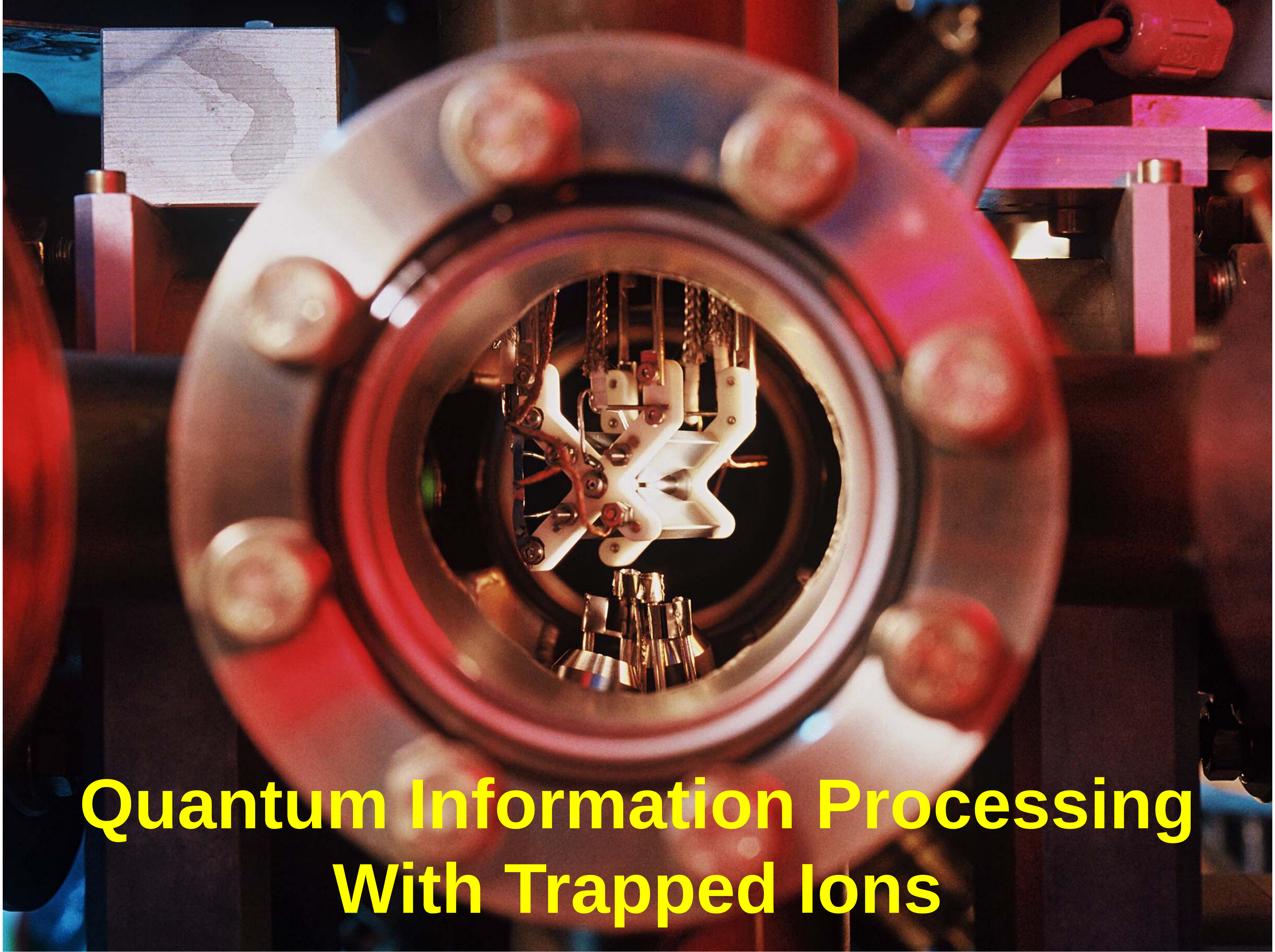


Quantum Information Processing With Trapped Ions

Quantum Information Processing with Trapped Ions

- Quantum Information Processing
- Quantum computer with trapped ions
- Cirac-Zoller CNOT gate operation
- State and process tomography
- Generation of Bell, GHZ and W states
- Teleportation with trapped ions
- Scaling the ion trap quantum computer

Rainer Blatt

Institute for Experimental Physics, University of Innsbruck,
Institute for Quantum Optics and Quantum Information,
Austrian Academy of Science

The requirements for quantum information processing

D. P. DiVincenzo, Quant. Inf. Comp. **1** (Special), 1 (2001)

I. Scalable physical system, well characterized qubits

II. Ability to initialize the state of the qubits

III. Long relevant coherence times, much longer than gate operation time

IV. "Universal" set of quantum gates

V. Qubit-specific measurement capability

VI. Ability to interconvert stationary and flying qubits

VII. Ability to faithfully transmit flying qubits between specified locations

The seven commandments for QIP !!

Quantum bits and quantum registers

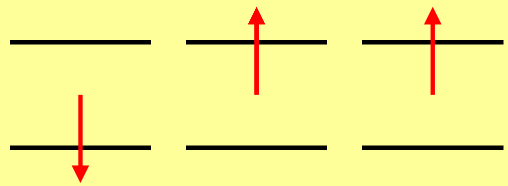
- classical bit: physical object in state 0 or 1
- register: bit rows 0 1 1 . . .
- quantum bit (qubit): superposition of two orthogonal quantum states

$$|\Psi\rangle = c_0|0\rangle + c_1|1\rangle$$

- quantum register: L 2-level atoms, 2^L quantum states

  2^L states correspond

  to numbers 0,....., $2^L - 1$



$$|0\rangle \otimes |1\rangle \otimes |1\rangle$$

- most general state of the register is the superposition

$$|\Psi\rangle = c_{000}|000\rangle + c_{001}|001\rangle + \cdots + c_{110}|110\rangle + c_{111}|111\rangle \quad \text{(binary)}$$

$$= c_0|1\rangle + c_1|1\rangle + \cdots + c_6|110\rangle + c_7|7\rangle \quad \text{(decimal)}$$

Universal Quantum Gates

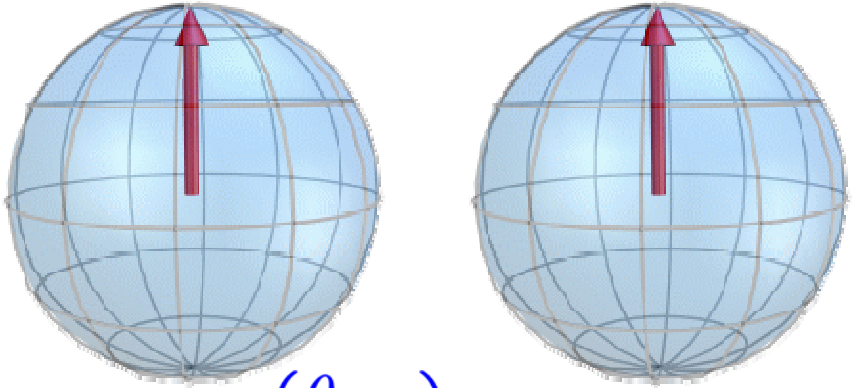
Operations with single qubit:
(1-bit rotations)

together universal !

Operations with two qubits:
(2-bit rotations)

CNOT – gate operation
(controlled-NOT)

analogous to XOR



$(\theta, \varphi) =$

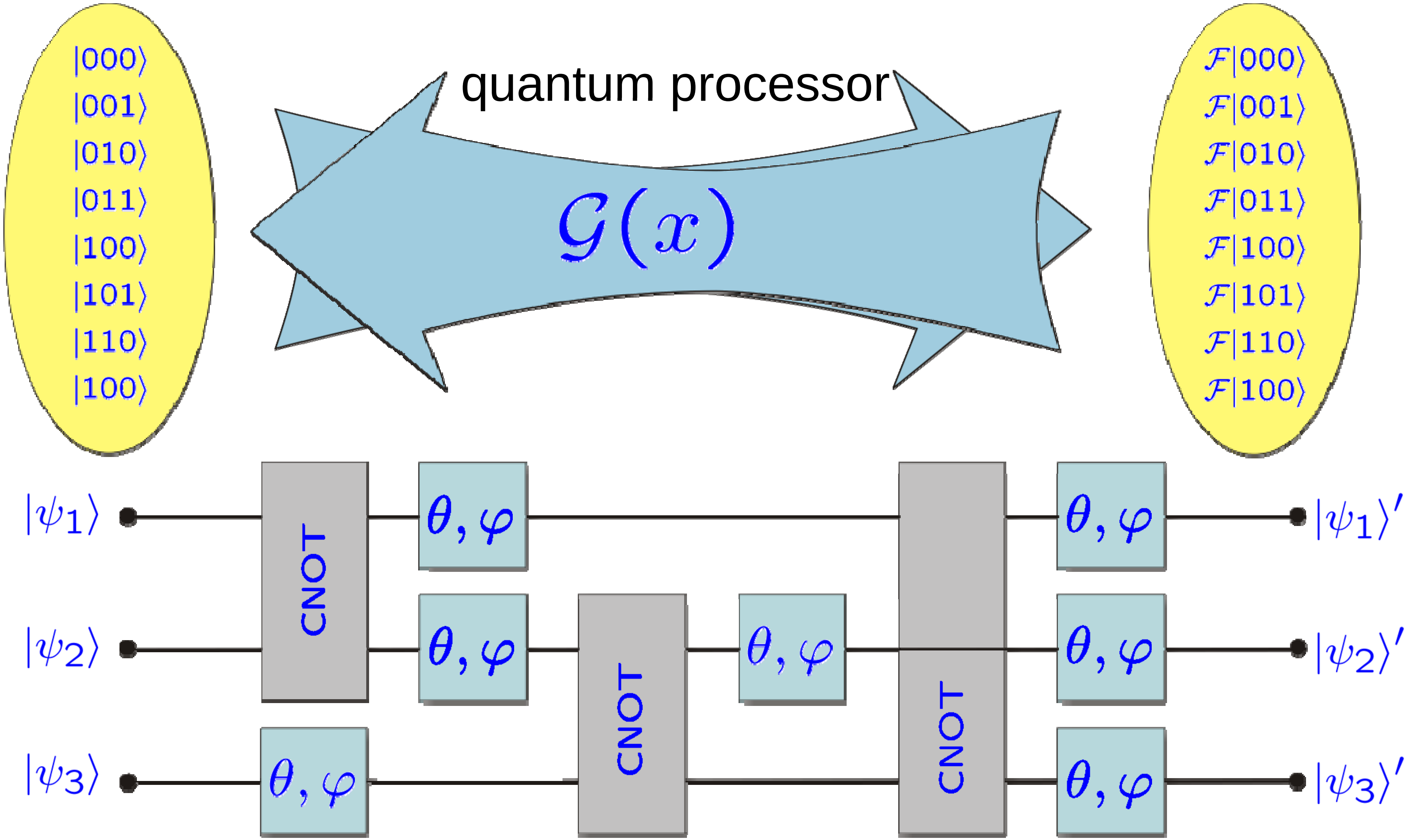
$(\pi, 0)$ $2/3(\pi, \pi)$

$$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$$
$$|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$$
$$|1\rangle|0\rangle \rightarrow |1\rangle|1\rangle$$
$$|1\rangle|1\rangle \rightarrow |1\rangle|0\rangle$$

control bit

target bit

How a quantum computer works



Input ⇨ computation: sequence of quantum gates ⇨ output

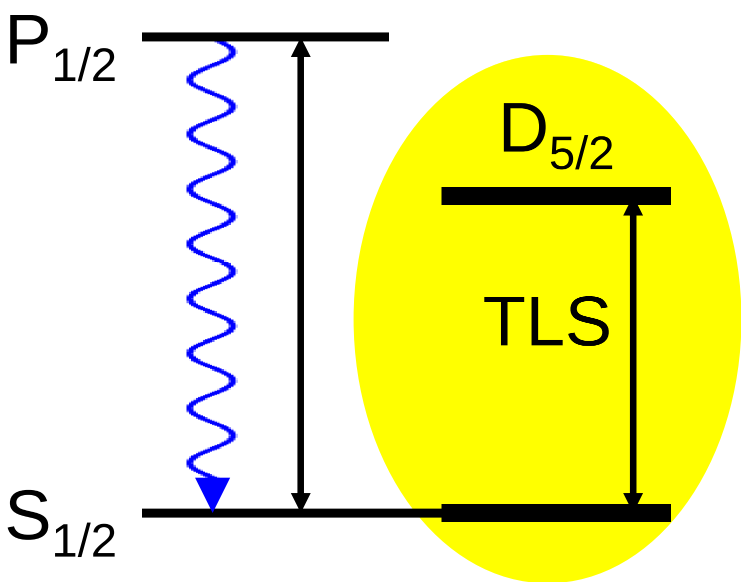
Meeting the DiVincenzo criteria with trapped ions

criterion	physical implementation	technique	
scalable qubits	internal atomic transitions (2-level-systems)	linear traps (trap arrays)	✓
initialization	laser cooling, state preparation	optical pumping, laser pulses	✓
long coherence times	narrow transitions (optical, microwave)	coherence time ~ ms - min	✓
universal quantum gates	single qubit operations, two-qubit operations	Rabi oscillations Cirac-Zoller CNOT	✓
qubit measurement	quantum jump detection	individual ion fluorescence	✓
convert qubits to flying qubits	coupling of ions with high finesse cavity	CQED, bad cavity limit	T E
faithfully transmit flying qubits	coupling of cavities via fiber (photonic channel)	coupling pulse sequences (CZKM)	T E

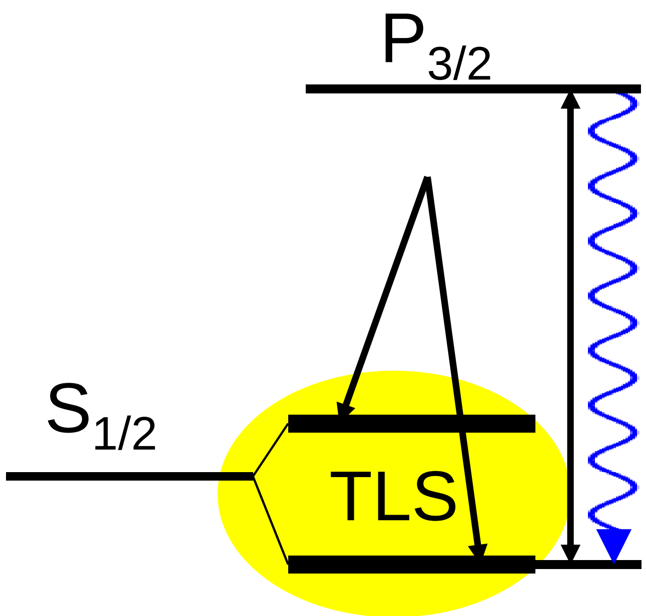
Qubits with trapped ions

Storing and keeping quantum information requires long-lived atomic states:

- optical transition frequencies (forbidden transitions, intercombination lines) S – D transitions in alkaline earths: Ca^+, Sr^+, Ba^+, Ra^+, (Yb^+, Hg^+) etc.
- microwave transitions (hyperfine transitions, Zeeman transitions) alkaline earths: $^9Be^+$, $^{25}Mg^+$, $^{43}Ca^+$, $^{87}Sr^+$, $^{137}Ba^+$, $^{111}Cd^+$, $^{171}Yb^+$



Innsbruck $^{40}Ca^+$



Boulder $^9Be^+$; Michigan $^{111}Cd^+$; Innsbruck $^{43}Ca^+$, Oxford $^{43}Ca^+$; Maryland $^{171}Yb^+$;

Quantum computation with trapped ions

VOLUME 74, NUMBER 20 PHYSICAL REVIEW LETTERS 15 MAY 1995

Quantum Computations with Cold Trapped Ions

J. I. Cirac and P. Zoller*

Institut für Theoretische Physik, Universiät Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria

(Received 30 November 1994)

A quantum computer can be implemented with cold ions confined in a linear trap and interacting with laser beams. Quantum gates involving any pair, triplet, or subset of ions can be realized by coupling the ions through the collective quantized motion. In this system decoherence is negligible, and the measurement (readout of the quantum register) can be carried out with a high efficiency.

PACS numbers: 89.80.+h, 03.65.Bz, 12.20.Fv, 32.80.Pj

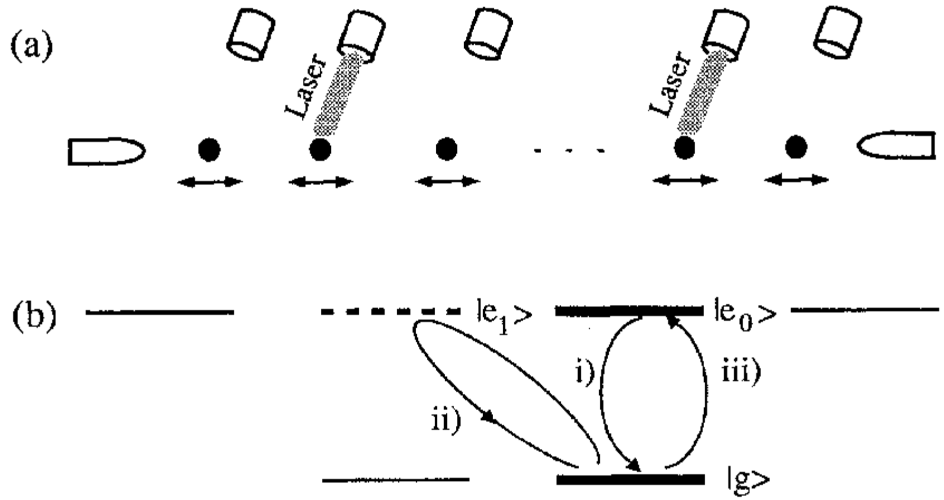


FIG. 1. (a) N ions in a linear trap interacting with N different laser beams; (b) atomic level scheme.

J. I. Cirac

P. Zoller

other gate proposals (and more):

- Cirac & Zoller
- Mølmer & Sørensen, Milburn
- Jonathan & Plenio & Knight
- Geometric phases
- Leibfried & Wineland

Quantum computation with trapped ions

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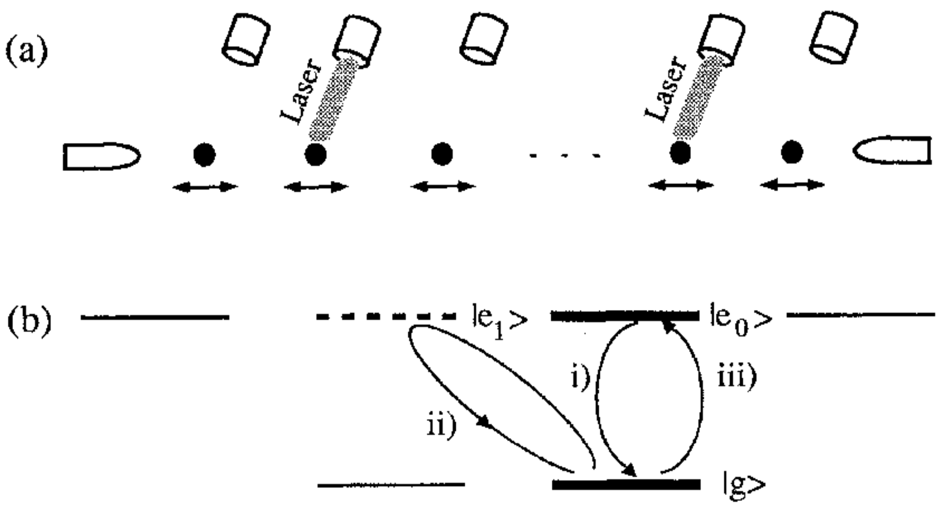


FIG. 1. (a) *N* ions in a linear trap interacting with *N* different laser beams; (b) atomic level scheme.

controlled – NOT :

$$|\varepsilon_1\rangle|\varepsilon_2\rangle \rightarrow |\varepsilon_1\rangle|\varepsilon_1 \oplus \varepsilon_2\rangle$$

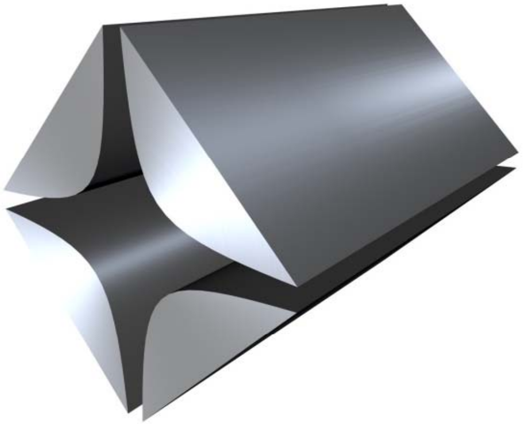
$$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$$
$$|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$$
$$|1\rangle|0\rangle \rightarrow |1\rangle|1\rangle$$
$$|1\rangle|1\rangle \rightarrow |1\rangle|0\rangle$$

control bit target bit

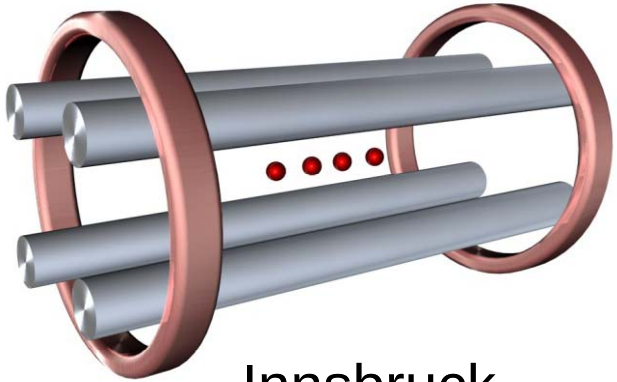
other gate proposals (and more):

- Cirac & Zoller
- Mølmer & Sørensen, Milburn
- Jonathan & Plenio & Knight
- Geometric phases
- Leibfried & Wineland

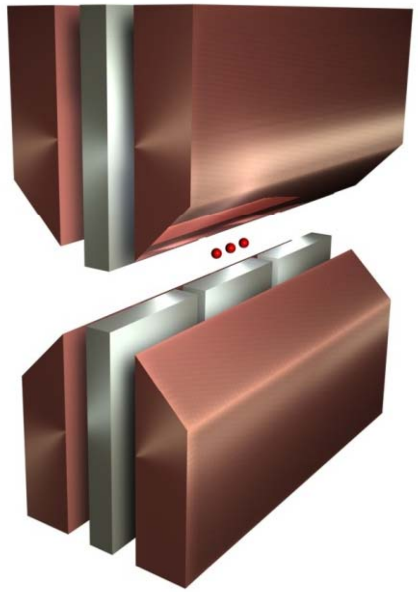
Linear Ion Traps



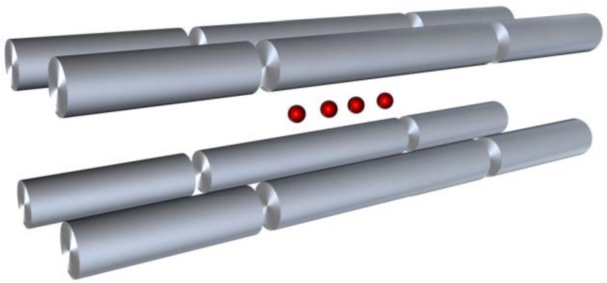
Paul mass filter



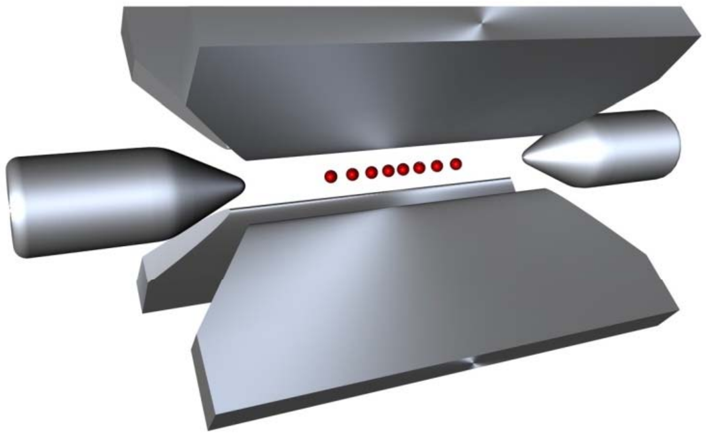
Innsbruck
Ann Arbor



München
Sussex

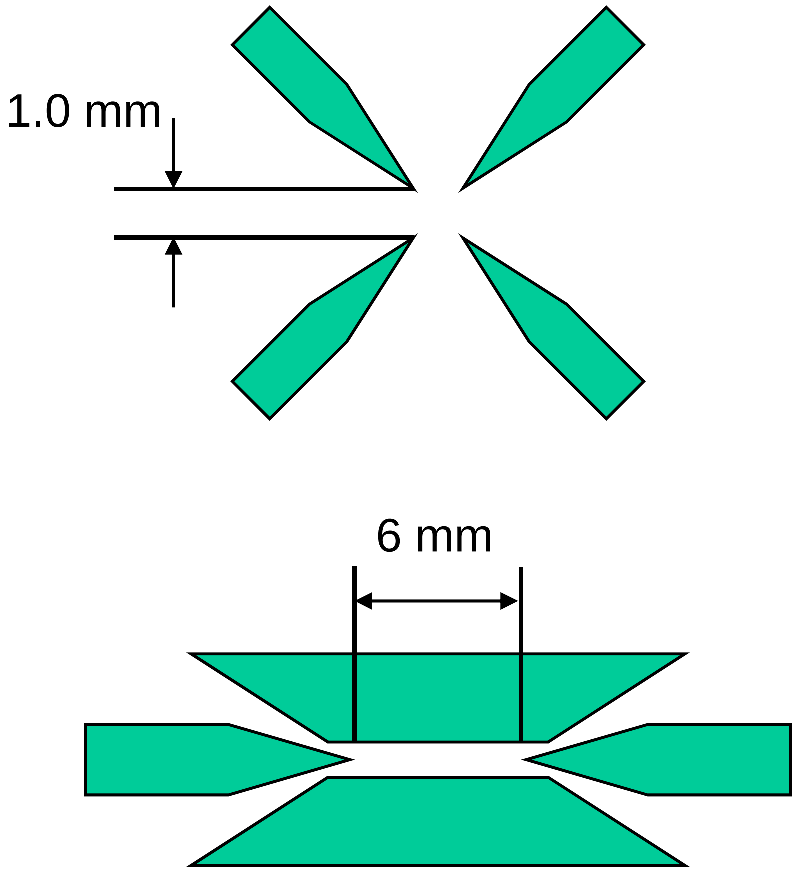


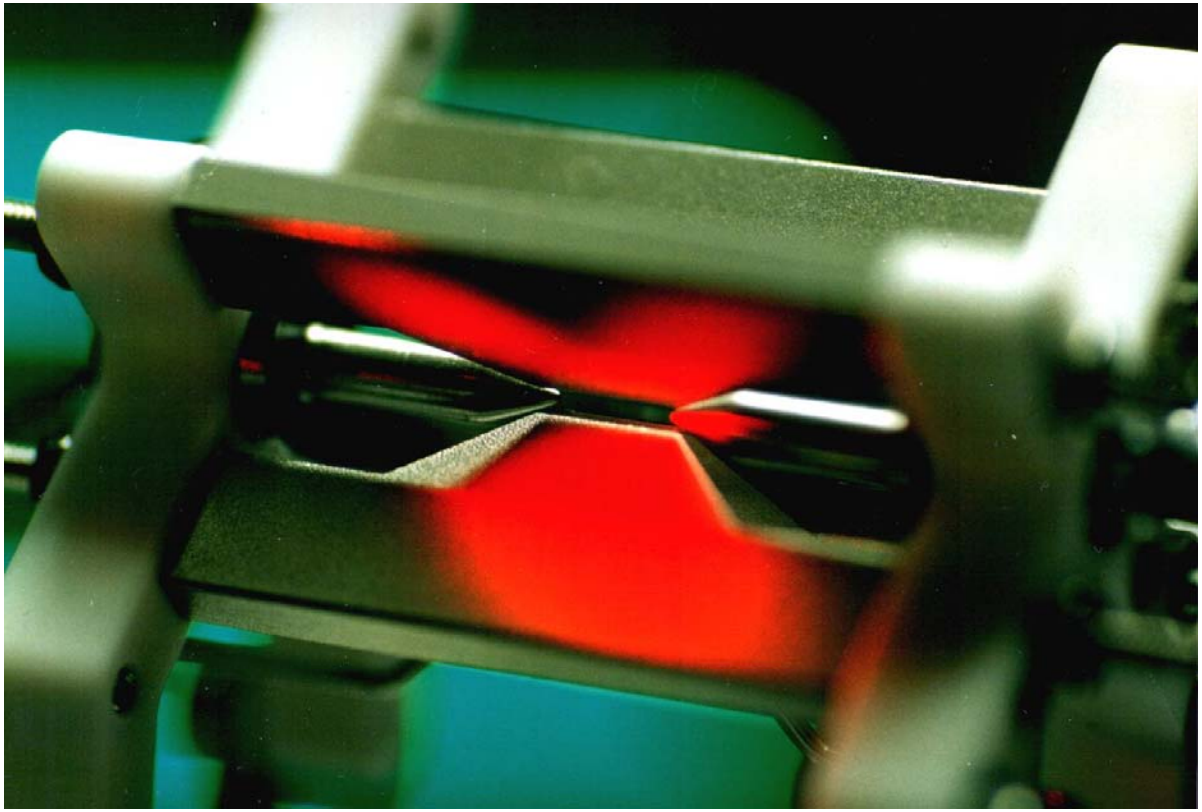
Boulder, Mainz, Aarhus



Innsbruck, Oxford

Innsbruck linear ion trap (2000)





$$\omega_z \approx 0.7 - 2\,\mathrm{MHz} \qquad \omega_{x,y} \approx 1.5 - 4\,\mathrm{MHz}$$

Quantum computer with trapped ions

J. I. Cirac, P. Zoller; Phys. Rev. Lett. **74**, 4091 (1995)

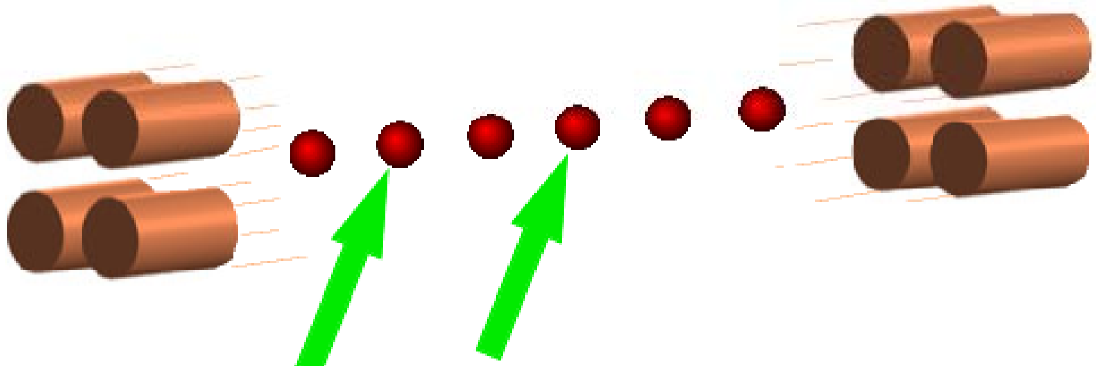
L Ions in linear trap

- quantum bits, quantum register
  - narrow optical transitions
  - groundstate Zeeman coherences

- state vector of quantum computer

$$|\Psi\rangle = \sum_{\underline{x}} c_{\underline{x}} |x_{L-1}, \ldots, x_0\rangle \otimes |0\rangle_{\mathrm{CM}}$$

