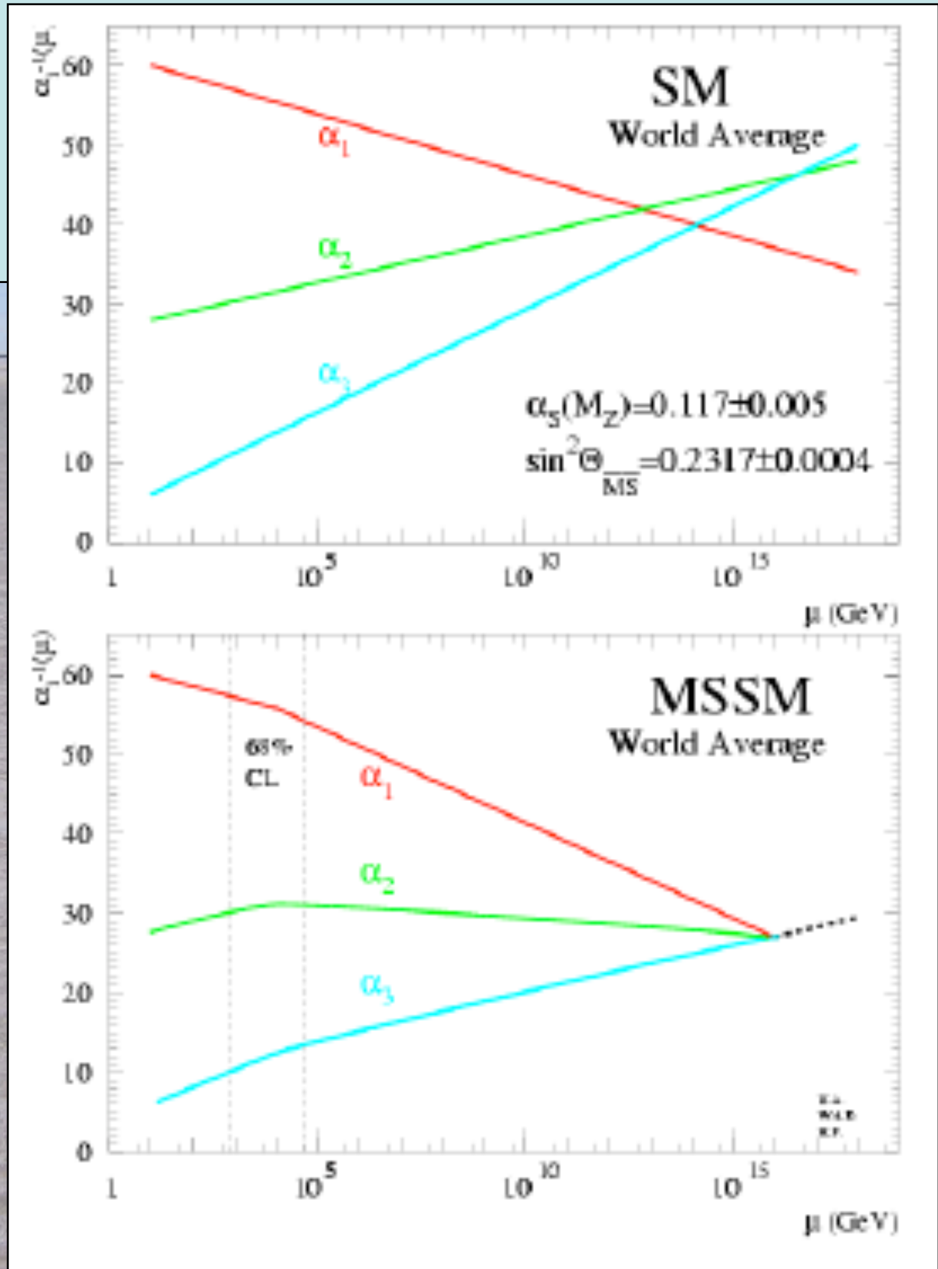

SM
World Average
α_1
α_2
α_3
$\alpha_s(M_Z)$=0.117±0.005
$\sin^2\Theta_{\overline{MS}}$=0.2317±0.0004
μ (GeV)
MSSM
World Average
68% CL
α_1
α_2
α_3
μ (GeV)

How to Grand Unify?

- Exploit logarithmic evolution of gauge couplings:

$$\frac{dg_a^2}{dt} = b_a \frac{g_a^4}{16\pi^2} + \dots \quad \rightarrow \quad \frac{m_{GUT}}{m_W} = \exp\left(\mathcal{O}\left(\frac{1}{\alpha_{em}}\right)\right)$$

- Combination measurable at low energies:

$$\sin^2\theta_W(m_Z) = \frac{g'^2}{g_2^2 + g'^2} = \frac{3}{5}\frac{g_1^2(m_Z)}{g_2^2(m_Z) + \frac{3}{5}g_1^2(m_Z)} = \frac{1}{1+8x}[3x + \frac{\alpha_{em}(m_Z)}{\alpha_3(m_Z)}] \quad x \equiv \frac{1}{5}(\frac{b_2 - b_3}{b_1 - b_2})$$

