

Lectures on Supersymmetry Breaking

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We review the subject of spontaneous supersymmetry breaking. First we consider supersymmetry breaking in a semiclassical theory. We illustrate it with several examples, demonstrating different phenomena, including metastable supersymmetry breaking. Then we give a brief review of the dynamics of supersymmetric gauge theories. Finally, we use this dynamics to present various mechanisms for dynamical supersymmetry breaking. These notes are based on lectures given by the authors in 2007, at various schools.

1. Introduction

With the advent of the LHC it is time to review old model building issues leading to phenomena which could be discovered, or disproved, by the LHC. Supersymmetry (SUSY) is widely considered as the most compelling new physics that the LHC could discover. It gives a solution to the hierarchy problem, leads to coupling constant unification and has dark matter candidates.

Clearly, the standard model particles are not degenerate with their superpartners, and therefore supersymmetry should be broken. To preserve the appealing features of supersymmetry, this breaking must be spontaneous, rather than explicit breaking. This means that the Lagrangian is supersymmetric, but the vacuum state is not invariant under supersymmetry.

Furthermore, as was first suggested by Witten [1], we would like the mechanism which spontaneously breaks supersymmetry to be dynamical. This means that it arises from an exponentially small effect, and therefore it naturally leads to a scale of supersymmetry breaking, M_s , which is much smaller than the high energy scales in the problem M_{cutoff} (which can be the Planck scale or the grand unified scale):

$$M_s = M_{cutoff} e^{-c/g(M_{cutoff})^2} \ll M_{cutoff}. \quad (1.1)$$

This can naturally lead to hierarchies. For example, the weak scale m_W can be dynamically generated, explaining why $m_W/m_{Pl} \sim 10^{-17}$.

In these lectures, we will focus on the key conceptual issues and mechanisms for supersymmetry breaking, illustrating them with the simplest examples. We will not discuss more detailed model building questions, such as the question of how the supersymmetry breaking is mediated to the MSSM, and what the experimental signatures of the various mediation schemes are. These are very important topics, which deserve separate sets of lectures. Also, we will not discuss supersymmetry breaking by Fayet-Iliopoulos terms [2].

We will assume that the readers (and audience in the lectures) have some basic familiarity with supersymmetry. Good textbooks are [3-7].

As seen from the supersymmetry algebra,

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2P_{\alpha\dot{\alpha}}, \quad (1.2)$$

the vacuum energy

$$\langle \psi | \mathcal{H} | \psi \rangle \propto \sum_{\alpha} |Q_{\alpha} | \psi \rangle|^2 + \sum_{\dot{\alpha}} | \bar{Q}_{\dot{\alpha}} | \psi \rangle|^2 \geq 0 \quad (1.3)$$

is an order parameter for supersymmetry breaking. Supersymmetry is spontaneously broken if and only if the vacuum has non-zero energy¹,

$$V_{vac} = M_s^4. \tag{1.4}$$

In the case of dynamical supersymmetry breaking (DSB), the scale M_s is generated by dimensional transmutation, as in (1.1).

As with the spontaneous breaking of an ordinary global symmetry, the broken supersymmetry charge Q does not exist in an infinite volume system. Instead, the supersymmetry current S exists, and its action on the vacuum creates a massless particle – the Goldstino. (The supercharge tries to create a zero momentum Goldstino, which is not normalizable.) In the case of supergravity, where the symmetry (1.2) is gauged, we have the standard Higgs mechanism and the massless Goldstino is “eaten” by the gravitino.

There are many challenges in trying to implement realistic realizations of dynamical supersymmetry breaking. A first challenge, which follows from the Witten index [8], is that dynamical supersymmetry breaking, where the true vacuum is static and has broken supersymmetry, seems non-generic, requiring complicated looking theories. On the other hand, accepting the possibility that we live in a metastable vacuum improves the situation. As even very simple theories can exhibit metastable dynamical supersymmetry breaking, it could be generic [9]. (Particular models of metastable supersymmetry breaking have been considered long ago, e.g. a model [10], which we review below.)

Another challenge is the relation [11] between R-symmetry and broken supersymmetry. Generically, there is broken supersymmetry if and only if there is an R-symmetry. As we will also discuss, there is broken supersymmetry in a metastable state if and only if there is an approximate R-symmetry. For building realistic models, an unbroken R-symmetry is problematic. It forbids Majorana gaugino masses. Having an exact, but spontaneously broken R-symmetry is also problematic, it leads to a light R-axion (though including gravity can help²). We are thus led to explicitly break the R-symmetry. Ignoring gravity, this then means that we should live in a metastable state!

¹ In these lectures we focus on global SUSY, $M_{pl} \rightarrow \infty$. In supergravity we can add an arbitrary negative constant to the vacuum energy, via $\Delta W = const$, so the cosmological constant can still be tuned to the observed value.

² Including gravity, the R-symmetry needs to be explicitly broken, in any case, by the $\Delta W = const.$, needed to get a realistic cosmological constant. It is possible that this makes the R-axion sufficiently massive [12].

The outline of these lectures is as follows. In the next section, we consider theories in which the supersymmetry breaking can be seen semiclassically. Such theories can arise as the low energy theory of another microscopic theory. Various general points about supersymmetry breaking (or restoration) are illustrated, via several simple examples.

In section 3, we give a lightning review of $\mathcal{N} = 1$ supersymmetric QCD (SQCD), with various numbers of colors and flavors. Here we will be particularly brief. The reader can consult various books and reviews, e.g. [6,7,13-16], for more details.

In section 4, we discuss dynamical supersymmetry breaking (DSB), where the supersymmetry breaking is related to a dynamical scale Λ , and thus it is non-perturbative in the coupling. Using the understood dynamics of SQCD, it is possible to find an effective Lagrangian in which supersymmetry breaking can be seen semiclassically. We will discuss only four characteristic examples, demonstrating four different mechanisms of DSB.

2. Semiclassical spontaneous supersymmetry breaking

In this section we consider theories with chiral superfields Φ^a , a smooth Kähler potential $K(\Phi, \bar{\Phi})$ and a superpotential $W(\Phi)$. For simplicity we will ignore the possibility of adding gauge fields. A detailed analysis of their effect will be presented in [17]. The Kähler potential leads to the metric on field space

$$g_{a\bar{a}} = \partial_a \partial_{\bar{a}} K, \tag{2.1}$$

which determines the Lagrangian of the scalars

$$\begin{aligned} \mathcal{L}_{scalars} &= g_{a\bar{a}} \partial_\mu \Phi^a \partial^\mu \bar{\Phi}^{\bar{a}} - V(\Phi, \bar{\Phi}) \\ V &= g^{a\bar{a}} \partial_a W \partial_{\bar{a}} \bar{W}. \end{aligned} \tag{2.2}$$

It is clear from the scalar potential V that supersymmetric ground states, which must have zero energy, are related to the critical points of W ; i.e. points where we can solve

$$\partial_a W(\Phi^a) = 0 \quad \forall a. \tag{2.3}$$

If no such point exists, it means that the system does not have supersymmetric ground states.

However, before we conclude in this case that supersymmetry is spontaneously broken we should also exclude the possibility that the potential slopes to zero at infinity. Roughly, in this case the system has “a supersymmetric state at infinity.” More precisely, it does not have a ground state at all!

2.1. The simplest example

Consider a theory of a single chiral superfield X , with linear superpotential with coefficient f (with units of mass square),

$$W = fX, \tag{2.4}$$

and canonical Kähler potential

$$K = K_{can} = \bar{X}X. \tag{2.5}$$

Supersymmetry is spontaneously broken by the expectation value of the F-component of X , $\bar{F}_X = -f$. Using (2.2) the potential is $V = |f|^2$. It is independent of X , so there are classical vacua for any $\langle X \rangle$.

Supersymmetric theories often have a continuous manifold of supersymmetric vacua which are usually referred to as “moduli space of vacua.” However, in the case where supersymmetry is broken, such a space is not robust: this nonsupersymmetric degeneracy of vacua is often lifted once radiative corrections are taken into account. Therefore, we prefer to refer to this space as a *pseudomoduli space of vacua*. The example we study here is free, and therefore the space of vacua remains present even in the quantum theory. We will see below examples of the more typical situation, in which the classical theory has a pseudomoduli space of nonsupersymmetric vacua, but the quantum corrections lift the degeneracy.

The exactly massless Goldstino is ψ_X , and its complex scalar partner X is the classically massless pseudomodulus. Note that there is a $U(1)_R$ symmetry, with $R(X) = 2$. For $\langle X \rangle \neq 0$ it is spontaneously broken, and the corresponding massless Goldstone boson is the phase of the field X .

Deforming (2.4) by any superpotential interactions, say a degree n polynomial in X , leads to $n - 1$ supersymmetric vacua. For example, if we add $\Delta W = \frac{1}{2}\epsilon X^2$, there is a vacuum with unbroken supersymmetry at $\langle X \rangle = -f/\epsilon$. This deformation lifts the pseudomoduli space by creating a potential $|f + \epsilon X|^2$ over it. We can also see that supersymmetry is not broken from the fact that ψ_X now has mass ϵ , so there is no massless Goldstino. Note also that any such ΔW deformations of (2.4) explicitly break the $U(1)_R$ symmetry; the fact that they lead to supersymmetric vacua illustrates a general connection between R-symmetry and supersymmetry breaking, which will be developed further below.

2.2. *The simplest example but with more general Kähler potential*

Consider again the theory of section 2.1 with superpotential (2.4), but with a general Kähler potential $K(X, \bar{X})$. Of course, this theory is not renormalizable. It should be viewed either as a classical field theory or as a quantum field theory with a cutoff Λ . More physically, such a theory can be the low energy approximation of another, microscopic theory, which is valid at energies larger than Λ .

The potential,

$$V = K_{X\bar{X}}^{-1} |f|^2 \tag{2.6}$$

lifts the degeneracy along the pseudomoduli space of the previous example. Let us suppose that the Kähler potential K is smooth. (Non-smooth K signals the need to include additional degrees of freedom, in the low-energy effective field theory at the singularity. An example of this case is discussed in the next subsection.) For smooth K , the potential (2.6) is non-vanishing, and thus there is no supersymmetric vacuum.

Before concluding that supersymmetry is spontaneously broken, we should consider the behavior at $|X| \rightarrow \infty$. If there is any direction along which $\lim_{|X| \rightarrow \infty} K_{X\bar{X}}$ diverges, then V slopes to zero at infinity and the system does not have a ground state. If $\lim_{|X| \rightarrow \infty} K_{X\bar{X}}$ vanishes in all directions, the potential rises at infinity and it has a supersymmetry breaking global minimum for some finite X . Finally, if there are directions along which $\lim_{|X| \rightarrow \infty} K_{X\bar{X}}$ is finite, the potential approaches a constant along these directions and the global minimum of the potential needs a more detailed analysis.

Consider the behavior of the system near a particular point, say $X \approx 0$. Let

$$K = X\bar{X} - \frac{c}{|\Lambda|^2} (X\bar{X})^2 + \dots, \tag{2.7}$$

with positive c .³ Then there is a locally stable nonsupersymmetric vacuum at $X = 0$. In this vacuum, the scalar component of X gets mass $m_X^2 = 4c|f|^2/|\Lambda|^2$. The fermion ψ_X is the exactly massless Goldstino. Note also that if $K(X, \bar{X})$ depends only on $X\bar{X}$, then there is a $U(1)_R$ symmetry, which is unbroken if the vacuum is at $X = 0$. This ground state can be the global minimum of the potential. Alternatively, it can be only a local

³ The parameter Λ in (2.7) determines the scale of the features in the potential. When this theory arises as the low energy approximation of another theory, this parameter Λ is typically the scale above which the more microscopic theory is valid.

minimum, with either another minimum of lower energy or no minimum at all if the system runs away to infinity.

If $X = 0$ is not the global minimum of the potential, the state at $X = 0$ is metastable. If the theory is sufficiently weakly coupled, the tunneling out of this vacuum can be highly suppressed and this vacuum can be very long lived. We see that it is easy to find examples where supersymmetry is broken in a long lived metastable state. (Though we have not yet demonstrated what physical dynamics leads to such features in the Kähler potential.)

Let us consider again the theory with Kähler potential (2.7), but deform the superpotential (2.4) to

$$W = fX + \frac{1}{2}\epsilon X^2, \quad (2.8)$$

taking ϵ as a small parameter. There is now a supersymmetric vacuum at

$$\langle X \rangle_{susy} = -f/\epsilon, \quad (2.9)$$

which is very far from the origin. On the other hand, for X near the origin, we find for the potential

$$V(X, \bar{X}) = (K_{X\bar{X}})^{-1} |f + \epsilon X|^2 = |f|^2 + \bar{f}\epsilon X + f\bar{\epsilon}\bar{X} + \frac{4c|f|^2}{|\Lambda|^2} |X|^2 + \dots \quad (X \approx 0, \epsilon \ll 1). \quad (2.10)$$

There is a local minimum, with broken supersymmetry, at

$$\langle X \rangle_{meta} = -\frac{\bar{\epsilon}|\Lambda|^2}{4c\bar{f}}. \quad (2.11)$$

For $|\epsilon| \ll \sqrt{c}|f/\Lambda|$, this supersymmetry breaking vacuum is very far from the supersymmetric vacuum (2.9). The metastable state (2.11) can thus be very long lived.

