

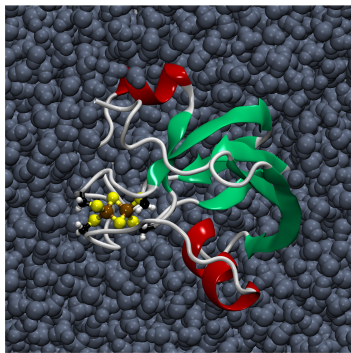
# Magnetic Exchange & Magnetostructural Dynamics in Ferredoxins

Nisanth N. Nair

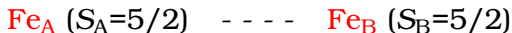
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Dec. 18, 2009, Bangalore

# Motivation



- Iron–sulfur proteins: electron transport in biology
- Ferredoxin: paramagnetic Fe centers bridged by S
- [2Fe–2S] type: Fe sites interact by “superexchange”; antiferromagnetic alignment of electrons



Beinert et al. (1997) *Science* 277:653

# Magnetic Coupling

- Magnetic exchange interactions
  - result to a low-spin ground state  $\Rightarrow$  **antiferromagnetic coupling**
  - result to a high-spin ground state  $\Rightarrow$  **ferromagnetic coupling**
- Magnetic coupling modeled using Heisenberg-Dirac-Van Vleck spin Hamiltonian

$$\hat{H} = -2J \hat{S}_A \cdot \hat{S}_B$$

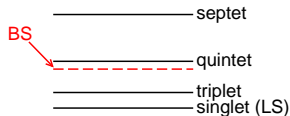
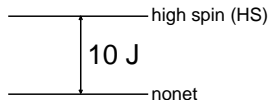
# Magnetic Coupling (2)

- Spin ladder:

$$E^S - E^{S-1} = -2JS$$

- Low spin ground state:  
multireference character

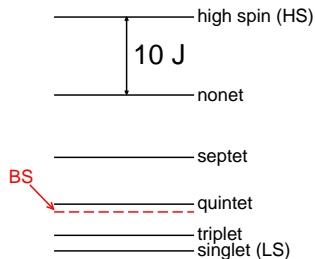
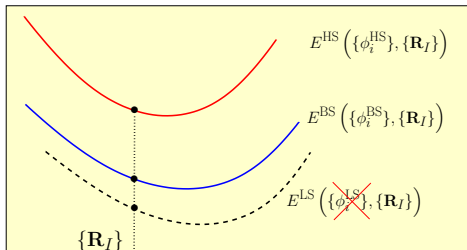
- Poor man's description:  
single singlet determinant (broken symmetry)  
 $\text{Fe}_A(\uparrow\uparrow\uparrow\uparrow) \cdots \text{Fe}_B(\downarrow\downarrow\downarrow\downarrow)$
- Broken symmetry wavefunction is **not** the true ground state wavefunction
- **How to access structure and dynamics at low-spin state?**



# Acknowledgments

- Dr. Eduard Schreiner (Beckman Institute, USA)
- Prof. Dr. Dominik Marx (RUB, Germany)
  
- Dr. Rodolphe Pollet (CEA, France)
  
- Prof. Dr. Volker Staemmler (RUB, Germany)
- Dr. J. Ribas–Arino (RUB, Germany)
  
- E. Schreiner, N. N. Nair, R. Pollet, V. Staemmler, and D. Marx (2007) **Proc. Natl. Acad. Sci. USA**, 104:20725–20730
- N. N. Nair, E. Schreiner, R. Pollet, V. Staemmler, and D. Marx (2008) **J. Chem. Theory Comput.**, 4:1174–1188

# The Essence



- Estimate  $E^{\text{HS}}$  ( $S = S_{\text{max}}$ ) and  $E^{\text{BS}}$  ( $S = S_{\text{min}}$ )
- Compute  $J$  and thus spin ladder; For e.g.,

$$J = \frac{E^{\text{BS}} - E^{\text{HS}}}{S_{\text{max}}^2}$$

- Estimate  $E^{\text{LS}}$  from spin ladder (projection)
- Calculate  $\mathbf{F}_I^{\text{LS}}$  by similar projection  $\rightarrow$  dynamics

# General Expression for $J$

$$J = \frac{E^{\text{BS}} - E^{\text{HS}}}{S_{\text{max}}^2 - S_{\text{min}}^2 - \Theta^{\text{BS}} + \Theta^{\text{HS}}}$$

with

$$\Theta = N_{\text{nmag}}^{\beta} - \sum_i^N \sum_j^N f_i^{\alpha} f_j^{\beta} \langle \phi_i^{\alpha} | \phi_j^{\beta} \rangle^2$$

for unrestricted Hartree–Fock theory  
(overlap of magnetic orbitals and spin contamination)

Nair et al. (2008) **J. Chem. Theory Comput.**, 4:1174–1188

# General Expression for $J$

$$J = \frac{E^{\text{BS}} - E^{\text{HS}}}{S_{\text{max}}^2 - S_{\text{min}}^2 - \Theta^{\text{BS}} + \Theta^{\text{HS}}}$$

- Weak overlap limit:

$\Theta^{\text{BS/HS}} = 0$  and  $N^\alpha = N^\beta \rightarrow$  Noodleman formula

$$J = \frac{E^{\text{BS}} - E^{\text{HS}}}{S_{\text{max}}^2}$$

Noodleman L (1981) *J. Chem. Phys.* 74:5737

- Strong overlap limit:

$\Theta^{\text{HS}} = 0$ ,  $\Theta^{\text{BS}} = N_{\text{mag}}$  and  $N^\alpha = N^\beta \rightarrow$  Ruiz formula

$$J = \frac{E^{\text{BS}} - E^{\text{HS}}}{S_{\text{max}}(S_{\text{max}} + 1)}$$

Ruiz et al. (1999) *J. Comp. Chem.* 20:1391



# General Expression for Ground State Energy

$$E^{\text{LS}} = E^{\text{BS}} + J(S_{\text{max}} - S_{\text{min}} + \Theta^{\text{BS}})$$

$$\text{or } E^{\text{LS}} = (1 + c) E^{\text{BS}} - c E^{\text{HS}}$$

$$\text{where } c = \frac{S_{\text{max}} - S_{\text{min}} + \Theta^{\text{BS}}}{S_{\text{max}}^2 - S_{\text{min}}^2 - \Theta^{\text{BS}} + \Theta^{\text{HS}}}$$

- Accessing energy of the low-spin state *without* having the low-spin density
- Requirement: Heisenberg Hamiltonian must hold

# The Extended Broken Symmetry Approach

- Low spin energy

$$E^{\text{LS}} = (1 + c)E^{\text{BS}} - cE^{\text{HS}} = \mathcal{P}E^{\text{BS,HS}}$$

Low spin forces

$$\begin{aligned}\mathbf{F}_I^{\text{LS}} &= (1 + c)\mathbf{F}_I^{\text{BS}} - c\mathbf{F}_I^{\text{HS}} \\ &= -\nabla_{\mathbf{R}_I} \left[ (1 + c)E^{\text{BS}} \right] + \nabla_{\mathbf{R}_I} \left[ cE^{\text{HS}} \right]\end{aligned}$$

- Equations of motion

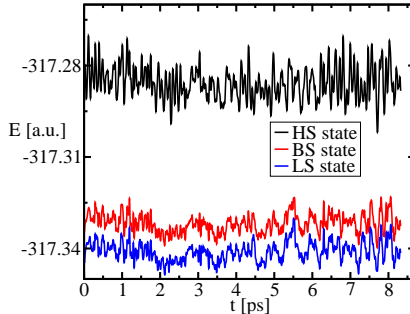
$$M_I \ddot{\mathbf{R}}_I = -\nabla_I \mathcal{P}E^{\text{BS,HS}}$$

⇒ MD, geometry opt. and vibrational anal.

# The Extended Broken Symmetry Approach

## Car-Parrinello Lagrangian for EBS

$$\begin{aligned}\mathcal{L}_{\text{CP}}^{\text{LS}} = & \frac{1}{2} \sum_I M_I \dot{\mathbf{R}}_I^2 + \frac{1}{2} \sum_i \mu \mathcal{P} \langle \dot{\psi}_i | \dot{\psi}_i \rangle^{\text{BS,HS}} \\ & - \mathcal{P} E^{\text{BS,HS}} + \sum_{i,j} \mathcal{P} \Lambda_{ij}^{\text{BS,HS}} \left( \langle \psi_i^{\text{BS,HS}} | \psi_j^{\text{BS,HS}} \rangle - \delta_{ij} \right)\end{aligned}$$



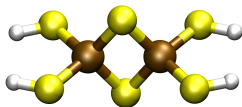
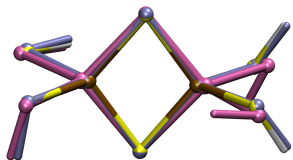
# The Extended Broken Symmetry Approach

Low spin QM/MM electrostatics

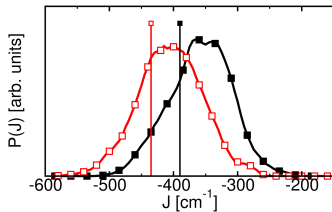
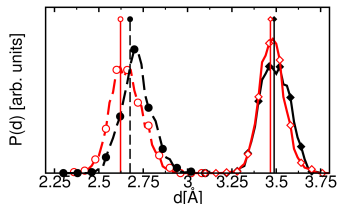
$$E_{\text{QM-MM}}^{\text{LS}} = \mathcal{P}E_{\text{SR}}^{\text{BS,HS}} + \mathcal{P}E_{\text{LR}}^{\text{BS,HS}}$$

# Broken Symmetry vs. Extended Broken Symmetry

- Minimum energy structure  $[\text{Fe}_2\text{S}_2(\text{SH})_4]^{2-}$   
(XC=PBE/PP=USPP/PW=30 Ry)

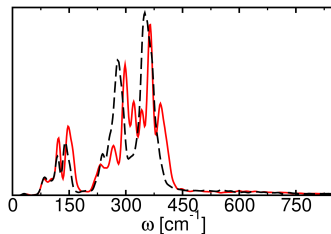


- Distribution function of structural parameters and  $J$   
(EBS vs. BS)



# Broken Symmetry vs. Extended Broken Symmetry (2)

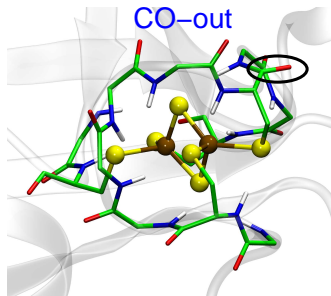
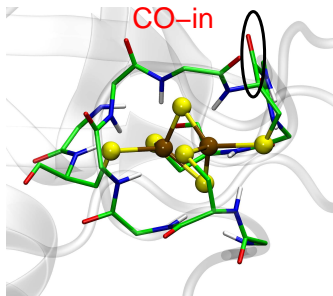
- Vibrational frequencies (**EBS** vs. BS)