- 2-qubit quantum gate

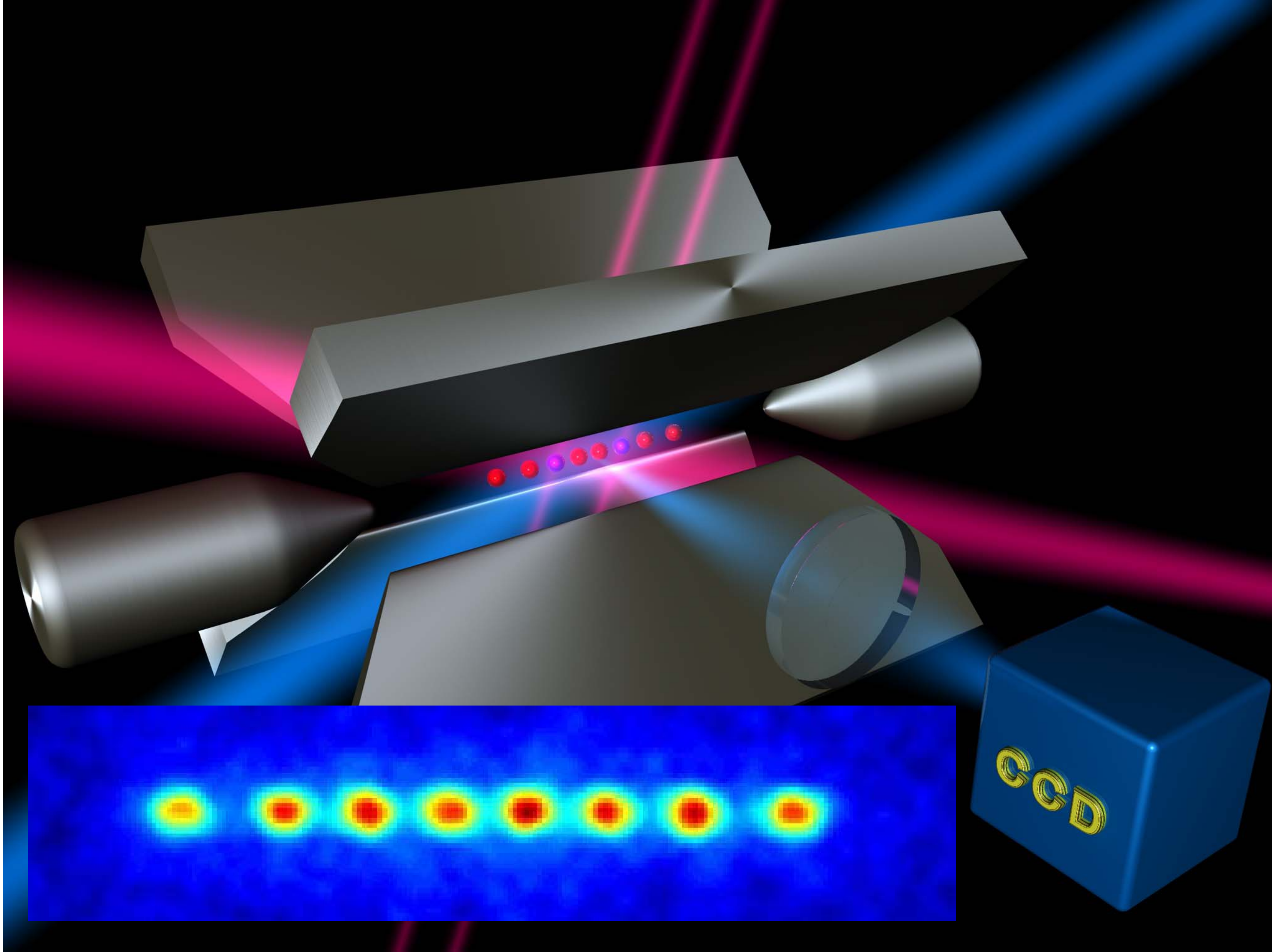


laser pulses entangle pairs of ions

$$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$$
$$|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$$
$$|1\rangle|0\rangle \rightarrow |1\rangle|1\rangle$$
$$|1\rangle|1\rangle \rightarrow |1\rangle|0\rangle$$

control bit target bit

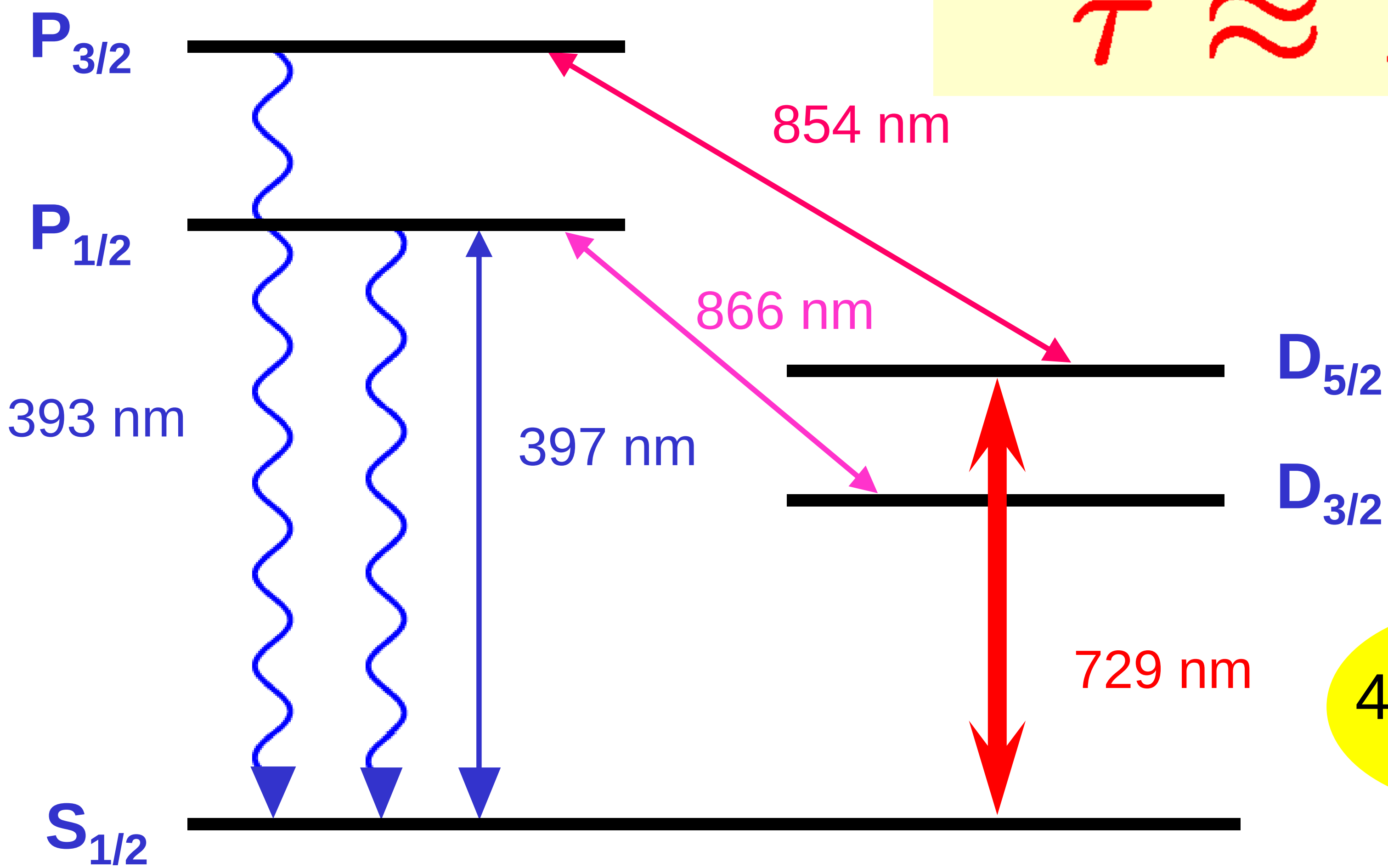
- needs individual addressing, efficient single qubit operations
- small decoherence of internal and motional states
- quantum computer as series of gate operations (sequence of laser pulses)


CCD

Level scheme of Ca^+

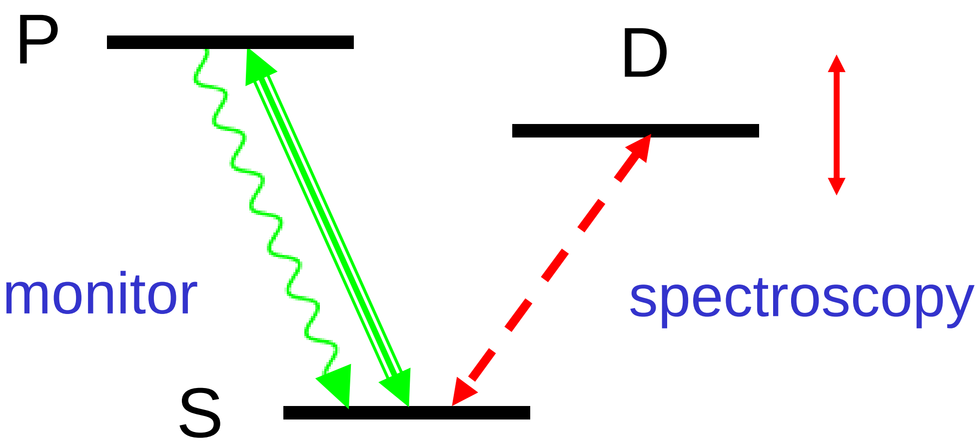
qubit on narrow S - D quadrupole transition

$$\tau \approx 1s$$

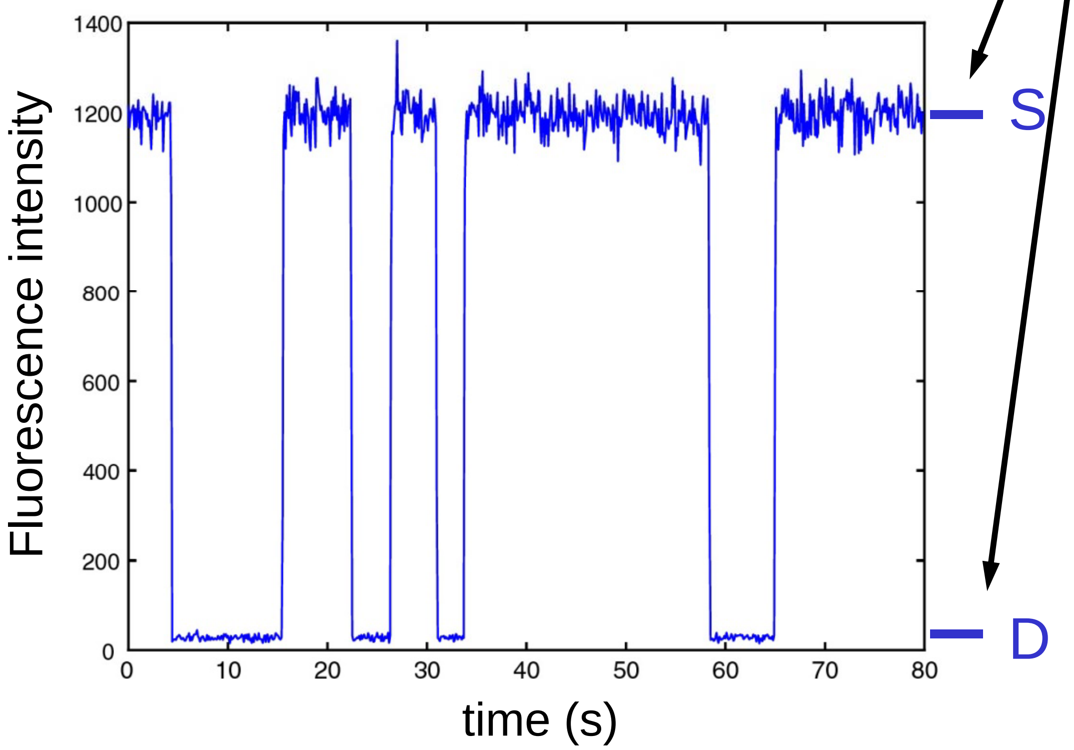


$^{40}Ca^+$

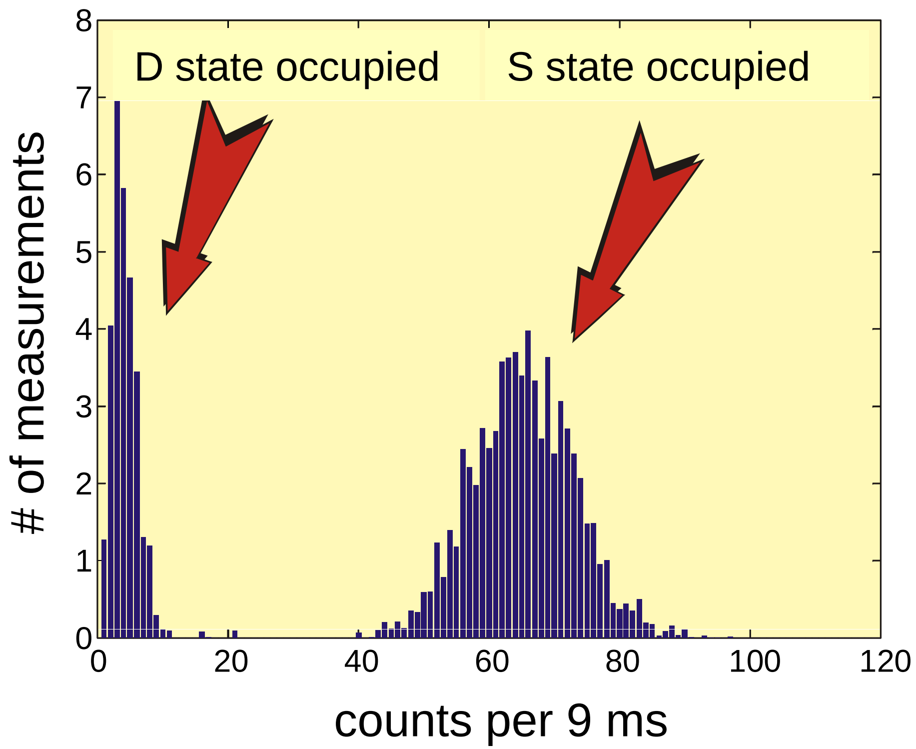
Spectroscopy with quantized fluorescence (quantum jumps)



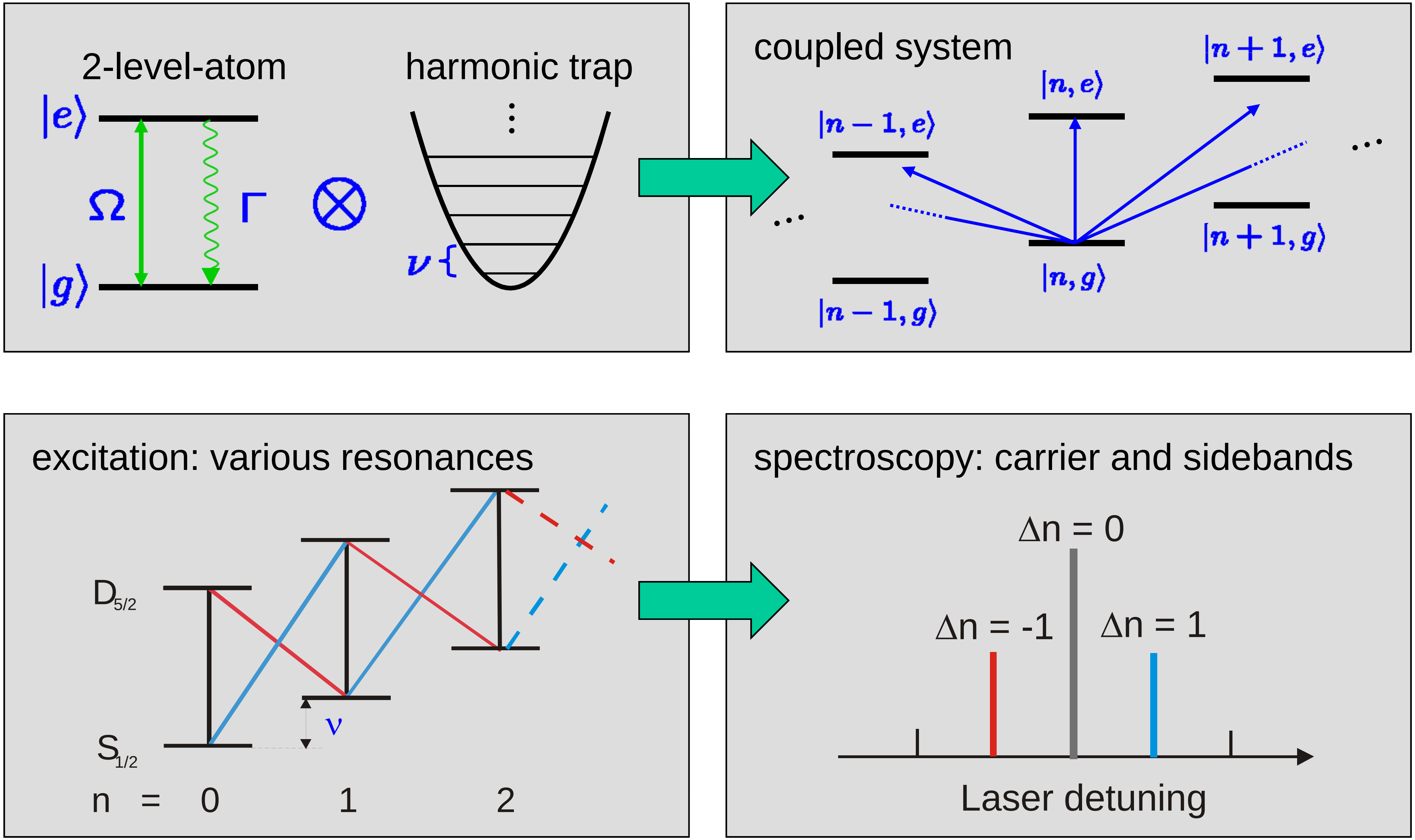
absorption and emission cause fluorescence steps (digital quantum jump signal)



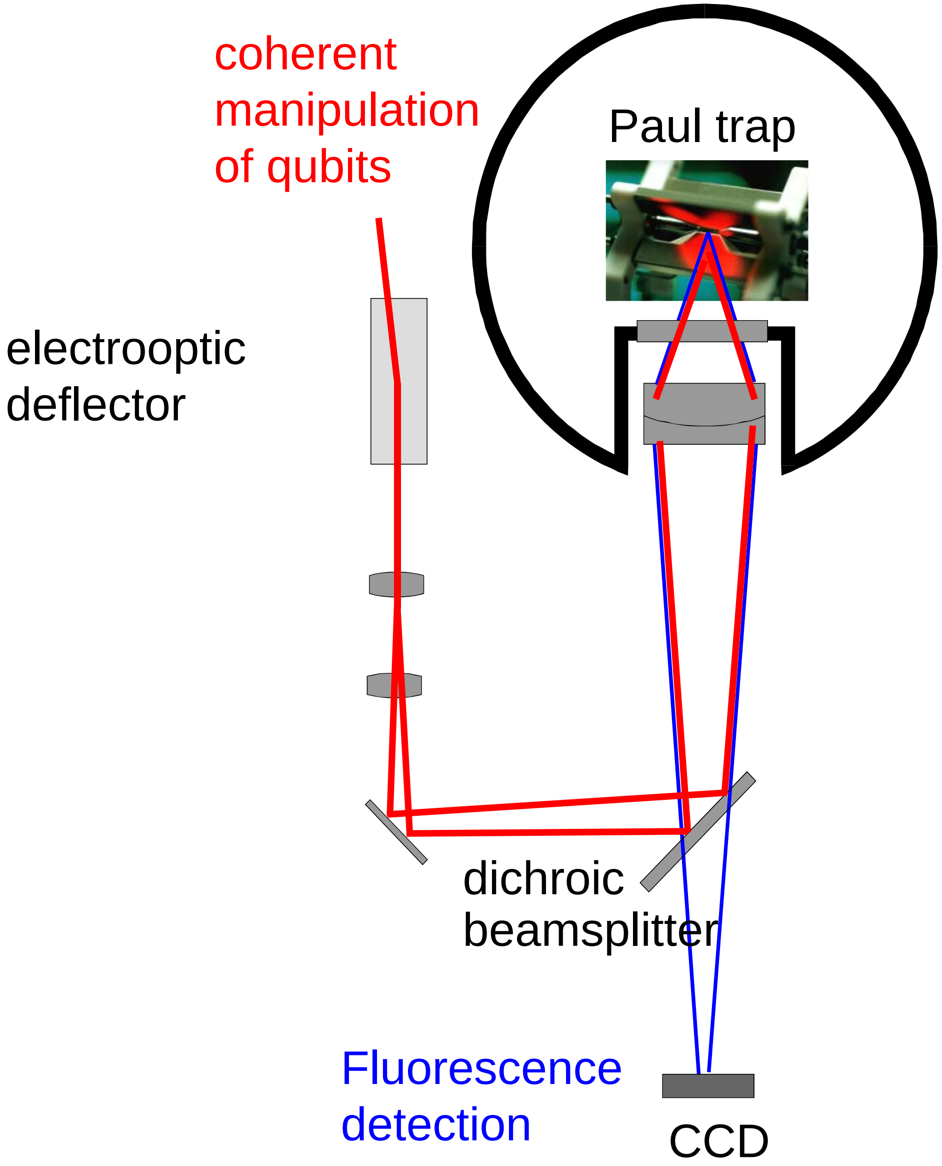
detection efficiency:
99.85%

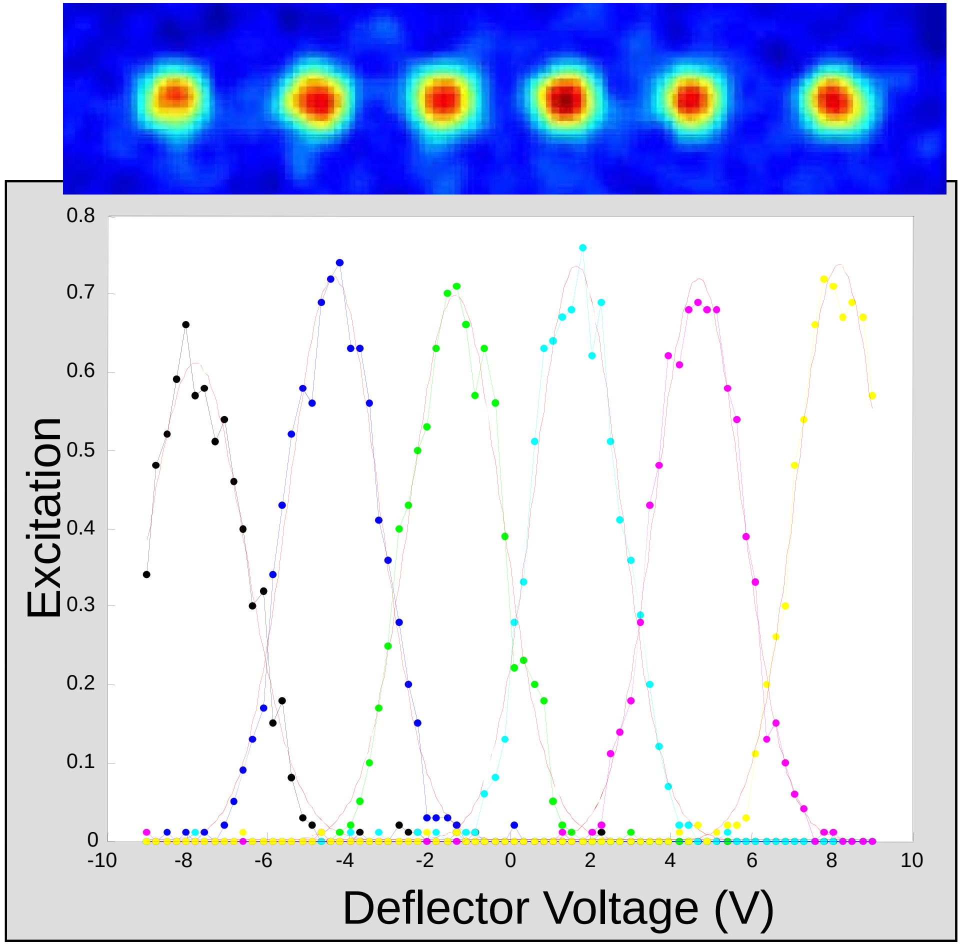


Quantized Ion motion



Addressing of individual ions

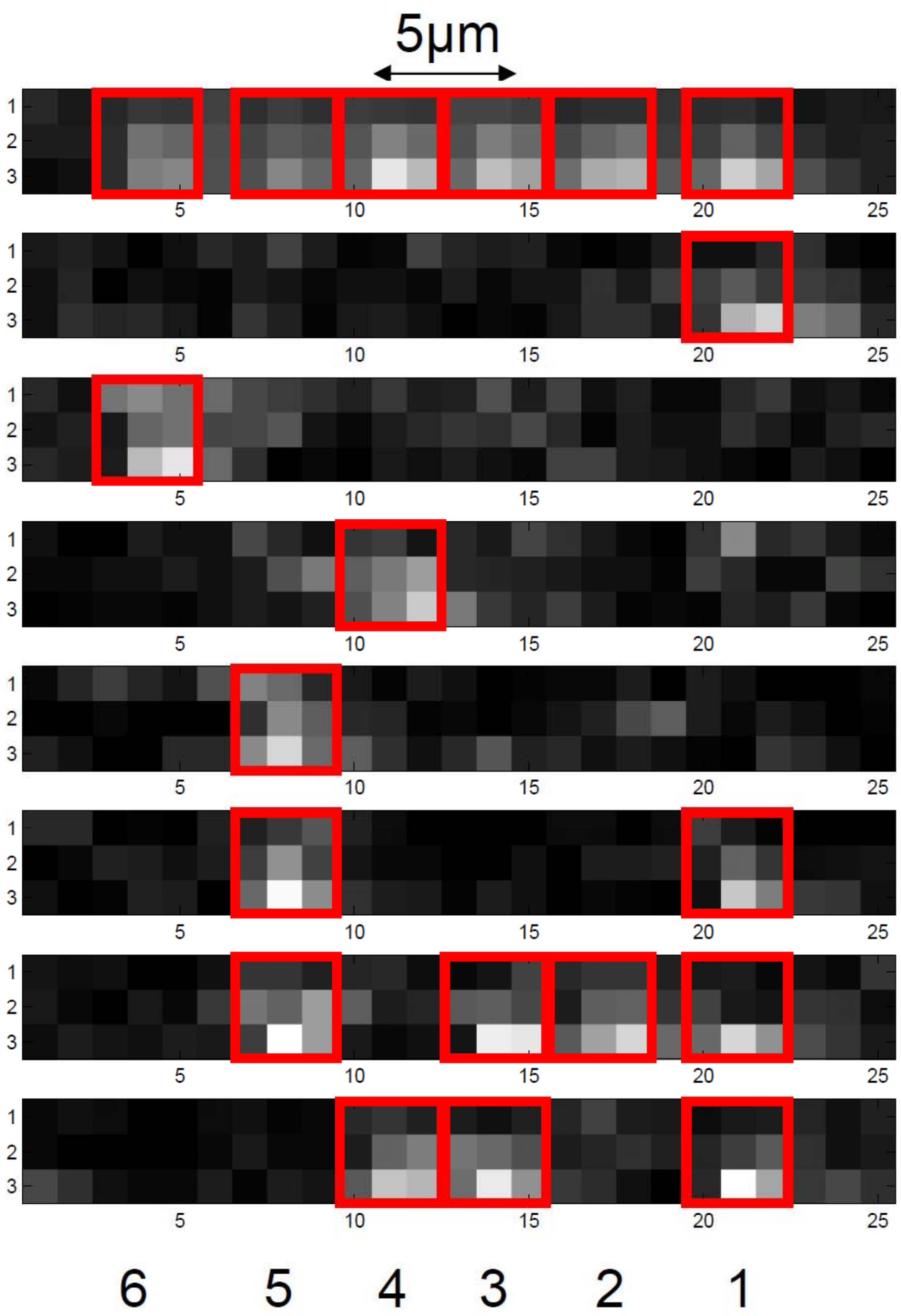




- inter ion distance: ~ 4 µm
- addressing waist: ~ 2.5 µm

< 0.1% intensity on neighbouring ions

Detection of 6 individual ions



state detection on a CCD camera

all ions in |S> $|SSSSSS\rangle$

ion 1 in |S> $|DDDDDS\rangle$

ion 6 in |S> $|SDDDDD\rangle$

ion 4 in |S> $|DDSDDD\rangle$

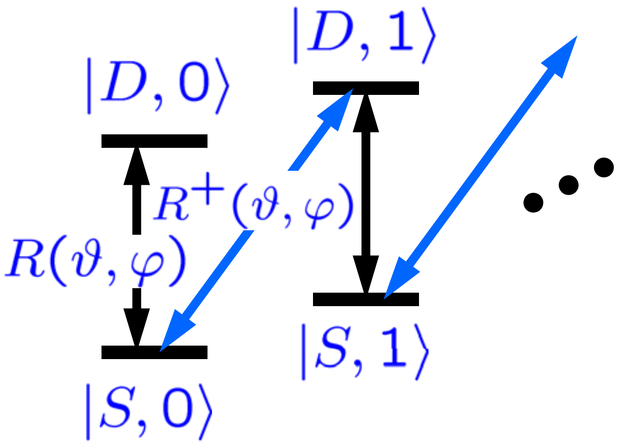
ion 5 in |S> $|DSDDDD\rangle$

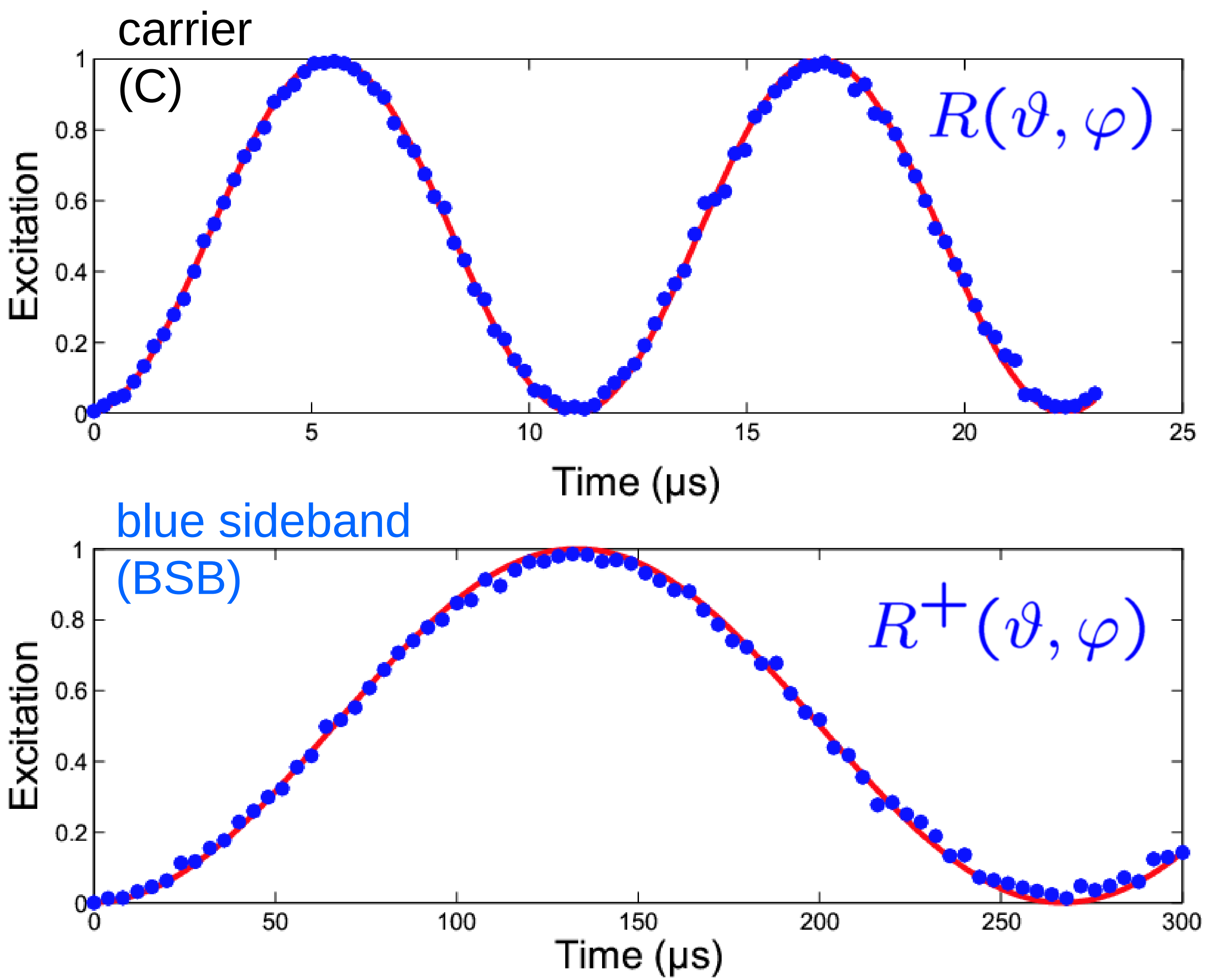
ions 1 and 5 in |S> $|DSDDDS\rangle$

ions 1,2,3, and 5 in |S> $|DSDSSS\rangle$

ions 1,3 and 4 in |S> $|DDSSDS\rangle$

Coherent state manipulation



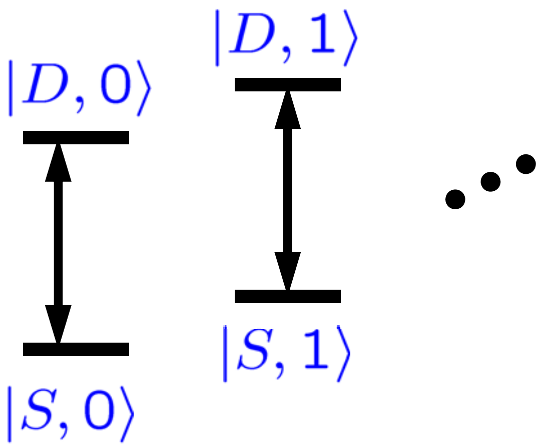


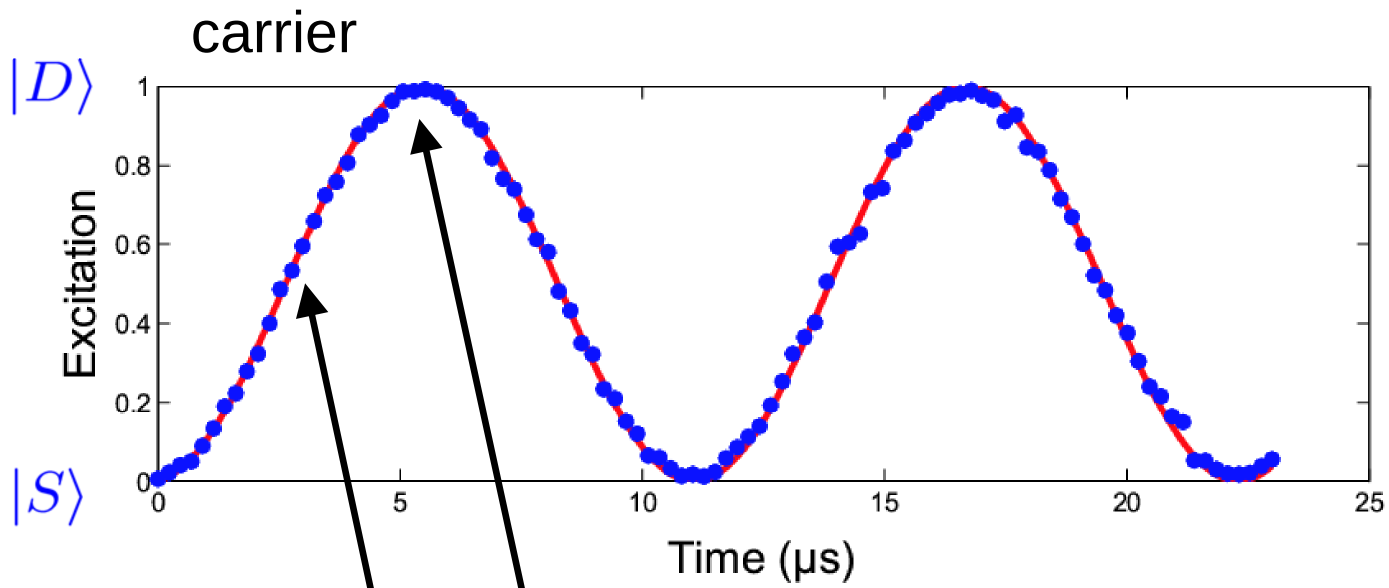
carrier and sideband
Rabi oscillations
with Rabi frequencies

$$\Omega, \quad \eta\Omega\sqrt{n+1}$$

$\eta = kx_0$ Lamb-Dicke parameter

Coherent state manipulation: carrier





$$|S\rangle \xrightarrow{\pi/2} |S\rangle + |D\rangle \qquad |S\rangle \xrightarrow{\pi} |D\rangle$$