At first glance, there is a small puzzle with the broken supersymmetry vacuum (2.11). The superpotential (2.8) gives a mass ϵ to the fermion ψ_X , whereas any vacuum with broken supersymmetry must have an *exactly massless* Goldstino. The Goldstino must be exactly massless, regardless of whether the supersymmetry breaking state is a local or global minimum of the potential. The resolution of the apparent puzzle is that

$$\int d^4\theta K \supset K_{X\bar{X}\bar{X}} \bar{F}_X \psi_X \psi_X \quad (2.12)$$

and evaluating this term in the vacuum (2.11), with $\bar{F}_X \approx -f$, exactly cancels the $\epsilon\psi\psi$ term coming from the superpotential. So there is indeed an exactly massless Goldstino, ψ_X , consistent with the supersymmetry breaking in the metastable state.

2.3. Additional degrees of freedom can restore supersymmetry

Let us consider a renormalizable theory of two chiral superfields, X and q , with canonical Kähler potential, $K = X\bar{X} + q\bar{q}$. We modify the example of section 2.1 by coupling the field X to the additional field q via

$$W = \frac{1}{2}hXq^2 + fX, \quad (2.13)$$

where h is the coupling constant. The field q gets a mass from an X expectation value (an added mass term $\Delta W = \frac{1}{2}Mq^2$ can be eliminated by a shift of X). There is a $U(1)_R$ symmetry, with $R(X) = 2$, and $R(q) = 0$, and also a \mathbf{Z}_2 symmetry $q \rightarrow -q$.

The potential

$$V = |hXq|^2 + \left|\frac{1}{2}hq^2 + f\right|^2 \quad (2.14)$$

does not break supersymmetry. There are two supersymmetric vacua, at

$$\langle X \rangle_{susy} = 0, \quad \langle q \rangle_{susy} = \pm \sqrt{-2f/h}. \quad (2.15)$$

The additional degrees of freedom, q , as compared with the example of section 2.1, have restored supersymmetry.

Note that the potential (2.14) also has a supersymmetry breaking pseudoflat direction with $\langle q \rangle = 0$, and arbitrary $\langle X \rangle$, with $V = |f|^2$. It reflects the fact that for large X the q fields are massive, can be integrated out, and the low energy theory is then the same as that of section 2.1. The spectrum of the massive q fields depends on X , and is given by

$$m_0^2 = |hX|^2 \pm |hf| \quad ; \quad m_{1/2} = hX. \quad (2.16)$$

We see, however, that this pseudomoduli space has a tachyon for

$$|X|^2 < \left| \frac{f}{h} \right|. \quad (2.17)$$

In the region (2.17), the potential can decrease along the $\langle q \rangle$ direction, down to the supersymmetric vacua (2.15).

2.4. *An example with a runaway [18]*

Consider a renormalizable theory of two chiral superfields, X and Y , with canonical Kähler potential, and superpotential

$$W = \frac{1}{2}hX^2Y + fX. \quad (2.18)$$

There is a $U(1)_R$ symmetry, with $R(X) = 2$, and $R(Y) = -2$. The potential is

$$V = \left| \frac{1}{2}hX^2 \right|^2 + |hXY + f|^2. \quad (2.19)$$

It is impossible for both terms to vanish, so the theory does not have supersymmetric ground states. As usual, before concluding that supersymmetry is spontaneously broken, we must examine for runaway directions. Indeed, taking $X = -f/hY$ the potential has a runaway direction as $Y \rightarrow \infty$:

$$V \rightarrow \left| \frac{f^2}{2hY^2} \right|^2 \rightarrow 0. \quad (2.20)$$

There is no static vacuum, but supersymmetry is asymptotically restored as $Y \rightarrow \infty$.

For large $|Y|$ the supersymmetry breaking is small, and the mass of X is large, so we can describe the theory by a supersymmetric low-energy effective Lagrangian with X integrated out. Integrating out X in (2.18) we find the effective superpotential

$$W_{eff} = -\frac{f^2}{2hY} \quad (2.21)$$

which is consistent with the R-symmetry, and leads to the potential (2.20).

2.5. *O’Raifeartaigh-type models*

Here we discuss models of supersymmetry breaking which arise in renormalizable field theories; i.e. unlike the example of section 2.2, we will examine classical theories with a canonical Kähler potential (for a recent analysis of such models see e.g. [19]).

The simplest version of this class of models has three chiral superfields, X_1 , X_2 , and ϕ , with canonical Kähler potential

$$K_{cl} = \bar{X}_1X_1 + \bar{X}_2X_2 + \bar{\phi}\phi \quad (2.22)$$

and superpotential

$$W = X_1g_1(\phi) + X_2g_2(\phi) \quad (2.23)$$

with quadratic polynomials $g_{1,2}(\phi)$. This theory has a $U(1)_R$ symmetry, with $R(X_1) = R(X_2) = 2$, and $R(\phi) = 0$. The tree-level potential for the scalars is

$$V_{tree} = |F_{X_1}|^2 + |F_{X_2}|^2 + |F_\phi|^2 \quad (2.24)$$

with

$$-\overline{F}_{X_1} = \partial_{X_1} W = g_1(\phi), \quad -\overline{F}_{X_2} = g_2(\phi), \quad -\overline{F}_\phi = X_1 g'_1(\phi) + X_2 g'_2(\phi). \quad (2.25)$$

We are interested in the minima of this potential.

We can always choose X_1 and X_2 to set $F_\phi = 0$. But, for generic functions $g_1(\phi)$ and $g_2(\phi)$, we cannot simultaneously solve $g_1(\phi) = 0$ and $g_2(\phi) = 0$, so F_{X_1} or F_{X_2} is non-zero, and hence supersymmetry is generically broken. There is a one-complex dimensional classical pseudomoduli space of non-supersymmetric vacua, since only one linear combination of X_1 and X_2 is constrained by the condition that $F_\phi = 0$. Setting $F_\phi = 0$ ensures that the vacuum satisfies the X_1 and X_2 equations of motion, $\partial_{X_i} V_{tree} = 0$. We still need to impose $\partial_\phi V_{tree} = 0$, which requires that $\langle \phi \rangle$ solve

$$\overline{g_1(\phi)} g'_1(\phi) + \overline{g_2(\phi)} g'_2(\phi) = 0. \quad (2.26)$$

Expanding to quadratic order in δX_1 , δX_2 , and $\delta \phi$ yields the mass matrix m_0^2 of the massive scalars; the eigenvalues of this matrix must all be non-negative, of course, if we are expanding around a (local) minimum of the potential. The fermion mass terms are given by

$$\mathcal{L} \supset (X_1 g''_1(\phi) + X_2 g''_2(\phi)) \psi_\phi \psi_\phi + (g'_1(\phi) \psi_{X_1} + g'_2(\phi) \psi_{X_2}) \psi_\phi. \quad (2.27)$$

It is easy to see that there is a massless eigenvector, corresponding to the massless Goldstino.

Example 1 – the basic O’Raifeartaigh model [20]

As a special case of the above class of models, consider⁴ $g_1(\phi) = \frac{1}{2} h \phi^2 + f$, $g_2(\phi) = m \phi$. It is characterized by the discrete \mathbf{Z}_2 symmetry under which ϕ and X_2 are odd.

⁴ If, instead, $g_{1,2}$ are even quadratic polynomials: $g_i(\phi) = \frac{1}{2} h_i \phi^2 + f_i$, a simple change of variables shows that the theory decouples to a free field which breaks supersymmetry as in section 2.1 and the example of section 2.3.

For convenience, let us also write it as

$$W = \frac{1}{2}hX\phi_1^2 + m\phi_1\phi_2 + fX, \quad (2.28)$$

where we denote $X = X_1$, $\phi_2 = X_2$, and $\phi_1 = \phi$. Note that, for $m \rightarrow 0$, the field ϕ_2 decouples, and what remains in (2.28) is the theory of section 2.3, which we have seen does not break supersymmetry. For $m \neq 0$, it does break supersymmetry, as in the general case discussed above, as there is no simultaneous solution of $g_1(\phi_1) = \frac{1}{2}h\phi_1^2 + f = 0$ and $g_2(\phi_1) = m\phi_1 = 0$. The potential rises for large ϕ_1 and ϕ_2 , so these fields do not have runaway directions. The minima of the potential form a one-complex dimensional pseudomoduli space of degenerate, non-supersymmetric vacua, with $\langle X \rangle$ arbitrary.

The equation (2.26) is a cubic equation for ϕ_1 . The solution with minimum energy depends on the parameter

$$y \equiv \left| \frac{hf}{m^2} \right|. \quad (2.29)$$

Consider the case $y < 1$. Then the potential is minimized⁵ by $F_{\phi_2} = 0$, with value

$$V_{min} = |F_X|^2 = |f|^2, \quad (2.30)$$

at $\phi_1 = \phi_2 = 0$ and arbitrary X .

The fermion ψ_X is the exactly massless Goldstino. The scalar component of X is a classical pseudomodulus. The classical mass spectrum of the ϕ_1 and ϕ_2 fields can be easily computed. For the two, two-component fermions, the eigenvalues are

$$m_{1/2}^2 = \frac{1}{4}(|hX| \pm \sqrt{|hX|^2 + 4|m|^2})^2, \quad (2.31)$$

and for the four real scalars the mass eigenvalues are

$$m_0^2 = \left(|m|^2 + \frac{1}{2}\eta|hf| + \frac{1}{2}|hX|^2 \pm \frac{1}{2}\sqrt{|hf|^2 + 2\eta|hf||hX|^2 + 4|m|^2|hX|^2 + |hX|^4} \right), \quad (2.32)$$

where $\eta = \pm 1$. We see that, as in (2.16), the spectrum changes along the pseudomoduli space parameterized by X ; these vacua are physically distinct.

The parameter y sets the relative size of the mass splittings, corresponding to supersymmetry being broken, between (2.31) and (2.32). For $y \ll 1$, the spectrum (2.31)

⁵ There is a second order phase transition at $y = 1$, where this minimum splits to two minima and a saddle point. Here we will not analyze the phase $y > 1$. See e.g. [9] for a detailed analysis.

and (2.32) is approximately supersymmetric, whereas for $y \sim 1$ supersymmetry is badly broken. (In particular, for $y = 1$, there is a massless real scalar in (2.32) for all X , whereas the fermions (2.31) are all massive.)

We can write (2.28) as $W = \frac{1}{2}M_{ij}\phi^i\phi^j + fX$, where $M = \begin{pmatrix} hX & m \\ m & 0 \end{pmatrix}$, and the supersymmetry breaking can be seen from the fact that $\det M = -m^2$ is non-zero and X independent. This can be generalized to similar models, with more fields ϕ^i , and M_{ij} such that $\det M$ is non-zero and independent of X [9].

Example 2 – supersymmetry breaking in a metastable state [10]

We noted above that the theory (2.23) breaks supersymmetry for generic functions $g_1(\phi)$ and $g_2(\phi)$, because we generically cannot solve $g_1(\phi) = g_2(\phi) = 0$. Let us consider the case of a *non-generic* superpotential, where there is a solution $\langle\phi\rangle_{susy}$ of $g_1(\phi) = g_2(\phi) = 0$. In this case, there are supersymmetric vacua. There can still, however, be metastable vacua with broken supersymmetry.

As a particular example, consider

$$g_1(\phi) = h\phi(\phi - m_1), \quad g_2(\phi) = m_2(\phi - m_1). \quad (2.33)$$

(This theory was first analyzed in [10] and was recently reexamined in [19].) There is a moduli space of supersymmetric vacua at

$$\langle\phi\rangle_{susy} = m_1 \quad ; \quad \langle X_2\rangle_{susy} = -\frac{hm_1}{m_2}\langle X_1\rangle_{susy}, \quad (2.34)$$

with arbitrary $\langle X_1\rangle_{susy}$. The equation (2.26) is a cubic equation for ϕ , and this moduli space of supersymmetric vacua corresponds to one root of this cubic equation. For $|hm_1/m_2|^2 > 8$, there is also a pseudomoduli space of supersymmetry violating minima of the potential at

$$\langle\phi_1\rangle_{meta} \approx \left|\frac{m_2}{hm_1}\right|^2 m_1 \quad , \quad \langle X_2\rangle_{meta} \approx \frac{hm_1}{m_2}\langle X_1\rangle_{meta} \quad \text{for} \quad \left|\frac{hm_1}{m_2}\right| \gg 1 \quad (2.35)$$

with arbitrary $\langle X_1\rangle_{meta}$. These metastable false vacua, in which supersymmetry is broken, become parametrically long lived as $|hm_1/m_2|$ is increased [10]. (The third root of the cubic equation (2.26) is a saddle point.)

2.6. Metastable SUSY breaking in a modified O’Raifeartaigh model [17]

Let us modify the original, basic O’Raifeartaigh model by adding to the superpotential (2.28) a small correction

$$W = \frac{1}{2}hX\phi_1^2 + m\phi_1\phi_2 + fX + \frac{1}{2}\epsilon m\phi_2^2 \quad (2.36)$$

with $|\epsilon| \ll 1$. This added term breaks the $U(1)_R$ symmetry. It has an interesting effect: it leads to metastable supersymmetry breaking. A similar model, but with the ϵ term in (2.36) replaced with $\frac{1}{2}\epsilon mX^2$ was considered in [21], with similar conclusions to ours here. (Note that adding $\Delta W = \frac{1}{2}b\phi_1^2$ has no physical effect; it can simply be eliminated by shifting X by an appropriate constant.)