- **EBS** and BS differ in their structure and dynamics

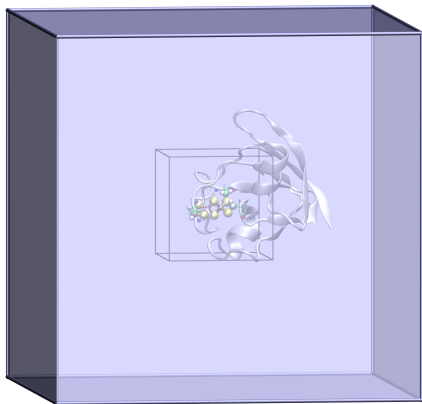
# Anabaena Ferredoxin

- Redox-induced conformational changes in protein



- EBS-QM/MM: CPMD-GROMOS/USPP/Amber FF/16 ps production

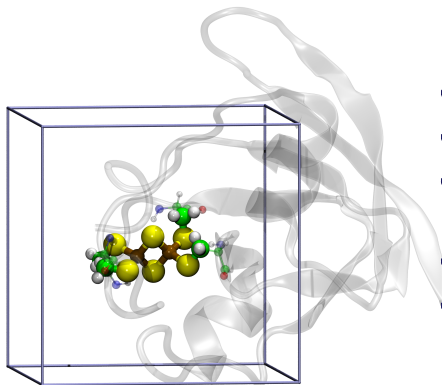
# Anabaena Ferredoxin: Modeling Aspects (1)



- *Anabaena* Fd (pdb: 1qt9) in CO-in and CO-out form
- 13265 H<sub>2</sub>O molecules  
23 Na<sup>+</sup> and 5 Cl<sup>-</sup> ions
- NVT
- 74 Å cubic box, 300 K
- AMBER94 Force Field
- CPMD/GROMOS interface

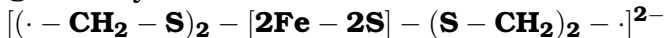


# Anabaena Ferredoxin: Modeling Aspects (2)



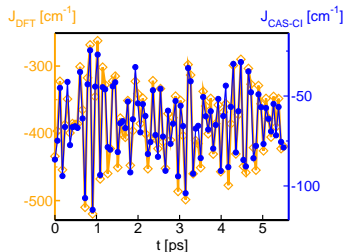
- Spin-polarized DFT
- PBE functional
- Vanderbilt USPP, 30 Ry cutoff
- 21 Å cubic QM box
- EBS-CPMD

- QM subsystem:

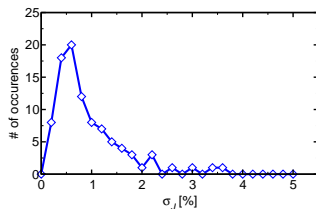


# Validity of the EBS

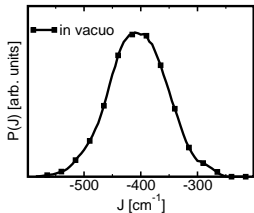
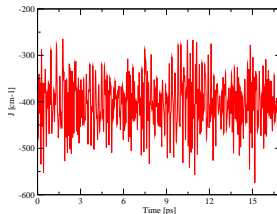
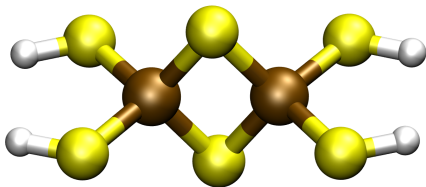
Same dynamics of  $J$  within  
EBS-DFT and CAS-CI



Heisenberg model holds well in  
asymmetric protein environments.



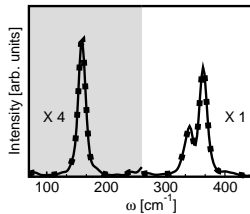
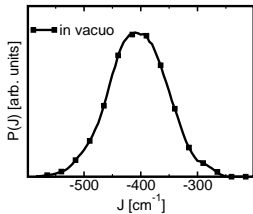
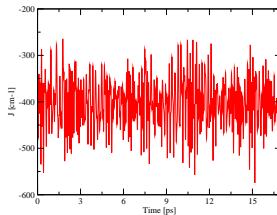
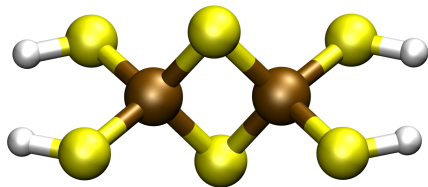
# Effect of Temperature and Environment on $J$



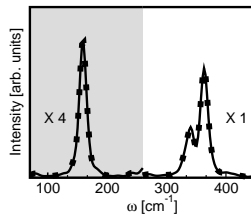
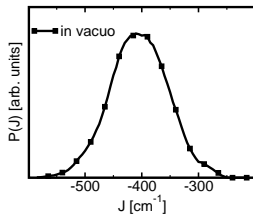
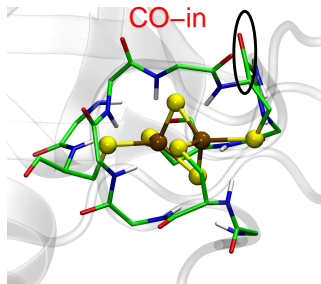
$$G_{\mathbf{PP}}(t) = \frac{\langle \mathbf{P}(t_0) \cdot \mathbf{P}(t_0 + t) \rangle_{t_0}}{\langle \mathbf{P}(t_0) \cdot \mathbf{P}(t_0) \rangle_{t_0}},$$

$$G_{\mathbf{PP}}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G_{\mathbf{PP}}(t) \exp(-i\omega t) dt$$

