Transport in strongly correlated systems

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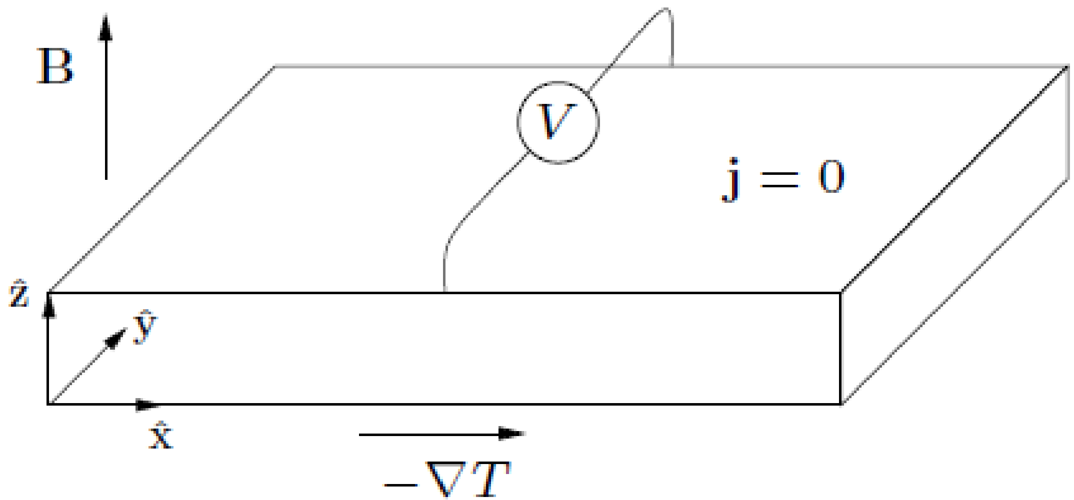
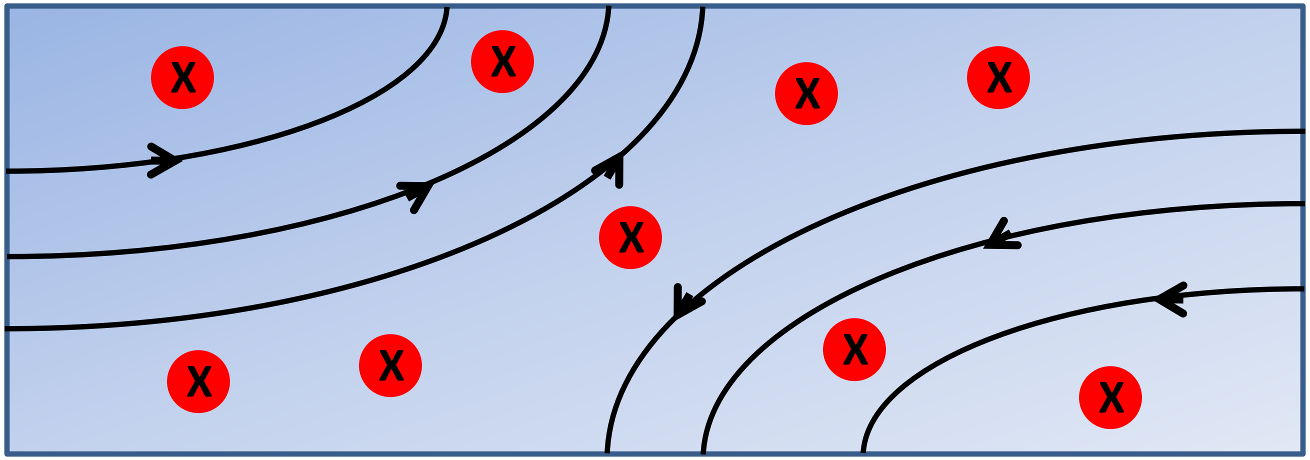
Lecture 5: Electro-magneto-thermal-transport (the Nernst effect)

- Vortices and the Nernst effect
- Experimental observations in high T_c systems
- Role of superconducting fluctuations

Nernst effect



$$\nu = \frac{1}{B}\left(\alpha_{xy}\sigma_{xx} - \sigma_{xy}\alpha_{xx}\right) / \left(\sigma_{xx}\sigma_{yy} + \sigma_{xy}\sigma_{yx}\right)$$