The potential is now

$$V_{tree} = |F_X|^2 + |F_{\phi_1}|^2 + |F_{\phi_2}|^2 \quad (2.37)$$

with

$$-\bar{F}_X = \frac{1}{2}h\phi_1^2 + f, \quad -\bar{F}_{\phi_1} = hX\phi_1 + m\phi_2, \quad -\bar{F}_{\phi_2} = m\phi_1 + \epsilon m\phi_2. \quad (2.38)$$

Because of the modification of the superpotential by the last term in (2.36) two new supersymmetric minima appear at

$$\langle \phi_1 \rangle_{susy} = \pm \sqrt{-2f/h}, \quad \langle \phi_2 \rangle_{susy} = \mp \frac{1}{\epsilon} \sqrt{-2f/h}, \quad \langle X \rangle_{susy} = \frac{m}{h\epsilon} \quad (2.39)$$

However, for small ϵ and $y = \left| \frac{hf}{m^2} \right| < 1$, the potential near the previous supersymmetry breaking minimum $\phi_1 = \phi_2 = 0$ is not modified a lot.

Strictly, this theory does not break supersymmetry – it has supersymmetric ground states at (2.39). However, the generalization of the eigenvalues (2.32), to include ϵ , remains non-tachyonic for

$$\left| X - \frac{m}{h\epsilon} \right|^2 > \left(\frac{1}{|\epsilon|^2} + 1 \right) \left| \frac{f}{h} \right|. \quad (2.40)$$

Therefore, most of the pseudomoduli space of vacua of the $\epsilon = 0$ theory remains locally stable, and the tachyon exists only in a neighborhood of the supersymmetric value (2.39). In particular, for small ϵ and $y < 1$, the region near $X = 0$ is locally stable.

As $\epsilon \rightarrow 0$ the supersymmetry preserving vacua (2.39) are pushed to infinity until finally, for $\epsilon = 0$ they are not present, and we are left with only the pseudomoduli space of non-supersymmetric vacua. A more detailed analysis will be presented in [17].

2.7. Supersymmetry breaking by rank condition [9]

Our final example in this section is more complicated. It involves several fields transforming under a large symmetry group. The fields X_i in (2.23) are replaced by a matrix of fields. Apart from the intrinsic interest in this example, it will also be useful in our discussion in section 4.

Consider a theory with fields $\varphi, \tilde{\varphi}, \Phi$, and parameters f , with global⁶ symmetries

	$SU(n)$	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_V$	$U(1)_R$	$U(1)_A$	
φ	\mathbf{n}	$\overline{\mathbf{N}}_f$	$\mathbf{1}$	1	0	1	
$\tilde{\varphi}$	$\overline{\mathbf{n}}$	$\mathbf{1}$	\mathbf{N}_f	-1	0	1	(2.41)
Φ	$\mathbf{1}$	\mathbf{N}_f	$\overline{\mathbf{N}}_f$	0	2	-2	
f	$\mathbf{1}$	$\overline{\mathbf{N}}_f$	\mathbf{N}_f	0	0	2	

We will take

$$n < N_f. \tag{2.42}$$

We take the Kähler potential K to be canonical, and the superpotential is

$$W = h \text{Tr} \Phi \varphi \tilde{\varphi}^T + \text{Tr} f \Phi, \tag{2.43}$$

where h is a coupling constant and the trace is over the global symmetry indices. The last term in (2.43) respects the symmetries in (2.41) because of the transformation laws of the parameter f . Alternatively, the parameter f breaks $SU(N_f) \times SU(N_f)$ to a subgroup, and breaks $U(1)_A$, but it does not break the $SU(n)$ symmetry or the R-symmetry.

Supersymmetry is broken when (2.42) is satisfied. Consider the F -component of Φ

$$-F_\Phi^\dagger = h \varphi \tilde{\varphi}^T + f \tag{2.44}$$

(here we use \dagger even in the classical theory because of the flavor indices of Φ). This is an $N_f \times N_f$ matrix relation. Because of (2.42), the first term is a matrix of rank n . On the other hand, we can take f to have rank larger than n , up to rank N_f . Therefore, if the rank of f is larger than n , and in particular if f is proportional to the unit matrix $\mathbb{1}_{N_f}$, then (2.44) cannot vanish, $F_\Phi \neq 0$, and supersymmetry is broken.

⁶ For our discussion in section 4, we will take the $SU(n)$ symmetry to be gauged, but IR free. In that case, the $U(1)_R$ symmetry below is anomalous (a linear combination of $U(1)_R$ and $U(1)_A$ is anomaly free, but broken by the parameter f), but is restored as an approximate, accidental symmetry in the IR. Also, the $SU(n)$ D -terms will vanish in the vacua. The results discussed here will be completely unaffected by the weak gauging of $SU(n)$ in section 4.

When (2.42) is not satisfied, there are supersymmetric vacua, as in the example (2.13), which is similar to the case $n = N_f = 1$. The difference is that, when (2.42) is satisfied, there are not enough additional degrees of freedom, φ and $\tilde{\varphi}$, at $\Phi = 0$ to restore supersymmetry.

For simplicity, we take $f \equiv -h\mu^2 \mathbb{I}_{N_f}$, proportional to the unit matrix. The minimum of the potential is then at

$$V = (N_f - n)|h\mu^2|^2 \quad (2.45)$$

and it occurs along the pseudomoduli space

$$\Phi = \begin{pmatrix} 0 & 0 \\ 0 & \Phi_0 \end{pmatrix}, \quad \varphi = \begin{pmatrix} \varphi_0 \\ 0 \end{pmatrix}, \quad \tilde{\varphi} = \begin{pmatrix} \tilde{\varphi}_0 \\ 0 \end{pmatrix}, \quad \text{with } \varphi_0 \tilde{\varphi}_0^T = \mu^2 \mathbb{I}_n, \quad (2.46)$$

and arbitrary Φ_0 , φ_0 and $\tilde{\varphi}_0$ (subject to the constraint in (2.46)). The first entries in (2.46) are the first n components, and the second are the remaining $N_f - n$ components, so e.g. Φ_0 is a $(N_f - n) \times (N_f - n)$ square matrix. The non-zero F terms are $F_{\Phi_0} = \bar{h}\bar{\mu}^2 \mathbb{I}_{N_f - n}$. The massless Goldstino comes from the fermionic components of Φ_0 .

2.8. One-loop lifting of pseudomoduli

As we have seen in the examples above, models of tree-level spontaneous supersymmetry breaking generally have classical moduli spaces of degenerate, non-supersymmetric, vacua. Indeed, the massless Goldstino is in a chiral superfield (for F -term breaking), whose scalar component is a classical pseudomodulus. The example of section 2.3 shows that this is the case even if this space of classical vacua becomes unstable in a region in field space. The example of section 2.7 (2.46) shows that there can be additional pseudomoduli. We said above that we should use the term ‘‘pseudomoduli’’ space for the space of classical non-supersymmetric vacua, because the degeneracy between these vacua is usually lifted once quantum corrections are taken into account. In this section, we review how this comes about.

We will be interested in the one-loop effective potential (the Coleman-Weinberg potential) for the pseudomoduli (such as X), which comes from computing the one-loop correction to the vacuum energy

$$\begin{aligned} V_{eff}^{(1)} &= \frac{1}{64\pi^2} \text{STr} \left(\mathcal{M}^4 \log \frac{\mathcal{M}^2}{M_{cutoff}^2} \right) \\ &\equiv \frac{1}{64\pi^2} \left[\text{Tr} \left(m_B^4 \log \frac{m_B^2}{M_{cutoff}^2} \right) - \text{Tr} \left(m_F^4 \log \frac{m_F^2}{M_{cutoff}^2} \right) \right], \end{aligned} \quad (2.47)$$

where m_B^2 and m_F^2 are the tree-level boson and fermion masses, as a function of the expectation values of the pseudomoduli, and M_{cutoff} is a UV cutoff. In (2.47), \mathcal{M}^2 stands for the classical mass-square matrix of the various fields of the theory.

We would like to make two comments about the divergences in this expression:

1. In non-supersymmetric theories the effective potential includes also a quartic divergent term proportional to $M_{cutoff}^4 \text{STr } \mathbb{1}$ and a quadratic divergent term proportional to $M_{cutoff}^2 \text{STr } \mathcal{M}^2$. They vanish in supersymmetric theories.
2. The logarithmic divergent term $(\log M_{cutoff}) \text{STr } \mathcal{M}^4$ in (2.47) can be absorbed into the renormalization of the coupling constants appearing in the tree-level vacuum energy V_0 (see below). In particular, $\text{STr } \mathcal{M}^4$ is independent of the pseudomoduli.

For completeness, we recall the standard expressions for these masses. For a general theory with k chiral superfields, Φ^a , with canonical classical Kähler potential, $K = \Phi^a \bar{\Phi}^a$, and superpotential $W(\Phi^a)$:

$$m_0^2 = \begin{pmatrix} \bar{W}^{ac} W_{cb} & \bar{W}^{abc} W_c \\ W_{abc} \bar{W}^c & W_{ac} \bar{W}^{cb} \end{pmatrix}, \quad m_{1/2}^2 = \begin{pmatrix} \bar{W}^{ac} W_{cb} & 0 \\ 0 & W_{ac} \bar{W}^{cb} \end{pmatrix}, \quad (2.48)$$

with $W_c \equiv \partial W / \partial Q^c$, etc., and m_0^2 and $m_{1/2}^2$ are $2k \times 2k$ matrices. Note that

$$\text{STr } \mathcal{M}^2 = 0 \quad (2.49)$$

We will be interested in situations where we integrate out some massive fields Φ^a whose superpotential is locally of the form

$$W = \frac{1}{2} \Phi^a M_{ab} \Phi^b + \dots, \quad (2.50)$$

where M_{ab} can depend on various massless fields X . Integrating out Φ^a leads to the one loop effective Kähler potential

$$K_{eff}^{(1)} = -\frac{1}{32\pi^2} \text{Tr}[M M^\dagger \log(M M^\dagger / M_{cutoff}^2)]. \quad (2.51)$$

If the supersymmetry breaking is small, we can use the effective Kähler potential to find the effective potential. For example, if M_{ab} depends on one pseudomodulus X , the effective potential is

$$V_{trunc} = (K_{eff}{}_{X, \bar{X}})^{-1} |\partial_X W|^2. \quad (2.52)$$

However, as we will discuss below, (2.52) gives the correct expression for the effective potential (2.47) only to leading order in $F_X = -\frac{\partial_X W}{K_{eff}{}_{X, \bar{X}}}$. (It is verified in [9] that (2.52)

and (2.47) agree to order $\mathcal{O}(F_X \overline{F}_X)$.) Higher powers of F_X arise from terms in the low energy effective Lagrangian with more superspace covariant derivatives, e.g. terms of the form

$$\int d^4\theta H(X, \overline{X})(DX)^2 + c.c. \quad (2.53)$$

for some function $H(X, \overline{X})$. They cannot be ignored when the supersymmetry breaking is large. The full effective potential (2.47) includes all these higher order corrections.

Example 1 – the theory of section 2.3

As a first application, we compute the one-loop potential on the supersymmetry breaking pseudomoduli space mentioned in section 2.3. Recall that this space exists for X outside of the range (2.17) where there is a tachyon, so we limit ourselves to $|X|^2 > |f/h|$. We treat the pseudomodulus X as a background, and use the masses (2.16) in (2.47). This yields

$$\begin{aligned} V^{(1)}(|X|) &= \frac{1}{64\pi^2} \left[-2|hf|^2 \log M_{cutoff}^2 - 2|hX|^4 \log |hX|^2 \right. \\ &\quad \left. + (|hX|^2 - |hf|)^2 \log(|hX|^2 - |hf|) + (|hX|^2 + |hf|)^2 \log(|hX|^2 + |hf|) \right] \\ &= \frac{|hf|^2}{32\pi^2} \left[\log \left| \frac{hX}{M_{cutoff}} \right|^2 + \frac{3}{2} + v(z) \right] \\ z &\equiv \left| \frac{f}{hX^2} \right| \\ v(z) &\equiv \frac{1}{2} (z^{-2}(1+z)^2 \log(1+z) + z^{-2}(1-z)^2 \log(1-z) - 3) = -\frac{z^2}{12} + \mathcal{O}(z^4), \end{aligned} \quad (2.54)$$

where the shift by $\frac{3}{2}$ is for later convenience.

The potential (2.54) lifts the degeneracy along the pseudomoduli space. It is an increasing function of $|X|$. It pushes X into the region (2.17); i.e. toward the region with a tachyon (where the expression (2.54) no longer makes sense). From there, the theory falls into its supersymmetric vacua (2.15).

We will now use this simple example, and result (2.54), to clarify and illustrate a number of technical points. Similar statements will apply to other examples.

Let us clarify the nature of the semiclassical limit. We take $h \rightarrow 0$ (the coupling h is IR free) with f , X , $q \sim h^{-1}$ (and therefore $z \sim h^0$). In this limit the classical Lagrangian, based on canonical Kähler potential and the superpotential (2.13), scales like h^{-2} . The

one loop corrections, in particular (2.54), are of order h^0 . We can neglect higher loop terms, which are order h^2 and higher.

Next, we want to understand the dependence on the UV cutoff M_{cutoff} . We define the running coupling

$$f(\mu) = f_{bare} \left(1 + \frac{|h^2|}{64\pi^2} \left(\frac{3}{2} + \log \frac{\mu^2}{M_{cutoff}^2} \right) + \mathcal{O}(h^4) \right), \quad (2.55)$$

where we have set an additive constant to a convenient value. In terms of this running f the potential (2.54) is independent of the UV cutoff M_{cutoff}

$$V(X) = |f(|hX|)|^2 \left(1 + \frac{|h^2|}{32\pi^2} v(z) + \mathcal{O}(h^4) \right). \quad (2.56)$$

Here $f(\mu = |hX|)$ is the running coupling (2.55) at the scale of the massive fields q .

