Quantum Optical Effects in Cold Atoms

G.S. Agarwal*
Department of Physics
Oklahoma State University
Stillwater, OK 74078-3072

*Email:agirish@okstate.edu,

Collaborators: B.Deb

Quantum Optical Effects In Bose Condensates

1. Coherence and photon-photon correlations – coherent states
2. Hanbury Brown Twiss correlations – quantum regime
3. Squeezed states
4. Mesoscopic states and superpositions, CAT states, GHZ states
5. Collapse and revival in quantum dynamics of nonlinear systems
6. Quantum entanglement
7. Using cavity QED to produce effective interactions between condensates
8. Measurement induced phases and entanglement
9. Multicomponent condensates – spin squeezing

COURSE OUTLINE

Nonrelativistic System of Bosons — Two Body Hamiltonian

Second Quantization — Fock Space; Annihilation Creation Operators

Bogoliubov Theory — approximate diagonalization; Bogoliubov transformation; spectrum, excitations

Harmonic Oscillator Potentials; their relevance in BEC; Optical dipole traps

Coherent States — properties and generation; Bogoliubov phonons in coherent states theory and exp

Bragg Scattering

Parametric Amplification; Squeezed States— Experimental Realization in Condensates

Quantum Entanglement

Measurement Induced Phases and Interference Between Independent Condensates

Spin Squeezing and its Observation in Condensates in Double Well Potentials

Coherent Superposition of Macroscopic States

Entangled States of Condensates

Collapse and Revival in BEC due to Self Interactions

Quantum Optical Effects in Spinor Condensates--

Bosonic systems

N Atoms

$$H \equiv \sum_i \frac{p_i^2}{2m} + \frac{1}{2}\sum_{i \neq j} V(|r_i - r_j|)$$

Free particles------ Second quantization

Fock space

Each plane wave momentum $\hbar\vec{k}$

$n_{\vec{k}}$: number of particles with momentum $\hbar\vec{k}$

$|0_{\vec{k}}\rangle$: zero particle

Annihilation and creation operators

$$[a_{k_i}, a^{+}_{k_j}] = \delta_{\vec{k}_i,\vec{k}_j}, \qquad [a_{k_i}, a_{k_j}] = 0$$

Bosons

$a_k^+ a_k$ number operator

$$a_k^+ a_k \left| n_{\vec{k}} \right\rangle = n_k \left| n_{\vec{k}} \right\rangle$$

$$a_k \left| n_{\vec{k}} \right\rangle = \sqrt{n_k} \left| n_{\vec{k}} - 1 \right\rangle, \qquad a_k \left| 0_{\vec{k}} \right\rangle = 0.$$

$$a_k^+ \left| n_{\vec{k}} \right\rangle = \sqrt{n_k + 1} \left| n_{\vec{k}} + 1 \right\rangle$$

$$\left| n_{\vec{k}} \right\rangle = \frac{a_k^n}{\sqrt{n!}} \left| 0_{\vec{k}} \right\rangle$$

$$H = \sum_{\vec{k}_i} \frac{\hbar^2 k_i^2}{2m} a_{k_i}^+ a_{k_i}$$

general state $\left| n_{k_1} n_{k_2} \cdots n_{k_i} \right\rangle$

$$\sum n_{k_i} = N$$

$$\left| \Psi \right\rangle = \sum C_{k_1 \cdots} \left| n_{k_1} \cdots \right\rangle$$

Field operator

$$\Psi(\vec{r}) = \int d^3k \, a_{\vec{k}} \frac{e^{i\vec{k}\cdot\vec{r}}}{\sqrt{(2\pi)^3}}$$

$$[\Psi(\vec{r}), \Psi^+(\vec{r}\,')] = \delta(\vec{r} - \vec{r}\,')$$

$\Psi^+(\vec{r})\Psi(\vec{r})$ particle density operator

Operators

$$\int d^3r \, \Psi^+(\vec{r}) f(\vec{r}) \Psi(\vec{r})$$

$$\frac{1}{2}\int d^3r \int d^3r' \, \Psi^+(\vec{r})\Psi^+(\vec{r}\,')V(\vec{r}-\vec{r}\,')\Psi(\vec{r})\Psi(\vec{r}\,')$$

$$H \equiv -\frac{\hbar^2}{2m}\int d^3r \, \Psi^+(\vec{r})\nabla^2\Psi(\vec{r}) + \frac{1}{2}\int\int d^3r \, d^3r' \, \Psi^+(\vec{r})\Psi^+(\vec{r}\,')V(\vec{r}-\vec{r}\,')\Psi(\vec{r})\Psi(\vec{r}\,')$$

Bogoliubov theory

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m} + \frac{g}{2} \sum_{i \neq j} \delta(r_i - r_j)$$

$$H_I = \frac{4\pi\hbar^2 a_s}{2mV} \sum_{k_3, k_4, k_5, k_6} \hat{a}_{k_3}^{+} \hat{a}_{k_4}^{+} \hat{a}_{k_5} \hat{a}_{k_6} \delta_{k_3+k_4, k_5+k_6}$$

Rb condensate $a_s \sim 100$ Bohr radius

$$\hat{a}_0, \hat{a}_0^+ \to \sqrt{N_0} \Rightarrow$$

$$H \approx \sum_{k\neq 0} (\varepsilon_k + \mu)(a_k^+ a_k + a_{-k}^+ a_{-k}) + \mu \sum_{k\neq 0} (a_k^+ a_{-k}^+ + a_k a_{-k})$$

$$\mu \equiv \frac{\hbar^2}{2m} \frac{8\pi a_s a_0^2}{V}$$

Variety of classical and quantum effects with matter waves

Nonlinearities $\xrightarrow[\text{Level}]{\text{Classical}}$

- parametric amplification
- four wave mixing

$\xrightarrow[\text{Level}]{\text{Quantum}}$

- collapse and revivals of matter wave packets
- Mott transition

$\xrightarrow[\text{Different Limits}]{\text{Quantum Level}}$

- coherent phonons
- squeezed and entangled phonons
- spin squeezing in multicomponent condensates

Bogoliubov Transformation

$$H = \omega a_1^+ a_1 + \omega a_2^+ a_2 + g a_1^+ a_2^+ + g^* a_1 a_2$$

Write $a_1 = c_1 \cosh r + c_2^+ e^{-i\theta} \sinh r,$

$$a_2 = c_2 \cosh r + c_1^+ e^{-i\theta} \sinh r.$$

Reduce H to $\Omega(c_1^+ c_1 + c_2^+ c_2) + \text{constant}.$

Find $\Omega, r, \theta.$

Solution : $H = \sqrt{\omega^2 - |g|^2}\,(c_1^+ c_1 + c_2^+ c_2) + \sqrt{\omega^2 - |g|^2} - \omega,$

$$\Omega = \sqrt{\omega^2 - |g|^2},$$

$$r = \frac{1}{4} \ln \frac{\omega - |g|}{\omega + |g|}, \qquad \theta = i \ln \sqrt{\frac{g}{g^*}}.$$

Bogoliubov's quasiparticle

$$H = \hbar\omega_q^B(\alpha_q^+\alpha_q + \alpha_{-q}^+\alpha_{-q})$$

$$\omega_q^B = \left[(\omega_q + \frac{\mu}{\hbar})^2 - \frac{\mu^2}{\hbar^2}\right]^{1/2} \qquad \mu = \frac{\hbar^2\xi^{-2}}{2m}$$

$$\alpha_q \equiv u_q a_q - v_q a_{-q}^+$$

$$u_q^2 - v_q^2 = 1$$

$$v_q^2 = \frac{1}{2}\left(\frac{\omega_q + \frac{\mu}{\hbar}}{\omega_q^B} - 1\right)$$

$$\alpha_q \to a_q \qquad \text{if } \nu \to 0,\ \xi \to 0.$$

Dipole Trap: Basic Physics

$$E = Ee^{-i\omega t} + c.c. \qquad p = \chi E e^{-i\omega t} + c.c.$$

$$\text{Energy} = -\frac{1}{2}\vec{p}\cdot\vec{E}$$

$$\text{Time average} = -(\mathrm{Re}\chi)\left|E\right|^2$$

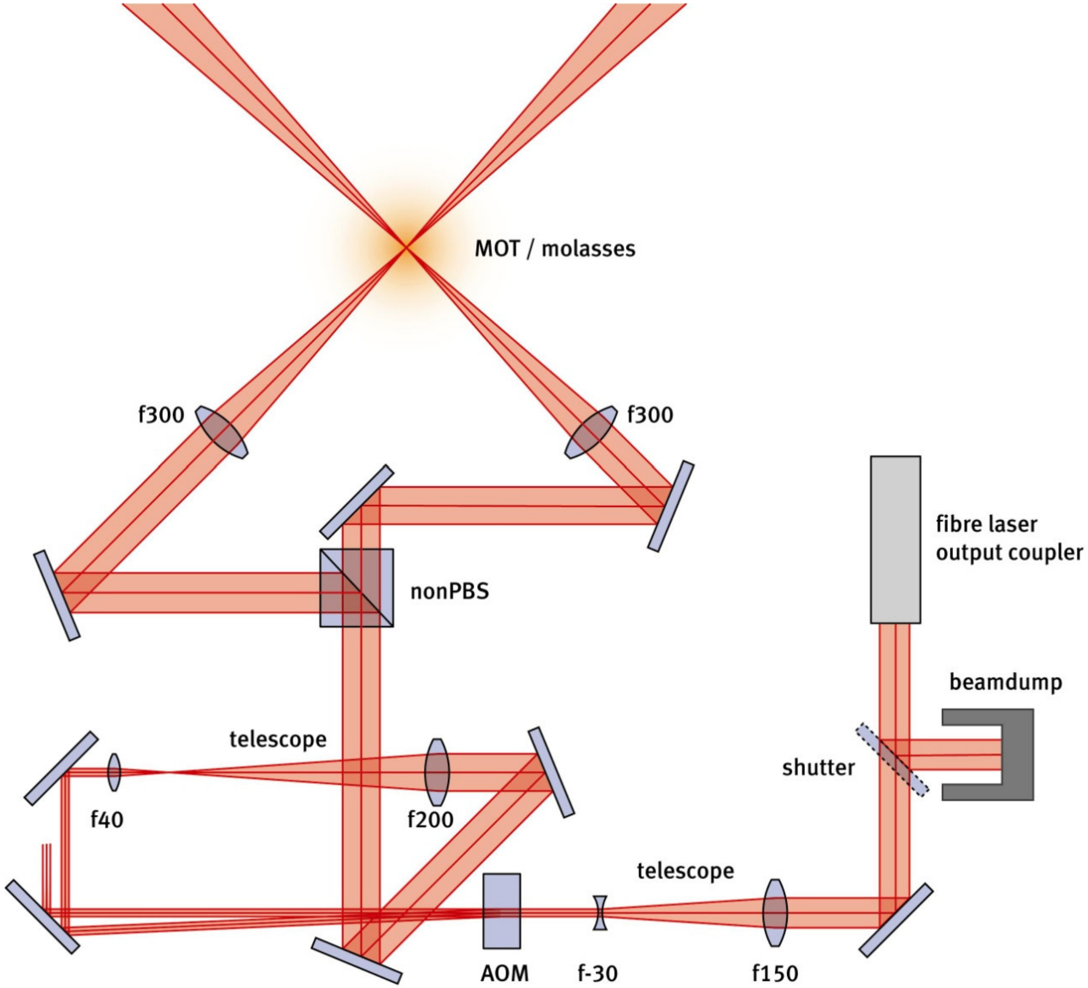
$$\text{Flux } \frac{c}{2\pi}\left|E\right|^2 \equiv I$$

$$\chi \equiv \frac{e^2}{m}\cdot\frac{1}{\omega^2-\omega_L^2}$$

$$\text{Energy} \equiv \frac{-2\pi e^2}{mc(\omega^2-\omega_L^2)}\cdot I; \qquad I:\ \text{standing wave} \Rightarrow \text{optical lattices}$$

$$\text{Gaussian beam } I \equiv \frac{I_0}{1+\left(\frac{z}{z_0}\right)^2}\exp\left\{\frac{-2r^2}{(1+\frac{z}{z_0})^2\,\omega_0^2}\right\}$$

Experimental Setup – Dipole Trap



The fiber laser for dipole traps:

- Max power = 35 W
- Wavelength = 1064 nm

(far-detuned from the sodium D2 line @ 589.159nm)

- not single frequency
- Random polarized
- waist = 55 µm
- Initial trap depth :1mK

•Initially load ~10^7 atoms directly from MOT

•Reduce dipole trap beam intensity to evaporate

•After 6 seconds evaporation, create a BEC with 50,000 ~ 200,000 |1,-1> sodium atoms

•Final trap oscillation freq:

220 (1, 1, 1.4) Hz

Anharmonicity; Collapse and Revival

Interaction of type $\hbar\chi a^{\dagger 2}a^2$,

Quantum Dynamics ~ Phases exp(-i♍n^2t)

Dephasing ; Rephasing; t = πp/♍q

$$|\alpha\rangle \rightarrow \quad \left|\Psi(t=\frac{\pi}{m\chi})\right\rangle = \sum f_q^{(e)}\left|\alpha e^{-2\pi iq/m}e^{i\pi/m}\right\rangle$$

$$f_q^{(e)} = \frac{1}{m}\sum_{N=0}^{m-1}\exp\left[\frac{2\pi iq}{m}N\right]\exp\left\{\frac{-i\pi}{m}N^2\right\}$$

Tara et. al. PRA **47**, 5024 (1993)