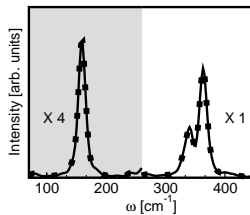
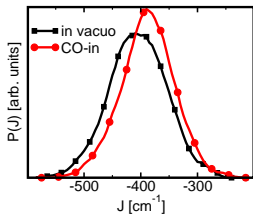
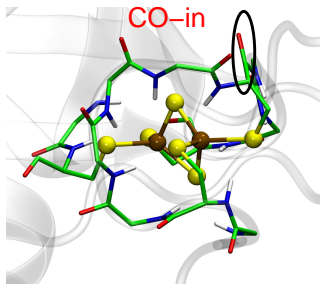
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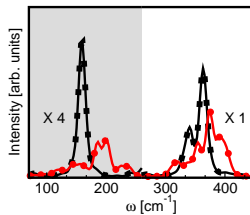
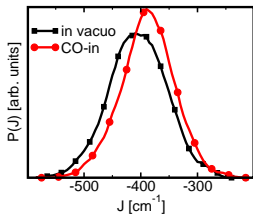
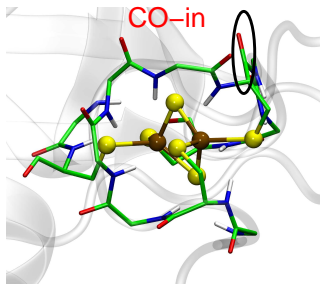
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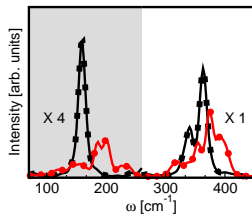
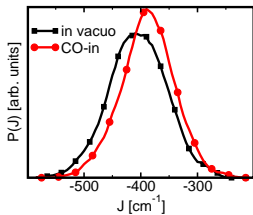
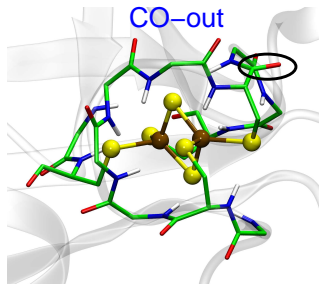
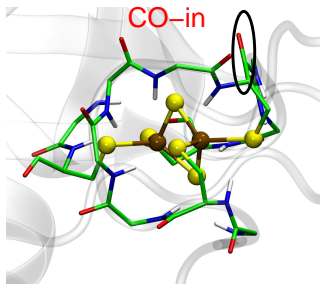
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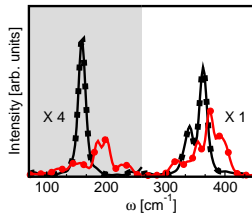
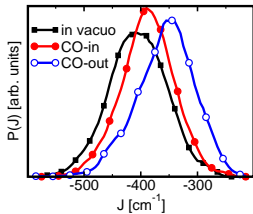
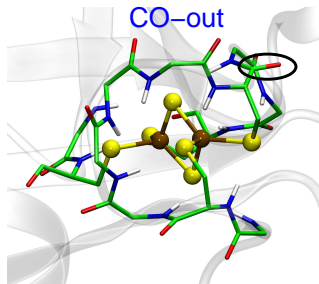
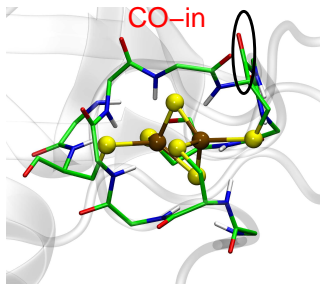


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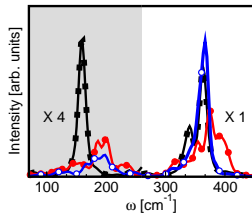
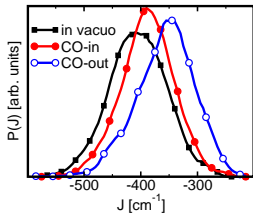
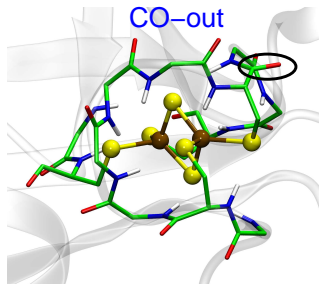
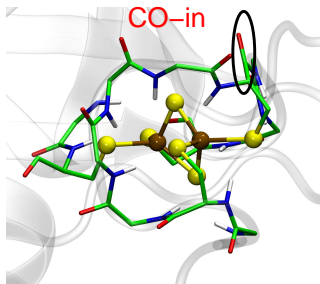




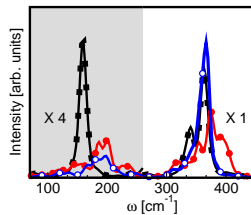
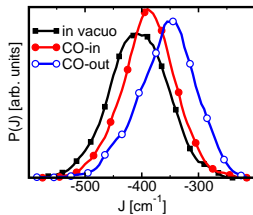
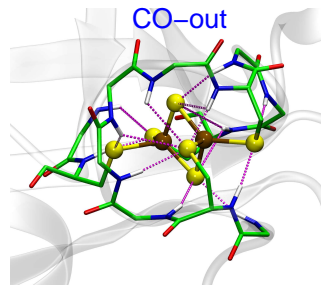
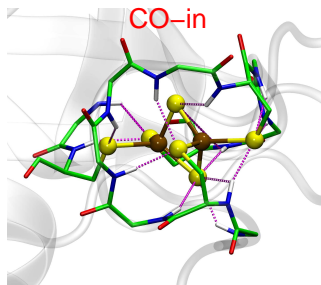
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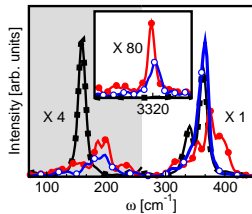
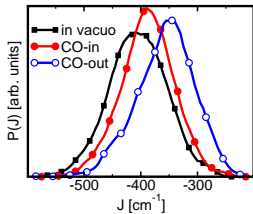
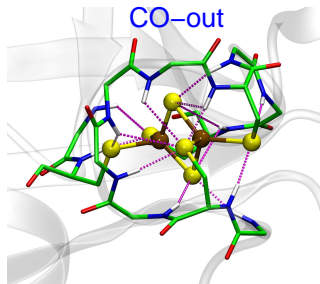
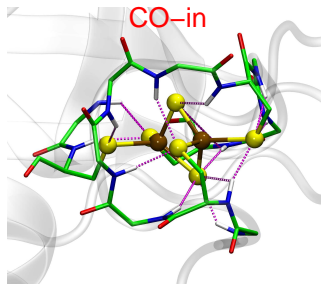
# Effect of Temperature and Environment on $J$



