Basics of Ion Traps

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Why trapping?

"A single trapped particle floating forever at rest in free space would be the ideal object for precision measurements" (H.Dehmelt)

Ion traps provide the closest approximation to this ideal

Pioneers of ion trapping: Hans Dehmelt and Wolfgang Paul (Nobel price 1989)





Trapping of charged particles by electromagnetic fields

Required: 3-dimensional force towards center $\mathbf{F} = -\mathbf{e} \ \mathbf{grad} \ \mathbf{U}$ Convenience: harmonic force $\mathbf{F} \propto \mathbf{x}, \mathbf{y}, \mathbf{z}$ \rightarrow $\mathbf{U} = \mathbf{a}\mathbf{x}^2 + \mathbf{b}\mathbf{y}^2 + \mathbf{c}\mathbf{z}^2$ Laplace equ.: $\Delta(\mathbf{eU}) = \mathbf{0}$ \rightarrow $\mathbf{a}, \mathbf{b}, \mathbf{c}$ can not be all positiveConvenience: rotational symmetry

 $U = (U_0/r_0^2)(x^2+y^2-2z^2)$

Quadrupole potential

Equipotentials: Hyperboloids of revolution

<u>Problem:</u> No 3-dimensional potential minimum because of different sign of the coefficients in the quadrupole potential

Solutions:

- Application of r.f. voltage: dynamical trapping
 Paul trap
- d.c. voltage + magnetic field in z-direction:

Penning trap

The ideal 3-dimensional Paul trap

Potential: $U=(U_0+V_0\cos\Omega t)(r^2-2z^2)/d_0^2$ Using dimensionless parameters a, q

 $a_{z} = \frac{8 eU_{o}}{md_{o}^{2} \Omega^{2}} = -2 a_{r}$ $q_{z} = \frac{4 eV_{o}}{md_{o}^{2} \Omega^{2}} = 2 q_{r}$ u = r, z $\tau = \Omega t / 2$ $U: DC_{V: RF}$ $w: ma_{e: cha}$ $d: dist_{d: dist_{d}}$ u: r, z $\Omega: RF$

U: DC V: RF m: mass of particle e: charge of particle d: distance between ele

$$d = \sqrt{\frac{1}{2}r_0^2 + z_0^2}$$

Ω: RF frequency

We can write the equation of motion from F=∆(eU) and obtain

$$\int \frac{d^2 u}{dt^2} + (a - 2q \cos 2\tau)u = 0$$

Mathieu differential equation

Electrode configuration for Paul traps



Paul trap of 1 cm diameter (Univ. of Mainz)



Wire trap with transparent electrodes (UBC Vancouver)



Mathieu differential equation:

The solutions are well known. Depending on the size of the parameters a and q, the amplitude u remains finite in time or goes to infinity.



Stability in 3 dimension when radial and axial domains overlap

Overlapping areas of stable confinemt in axial and radial direction for a Paul trap.

Defines operational parameter for stable trapping.



First stable region of a Paul trap



First region of stability of the Paul trap.

The shows a stable parameter set in RF-only mode (hence with no DC off-set)



solution of the equation of motion:

$$u(t) = A \sum_{n=0}^{\infty} c_{2n} \cos(\beta + 2n)(\Omega t/2)$$
$$\beta = \beta (a, q)$$
$$c_{2n} = f (a, q)$$

Approximate solution for a,q<<1:

 $u(t) = A[1 - (q/2)\cos\omega t]\cos\Omega t$

$$\omega = \beta/2\Omega$$
 $\beta^2 = a + q^2/2$

This is a harmonic oscillation at frequency Ω (micromotion) modulated by an oscillation at frequency ω (macromotion)

