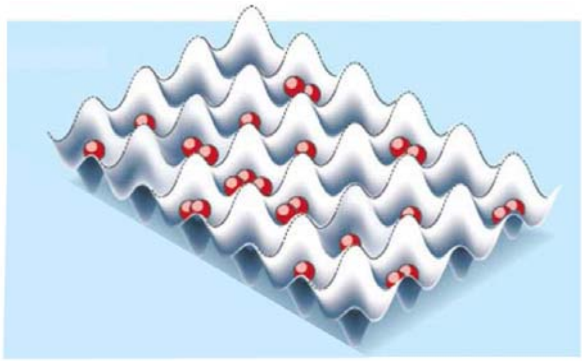


Rotation: Vortex lattice

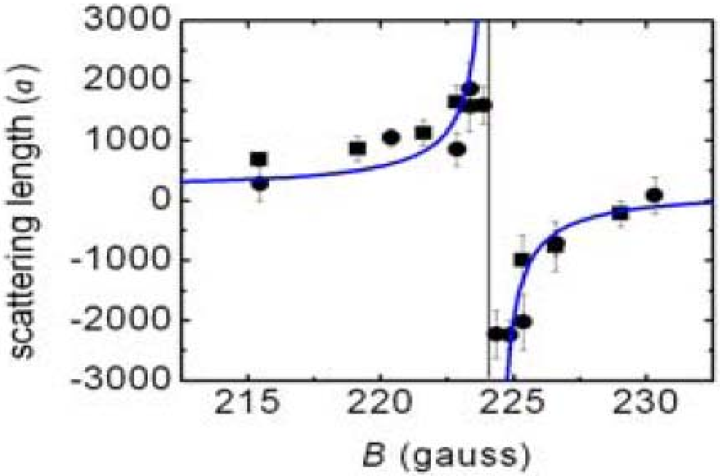


New era in ultracold atoms: focus on systems with strong interactions

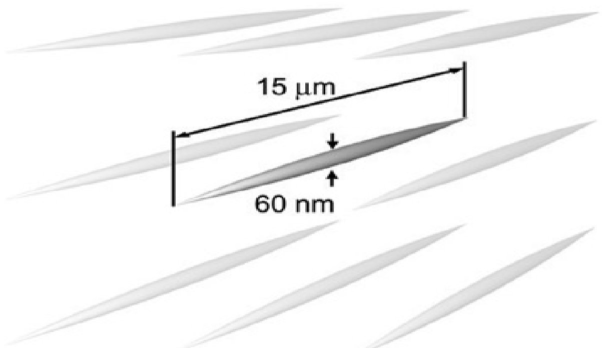
- Optical lattices



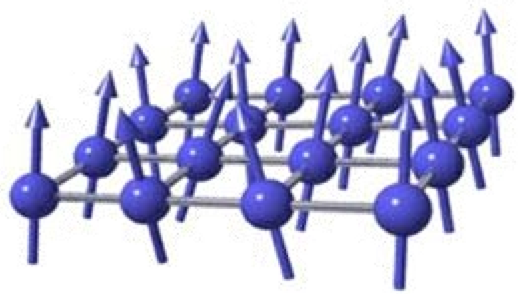
- Feshbach resonances



- Low dimensions



- Large dipole moments



Lecture outline

- Quantum noise interferometry:

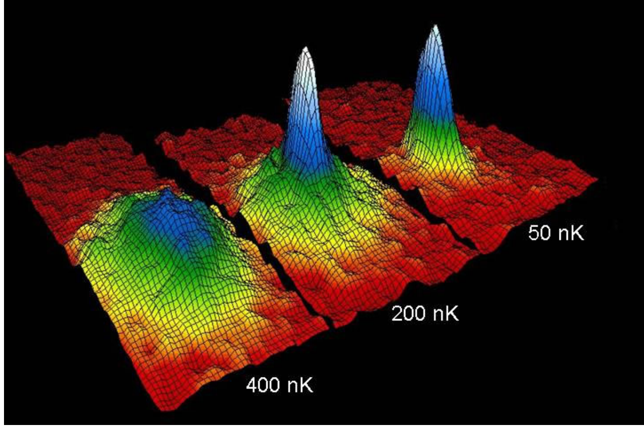
  A probe of broken symmetry states and algebraic correlations

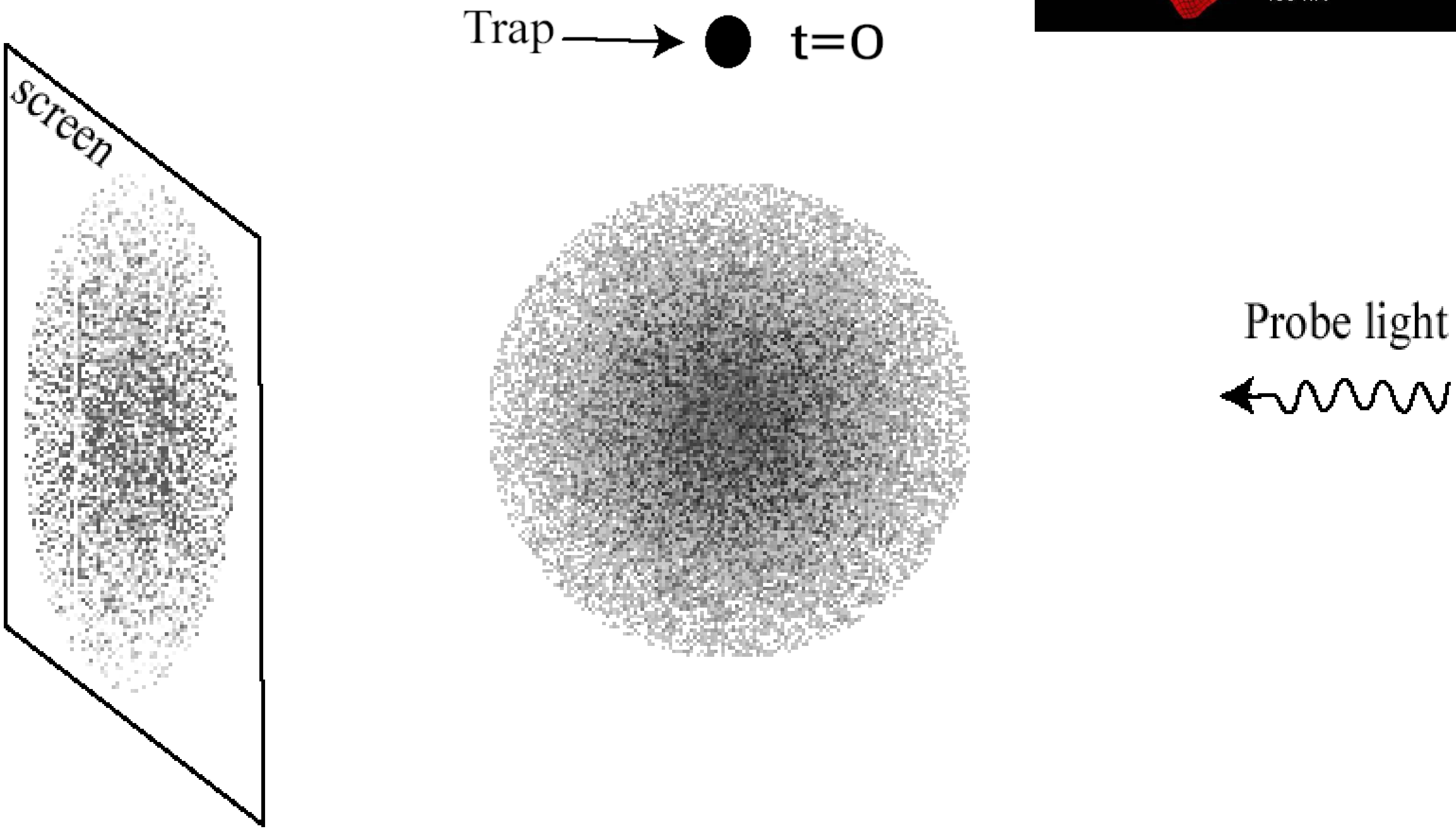
- Interference of independent condensates

  A probe of fluctuating condensates in low dimensions.

  Full statistics of interference fringe amplitude as a new type of quantum simulator

Time of flight imaging





$$\langle n(\mathbf{r}) \rangle_t \sim \langle n_{\mathbf{k}} \rangle_{t=0}$$

Digression

$$\langle n(r)\rangle_t = \langle \psi^\dagger(r,t)\psi(r,t)\rangle_0$$

$$\psi(\bar r,t) = \int d^d R\, G(\bar r-\bar R,t)\, a(R)$$

Assuming free expansion:

$$G(r-R,t) \sim \frac{1}{(\bar r-\bar R)^{d-1}}\frac{1}{t^d}\, e^{\frac{im}{2\hbar t}(\vec r-\vec R)^2}$$

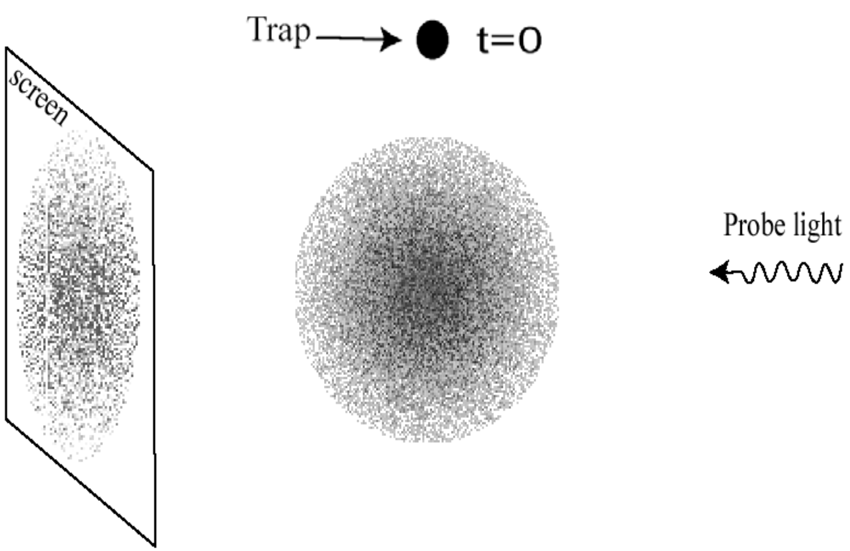
"Far field" limit ($r >> R$)

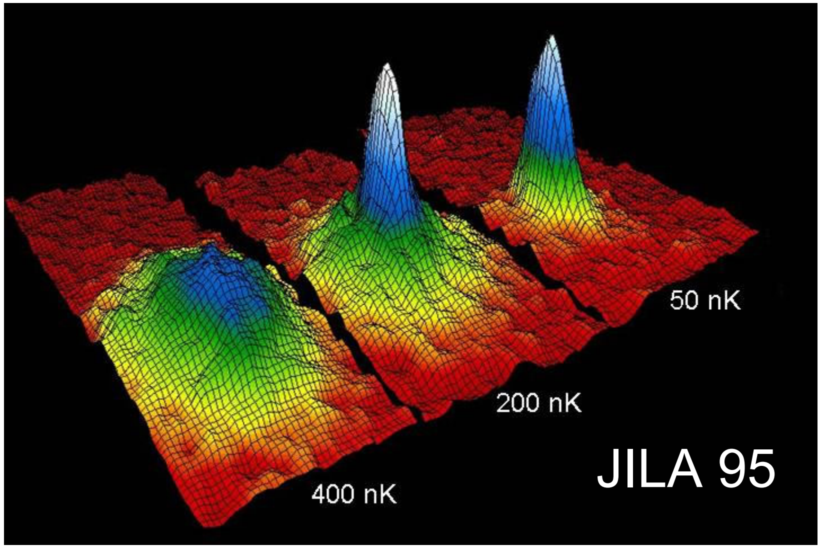
$$\sim \frac{1}{r^{d-1}}\frac{1}{t^d}\, e^{-i\frac{m\vec r}{\hbar t}\cdot\vec R}\, e^{i\frac{m}{2\hbar t}r^2}$$

$$\psi(\vec r,t) \sim \int d^d R\, e^{-i\frac{m\vec r}{\hbar t}\cdot\vec R}\, a(R) = a_{\vec k}$$

$$\langle n(\vec r)\rangle_t \sim \langle a^\dagger_{\vec k} a_{\vec k}\rangle_0 \qquad \vec k = \frac{m\vec r}{\hbar t}$$

Time of flight imaging





$$\langle n(\mathbf{r})\rangle_t \sim \langle n_{\mathbf{k}}\rangle_{t=0} \qquad \mathbf{r} = \frac{\hbar \mathbf{k}}{m} t$$