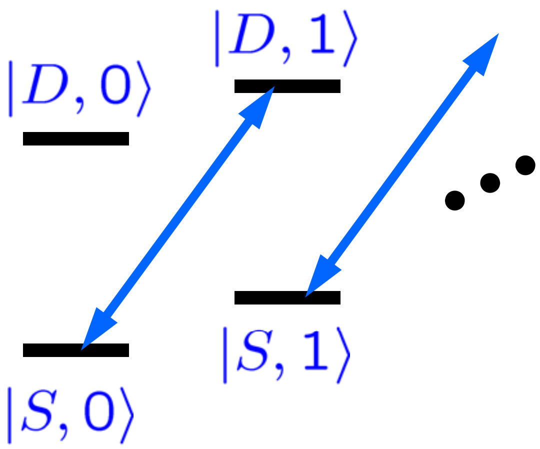
Carrier transitions leave the motion unchanged

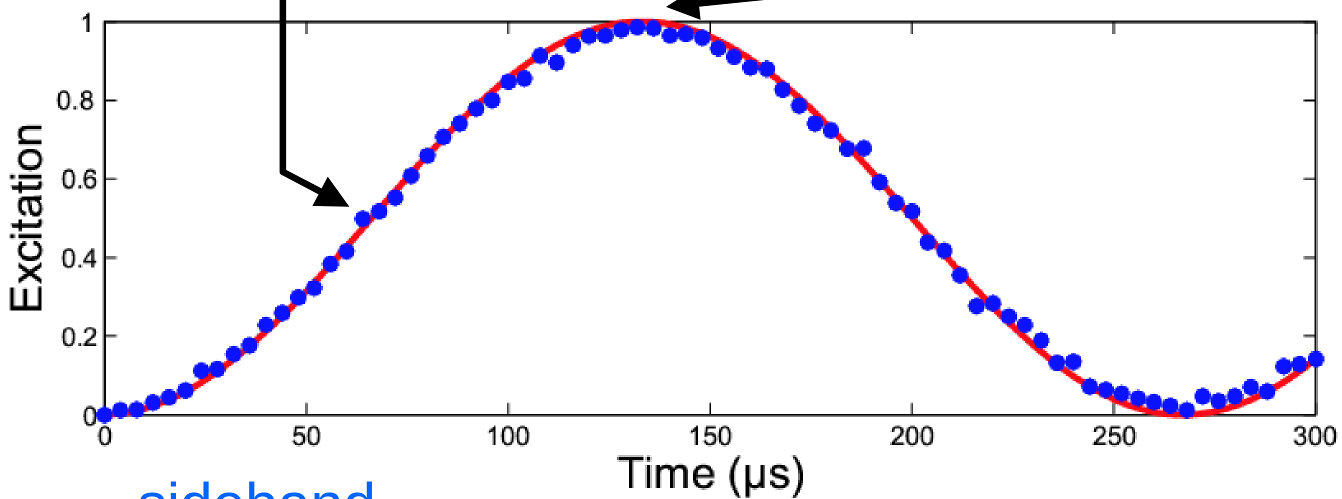
Coherent state manipulation: sideband

Sideband transitions entangle motion and internal excitation

$$|S,0\rangle \xrightarrow{\pi/2} |S,0\rangle + |D,1\rangle$$

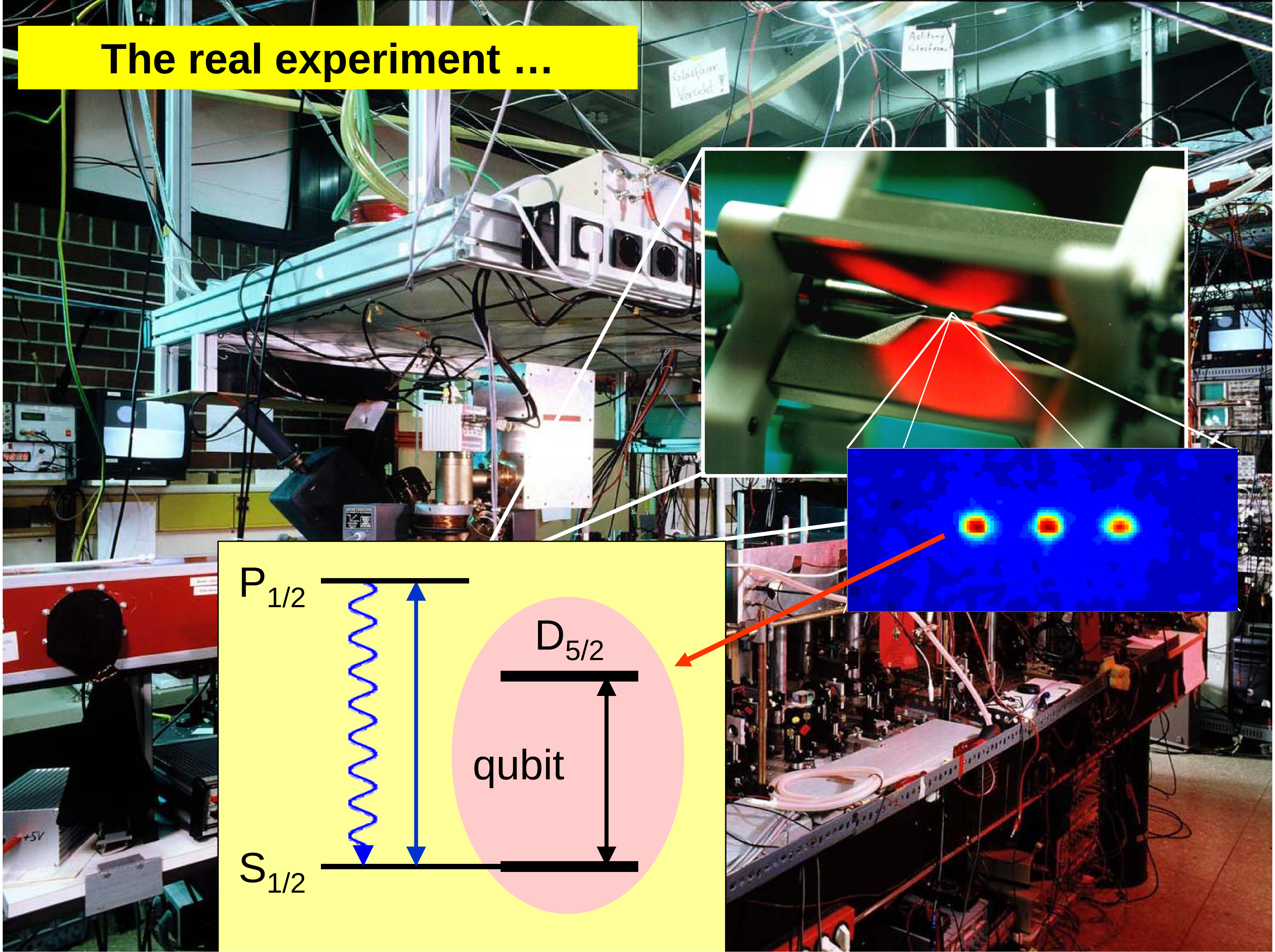
$$|S,0\rangle \xrightarrow{\pi} |D,1\rangle$$





sideband

The real experiment …



The Cirac-Zoller CNOT gate operation with 2 ions

allows the realization of a
universal quantum computer !

$$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$$
$$|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$$
$$|1\rangle|0\rangle \rightarrow |1\rangle|1\rangle$$
$$|1\rangle|1\rangle \rightarrow |1\rangle|0\rangle$$

control bit   target bit

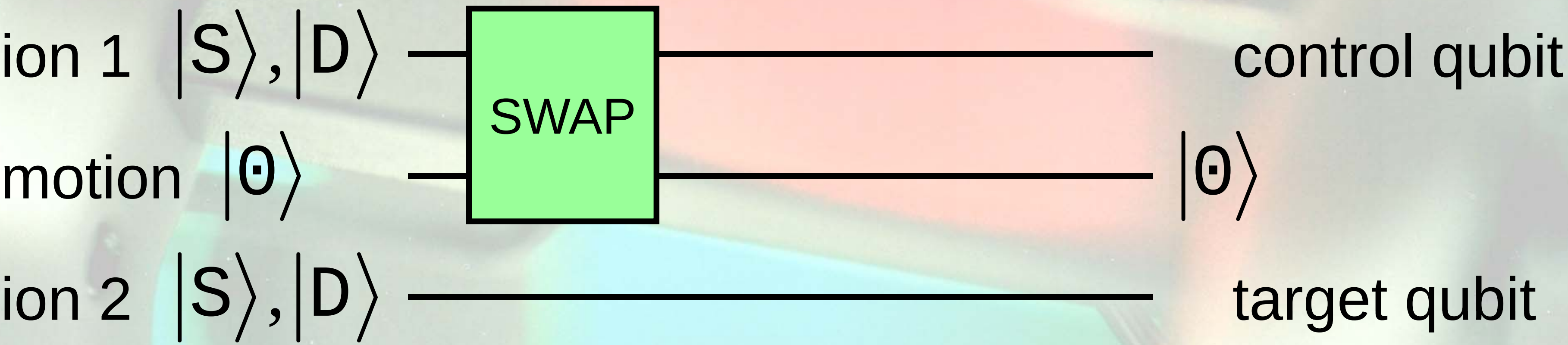
F. Schmidt-Kaler et al., Nature **422**, 408 (2003)

Cirac - Zoller two-ion controlled-NOT operation

$|\varepsilon_1\rangle \; |\varepsilon_2\rangle$

$$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$$
$$|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$$
$$|1\rangle|0\rangle \rightarrow |1\rangle|1\rangle$$
$$|1\rangle|1\rangle \rightarrow |1\rangle|0\rangle$$

control bit

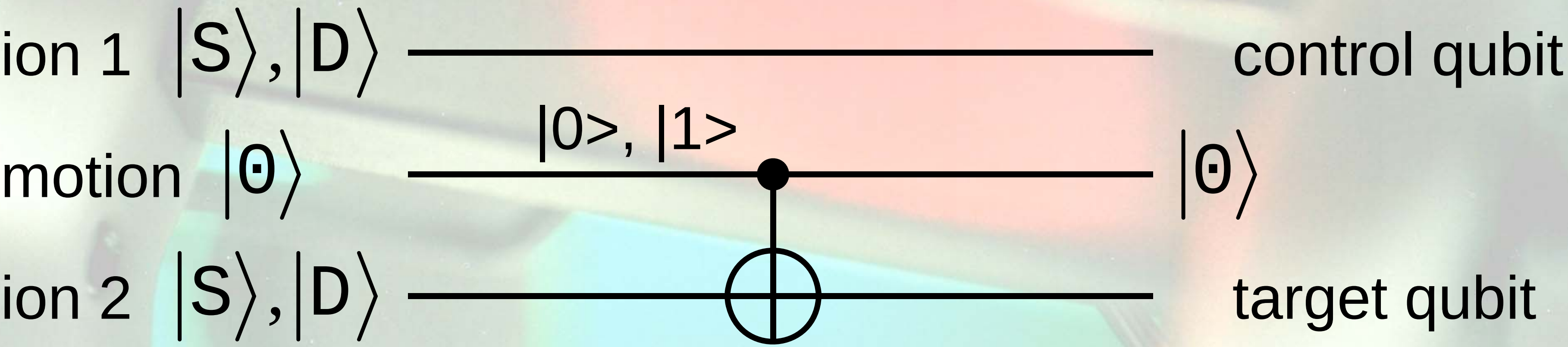


Cirac - Zoller two-ion controlled-NOT operation

$|\varepsilon_1\rangle \quad |\varepsilon_2\rangle$

$$
\begin{aligned}
|0\rangle|0\rangle &\rightarrow |0\rangle|0\rangle \\
|0\rangle|1\rangle &\rightarrow |0\rangle|1\rangle \\
|1\rangle|0\rangle &\rightarrow |1\rangle|1\rangle \\
|1\rangle|1\rangle &\rightarrow |1\rangle|0\rangle
\end{aligned}
$$

target bit



Cirac - Zoller two-ion controlled-NOT operation

$|\varepsilon_1\rangle \; |\varepsilon_2\rangle$

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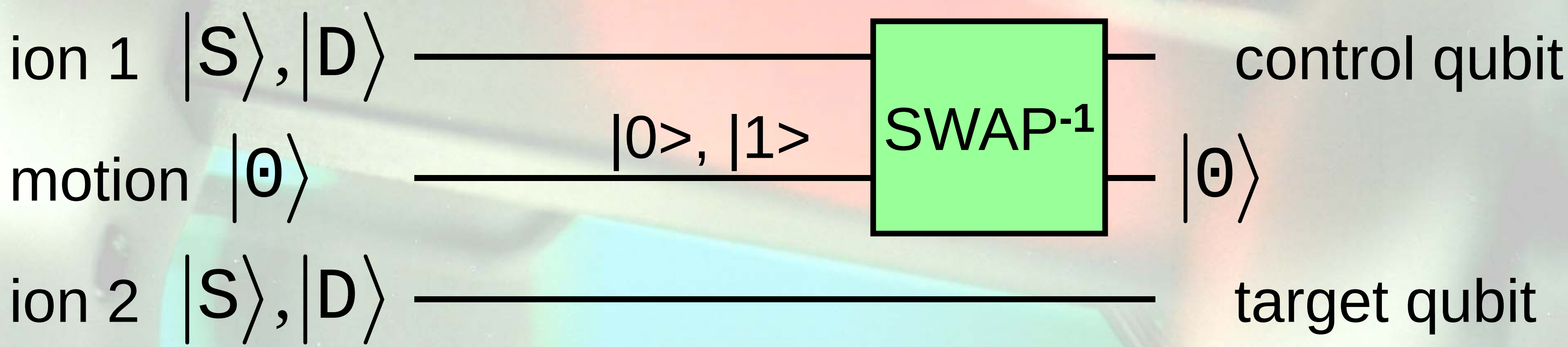
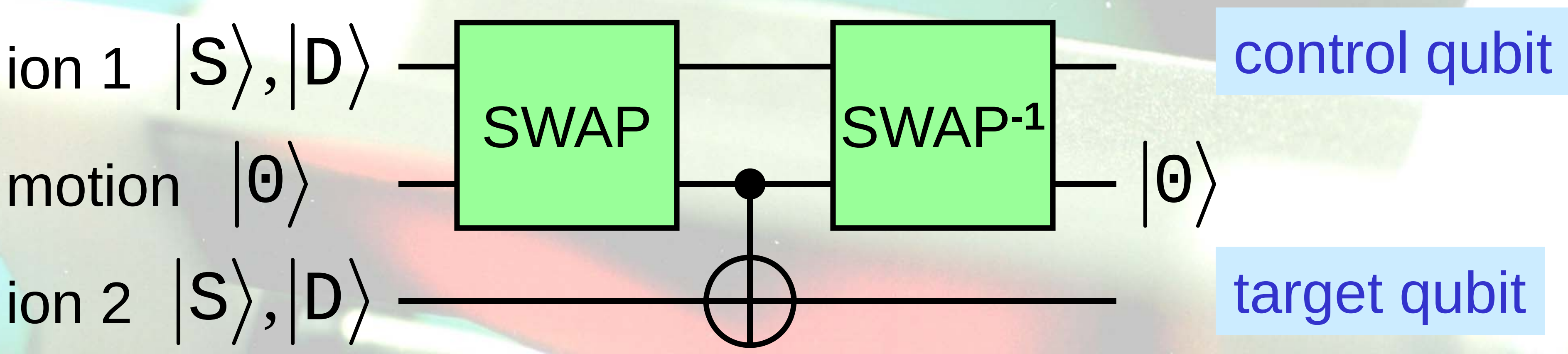
$$|1\rangle|0\rangle \rightarrow |1\rangle|1\rangle$$
$$|1\rangle|1\rangle \rightarrow |1\rangle|0\rangle$$

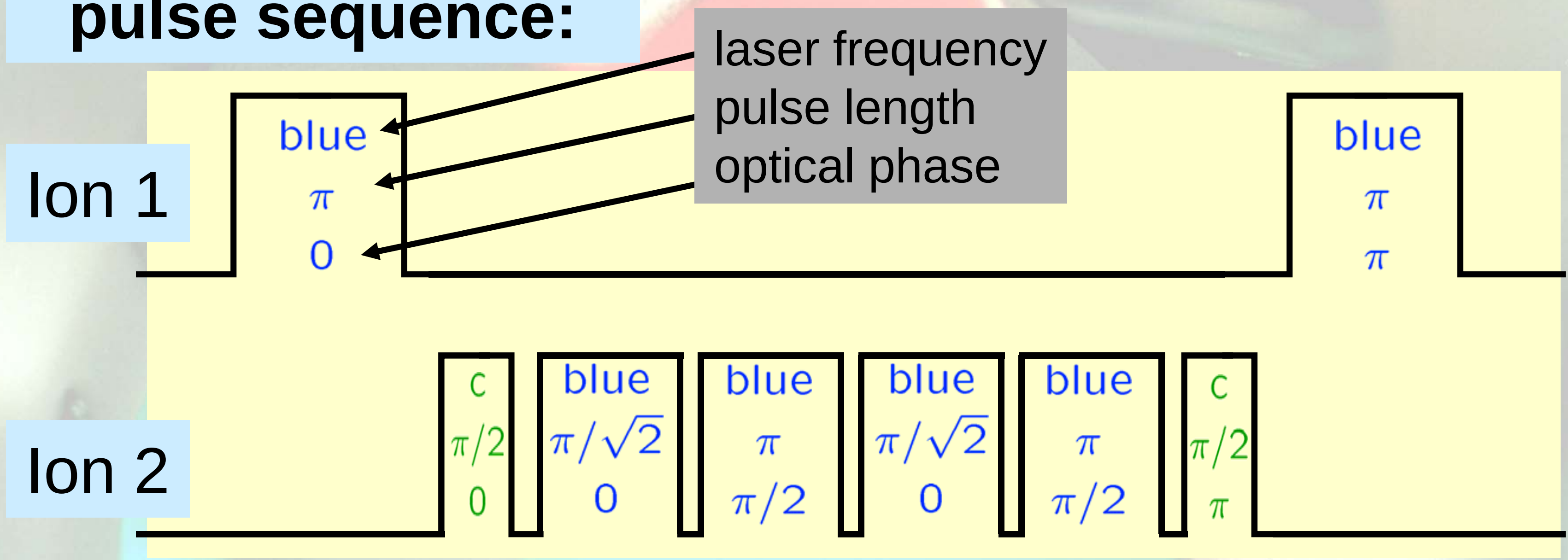
control bit



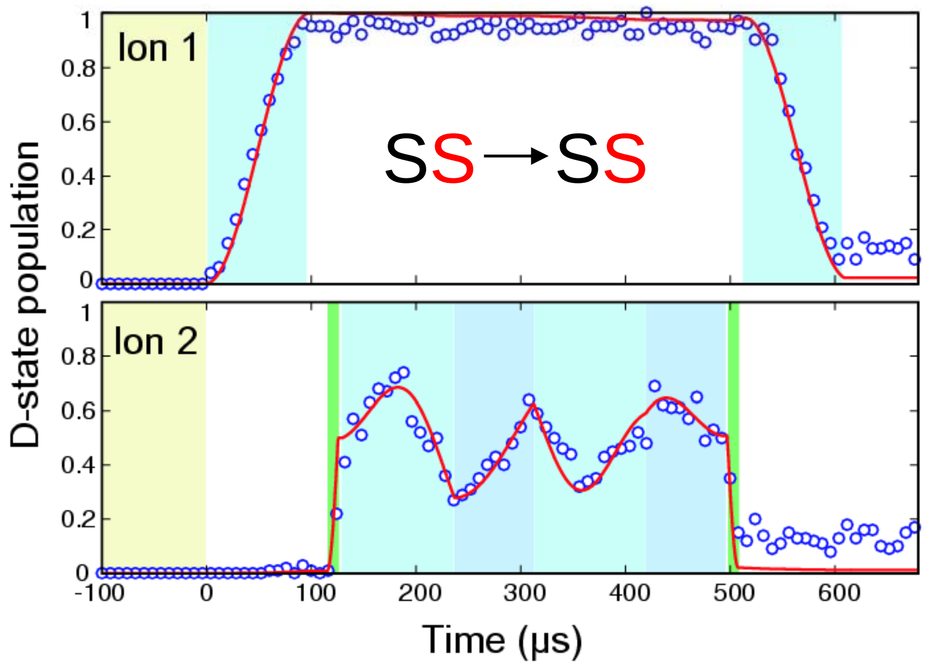
Cirac - Zoller two-ion controlled-NOT operation



pulse sequence:

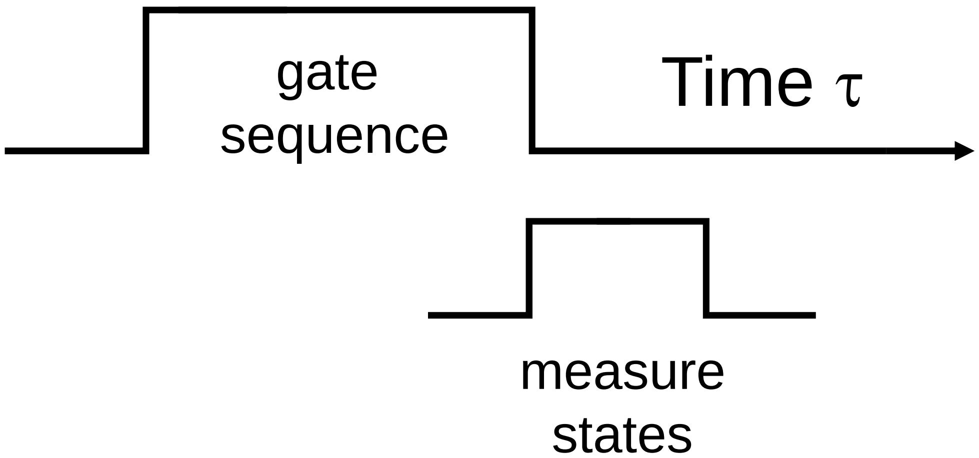


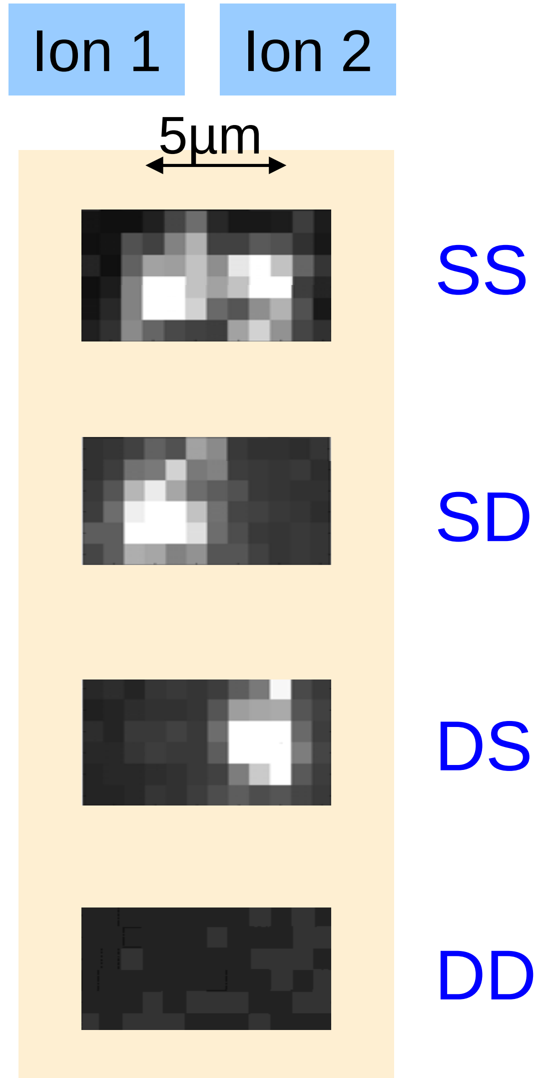
Individual ion detection



control qubit

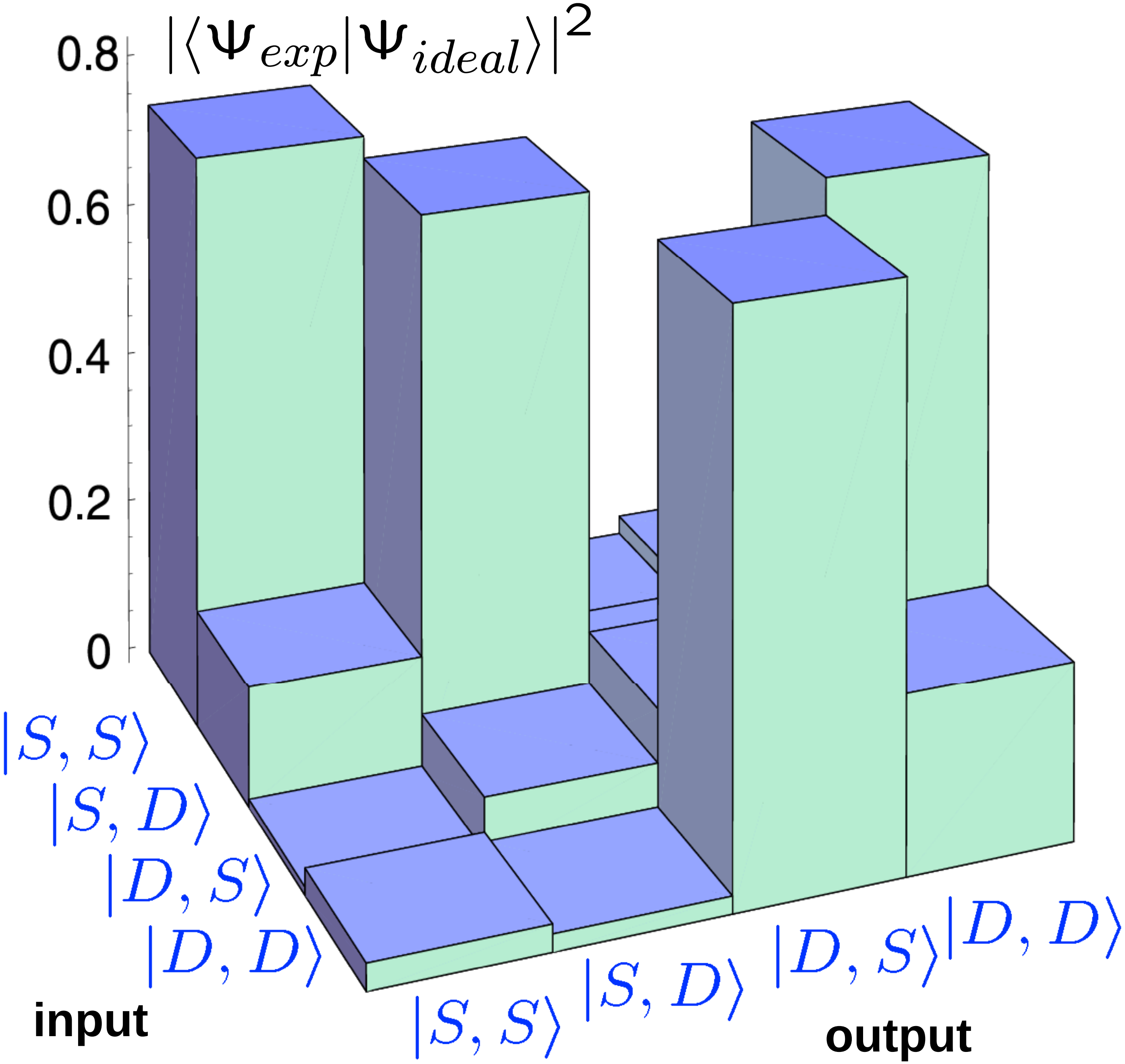
target qubit





Individual ion detection on CCD camera

Experimental fidelity of Cirac-Zoller CNOT operation



truth table:

$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$

$|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$

$|1\rangle|0\rangle \rightarrow |1\rangle|1\rangle$

$|1\rangle|1\rangle \rightarrow |1\rangle|0\rangle$

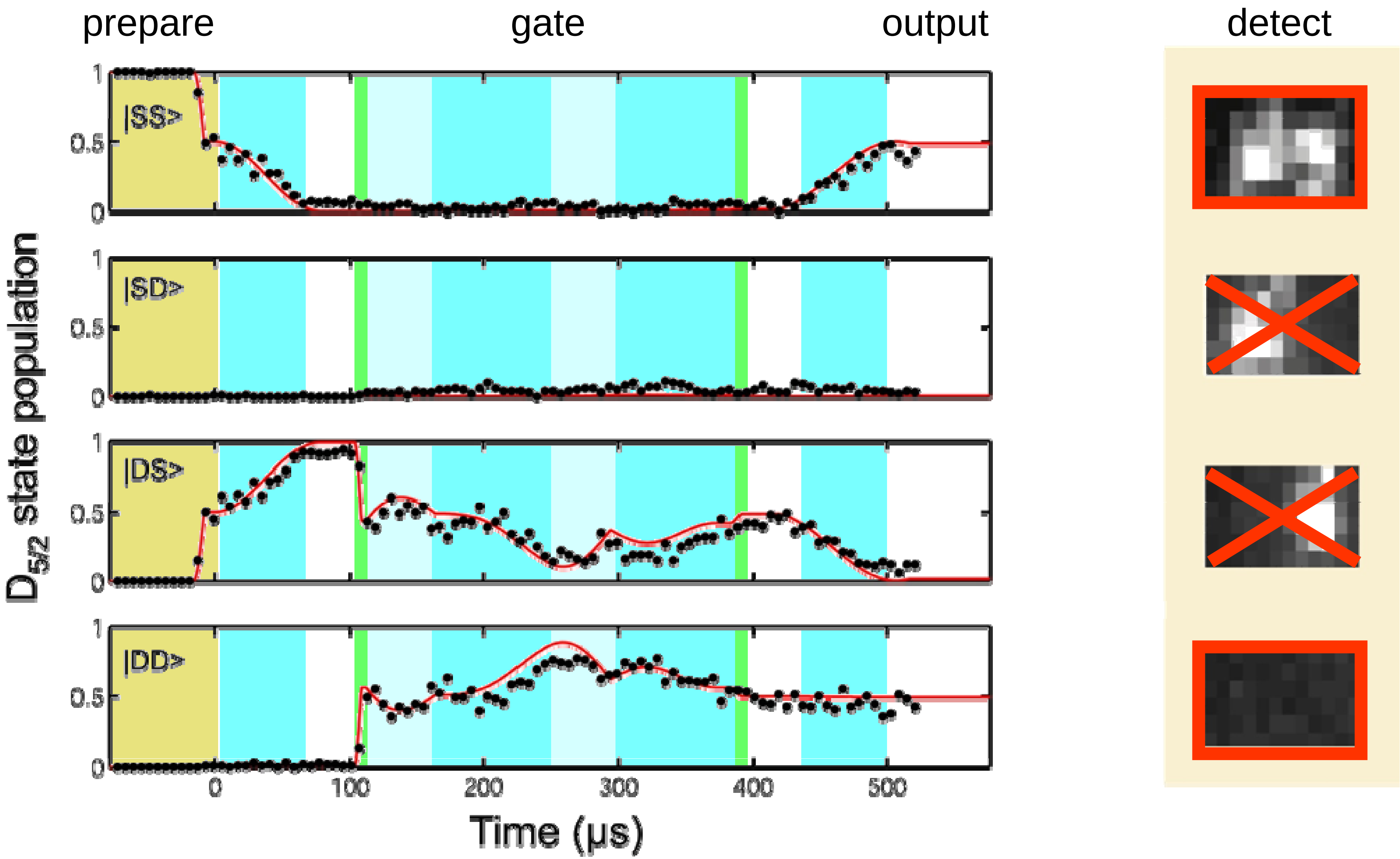
control bit

target bit

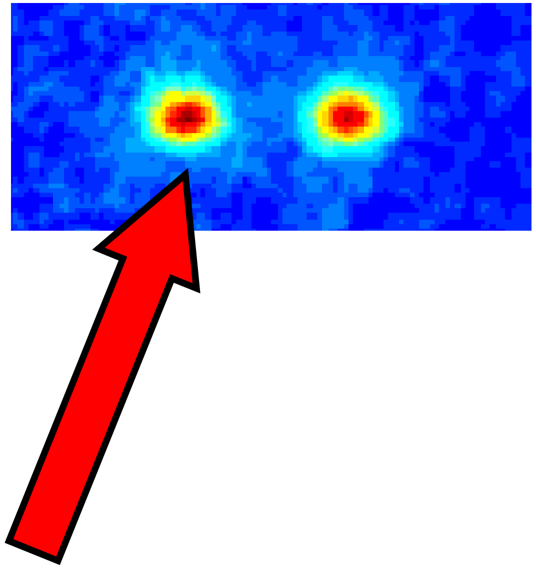
F. Schmidt-Kaler et al., Nature **422**, 408 (2003)

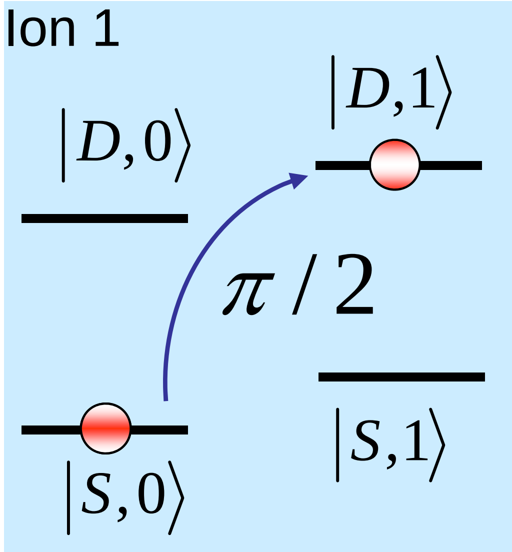
Superposition as input to CNOT gate operation

$$|D+S\rangle|S\rangle \longrightarrow |DD+SS\rangle$$



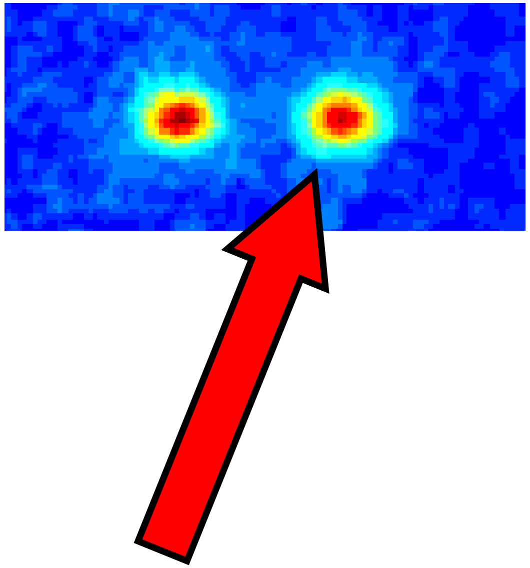
Bell states

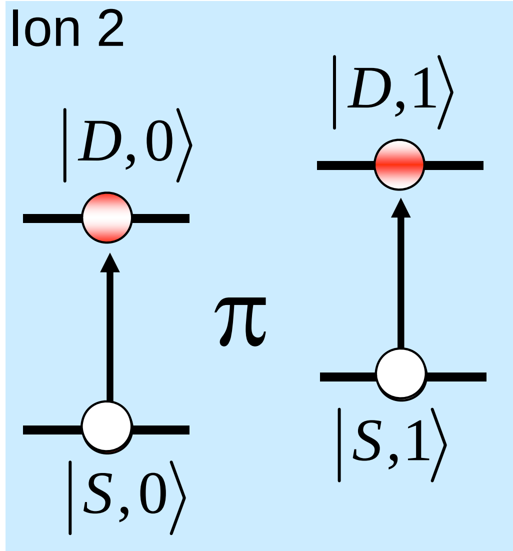




$$|SS\rangle|0\rangle \xrightarrow{\pi/2 \text{ pulse, BSB}} |SS,0\rangle + |DS,1\rangle$$

Bell states

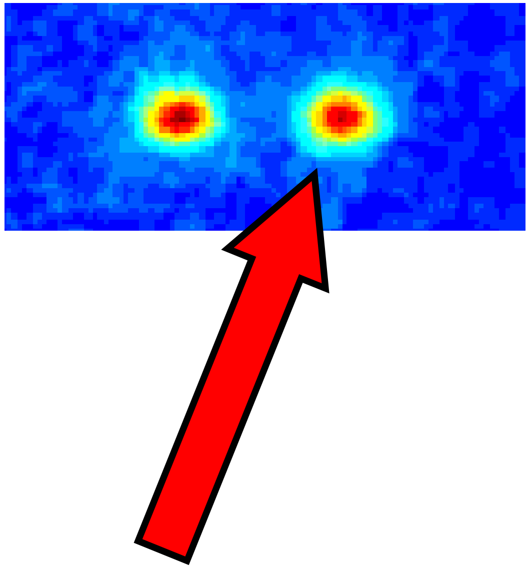


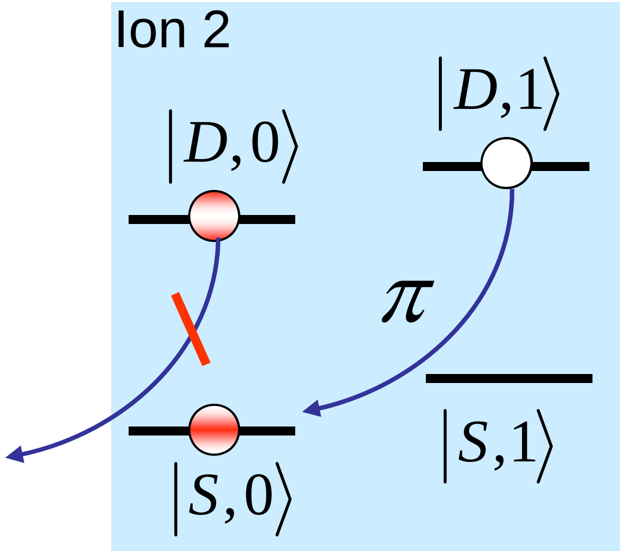


$$|SS\rangle|0\rangle \xrightarrow{\pi/2 \text{ pulse, BSB}} |SS,0\rangle + |DS,1\rangle$$

$$\xrightarrow{\pi \text{ pulse, carrier}} |SD,0\rangle + |DD,1\rangle$$

Bell states



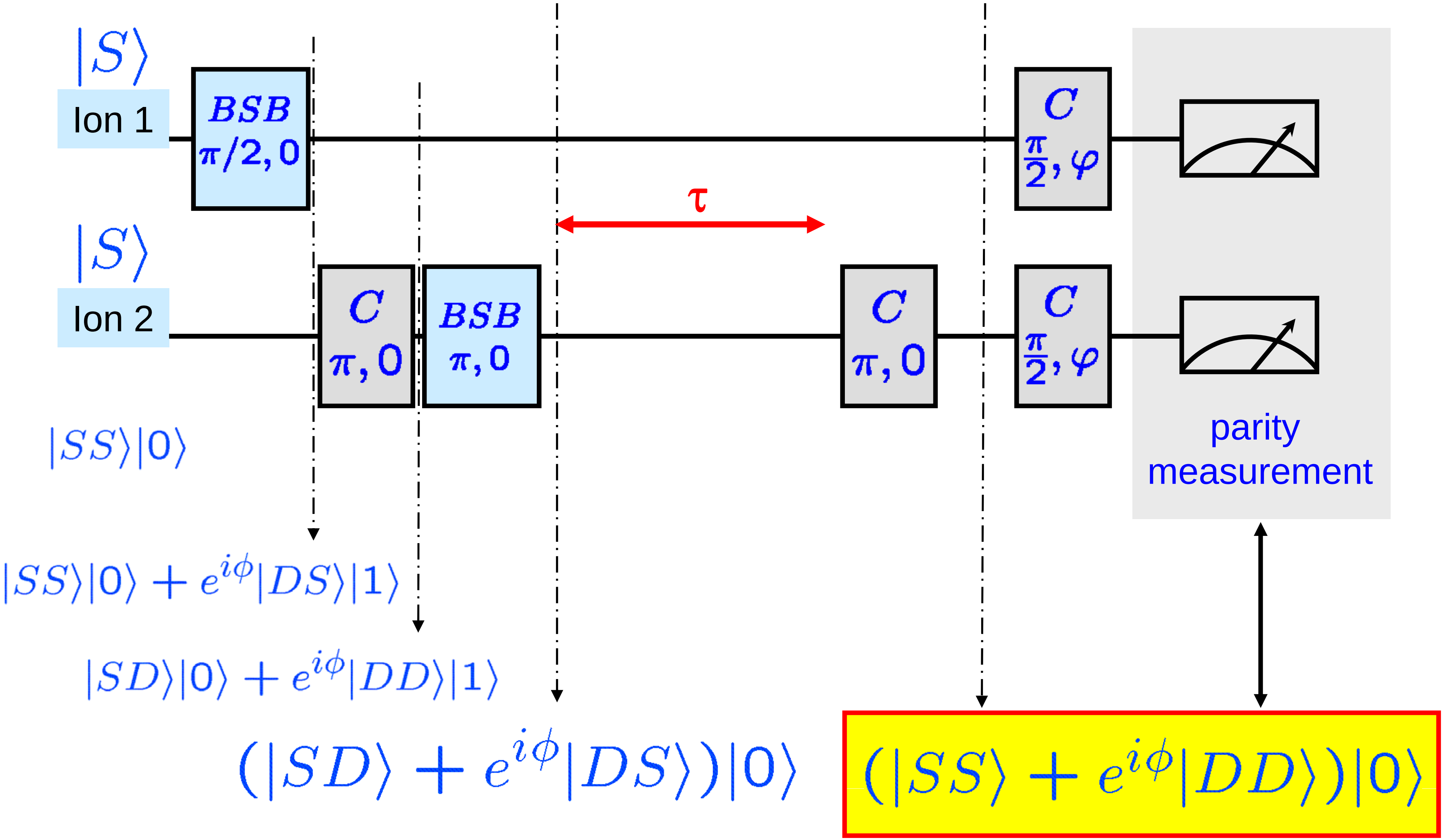


$$|SS\rangle|0\rangle \xrightarrow{\pi/2 \text{ pulse, BSB}} |SS,0\rangle + |DS,1\rangle$$

$$\xrightarrow{\pi \text{ pulse, carrier}} |SD,0\rangle + |DD,1\rangle$$

$$\xrightarrow{\pi \text{ pulse, BSB}} |SD\rangle|0\rangle + |DS\rangle|0\rangle$$

Preparation of Bell states and measurement



C. Roos et al., Phys. Rev. Lett. **92**, 220402 (2004)

Reconstruction of a density matrix ρ

Representation of ρ as a sum of orthogonal observables A_j :

$$\rho = \sum_i \lambda_i A_i \quad \text{with} \quad Tr(A_i A_j) = \delta_{ij}$$

ρ is completely determined by the expectation values $\langle A_j \rangle$:

$$\langle A_j \rangle = Tr(\rho A_j) = \sum_i \lambda_i Tr(A_i A_j) = \lambda_j$$

For a two-qubit system : $A_i \in \{\sigma_i^{(1)} \otimes \sigma_j^{(2)}, \sigma_i \in \{I, \sigma_x, \sigma_y, \sigma_z\}\}$

⟹ Joint measurements of all spin components $\sigma_i^{(1)} \otimes \sigma_j^{(2)}$

$$\rho_R = \sum_{i=1}^{16} \langle A_i \rangle \, A_i$$

Maximum likelihood estimation

The reconstructed $\rho_R = \sum_i \langle A_i \rangle \, A_i$ is not necessarily positive semidefinite

obtain ρ from **maximum likelihood estimation**:

Z. Hradil, Phys. Rev. **A 55**, R1561 (1997),
K. Banaszek et al., Phys. Rev. **A 61**, 010304 (1999)

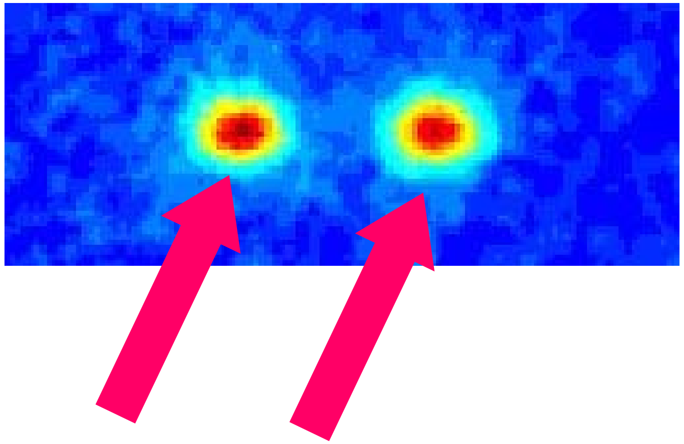
choose ρ such that

$$L(\rho) = \sum_i \frac{(\langle A_i \rangle_{exp} - Tr(\rho A_i))^2}{\sigma^2_{A_i,\rho}}$$

is minimized

⟹ optimization of 15 parameters A_i

Experimental tomography procedure



two $^{40}Ca^{+}$ ions trapped in linear trap

$$\omega_z = (2\pi)\,1.2\,\mathrm{MHz}$$
$$\omega_{x,y} = (2\pi)\,4.5\,\mathrm{MHz}$$

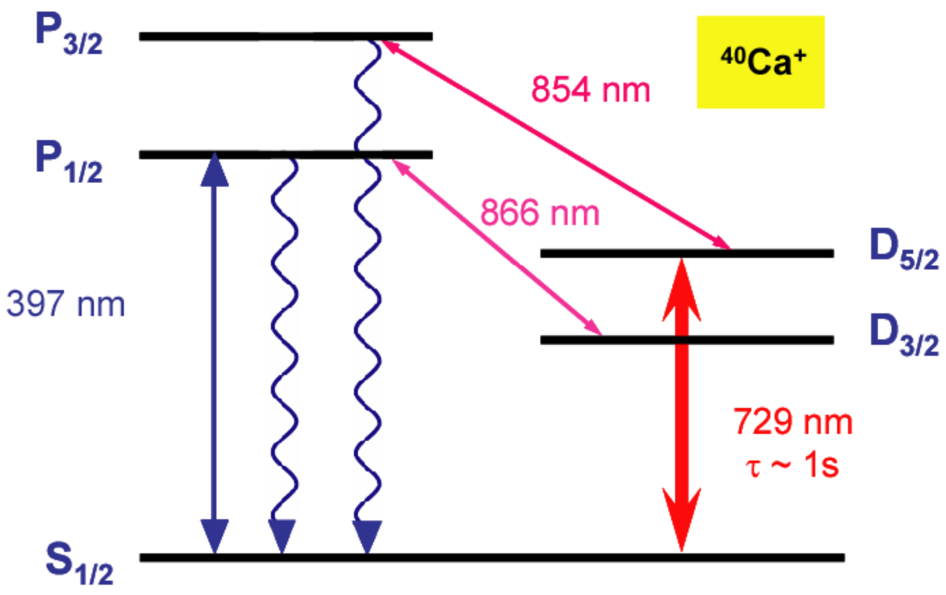
Individual qubit operations $\sigma_i^{(1)}, \sigma_j^{(2)}, \quad i = x, y, z$

Experimental cycle (~20 ms):

1. Laser cooling to the motional ground state
2. Quantum state preparation
3. Application of tomography pulses
4. State detection

} 100-200 experiments

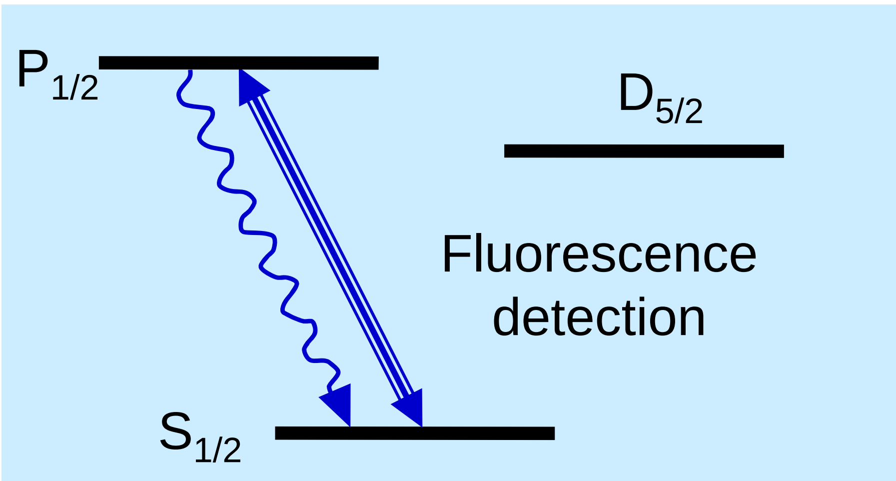
Experimental tomography procedure



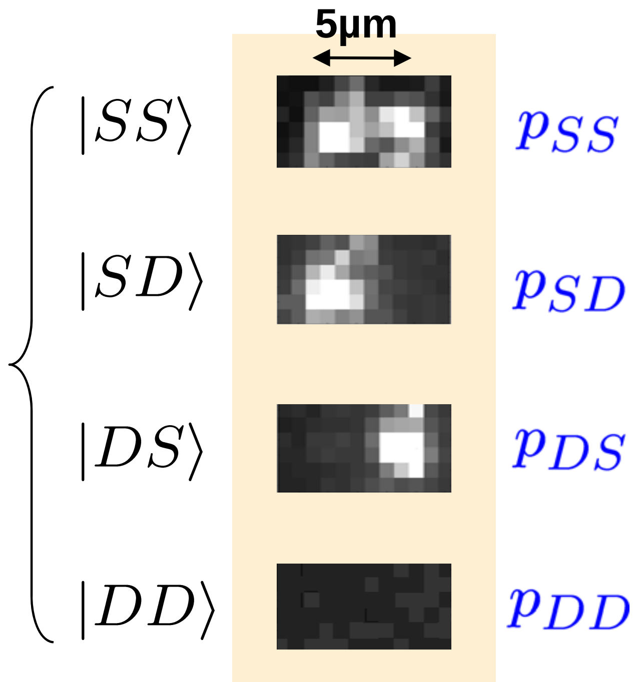
1. Initialization in |SS0>
   10 ms of Doppler and sideband cooling

2. Quantum state manipulation on the $S_{1/2} – D_{5/2}$ transition

3. Quantum state measurement by fluorescence detection



Detection with CCD camera:



Qubit transition : $|S_{1/2}, -1/2\rangle \leftrightarrow |D_{5/2}, -1/2\rangle$

Measurement of the density matrix

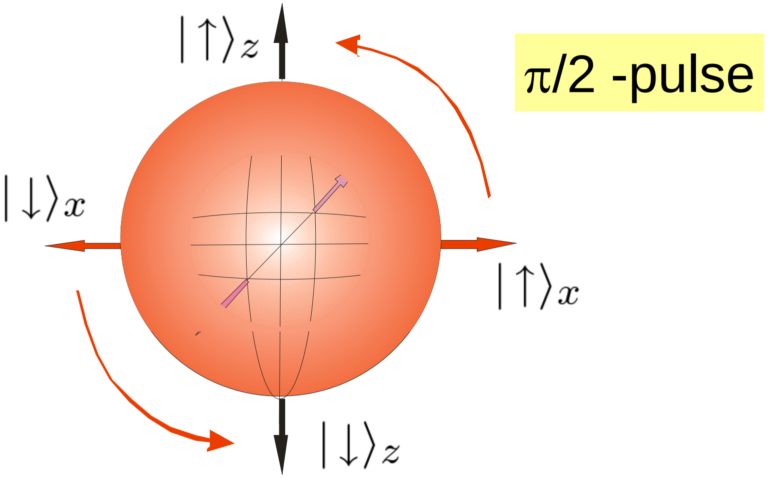
Measurement of

$\langle\sigma_z^{(1)}\rangle,\ \langle\sigma_z^{(2)}\rangle,\ \langle\sigma_z^{(1)}\sigma_z^{(2)}\rangle\ :$

$$\begin{aligned}\langle\sigma_z^{(1)}\rangle &= p_{DD}+p_{DS}-p_{SD}-p_{SS}\\ \langle\sigma_z^{(2)}\rangle &= p_{DD}-p_{DS}+p_{SD}-p_{SS}\\ \langle\sigma_z^{(1)}\sigma_z^{(2)}\rangle &= p_{DD}-p_{DS}-p_{SD}+p_{SS}\end{aligned}$$

$\langle\sigma_x^{(1)}\rangle,\ \langle\sigma_x^{(1)}\sigma_z^{(2)}\rangle$ etc. :

Rotation of Bloch sphere prior to measurement:



prepare Bell state
no rotation
measure
200 repetitions

↓

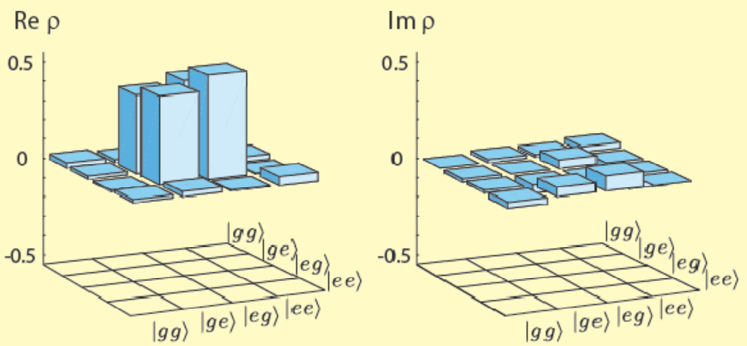
prepare Bell state
ion #1, σ_x rotation
ion #2, identity
measure
200 repetitions

↓
⋮
↓

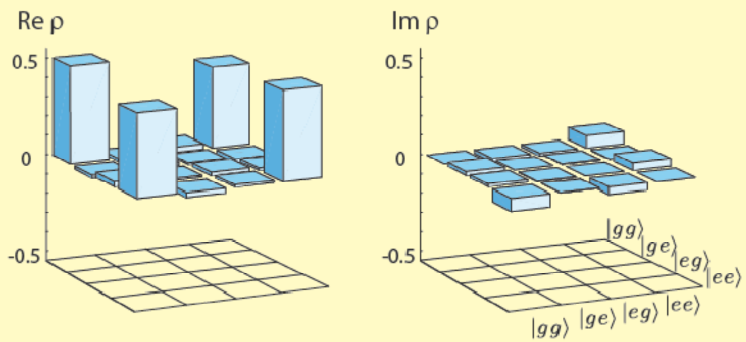
9 different settings

prepare Bell state
ion #1, σ_y rotation
ion #2, σ_y rotation
measure
200 repetitions

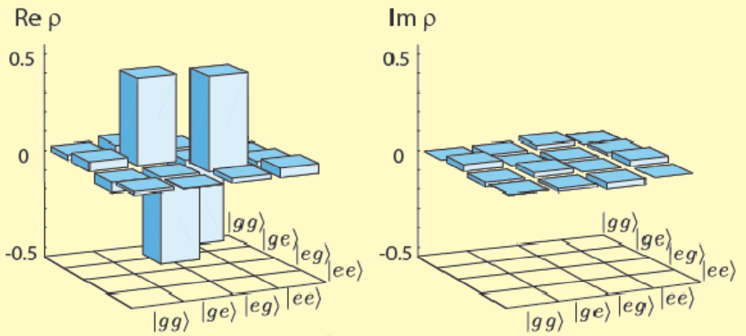
Push-button preparation and tomography of Bell states



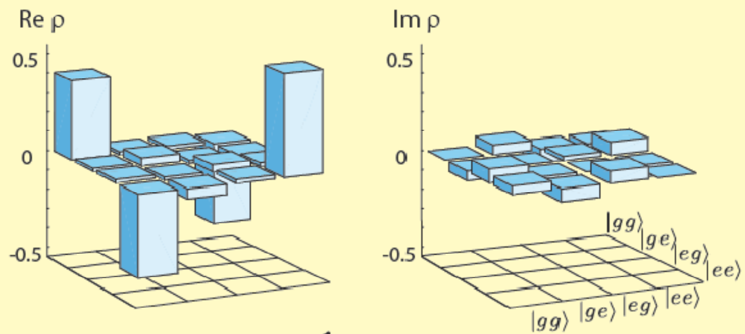
$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|ge\rangle + |eg\rangle)$$



$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|gg\rangle + |ee\rangle)$$



$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|ge\rangle - |eg\rangle)$$



$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|gg\rangle - |ee\rangle)$$

Fidelity:

F = 0.91

Entanglement of formation:

$E(\rho_{exp}) = 0.79$

Violation of Bell inequality:

$S(\rho_{exp}) = 2.53(6)$
> 2

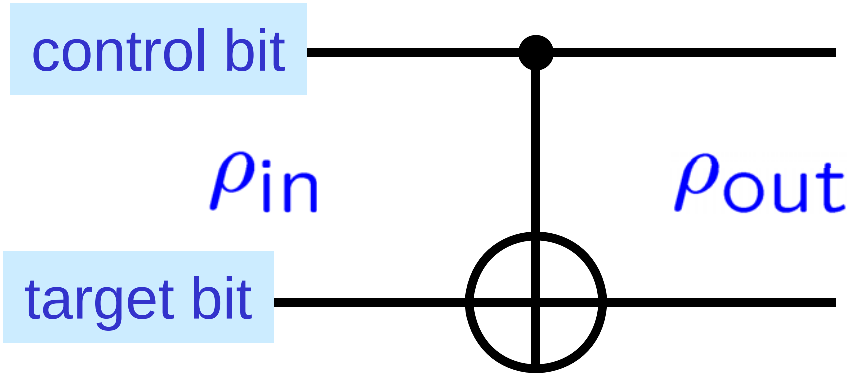
C. Roos et al., Phys. Rev. Lett. **92**, 220402 (2004)

Quantum Process Tomography

$$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$$
$$|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$$
$$|1\rangle|0\rangle \rightarrow |1\rangle|1\rangle$$
$$|1\rangle|1\rangle \rightarrow |1\rangle|0\rangle$$

target bit

control bit



$$\rho_{\text{out}} = \sum \chi_{ij} E_i \rho_{\text{in}} E_j^\dagger$$

$$E_i = A_i \otimes A_j$$

$$A_i \in \{I, \sigma_x, \sigma_y, \sigma_z\}$$

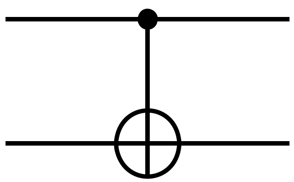
$$\equiv \{I, X, iY, Z\}$$

$$\chi_{ij}$$

characterizes gate operation completely

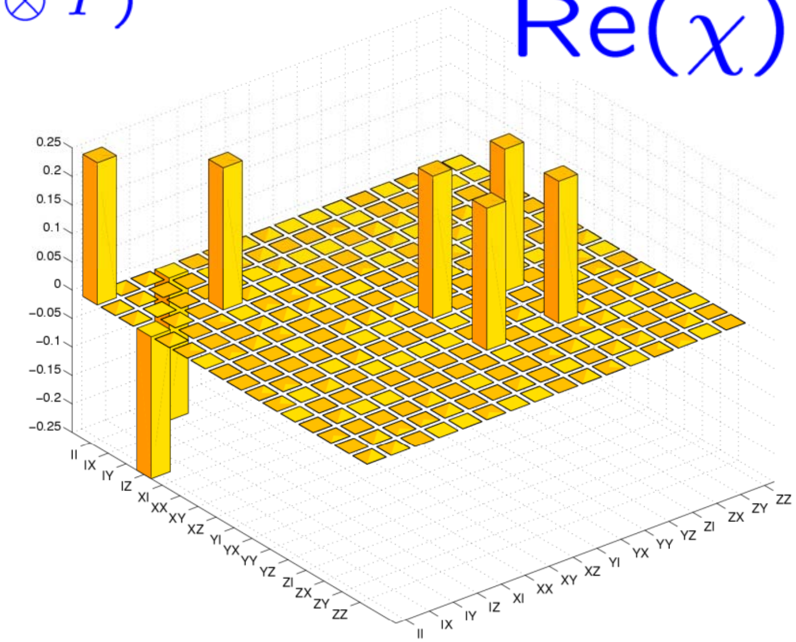
χ – matrix for ideal CNOT gate operation