Anharmonic Interactions Quantum Dynamics

Correlation Function

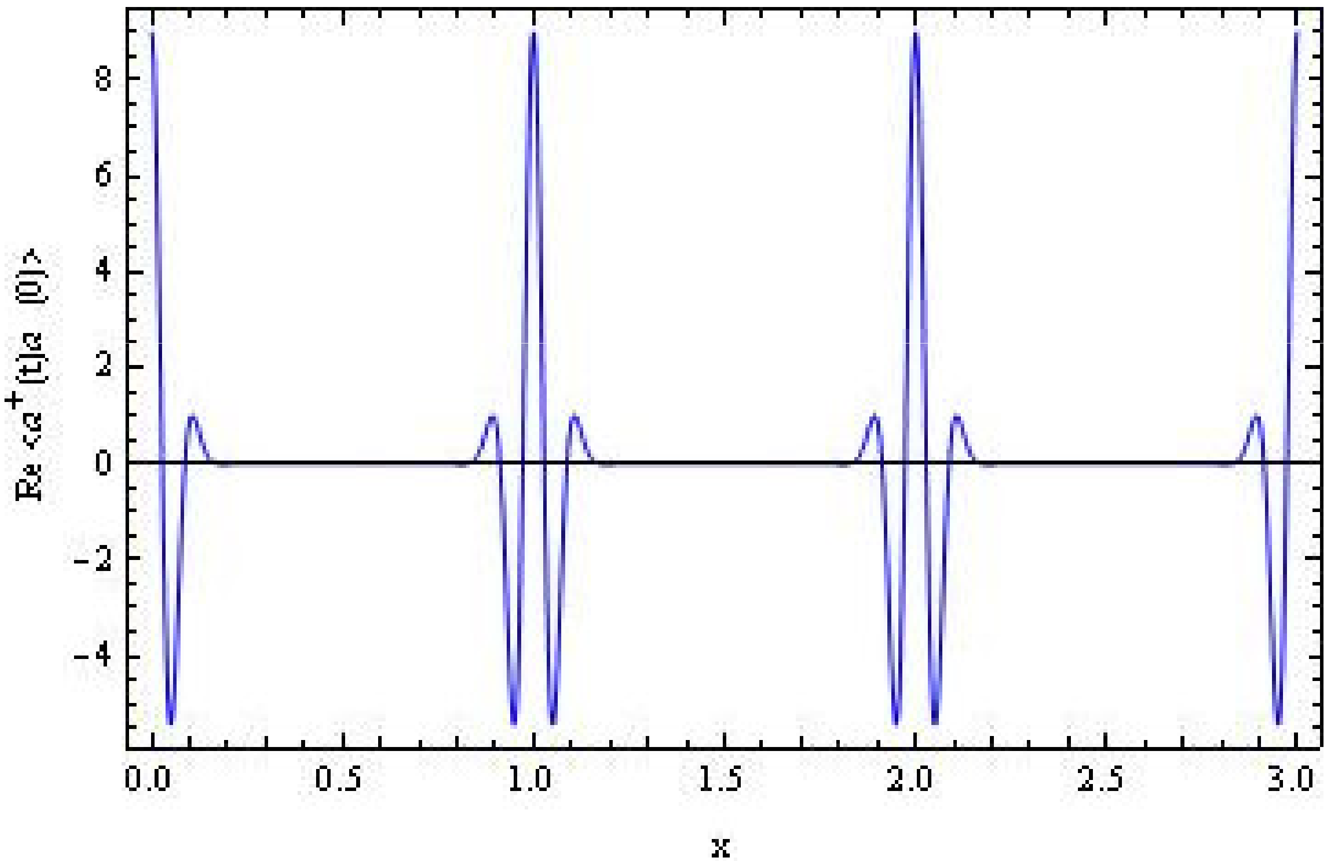
$$\left\langle a^{+}(t)a(0)\right\rangle = |\alpha|^{2}\sum_{nm}\frac{\alpha^{*n}\alpha^{m}}{\sqrt{n!m!}}e^{-|\alpha|^{2}}\frac{e^{i(\varepsilon_n-\varepsilon_m)t/\hbar}}{\alpha^{*}}\langle n|m+1\rangle\sqrt{m+1}$$

$$\frac{\varepsilon_n t}{\hbar} \equiv \pi x n(n-1)$$

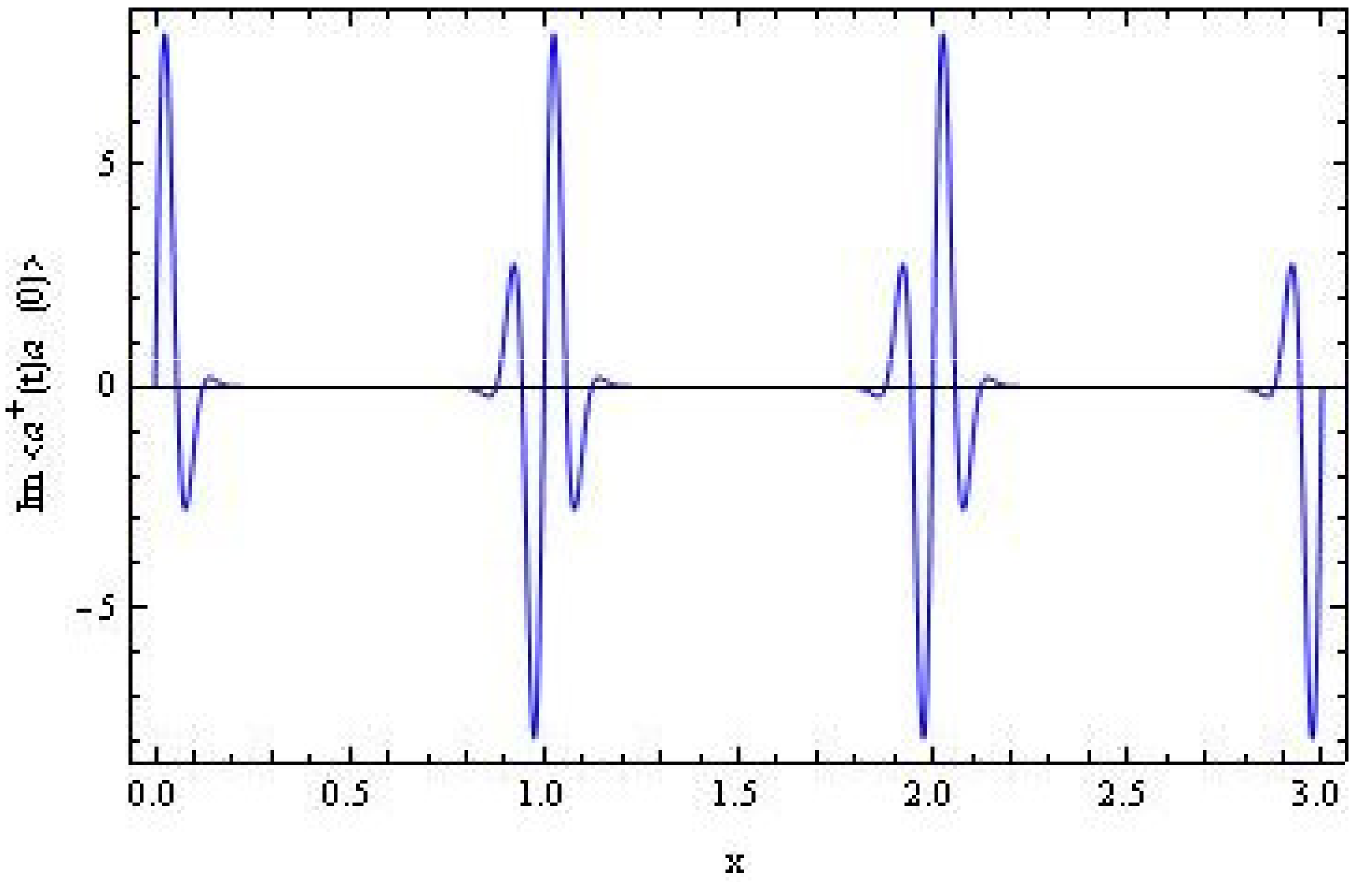
Coherent state

Say $\alpha = 3,\ m_{\max} = 20,$

$$\mathrm{Re}\left\langle a^{+}(t)a(0)\right\rangle$$



$$\mathrm{Im}\langle a^{+}(t)a(0)\rangle$$



Markus Greiner, Olaf Mandel
T. W. Hansch & I. Bloch,
Nature **419**, 51 (2002)

Collapse and revival of the matter wave field of a Bose–Einstein condensate

Markus Greiner, Olaf Mandel, Theodor W. Hänsch & Immanuel Bloch

Sektion Physik, Ludwig-Maximilians-Universität, Schellingstrasse 4/III, D-80799 Munich, Germany, and Max-Planck-Institut für Quantenoptik, D-85748 Garching, Germany

A Bose–Einstein condensate represents the most 'classical' form of a matter wave, just as an optical laser emits the most classical form of an electromagnetic wave. Nevertheless, the matter wave field has a quantized structure owing to the granularity of the discrete underlying atoms. Although such a field is usually assumed to be intrinsically stable (apart from incoherent loss processes), this is no longer true when the condensate is in a coherent superposition of different atom number states[1–6]. For example, in a Bose–Einstein condensate confined by a three-dimensional optical lattice, each potential well can be prepared in a coherent superposition of different atom number states, with constant relative phases between neighbouring lattice sites. It is then natural to ask how the individual matter wave fields and their relative phases evolve. Here we use such a set-up to investigate these questions experimentally, observing that the matter wave field of the Bose–Einstein condensate undergoes a periodic series of collapses and revivals; this behaviour is directly demonstrated in the dynamical evolution of the multiple matter wave interference pattern. We attribute the oscillations to the quantized structure of the matter wave field and the collisions between individual atoms.

Markus Greiner *et. al.* Nature **419**, 51 (2002)

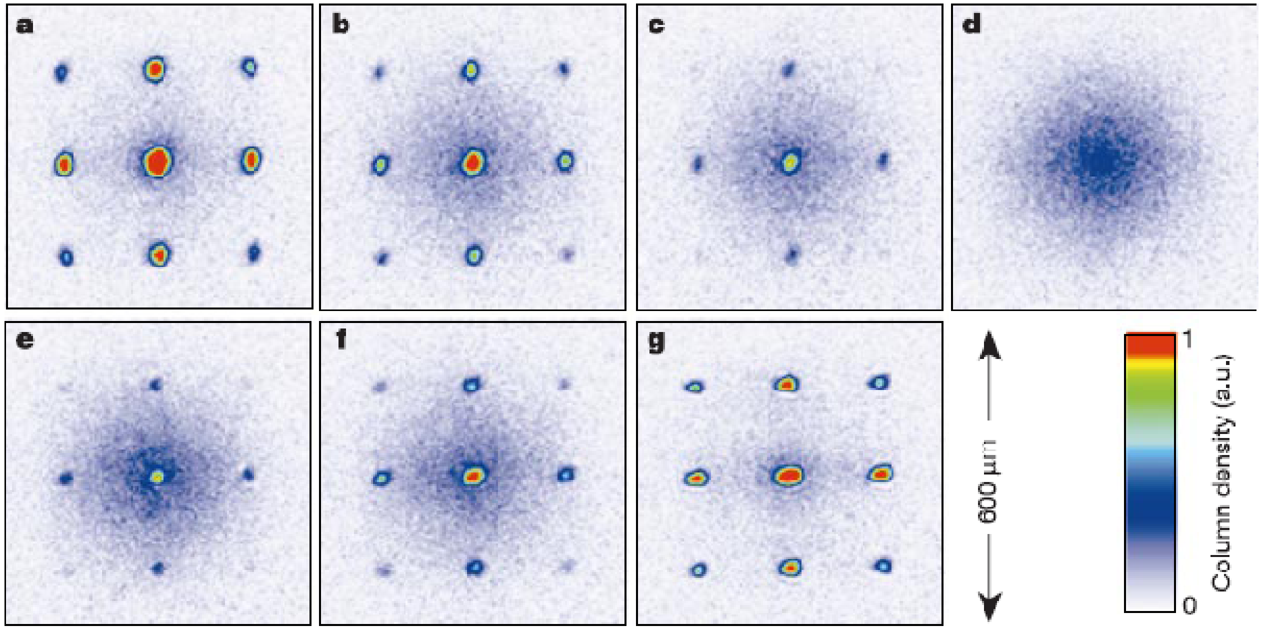
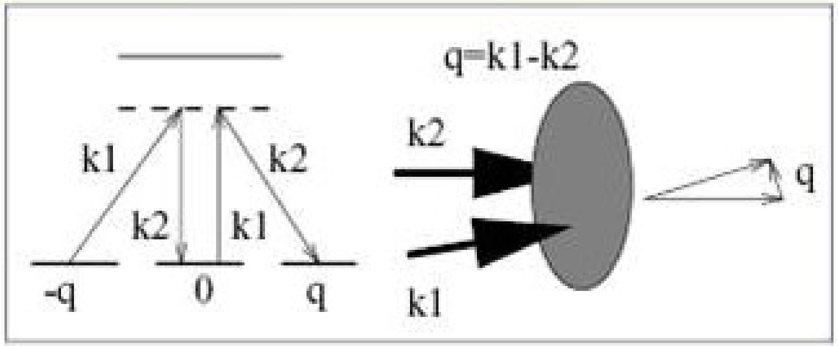


Figure 2 Dynamical evolution of the multiple matter wave interference pattern observed after jumping from a potential depth $V_A = 8\ E_r$ to a potential depth $V_B = 22\ E_r$ and a subsequent variable hold time t. After this hold time, all trapping potentials were shut off and absorption images were taken after a time-of-flight period of 16 ms. The hold times t were **a**, 0 μs; **b**, 100 μs; **c**, 150 μs; **d**, 250 μs; **e**, 350 μs; **f**, 400 μs; and **g**, 550 μs. At first, a distinct interference pattern is visible, showing that initially the system can be described by a macroscopic matter wave with phase coherence between individual potential wells. Then after a time of ~250 μs the interference pattern is completely lost. The vanishing of the interference pattern is caused by a collapse of the macroscopic matter wave field in each lattice potential well. But after a total hold time of 550 μs (**g**) the interference pattern is almost perfectly restored, showing that the macroscopic matter wave field has revived. The atom number statistics in each well, however, remains constant throughout the dynamical evolution time. This is fundamentally different from the vanishing of the interference pattern with no further dynamical evolution, which is observed in the quantum phase transition to a Mott insulator, where Fock states are formed in each potential well. From the above images the number of coherent atoms N_{coh} is determined by first fitting a broad two-dimensional gaussian function to the incoherent background of atoms. The fitting region for the incoherent atoms excludes 130 μm × 130 μm squares around the interference peaks. Then the number of atoms in these squares is counted by a pixel-sum, from which the number of atoms in the incoherent gaussian background in these fields is subtracted to yield N_{coh}. a.u., arbitrary units.