Idealy suited for detection of superfluid order

$$\langle b^\dagger(\mathbf{x}) b(\mathbf{x}')\rangle \longrightarrow |\psi|^2$$

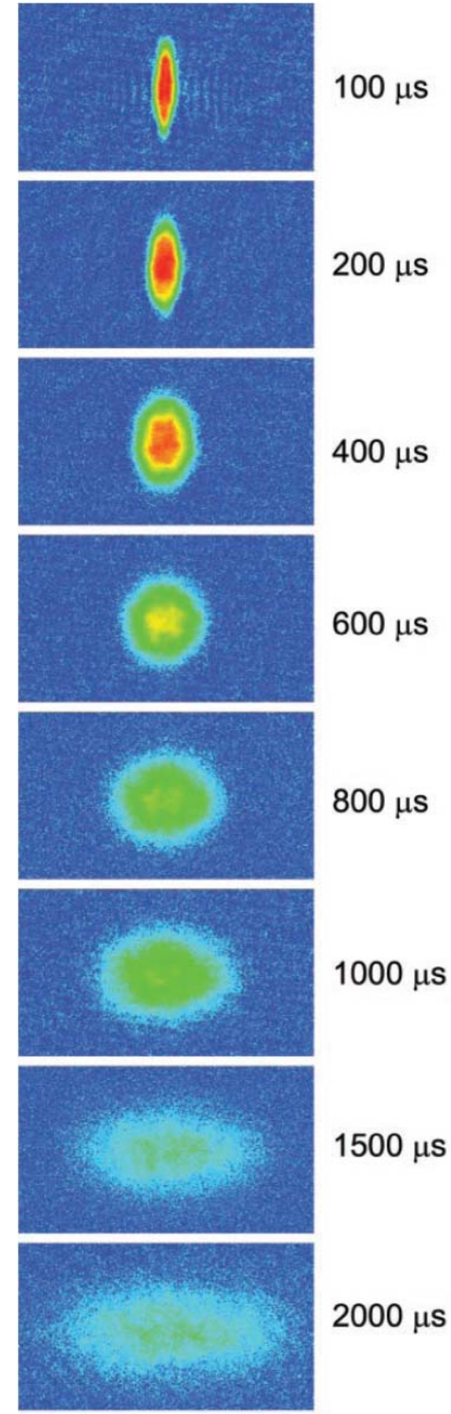
$$\langle n_{\mathbf{k}}\rangle = \frac{1}{V}\int d\mathbf{x} d\mathbf{x}' e^{i(\mathbf{x}-\mathbf{x}')\cdot\mathbf{k}} \langle b^\dagger(\mathbf{x}) b(\mathbf{x}')\rangle = |\psi|^2 \delta(\mathbf{k}) + \ldots$$

Aside

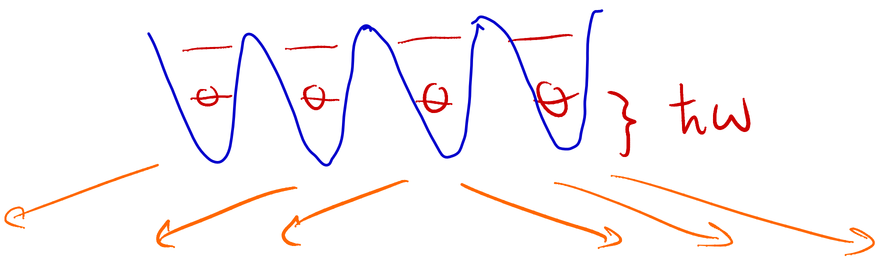
The ballistic expansion assumption can fail

In strongly interacting gasses it can be hydrodynamic for a significant length of time

e.g. Fermions near unitarity
Duke group, Science 2004



However,
expansion from a deep optical lattice is ballistic



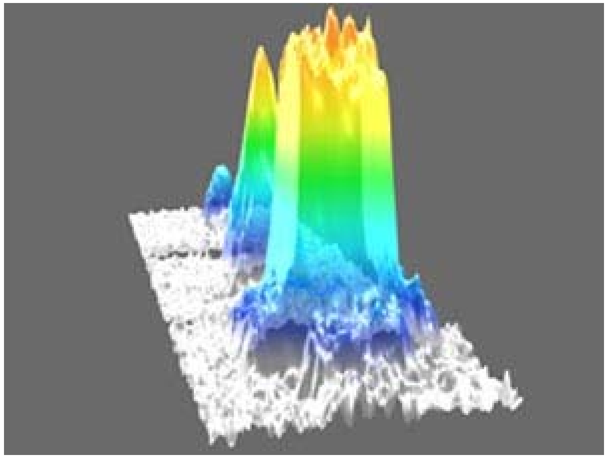
Dominated by the large kinetic energy associated with the localized wannier states in the wells.

$$E_R = \frac{1}{m\lambda^2} >> \frac{a}{m}\rho \approx \frac{a}{m}\frac{1}{\lambda^3}$$

How to identify and characterize states with no LRO in the single particle density matrix ?

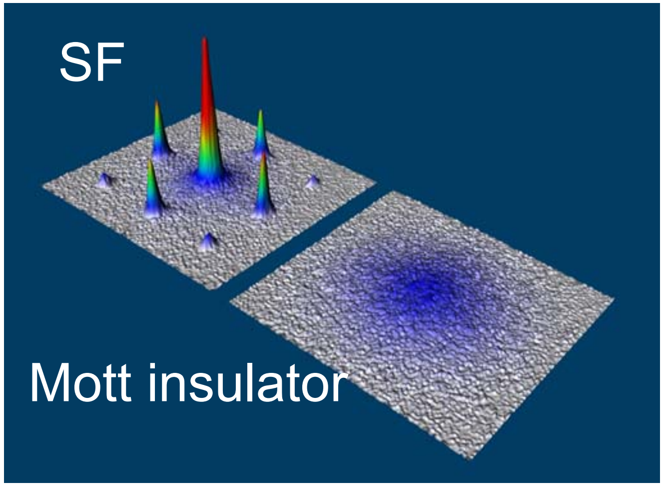
Fermions

Kohl et. al. (ETH), PRL 2005



Bosons

Greiner et al., Nature 426 (2003)

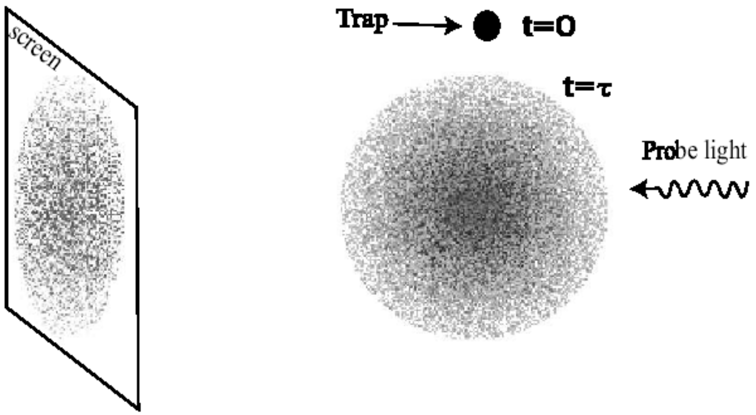


No condensate in the usual sense but other types of broken symmetry allowed: e.g. Spin density wave (SDW), Charge density wave (CDW), Pairing, etc …

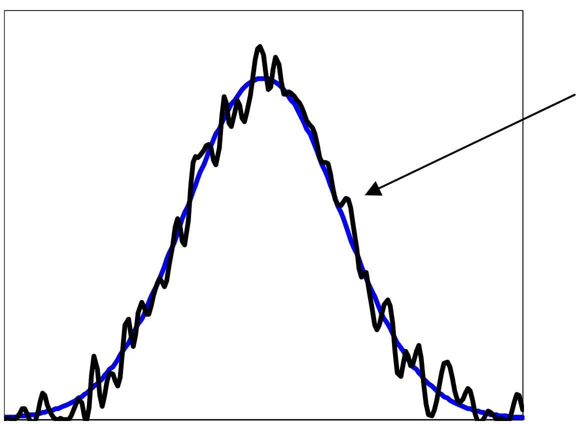
How to probe?

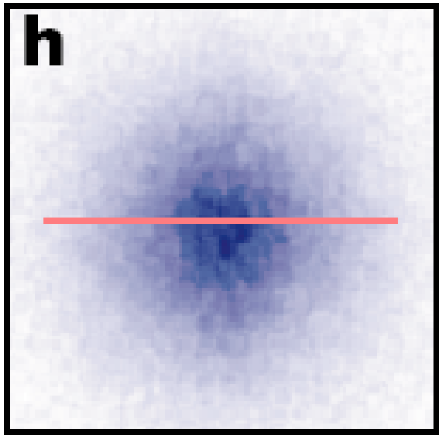
Quantum noise interferometry

EA , Demler and Lukin, PRA 2004



$\langle n(\mathbf{r})\rangle_t \sim \langle n_{\mathbf{k}}\rangle_{t=0}$



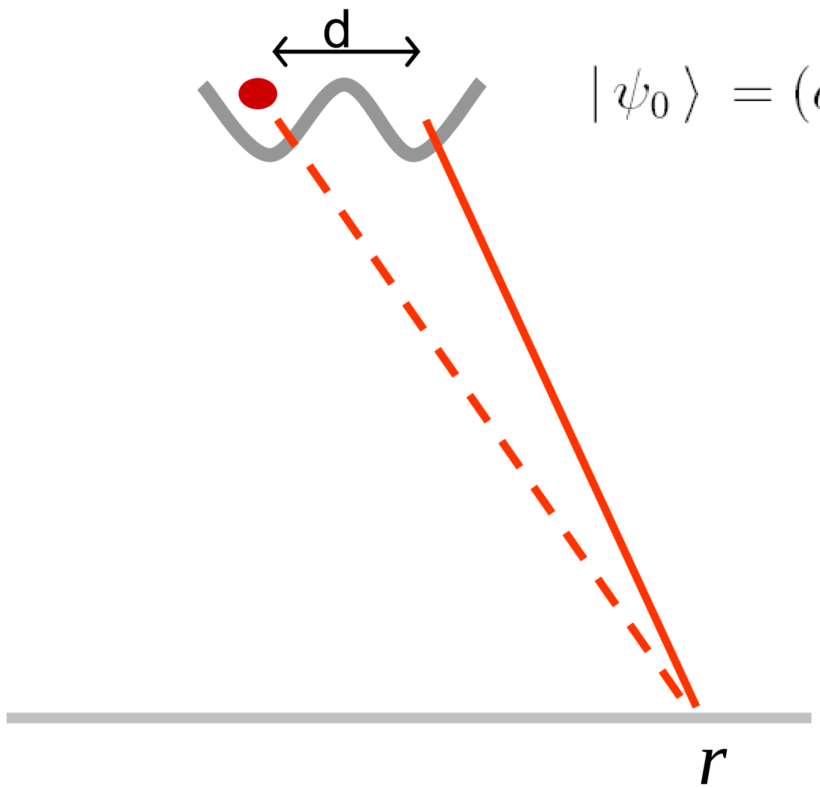


Correlations: $\langle n_{\mathbf{k}} n_{\mathbf{k'}}\rangle_{con} = \langle n_{\mathbf{k}} n_{\mathbf{k'}}\rangle_0 - \langle n_{\mathbf{k}}\rangle_0 \langle n_{\mathbf{k'}}\rangle_0$

number correlations in momentum space

Digression: single versus two-particle interference

Single particle interference:



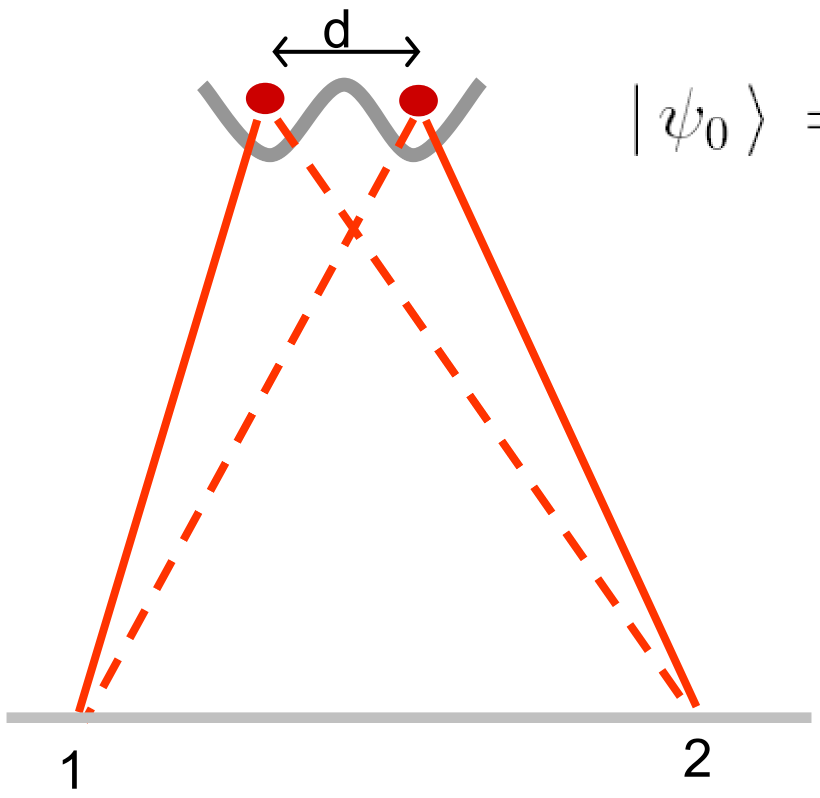
$$|\psi_0\rangle = (a_1^\dagger + e^{i\varphi} a_2^\dagger)\,|0\rangle$$

$$k = \frac{m}{\hbar t} r$$

$$\langle \rho(x) \rangle = \cos\left[k \cdot d + \varphi\right]$$

Digression: single versus two-particle interference

Two particle interference:



$$|\psi_0\rangle = a_1^\dagger a_2^\dagger |0\rangle$$

$$k_{1,2} = \frac{m}{\hbar t} r_{1,2}$$

Quantum interference between a pair of two-particle paths

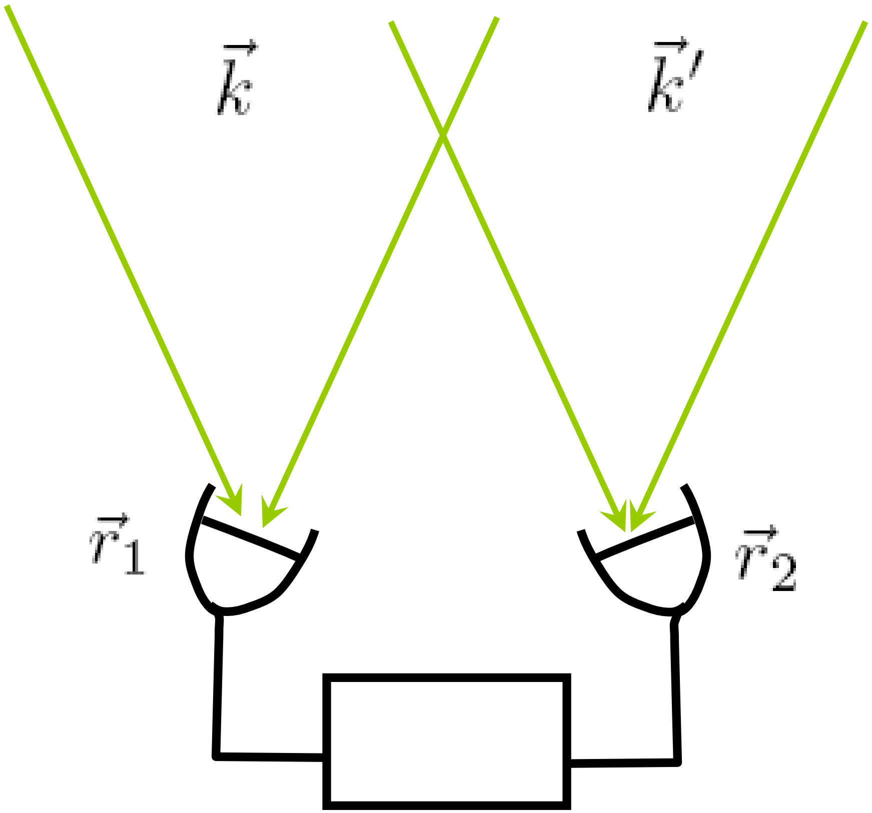
$$\langle \rho_1 \rho_2 \rangle - \langle \rho_1 \rangle \langle \rho_2 \rangle = \pm \cos\left[d\left(k_1 - k_2\right)\right]$$

(−) for fermions

Bunching of bosons vs. anti bunching of fermions

Classical limit of two particle interference

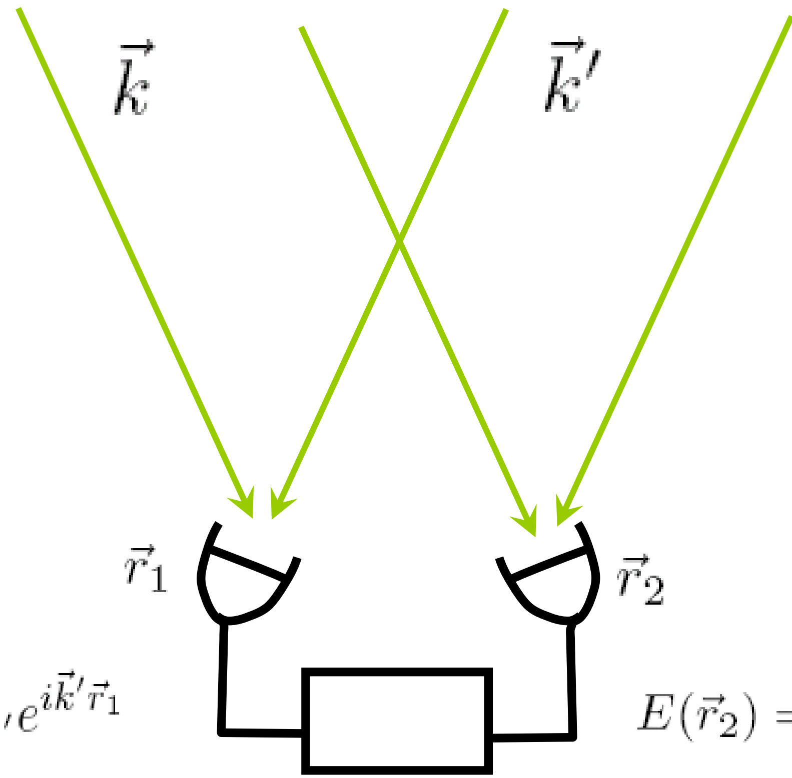
Hanburry Brown and Twiss (1954)



$$\langle I(\vec{r}_1)\ I(\vec{r}_2)\rangle = A + B\ \cos\Big((\vec{k}-\vec{k}')\ (\vec{r}_1-\vec{r}_2)\Big)$$

This was used to measure the angular diameter of Sirius!

Hanburry-Brown-Twiss interferometer

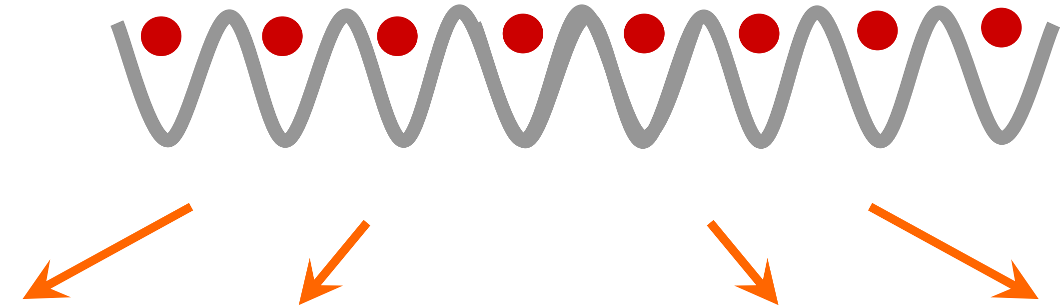


$$E(\vec{r}_1) = E_k e^{i\vec{k}\vec{r}_1} + E_{k'} e^{i\vec{k}'\vec{r}_1}$$

$$E(\vec{r}_2) = E_k e^{i\vec{k}\vec{r}_2} + E_{k'} e^{i\vec{k}'\vec{r}_2}$$

$$\begin{aligned}
\langle I(r_1) I(r_2) \rangle &= \langle |E(r_1)|^2 |E(r_2)|^2 \rangle \\
&= \langle \left\{ |E_k|^2 + |E_{k'}|^2 + (E_k E_{k'}^* e^{i(\vec{k}-\vec{k}')\vec{r}_1} + \text{c.c.}) \right\} \left\{ |E_k|^2 + |E_{k'}|^2 + (E_k E_{k'}^* e^{i(\vec{k}-\vec{k}')\vec{r}_2} + \text{c.c.}) \right\} \rangle \\
&= \langle (|E_k|^2 + |E_{k'}|^2)^2 \rangle + \langle |E_k|^2 |E_{k'}|^2 \rangle [e^{i(\vec{k}-\vec{k}')\vec{r}_1} + \text{c.c.}]
\end{aligned}$$

Lattice:



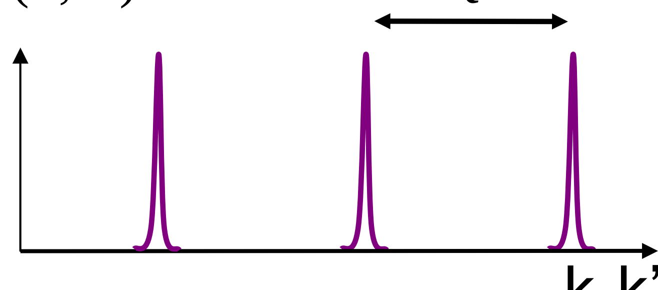
Atoms with **lattice-momentum** q expand as plane waves with wave-vectors $k = q, q + Q, q + 2Q, q + 3Q, \ldots$

$$k_{1,2} = \frac{m}{\hbar t} r_{1,2}$$

If k_1-$k_2 = nQ$ then the two particles originate from the same lattice-momentum q :

⇨ Enhanced correlation (bunching - bosons)

⇨ antibunching – fermions

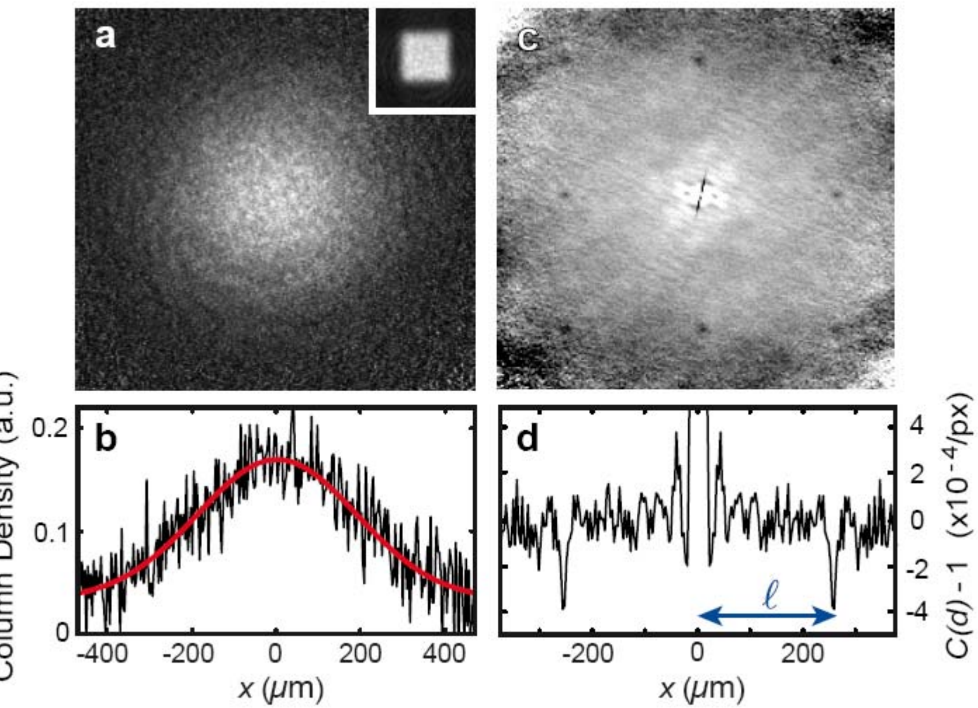


Bosons or Fermions on optical lattices

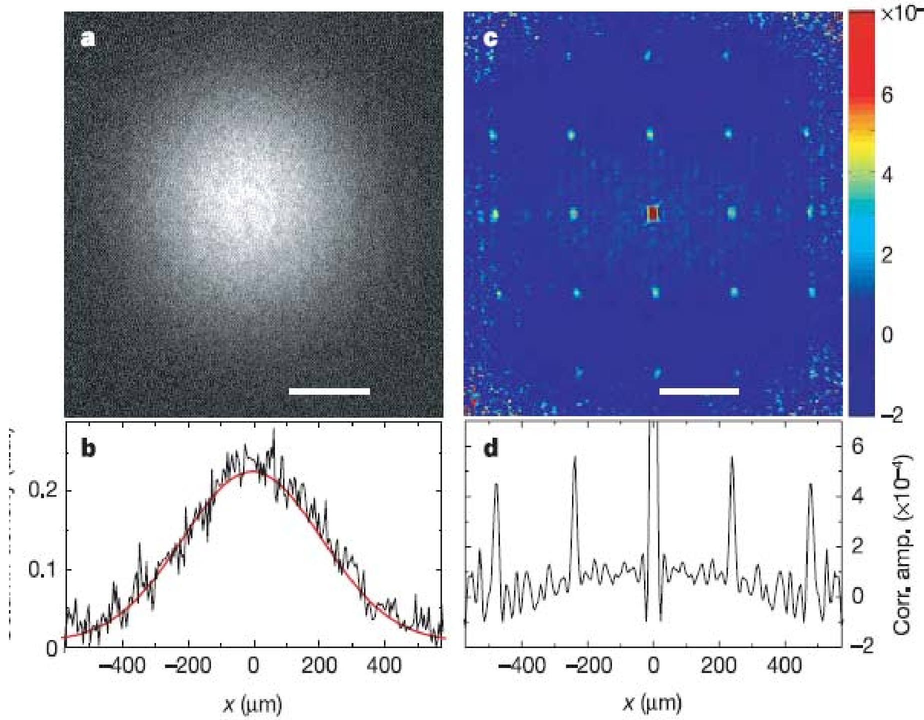
Theory: EA , Demler and Lukin, PRA 2004

$$\langle n_{\mathbf{k}} n_{\mathbf{k}'} \rangle_{con} = \langle n_{\mathbf{k}} n_{\mathbf{k}'} \rangle_0 - \langle n_{\mathbf{k}} \rangle_0 \langle n_{\mathbf{k}'} \rangle_0$$