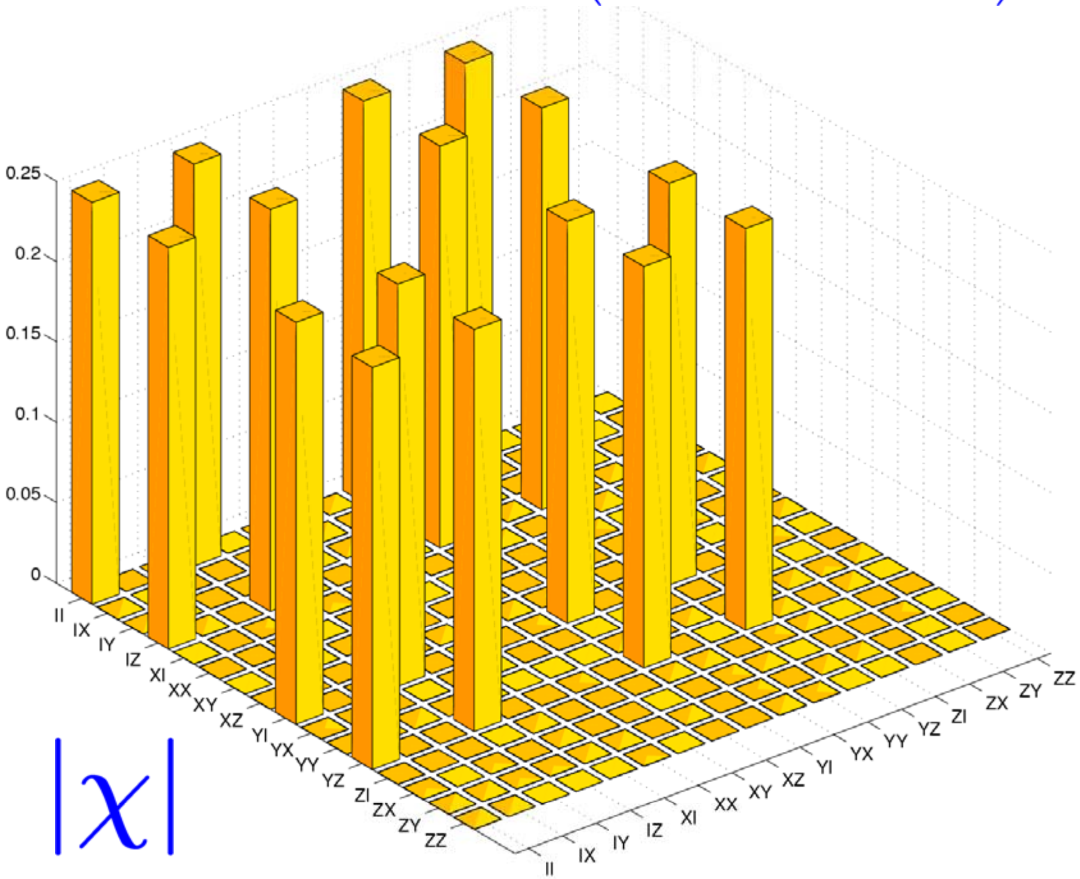
$$U^{12}_{\mathrm{CNOT}} = -\frac{1}{2}(I \otimes I + iI \otimes Y - Z \otimes I + iZ \otimes Y)$$



$$= \begin{pmatrix} 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

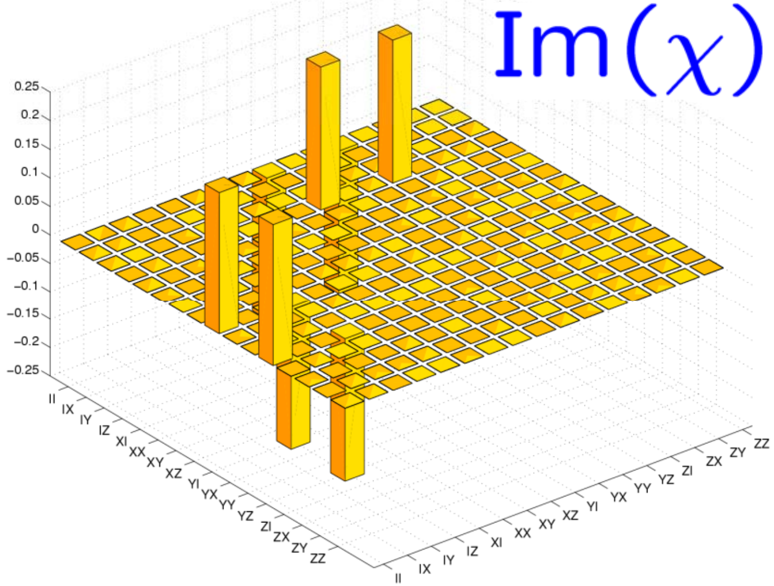
Re(χ)





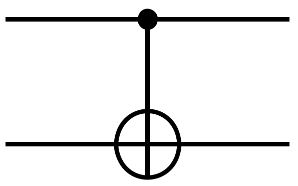
$|\chi|$

Im(χ)



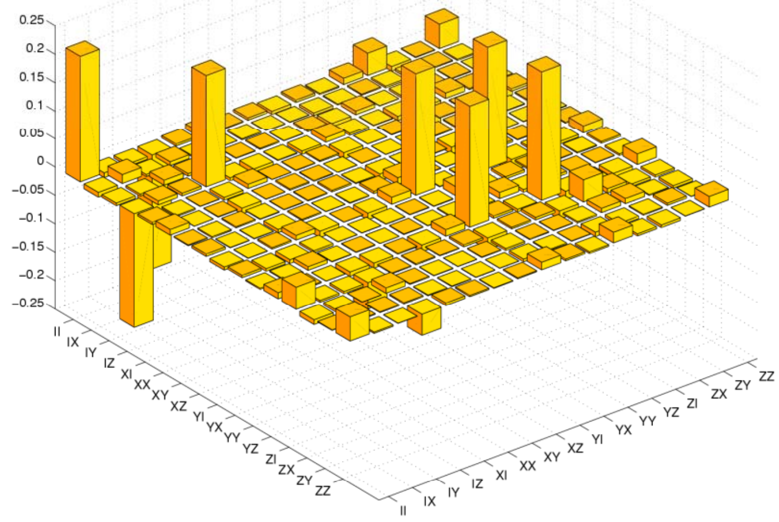
χ – matrix for observed CNOT gate operation

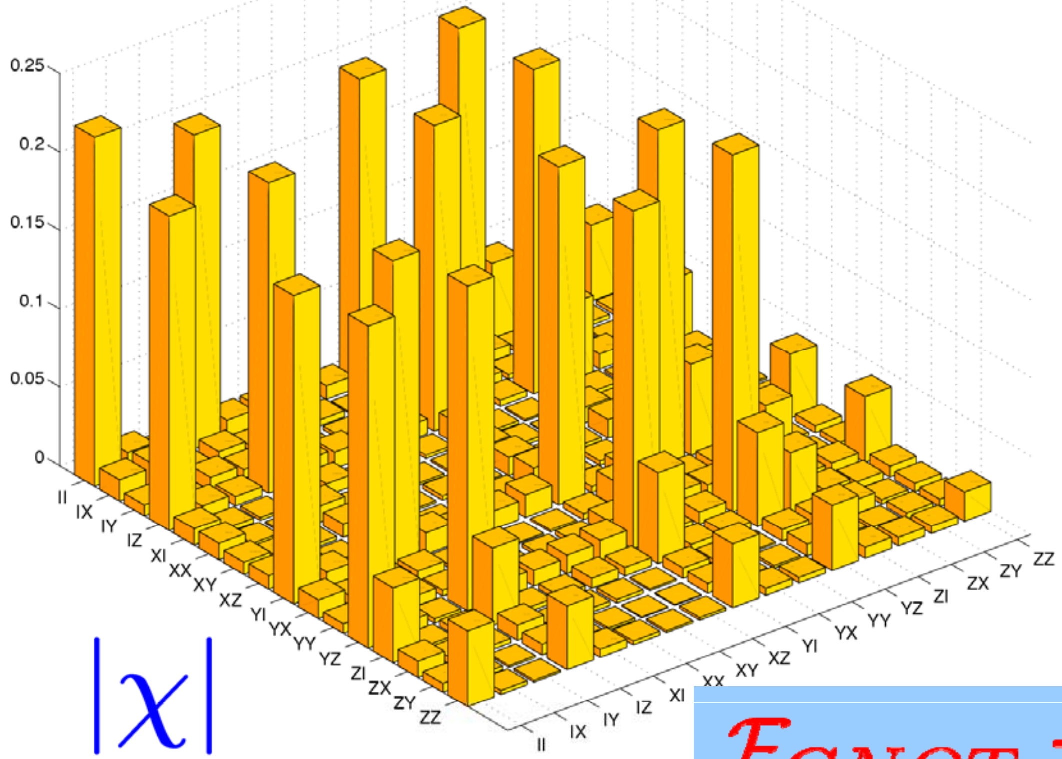
$$U^{12}_{\mathrm{CNOT}} = -\frac{1}{2}((I - Z) \otimes I + (I + Z) \otimes iY)$$



$$= \begin{pmatrix} 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

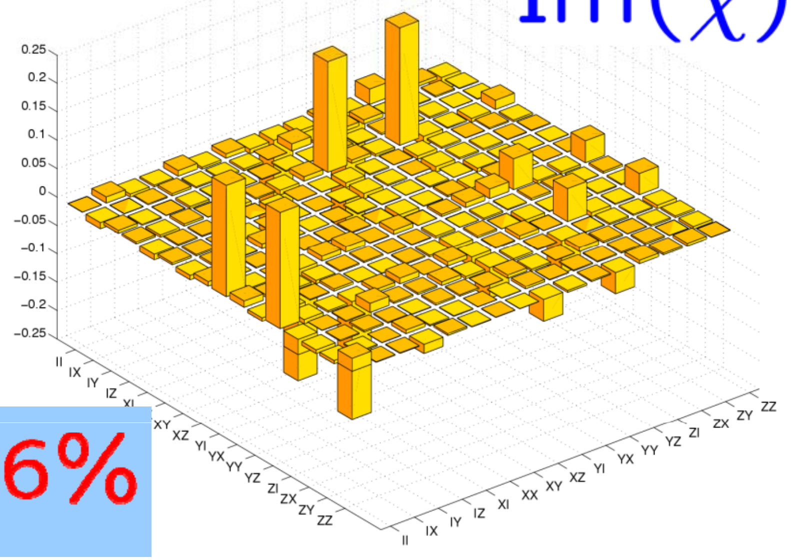
Re(χ)





$|\chi|$

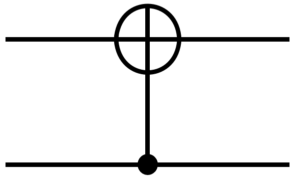
Im(χ)



$\mathcal{F}_{CNOT} = 86\%$

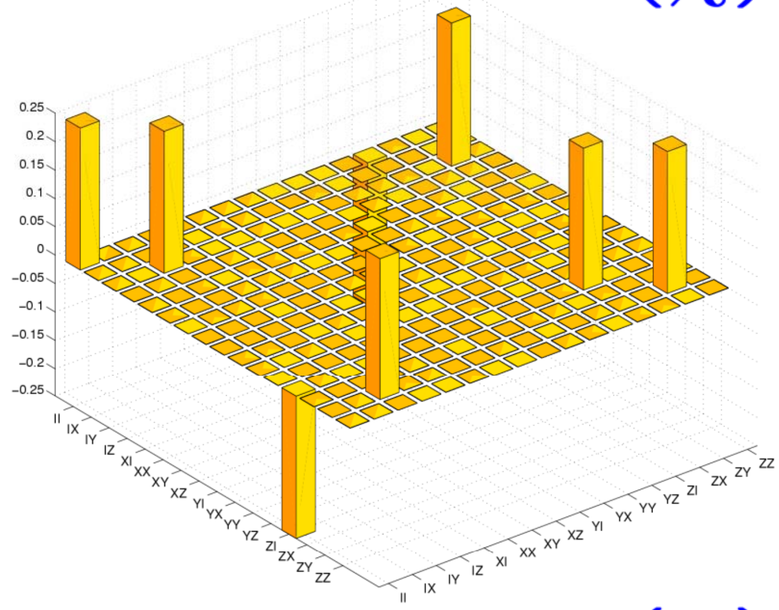
χ – matrix for ideal CNOT gate operation

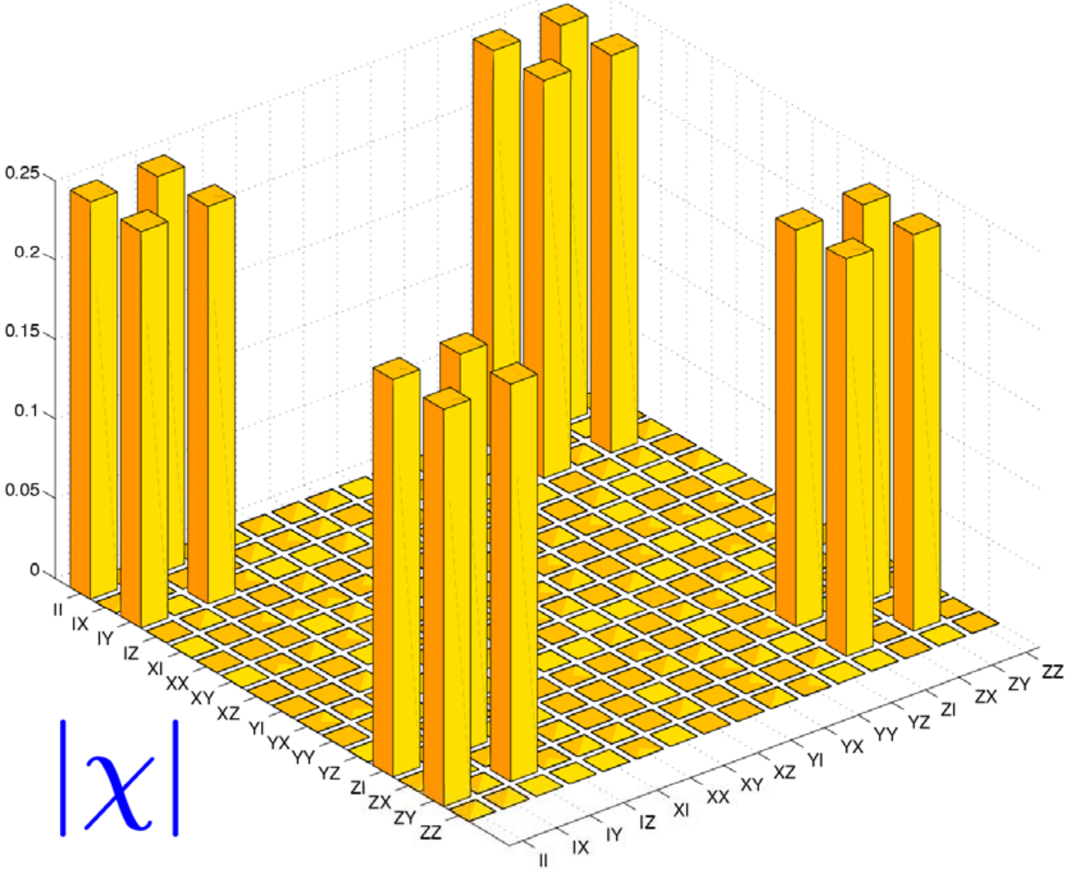
$$U_{\mathrm{CNOT}}^{21} = -\frac{1}{2}(I \otimes I + iY \otimes I - I \otimes Z + iY \otimes Z)$$



$$= \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & -1 & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

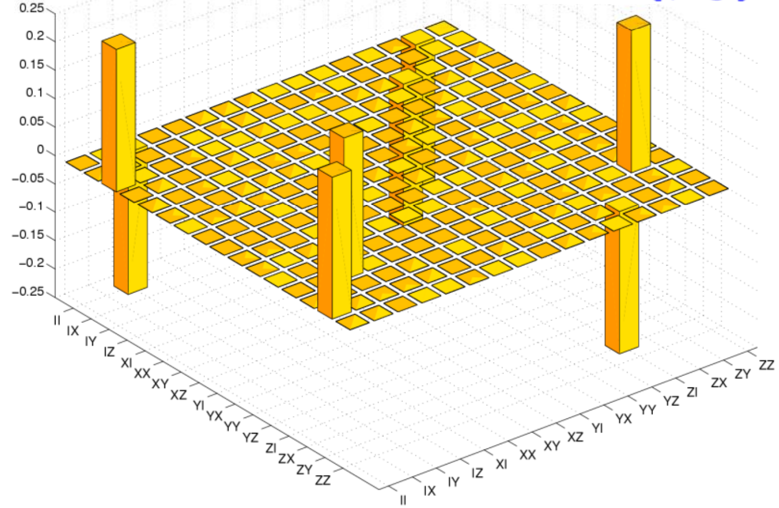
Re(χ)





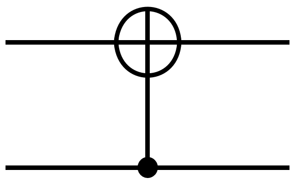
$|\chi|$

Im(χ)



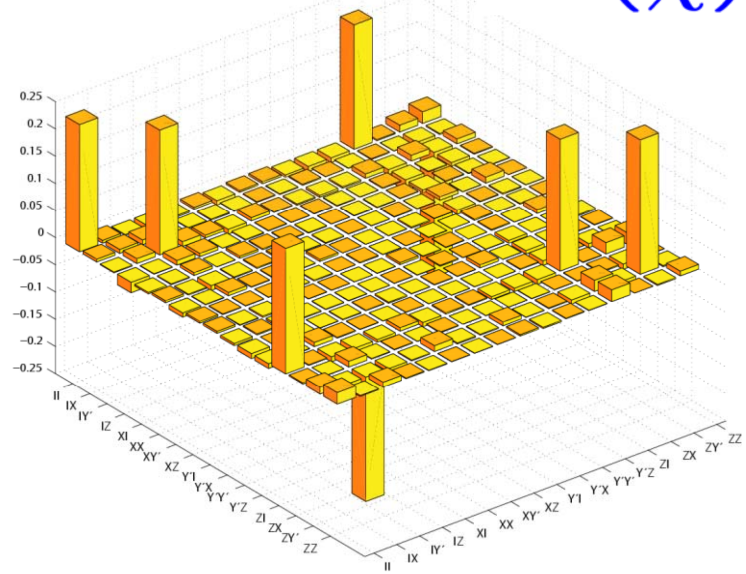
χ – matrix for observed CNOT gate operation

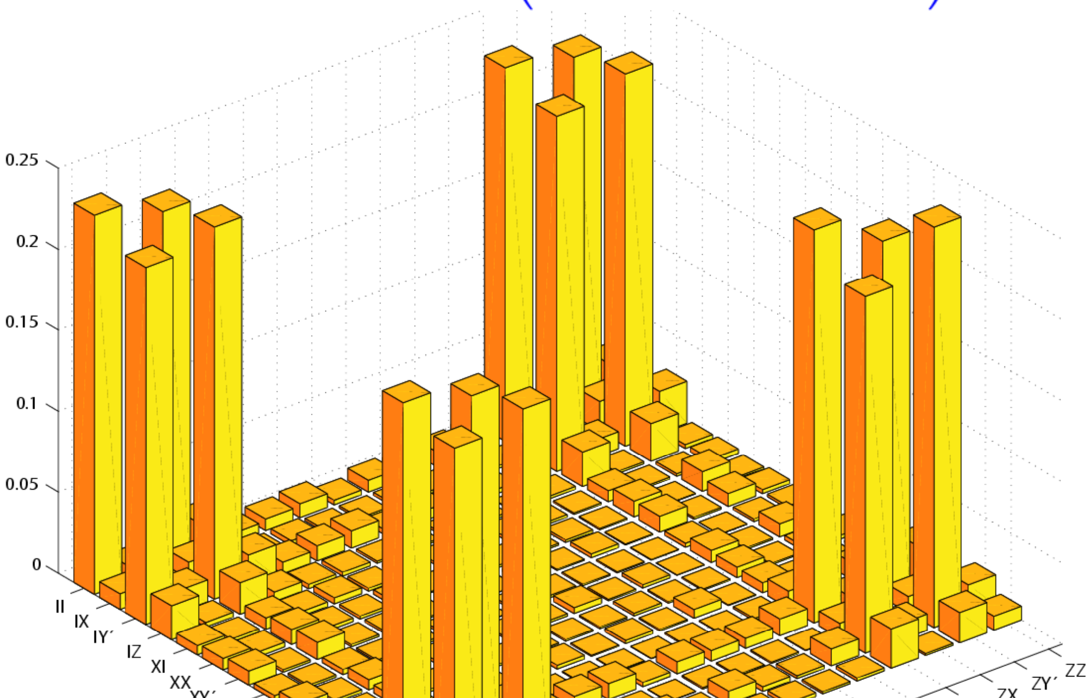
$$U^{21}_{\mathrm{CNOT}} = -\frac{1}{2}\left(I \otimes (I - Z) + iY \otimes (I + Z)\right)$$



$$= \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & -1 & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

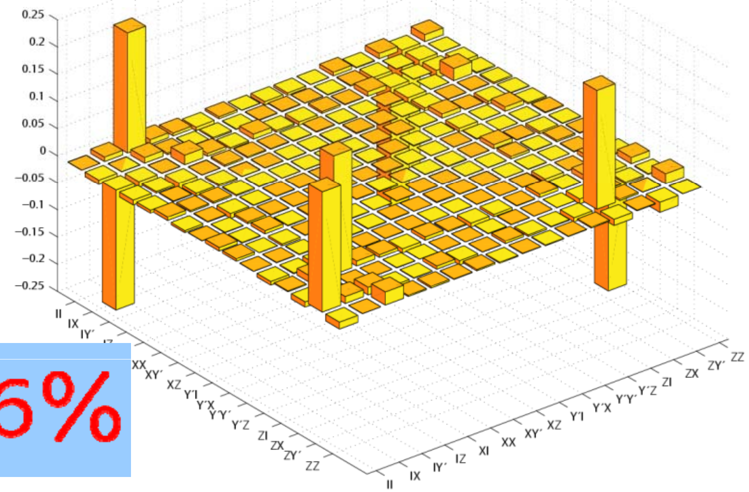
Re(χ)





$|\chi|$

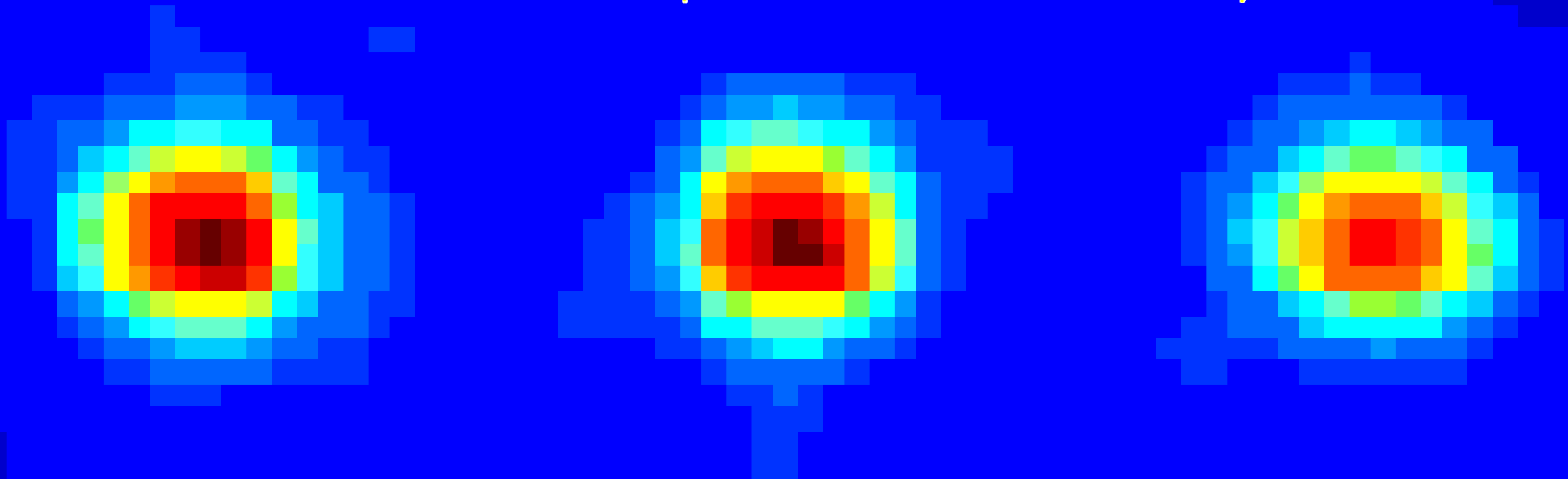
Im(χ)



$$\mathcal{F}_{CNOT} = 92.6\%$$

Entangled states with three ions

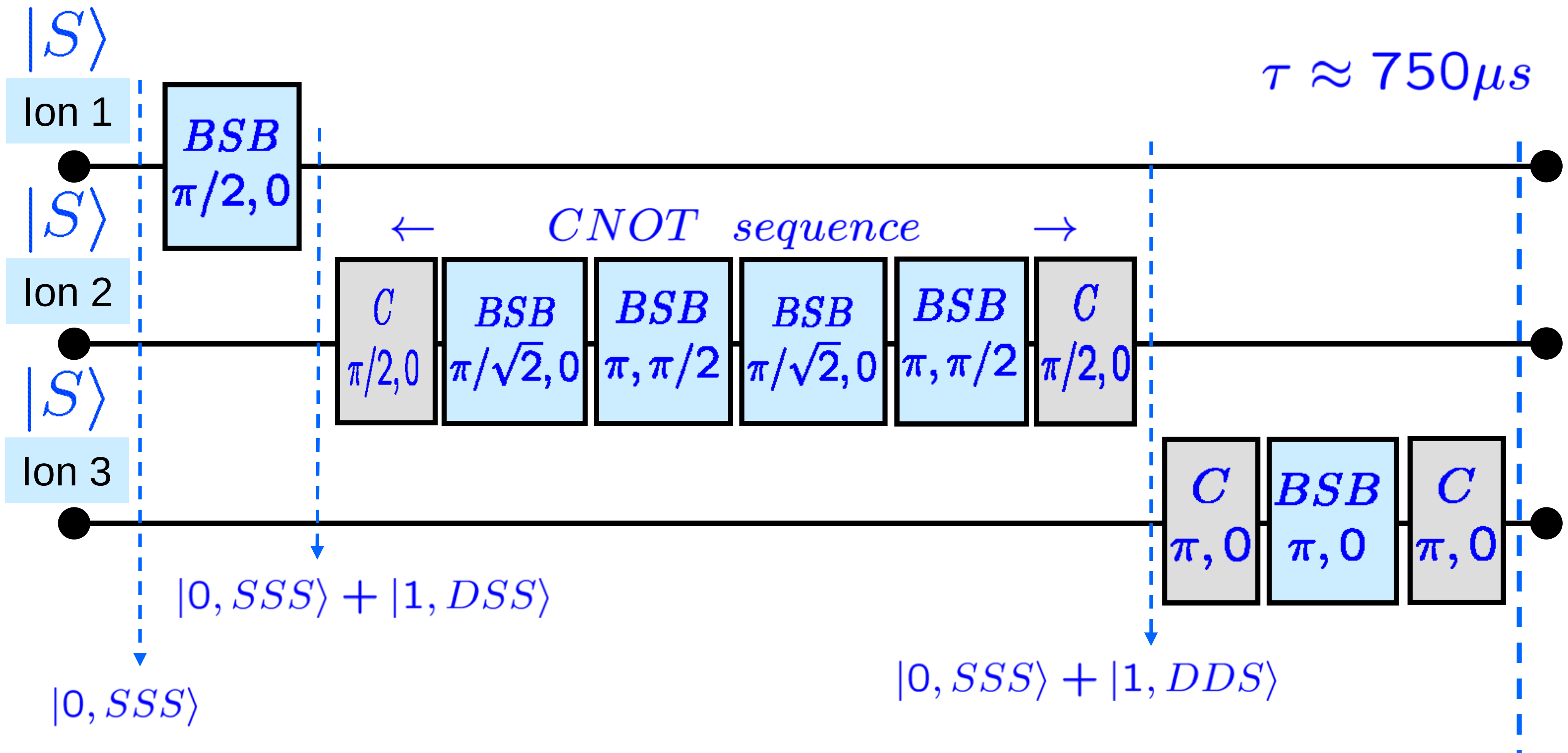
GHZ states: $|SSS + DDD\rangle$



W states: $|SDD + DSD + DDS\rangle$

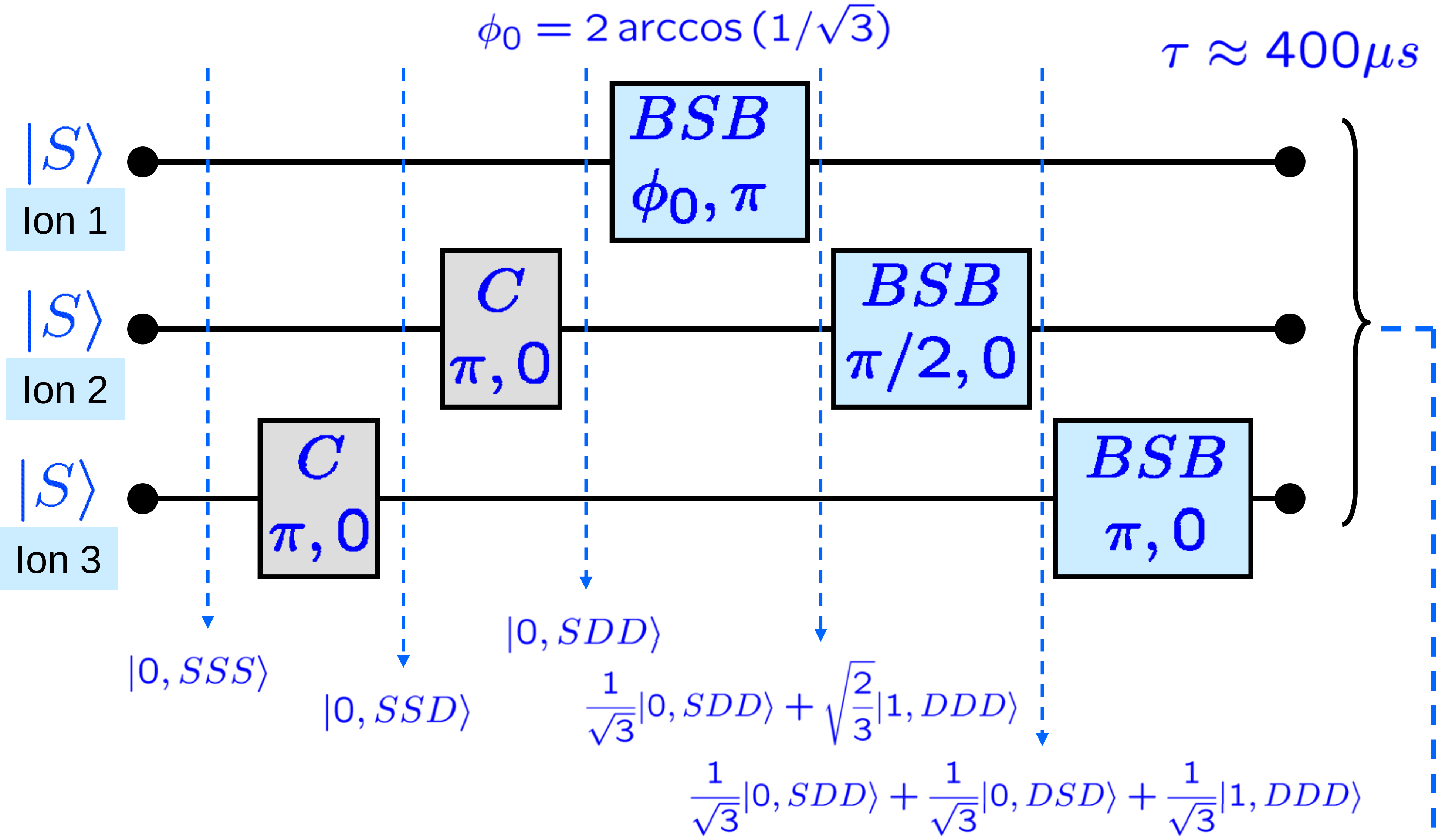
C. Roos et al., Science **304**, 1478 (2004)

Preparation of a GHZ state



$$|\Psi\rangle_{GHZ} = \frac{1}{\sqrt{2}}|0, DDD\rangle + \frac{1}{\sqrt{2}}|0, SSS\rangle$$