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# Dynamical Magnetostructural Analysis

Define a basis set

normal modes  $\{\mathbf{q}_\xi\}$  of  $[2\text{Fe}2\text{S}]$  based on  $D_{2h}$  symmetry



B<sub>2u</sub>



B<sub>3u</sub>



A<sub>g,D</sub>



A<sub>g,A</sub>



B<sub>1g</sub>



B<sub>1u</sub>

# Dynamical Magnetostructural Analysis

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Project trajectory of [2Fe2S] on each mode  $\{\mathbf{q}_\xi\}$

$$q_\xi(t) = \mathbf{R}_c(t) \cdot \mathbf{q}_\xi$$

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Minimize coupling between modes (modify basis set)

$$G_{\xi,\xi'}(t) = \frac{\langle q_\xi(t_0) q_{\xi'}(t_0 + t) \rangle_{t_0}}{\langle q_\xi(t_0) q_{\xi'}(t_0) \rangle_{t_0}}; \quad \xi \neq \xi'; \quad \xi = 1, \dots, 6$$

$$G_{\xi,\xi'}(t) \rightarrow G_{\xi,\xi'}(\omega); \quad \text{minimize } \int |G_{\xi,\xi'}(\omega)| d\omega$$

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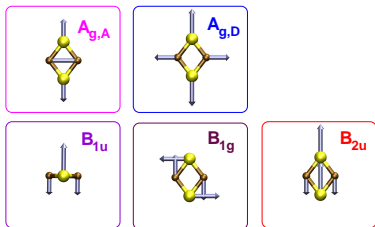
$$G_{\xi,\xi'}(t) \rightarrow G_{\xi,\xi'}(\omega); \quad \text{minimize } \int |G_{\xi,\xi'}(\omega)| d\omega$$

Cross Correlation  $G_{\xi,J}(\omega)$  with the new basis set

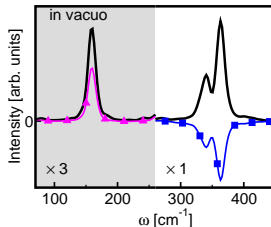
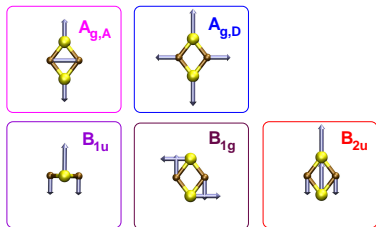
$$G_{\xi,J}(t) \rightarrow G_{\xi,J}(\omega)$$



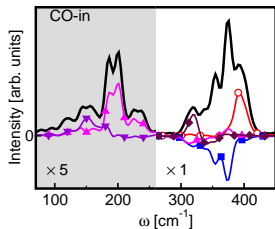
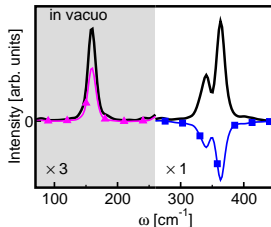
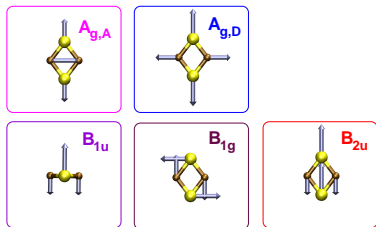
# Dynamical Magnetostructural Analysis: Results



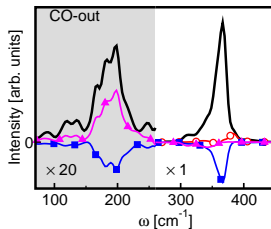
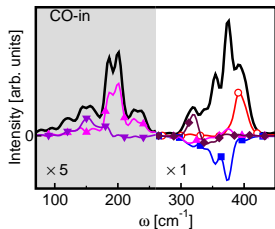
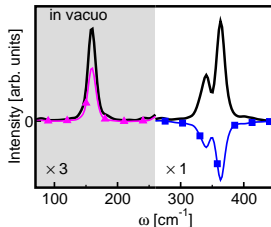
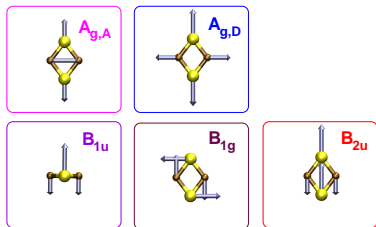
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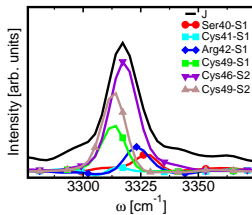
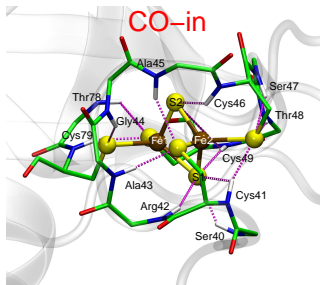
# Dynamical Magnetostructural Analysis: Results