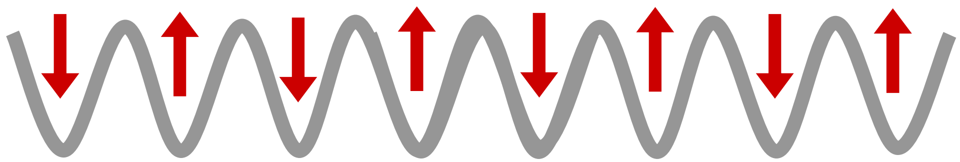
Lattice of fermions:
T. Rom et al. Nature (2006)



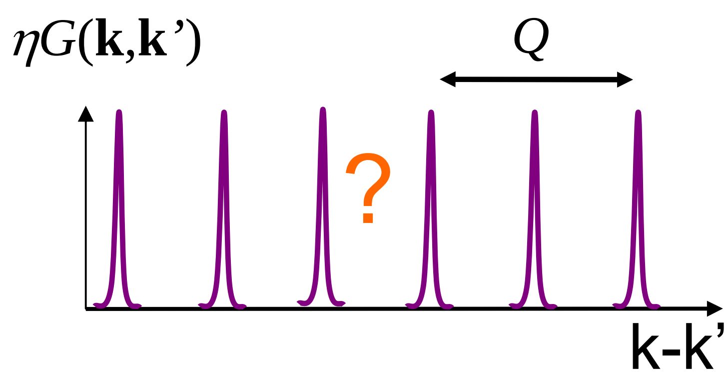
Lattice of bosons:
Foelling et al. Nature (2005)



Quiz for the students:



$\eta = \pm 1 \quad \text{bosons/fermions}$



More generally:

$$G(\mathbf{k},\mathbf{k}') \sim \eta \sum_{\mathbf{G}} \delta(\mathbf{k}-\mathbf{k}'+\mathbf{G}) + \eta \sum_{ij} e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{R}_{ij}} \langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle$$

Quantum noise interferometry beyond HBT

A unified detection scheme for broken symmetry states

We shall demonstrate this scheme for Fermi systems

Generalized fermion condensates

Mean field states:

Order parameter

$$| \Psi_{SC} \rangle = \prod_{\mathbf{k}} (u_{\mathbf{k}} + v_{\mathbf{k}} \psi^{\dagger}_{\mathbf{k}\uparrow} \psi^{\dagger}_{-\mathbf{k}\downarrow}) | 0 \rangle$$

particle-particle condensate

$$|\Delta| e^{i\varphi} = \sum_{\mathbf{k}} \langle \psi_{\downarrow,-\mathbf{k}} \psi_{\uparrow,\mathbf{k}} \rangle$$

Broken U(1) symmetry

$$| \Psi_{CDW} \rangle = \prod_{\mathbf{k}} (u_{\mathbf{k}} + v_{\mathbf{k}} \psi^{\dagger}_{s,\mathbf{k}+\mathbf{q}} \psi_{s,\mathbf{k}}) | FS \rangle$$

$$\rho_{\mathbf{q}} = \sum_{s\mathbf{k}} \langle \psi^{\dagger}_{s,\mathbf{k}+\mathbf{q}} \psi_{s,\mathbf{k}} \rangle$$

Broken translation symmetry

particle-hole condensates

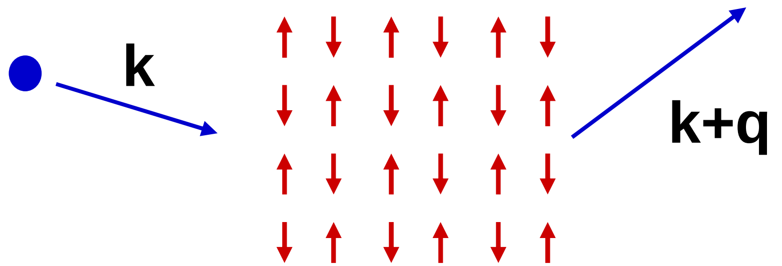
$$| \Psi_{SDW} \rangle = \prod_{\mathbf{k}} [u_{\mathbf{k}} + v_{\mathbf{k}} (\psi^{\dagger}_{s,\mathbf{k}+\mathbf{q}} \vec{\sigma}_{ss'} \psi_{s',\mathbf{k}}) \cdot \mathbf{n}] | FS \rangle$$

$$\vec{m}_{\mathbf{q}} = \sum_{\mathbf{k}} \langle \psi^{\dagger}_{s,\mathbf{k}+\mathbf{q}} \vec{\sigma}_{ss'} \psi_{s',\mathbf{k}} \rangle$$

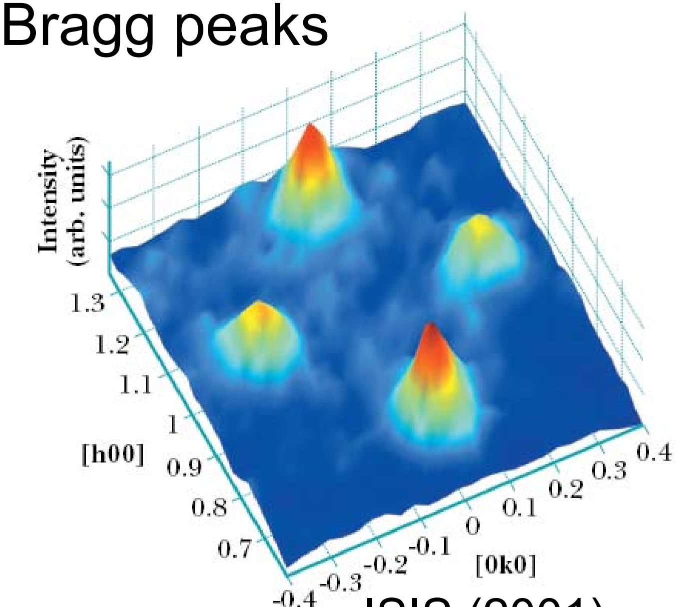
Broken spin symmetry
and translational symmetry

Standard probe of order in solids:

Elastic neutron scattering:



Magnetic Bragg peaks



ISIS (2001)

Measures:

$$\langle \vec{m}_{-q} \cdot \vec{m}_{q} \rangle , \quad \langle \rho_{-q}\, \rho_{q} \rangle$$

Neutrons (like any external probe) couple only to local densities (spin or charge). Cannot detect pairing correlations!

Noise correlations offer a more general scheme for atoms!

Noise correlations in ordered states

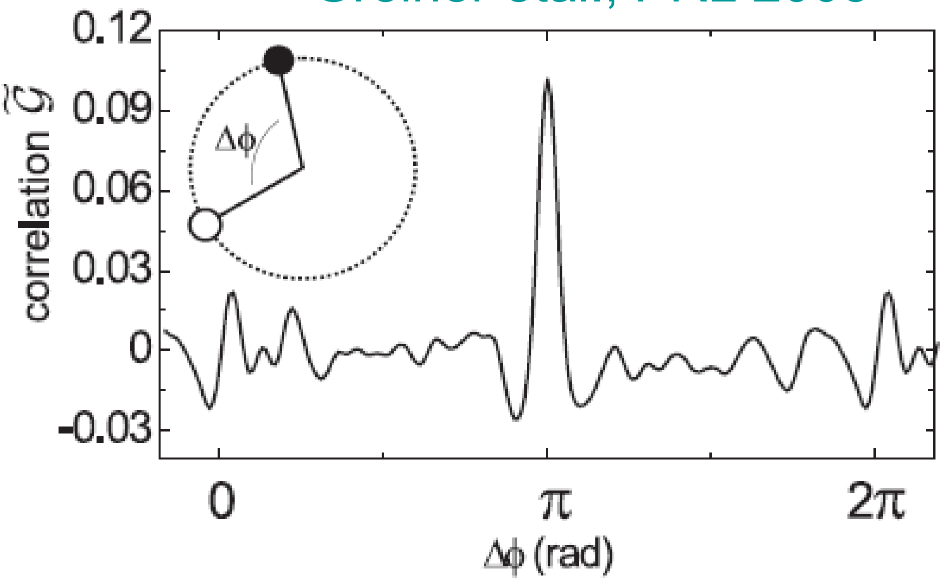
Greiner etal., PRL 2005

Superconductivity (p-p):

$$\prod_{\mathbf{k}} (u_{\mathbf{k}} + v_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow}) \,|\, 0 \,\rangle$$

$$\langle n_{\mathbf{k}} n_{\mathbf{k}'} \rangle_{con} = \sum_{\mathbf{G}} F_{\mathbf{G}} |u^{\star}_{\mathbf{k}} v_{\mathbf{k}}|^2 \delta(\mathbf{k} + \mathbf{k}' -$$

(correlation → Pea

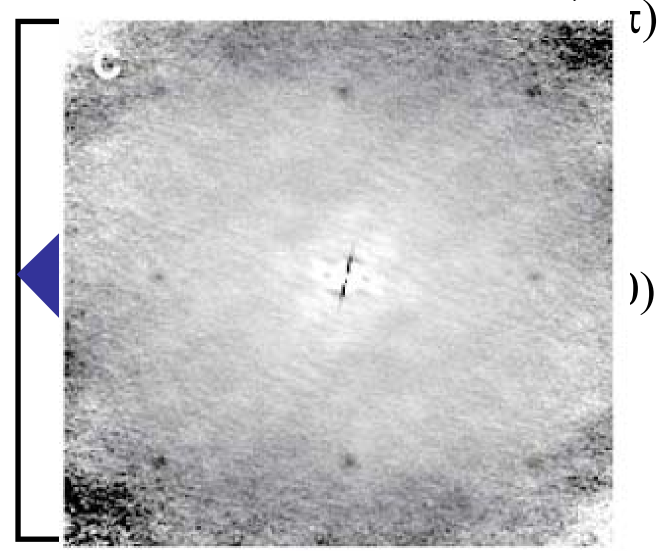


Spin/charge density wave (p-h):

$$\prod_{\mathbf{k}} (u_{\mathbf{k}} + v_{\mathbf{k}} c^{\dagger}_{\mathbf{k}+\mathbf{q}\sigma} c_{\mathbf{k}\sigma'}) \,|\, FS \,\rangle$$