Stimulated light scattering



$$H_{AF} = \hbar\Omega \hat{c}_{k_2}^{+} e^{-i\omega_1 t} \sum_k (\hat{a}_{q+k}^{+}\hat{a}_k + \hat{a}_{-q+k}\hat{a}_k^{+}) + \mathrm{H.c.}$$

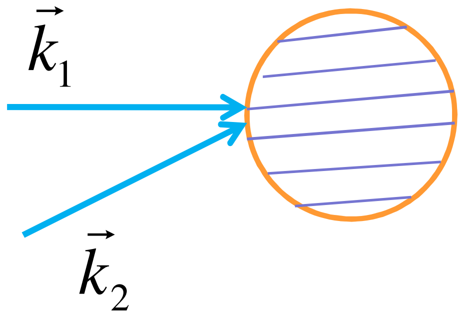
$$k = 0, \hat{a}_0 \to \sqrt{N}, \hat{a}_0^{+} \to \sqrt{N} \Rightarrow$$

$$H_{AF} \approx \hbar\sqrt{N}\Omega e^{-i\omega_1 t} \hat{c}_{k_2}^{+}(\hat{a}_q^{+} + \hat{a}_{-q}) + \mathrm{H.c.}$$

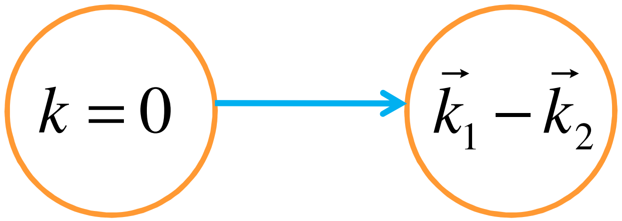
Bragg Scattering

Light field classically, displacement of phonons

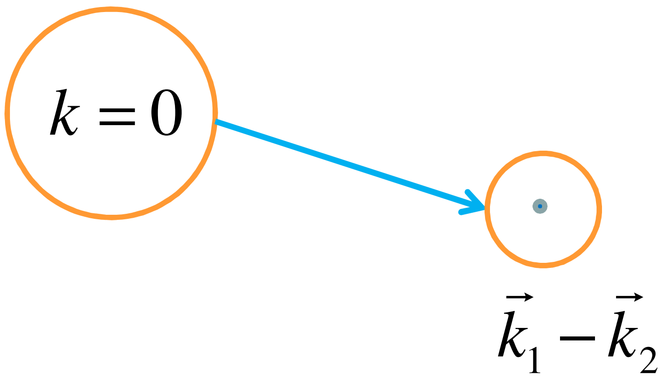
Bragg scattering



Strength of "pulses" effectively like 2 level transitions



Application of $\frac{\pi}{2}$ pulse



Coherent states

Definition

$$a|\alpha\rangle = \alpha|\alpha\rangle, \qquad \alpha \ \text{complex}$$

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

$$\langle\alpha|a|\alpha\rangle = \alpha,$$

Well defined

$$\Delta x \Delta p = \left(\frac{\hbar}{2}\right)^2$$

Minimum uncertainty

Photon number distribution

$$p(n) \equiv \frac{|\alpha|^{2n}}{n!} e^{-|\alpha|^2}$$

Poissonian

Fluctuations

$$(\Delta n)^2 = \bar{n}, \qquad \bar{n} = |\alpha|^2.$$

$$\left|\alpha\right\rangle = e^{a^{+}\alpha - a\alpha^{*}}\left|0\right\rangle$$

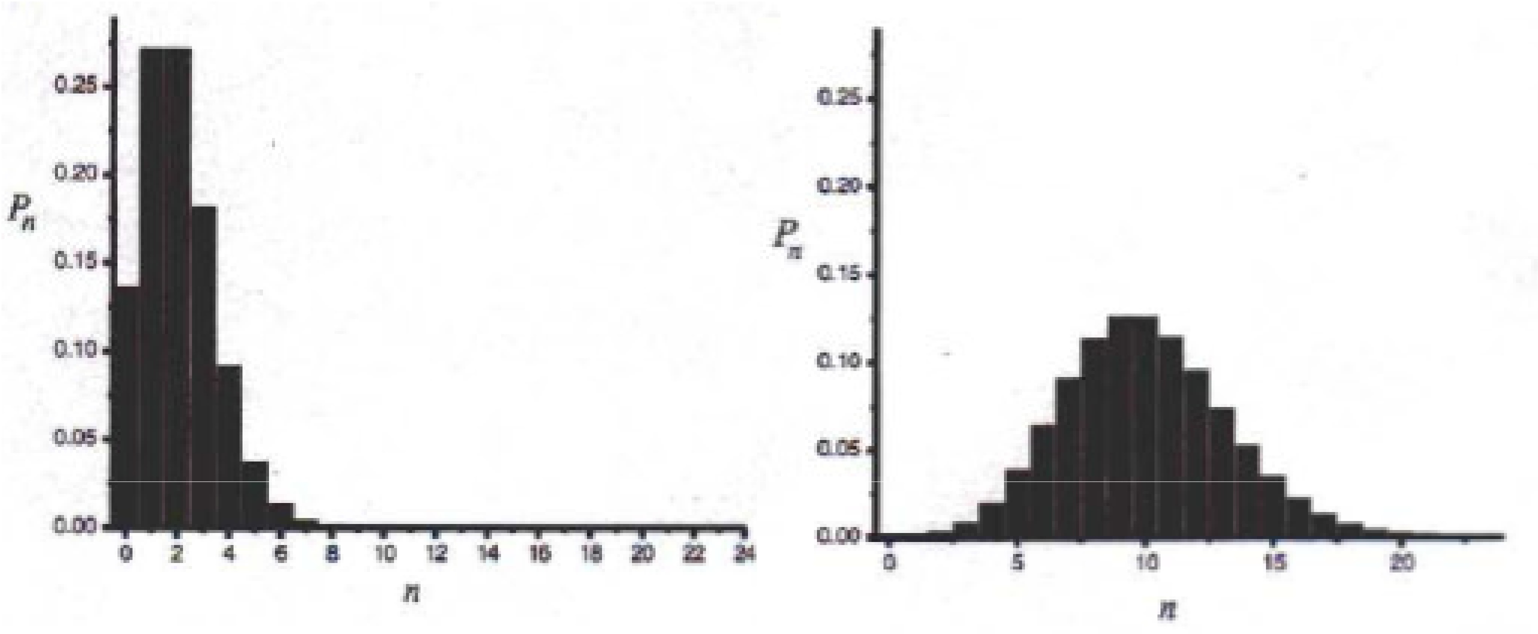
$$H_1 \Rightarrow fa^{+} + f^{*}a$$

Phase distribution

$$p(\varphi) \equiv \frac{1}{2\pi}\left|\left\langle\varphi|\alpha\right\rangle\right|^{2}$$

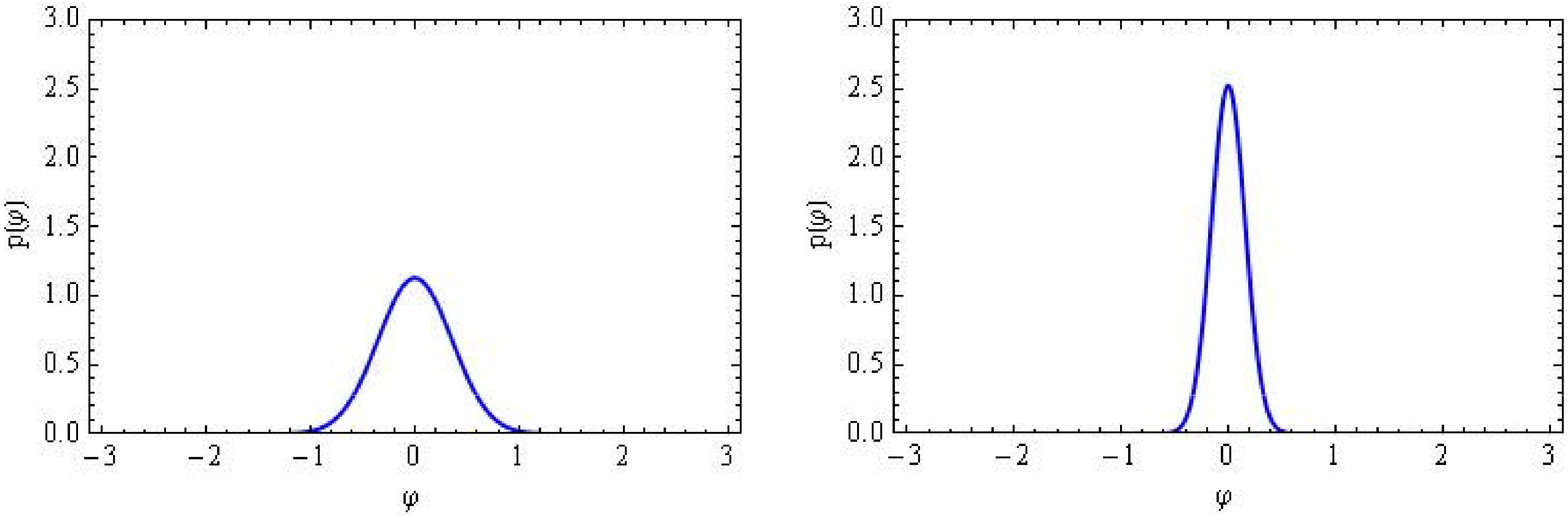
$$\approx \left(\frac{2|\alpha|^{2}}{\pi}\right)^{1/2} e^{-2|\alpha|^{2}(\varphi-\theta)^{2}}$$

$$\Delta n\Delta\varphi \sim 1, \Delta\varphi \sim \frac{1}{\Delta n} \sim \frac{1}{|\alpha|} = \frac{1}{\sqrt{\bar{n}}}$$



(a) (b)

Coherent state photon number probability distributions for (a) $\bar{n} = 2$ and (b) $\bar{n} = 10$.



(a) (b)

Phase distributions for coherent states with $\theta = 0$ for (a) $\bar{n} = 2$ and (b) $\bar{n} = 10$.

Experimental Observation of the Bogoliubov Transformation for a Bose-Einstein Condensed Gas

J. M. Vogels, K. Xu, C. Raman,* J. R. Abo-Shaeer, and W. Ketterle†

Department of Physics, MIT-Harvard Center for Ultracold Atoms, and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

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Phonons with wave vector $q/\hbar$ were optically imprinted into a Bose-Einstein condensate. Their momentum distribution was analyzed using Bragg spectroscopy with a high momentum transfer. The wave function of the phonons was shown to be a superposition of $+q$ and $-q$ free particle momentum states, in agreement with the Bogoliubov quasiparticle picture.

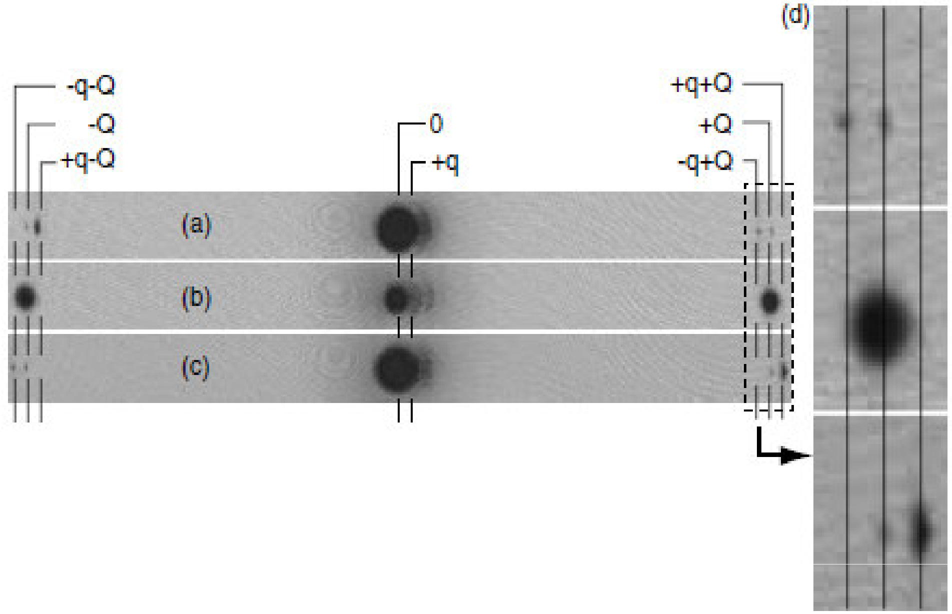


FIG. 1. Momentum distribution of a condensate with phonons. After imprinting $+q$ phonons into the condensate, momentum analysis via Bragg spectroscopy transfers a momentum $\pm Q$ (two-photon recoil) to the atoms. Absorption images after 40 ms time of flight in (a), (b), and (c) show the condensate in the center and outcoupled atoms to the right and left for probe frequencies of 94, 100, and 107 kHz, respectively. The small clouds centered at $+q$ are phonons that were converted to free particles. The size of the images is 25×2.2 mm. (d) The outlined region in (a)–(c) is magnified, and clearly shows outcoupled atoms with momenta $Q \pm q$, implying that phonons with wave vector $q/\hbar$ have both $+q$ and $-q$ free particle momentum components.

$$\left(c_k^+ a_q^+ + H.C.\right)$$

Parametric Hamiltonian

Simultaneous generation of probe excitation and phonon collective excitation

Entangled States of phonons and photons

Quantum State Transfer

Higher order scattering --- Two phonon processes

$$\left(a_q^+ a_{q'}^+ + H.C.\right) \longrightarrow$$

Possibility of using standing pump and probe waves

Deb, GSA PRA **65**, 063618 (2002); **67**, 023603 (2003)

Bogoliubov Transformations, Quantum Squeezing, Entanglement, Parametric Generation etc.

Effective Interaction

$$H = \hbar\xi a^+ b^+ + \hbar\xi^* ab$$

a, b bosons created out of vacuum

$$\langle a^+ a \rangle = \langle b^+ b \rangle = \sinh^2 |\xi| t \to e^{2|\xi|t} / 4$$

Exponential growth

States Generated from Vacuum $|0,0\rangle$

$$S(z)|0,0\rangle, \qquad S(z) = \exp[z a^+ b^+ - z^* ab], \qquad z = -i\xi t$$

Heisenberg operator evolution:

Bogoliubov transformation on the two modes

$$S^{+}(z)\begin{pmatrix} a \\ b^{+} \end{pmatrix} S(z) = \begin{pmatrix} \cosh r & \sinh r e^{i\theta} \\ \sinh r e^{-i\theta} & \cosh r \end{pmatrix} \begin{pmatrix} a \\ b^{+} \end{pmatrix}, \quad z = re^{i\theta}$$

Operators

$$K_{-} = ab, \; K_{+} = a^{+}b^{+} \text{ and } K_{3} = \frac{1}{2}(a^{+}a + b^{+}b + 1)$$

Realisation of the su(1,1) algebra with $K_0 = \frac{1}{2}(a^{+}a - b^{+}b)$

as the Casimir operator.

Disentanglement of $S(z)$

$$S(z) = \exp\{e^{i\theta} \tanh r\, K_+\} \exp\{-\ln[\cosh r\,(2K_3)]\}$$
$$\times \exp\{-e^{-i\theta} \tanh r\, K_-\}$$

Two mode squeezed vacuum

The two mode squeezed vacuum $|z\rangle_{sq}$ is defined by

$$|z\rangle_{sq} = \exp\{z a^+ b^+ - z^* ab\}|0,0\rangle = S(z)|0,0\rangle$$
$$= \frac{1}{\cosh r} \sum_{n=0}^{\infty} e^{in\theta} (\tanh r)^n |n,n\rangle$$

The probability $P(n,n)$ of finding n photons in each of the two modes:

$$P(n,n) = (\tanh r)^{2n} / \cosh^2 r$$

On defining $\bar{n} = \sinh^2 r$,

$$P(n,n) = \frac{\bar{n}^n}{(1+\bar{n})^n}$$ Bose-Einstein distribution

The reduced density matrix for either of the two modes, say, that for the mode a

$$\rho^{(a)} = Tr_b |\rho\rangle\langle\rho|$$

Obtained by tracing over the mode b is given by

$$\rho^{(a)} = \sum_{n=0}^{\infty} \frac{\bar{n}^n}{(1+\bar{n})^n} |n\rangle\langle n|$$ a thermal state

Quantum Entanglement

$$\left|z\right\rangle_{sq} = \frac{1}{\cosh r}\sum_{n=0}^{\infty} e^{in\theta}(\tanh r)^n \left|n,n\right\rangle$$

Does not factorize as a product of the wave function for mode a and the wave function for mode b.