Time averaged potential depth:

$$\overline{D_i} = \frac{m}{8} \Omega^2 r_0^2 \beta_i^2$$

Numerical example: m=50 $\Omega/2\pi=1$ MHz $r_0=1$ cm $\beta=0.3$

 $\mathbf{D} = \mathbf{25} \ \mathbf{eV}$

Maximum ion density, when space charge potential equals trapping potential depth

 $n_{max} \approx 10^6 \text{ cm}^{-3}$

Ion oscillation frequencies in a Paul Trap

$$\omega_z \approx (a_z + \frac{q_z^2}{2})\Omega$$

$$\omega_x = \omega_y = \omega_r \approx (a_r + \frac{q_r^2}{2})\Omega$$

$$\omega_{z,r} = m\Omega \pm n\omega_{z,r}$$
$$m, n = 0, 1, 2...$$

Mechanical analogon: Rotating saddle



Figure 1.1: The effect of rotating the potential in part A, is a pseudopotential well which is illustrated in part B. The particle motion in the pseudopotential is indicated by the black line. The motion is a combination of a secular motion in the pseudopotential and a small amplitude micromotion at the frequency of rotation ω_f .

Ion motion at different operating conditions









Trajectory of a single microscopic particle (Wuerker 1959)



Observed ion oscillation frequencies

Excitation of motion by an additional weak rf field leads to ion loss at resonances



Frequency (kHz)

Excitation of a motional frequency with different rf amplitudes







Ion number (arb. Units)

Interaction in ion cloud characterized by plasma parameter

$$\Gamma = \frac{1}{4\pi\varepsilon_0} \frac{q^2/r}{k_B T}$$

- F<1: gaseous behaviour→ Individual particles model with small perturbation</p>
- Γ>1: liquid like behaviour
- **Γ>176**: strong interaction regime: crystallization

Calculated spatial distribution of an ion cloud at different temperatures



Measured density distribution of trapped ion cloud



Maximum density limited by space charge to ~ 10⁶ cm⁻³

Influence of trap imperfections

expansion of the trapping potential in spherical harmonics

$$\Phi(|\mathbf{r}|) = U_0 \sum_{k=1}^{\infty} C_k \left(\frac{r}{r_0}\right)^k P_k(\cos\theta)$$

Consequence: Asymmetry of motional resonances



Instability of ion motion

Ion trajectories become unstable when the axial and radial macromotion frequencies ω_r , ω_z and the micromotion frequency Ω are related as

 $\mathbf{n}_{\mathbf{r}}\omega_{\mathbf{r}} + \mathbf{n}_{\mathbf{z}}\omega_{\mathbf{z}} = \mathbf{m}\Omega$

 n_r, n_z, m integer, $n_r + n_z = k$, 2k=order of perturbing potential



caused by higher order contributions to the quadrupolar trap poten

Measured ion density in a real Paul trap



Influence of buffer gas collisions

- m_{gas} >m_{ion}: ion loss
- m_{gas} < m_{ion}: cooling



Temperature of Ca⁺ ion cloud with He buffer gas

Trap loading

Creation of ions inside trap by

- Electron bombardement of atoms
- Laser ionisation (element and isotope selectice)



Fig. 4.1: Level scheme showing levels in calcium relevant for the photo-ionization of neutral calcium. The ionization threshold is 6.11 eV above the ground state in calcium [50]. The lifetimes of the excited levels are also indicated.

Injection from outside requires friction

(mean free path between ion-neutral collision ≈trap size)



Ba ⁺ ions injected at 4 keV into 2 cm radius trap

Ion Detection

Destructive detection:

Ejection from trap by high voltage puls Sensitivity: single ion

Non-destructive detection: Absorbtion of energy from tank circuit Sensitivity: 10³ ions (at room temperature single ion (cryogenic)

optical detection: Sensitivity: single ion

Destructive ion detection



Separation of different mass ions by different time-of-flight



Nondestructive detection of ion clouds



Ramping of d.c. trap voltage. $\omega_z \sim V^{1/2}$. When $\omega_z = \omega_{Res}$, ions absorb energy from circuit \Rightarrow damping of circuit

Damping signal from ca. 10⁴ ions

Optical detection

Fluorescence strength from a single trapped ion continuously excited at saturation intensity on an allowed electric dipole transition: ca.10⁸ Photons/s

- Detection efficiency (solid angle, transmission losses, detector quantum efficiency): ca. 10⁻⁵
- $\rightarrow 10^3$ detected Photons from a single ion