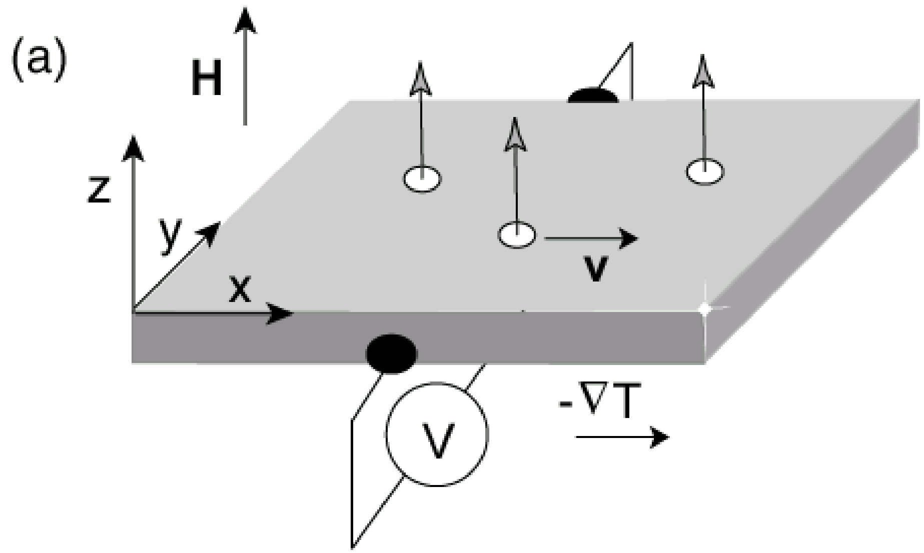


$-\nabla T \rightarrow$

Sondheimer cancellation in Drude theory $\nu = 0$

Need either $\tau(\epsilon)$ or particles and holes to get $\nu \neq 0$

or vortices will do the trick



Vortices induced by magnetic field

Move from hot end to cold end without transporting charge

Induce transverse voltage through phase slips

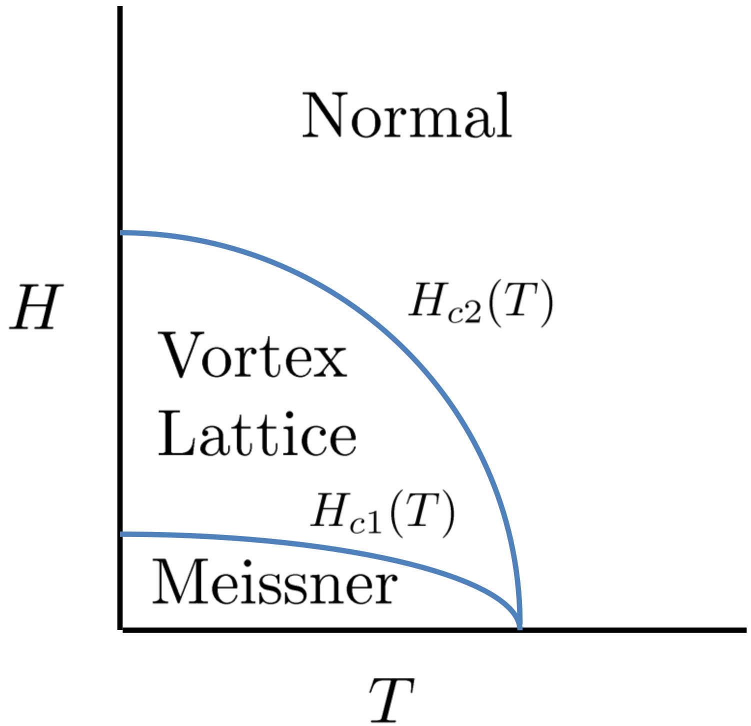
To see a Nernst effect due to vortices

Need a type II superconductor

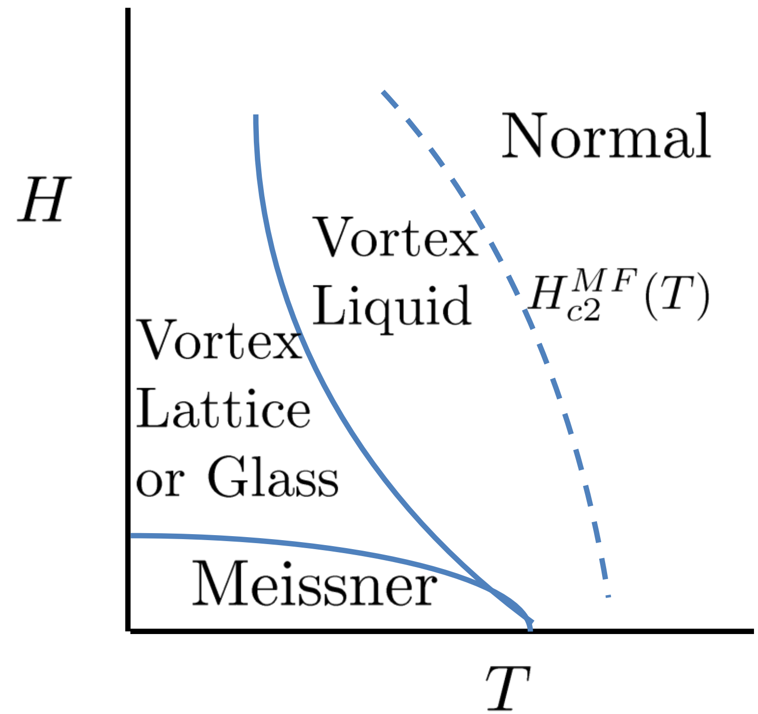
Need mobile vortices

Mobile vortices exist in the “vortex liquid” phase

Distinct from a “vortex solid” phase where vortices are immobile



Mean field phase diagram



Phase diagram with fluctuations

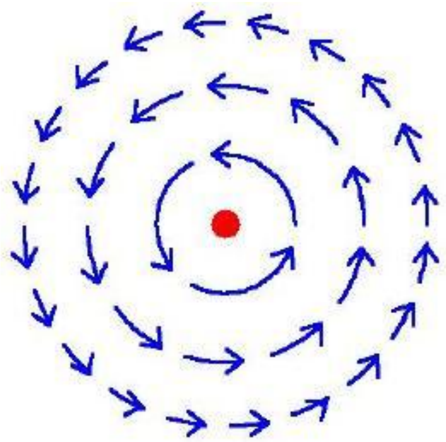
Vortex liquid → Normal state is a crossover

[D. S. Fisher, M. P. Fisher and D. A. Huse, *Phys. Rev. B* **43** 130 (1991)]

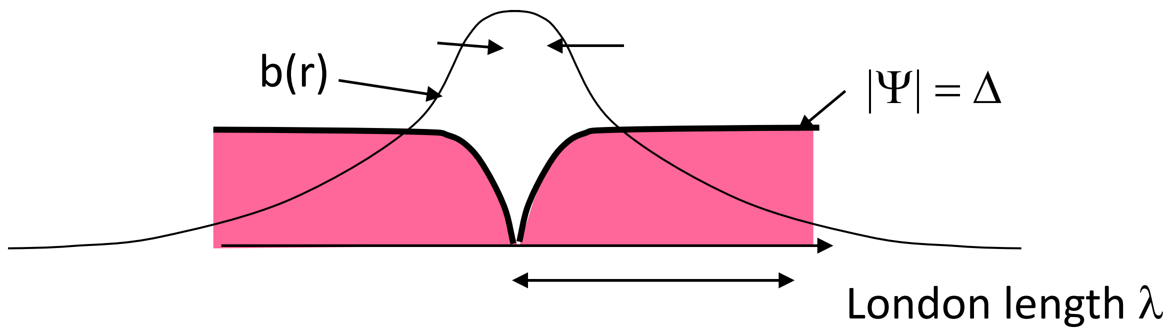
What is a vortex?

Superconducting order parameter $\Psi = |\Psi| e^{i\theta}$

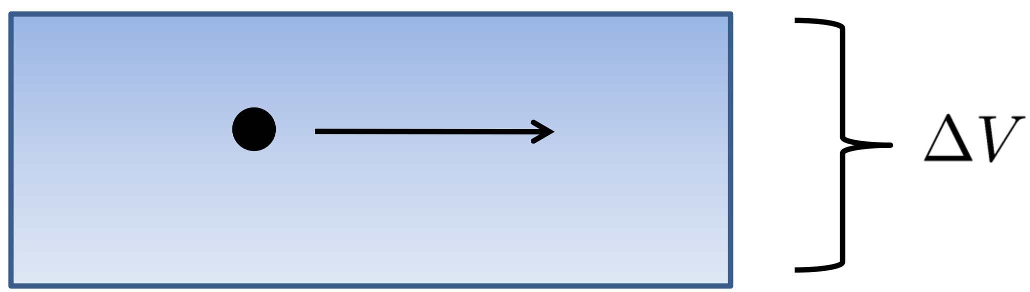
A vortex corresponds to a winding of θ by 2π going around a loop in space



Magnetic fields penetrate superconductors by creating vortices



The density of vortices is proportional to the field



Phase slips by 2π every time a vortex passes

$$\Delta V = \frac{\hbar}{2e}\dot{\theta} \text{ (Josephson relation)}$$

Moving vortices destroy phase coherence (superconductivity)

Phase coherence destroyed but $|\Psi|$ does not fluctuate strongly at $T \approx T_c$