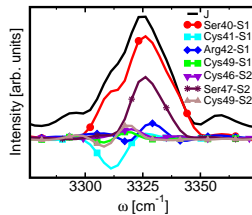
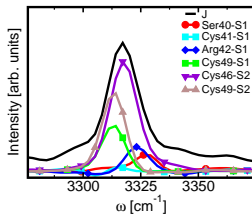
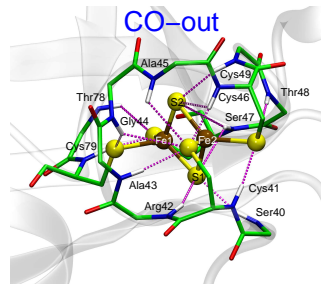
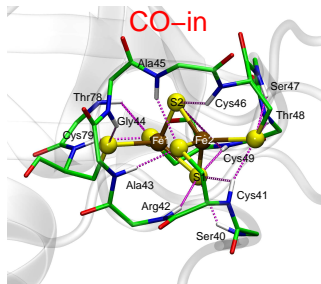
# Dynamical Magnetostructural Analysis: Results



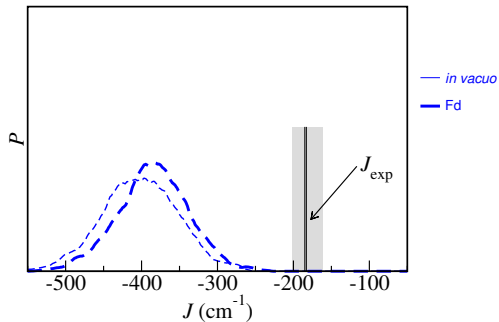
# The Role of Hydrogen Bonds



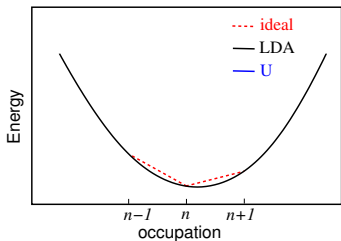
# The Role of Hydrogen Bonds



# Why $J$ is overestimated?



# Hubbard $U$ correction; Linear Response approach

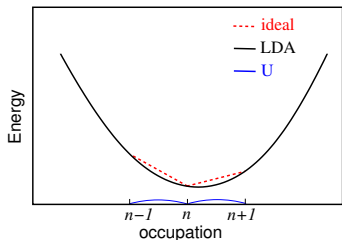


- Self interaction stabilizes fractional occupation

- Delocalization of electrons to ligands  $\Rightarrow$  stronger bond
- Increases the value of  $J$
- Solution: Minimize delocalization



# Hubbard $U$ correction; Linear Response approach



- Self interaction stabilizes fractional occupation

Cococcioni & de Gironcoli PRB 2005

Marzari et al. PRL 2006

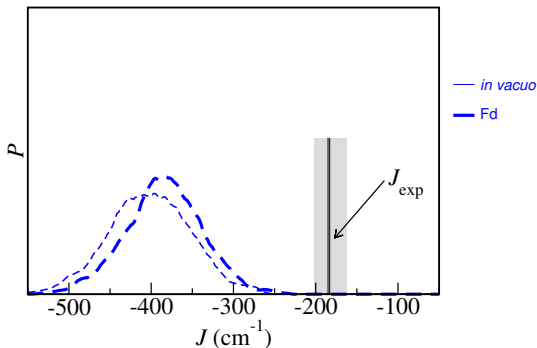
- Hubbard correction to retain linearity

$$E_U = \frac{1}{2} \sum_{I,\sigma} U \text{Tr} [\mathbf{n}^{I,\sigma} (\mathbf{1} - \mathbf{n}^{I,\sigma})]$$

- $U$  is curvature of energy–occupation curve  $\rightarrow$  from linear response
- $V^{\text{GGA}}(\mathbf{r}) + V^U(\mathbf{r}) \rightarrow$  new density  $\Rightarrow$  a self-consistent procedure

# EBS+ $U$ Car–Parrinello Simulations (1)

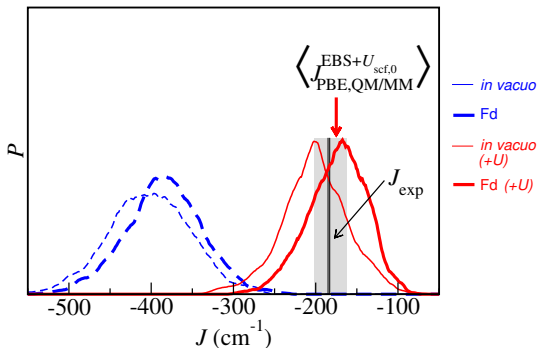
- QM–MM/EBS+ $U$  implementation in CPMD (+USPPs)  
→ +6% CPU time overhead



N. N. Nair, J. Ribas–Arino, V. Staemmler, and D. Marx,  
*submitted*

# EBS+U Car–Parrinello Simulations (1)

- QM–MM/EBS+U implementation in CPMD (+USPPs)  
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N. N. Nair, J. Ribas–Arino, V. Staemmler, and D. Marx,  
*submitted*

# EBS+*U* Car–Parrinello Simulations (2)

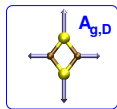
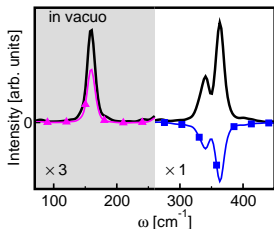
	Fd (EBS)	Fd (EBS+ <i>U</i> )	Xray–Fd
$r(\text{Fe1–Fe2})$	2.62	2.74	2.75
$r(\text{Fe–S})$	2.22	2.27	2.29
$\theta(\text{Fe–S–Fe})$	72.2	74.0	75.1

# Parameterization of $J$

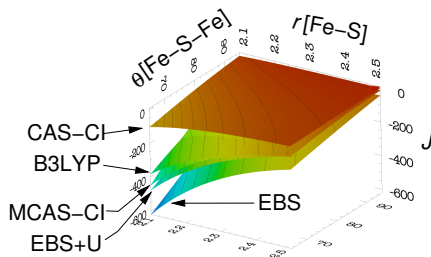
- Force-field for complex [2Fe-2S] prosthetic group is now possible
- BUT... $J$  in classical force-fields??  $\Rightarrow$  parameterization of  $J$