Entanglement entropy

$$S \propto (\overline{n}+1)\ln(\overline{n}+1) - \overline{n}\ln\overline{n}$$

Case of Gaussian entanglement

$$\Psi(x,y) \Rightarrow \text{Gaussian}$$

Strong correlation between modes

$$\left\langle ab\right\rangle - \left\langle a\right\rangle\left\langle b\right\rangle = \cosh r \sinh r e^{i\theta}$$

“ Quantum Entanglement ”

First introduced by **E. Schrödinger**

[Naturwissenschaften, **23**, 807 (1935)]

To explain intriguing features of a composite system

“Any state of a composite system is said to be entangled if it cannot be expressed as a direct product of the state of each subsystem .”

$$\left|\Psi_{AB}\right\rangle = \left|\Phi_{A}\right\rangle \otimes \left|\chi_{B}\right\rangle \equiv \text{ Not entangled}$$

$$\left|\Psi_{AB}\right\rangle = \sum_{ij} C_{ij} \left|\Phi_{A}^{i}\right\rangle \otimes \left|\chi_{B}^{j}\right\rangle \equiv \text{ Entangled} \qquad \text{if } C_{ij} \neq C_i C_j$$

Basic bipartite entangled states :

$$\left.\begin{aligned} \left|\Psi^{\pm}\right\rangle_{AB} &= \frac{1}{\sqrt{2}}\left(\left|0\right\rangle_A \left|1\right\rangle_B \pm \left|1\right\rangle_A \left|0\right\rangle_B\right) \\ \left|\Phi^{\pm}\right\rangle_{AB} &= \frac{1}{\sqrt{2}}\left(\left|0\right\rangle_A \left|0\right\rangle_B \pm \left|1\right\rangle_A \left|1\right\rangle_B\right) \end{aligned}\right\} \text{ (Bell States)}$$

Spin $\uparrow\downarrow$; Polarization - right, left circular/x, y; BEC;

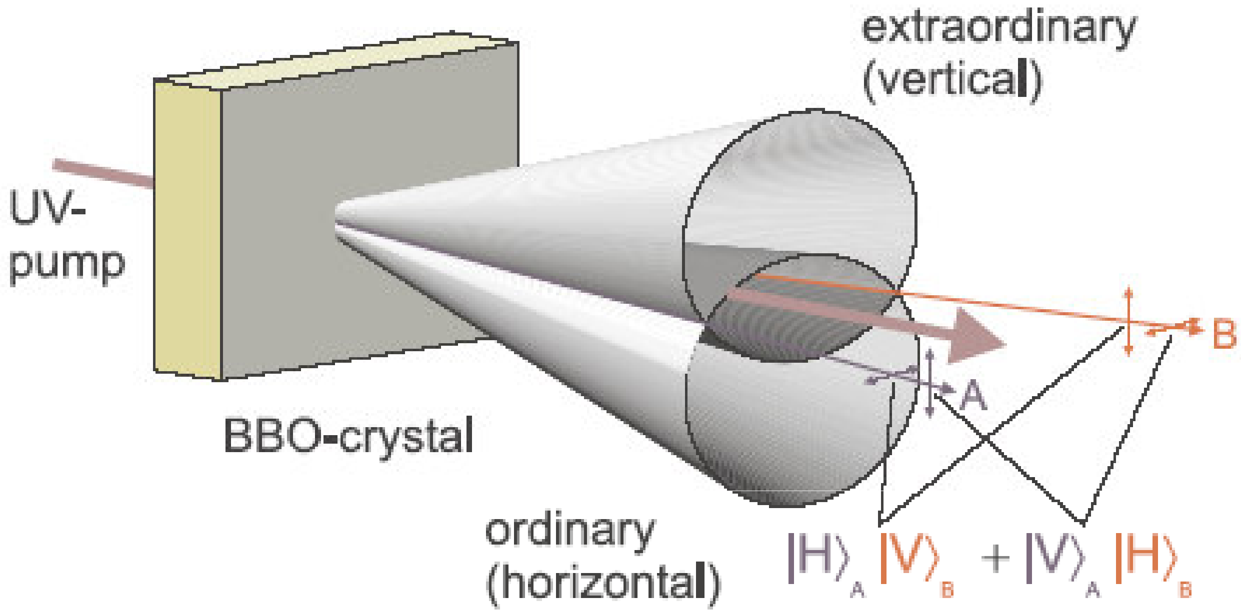
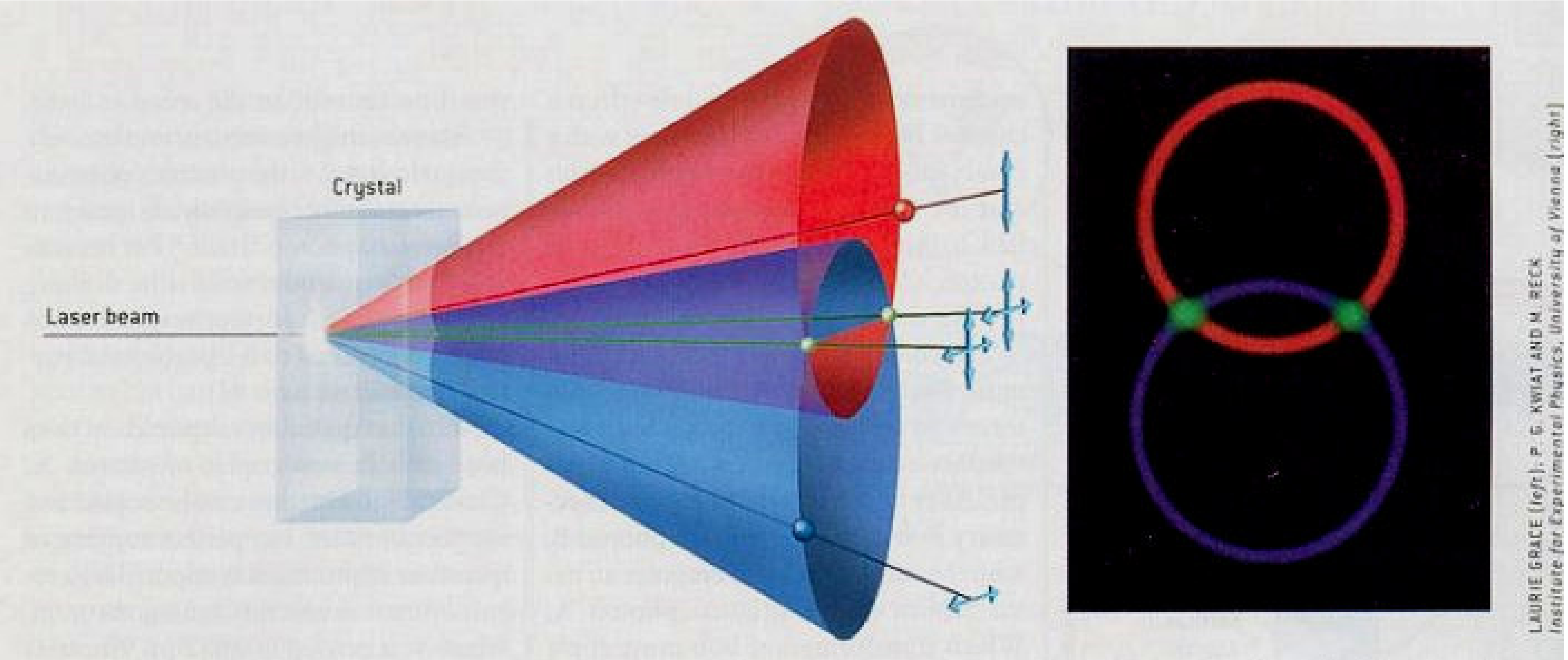


Figure 2.8: *Principle of type-II parametric down-conversion. Inside a nonlinear crystal(here, BBO), an incoming pump photon can decay spontaneously into two photons. Two down-converted photons arise polarized orthogonally to each other. Each photon is emitted into a cone, and the photon on the top cone is vertically polarized while its exactly opposite partner in the bottom cone is horizontally polarized. Along the directions where the two cones intersect, their polarizations are undefined; all that is known is that they have to be different, which results in polarization entanglement between the two photons in beams A and B.*



ENTANGLED PHOTON PAIRS are created when a laser beam passes through a crystal such as beta barium borate. The crystal occasionally converts a single ultraviolet photon into two photons of lower energy, one polarized vertically (*on red cone*), one polarized horizontally (*on blue cone*). If the photons happen to travel along the cone intersections (*green*), neither photon has a definite polarization, but their relative polarizations are complementary; they are then entangled. Colorized image (*at right*) is a photograph of down-converted light. Colors do not represent the color of the light.

Applications

- Foundation Issues
- Nonclassicality
- Precision Measurements

Measurements at Heisenberg limit

Quantum Lithography
Quantum Imaging
Quantum Magnetometry

- Simulation of Quantum Systems
- Quantum Teleportation, Cryptography
- Quantum Logic Gates
- Quantum Algorithms

Quantum entanglement in a two-body system

- Consider two coupled oscillators :

$$H = \frac{1}{2}\left[\left(\frac{p_1^2}{m} + m\omega^2 x_1^2\right) + \left(\frac{p_2^2}{m} + m\omega^2 x_2^2\right) + v(x_1 - x_2)^2\right]$$

- Initial state : two uncoupled oscillators

$$\Psi(x_1, x_2, 0) \propto \exp\left[-\frac{1}{2}(x_1^2 + x_2^2)\right]$$

$$\Psi(x_1, x_2, t) \propto \exp\left[-\left\{\frac{\left[(\omega_- + 1)^2 - (\omega_- - 1)^2 e^{-2i\omega_- t}\right](x_1^2 + x_2^2) - (\omega_- - 1)(1 - e^{-2i\omega_- t})x_1 x_2}{4[\omega_- + 1 + (\omega_- - 1)e^{-2i\omega_- t}]}\right\}\right]$$

$$\sqrt{1 + \frac{v}{m\omega^2}}$$

x_1x_2 term : Correlation arises between two modes 1 and 2

Two particle Wigner function becomes a periodic one with frequency ω_-

Two parts will become entangled and disentangled alternatively

S12 □ two-particle correlation entropy

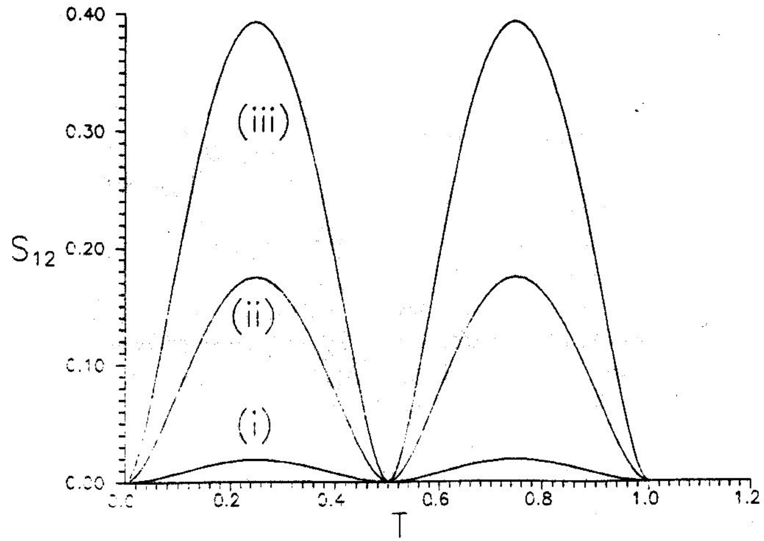
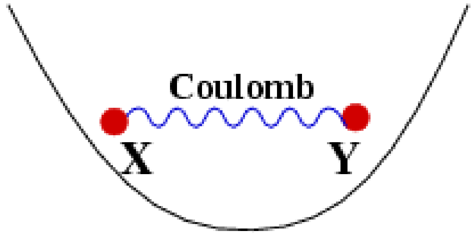


FIG. 1. Two-particle correlation entropy calculated from (A3) and (5.21) within one cycle of the normal mode ω_-: (i) $\omega_-=1.1$, (ii) $\omega_-=1.5$, and (iii) $\omega_-=2.0$.

*H. Huang and G. S. Agarwal, PRA **49**, 52 (1994)*



- Potential energy of the bound ions at **X** and **Y** $\mathbf{U = \frac{1}{2} M\omega^2 \left(X^2 + Y^2\right) + \frac{C}{|X - Y|}}$
- Equilibrium conditions : $\mathbf{\frac{\partial U}{\partial X} = 0, \quad \frac{\partial U}{\partial Y} = 0}$
- Equilibrium solutions : $X = L/2, Y = -L/2$

$$\text{L : characteristic length scale} = \left(\frac{2C}{M\omega^2}\right)^{1/3}$$

➢ Consider deviation from the equilibrium : $X = \frac{L}{2} + \delta X, Y = -\frac{L}{2} + \delta Y$

$$\mathbf{U = M\omega^2 \left[\frac{3}{4} L^2 + \delta X^2 + \delta Y^2 - \delta X.\delta Y\right] + O\left[(\delta X)^3\right]}$$

X and Y motions are entangled

❑ Choose :

$$\delta X = \frac{1}{\sqrt{2}}(Q+R), \quad \delta Y = \frac{1}{\sqrt{2}}(Q-R)$$

$$\Downarrow$$

$$U = \frac{1}{2}M\omega^2\left(Q^2 + 3R^2 + \frac{3}{2}L^2\right)$$

U is sum of two harmonic oscillators with frequencies :

ω for Q-motion (center of mass)

$\sqrt{3}\omega$ for R-motion (relative coordinate)

Entanglement Entropy

$\Psi(x_1, x_2)$ nonfactorized pure state

Entropy is zero

Single particle space

$$\rho(x_1, x_1') = \int \Psi^*(x_1, x_2)\Psi(x_1', x_2)dx_2$$

Mixed state

Non zero entropy S_1

Entanglement entropy $2S_1$

Entanglement Entropy Gaussian States

$$\rho \sim \exp[\text{Gaussian in x, p}]$$

$$S / R = (\sigma + 1)\ln(\sigma + 1) - \sigma \ln \sigma$$

GSA-PRA **3**, 828 (1971).

$$\sigma = \sqrt{\alpha\beta - \gamma^2} - \frac{1}{2}$$

$$\alpha = \langle p^2 \rangle, \quad \beta = \langle x^2 \rangle$$

$$\gamma = \frac{1}{2}\langle xp + px \rangle$$

Parametric Amplification of Scattered Atom Pairs

Gretchen K. Campbell, Jongchul Mun, Micah Boyd, Erik W. Streed, Wolfgang Ketterle, and David E. Pritchard*

MIT-Harvard Center for Ultracold Atoms, Research Laboratory of Electronics and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA

(Received 12 September 2005; published 19 January 2006)

We have observed parametric generation and amplification of ultracold atom pairs. A ^{87}Rb Bose-Einstein condensate was loaded into a one-dimensional optical lattice with quasimomentum k_0 and spontaneously scattered into two final states with quasimomenta k_1 and k_2. Furthermore, when a seed of atoms was first created with quasimomentum k_1 we observed parametric amplification of scattered atoms pairs in states k_1 and k_2 when the phase-matching condition was fulfilled. This process is analogous to optical parametric generation and amplification of photons and could be used to efficiently create entangled pairs of atoms. Furthermore, these results explain the dynamic instability of condensates in moving lattices observed in recent experiments.