3 Ions: Preparation of W – states

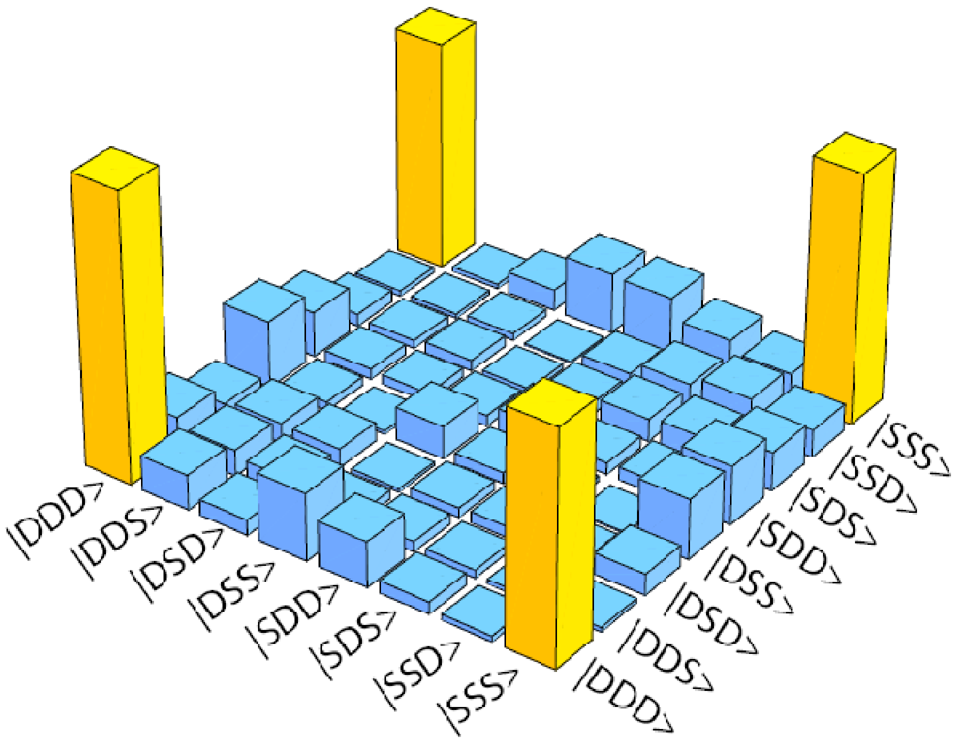


$$|\Psi\rangle_W = \frac{1}{\sqrt{3}}\left(|0, SDD\rangle + |0, DSD\rangle + |0, DDS\rangle\right)$$

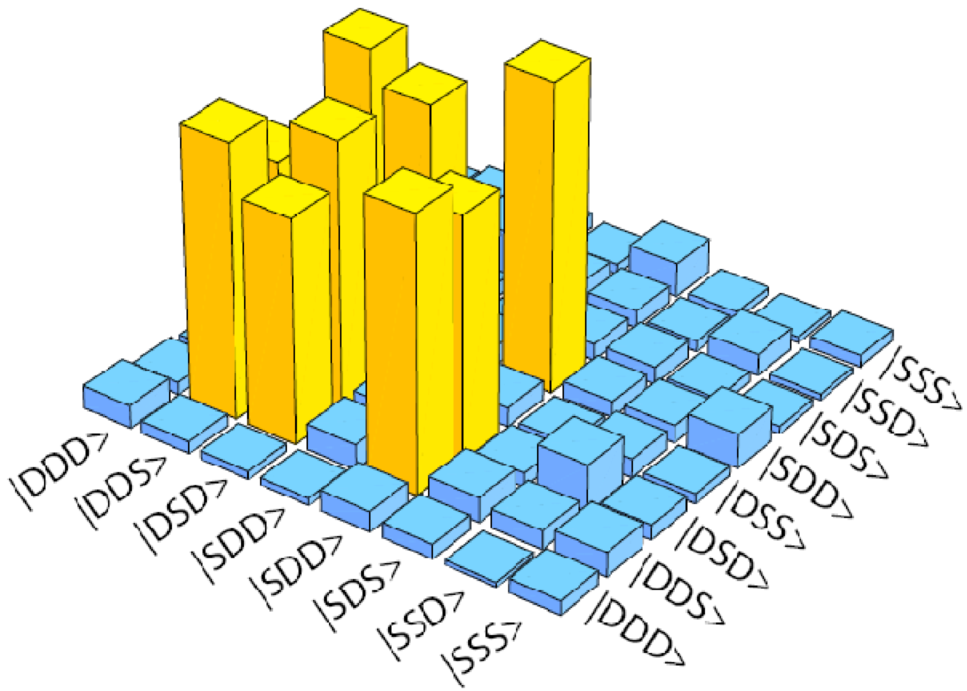
State tomography of GHZ and W states

C. Roos et al., Science **304**, 1478 (2004)

GHZ - state



W - state



Fidelity: **78 %**

Fidelity: **85 %**

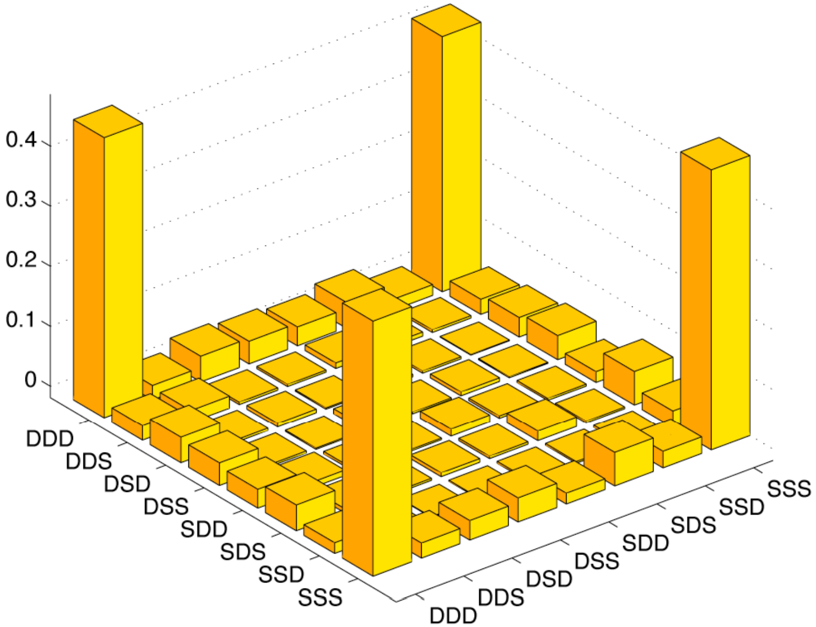
reconstruction time: ~ 200 s

Scalable push-button generation of GHZ states

$$|\Psi\rangle_{\mathrm{GHZ}_4} = \frac{1}{\sqrt{2}}(|SSSS\rangle + |DDDD\rangle)$$

Fidelity

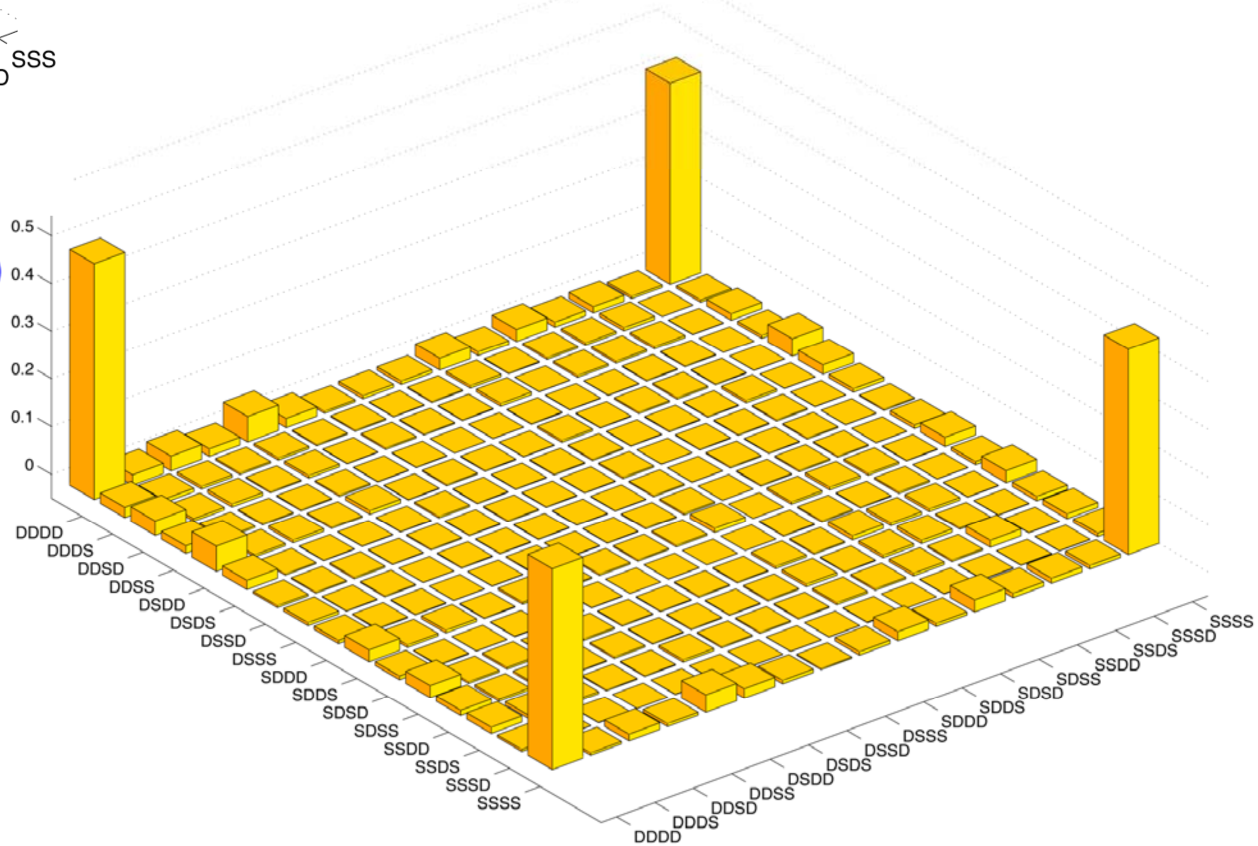
$$F_{\mathrm{GHZ}_4} = 88(1)\%$$



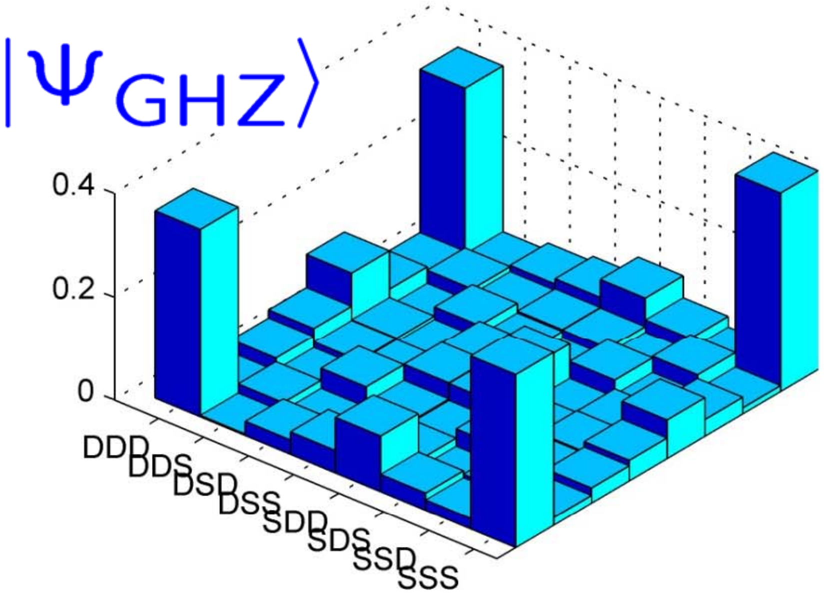
$$|\Psi\rangle_{\mathrm{GHZ}_3} = \frac{1}{\sqrt{2}}(|SSS\rangle + |DDD\rangle)$$

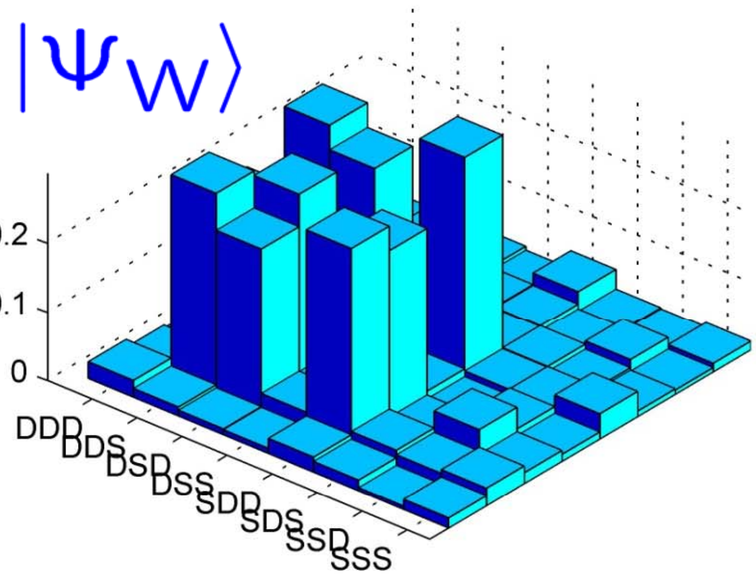
Fidelity

$$F_{\mathrm{GHZ}_3} = 89(1)\%$$



Measuring GHZ and W states

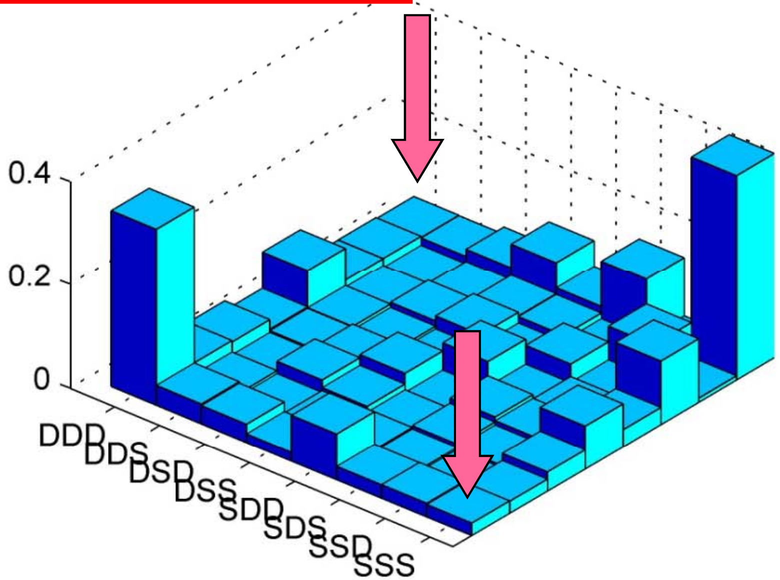


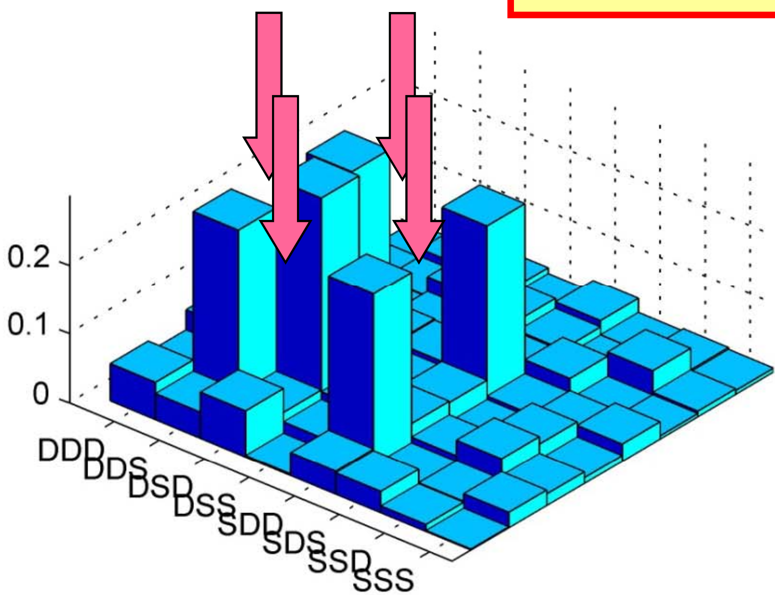


coherence destroyed

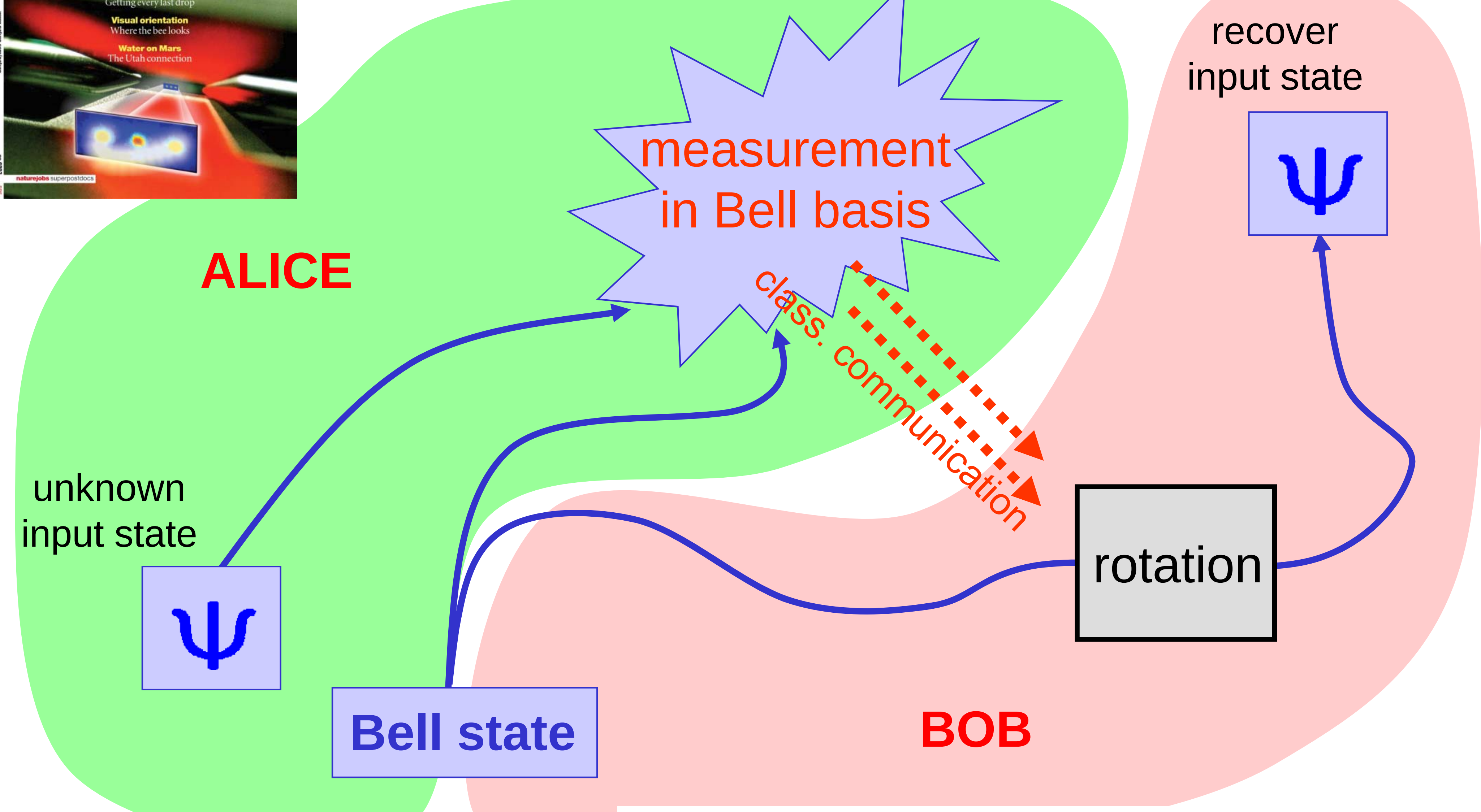
projection of the center ion

Bell state survives !





Teleportation



M. Riebe et al., Nature **429**, 734 (2004)

M.D. Barrett et al., Nature **429**, 737 (2004)

Teleportation

M. Riebe et al., New J. Phys. **9**, 211(2007)

Alice‘s input state $|\Psi\rangle_{\text{in}} = \alpha|0\rangle + \beta|1\rangle$

Alice and Bob share the state $|\Psi\rangle^{+} = \frac{1}{\sqrt{2}}(|0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B)$

joint three-qubit quantum state can be written as

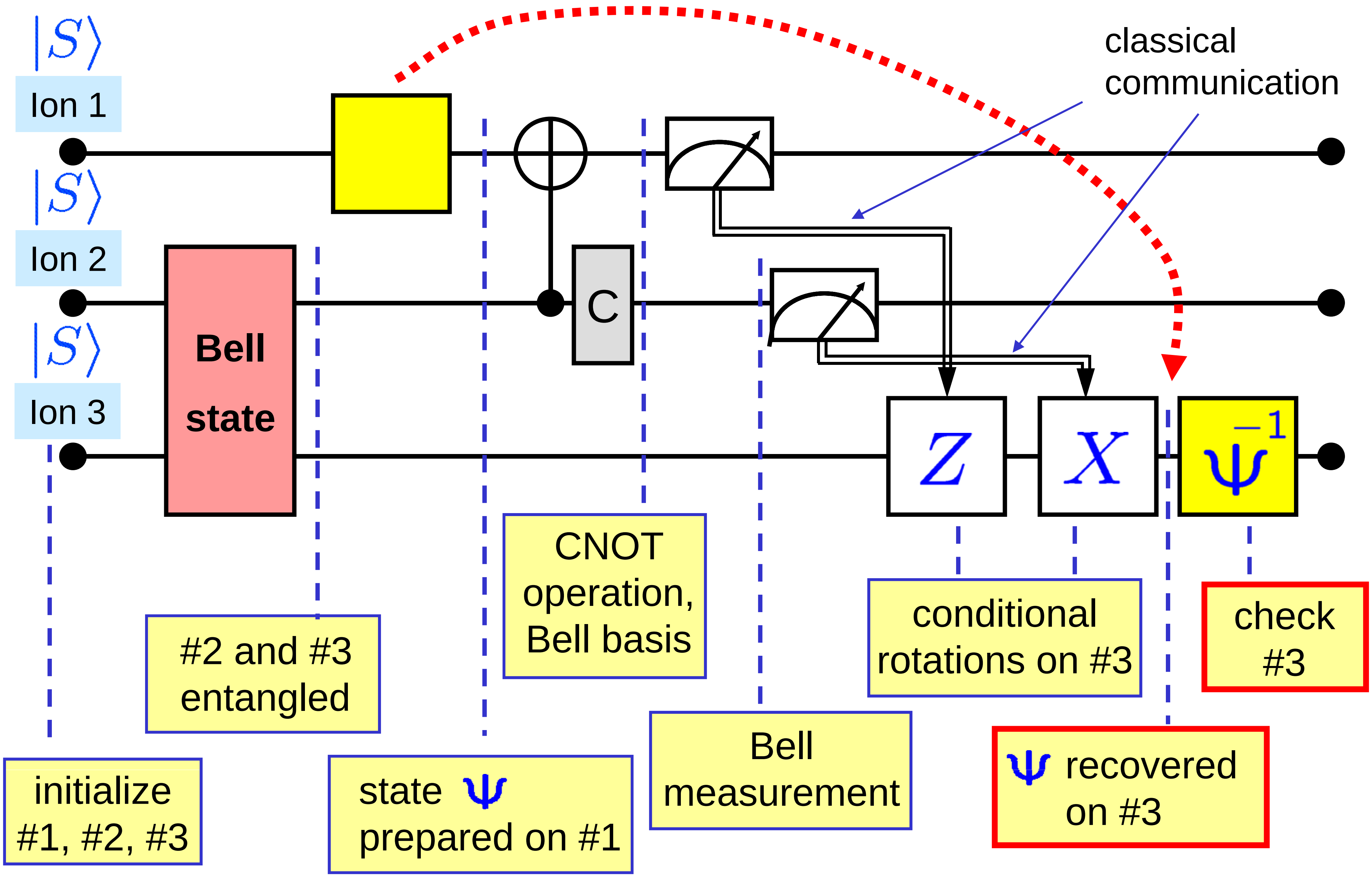
$$|\Psi\rangle_{AB} = \frac{1}{\sqrt{2}}(\alpha|00\rangle_A|1\rangle_B + \beta|10\rangle_A|1\rangle_B + \alpha|01\rangle_A|0\rangle_B + \beta|11\rangle_A|0\rangle_B)$$

rearrange

$$|\Psi\rangle_{AB} = \frac{1}{2}(\Phi_A^+ \underbrace{(\alpha|1\rangle + \beta|0\rangle)_B}_{\sigma_x \cdot \Psi_{in}} + \Phi_A^- \underbrace{(\alpha|1\rangle - \beta|0\rangle)_B}_{\sigma_z \cdot \sigma_x \cdot \Psi_{in}} + \Psi_A^+ \underbrace{(\alpha|0\rangle + \beta|1\rangle)_B}_{\Psi_{in}} + \Psi_A^- \underbrace{(\beta|1\rangle - \alpha|0\rangle)_B}_{-\sigma_z \cdot \Psi_{in}})$$

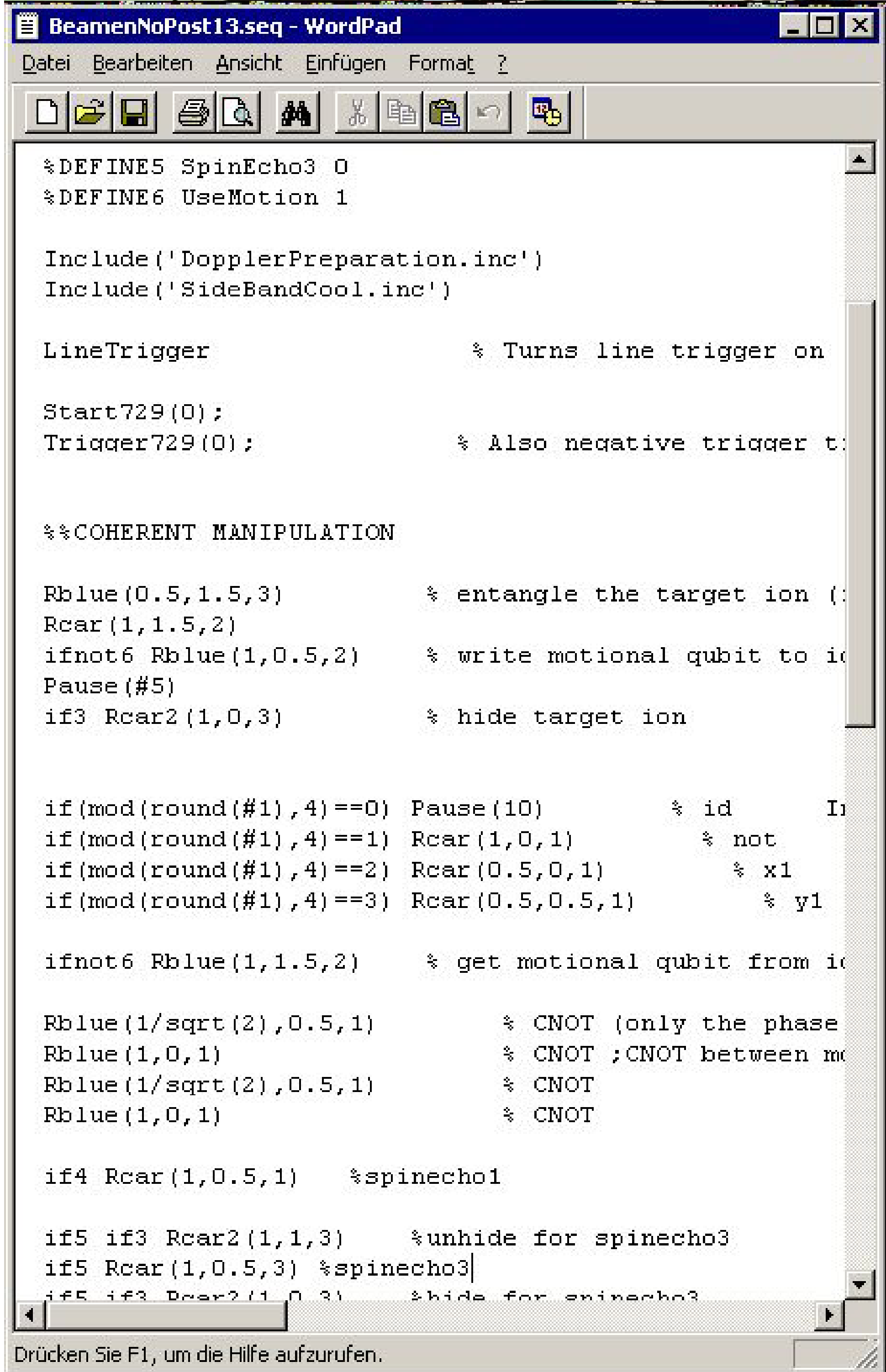
with $\Psi^{\pm} = (|10\rangle \pm |01\rangle)/\sqrt{2},\ \Phi^{\pm} = (|00\rangle \pm |11\rangle)/\sqrt{2}$

Quantum teleportation protocol

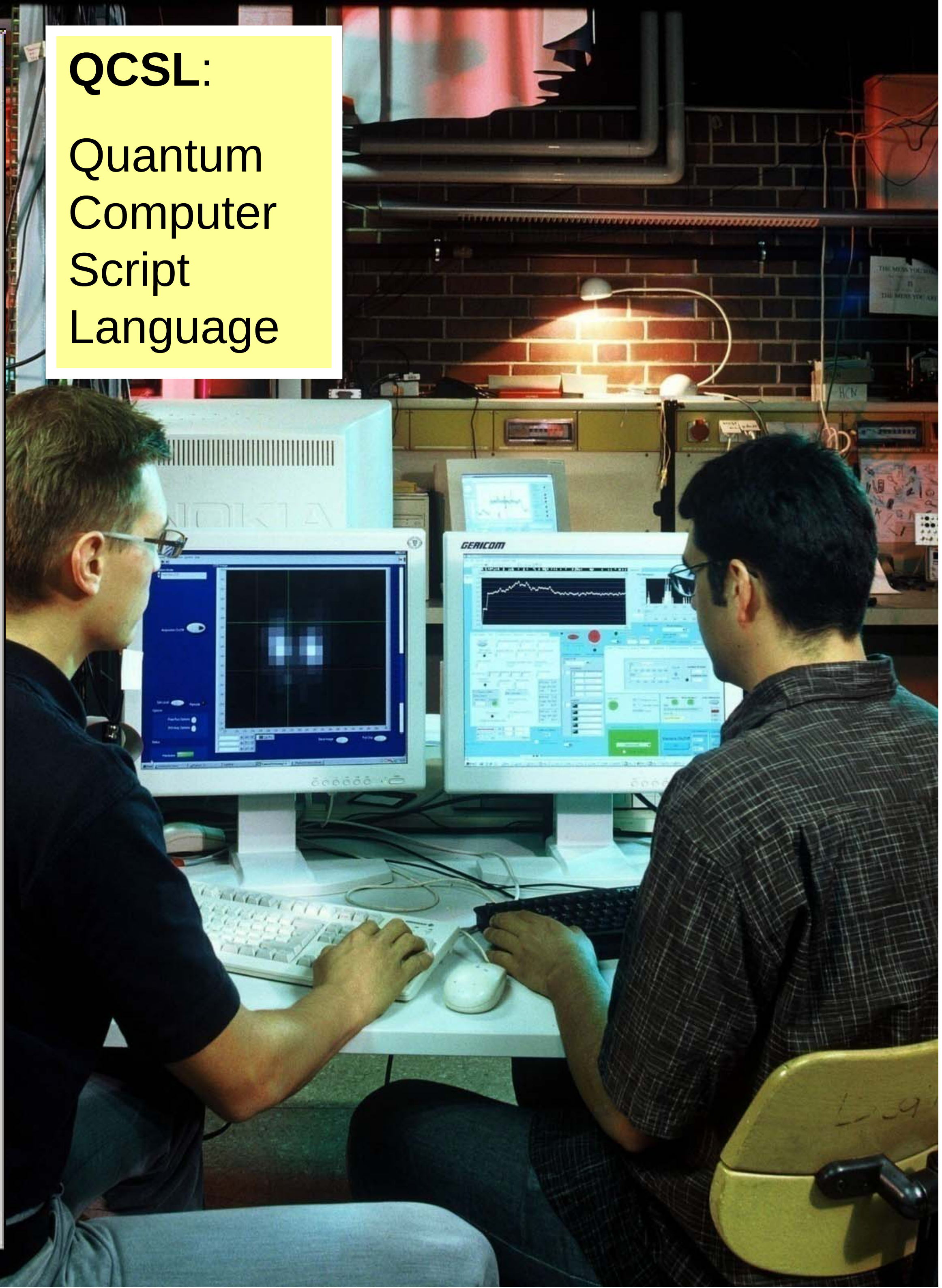


QCSL:

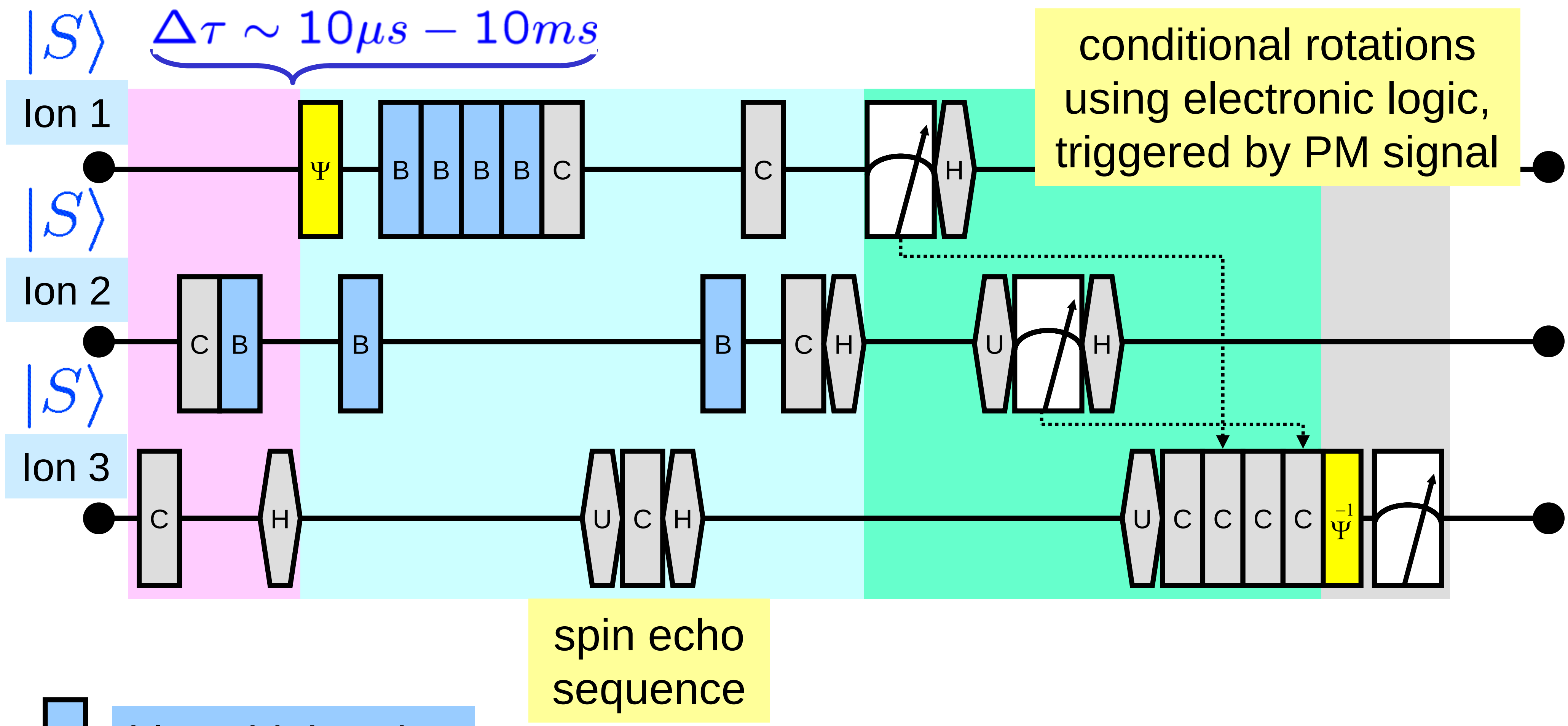
Quantum
Computer
Script
Language




```
%DEFINE5 SpinEcho3 0
%DEFINE6 UseMotion 1

Include('DopplerPreparation.inc')
Include('SideBandCool.inc')

LineTrigger                     % Turns line trigger on

Start729(0);
Trigger729(0);                 % Also negative trigger t:


%%COHERENT MANIPULATION

Rblue(0.5,1.5,3)          % entangle the target ion (:
Rcar(1,1.5,2)
ifnot6 Rblue(1,0.5,2)     % write motional qubit to ic
Pause(#5)
if3 Rcar2(1,0,3)          % hide target ion


if(mod(round(#1),4)==0) Pause(10)          % id       I:
if(mod(round(#1),4)==1) Rcar(1,0,1)          % not
if(mod(round(#1),4)==2) Rcar(0.5,0,1)          % x1
if(mod(round(#1),4)==3) Rcar(0.5,0.5,1)          % y1

ifnot6 Rblue(1,1.5,2)     % get motional qubit from ic

Rblue(1/sqrt(2),0.5,1)         % CNOT (only the phase
Rblue(1,0,1)                   % CNOT ;CNOT between mc
Rblue(1/sqrt(2),0.5,1)         % CNOT
Rblue(1,0,1)                   % CNOT

if4 Rcar(1,0.5,1)    %spinecho1

if5 if3 Rcar2(1,1,3)     %unhide for spinecho3
if5 Rcar(1,0.5,3) %spinecho3
if5 if3 Rcar2(1,0,3)     %hide for spinecho3
```

Quantum teleportation protocol, details



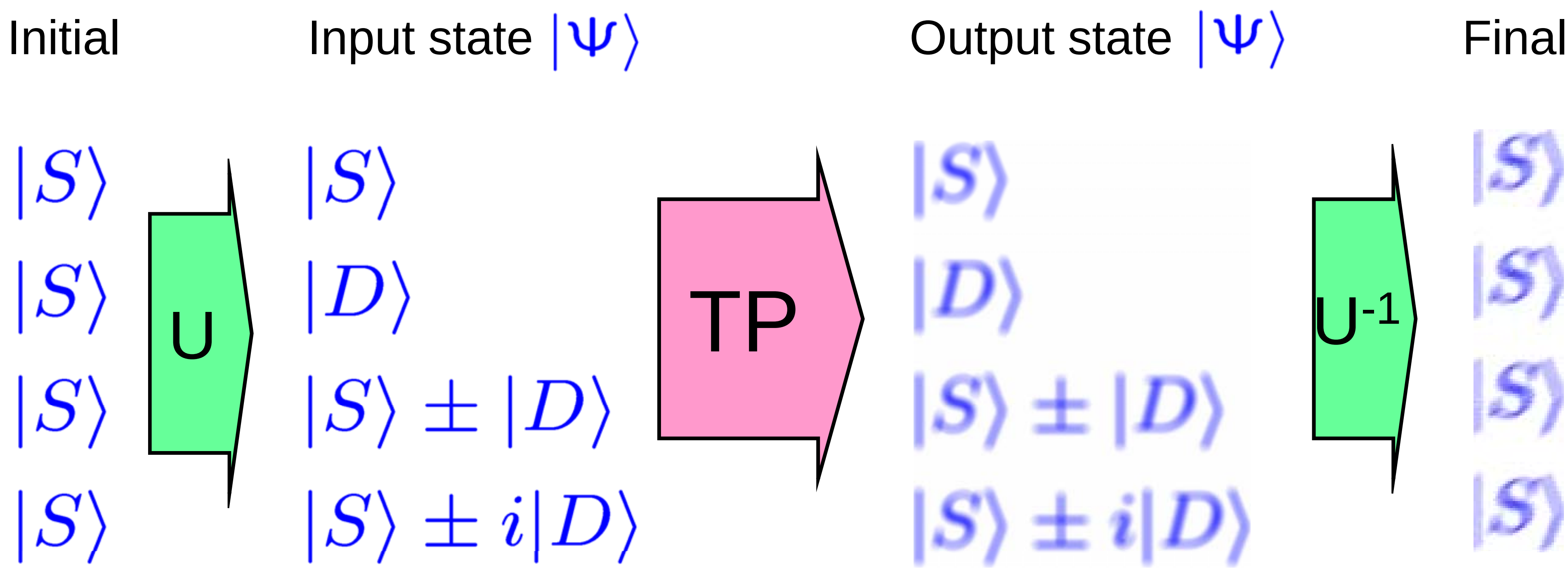
B: blue sideband pulses $R^{+}(\theta, \varphi)$

C: carrier pulses $R(\theta, \varphi)$

$\tau \approx 2ms$

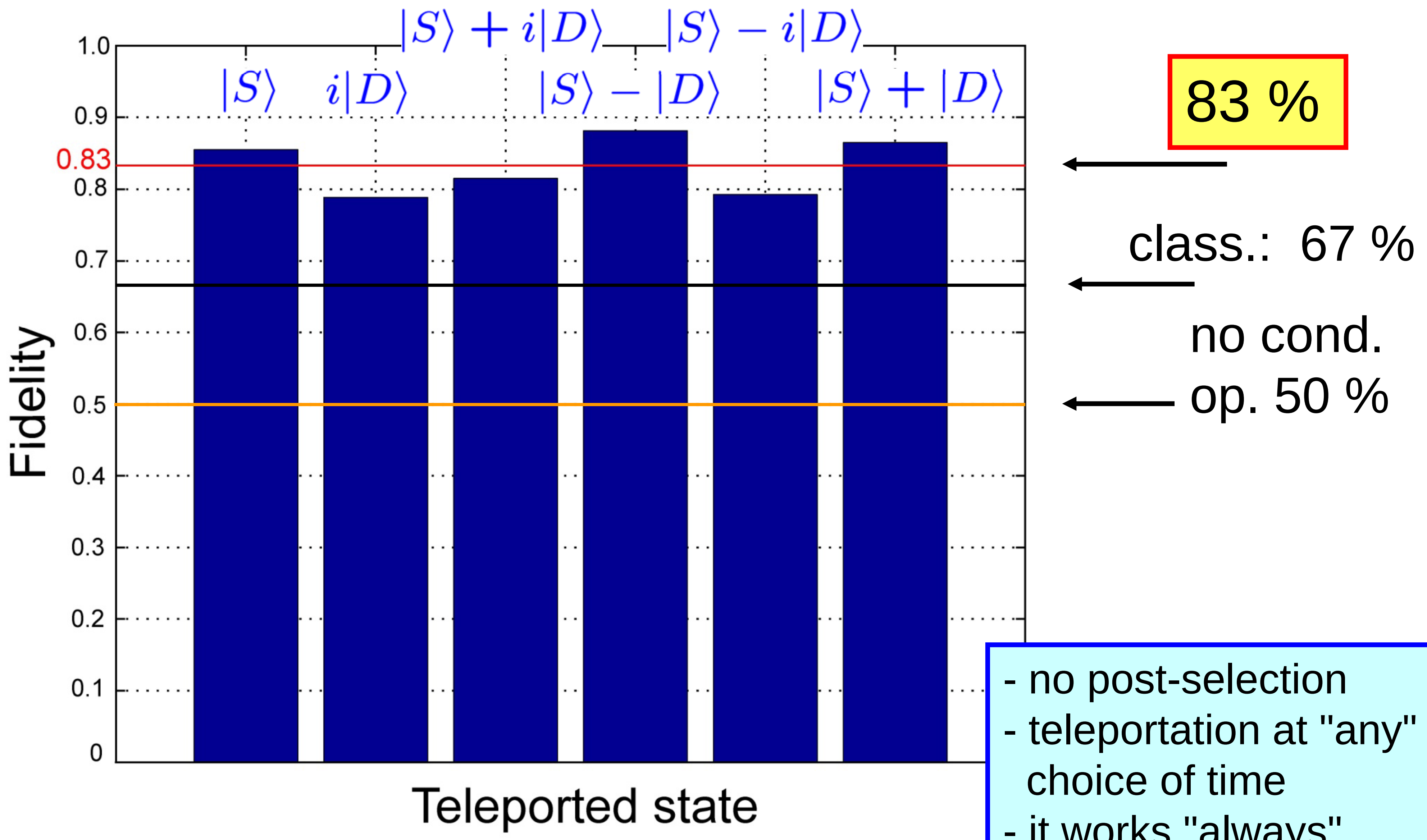
full sequence:
26 pulses + 2 measurements

Teleportation procedure, analysis



→ **Fidelities**

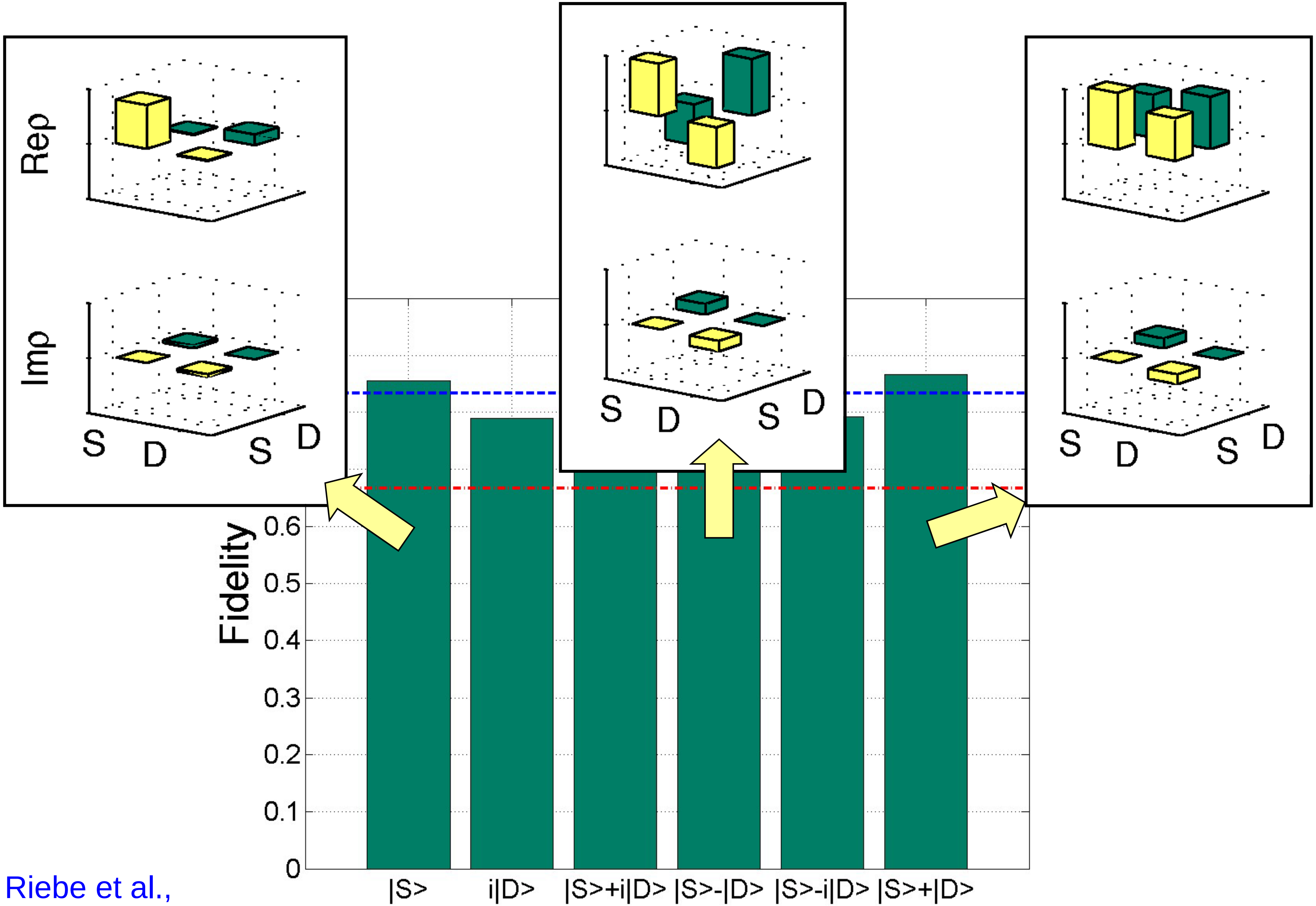
Teleportation with atoms: results



- no post-selection
- teleportation at "any" choice of time
- it works "always"
- only 10 μm

M. Riebe et al., New J. Phys. **9**, 211(2007)

Process tomography of quantum teleportation

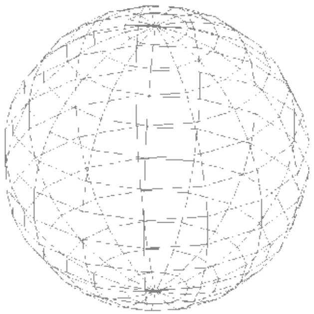


M. Riebe et al.,
New J. Phys. **9**, 211(2007)

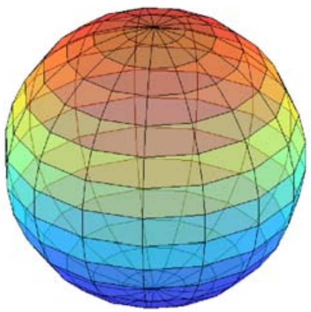
Process tomography of quantum teleportation

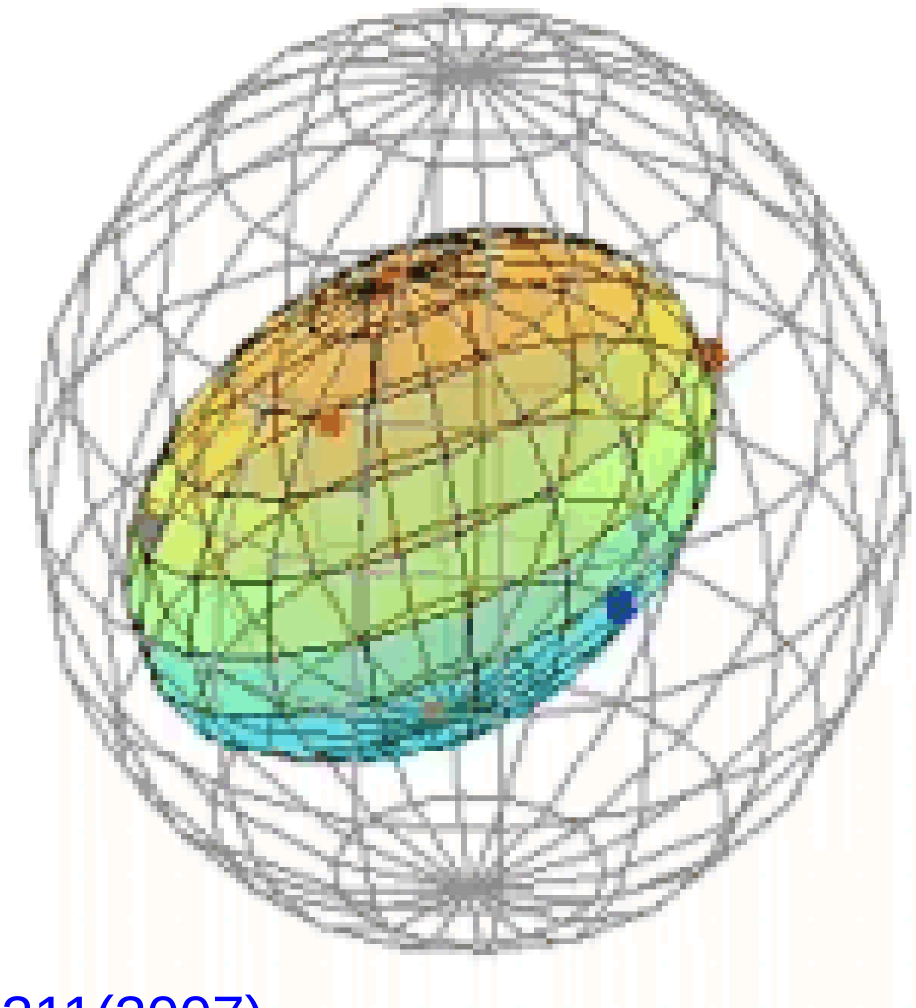
represent input/output states with Bloch spheres:

input sphere



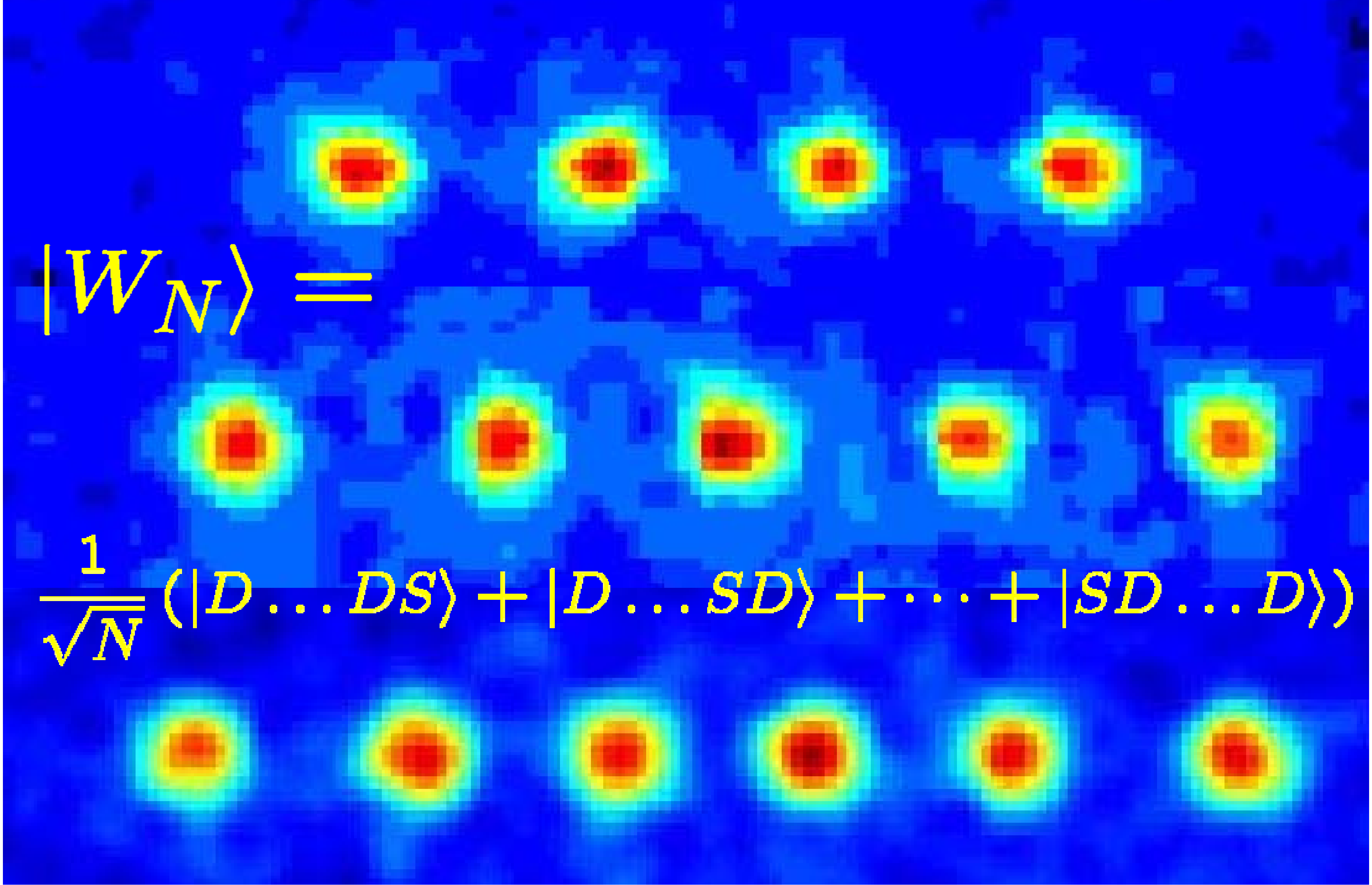
output sphere





M. Riebe et al., New J. Phys. **9**, 211(2007)

Multi - particle entanglement

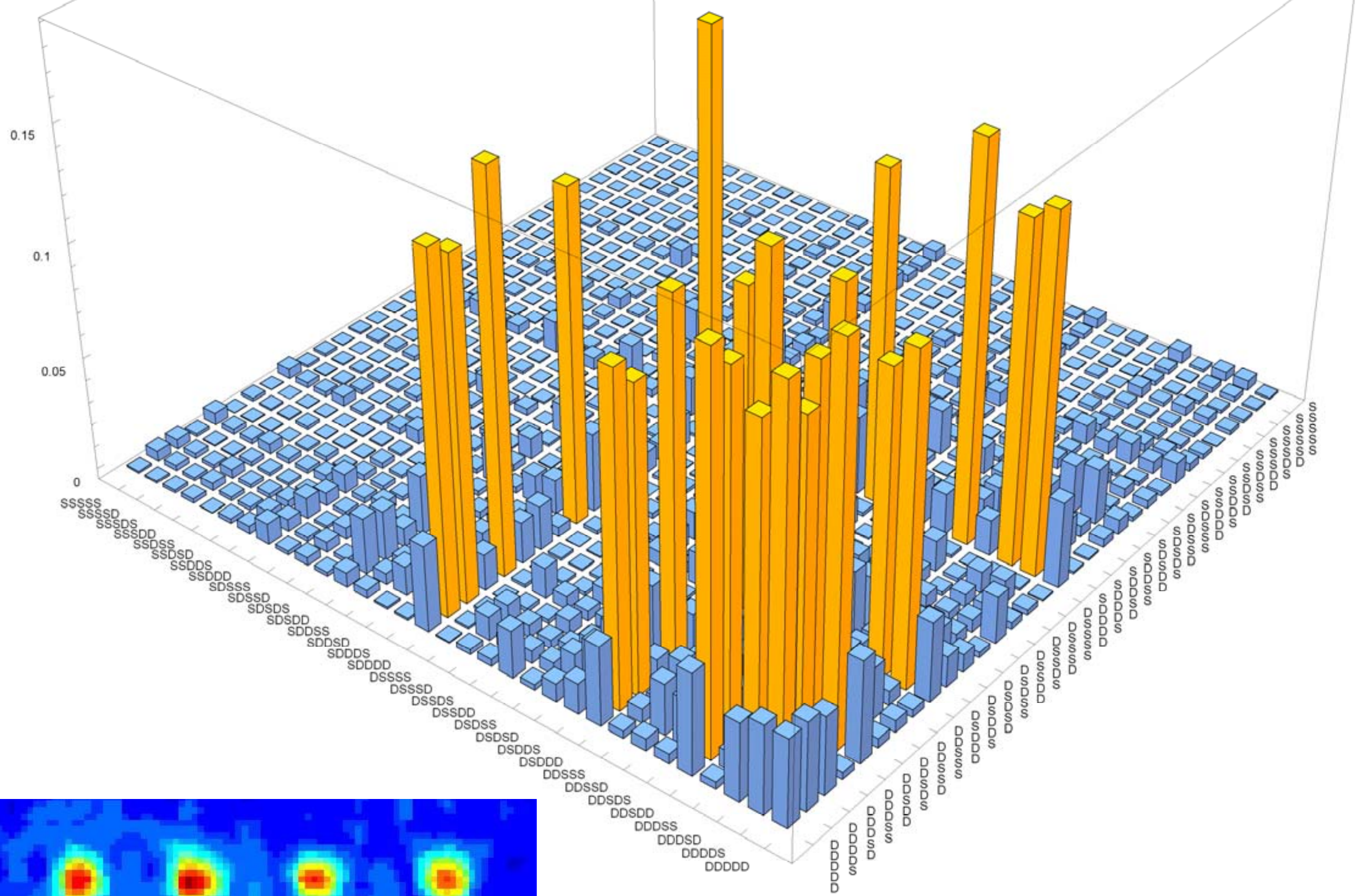


$$|W_N\rangle = \frac{1}{\sqrt{N}}\left(|D \ldots DS\rangle + |D \ldots SD\rangle + \cdots + |SD \ldots D\rangle\right)$$

Five-ion W state

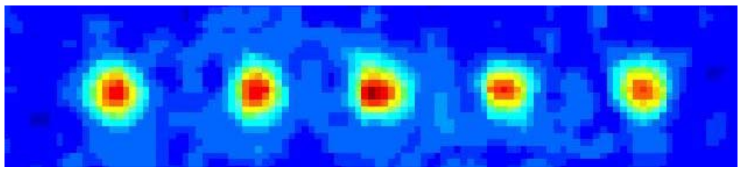
$$\Psi_5 = \frac{1}{\sqrt{5}}(|DDDDS\rangle + |DDDSD\rangle + |DDSDD\rangle + \\ + |DSDDD\rangle + |SDDDD\rangle)$$

$$|\rho_{ij}|$$



Fidelity:
76 %

15.4.2005



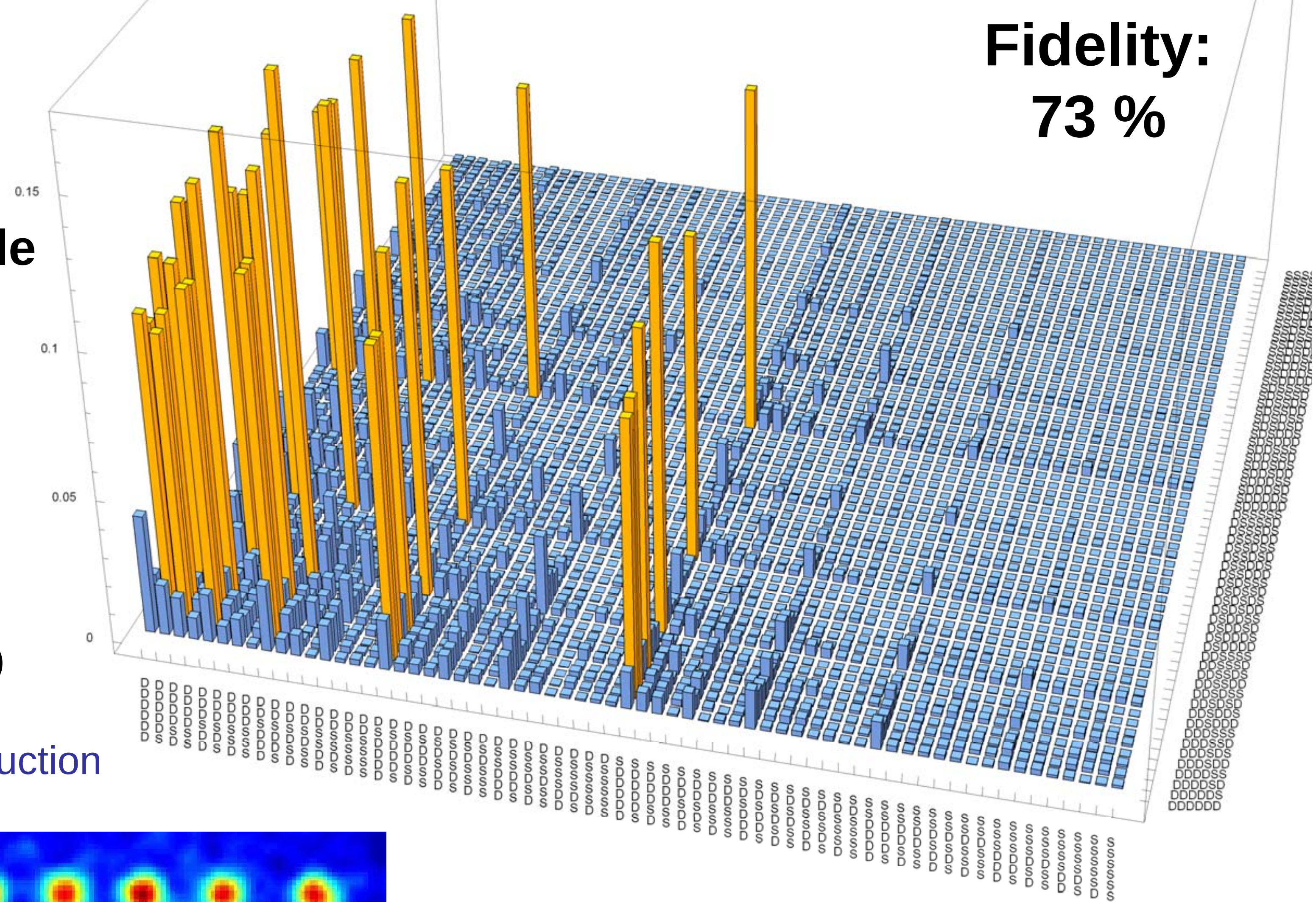
measurement time ~ 11 min.

Six-Ion W-state

$$\Psi_6 = \frac{1}{\sqrt{6}}(|DDDDDS\rangle + |DDDDSD\rangle + |DDDSDD\rangle + |DDSDDD\rangle + |DSDDDD\rangle + |SDDDDD\rangle)$$

$$|\rho_{ij}|$$

Fidelity: 73 %

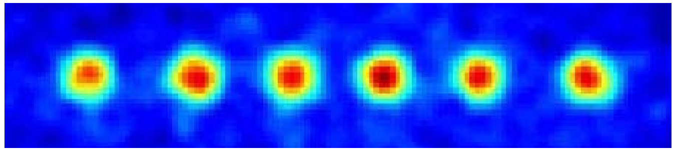


Genuine 6-particle entanglement !