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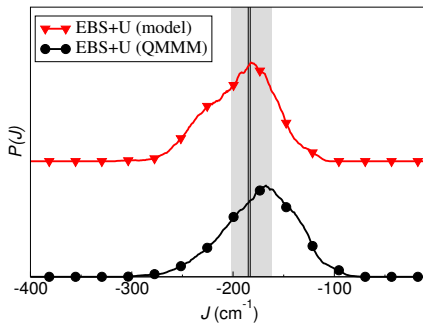


# Parameterization of $J$ (2)



$$J(r, \theta) = A \exp[-\alpha(r - r_0)] + \\ B \exp[-\beta(r - r_0)] \cos \theta + \\ C \exp[-\gamma(r - r_0)] \cos 2\theta$$

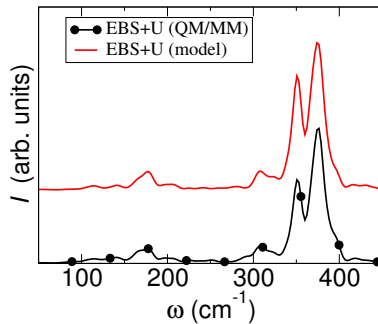
# Parameterization of $J$ (3)



S. A. Fiethen, V. Staemmler, N. N. Nair, J. Ribas-Arino, E. Schreiner and D. Marx, *submitted*

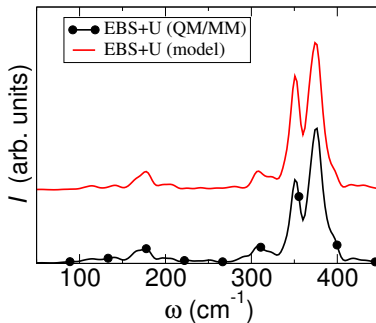


# Parameterization of $J$ (3)



S. A. Fiethen, V. Staemmler, N. N. Nair, J. Ribas-Arino, E. Schreiner and D. Marx, *submitted*

# Parameterization of $J$ (3)



- Parameterization of  $J$  reproduces  $P(J)$  and  $J(\omega)$

S. A. Fiethen, V. Staemmler, N. N. Nair, J. Ribas-Arino, E. Schreiner and D. Marx, *submitted*

# Summary

- Dynamics at the low-spin ground state or “any” spin-state by the extended broken symmetry scheme
- Accurate prediction of  $J$  values (using DFT)
- Thermal fluctuations change  $J$
- Dynamical magnetostructural analysis
- Complex influence of the protein environment on  $J(\omega)$  can be understood
- Parameterization of [2Fe-2S] core and  $J$  for classical MD

# Thank You For Your Attention

## Other Magnetic Systems

- 1 Rieske proteins
- 2 Multicenter magnetic clusters/embedded proteins

## Generalize the Computational Tool

- 1 General projectors for GGA+ $U$  method

# Two particle density matrix

$$\Gamma(\mathbf{x}'_1, \mathbf{x}'_2 | \mathbf{x}_1, \mathbf{x}_2) = \frac{N(N-1)}{2} \times \int \Psi^*(\mathbf{x}'_1, \mathbf{x}'_2, \dots, \mathbf{x}_N) \Psi(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N) d\mathbf{x}_3 \dots d\mathbf{x}_N,$$

having  $\mathbf{x}_i = (\mathbf{r}_i, \sigma_i)$

How to measure  $P(J)$ ?

? inelastic neutron scattering (INS)

How to access magnetostructural correlations?

Idea: excite specific vibrations and measure  $J$  simultaneously  $\Rightarrow$  combination of

? inelastic neutron scattering + resonance Raman

? inelastic neutron scattering + nuclear inelastic scattering (INS+NIS)

$\Rightarrow$  only Mößbauer active nuclei

# General Expression for Ground State Energy

- Heisenberg-Dirac-Van Vleck spin Hamiltonian

$$\hat{H} = -2J \hat{S}_A \cdot \hat{S}_B = -J(\hat{S}^2 - \hat{S}_A^2 - \hat{S}_B^2)$$

- Energy of a spin state

$$E^S = -J[S(S+1) - S_A(S_A+1) - S_B(S_B+1)]$$

- For practical purpose employing  $\langle \hat{S}^2 \rangle$

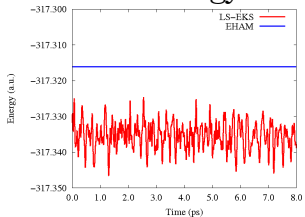
$$E^S = -J[\langle \hat{S}^2 \rangle - S_A(S_A+1) - S_B(S_B+1)]$$
$$J = \frac{E^{BS} - E^{HS}}{\langle \hat{S}^2 \rangle_{HS} - \langle \hat{S}^2 \rangle_{BS}}$$

Yamaguchi et al. (1988) *Chem. Phys. Lett.* 149:537

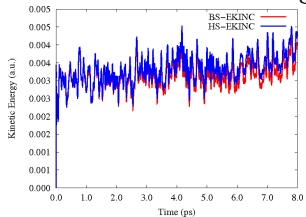


# Stability of EBS CP Dynamics

## Total energy



## Electronic kinetic energy



# Average Structure Parameters & $J$ in Protein

	$\text{Fe}_2\text{S}_2(\text{SH})_4^{2-}$ <sup>a</sup>	$\text{Fd}_{\text{in}}$ <sup>b</sup>	$\text{Fd}_{\text{out}}$ <sup>b</sup>	$\text{Fd}_{\text{in}}$ <sup>c</sup>
distance [Å]				
Fe-Fe	2.64	2.62	2.62	2.75
Fe1-S1	2.18	2.22	2.21	2.28
Fe2-S2	2.18	2.17	2.18	2.18
$J$ [ $\text{cm}^{-1}$ ]	-403	-386	-360	-