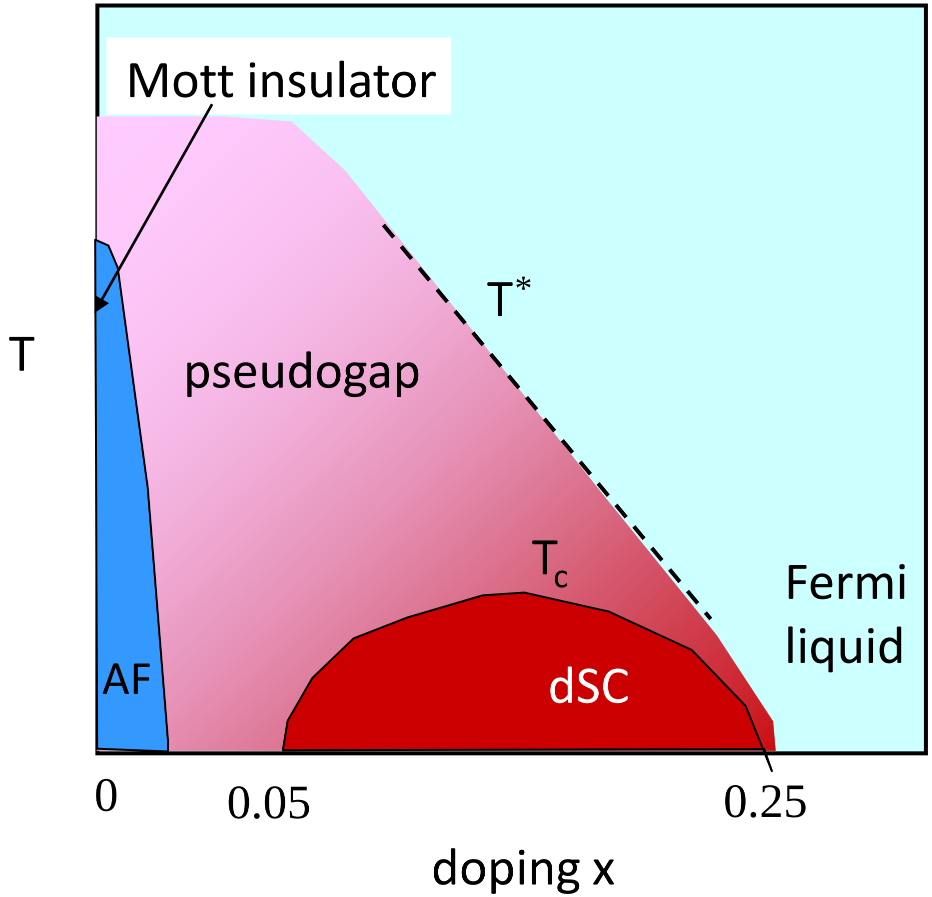
Fluctuations in $|\Psi|$ increase as T increases

At T_c^{MF}, $|\Psi| = 0$ in MF theory
(vortex liquid crosses over to normal state)

$|\Psi|$ fluctuates strongly $T \approx T_c^{MF}$ and above

$T > T_c^{MF}$ soup of phase and amplitude fluctuations
(Gaussian fluctuations)

Why do we care about T_c^{MF}?



Pseudogap temperature T^*

Large change in spin susceptibility, specific heat etc.

Fermi arcs

Can the physics of the pseudogap be mostly attributed to strong phase fluctuations?

[V. J. Emery and S. A. Kivelson, *Nature* **374** 434 (1995)]

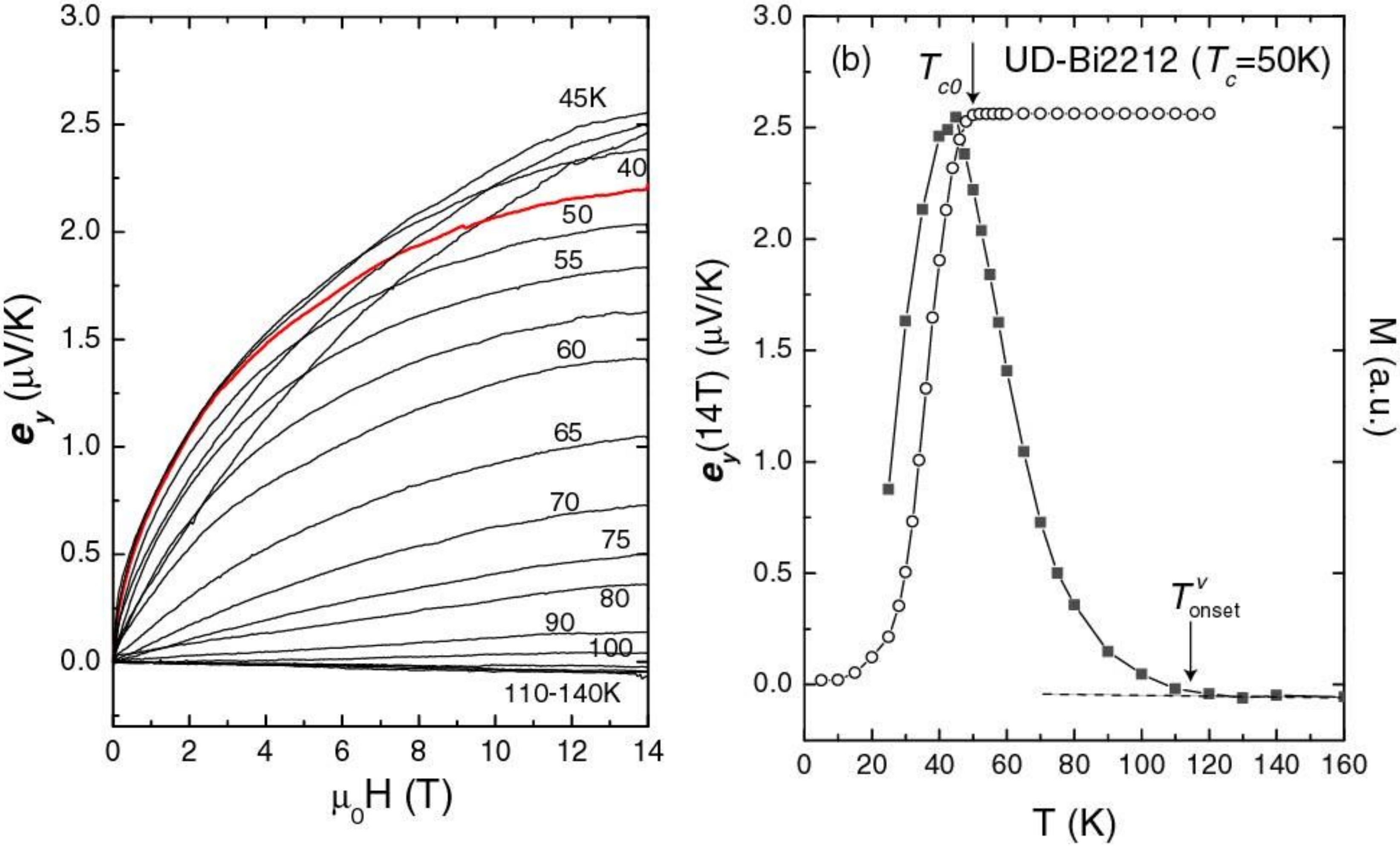
If so, how is T^* related to T_c^{MF}?