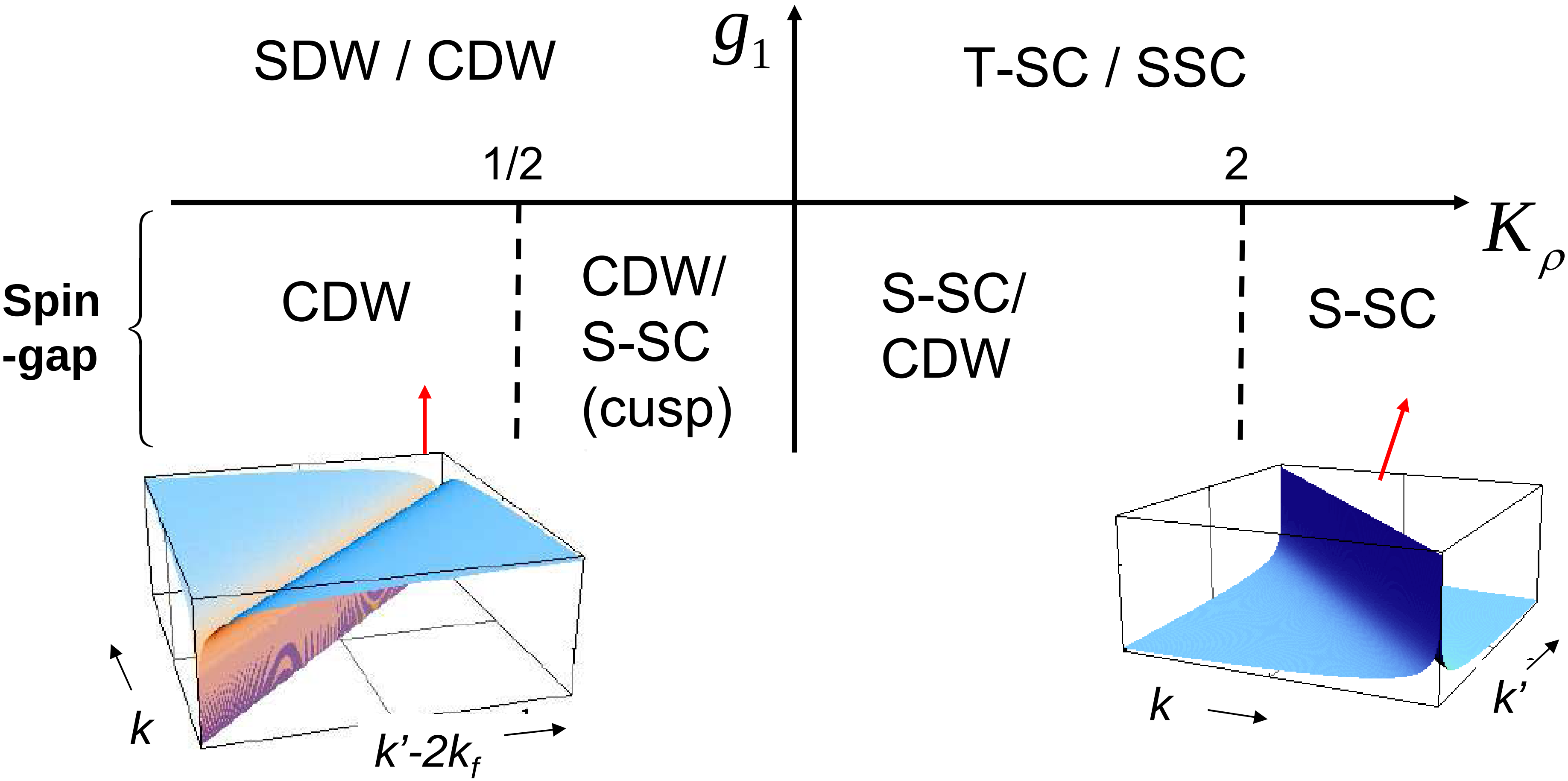
$$\langle n_{\mathbf{k}} n_{\mathbf{k}'} \rangle_{con} = -\sum_{\mathbf{G}} F_{\mathbf{G}} |u^{\star}_{\mathbf{k}} v_{\mathbf{k}}|^2 \delta(\mathbf{k} - \mathbf{k}' - 2\mathbf{k}_f + \mathbf{G})$$

(anticorrelation → dips)



T. Rom et al. Nature (2006)

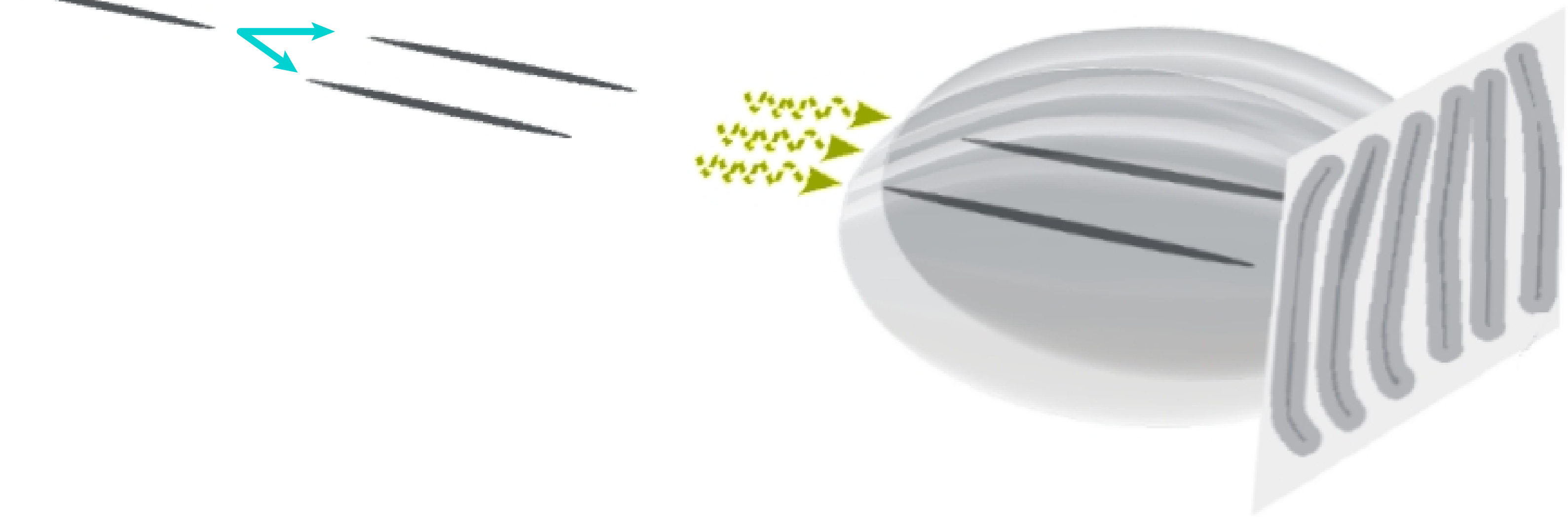
Beyond mean field: itinerant fermions in 1d



$$\langle n_{\sigma\mathbf{k}} n_{\sigma\mathbf{k}'}\rangle_{con} \sim -|k-k'-2k_f|^{K_\rho-1} \qquad \langle n_{\uparrow\mathbf{k}} n_{\downarrow\mathbf{k}'}\rangle_{con} \sim |k+k'|^{-1+1/K_\rho}$$

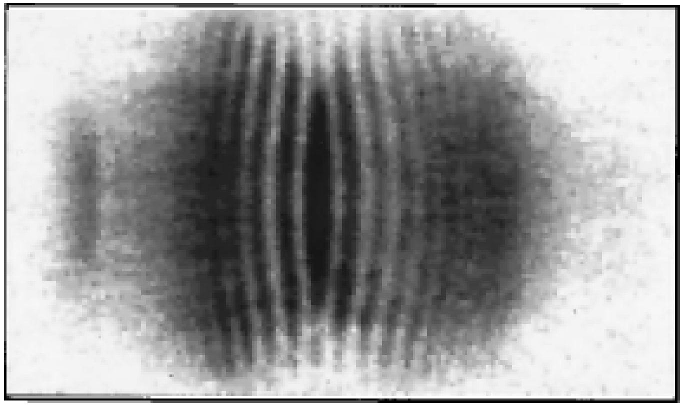
L. Mathey, EA, A. Vishwanath cond-mat/0507108

Interference of independent condensates



Interference of independent perfect condensates

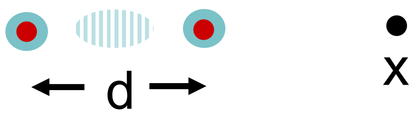
Andrews et. al., Science (1997)



We need a formula for the pattern we see.
Essentially this is a snapshot of the density after long time of expansion.
So, let us start by computing the expectation value of the density.

Interference of two independent perfect condensates

$$\psi(x,t) \sim a_1 e^{idm/2\hbar t} + a_2 e^{-idm/2\hbar t}$$



$$\hat{\rho}(x,t) \sim 2N + \left[e^{i\frac{md}{\hbar t}x} a_2^\dagger a_1 + H.c. \right]$$

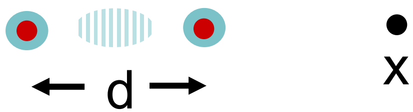
Average fringe amplitude:

$$\langle A_{fr} \rangle \equiv \langle a_2^\dagger a_1 \rangle = N \langle e^{i(\varphi_2 - \varphi_1)} \rangle = 0 \qquad \langle \hat{\rho}(x) \rangle \sim 2N$$

Clouds 1 and 2 do not have a well defined phase difference.
Does this mean we should not see an interference pattern?

Answer: Javanainen-Yoo (96); Castin-Dalibard (97) , …
I will show a different formulation, easily generalizable to more complex cases

Interference of two perfect independent condensates



$$\hat{\rho}(x) \sim 2N + \left[e^{i\frac{md}{\hbar t}x} a_2^\dagger a_1 + H.c. \right]$$

Single experiment = one possible measurement outcome (not <ρ>)!

We can measure an expectation value of an operator in a single experiment <u>only if</u> the relative fluctuation of that operator is small.

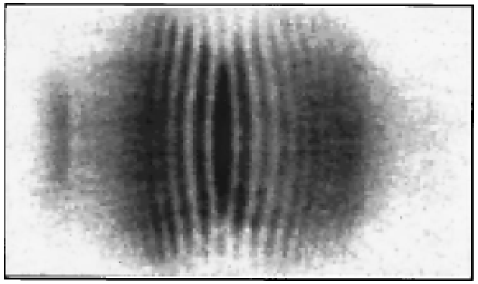
Fluctuations in density:

$$\langle \delta\rho(x)\delta\rho(x') \rangle \sim \underbrace{\langle a_1^\dagger a_2 a_2^\dagger a_1 \rangle}_{\langle |A_{fr}|^2 \rangle \sim N^2} e^{i\frac{md}{\hbar t}(x-x')} + c.c. = N(N+1)\cos\left[\frac{md}{\hbar t}(x-x')\right]$$

Interference of two perfect independent condensates

$$\hat{\rho}(x) \sim 2N + \left[e^{i\frac{md}{\hbar t}x} a_2^\dagger a_1 + H.c. \right]$$

$$\langle \delta\rho(x)\delta\rho(x')\rangle \sim N^2 \cos\left[\frac{md}{\hbar t}(x - x')\right]$$



$$\frac{\sqrt{\langle \delta\rho^2 \rangle}}{\langle \rho \rangle} \sim 1$$ → <ρ(x)> not measured in a single shot !

$$\frac{\sqrt{\langle [\delta\rho\,\delta\rho']^2 \rangle - \langle \delta\rho\,\delta\rho' \rangle^2}}{\langle \delta\rho\,\delta\rho' \rangle} \sim \frac{1}{\sqrt{N}}$$ → But <δρ(x)δρ(x')> is !

Interference fringes = oscillations in the density-density correlation.

The phase of the oscillations is undetermined (random from shot to shot)

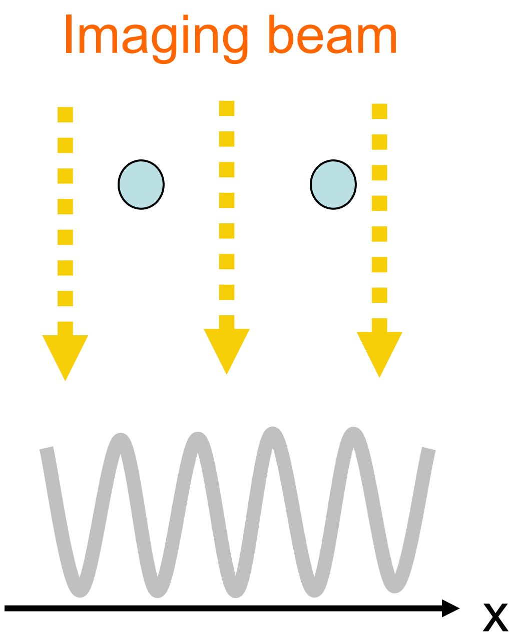
But their magnitude is the same every time (up to corrections ~ $1/N^{1/2}$)

Distinction between coherent states and a pair of number states betrayed by the pre-factor of the $1/N^{1/2}$ correction, Polkovnikov EPL (2007)

Interference of two perfect independent condensates summary

In each experimental run:

$$\rho(x) \sim N + N\cos\left(\frac{md}{\hbar t}x + \Delta\phi\right)$$



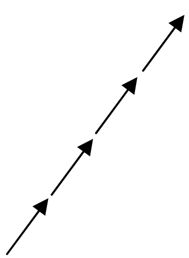
The phase changes randomly from run to run

What if the two condensates are not point like?

