Multiscale Exploration of Conformational Space Using the MOLS Technique
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## - Introduction - the problem

- Applications to -
- Peptide structure
- Energy landscapes
- Protein structure
- Multiscale modelling


## - Mean Field Theory

## - Mutually Orthogonal Latin Squares

## Introduction

## The Problem

- Exploring and understanding the complex conformational space of molecules (proteins, peptides...)
- Large number of dimensions
- Energy landscape is rugged
- Multiple minima


## Questions

- How rugged?
- How many minima?
- How deep are they?



## Introduction

- Conformational space can be explored by MD simulation at high temperature + quenching
- Also by the MOLS technique
- MOLS is essentially an optimization technique
- MOLS may be explained as a variant of Mean Field Technique


## Mean Field Technique

- Mean field technique has been applied, e.g. to predict protein side chain structure
- $\Phi$ is the conformational search space
- This is divided into a number of subspaces $\varphi_{i}$
- Each such subspace has a number of states $\varphi_{\mathrm{ij}}$ each with a probability of occurrence $\rho_{\mathrm{ij}}$


## Mean Field Technique

$\Phi$ is the conformational search space

$\pm$

## Mean Field Technique

$\Phi$ is divided into a number of subspaces $\varphi_{i}$


## Mean Field Technique

- Each such subspace has a number of states $\varphi_{i j}$ each with a probability of occurrence $\rho_{\mathrm{ij}}$


Probability $=\rho_{11}$

## Mean Field Technique

- Each such subspace has a number of states $\varphi_{i j}$ each with a probability of occurrence $\mathrm{p}_{\mathrm{ij}}$


Probability $=\rho_{12}$

## Mean Field Technique

- Each such subspace has a number of states $\varphi_{i j}$ each with a probability of occurrence $\mathrm{p}_{\mathrm{ij}}$


Probability $=\rho_{13}$

## Mean Field Technique

- First $\rho_{\mathrm{ij}}=1 / n_{i}$
- $n_{i}$ is number of states of subspace $i$


## Mean Field Technique

Next, the effective potential due to a state $\varphi_{r s}$ of a subspace $\varphi_{r}$ is evaluated as

$$
\operatorname{Veff}\left(\varphi_{12}\right)=\sum_{i, j} \rho_{\mathrm{ij}} \mathrm{~V}\left(\varphi_{12}, \varphi_{\mathrm{ij}}\right)
$$



$$
\begin{array}{rrr}
\mathrm{r}=1, \mathrm{~s}=2 & \mathrm{i}=3, \mathrm{j}=1 & \\
\Phi_{12}, \rho_{12} & \Phi_{31}, \rho_{31} & \rho_{31} \mathrm{~V}\left(\varphi_{12}, \varphi_{31}\right)
\end{array}
$$

## Mean Field Technique

- Next, all the probabilities $\rho_{\mathrm{ij}}$ re-evaluated from the respective effective potential as

$$
\rho_{\mathrm{ij}}=\exp \left\{-V^{\operatorname{eff}}\left(\varphi_{\mathrm{ij}}\right) / R T\right\} / \Sigma_{\mathrm{q}} \exp \left\{-\mathrm{Veff}\left(\varphi_{\mathrm{ij}}\right) / R T\right\}
$$

- The cycle is repeated -------
- Evaluate effective potential based on probability
- Evaluate probability based on effective potential
----- until convergence


## Mean Field Technique

- At convergence we have determined all the probabilities $\rho_{\mathrm{ij}}$ of all the states of every subspace
- Finally, side-chain conformations (subspace states) with highest probability represent the 'true state' of the side-chain conformational space $\Phi$

Application to peptide/protein (backbone) structure
------ Backbone torsion angles as subspaces?