Do only mobile vortices or phase fluctuations produce a "large" Nernst effect?

If so, can it be used to bound the "vortex liquid" by extracting a temperature scale?

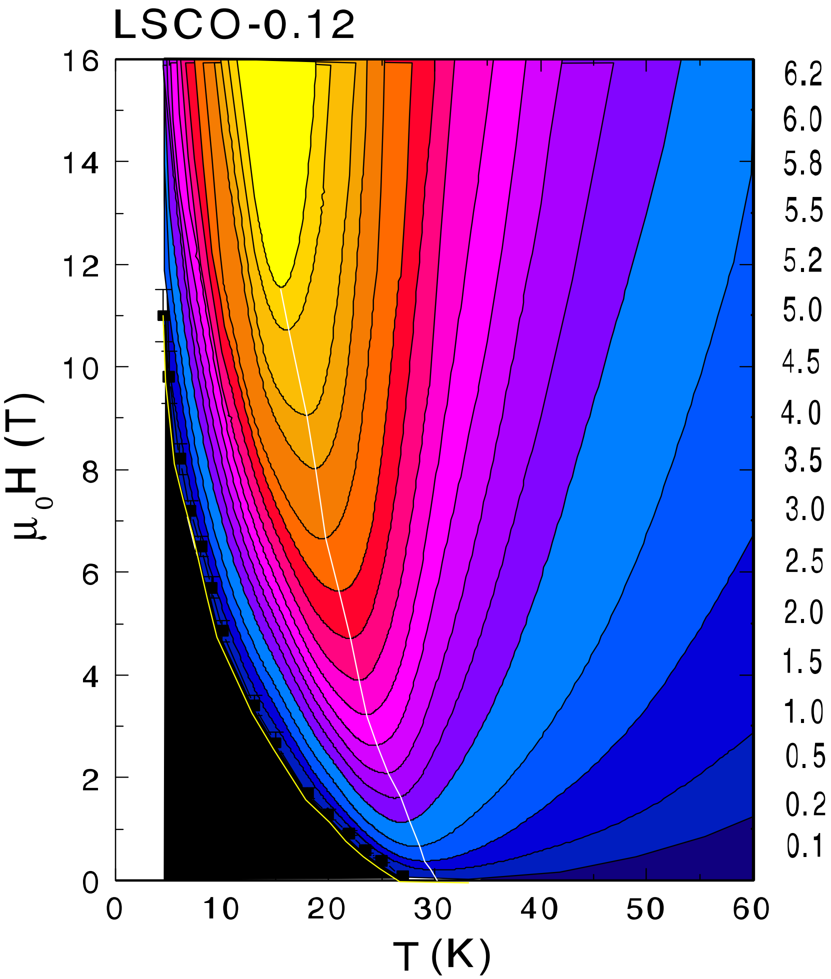
Can Gaussian fluctuations give you a "large" Nernst effect?

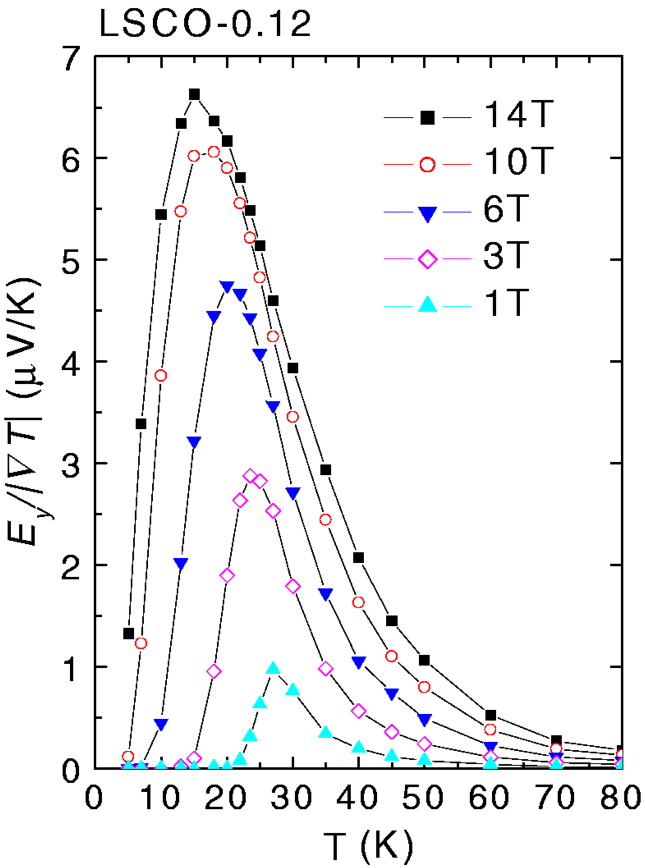
Underdoped BSSCO (2212) $T_c = 50$K



[Y. Wang, L. Li and N. P. Ong, *Phys. Rev. B* **73** 024510 (2006)]