Laser induced fluorescence from individual ions





Paul-Straubel trap:



Linear Traps



$$m\frac{d^2}{dt^2} \begin{bmatrix} X_x \\ X_y \\ X_z \end{bmatrix} = -Q \begin{bmatrix} \left(-\frac{\kappa V_{\text{sc}}}{z_0^2} + \frac{V_{rf}}{r_0^2}\cos(\omega_{rf}t)\right)x \\ \left(-\frac{\kappa V_{\text{sc}}}{z_0^2} - \frac{V_{rf}}{r_0^2}\cos(\omega_{rf}t)\right)y \\ \left(2\frac{\kappa V_{\text{sc}}}{z_0^2}\right)z \end{bmatrix}$$

Stability diagram for radial motion


Radial potential of a linear quadrupole trap



Multipole traps



Ion trajectories in a 32-pole trap



Linear trap constructions



NIST, Boulder



Univ. of Michigan

Blade trap with segmented electrodes (Univ. Ulm, Innsbruck)



MEMS Gallium-Arsenide ion trap in a microchip

D. Stick W. K. Hensinger S. Olmschenk M. Madsen K. Schwab C. Monroe

60 µm





MEMS Silicon ion traps







NIST Planar Trap Chip

Gold on fused silica



NIST Planar Trap Chip

Magnified trap electrodes



CCD pictures of strings of Mg⁺ ions (trapped 40 mm above surface)





Monolithic microfabricated ion trap chip design for scaleable quantum processors M. Brownnutt, G. Wilpers, P. Gill, R.C. Thompson, A.G. Sinclair, New J. Phys. 8, 232(2006).

NIST two layer cross trap



Mg⁺ and Be⁺ moved through junction > 10⁴ times without ion loss

Characterization of heating during shuttling in progress



Brad Blakestad, Didi Leibfried and David Wineland

2 wafers of alumina (0.2 mm thick) gold conducting surfaces (2 µm)



18 zones, 1 cross



Ulm linear multizone trap



Stefan Schulz, Ulrich Poschinger, Frank Ziesel and Ferdinand Schmidt-Kaler

3-dim. stack of single ion traps

K. Ravi, S. Rangwala et al, 2009



Trap applications without ion cooling:

Hyperfine spectroscopy

- No limitation of linewidth fromHeisenberg uncertainty because of ,,infinite" long coherence time
- Spectral resolution limited mainly by technical reasons (collisions with background gas molecules, fluctuations of residual magnetic field, finite spectral purity of exciting radio-frequency field)
- No first order Doppler effect even for uncooled ions because of Dicke effect (wavelength of radiation large compared to ion oscillation amplitude)
- Limitation in accuracy by second order Doppler effect $\Delta \omega / \omega = (v^2/2c^2)$ ($\approx 5 \ 10^{-10}$ at T = 10 000 K) ($\approx 5 \ 10^{-14}$ at T = 1 K)

Experimental method: Laser-microwave double resonance

- Selective excitation of one hyperfine level by laser excitation leads to population difference between hyperfine levels (optical pumping)
- Inducing transitions between hyperfine levels by microwave field (selection rule: $\Delta F = 0,1$; $\Delta m_F = 0,1$)
- Detection of resonant excitation by observation of change in fluorescence intensity





More complex ion: Eu⁺



Optical excitation spectrum in a mixture of natural isotopes



Micowave induced $\Delta m_F = 0$ hyperfine transitions between the F=11/2 and F=13/2 hyperfine levels in the 9S_4 ground state of ${}^{151}Eu^+$





Features of ion traps

• Storage time:

minutes - days

(depending on background gas conditions)

• Detection sensitivity:

Single ion

• Storage Capacity:

 $\sim 10^6$ ions/cm³

(limited by space charge)

• Ion oscillation amplitude:

mm - µm

(depending on ion temperature)

• Ion temperature:

 10^{4} K - 0.1 mK

(depending on cooling method)