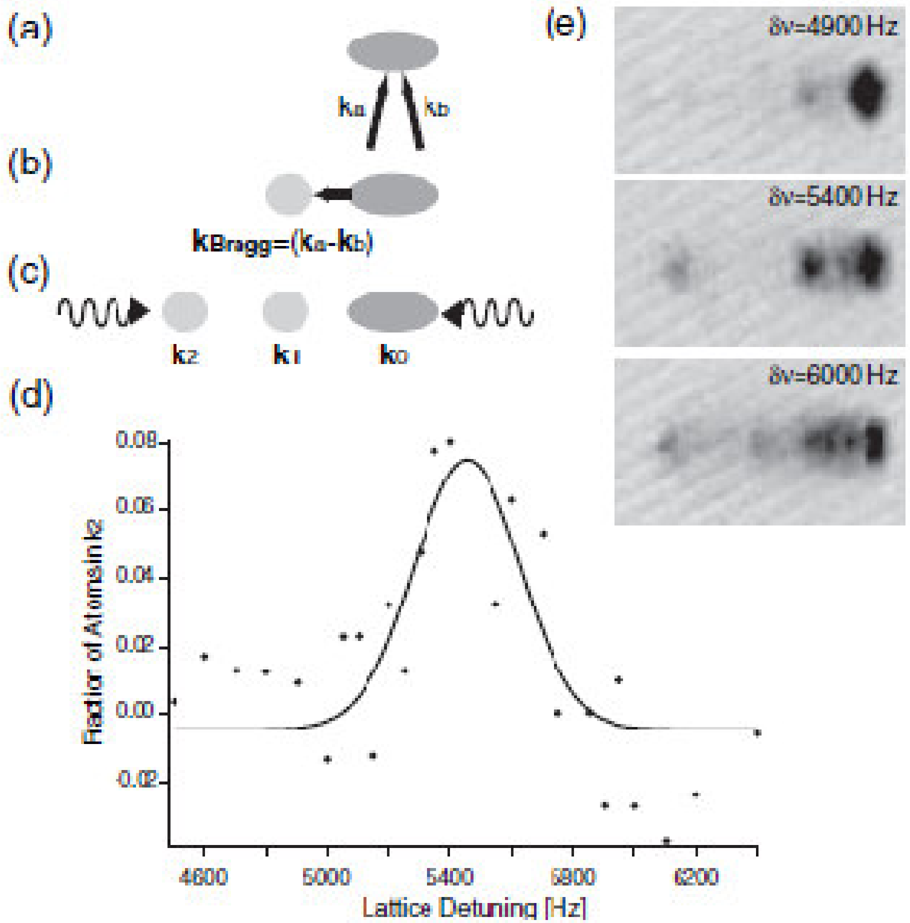


FIG. 2. Parametric amplification of scattered atom pairs in a 1D optical lattice. (a) First, a 2 ms Bragg pulse was applied to the condensate. (b) The Bragg pulse seeded atoms along the long axis of the condensate with momentum $k_{\text{Bragg}} = (k_a - k_b)$ in the lab frame. (c) The optical lattice was then adiabatically ramped on and applied for 10 ms. When the phase-matching condition was fulfilled, parametric amplification of atoms in the seeded state k_1 and its conjugate momentum state k_2 was observed. (d) Resonance curve showing amplification of k_2, when k_1 was seeded. Amplification occurred only when the phase-matching condition was met. For a fixed k_{Bragg}, the resonance condition was found by varying the detuning $\delta\nu$ of the lattice. The data was taken for $k_{\text{Bragg}} = 0.43k_L$. The fraction of amplified atoms was obtained by subtracting images with and without the seed pulse. (e) Absorption images showing amplification of k_1 and k_2 when the phase-matching condition is met. The center of the resonance was at $\delta\nu \approx 5450$ Hz, close to the calculated value of $\delta\nu \approx 5350$ Hz. The width of the resonance is determined by the Fourier width of the Bragg pulse. Most of the scattered atoms in the third image were independent of the seed pulse.

Generation of Macroscopic Pair-Correlated Atomic Beams by Four-Wave Mixing in Bose-Einstein Condensates

J. M. Vogels, K. Xu, and W. Ketterle*

Department of Physics, MIT-Harvard Center for Ultracold Atoms,
Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 13 March 2002; published 24 June 2002)

By colliding two Bose-Einstein condensates, we have observed strong bosonic stimulation of the elastic scattering process. When a weak input beam was applied as a seed, it was amplified by a factor of 20. This large gain atomic four-wave mixing resulted in the generation of two macroscopically occupied pair-correlated atomic beams.

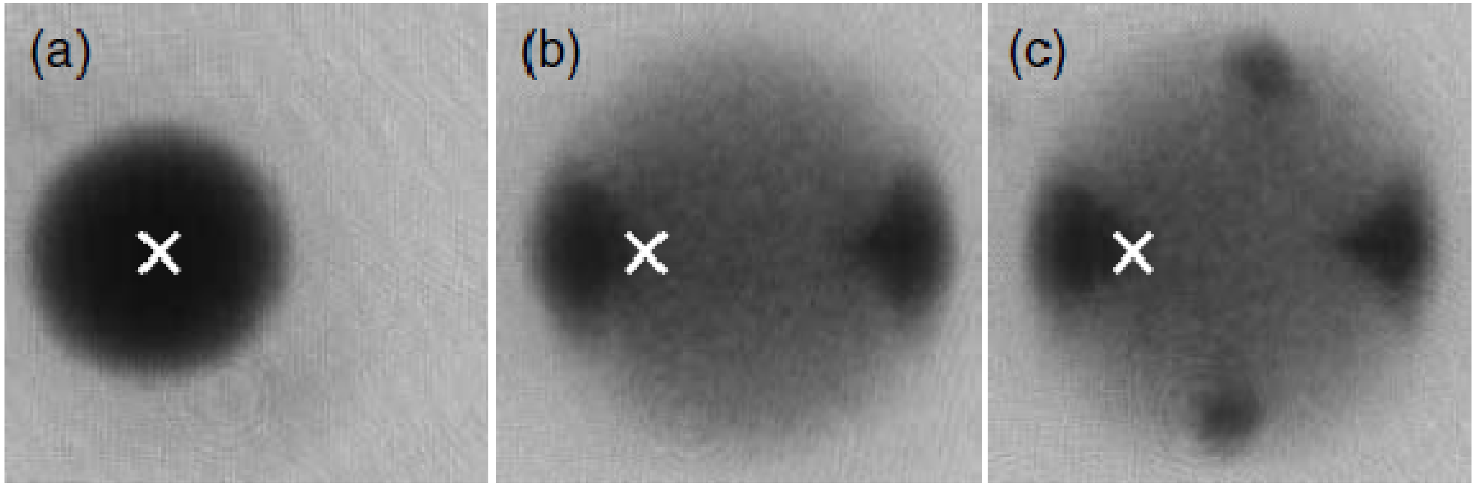


FIG. 2. High-gain four-wave mixing of matter waves. The wave packets separated during 43 ms of ballistic expansion. The absorption images [1] were taken along the axis of the condensate. (a) Only a 1% seed was present (barely visible), (b) only two source waves were created and no seed, and (c) two source waves and the seed underwent the four-wave mixing process where the seed wave and the fourth wave grew to a size comparable to the source waves. The gray circular background consists of spontaneously emitted atom pairs that were subsequently amplified to around 20 atoms per mode. The crosses mark the center position of the unperturbed condensate. The field of view is 1.8 mm wide.

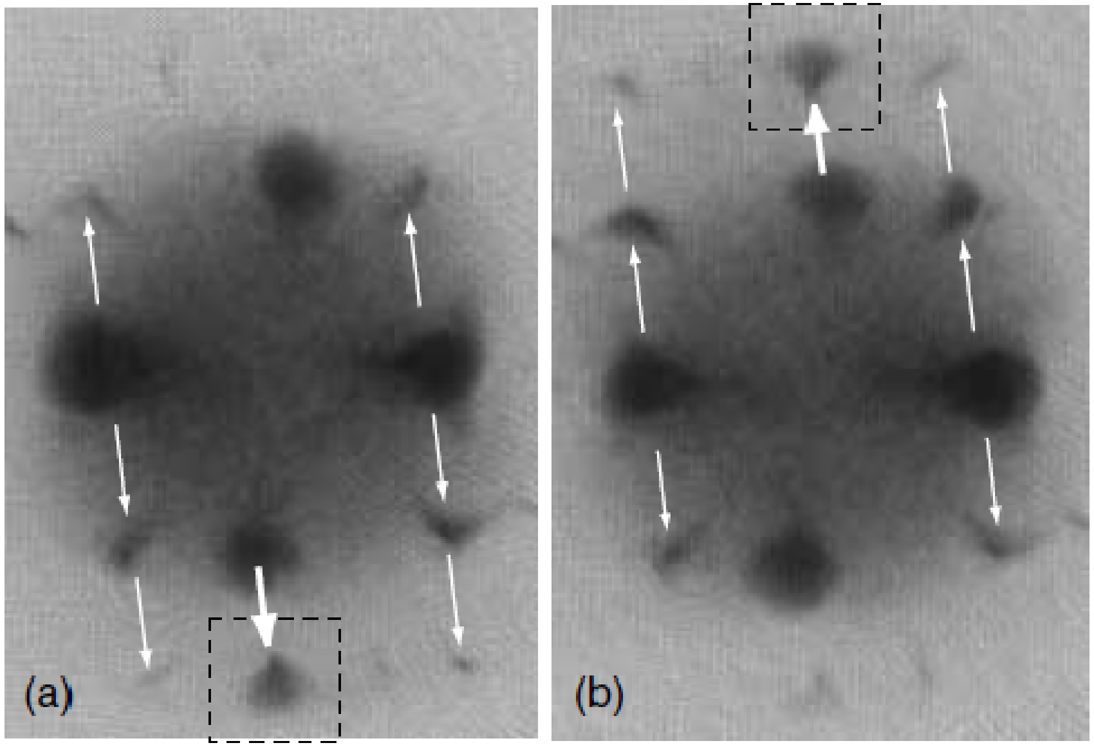


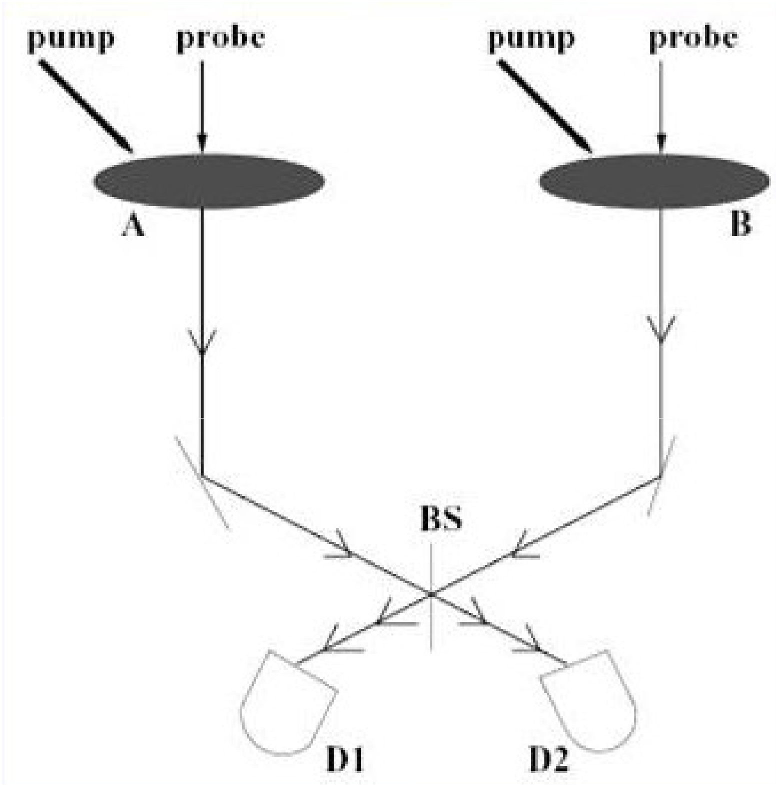
FIG. 3. Absorption images after a readout pulse was applied to (a) the seed wave and (b) the fourth wave. The thick arrows indicate the readout process. The readout pulse was kept short (40 μs), resulting in a large Fourier bandwidth and off-resonant coupling to other wave packets indicated by the narrow arrows. However, this did not affect the readout signal (atoms in the dashed box).

Light Scattering to Determine the Relative Phase of Two Bose-Einstein Condensates

M. Saba,* T. A. Pasquini, C. Sanner, Y. Shin, W. Ketterle, D. E. Pritchard

We demonstrated an experimental technique based on stimulated light scattering to continuously sample the relative phase of two spatially separated Bose-Einstein condensates of atoms. The phase measurement process created a relative phase between two condensates with no initial phase relation, read out the phase, and monitored the phase evolution. This technique was used to realize interferometry between two trapped Bose-Einstein condensates without need for splitting or recombining the atom cloud.

Entangling two condensates by interference



HBT effect: Coincident detection of two photons

Deb and GSA, PRA **78**, 013639 (2008)

Interference between two independent condensates

Probe field after interaction with condensates quasiparticles-phonons

$$\hat{c}_a(t) = a_q(t)\hat{\alpha}_q^+ + a_{-q}(t)\hat{\alpha}_{-q} + a_c(t)\hat{c}_a(0),$$

$$\hat{c}_b(t) = b_q(t)\hat{\beta}_q^+ + b_{-q}(t)\hat{\beta}_{-q} + b_c(t)\hat{c}_b(0),$$

Fields after beam splitters

$$\hat{C}_{D_1} = t'\hat{c}_b + r\hat{c}_a,$$

$$\hat{C}_{D_2} = t\hat{c}_a + r'\hat{c}_b,$$

Initial state $\left|\Psi_0\right\rangle = \left|0,0\right\rangle_{AB} \left|\alpha,\beta\right\rangle_{\text{fields}}$

STATE FOLLOWING MEASUREMENT

Joint state of A and B ρ_{AB}

Measurement on B, measured observable D

Von Neumann protection postulate

$$\frac{Tr_B D\rho_{AB} D^+}{Tr_B DD^+}$$

Conditional state of the system A

Reduced state of the two condensate after detection of a photon

$$\propto Tr_{\text{field}}(\hat{C}_{D_1}\rho\hat{C}^{+}_{D_1})$$

$$|\Psi\rangle \Rightarrow \underset{\alpha_q}{(\ )}|1_A,0_B\rangle + \underset{\beta_q}{(\ )}|0_A,1_B\rangle + \underline{(\ )|0_A,0_B\rangle}$$

drop by phase choice

Interference possible

Interferences also in measurements of transmitted intensity

Entanglement between two distant condensates

Deb and GSA, PRA **78**, 013639 (2008).