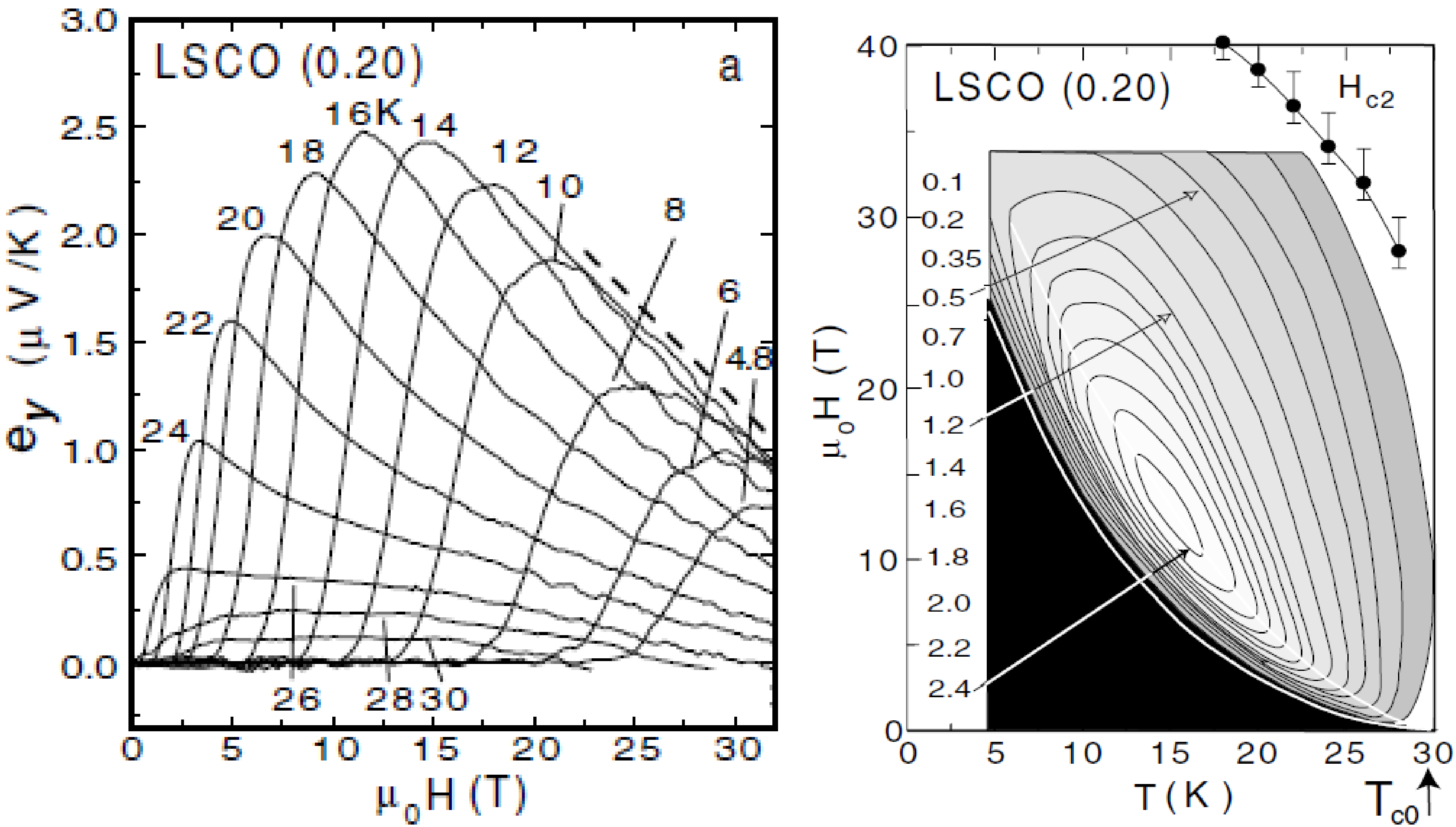
Underdoped LSCO $T_c = 29\text{K}$



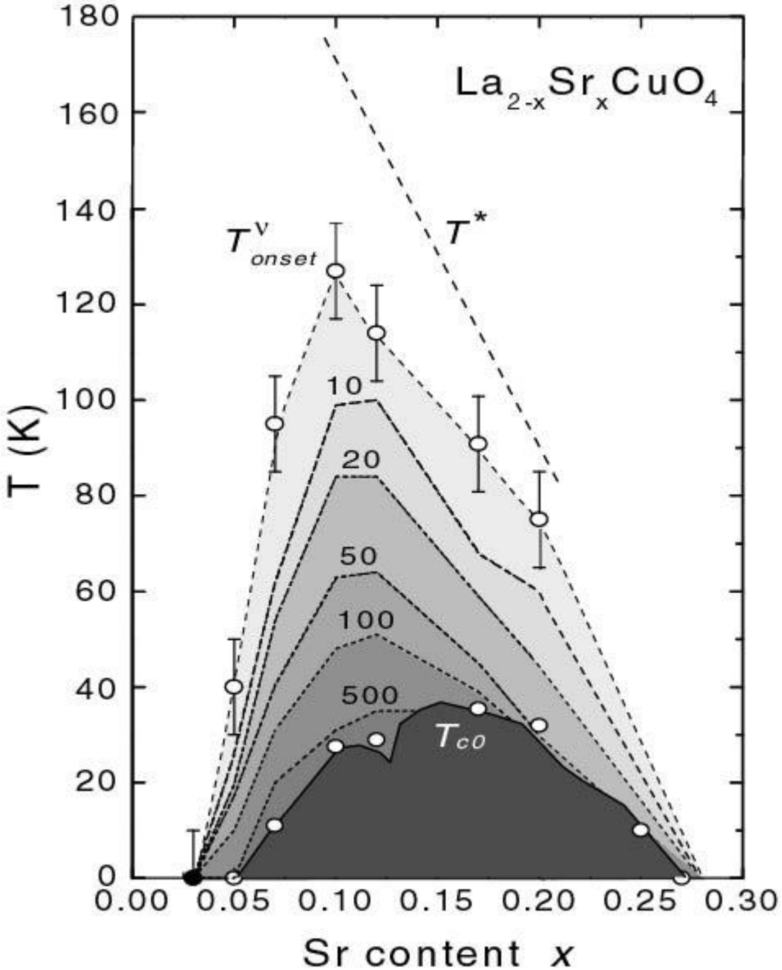


[Z. A. Xu *et. al*, *Nature* **406** 486 (2000)]

Overdoped LSCO $T_c = 29\text{K}$



[Y. Wang *et. al*, *Phys. Rev. B* **64** 224519 (2001)]



Experimental temperature scale from Nernst effect T_{onset}

T^* determined from other experiments (NMR relaxation etc.)

$T_{onset} < T^*$

At T_{onset}, Nernst signal starts getting larger than expected due to quasiparticle background

[Y. Wang, L. Li and N. P. Ong, *Phys. Rev. B* **73** 024510 (2006)]

What about the relation between T_c^{MF} and T_{onset}?

In some sense both can be used to define the bound on the vortex liquid region.

Neither is a particularly sharp scale

T_C^{MF} is a parameter in a Ginzburg-Landau (GL) theory

Calculate the Nernst effect from a GL theory

If such a theory fits experiment, can it be used to extract T_c^{MF} which can then be compared to T^* and T_{onset}?

The Nernst effect is a dynamic phenomenon

Need dynamics

Simplest dynamics: Time Dependent Ginzburg Landau (TDGL)

Simple relaxational dynamics that gives canonical distribution in equilibrium

No conserved quantities

Also known as model A dynamics

Ψ is a broken symmetry field

It is different from conserved densities like $n(\mathbf{r})$ and $h(\mathbf{r})$

$$\mathcal{F} = \int d\mathbf{r} \left[\frac{\hbar^2}{2m^*} \left| \left(\nabla - i\frac{e^*}{\hbar}\mathbf{A} \right) \Psi \right|^2 + a(T)|\Psi|^2 + b|\Psi|^4 \right]$$

$$\tau \left(\frac{\partial}{\partial t} + i\frac{e^*}{\hbar}\phi \right) \Psi = -\frac{\delta \mathcal{F}}{\delta \Psi^*} + \zeta$$

$$\langle \zeta^*(\mathbf{r}, t)\zeta(\mathbf{r}', t') \rangle = 2T\Re e(\tau)\delta(\mathbf{r} - \mathbf{r}')\delta(t - t')$$

No dynamics for $\mathbf{A}$ since $\kappa = \lambda/\xi$ is large

$$\nu = \frac{1}{B}\left(\alpha_{xy}\sigma_{xx} - \sigma_{xy}\alpha_{xx}\right) / \left(\sigma_{xx}\sigma_{yy} + \sigma_{xy}\sigma_{yx}\right)$$

$$\approx \frac{1}{B}\frac{\alpha_{xy}}{\sigma_{xx}} \text{ (Experimentally } \frac{\sigma_{xy}}{\sigma_{xx}} \text{ is small)}$$