- Values in SM and MSSM:

$$\begin{aligned}
\frac{4}{3}N_G - 11 &\leftarrow b_3 \rightarrow 2N_G - 9 = -3 \\
\frac{1}{6}N_H + \frac{4}{3}N_G - \frac{22}{3} &\leftarrow b_2 \rightarrow \frac{1}{2}N_H + 2N_G - 6 = +1 \\
\frac{1}{10}N_H + \frac{4}{3}N_G &\leftarrow b_1 \rightarrow \frac{3}{10}N_H + 2N_G = \frac{33}{5} \\
\frac{23}{218} = 0.1055 &\leftarrow x \rightarrow \frac{1}{7}\,.
\end{aligned}$$

- Experiment:

$$\alpha_{em} = \frac{1}{128};\ \alpha_3(m_Z) = 0.119 \pm 0.003,\ \sin^2\theta_W(m_Z) = 0.2315 \quad \rightarrow \quad x = \frac{1}{6.92 \pm 0.07}$$

MSSM Calculation

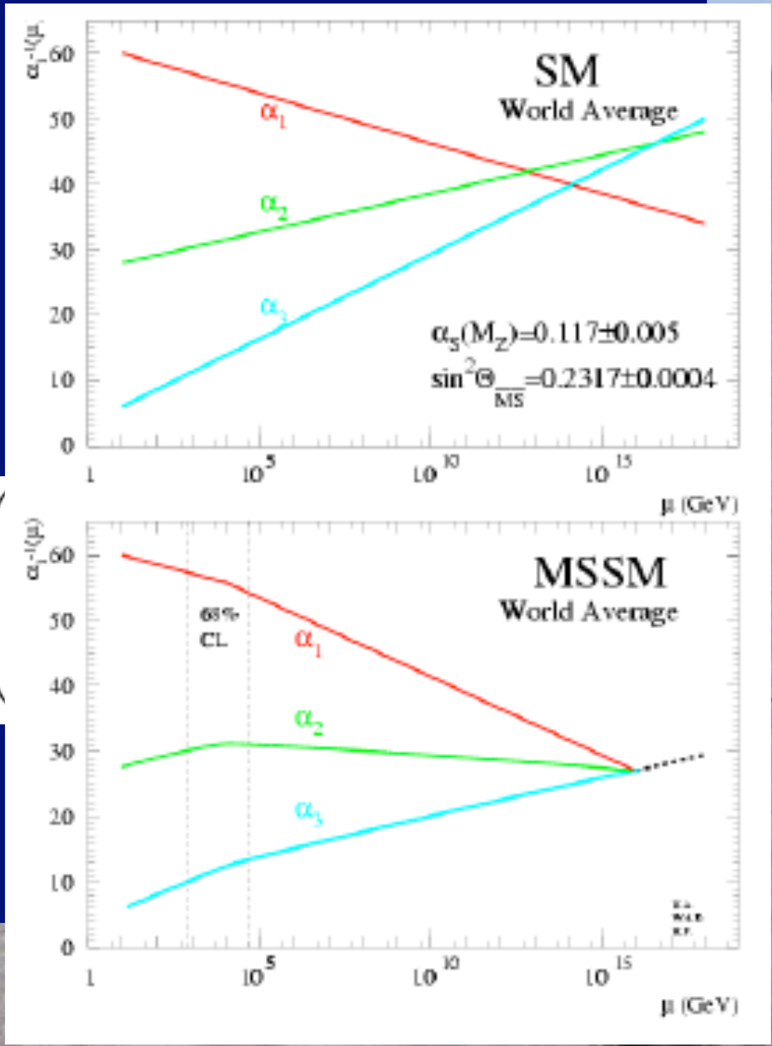
- At one loop:

$$b_i = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + N_g \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} + N_H \begin{pmatrix} \frac{3}{10} \\ \frac{1}{2} \\ 0 \end{pmatrix}$$

- Two loops:

$$b_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -24 & 0 \\ 0 & 0 & -54 \end{pmatrix} + N_g \begin{pmatrix} \frac{38}{15} & \frac{6}{5} & \frac{88}{15} \\ \frac{2}{5} & 14 & 8 \\ \frac{11}{5} & 3 & \frac{68}{3} \end{pmatrix} + N_H \Bigg($$

- Results are stable



Choice of GUT Group

- Should accommodate the known fermions:

$$(\nu, e)_L \in (1,2)\ ,\quad (u,d)_L \in (3,2)\ ,\quad e^c_L \in (1,1)\ ,\quad u^c_L\ ,\quad d^c_L \in (\bar{3},1)$$

- Need group with complex representations
- Preferably irreducible: $\sum_{q,\ell} Q_i = 3Q_u + 3Q_d + Q_e = 0$
- List of candidate groups of rank 4:

$$Sp(8)\ ,\quad SO(8)\ ,\quad SO(9)\ ,\quad F_4\ ,\quad SU(3)\times SU(3)\ ,\quad SU(5)$$

- BUT: real, real, real, real, $\Sigma_q Q_q \neq 0$, OK!