Coherence, Correlations, and Interference

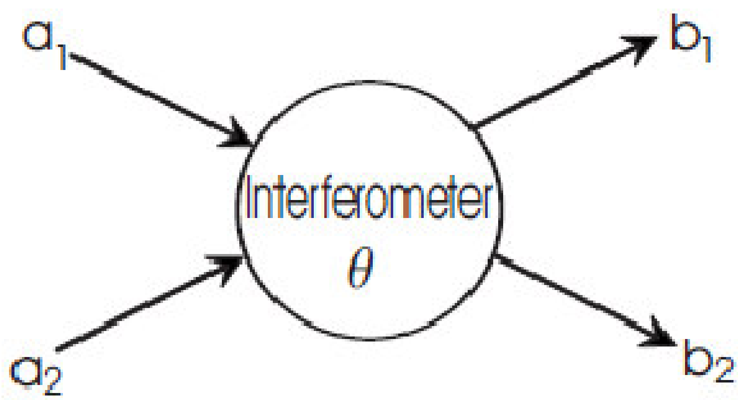


FIG. 1. Schematic of a two-beam interferometer.

a_1, a_2 → Annihilation operator for the input modes

Total field operator after the two beams are made to interfere :

$$b_1 = (a_1 + e^{i\theta} a_2) / \sqrt{2}$$

Coherence, Correlations, and Interference

The mean intensity of the output b_1 is :

$$\langle b_1^+ b_1 \rangle = \frac{\langle a_1^+ a_1 \rangle + \langle a_2^+ a_2 \rangle + e^{i\theta} \langle a_1^+ a_2 \rangle + e^{-i\theta} \langle a_2^+ a_1 \rangle}{2} \qquad (1)$$

For interference to occur we need

$$\langle a_1^+ a_2 \rangle \neq 0$$

If the two beams are in coherent states, then

$$\langle a_1^+ a_2 \rangle = \langle a_1^+ \rangle \langle a_2 \rangle$$

Thus interference obviously occurs for coherent state input

Coherence, Correlations, and Interference

For the input Fock state $\left|n_1, n_2\right\rangle$

$$\left\langle a_1^+ a_2 \right\rangle = 0$$

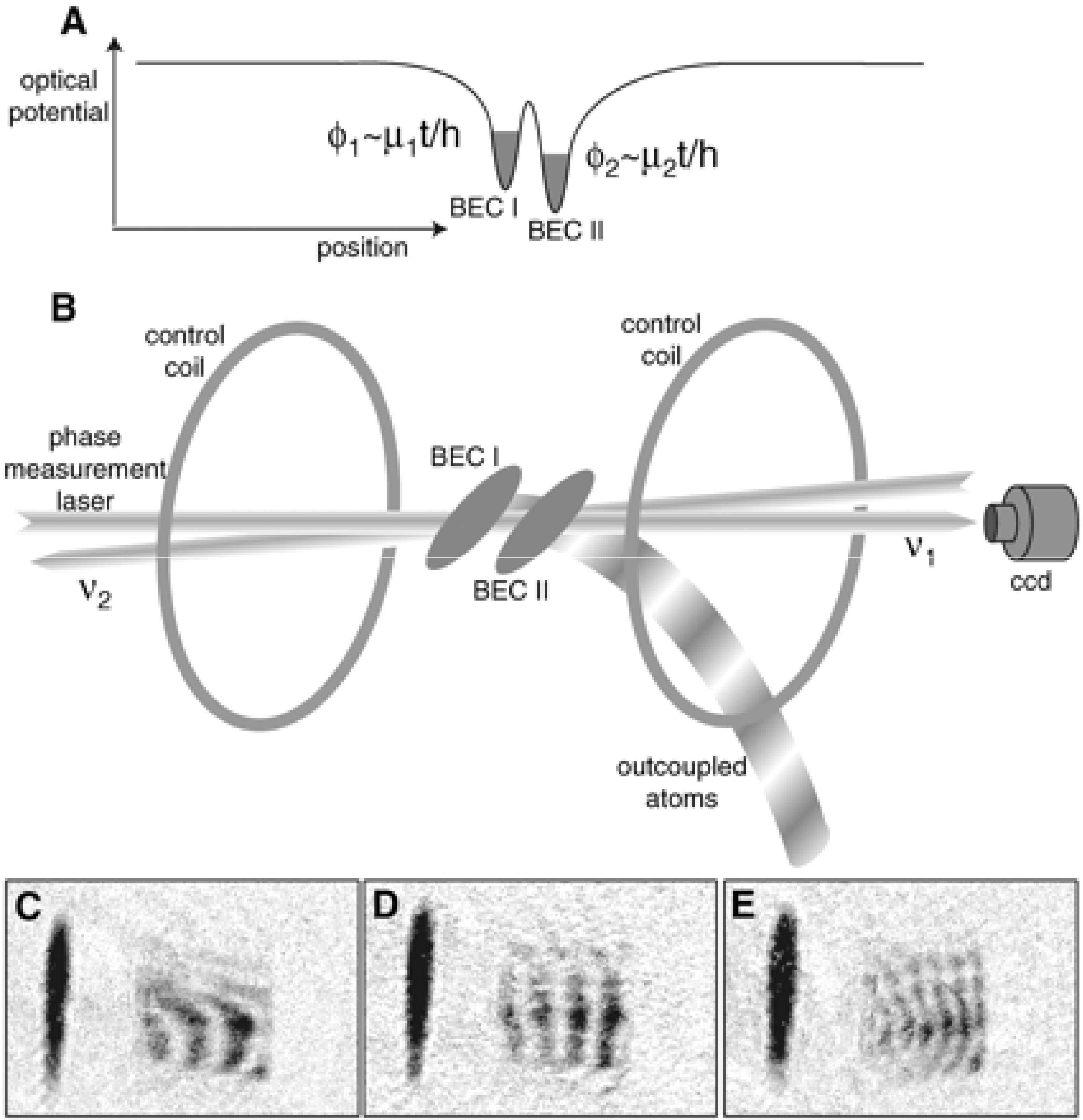
Thus no interference occurs in the mean intensity given by Eq. (1)

Consider an entangled state of the form

$$\left|\Psi\right\rangle = \frac{\left|1,0\right\rangle + \left|0,1\right\rangle}{\sqrt{2}}$$

then $$\left\langle a_1^+ a_2 \right\rangle = \frac{1}{2}$$

Therefore for the observation of interference at the level of mean intensity one needs to have nonzero correlation which is possible with a state like $\left|\Psi\right\rangle$ unless one is dealing with coherent beams of light.


A
optical potential
$\phi_1 \sim \mu_1 t/h$
$\phi_2 \sim \mu_2 t/h$
BEC I
BEC II
position
B
control coil
control coil
phase measurement laser
BEC I
BEC II
ν_2
ν_1
ccd
outcoupled atoms
C
D
E

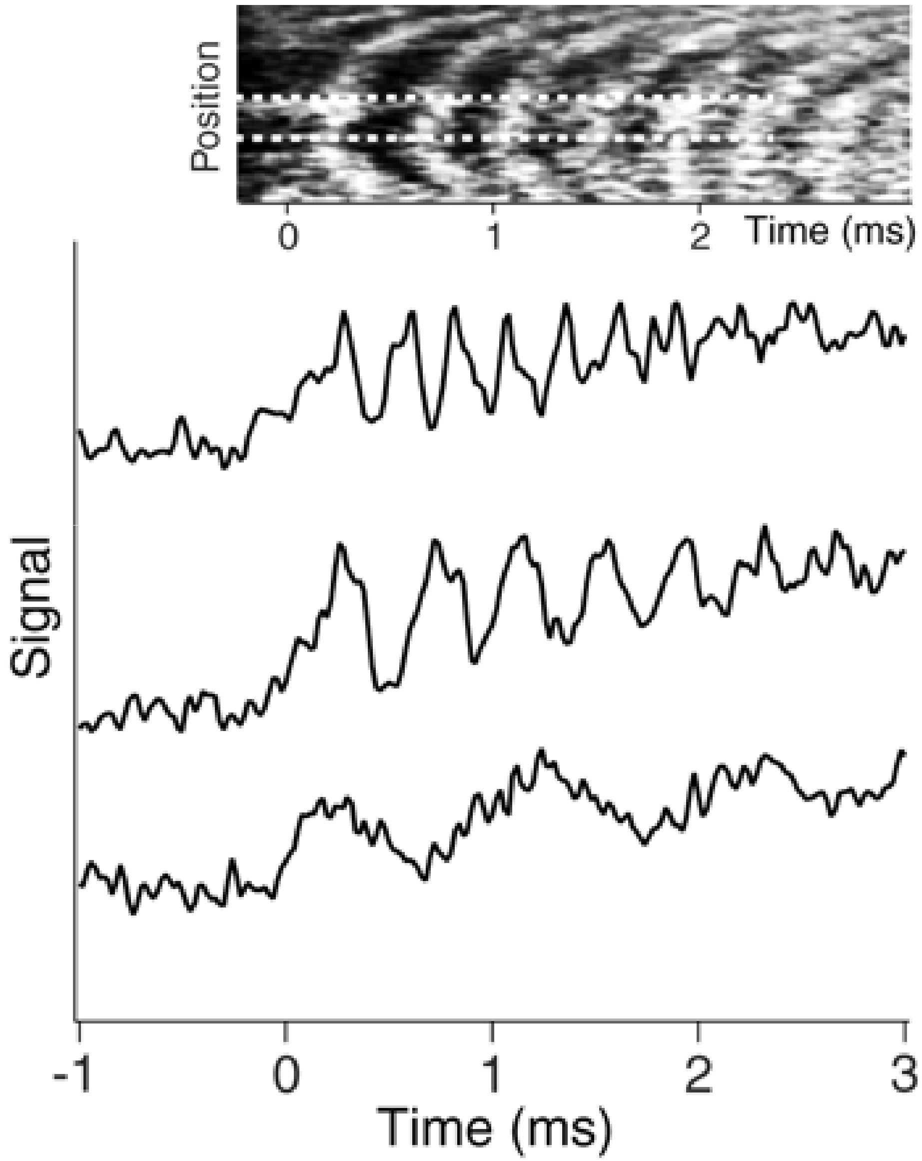


Fig. 2. Continuous optical readout of the relative phase of two condensates. In the upper panel is the optical signal image detected by streaking the CCD camera (*24*). The traces, offset vertically for clarity, are cross sections of the images (the central trace corresponds to the upper image integrated between the dashed lines). Bragg scattering starts at $t = 0$ when the second beam is turned on. The relative depth of the two wells was different for the three traces, generating a difference in the oscillation frequency. The overall slope on the traces was due to spontaneous Rayleigh scattering of the light from the atoms in the condensates. As the time went on, the condensates were depleted and the Rayleigh scattering was reduced. Excitations in the condensates appeared as tilted or curved fringes in the streak image; in such cases, we took cross sections from portions of the images where fringes were vertical and therefore the phase evolution was less perturbed.

Quantum squeezing

$$d = \frac{a + be^{i\varphi}}{\sqrt{2}}, \qquad [d, d^+] = 1.$$

The second order moments of d

$$\langle d^2 \rangle = \cosh r \sinh r e^{i(\theta+\varphi)}, \quad \langle d^+ d \rangle = \sinh^2 r.$$

Quadratures associated with $de^{-i\frac{\theta+\varphi}{2}}$

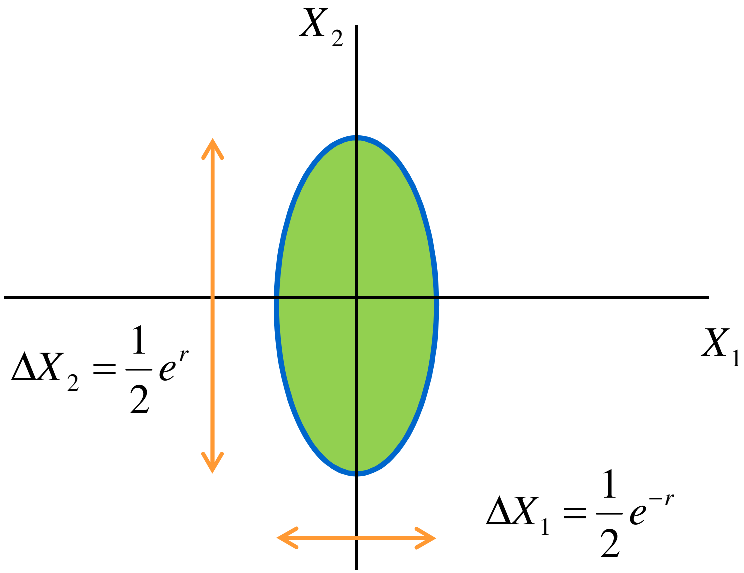
$$Q = (de^{-i\frac{\theta+\varphi}{2}} + d^+ e^{i\frac{\theta+\varphi}{2}}) / \sqrt{2},$$
$$P = (de^{-i\frac{\theta+\varphi}{2}} - d^+ e^{i\frac{\theta+\varphi}{2}}) / (\sqrt{2} i),$$

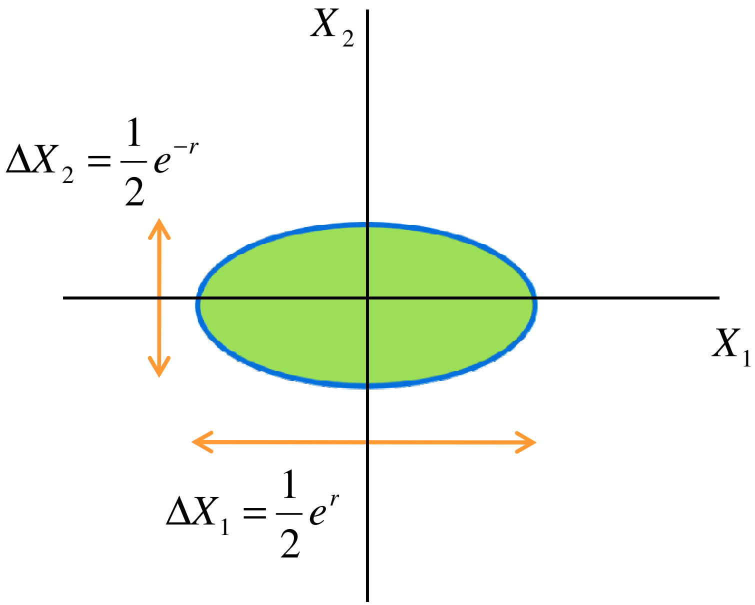
Uncertainties:

$$S_Q = e^{2r}, \qquad S_P = e^{-2r}.$$

Quadrature P squeezed.

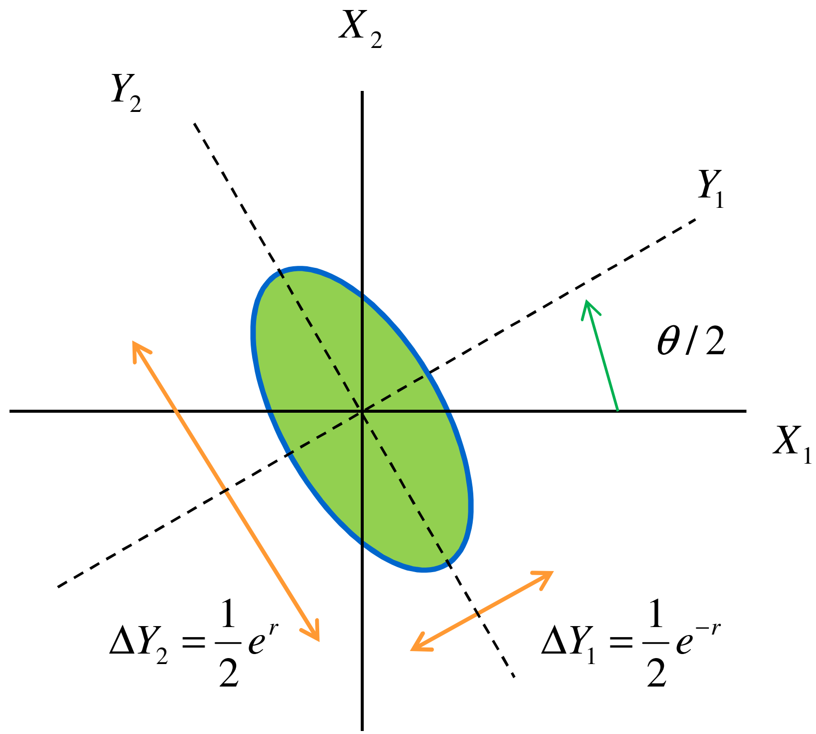
No squeezing if a and b were uncorrelated.