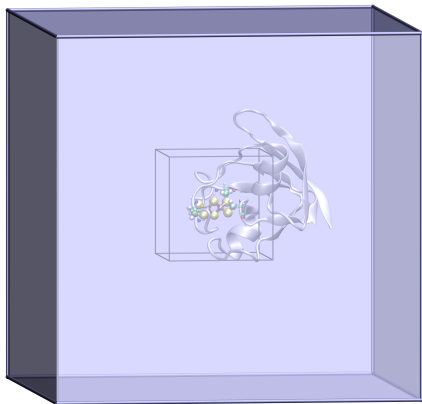
<sup>a</sup> *in vacuo* ;

<sup>b</sup> solvated protein;

<sup>c</sup> X-ray diffraction data

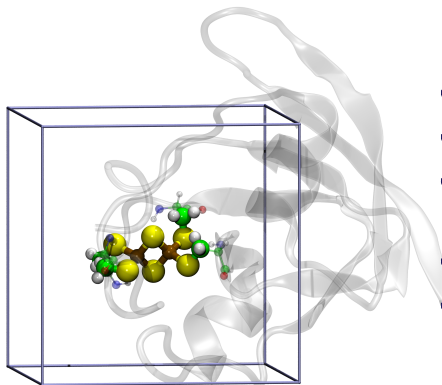
Experimental  $J$  for Spinach ferredoxin:  $-183 \text{ cm}^{-1}$

# Modeling Aspects: MM



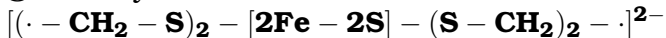
- *Anabaena* Fd (pdb: 1qt9) in CO-in and CO-out form
- 13265 H<sub>2</sub>O molecules  
23 Na<sup>+</sup> and 5 Cl<sup>-</sup> ions
- NVT
- 74 Å cubic box, 300 K
- AMBER94 Force Field
- CPMD/GROMOS interface

# Modeling Aspects: QM



- Spin-polarized DFT
- PBE functional
- Vanderbilt USPP, 30 Ry cutoff
- 21 Å cubic QM box
- EBS-CPMD

- QM subsystem:



# General Expression for $J$

$$J = \frac{E^{\text{BS}} - E^{\text{HS}}}{S_{\text{max}}^2 - S_{\text{min}}^2 - \Theta^{\text{BS}} + \Theta^{\text{HS}}}$$

with

$$\Theta = N_{\text{nmag}}^{\beta} + 2 \int \Gamma(\mathbf{r}_1\alpha, \mathbf{r}_2\beta | \mathbf{r}_1\beta, \mathbf{r}_2\alpha) d\mathbf{r}_1 d\mathbf{r}_2$$

$\Gamma$  is the two particle density matrix

Löwdin P. O. (1955) *Phys. Rev.* 97:1474

Wang et al. (1995) *J. Chem. Phys.* 102:3477

# General Expression for $J$

$$J = \frac{E^{\text{BS}} - E^{\text{HS}}}{S_{\text{max}}^2 - S_{\text{min}}^2 - \Theta^{\text{BS}} + \Theta^{\text{HS}}}$$

with

$$\Theta = N_{\text{mag}}^{\beta} - \sum_i^N \sum_j^N f_i^{\alpha} f_j^{\beta} \langle \phi_i^{\alpha} | \phi_j^{\beta} \rangle^2$$

for unrestricted Hartree–Fock theory  
(overlap of magnetic orbitals and spin contamination)

# General Expression for $J$

$$J = \frac{E^{\text{BS}} - E^{\text{HS}}}{S_{\text{max}}^2 - S_{\text{min}}^2 - \Theta^{\text{BS}} + \Theta^{\text{HS}}}$$

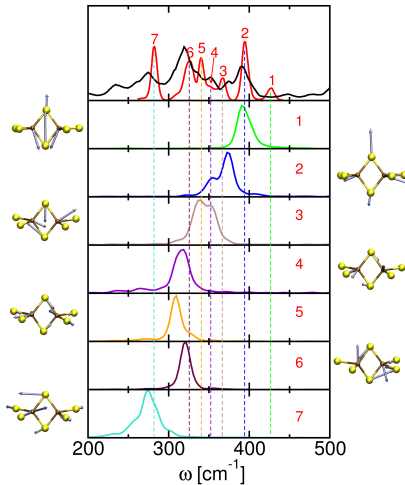
For a pure spin state:

$$\Theta = 0 \implies \langle \hat{S}^2 \rangle = S(S + 1)$$

Nair et al. (2008) **J. Chem. Theory Comput.**, 4:1174–1188

# Comparison with Experiment

- Modes from principal component analysis on the QM/MM trajectory





# Effect of Overlap of Magnetic Orbitals

$$\mathbf{F}_I^{\text{LS}} = (1 + c)\mathbf{F}_I^{\text{BS}} - c\mathbf{F}_I^{\text{HS}}$$
$$c = \frac{S_{\text{max}} - S_{\text{min}} + \Theta^{\text{BS}}}{S_{\text{max}}^2 - S_{\text{min}}^2 - \Theta^{\text{BS}} + \Theta^{\text{HS}}}$$

$$\Theta^{\text{HS}} = 0.0$$

$\Theta^{\text{BS}}$	0.000	-0.313	-1.250	-2.813	-5.000
$c$	0.200	0.185	0.143	0.079	0.000
$S^{\alpha\beta}$	0.00	0.25	0.50	0.75	1.00
$\Delta r(\text{Fe-Fe}) [\text{\AA}]$	0.000	0.004	0.015	0.033	0.054
$\Delta r(\text{Fe-S}) [\text{\AA}]$	0.000	0.002	0.007	0.016	0.025
$\Delta J [\text{cm}^{-1}]$	0	9	33	69	110