Equivalently, we can remember that in supersymmetric theories there is only wavefunction renormalization. The potential arises from F_X , and therefore at the leading order only Z_X can affect the potential. The renormalization of f in (2.55) can be understood as coming from Z_X , as

$$V = Z_X^{-1} |\partial_X W|^2 + \text{finite} = Z_X^{-1} |f|^2 + \text{finite}. \quad (2.57)$$

We thus have

$$-\frac{\partial V}{\partial \ln M_{cutoff}^2} = \gamma_X |f|^2 = \frac{1}{64\pi^2} \text{Str} \mathcal{M}^4 + \mathcal{O}(h^2), \quad (2.58)$$

where we recognize γ_X as the anomalous dimension of X .

A special situation arises when the supersymmetry breaking mass splittings are effectively small. This happens when $z \equiv |f/hX^2| \ll 1$; i.e. either for small $|f|$, or for large $|X|$. Expanding (2.54) we find

$$V \approx |f|^2 + \frac{|hf|^2}{32\pi^2} \left[\log \left| \frac{hX}{M_{cutoff}} \right|^2 + \frac{3}{2} \right] + \mathcal{O}(h^4) = |f(hX)|^2. \quad (2.59)$$

This can be interpreted as arising from renormalization of the Kähler potential

$$K_{ren} = |X|^2 - \frac{|hX|^2}{32\pi^2} \left(\log \left| \frac{hX}{M_{cutoff}} \right|^2 - \frac{1}{2} \right) + \mathcal{O}(|h|^4). \quad (2.60)$$

Note that this expression for the renormalized K is valid also for $f = 0$, where supersymmetry is not broken along the moduli space parameterized by $\langle X \rangle$.

We should also comment that since as $X \rightarrow 0$ the coupling constant h is renormalized to zero, the expression (2.60) becomes accurate for small X (though still outside of the tachyonic range (2.17)).

We have just seen that for small z we can study a supersymmetric low energy theory with superpotential $W = fX$ and an effective Kähler potential given by (2.60). This is a special case of the discussion above about the Kähler potential (2.51). Using $M = hX$ in (2.51) and $W = fX$, the approximate effective potential (2.52) agrees with (2.59).

As discussed around (2.52), the supersymmetric effective potential (2.52) is valid only when the supersymmetry breaking is small. The correct one-loop effective potential is given by (2.47) (which in our simple example is given by (2.54)), whether or not the supersymmetry breaking is small. In general, additional contributions which are not included in (2.52) are higher orders in $|f|$ in (2.54) (i.e. the function $v(z)$ in (2.54)).

Example 2 – the basic O’Raifeartaigh model (section 2.5)

We now compute the one loop correction to the pseudomodulus potential in the O’Raifeartaigh model, example 1 of section 2.5. The classical flat direction of the classical pseudomodulus X is lifted by a quantum effective potential, $V_{eff}(X)$ [22].

We again treat the pseudomodulus X as a background. The one-loop effective potential $V_{eff}(X)$ is given by the expression (2.47), using the classical masses (2.31) and (2.32). As follows from the R-symmetry, $V_{eff}(X)$ depends only on $|X|$. We find that the potential $V_{eff}(X)$ is a monotonically increasing function of $|X|$, with the following asymptotic behavior at small and large $|X|$:

$$V_{eff}(X) = \begin{cases} V_0 + m_X^2 |X|^2 + \mathcal{O}(|X|^4) & X \approx 0 \\ |f|^2 \left(1 + \gamma_X \left(\log \left| \frac{hX}{M_{cutoff}} \right|^2 + \frac{3}{2} \right) + \mathcal{O}(h^4, \frac{\log |X|}{|X|^4}) \right) & X \rightarrow \infty \end{cases} \quad (2.61)$$

where the constants are

$$\begin{aligned} V_0 &= |f|^2 \left[1 + \frac{|h^2|}{32\pi^2} \left(\log \frac{|m|^2}{M_{cutoff}^2} + \frac{3}{2} + v(y) \right) + \mathcal{O}(h^4) \right] \\ y &= \left| \frac{hf}{m^2} \right| \\ v(y) &= \frac{1}{2} (y^{-2}(1+y)^2 \log(1+y) + y^{-2}(1-y)^2 \log(1-y) - 3) = -\frac{y^2}{12} + \mathcal{O}(y^4) \\ m_X^2 &= \frac{1}{32\pi^2} \left| \frac{h^4 f^2}{m^2} \right| \nu(y) + \mathcal{O}(h^4) \\ \nu(y) &= y^{-3} ((1+y)^2 \log(1+y) - (1-y)^2 \log(1-y) - 2y) = \frac{2}{3} + \mathcal{O}(y^2) \\ \gamma_X &= \frac{|h|^2}{32\pi^2} + \mathcal{O}(h^4). \end{aligned} \quad (2.62)$$

The function $v(y)$ is as in (2.54) but its argument here, y , depends only on the coupling constants, and is independent of the pseudomodulus X . Recall that we take the parameter y , defined in (2.29), to be in the range $0 \leq y \leq 1$.

As in the previous example, the semiclassical limit is $h \rightarrow 0$ (the coupling h is IR free) with $f, X, \phi_{1,2} \sim h^{-1}$ and $m \sim h^0$ (and therefore $y \sim h^0$).

Also, as in that example, the running coupling constant

$$f(\mu) = f_{bare} \left(1 + \frac{|h^2|}{64\pi^2} \left(\frac{3}{2} + \log \frac{\mu^2}{M_{cutoff}^2} \right) + \mathcal{O}(h^4) \right), \quad (2.63)$$

removes the dependence on the UV cutoff M_{cutoff}

$$V(x) = \begin{cases} V_0 + m_X^2 |X|^2 + \mathcal{O}(|X|^4) & X \approx 0 \\ |f(hX)|^2 + \dots & X \rightarrow \infty \end{cases} \quad (2.64)$$

$$V_0 = |f(m)|^2 \left(1 + \frac{|h^2|}{32\pi^2} v(y) + \mathcal{O}(h^4) \right).$$

Let us discuss the effective potential in the two limits $X \approx 0$ and $|X| \rightarrow \infty$. The sign of the mass square in (2.62) is positive, signaling that the potential has a minimum at $X = 0$. The behavior for large X is dominated by the renormalization group running of the effective coupling constant at the scale $|hX|$, which is the scale of the masses in the problem. Finally, it is easy to show using the full expression from (2.47) that the one loop potential is monotonic between these two limits, and therefore $X = 0$ is the global minimum of the potential.

Again, as in the previous example, for $y \equiv |hf/m^2| \ll 1$, the supersymmetry breaking is small. Then, the effective potential can alternatively be computed in the supersymmetric low-energy effective theory, with K given by (2.51) and $W = fX$, leading to the effective potential (2.52). The potential (2.47) applies more generally.

For example, expanding around the minimum at $X = 0$, (2.52) only reproduces the leading order term in the expansion in $y \ll 1$ for m_X^2 in (2.62). It fails to reproduce the answer for larger values of y , e.g.

$$m_X^2 = \frac{|h^3 f|}{16\pi^2} (\log 4 - 1) \quad \text{for} \quad |hf| = |m|^2 \quad ; \quad y = 1. \quad (2.65)$$

On the other hand, even if y is not small, the higher order F terms are insignificant far from the origin of the pseudomoduli space, and indeed there the truncated potential (2.52) agrees with the full effective potential (2.61):

$$V^{(1)} \rightarrow \gamma_X^{(1)} \log \left(\frac{|hX|^2}{M_{cutoff}^2} \right) |f|^2 \quad \text{for } hX \text{ large.} \quad (2.66)$$

Let us now consider the modified model of section 2.6, where we add $\frac{1}{2}h\epsilon\phi_2^2$ to the superpotential (2.36). As we saw, there are then two supersymmetric states at (2.39), and there can also be a metastable state near $X = 0$. Including the ϵ correction to the mass eigenvalues, the one-loop potential (2.47) now has a linear term in X (a tadpole) at $X = 0$, with coefficient $\mathcal{O}(\epsilon)$. The quadratic term in X is not much changed by the $O(\epsilon)$ correction, so the upshot is a local minimum of the one-loop potential at $X \sim \epsilon$.

To summarize this example, we found in section 2.6 that the theory with nonzero f and ϵ has a classical pseudomoduli space of nonsupersymmetric vacua, which is sensible in the range (2.40) (which includes the region around $X = 0$), where there are no tachyonic modes. Now we have shown that the one-loop effective potential lifts this pseudomoduli space, and stabilizes X near the origin. For $\epsilon \ll 1$, the tachyonic direction down to the supersymmetric vacua (2.39) only appears at large X , so the metastable vacuum near the origin, with broken supersymmetry, can be parametrically long lived.

It is straightforward to repeat the computation of the one-loop effective potential for the model where supersymmetry is broken by the rank condition (section 2.7). Again, we set $f = -h\mu^2\mathbb{I}$, and then we find that most of the degeneracy along the classical pseudomoduli space (2.46) is removed by the one-loop effective potential (2.47). The masses of the fluctuations of Φ , φ and $\tilde{\varphi}$, as a function of the pseudomoduli in (2.46), are found to be similar to those of the O’Raifeartaigh model given in (2.31) and (2.32), with $m^2 = hf \equiv -h\mu^2$ (so $y = 1$ in (2.29)). The $SU(n)$ gauge fields do not contribute to (2.47), since their spectrum is supersymmetric to this order. Up to symmetry transformations, the vacua are found to be at

$$\Phi = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad \varphi = \tilde{\varphi} = \begin{pmatrix} \mu\mathbb{I}_n \\ 0 \end{pmatrix}. \quad (2.67)$$

The vacua (2.67) spontaneously break the global symmetry, $G \rightarrow H$. Associated with that, the vacua (2.67) actually form a compact moduli space of vacua, $\mathcal{M}_{vac} = G/H$, parameterized by the massless Goldstone bosons. Since this space of vacua is associated with an exact global symmetry breaking it is robust, and the degeneracy is not lifted by higher order corrections. In particular, these vacua cannot become tachyonic. The one-loop potential computed from (2.47) gives non-tachyonic masses to *all* other pseudomoduli, so the vacua (2.67) are true local minima of the effective potential [9].

2.9. Relation to R-symmetry [11]

Consider a generic theory and ask for a condition for broken supersymmetry. This means that we cannot solve all the equations

$$\partial_a W(\Phi) = 0 \quad \text{for all } a = 1 \dots k. \quad (2.68)$$

But if W is a generic superpotential, then (2.68) involves k equations for the k quantities Φ^a , so generally they can all be solved. Non-R flavor symmetries do not help. Consider for example a global non-R $U(1)$ symmetry. Then, the equations (2.68) can be written as $k - 1$ independent equations for $k - 1$ independent unknowns, as seen by writing

$$W = W(t^a = \Phi^a \Phi_1^{-q_a/q_1}) \quad a = 2 \dots k. \quad (2.69)$$

(q_a is the $U(1)$ charge of Φ^a). But if there is an R-symmetry, then we can write

$$W = T f(t^a = \Phi^a \Phi_1^{-r_a/r_1}) \quad T = \Phi_1^{2/r_1}, \quad (2.70)$$

(r_a is the R-charge of Φ^a), and then in terms of T and t^a for generic f the equations (2.68) set $T = 0$ which is a singular point. Away from $T = 0$ the equations are over-constrained: they are k equations for $k - 1$ independent unknowns, so generically they cannot be solved. Exceptions occur either for a non-generic f , or when a solution with $T = 0$ and therefore $\Phi_1 = 0$ is allowed. This is the case when $r_1 = 2$ and all other $r_a = 0$. Then there is a $k - 2$ dimensional space of supersymmetric vacua, at $\Phi_1 = 0$, $f(\Phi_a) = 0$. (More generally, there are exceptional cases with supersymmetry unbroken for fields at the origin, when all fields, for which the Kähler potential is smooth, have non-negative R-charges less than 2.)

These observations about the relation between R-symmetry and supersymmetry breaking fit with the examples above.

The simplest theory (section 2.1) with $W = fX$ has an R-symmetry and broken supersymmetry. Adding e.g. $\Delta W = \frac{1}{2}\epsilon X^2$ breaks the R-symmetry, and restores supersymmetry.

This is also true for its generalization with more complicated K of section 2.2, which depends only on $X\bar{X}$. If K depends separately on X and \bar{X} (not only through the combination $X\bar{X}$), the theory does not have an R-symmetry but supersymmetry is still broken. This shows that we can have broken supersymmetry without R-symmetry. Here it happens because the superpotential is not a generic function of X .

The addition of light fields as in section 2.3 preserves the R-symmetry, but restores supersymmetry. This demonstrates that having an R-symmetry does not guarantee that

supersymmetry is broken. This example realizes the exceptional case, $r_1 = 2$, $r_{a \neq 1} = 0$, mentioned above.

The example of section 2.4 has a $U(1)_R$ symmetry, and indeed there is no static supersymmetric vacuum. But there is a runaway direction, along which supersymmetry is asymptotically restored. This illustrates the need to still check for runaway directions.

The O’Raifeartaigh type models of section 2.5 have an R-symmetry, and broken supersymmetry for generic $g_1(\phi)$ and $g_2(\phi)$. The example 2 there, with non-generic $g_1(\phi)$ and $g_2(\phi)$, illustrates that having an R-symmetry does not guarantee broken symmetry, if the superpotential is not generic.

The deformation (2.36) of the O’Raifeartaigh model in section 2.6 breaks the R-symmetry, and indeed restores supersymmetry. However, for small ϵ there is an approximate R-symmetry which is related to supersymmetry breaking in the metastable state.