Interaction between a pair of subspaces is not independent of other subspaces

- Extension to torsion angle space is not straightforward
- The interaction between a pair of subspaces (when the subspaces are the torsion angles) does not depend only on their respective states, but is a function of the states of all other subspaces.
- i.e. $\mathrm{V}\left(\varphi_{\mathrm{rs}}, \varphi_{\mathrm{ij}}\right)$ is not meaningful - we need

$$
\mathrm{V}\left(\varphi_{\mathrm{rs}} \varphi_{\mathrm{ij}}, \ldots \ldots\right)
$$

- Combinatorial explosion! Complexity is NP

Mutually Orthogonal Latin Squares

- To avoid combinatorial explosion we use a small sample ( $\sim n^{2}$ ) of the possible ( $\mathrm{m}^{\mathrm{n}}$ ) combinations
- We use mutually orthogonal Latin squares (MOLS) to identify the sample
- e.g. 3 torsion angles, 5 values each -


$$
\varphi_{\mathrm{ij}}, \quad \mathrm{i}=1,3 ; \mathrm{j}=1,5
$$

Totally $5^{3}=125$ conformations

## 3 MOLS of order 5

| $\begin{gathered} \varphi_{11,}, \varphi_{21} \\ \varphi_{31} \end{gathered}$ | $\begin{gathered} \varphi_{12}, \varphi_{22} \\ \varphi_{32} \end{gathered}$ | $\begin{gathered} \varphi_{13,}, \varphi_{23} \\ \varphi_{33} \end{gathered}$ | $\begin{gathered} \varphi_{14,}, \varphi_{24} \\ \varphi_{34} \end{gathered}$ | $\begin{gathered} \varphi_{15,}, \varphi_{25} \\ \varphi_{35} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \varphi_{15,}, \varphi_{24} \\ \varphi_{33} \end{gathered}$ | $\begin{gathered} \varphi_{11}, \varphi_{25} \\ \varphi_{34} \end{gathered}$ | $\begin{gathered} \varphi_{12,}, \varphi_{21} \\ \varphi_{35} \end{gathered}$ | $\begin{gathered} \varphi_{13,}, \varphi_{22} \\ \varphi_{31} \end{gathered}$ | $\begin{gathered} \varphi_{14,}, \varphi_{23} \\ \varphi_{32} \end{gathered}$ |
| $\begin{gathered} \varphi_{14,} \varphi_{22} \\ \varphi_{35} \end{gathered}$ | $\begin{gathered} \varphi_{15,}, \varphi_{23} \\ \varphi_{31} \end{gathered}$ | $\begin{gathered} \varphi_{11,}, \varphi_{24} \\ \varphi_{32} \end{gathered}$ | $\begin{gathered} \varphi_{12}, \varphi_{25} \\ \varphi_{33} \end{gathered}$ | $\begin{gathered} \varphi_{13,}, \varphi_{21} \\ \varphi_{34} \end{gathered}$ |
| $\varphi_{13}, \varphi_{25}$ | $\varphi_{14} \varphi$ | 15, | 11, |  |

- Each sub square corresponds to one conformation of the molecule
- The energy V is calculated for each of the $n^{2}$ conformations


## Mutually Orthogonal Latin Squares

-The effective energy is now

$$
\operatorname{Veff}^{\operatorname{lofs}}\left(\varphi_{\mathrm{rs}}\right)=\Sigma_{\mathrm{q}} \mathrm{w}_{\mathrm{q}} \mathrm{~V}_{\mathrm{q}}\left(\varphi_{\mathrm{rs}} \ldots\right)
$$

- The summation is over all the points in the MOLS grid in which $\varphi_{\text {rs }}$ occurs e.g. for $\varphi_{11}$

- The effective energy is now

$$
\begin{gathered}
\operatorname{Veff}\left(\varphi_{r s}\right)=\Sigma_{\mathrm{q}} \mathrm{w}_{\mathrm{q}} \mathrm{~V}_{\mathrm{q}}\left(\varphi_{\mathrm{rs}} \cdots\right) \\
\mathrm{w}_{\mathrm{q}}=\exp \left\{-\mathrm{V}_{\mathrm{q}}\left(\varphi_{\mathrm{rs}} \cdots\right) / \mathrm{RT}\right\} / \Sigma_{\mathrm{q}} \exp \left\{-\mathrm{V}_{\mathrm{q}}\left(\varphi_{\mathrm{rs}} \ldots\right) / R T\right\}
\end{gathered}
$$

- Note: $\mathrm{w}_{\mathrm{q}}$ is calculated from $\mathrm{V}_{\mathrm{q}}$ not from Veff