Error ellipse for a squeezed vacuum state where the squeezing is in the X_1 quadrature.

Error ellipse for a squeezed vacuum state where the squeezing is in the X_2 quadrature.



Rotated error ellipse for the squeezed vacuum.
The squeezing is along the $\theta/2$ direction.

dB scale

$$10\log_{10}\left(\frac{Power_{vac}}{P}\right) \equiv A$$

$$10\log_{10}\left(\frac{Power_{sq}}{P}\right) \equiv B$$

$$B - A \equiv 10\log_{10}\left(\frac{Power_{sq}}{Power_{vac}}\right)$$

$$\equiv 10\log_{10}\left(\frac{x^2}{\left\langle x^2\right\rangle_{vac}}\right)$$

Now for 50% squeezing $x^2/\left\langle x^2\right\rangle$ is 1/2 and hence we have

$$B - A \equiv 10\log_{10}\left(\frac{1}{2}\right)$$

$$\equiv -10\log_{10}2 = -3db$$

Spin squeezing

A. Via Quantum State Transfer

GSA and Puri (Opt commun. **69**, 267;
Phys. Rev. A **41**, 3782 (1990)).

B. Via Nonlinear Hamiltonians and Unitary Transformations

Kitagawa and Ueda, Phys. Rev. A **47**, 5138 (1992).

C. New Proposals using Projection Postulates

BEC: B+C

Theory A. Sørensen et al. Nature **409**, 63 (2001).

Exp J. Estève et al. Nature **455**,1216 (2008).

Two component condensates

$a, \ b$: Bosonic operators

Angular momentum algebra (Schwinger)

$$J^+ = a^+ b, \quad J^- = ab^+,$$
$$J_z = \frac{1}{2}(a^+ a - b^+ b).$$

$a^+ a + b^+ b$ Total number of particles-conserved

Squeezing parameter

$$\xi^2 = N(\Delta J_{n1})^2 / (\langle J_{n2} \rangle^2 + \langle J_{n3} \rangle^2)$$

n_i orthogonal

$\xi^2 < 1$; state entangled!

ξ^2 several orders less than unity in Na BEC,

$$\left| F = 1, M_F = \pm 1 \right\rangle$$

Effective interaction

$$U_{a,a} a^{+2} a^2 + U_{b,b} b^{+2} b^2 + 2U_{ab} a^+ b^+ ab$$

$$U_{\alpha\beta} = \frac{4\pi\hbar^2 a_{\alpha\beta}}{m}$$

Condition

$U_{aa} + U_{bb} \neq 2U_{ab}$, O.K. in Na condensate

Choose $\vec{n}_1 = (0, \cos\theta, \sin\theta)$

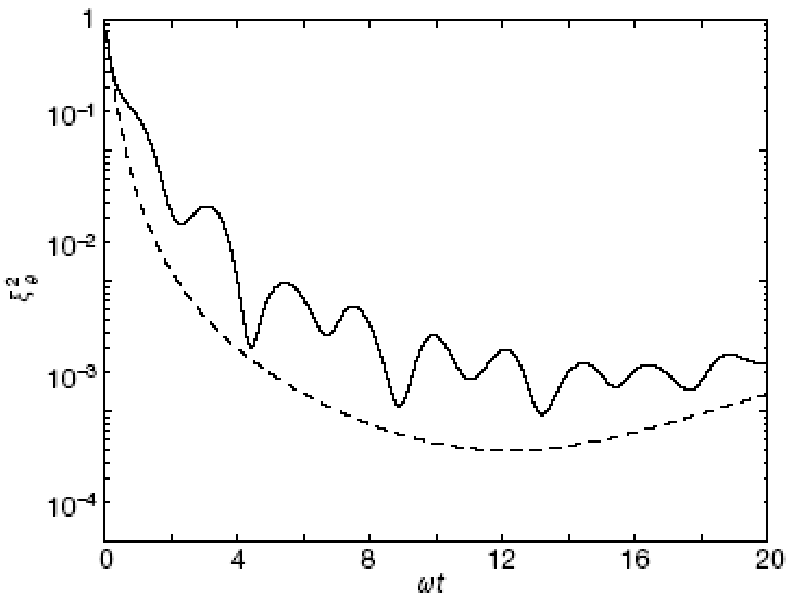


Figure 1 Reduction in the squeezing parameter ξ^2. A fast $\pi/2$ pulse between two internal states is applied to all atoms in the condensate. The subsequent free evolution with time results in a strong squeezing of the total spin. The angle θ is chosen such that ξ_θ^2 is minimal. The solid line shows the results of a numerical integration (see text). For numerical convenience we have assumed a spherically symmetric potential $V(r) = m\omega^2 r^2/2$. The parameters are $a_{aa}/d_0 = 6 \times 10^{-3}$, $a_{bb} = 2a_{ab} = a_{aa}$, and $N = 10^5$. The dashed curve shows the squeezing obtained from the hamiltonian $H_{spin} = \hbar\chi J_z^2$. The parameter χ is chosen such that the reduction of $\langle J_x \rangle$ obtained from the solution in ref. 7 is consistent with the results of ref. 11.

Sørensen et al. Nature **409**, 63 (2001).

Minimum obtainable squeezing parameter

$$\xi_\theta^2 = \frac{1}{2}\left(\frac{3}{N}\right)^{2/3}$$

Nature **455**, 1216 (2008)

Squeezing and entanglement in a Bose–Einstein condensate

J. Estève[1], C. Gross[1], A. Weller[1], S. Giovanazzi[1] & M. K. Oberthaler[1]

Entanglement, a key feature of quantum mechanics, is a resource that allows the improvement of precision measurements beyond the conventional bound attainable by classical means[1]. This results in the standard quantum limit, which is reached in today's best available sensors of various quantities such as time[2] and position[3,4]. Many of these sensors are interferometers in which the standard quantum limit can be overcome by using quantum-entangled states (in particular spin squeezed states[5,6]) at the two input ports. Bose–Einstein condensates of ultracold atoms are considered good candidates to provide such states involving a large number of particles. Here we demonstrate spin squeezed states suitable for atomic interferometry by splitting a condensate into a few parts using a lattice potential. Site-resolved detection of the atoms allows the measurement of the atom number difference and relative phase, which are conjugate variables. The observed fluctuations imply entanglement between the particles[7–9], a resource that would allow a precision gain of 3.8 dB over the standard quantum limit for interferometric measurements.

Spin squeezing was one of the first quantum strategies proposed to overcome the standard quantum limit, in a precision measurement[5,6] that triggered many experiments[10–17]. It applies to measurements where the final readout is done by counting the occupancy difference between two quantum states, as in interferometry or in spectroscopy. The name 'spin squeezing' originates from the fact that the N particles used in the measurement can be described by a fictitious spin $J = N/2$. In an interferometric sequence, the spin undergoes a series of rotations in which one of the rotation angles is the phase shift to be measured. A sufficient criterion for the input state, allowing for quantum-enhanced metrology, is given by $\xi_S < 1$, where $\xi_S^2 = 2J\Delta J_z^2/(\langle J_x\rangle^2 + \langle J_y\rangle^2)$ is the squeezing parameter introduced in ref. 6. The fluctuations of the spin in one direction have to be reduced below shot noise (here $\Delta J_z^2 < J/2$), and the spin polarization in the orthogonal plane, $\langle J_x\rangle^2 + \langle J_y\rangle^2$, has to be large enough to maintain the sensitivity of the interferometer. A pictorial representation of this condition is shown in Fig. 1b. The precision of such a quantum-enhanced measurement is $\xi_S/\sqrt{N}$, whereas the standard quantum limit set by shot noise is $1/\sqrt{N}$.

In this Letter, we report the observation of entangled squeezed states in a Bose–Einstein condensate of ^{87}Rb atoms. The particles are distributed over a small number of lattice sites (between two and six) in a one-dimensional optical lattice (Fig. 1a). The occupation number per site ranges from 100 to 1,100 atoms. The two modes supporting the squeezing are two states of the external atomic motion corresponding to the condensate mean-field wavefunctions

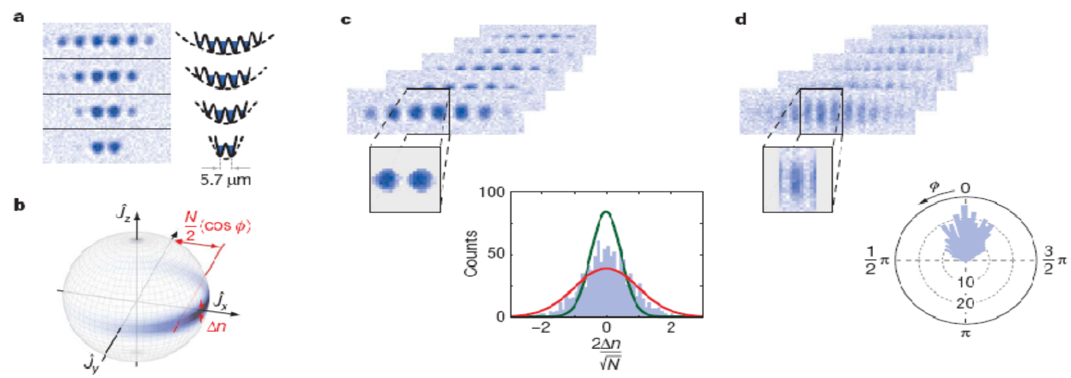


Figure 1 | Observing spin squeezing in a Bose–Einstein condensate confined in a double- or six-well trap. a, The atoms are trapped in an optical lattice potential superimposed on an harmonic dipole trap. The number of occupied sites is adjusted by changing the confinement in the lattice direction. High-resolution imaging allows us to resolve each site. **b,** Gain in quantum metrology is obtained for spin squeezed states exhibiting reduced fluctuations in one direction (z) and a sufficiently large polarization in the orthogonal plane (x, y) as depicted on the Bloch sphere. For our system, spin fluctuations in the z direction translate to atom number difference fluctuations Δn between two adjacent wells. The polarization of the spin in the x–y plane is proportional to the phase coherence, $\langle\cos\phi\rangle$, between the wells. **c,** The atom number fluctuations at each site are measured by integrating the atomic density obtained from absorption images. We compare a typical histogram showing sub-Poissonian fluctuations in the atom number difference with the binomial distribution (red curve). The green curve corresponds to the deduced distribution after subtracting the photon shot noise, leading to a number squeezing factor of $\xi_N^2 = -6.6$ dB. **d,** The phase coherence is inferred from the interference patterns between adjacent wells. The histogram shown corresponds to a phase coherence of $\langle\cos\phi\rangle = 0.9$.

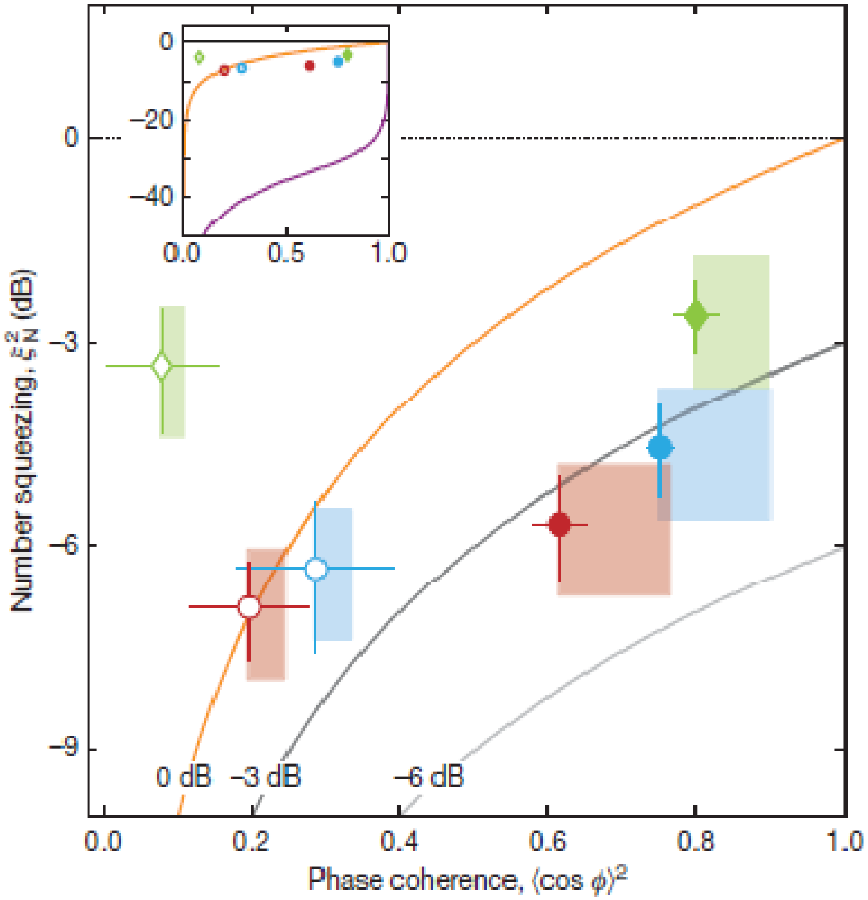
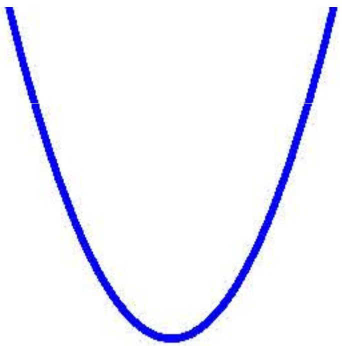


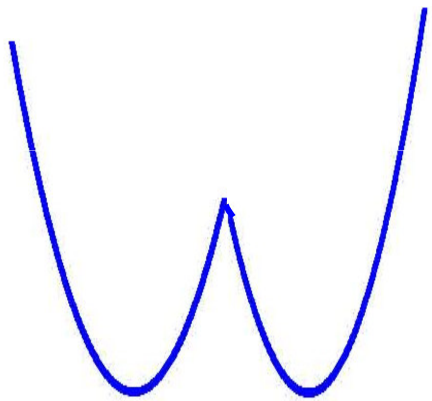
Figure 2 | Number squeezing and phase coherence as a criterion for quantum metrology and entanglement. Measurements are shown for the two main well pairs of a six-well lattice (red and blue circles) and for a double-well potential (green diamonds). The total atom number, N, in each pair is approximately 2,200 in the six-well case and 1,600 in the double-well case. Filled and open symbols respectively represent two different experimental regimes corresponding to different final lattice depths (see Fig. 3). For filled symbols, number squeezing and high phase coherence are simultaneously observed, whereas for the open symbols the phase coherence is degraded. The limits for different quantum metrology gains ξ_S^2 are plotted. In the coherent-number-squeezed region (below the orange line), directly usable entanglement is necessarily present in the system. The shaded areas show systematic error bounds due to a possible miscalibration of the atom number (±20%) and to an underestimation of the phase coherence caused by technical noise. The error bars indicate standard deviation deduced from at least 400 experimental realizations. The inset (same quantities as main panel) shows that our data are approximately 25 dB above the optimal allowed number squeezing (purple line), showing room for improvement[24].