Finally, the models based on the rank condition of section 2.7 have an R-symmetry and correspondingly they have broken supersymmetry, for $n < N_f$. (For $n \geq N_f$, supersymmetry is not broken, by a generalization of the comment following (2.70) about the case $r_1 = 2$, with all other $r_a = 0$.) As mentioned in footnote 6, we will later discuss this model with the $SU(n)$ symmetry gauged, but IR free. The $U(1)_R$ symmetry is then only an approximate symmetry. Correspondingly, the supersymmetry breaking (with $n < N_f$) will be in metastable vacua [9].

To summarize, generically there is broken supersymmetry if and only if there is an R-symmetry. There is broken supersymmetry in a metastable state if and only if there is an approximate R-symmetry. For realistic models of supersymmetry breaking, we need to break the R-symmetry, to get gaugino masses. To avoid having a massless R-axion if the symmetry is spontaneously broken it should also be explicitly broken. Gravity effects can help [12], but ignoring gravity, we conclude that realistic and generic models of supersymmetry breaking require that we live in a metastable state.

3. Supersymmetric QCD

In this section we will discuss the dynamics of supersymmetric QCD (SQCD) for various numbers of colors and flavors. This section will be brief. We refer the reader to the books and reviews of the subject, e.g. [6,7,13-16], for more details.

3.1. Super Yang-Mills theory – $N_f = 0$

A pure gauge theory is characterized by a scale Λ . At energy of order Λ , it confines and leads to nonzero gluino condensation, breaking a discrete R-symmetry.

For $SU(N_c)$ gauge theory we define the gauge invariant chiral operator

$$S \equiv -\frac{1}{32\pi^2} \text{Tr} W^\alpha W_\alpha = \frac{1}{32\pi^2} \text{Tr} (\lambda\lambda + \dots + \theta\theta(\frac{1}{2}F^{\mu\nu}F_{\mu\nu} + \dots)), \quad (3.1)$$

which can be interpreted as a “glueball” superfield. Here we follow the Wess and Bagger notation [3] where $\lambda\lambda \equiv \lambda^\alpha\lambda_\alpha$. The dynamics leads to gaugino condensation:

$$\langle S \rangle = \frac{1}{32\pi^2} \langle \text{Tr} \lambda\lambda \rangle = (\Lambda^{3N_c})^{\frac{1}{N_c}} \quad (3.2)$$

where branches of the fractional power in (3.2) represent the values in the N_c different supersymmetric vacua. The theory has an anomaly free \mathbf{Z}_{2N_c} discrete symmetry (left unbroken by instantons), and (3.2) implies that it is spontaneously broken to \mathbf{Z}_2 .

The N_c supersymmetric vacua with (3.2) are those counted by the Witten index, $\text{Tr}(-1)^F = N_c$ [8]. Since $\lambda\lambda$ is the first component of the chiral superfield S , the expectation values (3.2) do not break supersymmetry.

The relation (3.2) is exact. This can be seen by promoting Λ to an expectation value of a background chiral superfield [23,24], which is assigned charge $R(\Lambda) = 2/3$ to account for the anomaly. There is no correction to (3.2) compatible with this R charge assignment and holomorphy⁷.

The gaugino condensation can be represented as a nontrivial superpotential

$$W_{eff} = N_c (\Lambda^{3N_c})^{\frac{1}{N_c}}. \quad (3.3)$$

Comments:

1. The superpotential (3.3) is independent of fields. It is meaningful when coupling to supergravity, or if Λ is a background field source.
2. Equation (3.3) can be used to find the tension of domain walls interpolating between these vacua labelled by k_1 and k_2 [26]

$$T_{k_1, k_2} = \left| N_c (\Lambda^{3N_c})^{\frac{1}{N_c}} \left(e^{\frac{2\pi i k_1}{N_c}} - e^{\frac{2\pi i k_2}{N_c}} \right) \right|. \quad (3.4)$$

⁷ The non-zero value of the coefficient in (3.2) can be set to one in a particular renormalization scheme. See [25] for discussion, and comparison with various instanton calculations.

3. Thinking of $3N_c \log \Lambda$ as a source for the operator $S \sim \text{Tr} W_\alpha^2$ we can find

$$\langle S \rangle = \frac{1}{3N_c} \partial_{\log \Lambda} W_{eff} = (\Lambda^{3N_c})^{\frac{1}{N_c}}. \quad (3.5)$$

4. Using this observation we can perform a Legendre transform to derive the Veneziano-Yankielowicz superpotential [27]

$$W_{eff}(S) = N_c S (1 - \log S / \Lambda^3). \quad (3.6)$$

It should be stressed that S is not a light fields and therefore this expression is not a term in the Wilsonian effective action. It is a term in the 1PI action and therefore it can be used only to find $\langle S \rangle$ and tensions of domain walls. However, there is no particle-like excitation (e.g. a glueball) which is described by the field S .

3.2. Semiclassical SQCD

We consider $SU(N_c)$ gauge theory with N_f quarks Q and N_f anti-quarks \tilde{Q} .

The gauge and global symmetries are

	$SU(N_c)$	$[SU(N_f)_L$	$SU(N_f)_R$	$U(1)_B$	$U(1)_R$	$U(1)_A]$	
Q	\mathbf{N}_c	\mathbf{N}_f	$\mathbf{1}$	1	$1 - \frac{N_c}{N_f}$	1	(3.7)
\tilde{Q}	$\overline{\mathbf{N}}_c$	$\mathbf{1}$	$\overline{\mathbf{N}}_f$	-1	$1 - \frac{N_c}{N_f}$	1	

Here the global symmetries are denoted by [...]. The $U(1)_A$ symmetry is anomalous and the other symmetries are anomaly free. We also assign charges to the coupling constants: regarding them as background chiral superfields leads to useful selection rules [23],

	$SU(N_c)$	$[SU(N_f)_L$	$SU(N_f)_R$	$U(1)_B$	$U(1)_R$	$U(1)_A]$	
m	$\mathbf{1}$	$\overline{\mathbf{N}}_f$	\mathbf{N}_f	0	$2 \frac{N_c}{N_f}$	-2	(3.8)
$\Lambda^{3N_c - N_c}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	0	$2N_f$	

Here m is a possible mass term that we can add, $W_{tree} = \text{Tr} m \tilde{Q} Q$, and Λ is the dynamical scale, related to the running gauge coupling as

$$\Lambda^{3N_c - N_f} = e^{-\frac{8\pi^2}{g^2(\mu)} + i\theta} \mu^{3N_c - N_f}. \quad (3.9)$$

Instanton amplitudes come with the factor of $\Lambda^{3N_c - N_f}$, and their violation of the $U(1)_A$ symmetry is accounted for by the charge assignment in (3.8).

As seen from (3.9), the theory is UV free for $N_f < 3N_c$, i.e. $g^2(\mu) \rightarrow 0$ for $\mu \gg |\Lambda|$. On the other hand, for $N_f \geq 3N_c$, the theory is IR free, i.e. $g^2(\mu) \rightarrow 0$ for $\mu \ll |\Lambda|$ (for $N_f = 3N_c$ the beta function vanishes at one loop, but at two loops it is IR free).

In the rest of this subsection, we take $W_{tree} = 0$. The classical potential is then

$$V \sim \sum_a (D^a)^2 = \sum_a (\text{Tr}(QT^a Q^\dagger - \tilde{Q}^* T^a \tilde{Q}^T))^2 \quad (3.10)$$

(T^a are the $SU(N_c)$ generators). It leads to flat directions which we refer to as the classical moduli space of vacua \mathcal{M}_{cl} . As is always the case, \mathcal{M}_{cl} can be understood in terms of gauge invariant monomials of the chiral superfields, and the light moduli in \mathcal{M}_{cl} can be understood as the chiral superfields that are left uneaten by the Higgs mechanism.

For $N_f < N_c$ up to gauge and flavor rotations, \mathcal{M}_{cl} is given by [28]

$$Q = \tilde{Q} = \begin{pmatrix} a_1 & & & \\ & a_2 & & \\ & & \ddots & \\ & & & a_{N_f} \end{pmatrix}. \quad (3.11)$$

Its complex dimension is $\dim_{\mathbb{C}} \mathcal{M}_{cl} = N_f^2$. The gauge invariant description is $\mathcal{M}_{cl} = \{M_g^f = (\tilde{Q}Q^T)_g^f\}$, $f, \tilde{g} = 1 \dots N_f$. The gauge group is broken on \mathcal{M}_{cl} as $SU(N_c) \rightarrow SU(N_c - N_f)$. The classical Kähler potential on \mathcal{M}_{cl} is

$$K_{cl} = 2 \text{Tr} \sqrt{M^\dagger M}. \quad (3.12)$$

(To see that, write the D-term equations as $Q^\dagger Q = \tilde{Q}^T \tilde{Q}^*$, and use it find $M^\dagger M = Q^* \tilde{Q}^\dagger \tilde{Q} Q^T = (Q^* Q^T)^2$. Then the Kähler potential is $\text{tr} Q^\dagger Q + \text{tr} \tilde{Q}^\dagger \tilde{Q} = 2 \text{tr} \sqrt{M^\dagger M}$.) This is singular near the origin. As always, singularities in the low-energy effective theory signal new light fields, which should be included for a smooth description of the physics. Here the singularities of K_{cl} occur at subspaces where some of the $SU(N_c)/SU(N_c - N_f)$ gauge bosons become massless, and they need to be included in the description.

For $N_f \geq N_c$ we have $\dim_{\mathbb{C}} \mathcal{M}_{cl} = 2N_c N_f - (N_c^2 - 1)$. Up to gauge and flavor rotations [28],

$$Q = \begin{pmatrix} a_1 & & & \\ & a_2 & & \\ & & \ddots & \\ & & & a_{N_c} \end{pmatrix}, \quad \tilde{Q} = \begin{pmatrix} \tilde{a}_1 & & & \\ & \tilde{a}_2 & & \\ & & \ddots & \\ & & & \tilde{a}_{N_c} \end{pmatrix}, \quad |a_i|^2 - |\tilde{a}_i|^2 = \text{independent of } i. \quad (3.13)$$

The gauge invariant description is given by the fields $M = \tilde{Q}Q^T$, $B = Q^{N_c}$ (contracted with the epsilon-symbol), $\tilde{B} = \tilde{Q}^{N_c}$, subject to various classical relations,

$$\mathcal{M}_{cl} = \{M, B, \tilde{B} \mid C_i(M, B, \tilde{B}) = 0\}. \quad (3.14)$$

The functions C_i , giving the classical relations, are of course compatible with the symmetries (3.7), including $U(1)_A$. For example for $N_f = N_c$, we have [29]

$$\mathcal{M}_{cl} = \{M_g^f, B, \tilde{B} \mid \det M - B\tilde{B} = 0\}, \quad (3.15)$$

where the constraint follows from $\det M = \det Q \det \tilde{Q} = B\tilde{B}$. The spaces (3.14), for all $N_f \geq N_c$, are singular at the origin, $M = B = \tilde{B} = 0$, because it is possible to set all $C_i = 0$, and also all variations $\delta C_i = 0$ there. The classical interpretation is that the $SU(N_c)$ gauge fields, which are massless at the origin, need to be included for the low-energy effective theory to be non-singular.

For $N_f > N_c$, among other constraints, the $N_f \times N_f$ matrix $M = \tilde{Q}Q^T$ satisfies

$$\text{rank}(M) \leq N_c \quad \text{classically}. \quad (3.16)$$

3.3. Adding large quark mass terms

Consider adding quark masses, via the tree-level superpotential

$$W_{tree} = \text{Tr } m \tilde{Q}Q^T \equiv \text{Tr } mM. \quad (3.17)$$

For large m (more precisely, the eigenvalues of m are much larger than $|\Lambda|$) we can integrate out the quarks and the low energy theory is a pure gauge theory. Its scale Λ_L is determined at one loop as

$$\Lambda_L^{3N_c} = \det m \Lambda^{3N_c - N_f}. \quad (3.18)$$

Gluino condensation in this theory leads, as in (3.3), to

$$W_{eff} = N_c (\det m \Lambda^{3N_c - N_f})^{\frac{1}{N_c}}; \quad (3.19)$$

it follows from holomorphy and symmetries that (3.19) is the exact effective superpotential. The superpotential (3.19) can be interpreted as part of the generating functional for correlation functions, with the mass m in (3.17) acting as the source for the operator M ,

and $\log \Lambda^{3N_c - N_f}$ as the source for the operator $S \sim \text{Tr} W_\alpha W^\alpha$ [24,30]. We can thus use (3.19) to find

$$\begin{aligned}\langle M \rangle_{susy} &= \partial_m W_{eff} = (\det m \Lambda^{3N_c - N_f})^{\frac{1}{N_c}} \frac{1}{m} \\ \langle S \rangle_{susy} &= \partial_{\log \Lambda^{3N_c - N_f}} W_{eff} = (\det m \Lambda^{3N_c - N_f})^{\frac{1}{N_c}}.\end{aligned}\tag{3.20}$$

The subscript emphasizes that these are the expectation values in the supersymmetric vacua. Note that there are N_c solutions in (3.20), differing by a N_c -th root of unity phase, which correspond to the $\text{Tr}(-1)^F = N_c$ supersymmetric vacua of the low-energy super-Yang-Mills theory. The result (3.20) is valid for all N_f . It is interesting to note that, for $N_f > N_c$, the matrix $\langle M \rangle$ in (3.20) does not satisfy the classical constraint (3.16) of the theory with massless flavors; however, taking $m \rightarrow 0$ in (3.20) does bring $\langle M \rangle$ back to \mathcal{M}_{cl} .