## Mutually Orthogonal Latin Squares

- Since $\mathrm{w}_{\mathrm{q}}$ is calculated from $\mathrm{V}_{\mathrm{q}}$, not from Veff therefore $\mathrm{w}_{\mathrm{q}}$ is not $\rho_{\mathrm{ij}}$ and the procedure is not iterative
- For each torsion find value that gives Min(Veff)
- The set of Min(Veff) values is Minimum (Low) energy conformation
- Procedure repeated for another low energy structure


## Mutually Orthogonal Latin Squares

## Parameterize the search space

Use these to build a set of MOLS (chosen at random) to globally sample the space

Analyze the samples to obtain a low energy conformation (This is followed by gradient minimization)


Biophysical Jl., 84, 2897, 2003

## Mutually Orthogonal Latin Squares

- We obtain ~1500 low energy structures
- By clustering, we show these may be reduced to $\sim 50$ mutually dissimilar structures
e.g. 23 structures for Met-enkephalin

Biopolymers, 74, 4762004

- The search is exhaustive



## Plot of 'new' structure versus structure number

Sample Overlap

- First sample
- Second sample
(A)



## Applications - Peptide Structure

- Energy landscape, ECEPP/3 force field


Minimal energy envelope for Met-enkephalin
Jl. Phys. Chem. B, 108, 11196, 2004

Applications - Protein Structure

- Multiscale approach to protein structure prediction MOLS libraries + MOLS assembly (ECEPP/3)
(AMBER + 'hydrophobic')
Myoglobin

- Multiscale approach to protein structure prediction
- MOLS libraries + genetic algorithms (ECEPP/3) (AMBER + 'hydrophobic')


Applications - Protein Structure Left $\rightarrow$ Predicted; Right $\rightarrow$ Experimental


Avian Pancreatic Polypeptide 4.0 A


Villin Head Piece 5.2 A


Mellitin 4.3

c-MYB 6.1 A


Tryptophan zipper 1.8 A

BBRC, 342,424, 2006

## 遥

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Rab4 binding domain of Rabenosyn 5 46 residues
Backbone rmsd $3.6 \AA$

Engrailed Homeodomain 56 residues
Backbone rmsd 6.5 Å


Bovine Pancreatic Trypsin Inhibitor 58 residues
Backbone rmsd $10.2 \AA$

Applications - Protein loops

- Loops in protein crystal structures classified by size. Structure predicted using ECEPP/3
Blue - Predicted structure Orange - Crystal structure


Applications - Protein loops





- Ligand (drug) docking to proteins
- Redefine the search space as the conformational space of peptide ligand (i.e. $n$ torsion angles $\varphi_{1}$ to $\varphi_{n}$ ), plus the 'docking' space (i.e. the rotation and translation parameters of the peptide in receptor site, $r_{1}$ to $r_{6}$ ).
- Composite scoring function is now

$$
\mathrm{f}_{1}\left\{\varphi_{1} \text { to } \varphi_{\mathrm{n}}\right\} \quad+\quad \mathrm{f}_{2}\left\{\mathrm{r}_{1} \ldots \mathrm{r}_{6}\right\}
$$

conformational energy + 'docking' energy

PDB ID: 1a30, RMSD $=0.66 \AA$ Sequence: EDL


PDB ID: 1b32, RMSD $=0.51 \AA$
Sequence: KMK


Blue - predicted structure
Red - Crystal structure

PDB ID: 1sua, $\mathrm{RMSD}=1.50 \AA$
Sequence: ALAL

PDB ID: $1 \mathrm{dkx}, \mathrm{RMSD}=1.35 \AA$ Sequence: NRLLLTG


PDB ID: $8 \mathrm{gch}, \mathrm{RMSD}=1.04 \AA$ Sequence: GAW


PDB ID: 1awq, $\mathrm{RMSD}=1.50 \AA$ Sequence: HAGPIA


Applications - Docking

JCAMD, 22, 815, 2008


- Estimation of the numbers and density of low-energy structures in the conformational landscape of proteins.


The number of mutually dissimilar structures found at different rmsd cut-offs for each peptide in 10,000 MOLS structures using the ECEPP/3 potential.

$$
\mathrm{m}=\mathrm{a} \exp (\mathrm{bn})
$$



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CSIR; DST; DBT; UGC
www.unom.ac.in/Gautham_mols.pdf

## Thank You