This can be achieved in the dynamics by setting $\Im m(\tau) = 0$ (particle-hole symmetry, $\alpha_{xx} = \sigma_{xy} = \kappa_{xy} = 0$)

Further α_{xy} is independent of $\Re e(\tau)$

Calculate α_{xy} and compare to experiment

Experimentally α_{xy} can be obtained from the Nernst signal and magnetoresistance

Currents

$$\mathbf{J} = -i\frac{e^*\hbar}{2m^*}\left\langle \Psi^*\left(\nabla - i\frac{e^*}{\hbar}\mathbf{A}\right)\Psi\right\rangle + \text{ c.c.}$$

$$\mathbf{J}_E = \frac{\hbar^2}{2m^*}\left\langle\left(\frac{\partial}{\partial t} - i\frac{e^*}{\hbar}\phi\right)\Psi^*\left(\nabla - i\frac{e^*}{\hbar}\mathbf{A}\right)\Psi\right\rangle + \text{ c.c.}$$

Since model A has no conservation laws, usual derivation does not work

$\mathbf{J}$ and $\mathbf{J}_E$ can be obtained from hydrodynamic arguments

[A. Schmid, *Phys. Kondens. Mater* **5** 302 (1966)]

Also from within BCS theory

Magnetization currents

$$\mathbf{J}_{\mathrm{mag}}(\mathbf{r}) = -\frac{\partial \mathbf{M}}{\partial \mu}\times\nabla\mu - \frac{\partial \mathbf{M}}{\partial T}\times\nabla T - \mathbf{M}\times\nabla\psi$$

$$\mathbf{J}^{E}_{\mathrm{mag}}(\mathbf{r}) = -\frac{\partial \mathbf{M}^{E}}{\partial \mu}\times\nabla\mu - \frac{\partial \mathbf{M}^{E}}{\partial T}\times\nabla T - \mathbf{M}\times\nabla\phi - 2\mathbf{M}^{E}\times\nabla\psi$$

Subtract the right magnetization currents to get transport currents

Check Onsager relations

Magnetization currents are not small

Account for 2/3 of total heat current!!

Analytic results obtainable in the Gaussian approximation

Valid for $T > T_c^{MF}$. Ignore quartic term in $\mathcal{F}$

Equivalent to considering only Aslamazov-Larkin diagrams in BCS microscopics

[I. Ussishkin, *Phys. Rev. B* **68** 024517 (2003)]

$$\boxed{T_c = T_c^{MF} \text{ in this case}}$$

Main result

α_{xy} for small B

2D	3D	intermediate
$\frac{1}{6\pi}\frac{e^*}{\hbar}\frac{\xi^2}{l_B^2}$	$\frac{1}{12\pi}\frac{e^*}{\hbar}\frac{\xi}{l_B^2}$	$\frac{1}{6\pi}\frac{e^*}{\hbar}\frac{\xi_{ab}^2}{l_B^2}\frac{1}{\sqrt{1+(2\xi_c/s)^2}}$

$$l_B = (\hbar c/eB)^{1/2}$$

$$\xi \sim 1/\sqrt{(T - T_c)}$$

s - interlayer spacing

[I. Ussishkin, S. Sondhi and D. A. Huse, *Phys. Rev. Lett.* **89** 287001 (2002)]

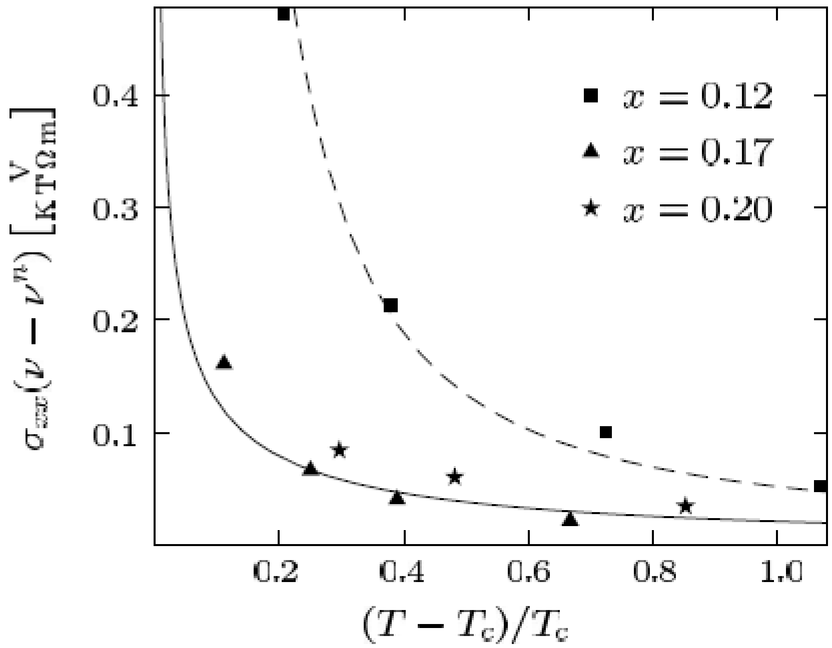


FIG. 1. Points are $\sigma_{xx}(\nu - \nu^n)$ for different samples of $La_{2-x}Sr_xCuO_4$ [22], with $x = 0.12$ (underdoped, $T_c = 29$ K), $x = 0.17$ (near optimal doping, $T_c = 36$ K), and $x = 0.2$ (overdoped, $T_c = 27$ K). The solid line is the theoretical value of α_{xy}^{SC}/B, using $\xi_{ab}^{(0)} = 30$ Å and an anisotropy of $\gamma = 20$. The dashed line is obtained using a Hartree approximation

[I. Ussishkin, S. Sondhi and D. A. Huse, *Phys. Rev. Lett.* **89** 287001 (2002)]

Gaussian fluctuations are able to explain the low field data above T_c for overdoped samples well

Large Nernst effect appears to be because of

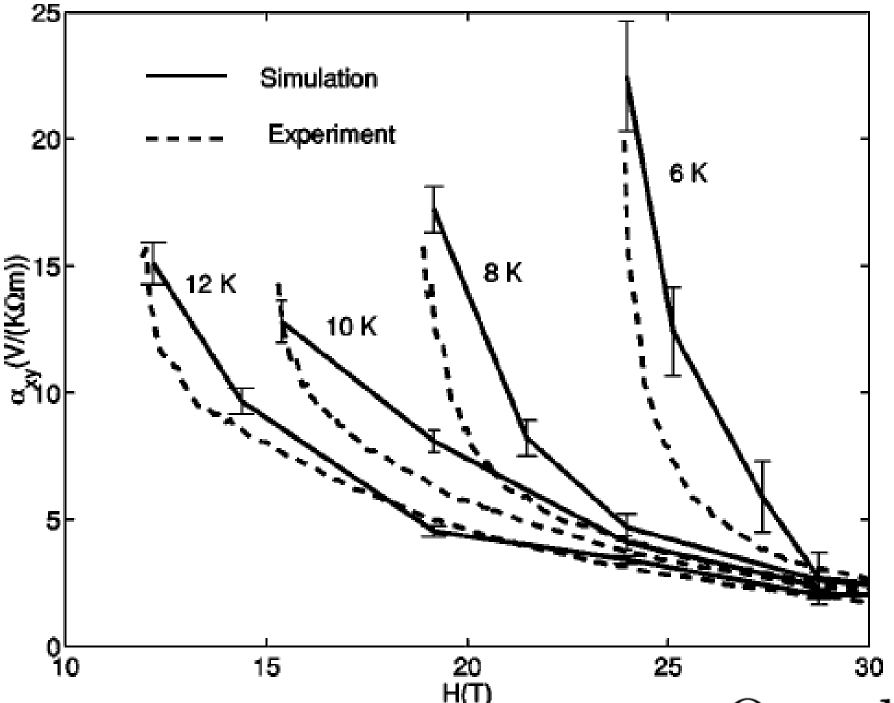
- Large anisotropy making the system 2D like
- Small σ_{xx} since normal state resistance is high