Performing a Legendre transform between m and M , we can use (3.19) to derive the 1PI effective action

$$W_{eff}(M) = (N_c - N_f) \left(\frac{\Lambda^{3N_c - N_f}}{\det M} \right)^{1/(N_c - N_f)} + \text{Tr} m M.\tag{3.21}$$

One might be tempted to interpret (3.21) also as a Wilsonian effective action for the light field M . However, as we will discuss below, this is not always correct.

Finally we can introduce the field S into (3.21) by performing a Legendre transform with respect to its source $\log \Lambda^{3N_c - N_f}$ to find [31]

$$W_{eff}(M, S) = S \left((N_c - N_f) - \log \frac{S^{N_c - N_f} \det M}{\Lambda^{3N_c - N_f}} \right) + \text{Tr} m M.\tag{3.22}$$

Again, this expression can be used to find the expectation values (3.20) and to study domain wall tensions, but it should not be viewed as a term in a Wilsonian effective action.

3.4. $N_f < N_c$ massless flavors [28]

We have seen that the classical theory has a moduli space of supersymmetric vacua \mathcal{M}_{cl} . We now explore the low energy effective Lagrangian along \mathcal{M}_{cl} and examine whether a superpotential can be generated there. The symmetries (3.7) constrain the superpotential to be of the form [32]

$$W_{dyn} \propto \left(\frac{\Lambda^{3N_c - N_f}}{\det M} \right)^{1/(N_c - N_f)}.\tag{3.23}$$

Therefore, we face a dynamical question of determining the coefficient in (3.23). Note that (3.23) is non-perturbative, because of the positive power of $\Lambda \sim \exp(-8\pi^2/(3N_c - N_f)g^2)$.

Recall that the gauge group is Higgsed to $SU(N_c - N_f)$ on the classical moduli space. For $N_f = N_c - 1$, the gauge group is completely Higgsed, and then there are finite action (constrained) instantons which generate (3.23). For $N_f < N_c - 1$, (3.23) is instead associated with gaugino condensation in the unbroken $SU(N_c - N_f)$ – that is the reason for the fractional power in (3.23). Finally, comparing with (3.21) we see that the coefficient in (3.23) must be $N_c - N_f$

$$W_{dyn} = (N_c - N_f) \left(\frac{\Lambda^{3N_c - N_f}}{\det M} \right)^{1/(N_c - N_f)}. \quad (3.24)$$

For $N_f \geq N_c$, (3.23) does not make sense. For $N_f = N_c$, the exponent diverges. For $N_f > N_c$, the constraint (3.16) implies $\det M = 0$. Therefore, for $N_f \geq N_c$ massless flavors, the quantum theory has a moduli space of inequivalent vacua.

3.5. $N_f = N_c$ massless flavors [29]

Here the vacuum degeneracy cannot be lifted by W_{dyn} , so the moduli space is still parameterized by the gauge invariant fields M , B and \tilde{B} . But the classical constraint (3.15) they satisfy is modified (consistent with the symmetries (3.7) and (3.8))

$$\mathcal{M}_{qu} = \{M_g^f, B, \tilde{B} | \det M - B\tilde{B} = \Lambda^{2N_c}\}. \quad (3.25)$$

Note that this is a nonperturbative effect, proportional to a positive power of Λ . So, as is appropriate, the deformation is important only near the origin, and is negligible at large fields, relative to Λ , where the theory is weakly coupled. Indeed, the power in (3.25) is precisely that associated with a one instanton correction to the constraint in (3.15). The constraint (3.25) can be seen from (3.20), which for $N_f = N_c$ has $\det M = \Lambda^{2N_c}$, independent of m . (One can introduce sources for the operators B and \tilde{B} , to get the full constraint (3.25).) The space \mathcal{M}_{cl} in (3.15) was singular at $M = B = \tilde{B} = 0$, but the space (3.25) is everywhere smooth. The only light degrees of freedom of the low-energy effective theory are the moduli of (3.25).

The theory with the modified constraint can be described using a Lagrange multiplier X and a superpotential

$$W = X(\det M - B\tilde{B} - \Lambda^{2N_c}), \quad (3.26)$$

but it should be stressed that this is not a term in a Wilsonian action. There is no light field X and similarly, the mode of M , B and \tilde{B} which is proportional to $\det M - B\tilde{B}$ are not light. However, (3.26) is still a useful way to implement the constraint.

3.6. $N_f > N_c$ [33]

The vacuum degeneracy of the theory with massless flavors again cannot be lifted by W_{dyn} . Moreover, for all $N_f > N_c$, the classical moduli space constraints (3.14) cannot be deformed because no deformation would be compatible with holomorphy and the symmetries in (3.7) and (3.8). So there is a quantum moduli space of vacua, coinciding with the classical moduli space (3.14), $\mathcal{M}_q = \mathcal{M}_{cl}$. The singularity of these spaces at the origin indicates additional, massless degrees of freedom there. Their nature is clarified by a duality.

The original $SU(N_c)$ theory, with N_f flavors, is dual to another gauge theory based on the gauge group $SU(n = N_f - N_c)$ with spectrum of fields and couplings

	$SU(n)$	$[SU(N_f)_L]$	$SU(N_f)_R$	$U(1)_B$	$U(1)_R$	$U(1)_A]$	
φ	\mathbf{n}	$\overline{\mathbf{N}}_f$	$\mathbf{1}$	$\frac{N_c}{n}$	$1 - \frac{n}{N_f}$	1	
$\tilde{\varphi}$	$\overline{\mathbf{n}}$	$\mathbf{1}$	\mathbf{N}_f	$-\frac{N_c}{n}$	$1 - \frac{n}{N_f}$	1	
Φ	$\mathbf{1}$	\mathbf{N}_f	$\overline{\mathbf{N}}_f$	0	$2\frac{n}{N_f}$	-2	(3.27)
f	$\mathbf{1}$	$\overline{\mathbf{N}}_f$	\mathbf{N}_f	0	$2 - 2\frac{n}{N_f}$	2	
Λ^{3n-N_f}	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	0	$2N_f$	

(again, the group in [...] is a global symmetry) with canonical K for the fields φ , $\tilde{\varphi}$, and Φ , and superpotential

$$W = h \text{Tr } \Phi \varphi \tilde{\varphi}^T + \text{Tr } f \Phi. \quad (3.28)$$

As we will discuss, the coupling f is proportional to the mass of the electric quarks. In particular, if $m = 0$ in the electric theory, then $f = 0$ in the magnetic theory. $U(1)_A$ in (3.27) is anomalous but the other symmetries are not. The scale $\tilde{\Lambda}$ of the magnetic theory can be taken to be the same as the Λ of the electric theory, as we indicate in (3.27).

We refer to the original theory (3.7) as electric and to (3.27) as magnetic. This duality between the electric and the magnetic theories states that these two different theories have the same IR behavior. Better agreement between the two theories is obtained if we modify the Kähler potential by higher order terms.

Comments:

1. The anomaly free symmetries of the electric and the magnetic theories are the same. All 'tHooft anomaly matching conditions of these symmetries are satisfied.
2. The relations between the variables of the electric and magnetic descriptions are

$$M = \tilde{Q} Q^T = \alpha \Lambda \Phi \quad , \quad B = Q^{N_c} = \beta^n \Lambda^{2N_c - N_f} \varphi^n \quad (3.29)$$

with some dimensionless constants α and β . (Below we will determine α .) It is easy to check that the identification of operators (3.29) is consistent with the anomaly free symmetries. (An alternative description was given in [13], where the scales of the electric and magnetic theories were taken to be different; the descriptions are equivalent, as reviewed, e.g. in [9].)

3. For $\frac{3}{2}N_c < N_f < 3N_c$, the electric and magnetic theories are both UV free, and they differ in the UV. The two different UV free starting points flow under the renormalization group (RG) to the same interacting RG fixed point in the IR. A detailed discussion of this RG flow can be found, e.g. in [16].
4. For $N_c + 2 \leq N_f \leq \frac{3}{2}N_c$ the magnetic theory is IR free, with irrelevant interactions. The UV free electric theory flows at long distance to the IR free magnetic theory.
5. For $N_f = N_c + 1$ we can still use the variables in (3.27) but without the magnetic gauge fields and with the addition of a term proportional to $\det \Phi$ to the superpotential [29].
6. Turning on mass terms $\text{Tr } mQ\tilde{Q} = \text{Tr } mM$ in the electric theory is described by adding to the magnetic superpotential $\Lambda\alpha\text{Tr } m\Phi$. We will analyze it in detail in the next subsection.

3.7. Adding small mass terms

We again add (3.17)

$$W_{tree} = \text{Tr } m\tilde{Q}Q^T = \text{Tr } mM \quad (3.30)$$

but this time we take the masses (eigenvalues of m) small compared with $|\Lambda|$. Now, we should be able to reproduce the expectation values (3.20) from our low energy effective theory.

For $N_f < N_c$, the low energy theory has $W_{exact} = W_{dyn} + W_{tree}$, which gives precisely the superpotential (3.21). The Legendre transform in (3.20) ensures that setting $F_M^\dagger = -\partial_M W_{exact} = 0$ yields the N_c supersymmetric vacua at $\langle M \rangle$ given in (3.20).

As we mentioned above, for $N_f \geq N_c$, (3.21) is not meaningful as a superpotential on the moduli space. Rather, it should be viewed as a superpotential on a larger field space, where M is arbitrary rather than subject to (3.16), and which is meaningful only for nonzero m . As we are going to discuss, the dual theory provides an interpretation of this.

For $N_f = N_c$ (3.21) does not make sense. Instead, we can find $\langle M \rangle$ using the superpotential (3.26).

For $N_f = N_c + 1$ we have to add (3.21) to the superpotential (as commented after (3.29)).

For $N_f > N_c + 1$ the meaning of (3.21) is slightly more subtle. Consider moving the field $\Phi \sim M$ away from its expectation value. The superpotential (3.28) gives masses to the dual quarks φ . Using an expression like (3.3) for gluino condensation in the magnetic gauge group leads to

$$W = n(h^{N_f} \det \Phi \Lambda^{3n-N_f})^{\frac{1}{n}}. \quad (3.31)$$

where we set the scales of the magnetic and electric theories to be the same Λ . This agrees with (3.21) provided

$$h^{N_f} \det \Phi \Lambda^{3n-N_f} = (-1)^{N_f-N_c} \frac{\det M}{\Lambda^{3N_c-N_f}} \quad (3.32)$$

which fixes the coefficient α in (3.29)

$$M = (-1)^{1-\frac{N_c}{N_f}} h \Lambda \Phi. \quad (3.33)$$

Correspondingly, the coefficient f in (3.28) is related to the electric mass by

$$f = \alpha \Lambda m = (-1)^{1+\frac{N_c}{N_f}} m h \Lambda. \quad (3.34)$$

4. Dynamical supersymmetry breaking

We will now consider four typical examples of DSB. The common feature of these examples is that at low energies they can be given a semiclassical supersymmetric description as in the examples in section 2. The first three examples which are based on the dynamics of $N_f < N_c$, $N_f = N_c$ and $N_f > N_c$ were found in the 80s, 90s and 00s respectively. The fourth example, which is based on the dynamics of $N_f = 0$, allows us to easily convert any example in section 2 to a model of DSB.

Many other examples of DSB are known. Some of them are strongly coupled and do not admit a semiclassical supersymmetric description involving an effective Kähler potential and an effective superpotential (examples are $SU(5)$ or $SO(10)$ gauge theories with a single generation of quarks and leptons [34,35]). In other situations the question of supersymmetry breaking is inconclusive (e.g. an $SU(2)$ gauge theory with matter in the four dimensional representation [36]). In addition, many variants of the examples below are known and they exhibit various interesting features (see, e.g. [37-47]). Additional review and references can be found in e.g. [48,49,6,7]

4.1. The (3,2) model [38]

The gauge group is

$$SU(3) \times SU(2) \tag{4.1}$$

and we have chiral superfields: Q in $(\mathbf{3}, \mathbf{2})$, \tilde{u} in $(\bar{\mathbf{3}}, \mathbf{1})$, \tilde{d} in $(\bar{\mathbf{3}}, \mathbf{1})$, L in $(\mathbf{1}, \mathbf{2})$. For $W_{tree} = 0$, the classical moduli space is given by arbitrary expectation values of the gauge invariants

$$X_1 = Q\tilde{d}L \quad , \quad X_2 = Q\tilde{u}L \quad , \quad Z = QQ\tilde{u}\tilde{d}. \tag{4.2}$$

Both gauge groups are Higgsed on this classical moduli space. We add to the model a tree level superpotential

$$W_{tree} = \lambda Q\tilde{d}L = \lambda X_1. \tag{4.3}$$

This theory has a $U(1)_R$ symmetry, with $R(Q) = -1$, $R(\tilde{u}) = R(\tilde{d}) = 0$, $R(L) = 3$. A crucial aspect of (4.3) is that it lifts all of the classical D-flat directions. Therefore, the theory does not have any runaway directions.

Using the global symmetries (including those under which the couplings, treated as background chiral superfields, are charged), the exact superpotential for the fields (4.2) is

$$W_{exact} = \frac{\Lambda_3^7}{Z} + \lambda X_1. \tag{4.4}$$

The first term in (4.4) is W_{dyn} , which is generated by an $SU(3)$ instanton. This theory dynamically breaks supersymmetry⁸.