Principle of the Penning trap



$$\Phi = \frac{U}{r_0^2} (x^2 + y^2 - 2z^2)$$

confinement in axial direction by electrostatic field

confinement in radial direction by strong homogeneous magnetic field

Trap geometries

Hyperbolical Penning trap







Electric quadrupole field and homogeneous magnetic field in axial direction

Force acting on charged particle in 3D:

$$\vec{F} = -e \nabla \Phi + e (\vec{v} \times \vec{B})$$

$$\Phi = \frac{U_0}{2 d^{-2}} (2 z^2 - x^2 - y^2)$$
Equations of motion:
$$\frac{d^2 x}{dt^2} - \omega_c \frac{dy}{dt} - \frac{1}{2} \omega_z^2 x = 0 \quad (1)$$

$$\frac{d^2 y}{dt^2} + \omega_c \frac{dx}{dt} - \frac{1}{2} \omega_z^2 y = 0 \quad (2)$$

$$\frac{d^2 z}{dt^2} + \omega_z^2 z = 0 \quad (3)$$

Axial direction: Harmonic oscillation: $z(t) = Z(0)e^{-i\omega_{z}t}$

 $\omega_{z} = \sqrt{\frac{eU}{md^{2}}}$

Radial direction: decouple (1) and (2) via: $u(t) = R_+ e^{-i\omega_+ t} + R_- e^{-i\omega_- t}$ Eigenfrequencies: u = x + iy

 $\omega_{+} = \frac{\omega_{c}}{2} + \sqrt{\frac{\omega_{c}^{2}}{4} - \frac{\omega_{z}^{2}}{2}} \quad \text{modified cyclotron frequency}$ $\omega_{-} = \frac{\omega_{c}}{2} - \sqrt{\frac{\omega_{c}^{2}}{4} - \frac{\omega_{z}^{2}}{2}} \quad \text{magnetron frequency}$ $\omega_{c} = \frac{e}{m}B$

Hierachy of frequencies: $\omega_+ \gg \omega_z \gg \omega_-$

Stability limit in a Penning trap

$$\omega_c^2 \ge 2\omega_z^2$$
$$\frac{e}{M}B^2 \ge \frac{8U}{r_0^2}$$

Numerical examples:

B=1T, r_0 =1 cm the maximum trap voltage for stable confinement for m=100 a.u. : 20 V for electrons at B=100 G, r_0 =1 cm: U_{max} = 400 V

Single particle motion in a Penning trap





Quantum mechanical eigenstates of particle motion

$$E = (n_{+} + 1/2)\hbar\omega_{+} - (n_{-} + 1/2)\hbar\omega_{-} + (n_{z} + 1/2)\hbar\omega_{z}$$

All three motions are independent of each other and can be described as harmonic oscillators.

Negative sign of the magnetron energy indicates the metastability of motion:

larger magnetron radius (higer quantum number) corresponds to smaller potential energy (important for cooling!)



Experimental motional frequencies in a Penning trap



Excitation of ion oscillations Leads to particle loss at resonance

Rotating wall compression of ion clouds (Univ. of Calif., San Diego)

Application of a rotating electric field to segments of the ring electrode:

Change of density due to additional centrifugal force



Effect of background gas collisions

Cyclotron orbit is reduced by damping force Magnetron orbit is increased due to metastability of magnetron motion

→ ion loss from trap

Coupling of magnetron and cyclotron motion by quadrupole field leads to aggregation of ions near trap center
Effect of mode coupling on ion trajectory

r.f.field at sum frequency of both oscillations applied between adjacent segments of ring electrode

$$\omega_{+} + \omega_{-} = \omega_{c}$$



Ion trajectories with buffer gas collisions:

Without mode coupling



With mode coupling

Imperfect Penning trap

Similar as in Paul traps:

- (a) Shift of eigenfrequencies proportional to
- higher order parts in trap potential
- magnetic field inhomogeneity
- ion energy
- Space charge density

(b) Instabilitiy of ion motion when

 $n\omega_z + m\omega_z = k\omega_+$

n,m,k integer

Space charge shift of motional frequencies



Observed Instabilities for electron confinemet



Instabilities in a Penning trap for different storage times



Trapping Voltage [V]