1. Two chains of condensates with perfect coherence within each chain

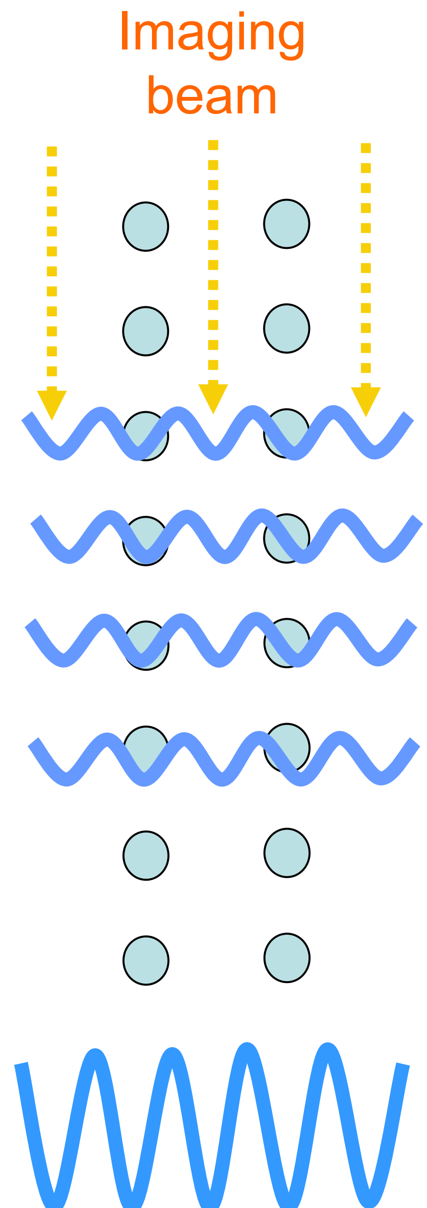
$$\Delta\phi_1 = \Delta\phi_2 = \ldots = \Delta\phi_L$$

$$\rho_{int}(x) = \sum_{i=1}^{L} N\cos(k_0 x + \Delta\phi_i)$$



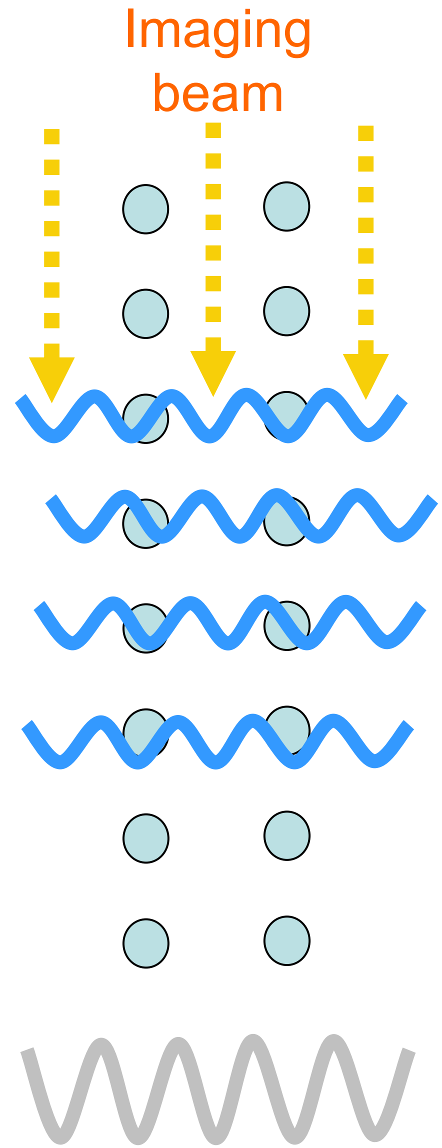
$$\sqrt{\langle |A_{fr}|^2\rangle} \propto L$$

$$\delta |A_{fr}| = 0$$

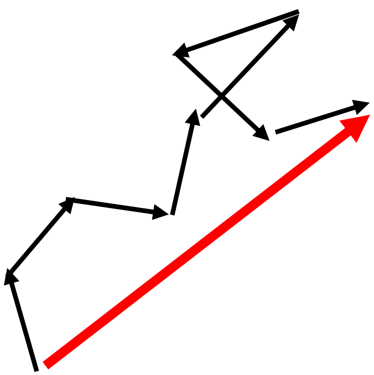


What if the two condensates are not point like?

2. Uncorrelated phases within the chains



$$\rho_{int}(x) = \sum_{i=1}^{L} N \cos(k_0 x + \Delta\phi_i)$$



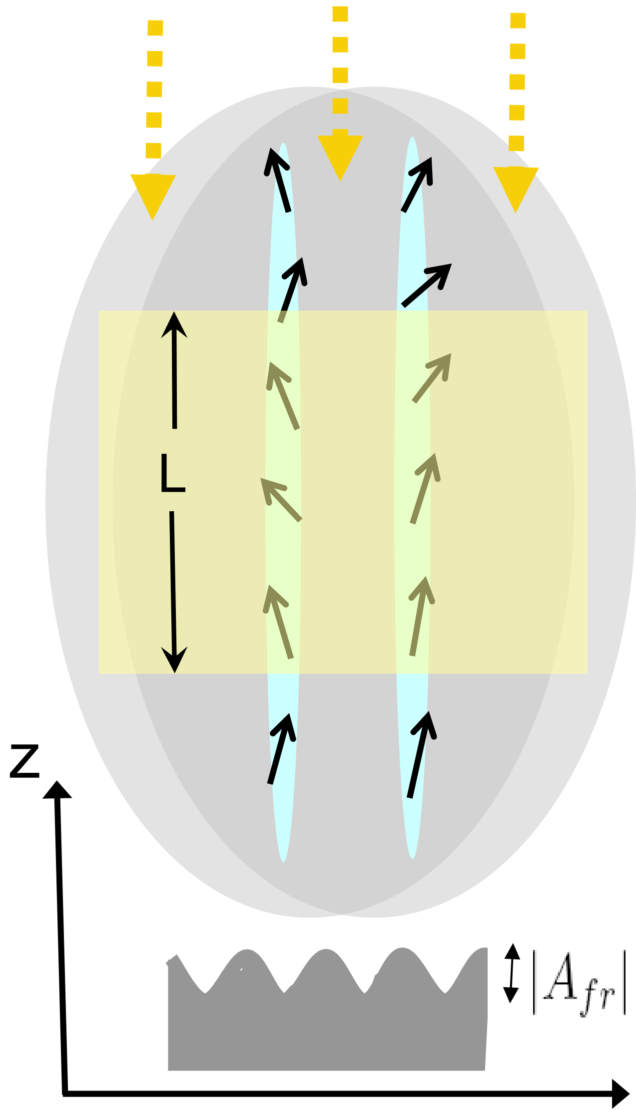
$$\sqrt{\langle |A_{fr}|^2 \rangle} \propto \sqrt{L}$$

$$\delta |A_{fr}| = \left[\langle |A_{fr}|^4 \rangle - \langle |A_{fr}|^2 \rangle^2 \right]^{1/4} \sim \sqrt{L}$$

Large fluctuations in fringe contrast.

More Generally:
Independent low dimensional condensates

$$\hat{A}_{fr} = \int_0^L dz e^{i\varphi_1(z) - i\varphi_2(z)} + H.c.$$



Fringe contrast (squared):

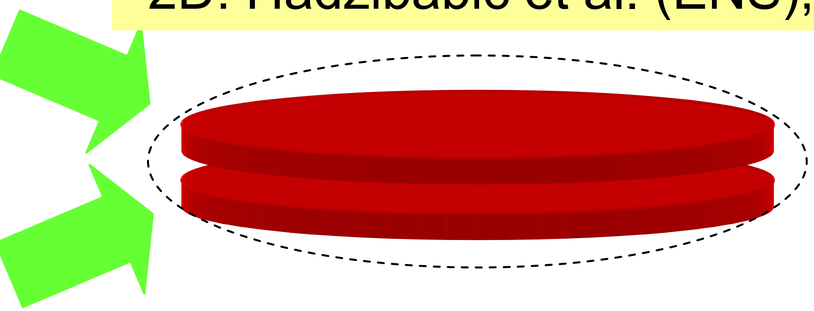
$$\langle |\hat{A}_{fr}|^2 \rangle = L \int_0^L dz \langle e^{i[\varphi_1(z) - \varphi_1(0)]} \rangle \langle e^{i[\varphi_2(0) - \varphi_1(z)]} \rangle$$

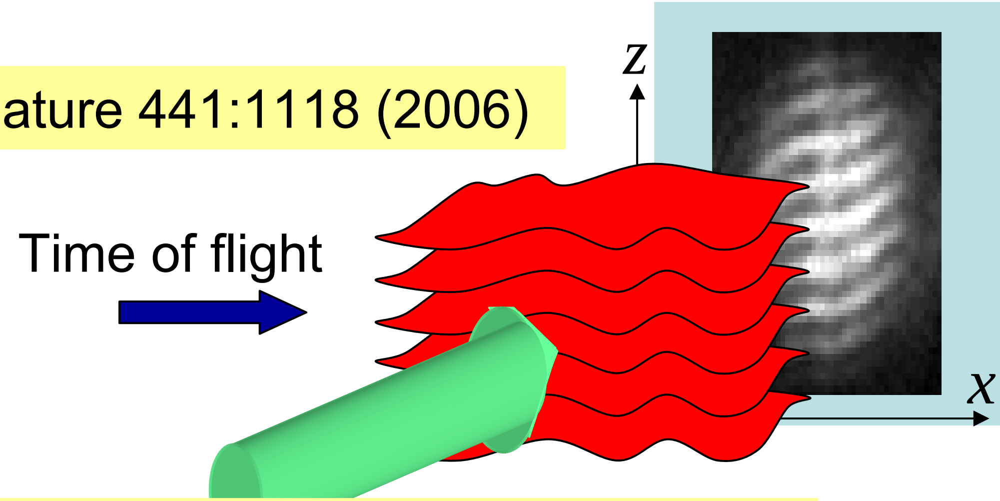
Contains information on the correlations along the condensates!

Polkovnikov, Altman and Demler PNAS (2006)

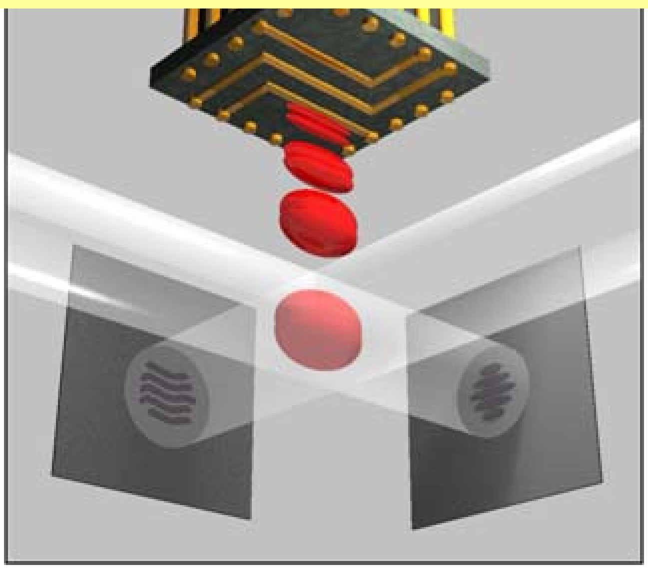
Experiments in low dimensions

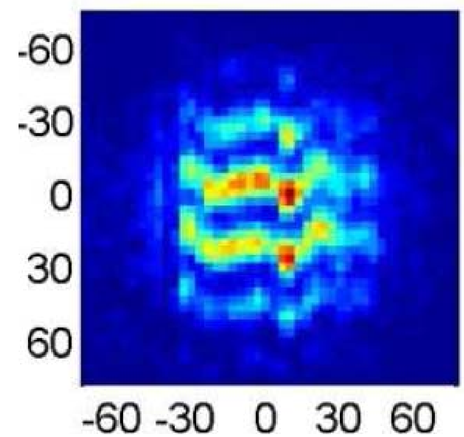
2D: Hadzibabic et al. (ENS), Nature 441:1118 (2006)





1D: Hofferberth et al. (Heidelberg → Vienna) arXiv:0710.1575





Simple examples of strongly correlated systems.
What can we learn about them from the fringe patterns and their fluctuations?