For $\lambda \ll 1$, the vacuum is at large expectation value for the fields. Since the gauge groups are Higgsed at a high energy scale, their running coupling is weak. Because the theory is weakly coupled for the fields in this limit, we have $K \approx K_{classical}$, so the Kähler potential is under control. It is then easy to find that the field expectation values and the vacuum energy density at the minimum are

$$v \sim \Lambda_3/\lambda^{1/7} \quad ; \quad V = M_S^4 \sim |\lambda^{10/7}\Lambda_3^4| \tag{4.5}$$

⁸ A quick way to see that is to note that W_{dyn} pushes Z away from the origin, which spontaneously breaks the $U(1)_R$ symmetry. There is thus a compact moduli space of vacua, whose modulus is the massless Goldstone boson. If supersymmetry were unbroken, the Goldstone boson would have a scalar superpartner, which would lead to a non-compact moduli space - but that cannot be the case, because W_{tree} lifts all of the classical flat directions [34].

(the precise coefficient can be computed, using $K = K_{cl}$). Note that, to justify $K \approx K_{cl}$, we need $v \gg \Lambda_3$ and also $v \gg \Lambda_2$, and the latter condition requires $\Lambda_3 \gg \lambda^{1/7} \Lambda_2$. In addition to the massless Goldstino, there is a massless Goldstone boson, because the vacuum spontaneously breaks the $U(1)_R$ symmetry.

The above analysis is valid when $\Lambda_3 \gg \Lambda_2$. As seen from the expressions above, in this limit the $SU(2)$ gauge dynamics scale Λ_2 does not appear directly in the approximate answers (4.5). The $SU(2)$ gauge group is weakly coupled at the scale Λ_3 , and the role of the $SU(2)$ gauge symmetry is simply to restrict the possible superpotential couplings, and its classical gauge potential lifts certain directions in field space thus avoiding runaway. The fact that Λ_2 does not enter into (4.4) fits with the fact that the $SU(2)$ gauge group has $N_f = N_c$. So, as reviewed in section 3.5, it does not contribute to W_{dyn} , but instead leads to the quantum modified moduli space constraint [29] of (3.25). The quantum modified moduli space is neglected in the analysis above, and that is justified when $\Lambda_3 \gg \Lambda_2$.

On the other hand, in the limit $\Lambda_2 \gg \Lambda_3$, the $SU(2)$ group becomes strong first in the RG flow to the IR, and it is then essential to include the quantum modified moduli space constraint. Below the scale Λ_2 , the light fields are $q = QL/\Lambda_2$, in the $\mathbf{3}$ of $SU(3)$, and $\tilde{q} = Q^2/\Lambda_2$, and \tilde{u} and \tilde{d} , all in the $\bar{\mathbf{3}}$, subject to the quantum constraint $q\tilde{q} = \Lambda_2^2$. The constraint breaks $SU(3)$ to $SU(2)' \subset SU(3)$, at the scale Λ_2 , and q and \tilde{q} are Higgsed. The fields \tilde{u} and \tilde{d} each decompose as $\mathbf{3}, \bar{\mathbf{3}} \rightarrow \mathbf{2} + \mathbf{1}$ under $SU(3) \rightarrow SU(2)'$, so we have $SU(2)'$ with $N_f = 1$ flavor, plus two singlets. In the limit, we obtain a superpotential which is similar to (4.4), but with a different interpretation of the terms. In particular, the λX_1 term is interpreted as $\lambda \Lambda_2^2 S_d$, where S_d is the $SU(2)'$ singlet from \tilde{d} . In the $\lambda^{1/7} \Lambda_2 \gg \Lambda_3$ limit, the $SU(2)' \subset SU(3)$ dynamics is insignificant, and we have $M_S^4 = \alpha |\lambda^2 \Lambda_2^4|$, where α is a positive $\mathcal{O}(1)$ Kähler potential coefficient, $K \supset \frac{1}{\alpha} S_d \bar{S}_d$ that cannot be directly calculated [50].

4.2. Modified moduli space example [50,51]

Consider the $SU(N_c)$ theory with $N_f = N_c$ and add fields $S_a^{\tilde{a}}$, b and \tilde{b} and a superpotential (up to coupling constants)

$$W_{tree} = \text{tr } S \tilde{Q} Q^T + b \det \tilde{Q} + \tilde{b} \det Q. \quad (4.6)$$

Classically $Q = \tilde{Q} = 0$. In the quantum theory we get the effective superpotential (see (3.26))

$$W_{effective} = \text{tr } S M + b \tilde{B} + \tilde{b} B + X(\det M - B \tilde{B} - \Lambda^{2N_c}) \quad (4.7)$$

which breaks SUSY. This breaking is dynamical. It depends on the IR confinement of the $N_f = N_c$ theory, from quarks and gluons in the UV, into the composite fields M and B and \tilde{B} in the IR and on the quantum deformation of the moduli space by Λ^{2N_c} in (3.25).

Let us specialize to $N_f = N_c = 2$, where the fundamentals and anti-fundamentals can be written as $2N_f = 4$ fundamentals Q^{fc} , $f = 1 \dots 4$, $c = 1, 2$. The gauge invariants are $U^{fg} = Q^{fc}Q^{gd}\epsilon_{cd}$, in the $\mathbf{6}$ of the global $SU(4) \cong SO(6)$ flavor symmetry. To emphasize that it is an $SO(6)$ vector we will also express it as

$$\vec{V} = (V^1 = \frac{1}{2}(U^{12} + U^{34}), V^2 = \frac{i}{2}(U^{12} - U^{34}), \dots). \quad (4.8)$$

The quantum moduli space constraint (3.26) for this case is [29]

$$\text{Pf } U = U^{12}U^{34} - U^{13}U^{24} + U^{14}U^{23} = \vec{V} \cdot \vec{V} = \Lambda^4. \quad (4.9)$$

We add singlets \vec{S} , also in the $\mathbf{6}$ of the global flavor $SO(6)$, with superpotential

$$W_{tree} = \frac{1}{2}hS_{fg}Q^{fc}Q^{gd}\epsilon_{cd} = 2h\vec{S} \cdot \vec{V}, \quad (4.10)$$

where S_{fg} is related to \vec{S} as in (4.8) and the factor of 2 arises from this change of notation. Unlike (4.6)(4.7), here we have explicitly exhibited the coupling constant h . There is a conserved $U(1)_R$ symmetry, with $R(Q) = 0$, and hence $R(\vec{V}) = 0$, and $R(\vec{S}) = 2$. Because $\overline{F}_{\vec{S}} = -2h\vec{V}$, the constraint (4.9) implies that $F_{\vec{S}} \neq 0$, so SUSY is broken.

Let us analyze it in more detail. We start with the classical theory. The superpotential coupling $\frac{1}{2}hS_{fg}Q^{fc}Q^{gd}\epsilon_{cd}$ lifts all the flat directions with nonzero Q . So the classical moduli space is the space of \vec{S} . Moving far out along these flat directions the fundamental quarks are massive and can be integrated out. The low energy $SU(2)$ gauge theory has scale $\Lambda_L^6 = \Lambda^4 h^2 \vec{S} \cdot \vec{S}$, and its gluino condensation generates

$$W_{low} = 2(\Lambda_L^6)^{1/2} = 2\left(h^2\Lambda^4\vec{S} \cdot \vec{S}\right)^{\frac{1}{2}}. \quad (4.11)$$

Using the symmetries and holomorphy it is easy to see that (4.11) is exact. Now it is clear that for any nonzero \vec{S} the superpotential is not stationary, and the point $\vec{S} = 0$ is singular and needs to be examined in detail.

Before we conclude that supersymmetry is broken away from the origin we have to examine the potential at infinity to make sure that there is no runaway. Using the classical Kähler potential for \vec{S} which is canonical, the superpotential (4.11) leads to

$$V_{cl} = 4|h\Lambda^2|^2 \frac{\vec{S} \cdot \vec{S}}{|\vec{S} \cdot \vec{S}|}. \quad (4.12)$$

Depending on the direction in the space this expression either diverges at infinity or asymptotes to a constant $4|h\Lambda^2|^2$. It is straightforward to include the one loop correction to this expression. This situation is very similar to the discussion around (2.54). The fundamental quarks Q are massive and their loop leads to logarithmic corrections to the potential which makes it grow at infinity. We conclude that the pseudoflat directions with broken supersymmetry in (4.12) is lifted and pushes the system to smaller values of \vec{S} .

When $|h\vec{S}| \ll |\Lambda|$ the superpotential (4.10) gives the quarks small masses and they cannot be integrated out so easily. But then we can use our understanding of the macroscopic theory, where the $SU(2)$ gauge fields and matter of the microscopic theory are replaced in the IR with the fields \vec{V} , subject to the constraint (4.9). We solve this constraint as

$$\vec{V} = \Lambda(\sqrt{\Lambda^2 - \vec{v}^2}, \vec{v}), \quad (4.13)$$

where \vec{v} is an $SO(5)$ vector. We will assume that $|\vec{v}| \ll |\Lambda|$. This assumption is valid up to symmetry transformations near the origin of the classical theory, where we expect to find our ground state. Similarly, we write $\vec{S} \equiv (S_1, \vec{s})$, where \vec{s} is an $SO(5)$ vector. Then (4.10) is

$$W = 2h\Lambda S_1 \sqrt{\Lambda^2 - \vec{v}^2} + 2h\Lambda \vec{v} \cdot \vec{s} \approx 2h\Lambda^2 S_1 - hS_1 \vec{v}^2 + 2h\Lambda \vec{v} \cdot \vec{s}. \quad (4.14)$$

The Kähler potential for the fields S_1 , \vec{s} , and \vec{v} is smooth, and can be taken to be

$$K = S_1 \bar{S}_1 + \vec{s} \cdot \bar{\vec{s}} + \frac{1}{\alpha} \vec{v} \cdot \bar{\vec{v}} + \mathcal{O}\left(\frac{1}{|\Lambda|^2}\right), \quad (4.15)$$

where α is an $\mathcal{O}(1)$ coefficient that we cannot determine.

Up to symmetry transformations, the vacua have arbitrary $\langle S_1 \rangle$, and $\vec{v} = \vec{s} = 0$. This leads to a seven real dimensional pseudomoduli space. Its dimensions include the two non-compact directions given by $\langle S_1 \rangle$, and five real Goldstone bosons living on $SO(6)/SO(5) \cong S^5$, coming from components of \vec{v} and \vec{s} .

We can integrate out the massive modes of \vec{v} to find an effective superpotential. For $\vec{s} = 0$ it is $W_{eff} = 2h\Lambda^2 S_1$, and more generally, it is given by $W_{eff} = 2 \left(h^2 \Lambda^4 \vec{S} \cdot \vec{S} \right)^{\frac{1}{2}}$ which agrees with (4.11).

Supersymmetry is broken by $-\bar{F}_{S_1} = 2h\Lambda^2 \neq 0$. Since F_{S_1} is generated by dimensional transmutation, the supersymmetry breaking is dynamical. The massless Goldstino comes from S_1 .

We should now examine how this pseudomoduli space is lifted in the quantum theory. This is easily done using the low energy theory based on the superpotential (4.14)

and the Kähler potential (4.15) by noticing that it is a multi-field analog of the $y = 1$ O’Raifeartaigh model. The one-loop potential (2.47) lifts the degeneracy and leads to a supersymmetry breaking minimum at $\vec{S} = 0$ [52]. At this vacuum the global $SO(6)$ symmetry is spontaneously broken to $SO(5)$ by the constraint (4.9), but the $U(1)_R$ symmetry is unbroken. So there is a five real dimensional, compact space of supersymmetry breaking vacua, given by the Goldstone boson manifold $SO(6)/SO(5) \cong S^5$.

For $h \ll 1$, we can have large S_1 and still use the low energy effective theory provided

$$|hS_1| \ll |\Lambda| \ll |S_1|. \quad (4.16)$$

In this limit, the behavior of the one-loop potential (2.47), computed in the low-energy effective field theory, asymptotes as in (2.66) to

$$V^{(1)} \rightarrow \gamma_{macro}^{(1)} \log \left(\frac{|2hS_1|^2}{M_{cutoff}^2} \right) |2h\Lambda^2|^2. \quad (4.17)$$

As we have reviewed, the dependence on M_{cutoff} can be absorbed into the renormalization of h . The coefficient in (4.17) is the anomalous dimension of the pseudomodulus, computed in the macroscopic theory. It depends on the $\mathcal{O}(1)$ unknown constant α in (4.15). Since $\gamma_{macro}^{(1)} > 0$, the potential (4.17) is an increasing function of $|S_1|$.

On the other hand, as we remarked above, if $|\Lambda| \ll |hS_1|$, then, we should instead use the microscopic theory. The result for the potential is similar to (4.17), though with a different, but again positive, numerical coefficient $\gamma_{micro}^{(1)}$ for the one-loop anomalous dimension of S_1 , computed from the microscopic Q fields running in the loop [53]. We cannot compute the potential in the intermediate range, $|hS_1| \sim |\Lambda|$, but in all calculable regions the potential slopes toward the origin, $S_1 = 0$.

Deforming the model

Consider adding a $U(1)_R$ breaking, but $SO(6)$ invariant, term

$$\Delta W = \frac{1}{2} \epsilon \vec{S}^2 \quad (4.18)$$

to (4.10). Adding this to (4.11) or (4.14), the theory has a five complex dimensional, non-compact, moduli space of supersymmetric vacua

$$\vec{S} = -\frac{2h}{\epsilon} \vec{V} \quad ; \quad \vec{V}^2 = \Lambda^4. \quad (4.19)$$

For $|\epsilon| \gg |\Lambda|$, the fields \vec{S} are heavy and can be integrated out. The low energy theory is simply the $SU(2)$ theory with four massless doublets and no superpotential (the cubic couplings of (4.10) do not lead to a quartic superpotential when \vec{S} is integrated out). This has a moduli space which is reproduced by (4.19).