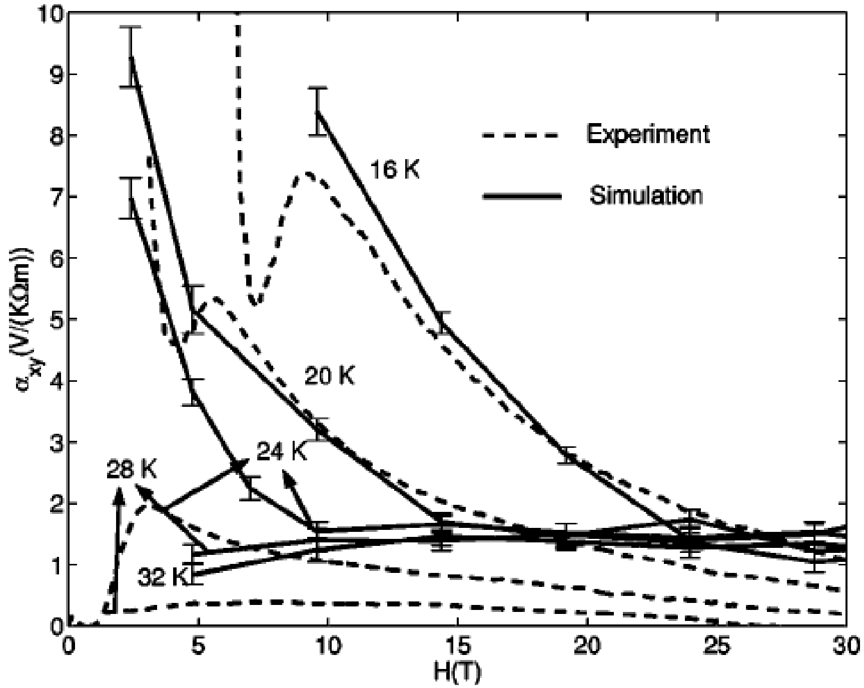
In the Gaussian calculation $T_c = T_c^{MF}$

How do we improve on this and look at high B and low T?

Keep $|\Psi|^4$ term in $\mathcal{F}$ and evaluate α_{xy} numerically using TDGL

$T_c \neq T_c^{MF}$ $\qquad$ T_c^{MF} now a fitting parameter





Overdoped LSCO

$T_c = 29\text{K}$, $T_c^{MF} = 39\text{K}$, $T_{onset} \approx 75\text{K}$, $T^* \approx 90\text{K}$

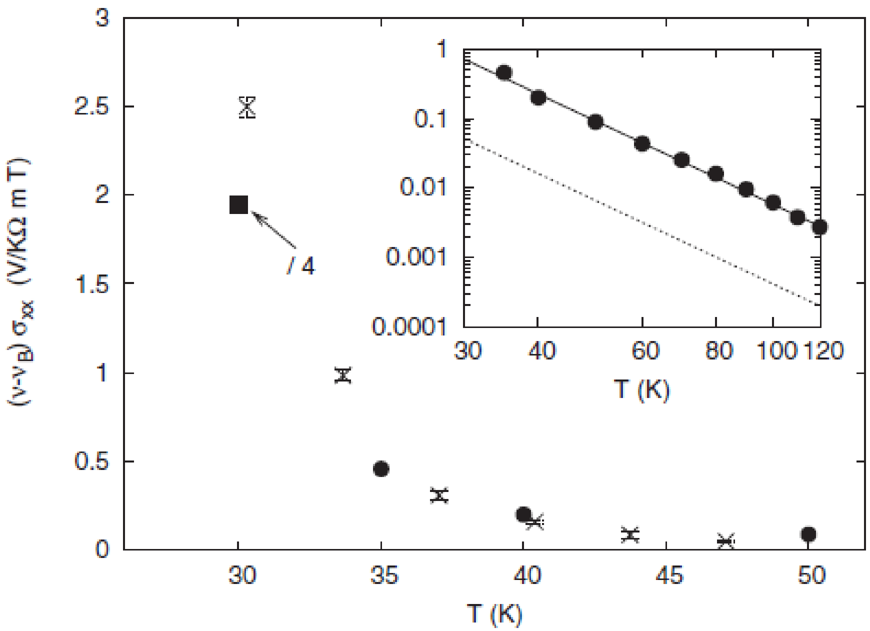
[S. Mukerjee and D. A. Huse, *Phys. Rev. B* **70** 014506 (2004)]

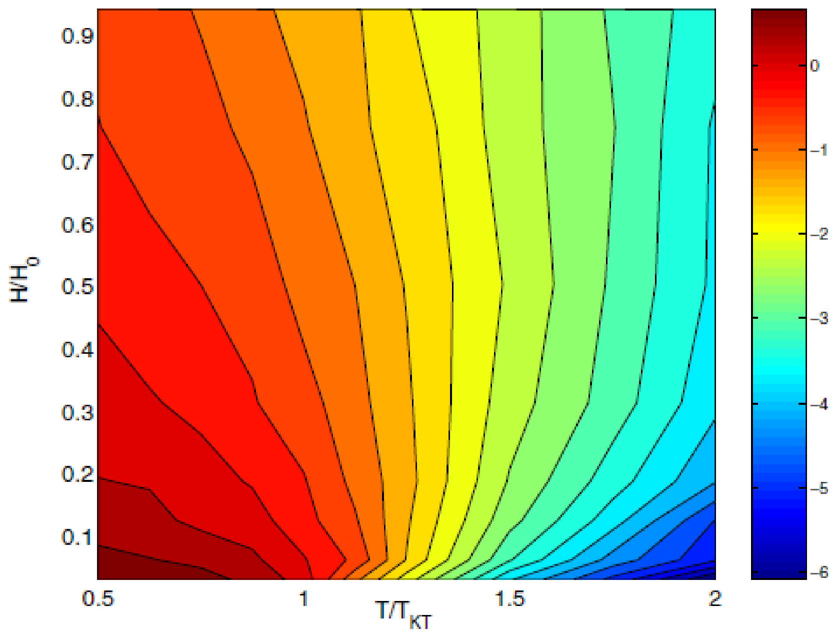
XY limit: opposite limit to Gaussian fluctuations