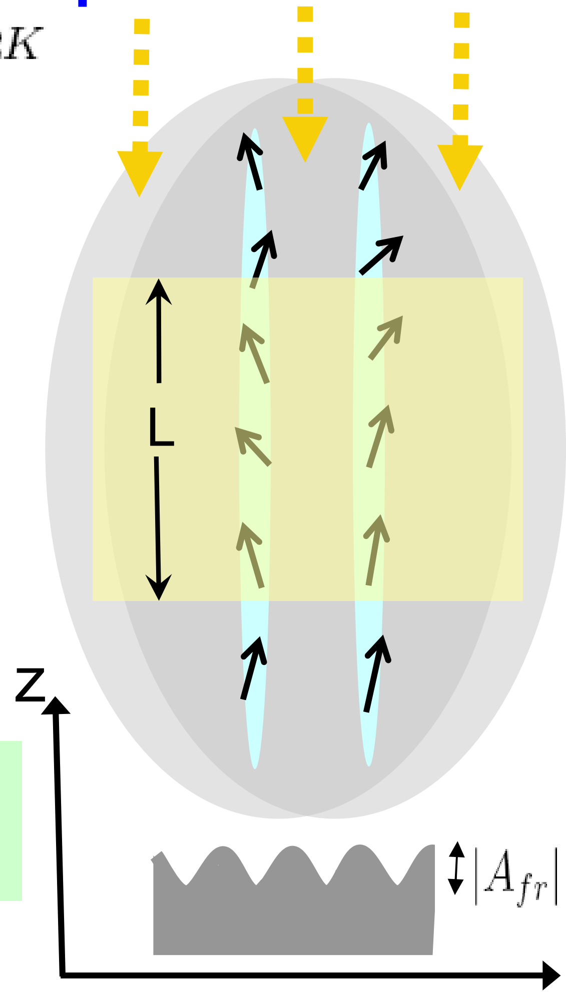
Fluctuations in a 1d Bose liquid

Quantum (T~0):

$$\langle e^{i[\varphi(z)-\varphi(0)]}\rangle \sim \left(\frac{\xi_h}{z}\right)^{1/2K}$$

$$\langle |\hat{A}_{fr}|^2\rangle = L\int_0^L dz\,\left|\langle e^{i[\varphi(z)-\varphi(0)]}\rangle\right|^2$$

$$\sim L\int_0^L \left(\frac{\xi_h}{z}\right)^{1/K} dz \sim L^{2-1/K}$$



Scaling of the fringe contrast with imaging length:
Power-law reveals the Luttinger parameter K

Non-interacting bosons $K = \infty$ $|A_{fr}| \sim L$

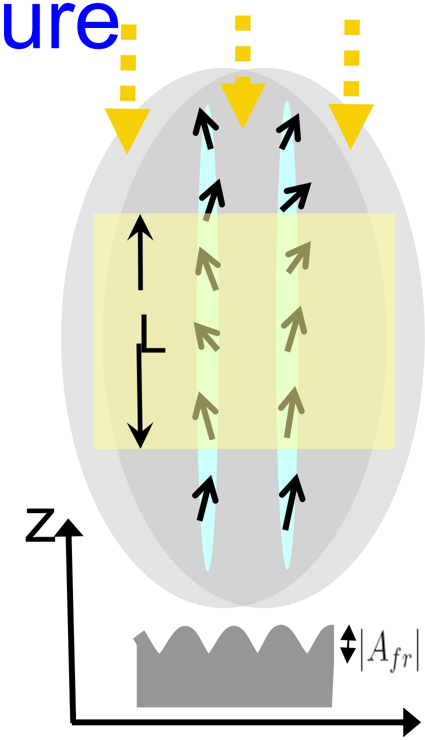
Impenetrable bosons $K = 1$ $|A_{fr}| \sim \sqrt{L}$

1d Bose liquid at finite temperature

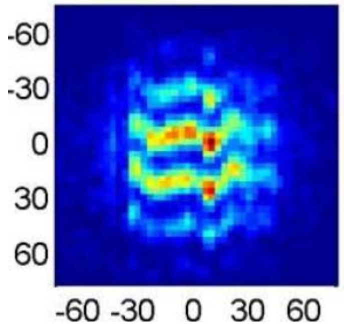
Phase correlations decay exponentially at long dist. !

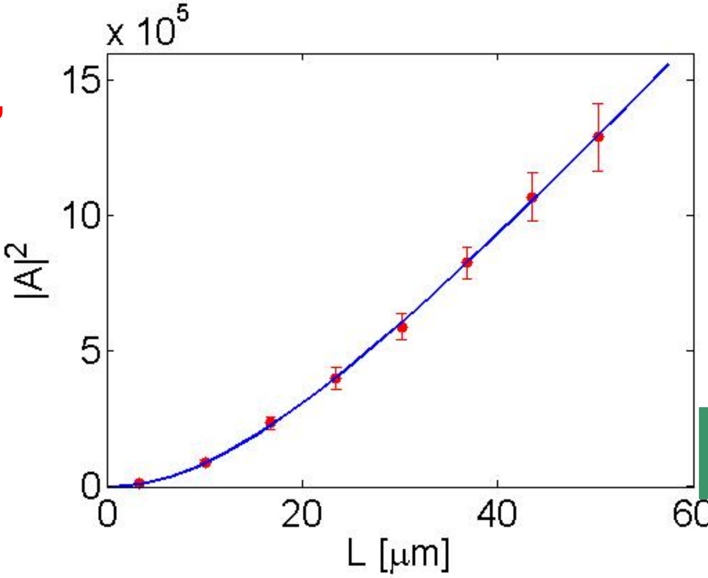
$$\langle e^{i[\varphi(z)-\varphi(0)]}\rangle \sim \left(\frac{\xi_h}{z}\right)^{1/2K} e^{-z/\xi_T} \qquad \xi_T = \frac{\hbar^2\rho}{mT}$$

$$\Rightarrow \quad \langle |A_{fr}|^2\rangle \sim L\xi_h\rho^2\left(\frac{\hbar^2\rho\xi_h}{mT}\right)^{1-\frac{1}{K}}$$



Experiments: Hofferberth, Schumm, Schmiedmayer





$n_{1d} = 60\mu\text{m}^{-1}$

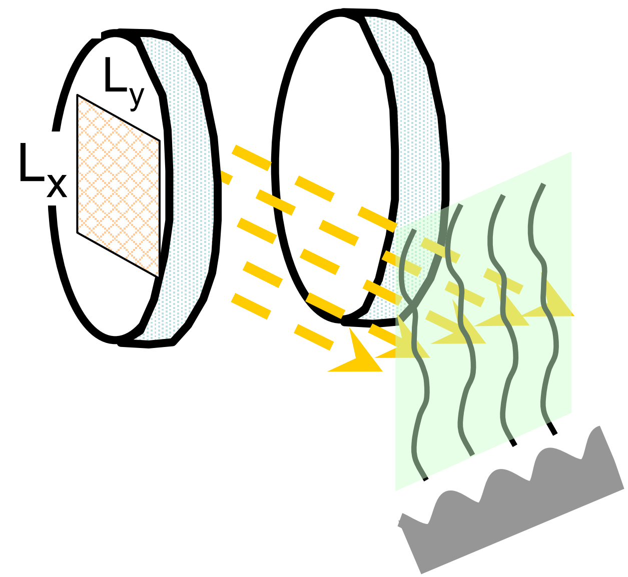
$K = 47$

$T_{fit} = 84 \pm 22$ nK

Crossover from quantum to thermal behavior as a function of length

2D - KT transition

$$\langle |A_{fr}|^2\rangle \sim L_x L_y \int_0^{L_x} dx \int_0^{L_y} dy \langle a^\dagger(x,y), a(0)\rangle^2$$



Below KT transition

$$\langle a^\dagger(\mathbf{r})a(0)\rangle \sim \rho\left(\frac{\xi_h}{r}\right)^\alpha$$

$$\langle |A_{\rm fr}|^2\rangle \sim (L_x L_y)^{2-\alpha}$$

Above KT transition

$$\langle a^\dagger(\mathbf{r})a(0)\rangle \sim e^{-r/\xi}$$

$$\langle |A_{\rm fr}|^2\rangle \sim L_x L_y$$

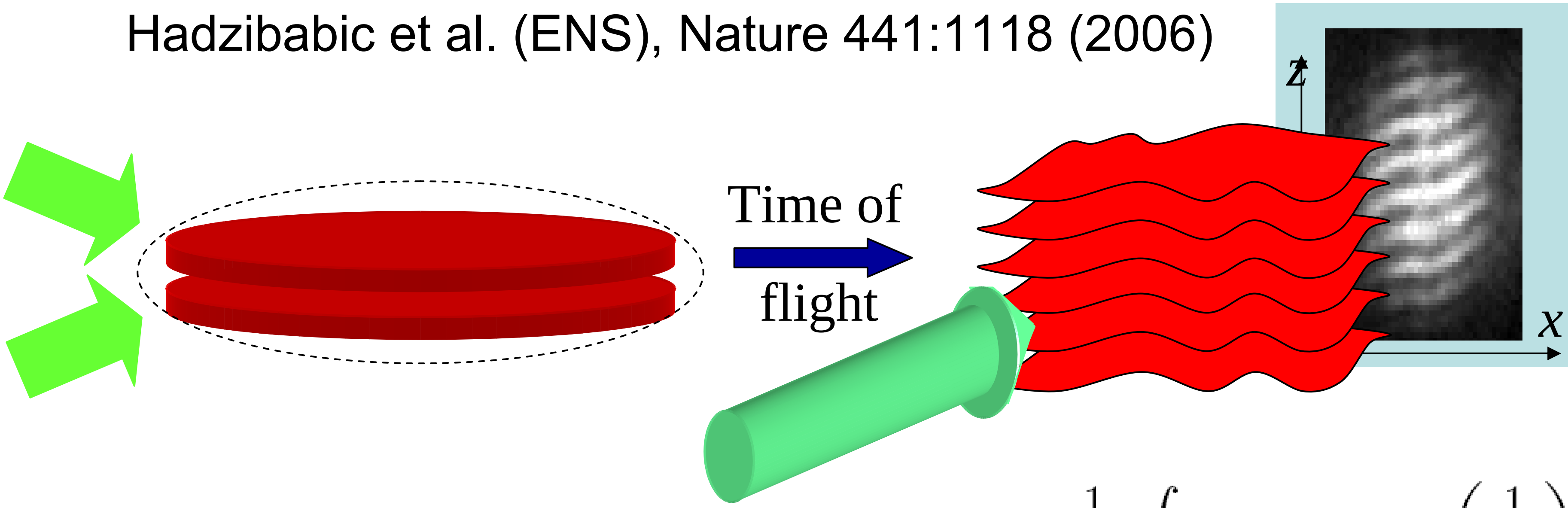
Universal jump in α at T_{KT}

$T = T_{KT}^-$ ➡ $\alpha_- = 1/4$

$T > T_{KT}$ ➡ $\alpha_+ = 1$

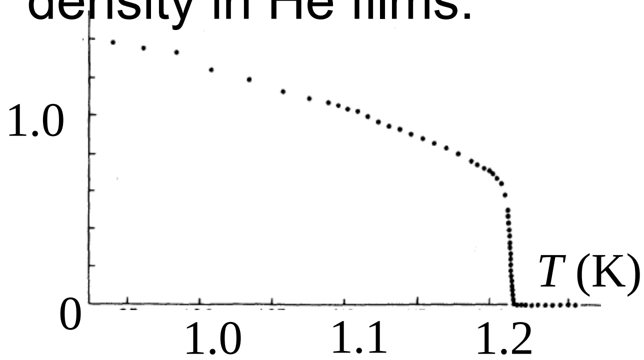
Experimental observation

Hadzibabic et al. (ENS), Nature 441:1118 (2006)

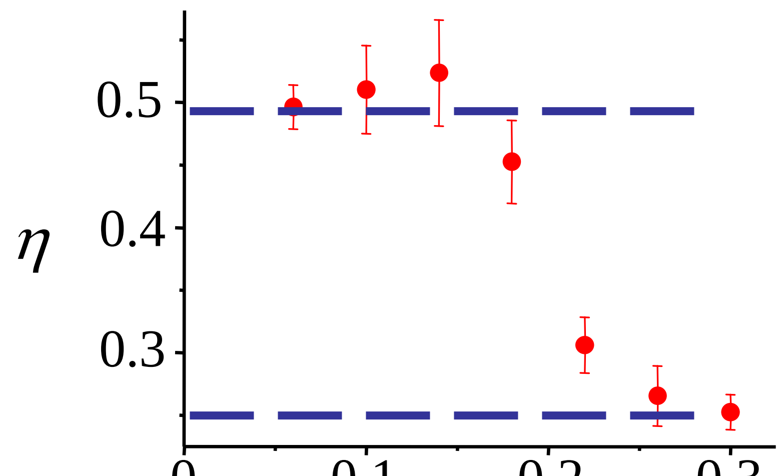


$$C^2 \approx \frac{1}{L_x} \int dx g(x,0)^2 \sim \left(\frac{1}{L_x}\right)^{2\eta}$$

Universal jump of superfluid density in He films:



Bishop and Reppy PRL (1978)



Full counting statistics of the interference amplitude

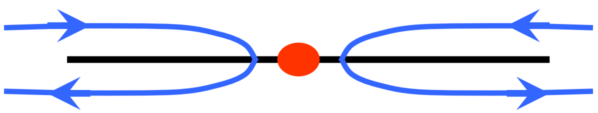
Gritsev, Altman, Demler and Polkovnikov Nature Phys. 2006

$A_{\rm fr}$ is a quantum operator. Measured value will fluctuate from shot to shot.