Ion Detection

Destructive detection:

Ejection from trap by high voltage puls Sensitivity: single ion

Non-destructive detection:

(a) Absorbtion of energy from tank circuit at room temperature Sensitivity: 10³ ions

(b) Induced noise in tank circuit Sensitivity: single ion (at 4 K)

(c) optical detection Sensitivity: single ion

Cylindrical Trap with detection circuit attached to endcap electrode



Detection of stored ions by induced voltage in endcap electrode



Induced noise from individual electrons Dehmelt 1987



Fourier transform of induced noise in ring electrode at the cyclotron frequency in an inhomogeneous magnetic field



Resistive cooling of a single ion (C⁵⁺)

Exponential energy dissipation through resistor

Time constant: $\tau = (2z_0/q)^2 (m/R)$





Cooling of an ion cloud:

Resistive cooling applies only to center-of-mass motion.

Individual ion oscillation is coupled to center-of-

mass through Coulomb interaction.



Cooling of an ion cloud: 2 time constants

Detection od a single ion in thermal equilibrium Fourier Transform of noise in axial circuit



Optical detection of ions in Penning trap



Fluorescence images taken for differents phases and amplitudes of the excitation voltage V_{ac} Bollinger et al., NIST



Observation by induced noise in trap electrodes

F. Anderegg et al., PRL <u>90</u>, 115001 (2003)



(a) (b) Amplitude (a) and phase (b) of the (2.0)-mode by Doppler imaging left: experiment; right: simulation



(9,0)-mode Mitchell et al, opt. Express <u>2</u>, 314 (1998)



Mitchell et al, opt. Express <u>2</u>, 314 (1998)



Top-view images of the differential intensities, proportional to the ion velocities, induced by a laser push beam (white spot) incident on a disk-shaped ion plasma rotating clockwise. (a) shows experimental results while (b) shows the prediction from theory.

Mitchell et al, opt. Express <u>2</u>, 314 (1998)



(a) Differential top-view image of a laser-induced wake in a clockwise rotating Be+ ion crystal.

(b)Average fractional change in fluorescence for the annular region between the white circles in (a). x = 0 is defined to be at the centre of the push beam. The solid curve is a fit to the data using a damped sinusoid

J.J. Bollinger et al., J. Phys. B: At. Mol. Opt. Phys. **36** (2003) 499–510

Alternative trap geometries

- Multiple traps
- Planar traps

Pentatrap (Max-Planck Inst., Heidelberg)



A planar Penning trap





Axial potential of the planar Penning trap (2 concentric ring electrodes, infinite ground)



Trap potentials for a 3 ring configuration and various voltage settings



Figure 3. (a) Electric potential $\Phi^{el}(z)$ for a trap with $R_0 = 300 \ \mu m$, $R_1 = 600 \ \mu m$, $R_2 = 900 \ \mu m$ and $U_0 = 0 \ V$, $U_1 = +0.5 \ V$, $U_2 = 0 \ V$ leading to a potential with a minimum near $z = 296 \ \mu m$ and an axial frequency of $\omega_z/(2\pi) = 89.9 \ MHz$ (dashed). Anharmonicities are minimized by changing the compensation voltage to $U_2 = -0.417 \ V$. The minimum is shifted to a distance of 234 μm for the axial frequency of 99.0 MHz (solid line). (b) Location of the equilibrium position above the surface for the case of a compensated potential. Th distance scales linearly as 0.78 R_0 .

Prototype planar trap (Univ. Mainz)





"Stylos Trap"

Maiwald et al., Nature Physics online (2009)





Pixel trap (Univ. Ulm/Mainz)



Pixel trap



Onion trap (Univ. Ulm/Mainz)



Multiple single particle traps on chip



Coupling of traps by induced image currents and superconducting wires



Summary of Penning trap properties

- Storage time nearly infinit
- Non-destructive single ion detection
- Space charge limit in ion clouds ~ $10^{6}/cm^{3}$
- Instability by background gas collisions
- Ion centering by mode coupling
- Instability by trap imperfections
- Non-neutral plasma effects