- 6-particle entanglement can be distilled from the state (W. Dür)
- Entanglement witness detects 6-particle entanglement (O. Gühne)

- error bars in the reconstruction process ?

22.4.2005

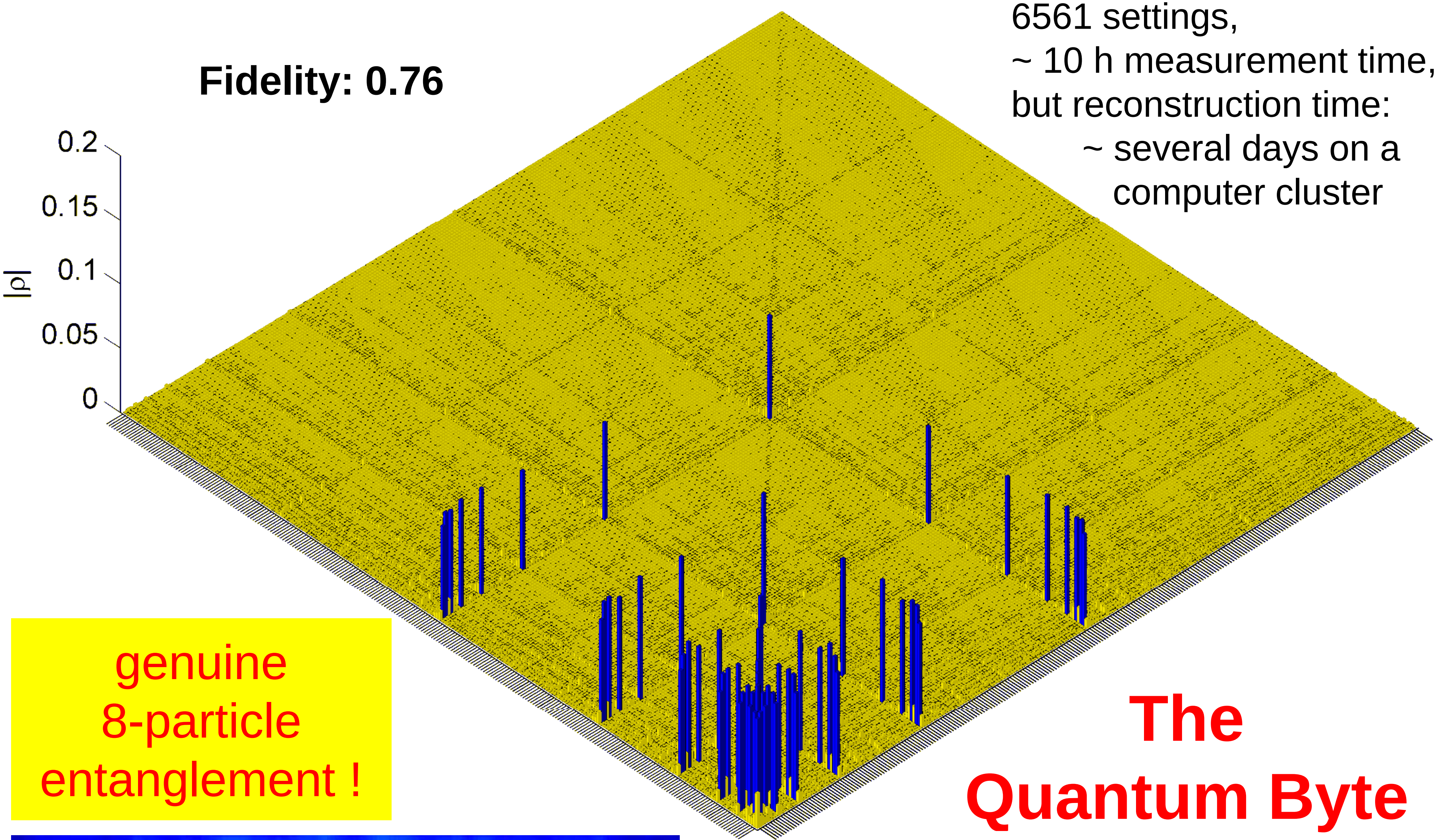


729 settings, measurement time ~ 40 min.

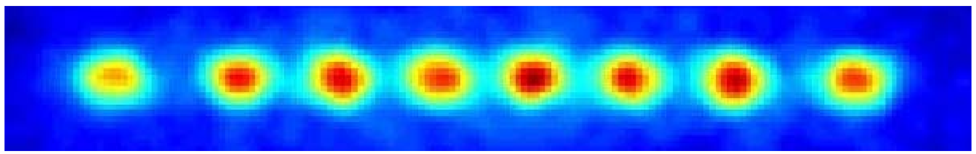
Eight entangled qubits: a quantum byte

Fidelity: 0.76

6561 settings,
~ 10 h measurement time,
but reconstruction time:
~ several days on a computer cluster



genuine
8-particle
entanglement !



The Quantum Byte

H. Häffner et al., Nature **438**, 643 (2005)

Scaling the ion trap quantum computer …

- more ions, larger traps, phonons carry quantum information

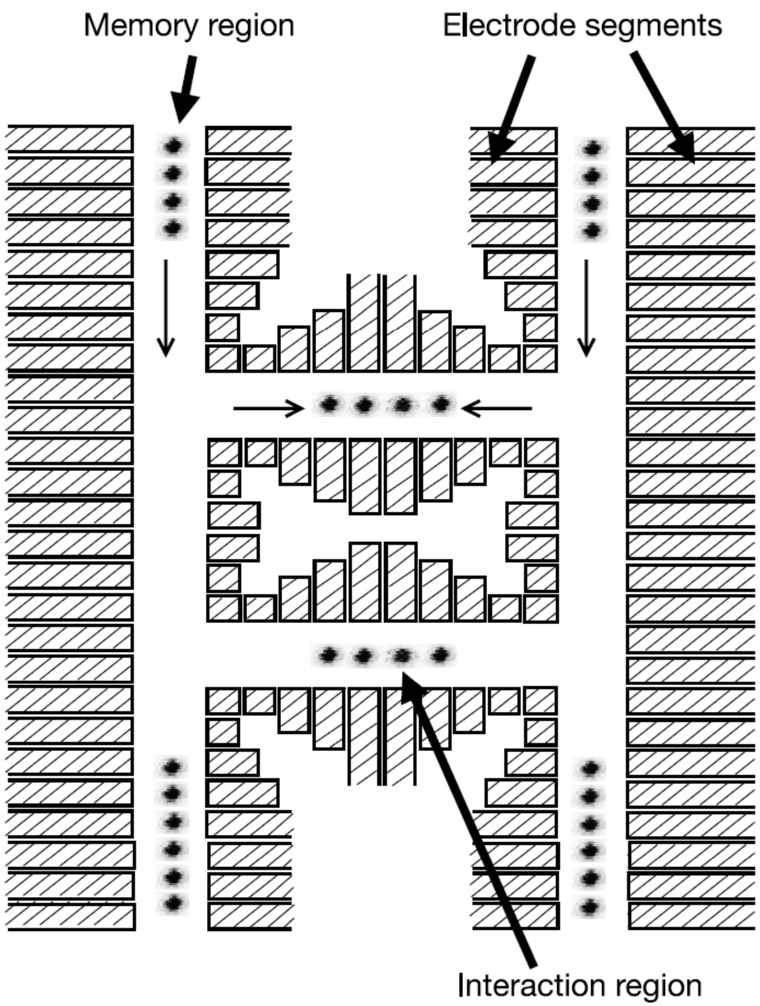
  Cirac-Zoller, slow for many ions (few 10 ions maybe possible)

- move ions, carry quantum information around

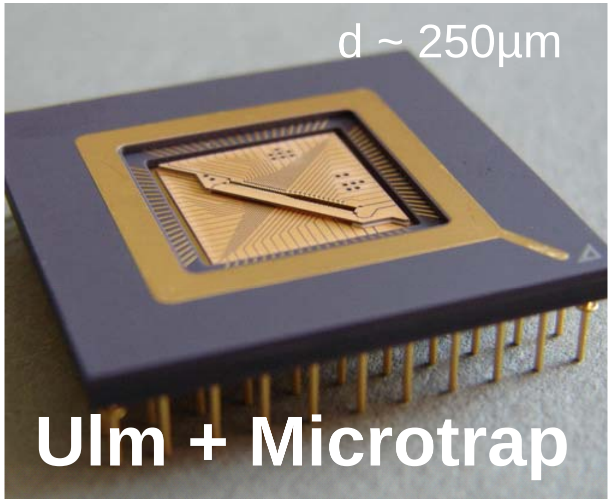
  Kielpinski et al.,
  Nature **417**, 709 (2002)

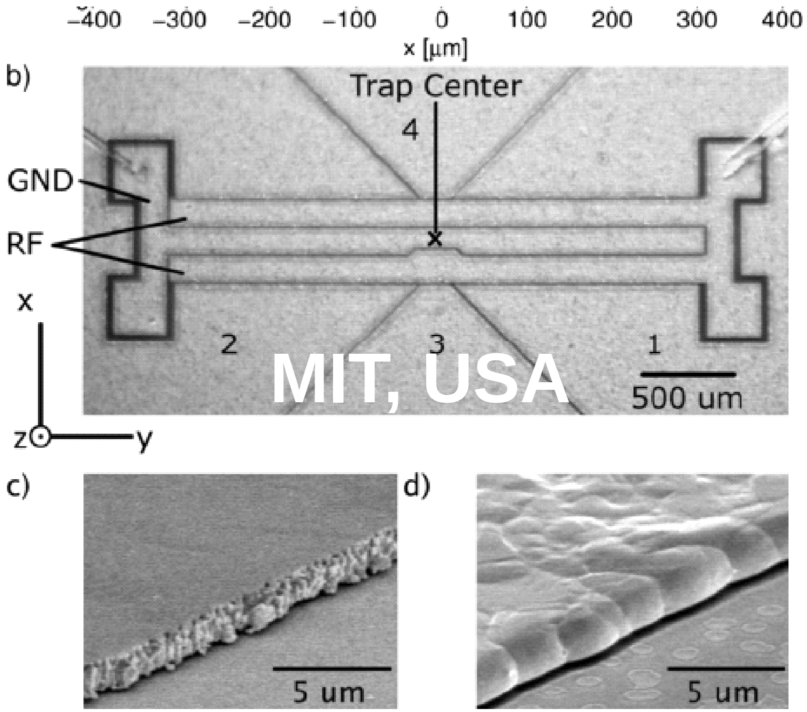
  requires small,
  integrated trap structures,

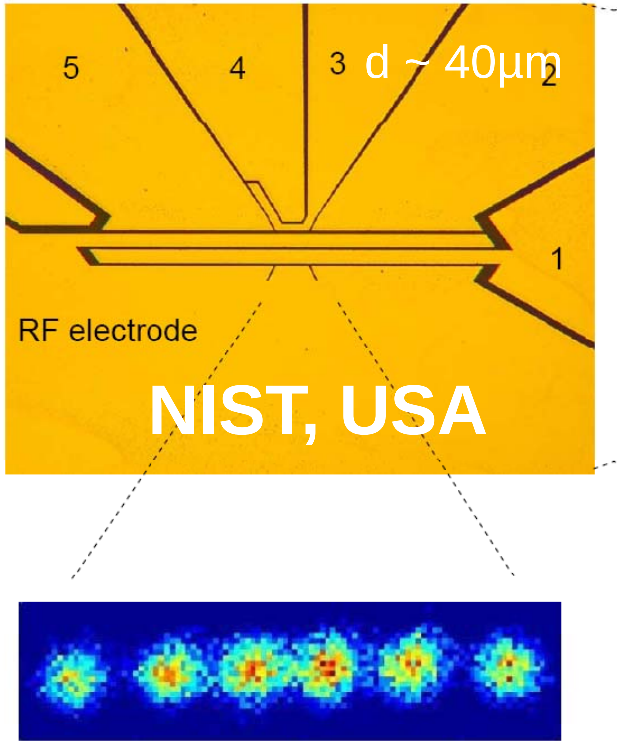
  miniaturized optics
  and electronics

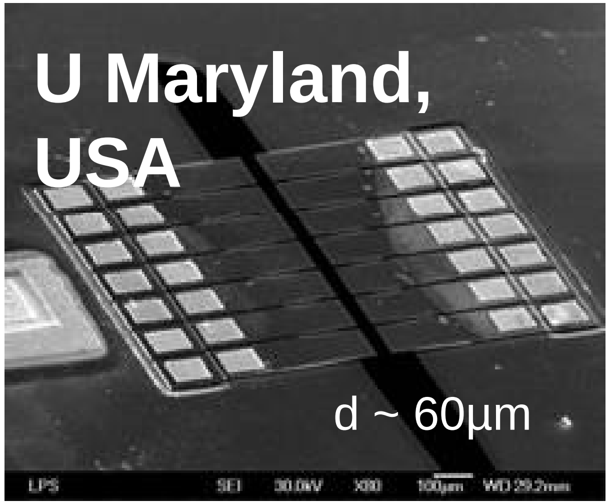


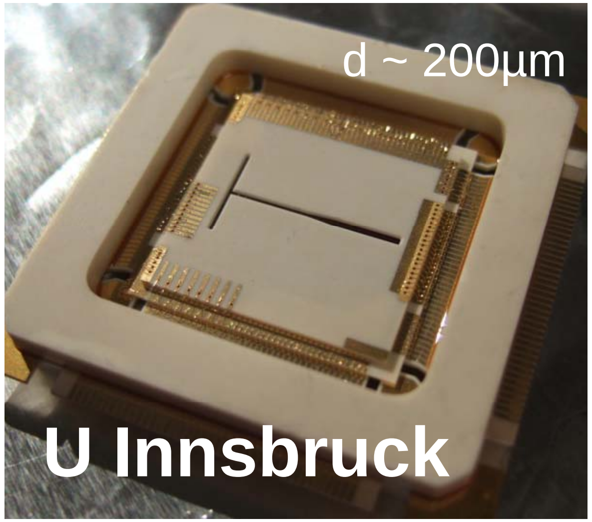
Ion Chip Trap Quantum Microprocessors: worldwide efforts





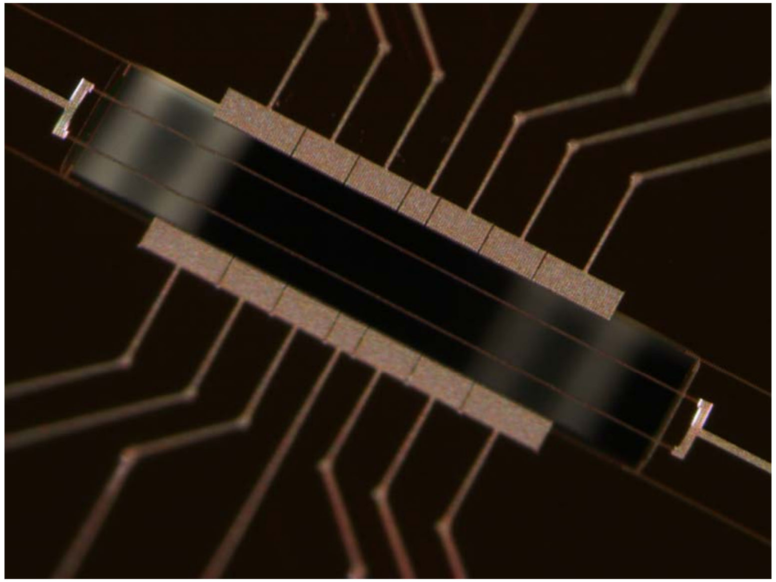


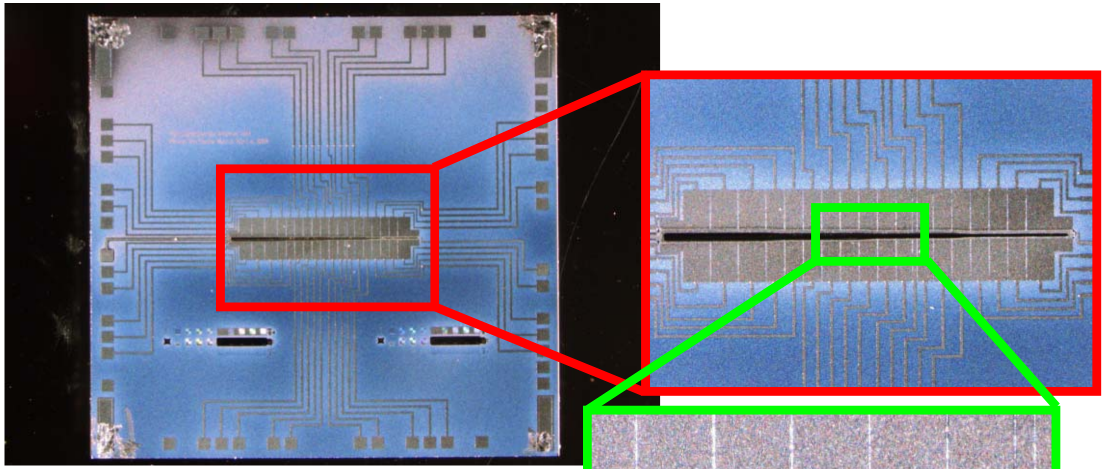


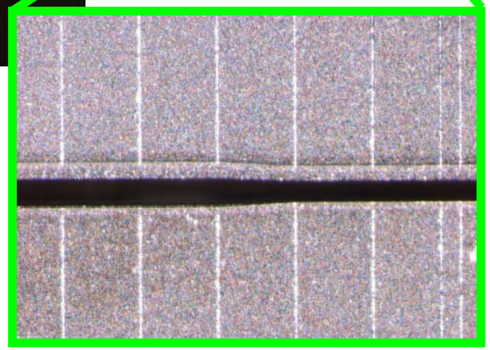


Chip traps in Innsbruck (2007)

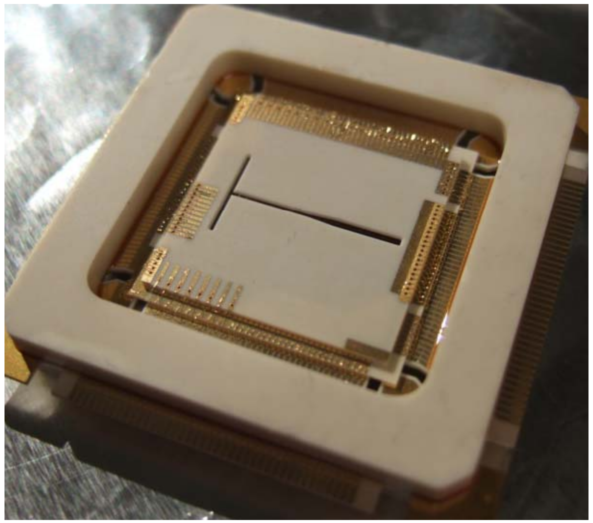
Sandia: planar trap structure





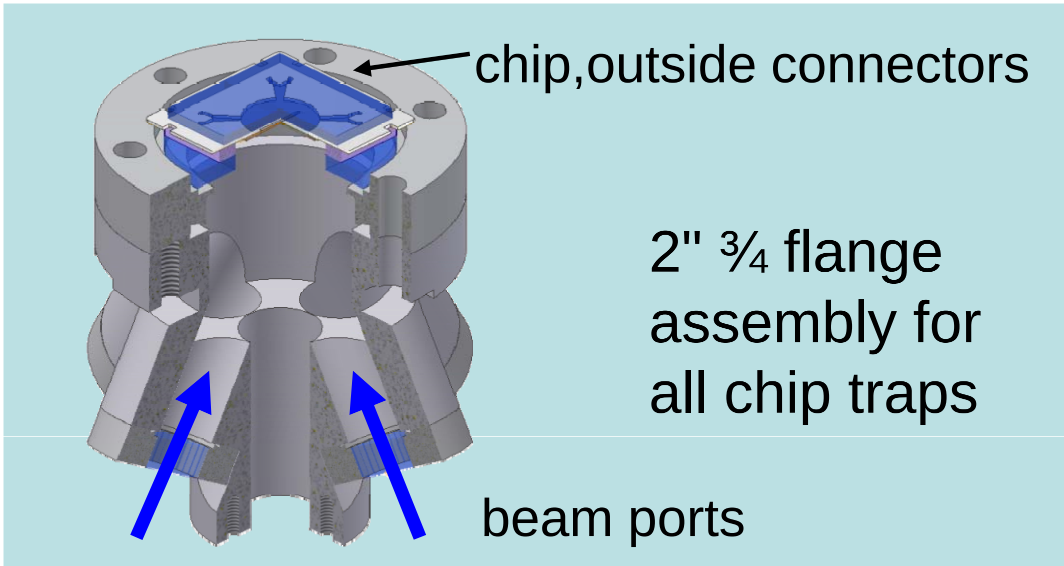


Lucent:
planar trap structure



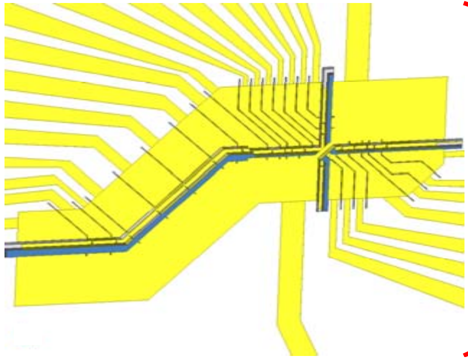
Innsbruck
design

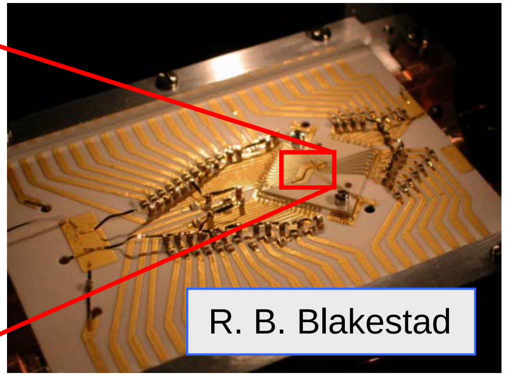
IOF
T - trap



Advanced chip traps at NIST

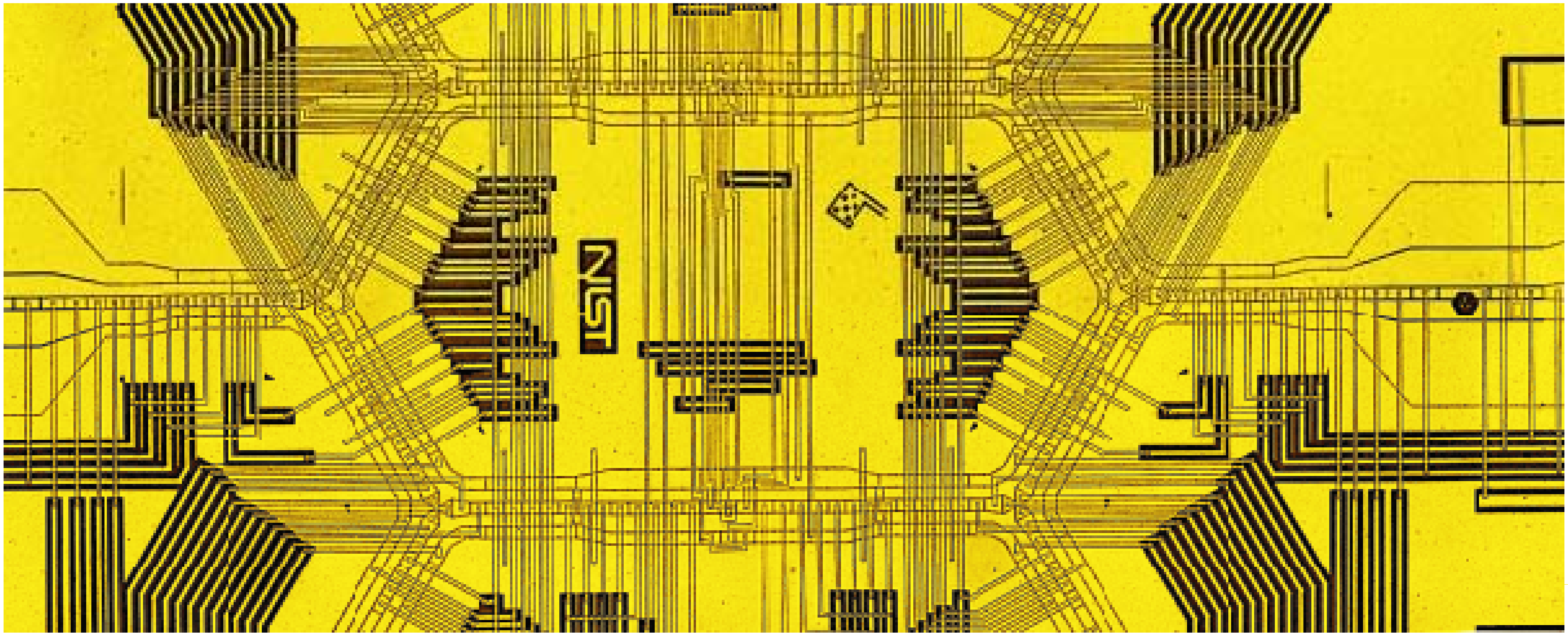
2-layer, 2-D, X-junction, 18 zones (Au on Al_2O_3)





- Transport through junction ($^9Be^+$,$^{24}Mg^+$)
  - ◊ minimal heating ~ 20 quanta
  - ◊ transport error < 3×10^{-6}

t

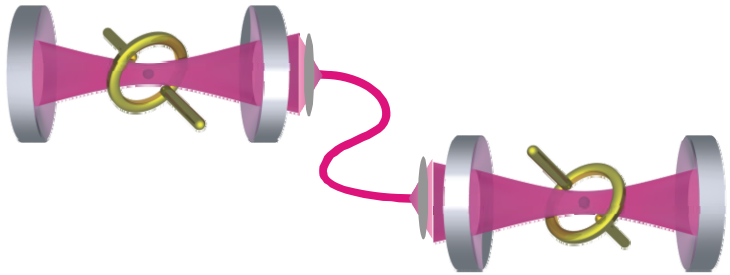


Scaling the ion trap quantum computer

- cavity QED: atom – photon interface, use photons for networking

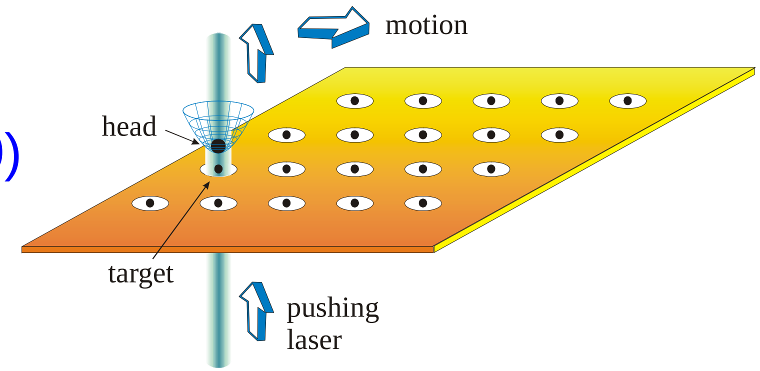
  J. I. Cirac et al., PRL **78**, 3221 (1997)

  P. Schmidt et al., Univ. Innsbruck



- trap arrays, using single ion as moving head

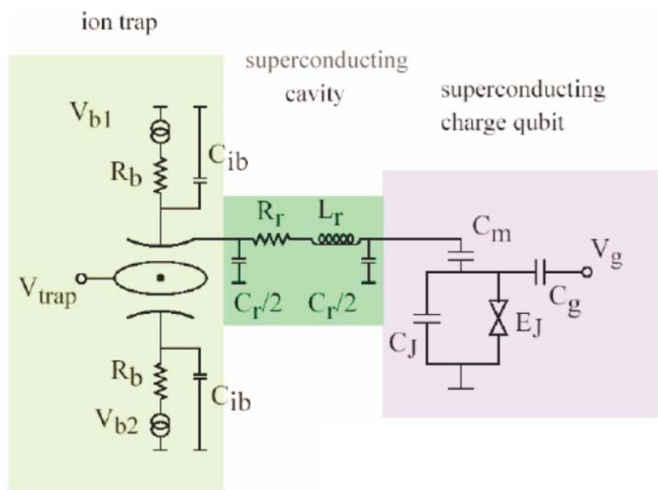
  I. Cirac und P. Zoller, Nature **404**, 579 (2000)



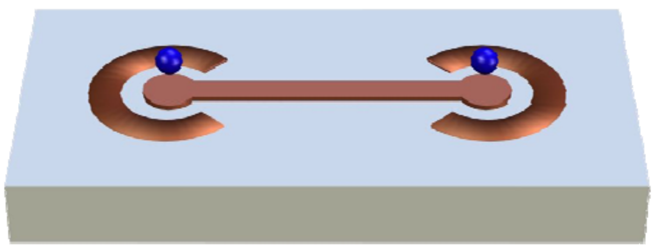
- ion – solid state qubits (e.g. charge qubit)

  L. Tian et al., PRL **92**, 247902 (2004)

  H. Häffner et al., Innsbruck



- …more ideas …?

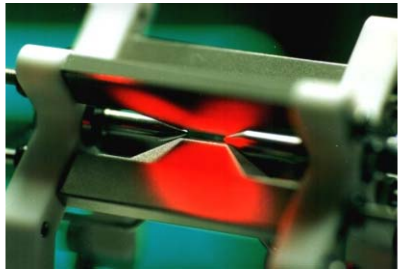


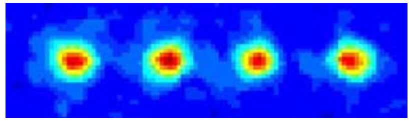
Quantum Computer – ion trap realization

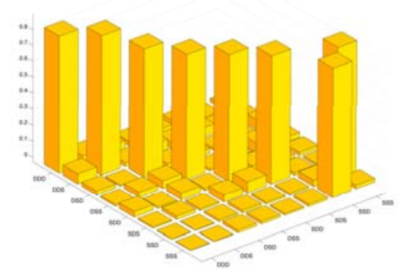
- Innsbruck: Ca^{+} experiments ($^{40}Ca^{+}$, $^{43}Ca^{+}$)
- Realization of two-ion Cirac-Zoller CNOT operation
- Quantum process tomography, gate operations
- Bell and GHZ measurements, tomography
- Multipartite entanglement, more gate operations
- Scaling the ion trap quantum computer

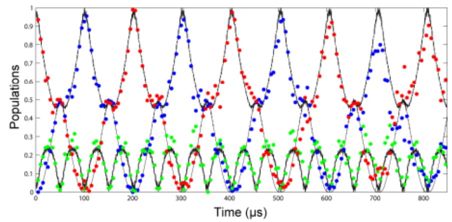
Future:

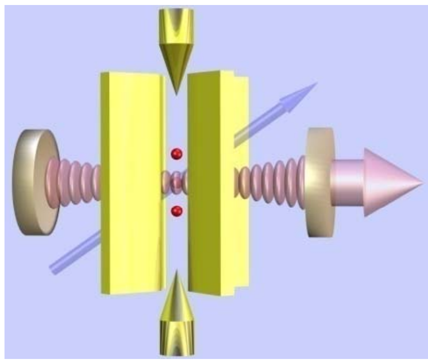
- further optimization of logic operations
- error correction protocols with three and five qubits
- implementation with $^{43}Ca^{+}$, logical qubits + scalability
- miniaturize traps, interface quantum information
- applications: metrology, quantum simulations



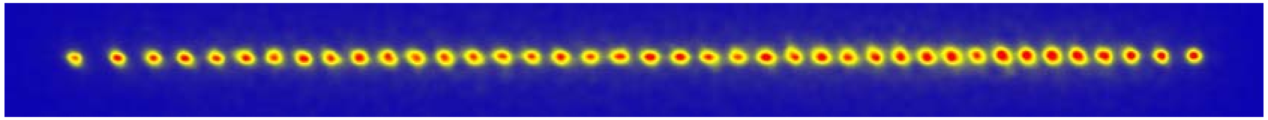








Future goals and developments



- more qubits (~20 – 50)
- better fidelities
- faster gate operations
- faster detection

} cryogenic trap, micro-structured traps

- development of 2-d trap arrays, onboard addressing, electronics etc.
- entangling of large(r) systems: characterization ?
- implementation of error correction, keep **„qubit alive“**
- applications
  - small scale QIP (e.g. repeaters)
  - quantum metrology, enhanced S/N, tailored atoms and states
  - quantum simulations
  - quantum computation

AG Quantenoptik und Spektroskopie

The international Team 2009

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