1d condensate T=0 :

$$\langle\, |A_{\rm fr}|^{2n}\,\rangle \;=\; \int_0^L dz_1 \ldots dz_n' \,|\,\langle\, e^{i\phi(z_1)} \ldots e^{i\phi(z_n)}\, e^{-i\phi(z_1')} \ldots e^{-i\phi(z_n')}\,\rangle|^2$$

Mapping to impurity in a 1d electron system:



$$S \,=\, \frac{\pi K}{2} \int dx\, d\tau\; \left[\, (\,\partial_\tau\, \phi\,)^2 \,+\, (\,\partial_x\, \phi\,)^2 \,\right] \,+\, 2g \int d\tau\; \cos\phi\,(\,x = 0\,,\,\tau\,)$$

$$Z_{imp}(K,g) = \sum_n \frac{g^{2n}}{(2n)!} Z_{2n}(K) = \sum_n \frac{g^{2n}}{(2n)!} \left(\frac{\langle A_{fr}^{2n} \rangle}{\langle |A_{fr}|^2 \rangle^n} \right)$$

Full distribution function of the interferece amplitude:

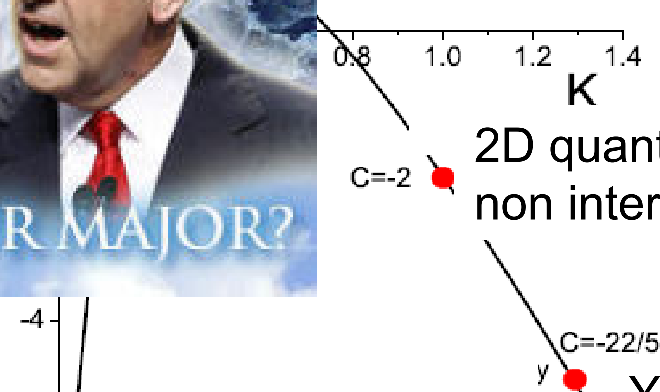
$$W(K,\alpha) = 2\int_0^\infty Z_{imp}(K,ig) J_0(2g\sqrt{\alpha}) g dg \qquad \alpha = |A_{fr}|^2 / \langle |A_{fr}|^2 \rangle$$

How to solve for $Z_{imp}(K,ig)$?

"Folks, I didn't major in math. I majored in miracles ..."
Mike Huckabee, on the campaign trail

Mapping to exotic conformal field theories relates Z_{imp} to the spectral determinant of a s… …dinger equation:

$$-\frac{d^2\Psi}{dx^2} + (x^{4K-2} + \ldots$$

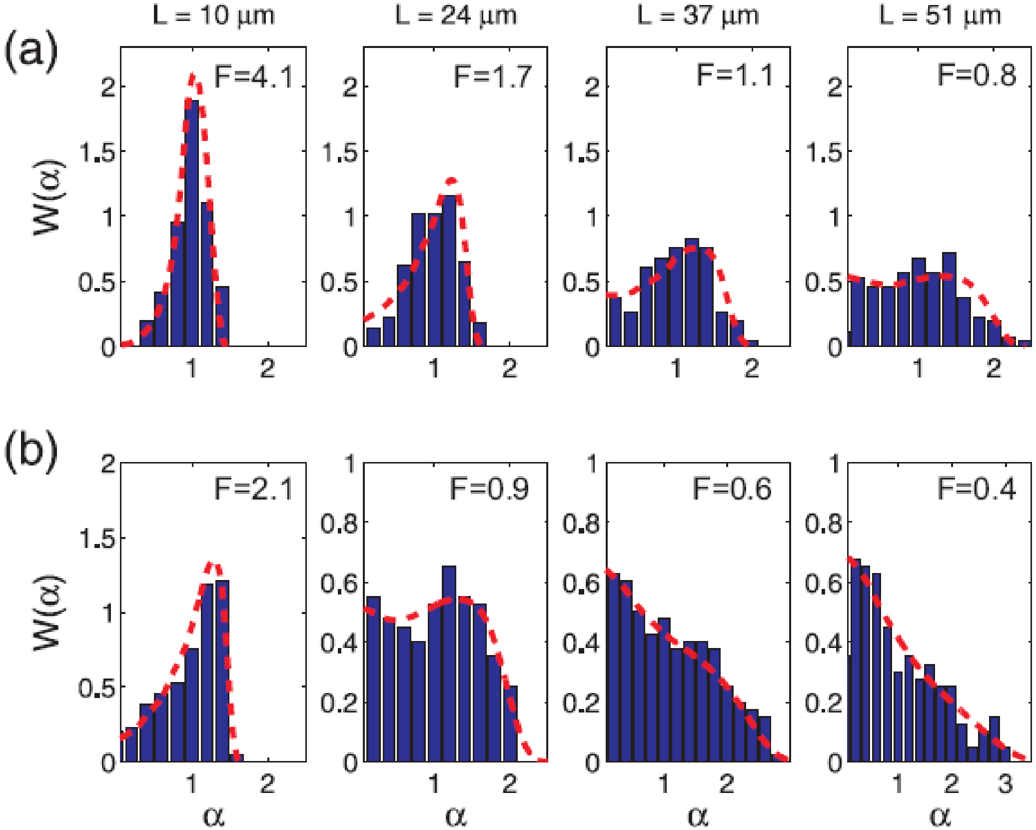


$$D(E) = \prod_{n=1}^{\infty} (1 - \ldots E_n$$

$$Z_{\rm imp}(K, ig) = D\left(\frac{g^2}{\pi^2}(4K)^{2-K^{-1}}[\Gamma(1-\frac{1}{2K})]^2 \sin^2(\frac{\pi}{2K})\right)$$

Distribution function of interference fringe contrast

Hofferberth et al., Nature Physics 4:489 (2008)



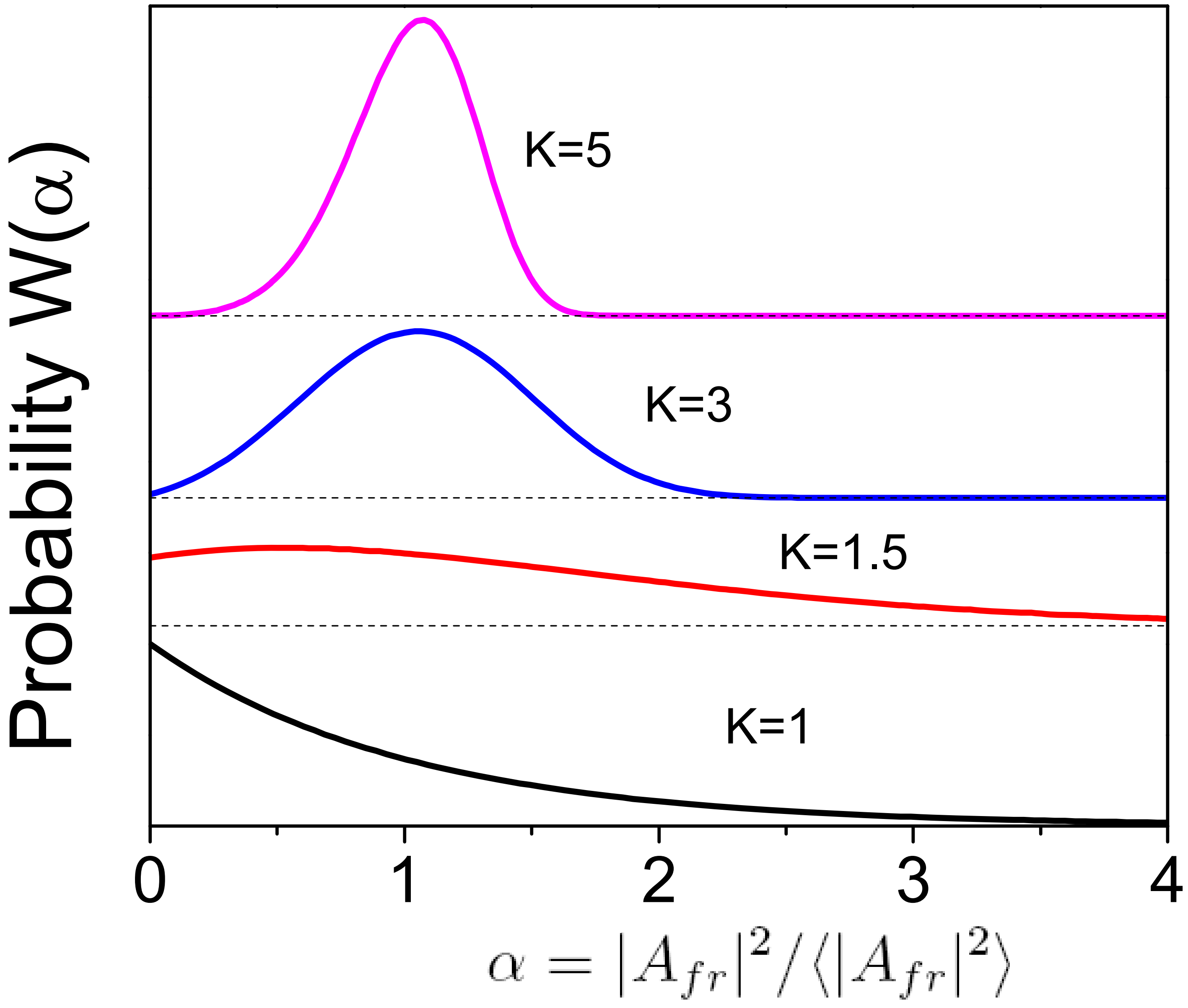
Quantum fluctuations dominate:
asymetric Gumbel distribution
(low temp. T or short length L)

Thermal fluctuations dominate:
broad Poissonian distribution
(high temp. T or long length L)

Intermediate regime:
double peak structure

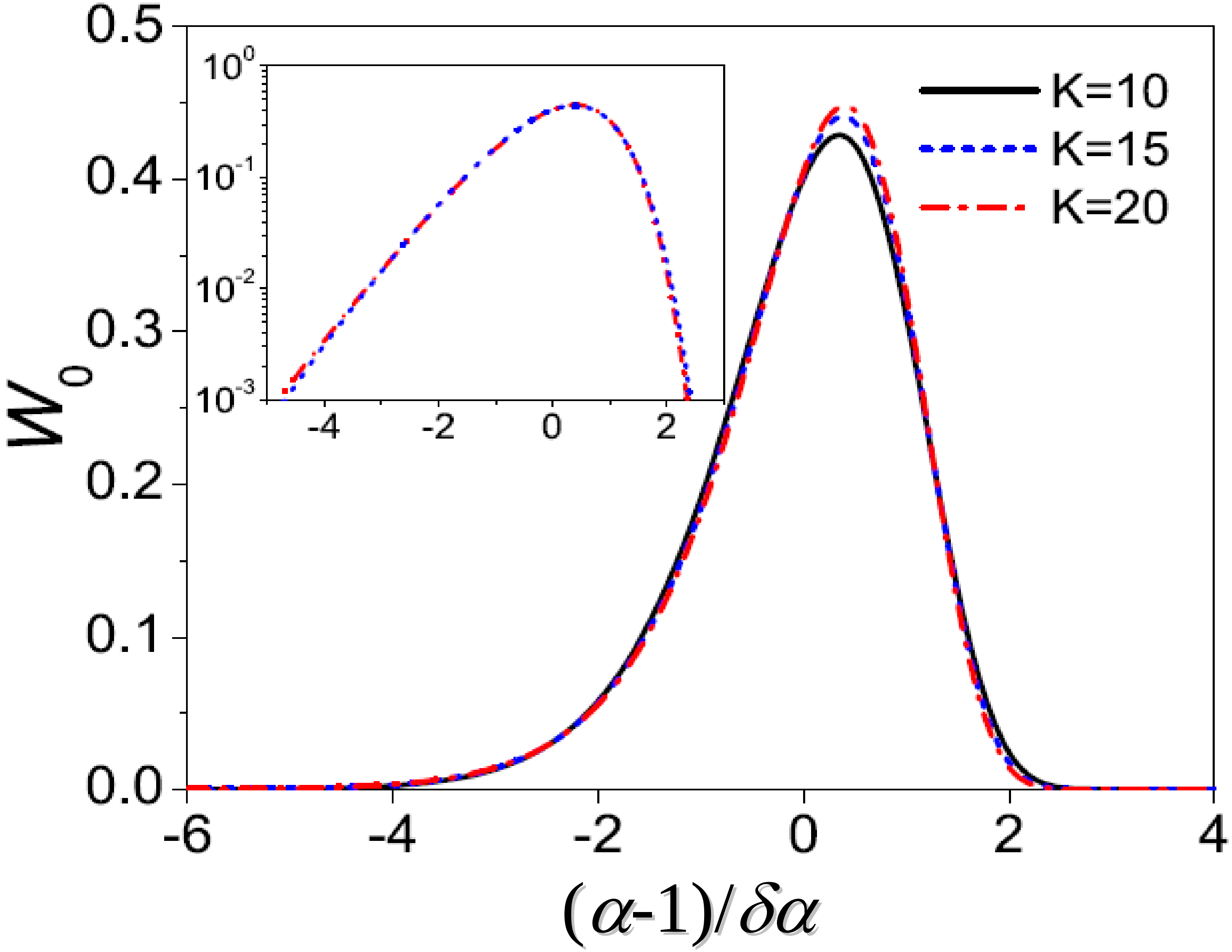
Comparison of theory and experiments: no free parameters
Higher order correlation functions can be obtained

Evolution of the distribution function.



Universal asymmetric distribution at large K

(Weak interactions)



$\delta\alpha \sim 1/K$

Summary

- Noise correlations:
  A unified scheme to detect broken symmetry as well as power-law correlations in both Bose and Fermi systems

- Interference of independent condensates:
  Easy to extract the spatial decay of the single particle density matrix.
  Statistics of fringe contrast give higher order correlations.

  Indirect quantum simulator of models in other fields:
  - Quantum impurity problems
  - Exotic conformal field theories (negative central charge)