Only phase fluctuations (amplitude kept constant)

Do TDGL with $H_{XY} = J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j - A_{ij})$

Now $T_c^{MF} = \infty$ and $T_c \sim J$





Fits underdoped data well after renormalization of J due to nodal quasiparticles

$$\boxed{T_c < T_{onset} < T^* < T^c_{MF}}$$

[D. Podolsky, S. Raghu and A. Vishwanath, *Phys. Rev. Lett.* **99** 117004 (2007)]

$\alpha_{xy} \sim 1/T^4$ for $T \gg T_c$ in the XY limit

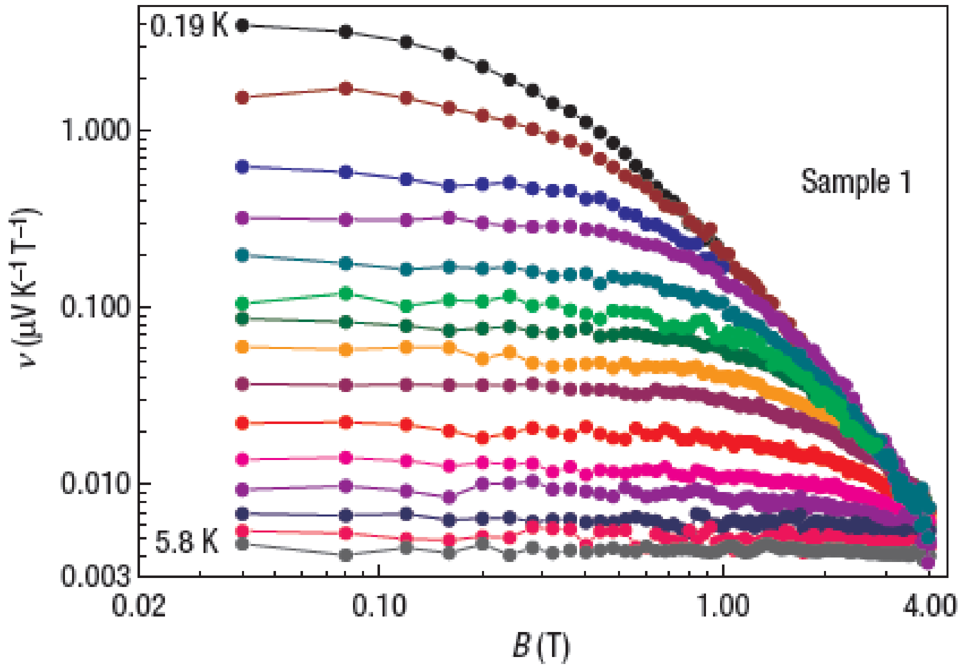
$\alpha_{xy} \sim 1/T$ for $T \gg T_c$ in the Gaussian limit

A reasonable theoretical definition of T_{onset} in the XY limit shows that it tracks T_c as seen in experiments

T_c^{MF} as obtained from GL theory appears to track T^*

Nevertheless the fact that T^* can be quite different from both T_{onset} and T_c^{MF} shows that it is likely due to something other than superconducting fluctuations

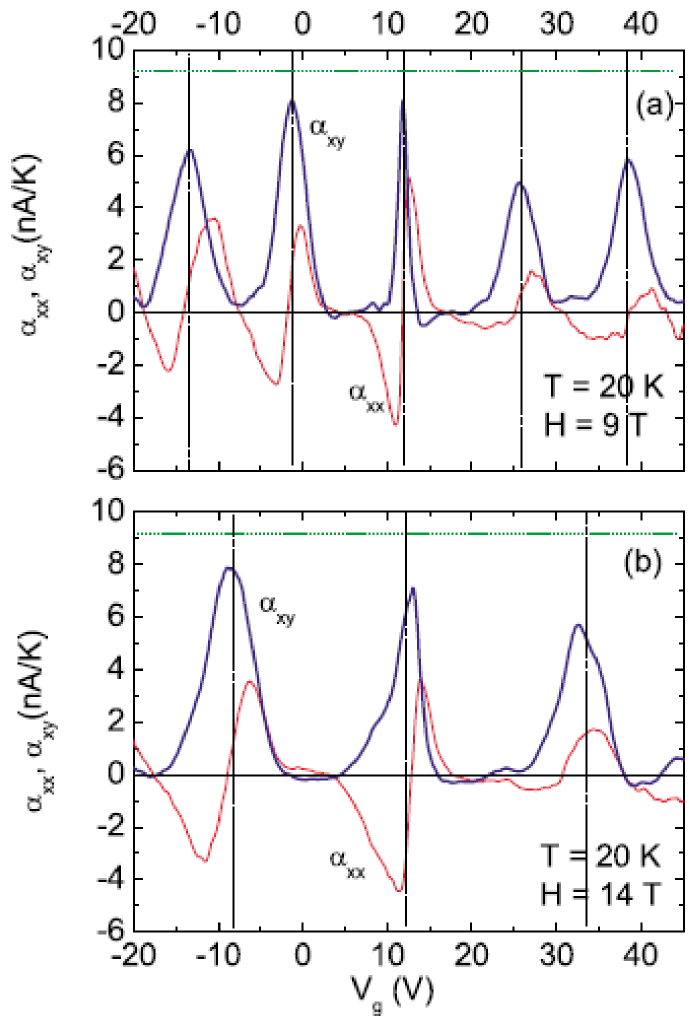
Dirty $Nb_{0.15}Si_{0.85}$



Gaussian fluctuations can account for the Nernst effect

[A. Pourret *et. al*, *Nat. Phys.* **2** 683 (2006)]

Nernst effect in graphene



Oscillations as a function of gate voltage

[J. Checkelsky and N. P. Ong, *Phys. Rev. B* **80** 081413(R) (2009)]

Nernst effect due to fluctuations about a quantum critical point (also in dyonic black holes)

[S. Hartnoll *et. al*, *Phys. Rev. B* **76** 144502 (2007)]

Nernst effect due to quasiparticles and *d*-density waves

[V. Oganesyan and I. Ussihskin, *Phys. Rev. B* **70** 054503 (2004)]

Nernst effect in Ferromagnets

Experiments [W-L. Lee *et. al.*, *Phys. Rev. Lett.* **93** 226601 (2004)]

Theory [D. Xiao *et. al.*, *Phys. Rev. Lett.* **97** 026603 (2006)]

Summary

1. The observed Nernst effect can be accounted for with a simple dynamical model of superconducting fluctuations

2. T_{onset} does not track T^* (experimental observation)

3. Pseudogap physics cannot be completely attributed to superconducting fluctuations

4. underdoped $T_c < T_{onset} < T_c^{MF} < (?)T^*$,
overdoped $T_c < T_c^{MF} < T_{onset} < T^*$

5. Seen in disordered superconductors in addition to high T_c. Can be accounted for by superconducting fluctuations