For $|\epsilon| \ll |\Lambda|$, the \vec{S} fields are light, and need to be included in the low energy theory; i.e. we add (4.18) to (4.14). As we take $\epsilon \rightarrow 0$, the SUSY vacua (4.19) run off to infinity. In addition to these supersymmetric ground states at large $|\vec{S}|$, we still have the compact moduli space of supersymmetry breaking vacua discussed following (4.14), with \vec{S} near the origin. For $|\epsilon| \ll |\Lambda|$ these metastable, supersymmetry breaking states are very long lived. Finally, as $\epsilon \rightarrow 0$ the supersymmetric states disappear from the Hilbert space and we are left with only the metastable states.

Note that these theories provide examples of nonchiral theories that dynamically break supersymmetry. How is that compatible with the Witten index [8]? The argument based on the Witten index relies on adding mass terms to the theory and tracking the supersymmetric states as the mass is removed. In this problem we can add two possible mass terms. First, we can add mass terms for the fundamental quarks. This is done in the effective theory by adding $\vec{m} \cdot \vec{V}$ to the superpotential. But this has no effect because \vec{m} can be absorbed in a shift of \vec{S} . Second, if we add (4.18), \vec{S} is massive. For large mass it leads to the non-compact moduli space of supersymmetric states (4.19). For small mass we also find the compact moduli space of supersymmetry breaking metastable states, and as $\epsilon \rightarrow 0$ the supersymmetric states disappear from the Hilbert space and supersymmetry is broken.

4.3. Metastable states in SQCD [9]

Consider SQCD with $N_c + 1 \leq N_f < \frac{3}{2}N_c$, with small quark masses

$$|\text{Eigenvalues}(m)| \ll |\Lambda|. \quad (4.20)$$

The range of N_f is such that the magnetic dual [33] of section 3.6 is the IR free, low-energy effective field theory. We thus analyze the groundstates in the magnetic dual, with superpotential

$$h\text{Tr } \Phi\varphi\tilde{\varphi} + \alpha\Lambda\text{Tr } m\Phi. \quad (4.21)$$

This is the same as the theory we studied in (2.41)(2.43) with the identification⁹

$$\alpha\Lambda m = f. \tag{4.22}$$

For simplicity, we will take m (and therefore also f) to be proportional to the unit matrix, thus preserving the global $SU(N_f)$.

As discussed following (2.41), this low energy theory has a supersymmetry breaking minimum (2.67). All non-Goldstone modes have non-tachyonic masses there, from the one-loop potential, which is computed via (2.47) in the low-energy dual theory. The fact that the magnetic theory is IR free ensures that higher loops are suppressed, and in particular cannot invalidate the results from the one-loop potential.

We thus conclude that SQCD has metastable dynamical supersymmetry breaking vacua. In terms of the microscopic electric SQCD theory, the DSB vacua (2.67) have zero expectation value for the meson fields, $\langle M \rangle = 0$, and non-zero expectation value of some baryon fields, $\langle B \rangle \neq 0$ and $\langle \tilde{B} \rangle \neq 0$, which follow from the non-zero $\langle \varphi \rangle$ and $\langle \tilde{\varphi} \rangle$ in (2.67). In terms of the IR dual magnetic theory, these vacua are semi-classical, but in terms of the microscopic, electric SQCD they are not, they are strongly quantum-mechanical.

As noted after (2.67), the supersymmetry breaking vacua (2.67) spontaneously break the global symmetries, from $G = SU(N_f) \times U(1)_B$ to $H = SU(N_f - N_c) \times SU(N_c) \times U(1)$. Associated with that, there is a compact moduli space of vacua, the manifold of massless Goldstone bosons¹⁰, $\mathcal{M}_{vac} = G/H$. Note that the DSB vacua have an assortment of massless fields: the G/H Goldstone bosons and a number of massless fermions including the Goldstino, which come from the fermionic components of the fields Φ_0 in (2.46). This is to be contrasted with the naive expectation that there should be no massless fields (and, in particular, no candidate Goldstino for DSB to occur), since the quarks Q all have a mass m , and the low-energy SYM gets a mass gap. The dual magnetic theory shows that this naive expectation is incorrect.

SQCD also has N_c supersymmetric vacua, with mass gap and $\langle M \rangle \sim \langle \Phi \rangle \neq 0$, and $\langle B \rangle = \langle \tilde{B} \rangle = 0$. These supersymmetric vacua arise from the effective interaction (3.31)

⁹ The global vector $U(1)$ symmetry in (2.41) is normalized differently than the baryon number symmetry in (3.27). Also, the $U(1)_R$ symmetry in (3.27) is anomaly free but it is broken by the mass term, while in (2.41) we took $U(1)_R$ to preserve the term linear in Φ but it is anomalous.

¹⁰ In various generalizations of this example, these compact moduli spaces of DSB vacua can support topological solitons, which can be (meta) stable, see [54] for a fuller discussion.

which, as explained earlier, are obtained from gluino condensation in the magnetic theory. Thus, in terms of the magnetic dual theory, supersymmetry is non-perturbatively restored, in a theory that breaks supersymmetry at tree-level. Indeed, from the point of view of the theory (2.41)(2.43), the R-symmetry is anomalous and is explicitly broken (this is manifest with the interaction (3.31)), and therefore supersymmetry is restored. As long as N_f is in the free magnetic range, $N_f < \frac{3}{2}N_c$, the supersymmetry restoring interaction (3.31) is irrelevant at the DSB vacua near $\Phi = 0$. Then the DSB and the SUSY vacua are sufficiently separated for the DSB vacua to be meaningful.

The small mass condition (4.20) has the following useful consequences:

1. It ensures that the analysis within the low-energy effective field theory (the magnetic dual) is valid: the superpotential coupling $f \sim m\Lambda$ is then safely below the UV cutoff, Λ , of the magnetic dual theory.
2. It ensures that effects from the microscopic (electric) theory do not invalidate the macroscopic analysis of supersymmetry breaking and the one loop stabilization of the vacua (2.67). A way to see this is to note that the one-loop potential gives all (non-Goldstone) pseudomoduli mass squares of order $|f| \sim |m\Lambda|$ (much as in (2.65)) which is non-analytic in the superpotential coupling $f \sim m\Lambda$. This reflects the fact that it comes from integrating out modes which become massless in this limit. On the other hand, any effects from the microscopic theory must be analytic in m , and then (4.20) ensures that such effects are subleading to (2.65).
3. The condition (4.20) also ensures that the supersymmetric vacua (3.20) can be seen in the magnetic effective theory, as then (3.20) is safely below its cutoff, $|\langle M \rangle| \ll |\Lambda|$.
4. It ensures that the metastable state is parametrically long lived. The tunneling probability is $\sim \exp(-S_{bounce})$, where $S_{bounce} \sim \Delta\Phi^4/V_{meta}$, with $\Delta\Phi$ the separation in field space between the metastable and the supersymmetric vacua, and $V_{meta} = M_s^4$. For small masses (4.20), S_{bounce} is parametrically large, and thus the metastable DSB vacua can be made parametrically arbitrarily long lived.

This kind of DSB appears generic. It exists also in similar $SO(N_c)$ and $SP(N_c)$ gauge theories [9], and many generalizations of it were found recently (see e.g. [55-64]). Also, the early universe favors populating the DSB vacua over the SUSY vacua. One reason for that is the large degeneracy of the Goldstone boson moduli space of DSB vacua, versus the discrete N_c mass gapped supersymmetric vacua. Another reason is that the DSB vacua are closer to the origin of the moduli space than the supersymmetric vacua, and that is favored by the thermal effective potential [65-68].

4.4. Naturalizing (retrofitting) models [21,60]

As we stressed in the introduction (around equation (1.1)), in order for a model of supersymmetry breaking to be fully natural, all scales which are much smaller than the UV cutoff M_{cutoff} should arise via dimensional transmutation. To be fully natural, the Lagrangian cannot have any super-renormalizable (relevant) operators, since they are naturally of order a positive power of M_{cutoff} . The Lagrangian should have only renormalizable (marginal) operators and non-renormalizable (irrelevant) operators, which are suppressed by inverse powers of M_{cutoff} . Any needed relevant operators should then arise dynamically, with exponentially suppressed coefficients, as in (1.1).

A simple way to achieve that is the following. Consider an “unnatural model” of supersymmetry breaking like one of the models in section 2, with superpotential terms like $W_{tree} \supset f\mathcal{O}_1 + m\mathcal{O}_2$, where \mathcal{O}_1 is some dimension one operator, \mathcal{O}_2 is a dimension two operator, and $f \equiv \mu^2$. We want the mass scales m and μ to be much less than M_{cutoff} . Such a model can easily be naturalized (or retrofitted) by removing these couplings from the theory and replacing them with interactions with the operator $S \equiv -\text{Tr} W_\alpha^2/32\pi^2$ of some added, but otherwise decoupled, pure Yang-Mills theory (with no charged matter):

$$\int d^2\theta \left[-\frac{8\pi^2}{g^2(M_{cutoff})} + \frac{a_1}{M_{cutoff}}\mathcal{O}_1 + \frac{a_2}{M_{cutoff}^2}\mathcal{O}_2 \right] S, \quad (4.23)$$

where $a_{1,2}$ are dimensionless coefficients of order one, so the couplings in (4.23) are natural.

The pure Yang-Mills theory entering in (4.23) has a dynamically generated scale Λ , which satisfies $\Lambda \ll M_{cutoff}$, as in (1.1). For energies below the scale Λ , the added Yang-Mills theory becomes strong and leads to gaugino condensation $\langle S \rangle = \Lambda^3$. Substituting this in (4.23) we find

$$\int d^2\theta \left[\frac{a_1\Lambda^3}{M_{cutoff}}\mathcal{O}_1 + \frac{a_2\Lambda^3}{M_{cutoff}^2}\mathcal{O}_2 \right]. \quad (4.24)$$

Thus we generate super-renormalizable couplings in the superpotential with $\mu^2 \sim \Lambda^3/M_{cutoff} \ll M_{cutoff}^2$ and $m \sim \Lambda^3/M_{cutoff}^2 \ll M_{cutoff}$. For example, the O’Raifeartaigh model of section 2.5 can be naturalized by replacing (2.28) with

$$\int d^2\theta \left[\frac{1}{2}hX\phi_1^2 + \left(-\frac{8\pi^2}{g^2(M_{cutoff})} + \frac{a_1}{M_{cutoff}}X + \frac{a_2}{M_{cutoff}^2}\phi_1\phi_2 \right) S \right]. \quad (4.25)$$

More generally, we can use couplings like (4.23) with different gauge groups or with couplings with higher powers of W_α . This way, every unnatural model can be easily naturalized.

This naturalization procedure is not unique. A given macroscopic theory can be naturalized in more than one way. Consider, for example, the macroscopic models based on the rank condition of section 2.6. One way to naturalize them is to replace the last term in (2.43) with $\frac{1}{M_{cutoff}} \text{Tr} \Phi \text{Tr} W'_\alpha{}^2$, where W'_α is the field strength of some other pure Yang-Mills theory, with scale Λ' ; this leads to $f \sim \Lambda'^3/M_{cutoff}$. Alternatively, we can first view this theory as the low energy approximation of a SQCD theory, as in section 4.3. This theory is not yet fully natural because of the existence of the quark mass term $m \text{Tr} \tilde{Q} Q^T$ in the Lagrangian. As in (4.22), this leads to $f \sim m\Lambda$, which is dynamical, but not yet fully natural because we need (4.20), $|m| \ll |\Lambda| \ll M_{cutoff}$. It can be made fully natural by replacing the mass term of the UV lagrangian with $\frac{1}{M_{cutoff}^2} \text{Tr} \tilde{Q} Q^T \text{Tr} W'_\alpha{}^2$ [63]. This leads to $m \sim \Lambda'^3/M_{cutoff}^2$, so $|m| \ll |\Lambda|$ is natural, and $f \sim \Lambda\Lambda'^3/M_{cutoff}^2$.

Throughout this analysis, we have viewed the theory in an expansion in powers of M_{cutoff}^{-1} . For example, in (4.25) we did not consider higher dimension operators like $\frac{X^2}{M_{cutoff}^2} W_\alpha^2$. As another example, gluino condensation in (4.25) does not simply replace $\left(-\frac{8\pi^2}{g^2} + \frac{X}{M_{cutoff}}\right) S$ with $\frac{X}{M_{cutoff}} \Lambda^3$. More precisely, following the analysis in section 3.1, for an $SU(N_c)$ gauge theory it replaces it with

$$N_c \Lambda^3 \exp\left(\frac{X}{N_c M_{cutoff}}\right) \approx N_c \Lambda^3 + \frac{X}{M_{cutoff}} \Lambda^3, \quad (4.26)$$

where we neglected higher order terms in M_{cutoff}^{-1} in the latter expression.

This expansion in powers of M_{cutoff}^{-1} is significant. It is well known that one can trigger supersymmetry breaking by coupling a chiral superfield to a Yang-Mills theory via higher dimension operators and using gluino condensation [69-71]. This usually leads to runaway behavior, as is clear from the first expression in (4.26). However, since we content ourselves with finding supersymmetry breaking only in a metastable state, we can focus on a particular region in field space and ignore possible vacua elsewhere in field space. This focusing on a region in field space is achieved by the expansion in M_{cutoff}^{-1} we mentioned above. Therefore, this naturalization procedure leads to acceptable, metastable, dynamical supersymmetry breaking.

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