Laser Field Created Potential Wells

Double Well Potential



Sudden Transformation



Mesoscopic States

Squeezed States – Typical Quantum Optical Models

$$(ga^2 + g^* a^{\dagger 2})$$

Strong modulations like δ(t) x^2

Double well potential
exactly soluble model
$(|x|-\alpha)^2$

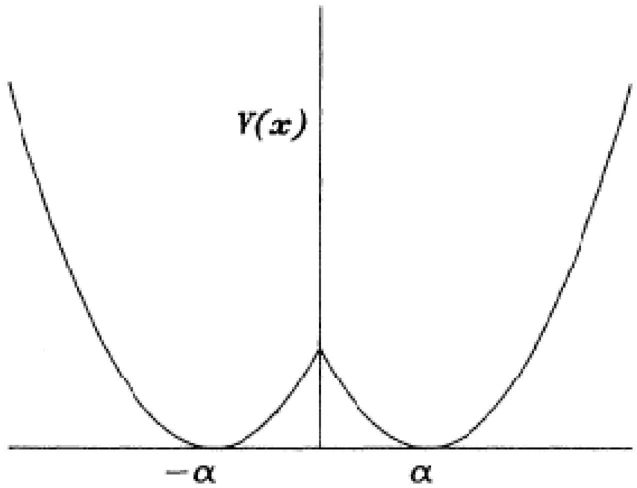


FIG. 2. Potential $V(x)$ for a double-harmonic oscillator. The two potential wells are centered at $\pm\alpha$, respectively.

G.V.Varada & G.S.Agarwal, PRA **48**, 4062 (1993)

Mesoscopic superposition of coherent states

$$|\Psi\rangle = N(|\alpha\rangle + e^{i\varphi}|-\alpha\rangle)$$

where N is the normalization constant $\quad N = \left\{2 + 2e^{-2|\alpha|^2}\cos\varphi\right\}^{-1/2}$

$$a^2|\Psi\rangle = \alpha^2|\Psi\rangle, \quad \langle\Psi|a^{+2}a^2|\Psi\rangle = |\alpha|^4$$

The mean number of photons in the state $|\Psi\rangle$ is

$$\langle\Psi|a^+a|\Psi\rangle = |\alpha|^2(1 - e^{-2|\alpha|^2}\cos\varphi)/(1 + e^{-2|\alpha|^2}\cos\varphi)$$

The Q - parameter for the state

$$Q = 4|\alpha|^2\cos\varphi\, e^{-2|\alpha|^2}/(1 - \cos^2\varphi\, e^{-4|\alpha|^2})$$

which is negative if $\dfrac{\pi}{2} < \varphi < \dfrac{3\pi}{2}$.

Thus the superposition of two coherent states can produce sub-Poissonian statistics.

$$\left\langle ae^{i\theta} + a^{+}e^{-i\theta} \right\rangle = \frac{2\alpha \sin\varphi \sin\theta \, e^{-2|\alpha|^2}}{1 + e^{-2|\alpha|^2}\cos\varphi}$$

$$\left\langle :(ae^{i\theta} + a^{+}e^{-i\theta})^2: \right\rangle = 2\alpha^2 \cos 2\theta + \frac{2|\alpha|^2 (1 - e^{-2|\alpha|^2}\cos\varphi)}{1 + e^{-2|\alpha|^2}\cos\varphi}$$

The squeezing parameter

$$S_\theta = \frac{2|\alpha|^2}{(1 + e^{-2|\alpha|^2}\cos\varphi)^2}[\cos 2\theta(1 + e^{-2|\alpha|^2}\cos\varphi)^2 + (1 - e^{-4|\alpha|^2}\cos\varphi) - 2\sin^2\theta \sin^2\varphi \, e^{-2|\alpha|^2}]$$

Clearly S_θ can become negative for a range of values θ and φ.

For example for $\theta = \pi/2$ and $\cos\varphi > 0$, S_θ is negative.

Mesoscopic superpositions

$$\left|N,0\right\rangle + \left|0,N\right\rangle$$

All atoms in one well or the other (CAT State)

Effective Interaction $\hbar\eta J_+ J_-$

Initial state

$$\left|\Psi(0)\right\rangle \equiv \left|\theta,\phi\right\rangle = \sum_{k=0}^{N} \sqrt{\frac{N!}{(N-k)!k!}} \exp[i(k-N)\phi] \sin^{N-k}\left(\frac{\theta}{2}\right) \cos^{k}\left(\frac{\theta}{2}\right) \left|\frac{N}{2}-k\right\rangle$$

can be prepared by rf pulse excitation; N=even

Use

$$\exp\left[\frac{i\pi}{m}k^2\right] = \sum_{q=0}^{m-1} f_q^{(e)} \exp\left[\frac{2\pi iq}{m}k\right]$$

$$f_q^{(e)} = \frac{1}{m}\sum_{k=0}^{m-1}\exp\left[-\frac{2\pi iq}{m}k\right]\exp\left[\frac{i\pi}{m}k^2\right]$$

$$|\Psi(t)\rangle \qquad \text{at } \eta\mathrm{t} = \frac{\pi}{2}$$

$$\exp[-iHt]|\theta,\phi\rangle = \frac{\exp[-iN^2\pi/2]}{\sqrt{2}}\left[\exp\left(\frac{i\pi}{4}\right)\left|\theta,\phi-\pi\frac{N-1}{2}\right\rangle + \exp\left(\frac{-i\pi}{4}\right)\left|\theta,\phi-\pi\frac{N-3}{2}\right\rangle\right]$$

GSA et al, PRA **56**, 2249 (1997).

$$N = \text{even}, \theta = \pi/2, \varphi_0 = \varphi - \frac{\pi(N-1)}{2},$$

$$\left|\Psi(t)\right\rangle = \frac{1}{\sqrt{2}}\left\{ e^{i\pi/4}\prod_j \frac{1}{\sqrt{2}}\left(\left|\uparrow\right\rangle e^{-i\varphi_0} + \left|\downarrow\right\rangle\right) + e^{-i\pi/4}\prod_j \frac{1}{\sqrt{2}}\left(e^{-i\varphi_0}\left|\uparrow\right\rangle - \left|\downarrow\right\rangle\right)\right\}$$

Rotation by pulses

$$R\left(\frac{\pi}{2}, \varphi_0\right) = \exp\left\{-\frac{i\pi}{2}\left(J_x \sin\varphi_0 - J_y \cos\varphi_0\right)\right\}$$

$$\frac{1}{\sqrt{2}}\left\{ e^{i\pi/4}\prod_j \left|\uparrow\right\rangle + e^{-i\pi/4}\prod_j \left|\downarrow\right\rangle \right\} = \frac{1}{\sqrt{2}}\left\{ e^{i\pi/4}\left|N,0\right\rangle + e^{-i\pi/4}\left|0,N\right\rangle \right\}$$

Schrödinger CAT

NOON State

Greenberger Horne Zeilinger State

Cavity QED Effects

Cavity Optomechanics with a Bose-Einstein Condensate

Science **322**, 235 (08)

Ferdinand Brennecke,* Stephan Ritter,* Tobias Donner, Tilman Esslinger†

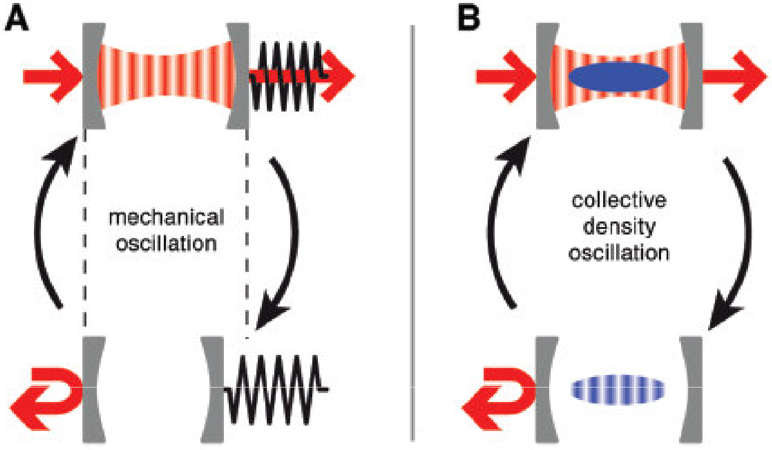


Fig. 1. (**A**) Cavity optomechanical model system. A mechanical oscillator, here one of the cavity mirrors, is coupled via radiation pressure to the field of a cavity whose length depends on the oscillator displacement. (**B**) Coupling a BEC dispersively to the field of an optical high-finesse cavity constitutes an equivalent system. Here, a collective density excitation of the condensate acts as the mechanical oscillator, which strongly couples to the cavity field. Feedback on the cavity field is accomplished by the dependence of the optical path length on the atomic density distribution within the spatially periodic cavity mode structure. In contrast to optomechanical systems presented so far, this mechanical oscillator is not based on the presence of an external harmonic potential (e.g., a spring); rather, it is provided by kinetic evolution of the condensate density excitation.

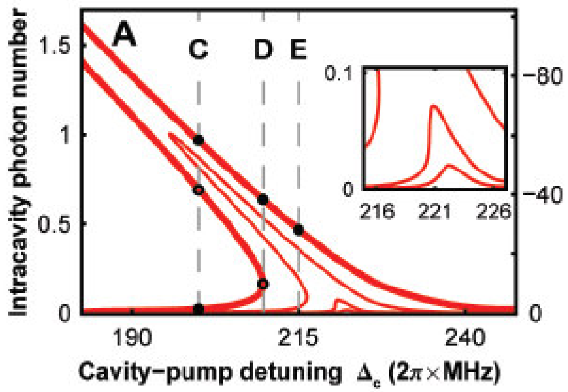
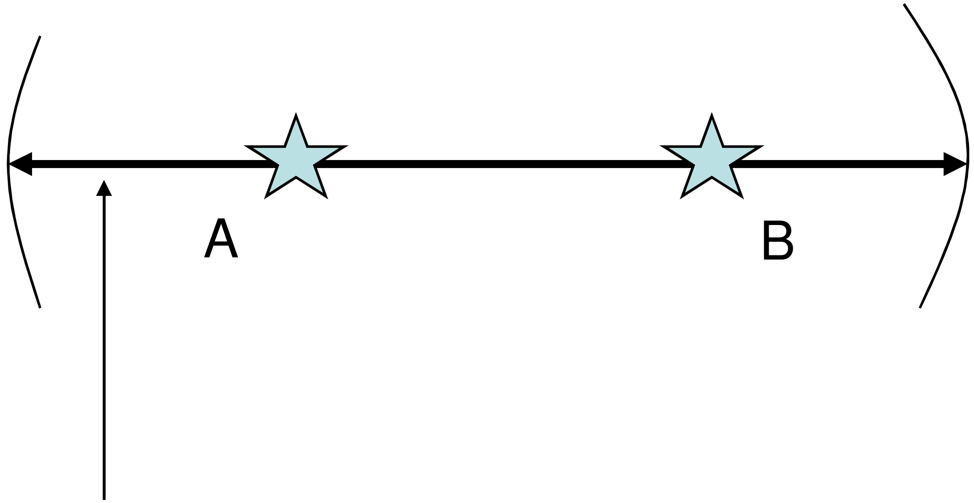


Fig. 3. Steady-state and dynamical behavior of the BEC-cavity system in the two-mode model. (**A**) Mean intracavity photon number and corresponding oscillator displacement X versus cavity-pump detuning Δ_c for the steady-state solutions of Eqs. 3 and 4. The curves correspond to mean intracavity photon numbers on resonance of $\eta^2/\kappa^2 = 0.02$, 0.07, 1, and 7.3, and a pump-atom detuning of $\Delta_a = 2\pi \times 32$ GHz. The inset highlights the bistable behavior for pump amplitudes larger than $\eta_{cr} \approx 0.27\kappa$. (**B**) Intracavity photon

Cavity QED Effects



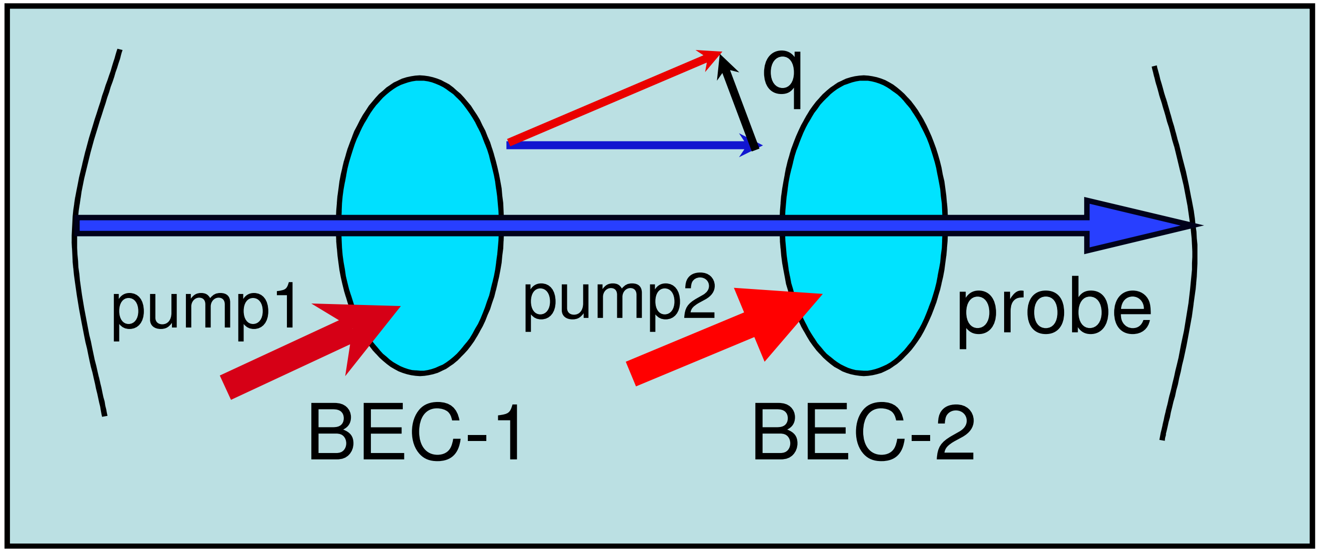
Cavity Mode; Atoms do not interact

Large detuning $\omega_A - \omega_c$ $\Longrightarrow$ Cavity field in vacuum state

Effective interaction between A & B :

$$v\left(S_A^+ S_B^- + H.C.\right); \qquad v \propto \frac{g^2}{(\omega_A - \omega_c)}$$

Cavity QED effects in Bose Condensates



Dispersive Cavity

$$c_{k_1} c_{k_2}^{+} \left(a_q^{+} + a_{-q} \right) + d_{k_1} d_{k_2}^{+} \left(b_q^{+} + b_{-q} \right)$$

Eliminate common probe field (cavity field) → variety of effective interaction depending on the role of pump and probe in scattering process

Quantum State Transfer

$$a_q^+ b_{q'}^+ + H.C$$

Entanglement between condensate

New Possibilities – parametric gain in “q” is already produced by using other Bragg beams.