Particles in SU(5)

- Gauge bosons:

$$\begin{pmatrix} & & & \vdots & \bar{X} & \bar{Y} \\ & g_{1,\dots,8} & & \vdots & \bar{X} & \bar{Y} \\ & & & \vdots & \bar{X} & \bar{Y} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ X & X & X & \vdots & & \\ & & & \vdots & W_{1,2,3} & \\ Y & Y & Y & \vdots & & \end{pmatrix}$$

- Matter particles: $\underline{\bar{5}} = (\bar{3}, 1) + (1, 2)$, $\underline{10} = (3, 2) + (\bar{3}, 1) + (1, 2)$

$$\bar{F} = \begin{pmatrix} d_R^c \\ d_Y^c \\ d_B^c \\ \cdots \\ -e^- \\ \nu_e \end{pmatrix}_L , \quad T = \begin{pmatrix} 0 & u_B^c & -u_Y^c & \vdots & -u_R & -d_R \\ -u_B^c & 0 & u_R^c & \vdots & -u_Y & -d_Y \\ u_Y^c & -u_R^c & 0 & \vdots & -u_B & -d_B \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ u_R & u_Y & u_B & \vdots & 0 & -e^c \\ d_R & d_Y & d_B & \vdots & e^c & 0 \end{pmatrix}_L$$

Higgs bosons in SU(5) GUT

- Adjoint 24-dimensional Higgs to break

  SU(5) → SU(3) x SU(2) x U(1) of SM

$$< 0|\Phi|0 >= \begin{pmatrix} 1 & 0 & 0 & \vdots & 0 & 0 \\ 0 & 1 & 0 & \vdots & 0 & 0 \\ 0 & 0 & 1 & \vdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \vdots & -\frac{3}{2} & 0 \\ 0 & 0 & 0 & \vdots & 0 & -\frac{3}{2} \end{pmatrix} \times \mathcal{O}(m_{GUT})$$

- 5-dimensional Higgs to break SU(2) x U(1)→ U(1)

$$< 0|\phi|0 >= (0, 0, 0, 0, 1) \times 0(m_W)$$

- Susy needed to prevent large GUT v.e.v. from leaking → small electroweak Higgs v.e.v.

Particle Masses in SU(5)

- Quarks and leptons in same GUT multiplet → relations between their masses
- Simple symmetry relations before renormalization

  e.g., $m_b = m_\tau$ in minimal SU(5) GUT
- Renormalized analogously to gauge couplings:

  non-susy case $\frac{m_b}{m_\tau} \simeq \left[\ln\left(\frac{m_b^2}{m_X^2}\right)\right]^{\frac{12}{33-2N_q}}$
- m_τ = 1.78 GeV used to predict m_b ~ 5 GeV a few weeks before its discovery!
- Different formula, similar number in susy SU(5)

Bigger GUT Models

- First look at groups of rank 5 with suitable complex representations
- Only suitable candidate is SO(10)
- Each generation in irreducible

$$16 = 10 + 5^* + 1 \text{ of } SU(5)$$

- Next step is rank 6: E_6 has suitable complex

$$27 = 16 + 10 + 1 \text{ of } SO(10)$$

Appears in String theory

Suitable for right-handed neutrino

New Interactions make Baryons Decay

- Exchanges of new X, Y bosons:

$$\left(\epsilon_{ijk} u_{R_k} \gamma_\mu u_{L_j}\right) \quad \frac{g_X^2}{8m_X^2} \quad \left(2e_R\ \gamma^\mu\ d_{L_i} + e_L\ \gamma^\mu\ d_{R_i}\right)$$

$$\left(\epsilon_{ijk} u_{R_k} \gamma_\mu d_{L_j}\right) \quad \frac{g_Y^2}{8m_X^2} \quad \left(\nu_L\ \gamma^\mu\ d_{R_i}\right), \qquad G_X \equiv \frac{g_X^2}{8m_X^2} \simeq G_Y \equiv \frac{g_Y^2}{8m_Y^2}$$

- Proton decay rate $\Gamma_B = c G_X^2 m_p^5$ lifetime: $\tau_p = \frac{1}{c}\frac{m_X^4}{m_p^5}$
- Preferred modes: $p \to e^+\pi^0\ ,\quad e^+\omega\ ,\quad \bar{\nu}\pi^+\ ,\quad \mu^+K^0\ ,\quad \ldots$
  $n \to e^+\pi^-\ ,\quad e^+\rho^-\ ,\quad \bar{\nu}\pi^0\ ,\quad \ldots$
- Estimate of X, Y masses: $m_X \simeq (1 \text{ to } 2) \times 10^{15} \times \Lambda_{QCD}$
- Lifetime too short:

$\tau(p \to e^+\pi^0) \simeq 2 \times 10^{31\pm1} \times \left(\frac{\Lambda_{QCD}}{400\ \text{MeV}}\right)^4\ y$ exp't: $\tau(p \to e^+\pi^0) > 1.6 \times 10^{33}\ y$

Proton Decay in Supersymmetric SU(5)

- Increase in GUT scale:

$$m_X \simeq 10^{16}\ \mathrm{GeV}$$

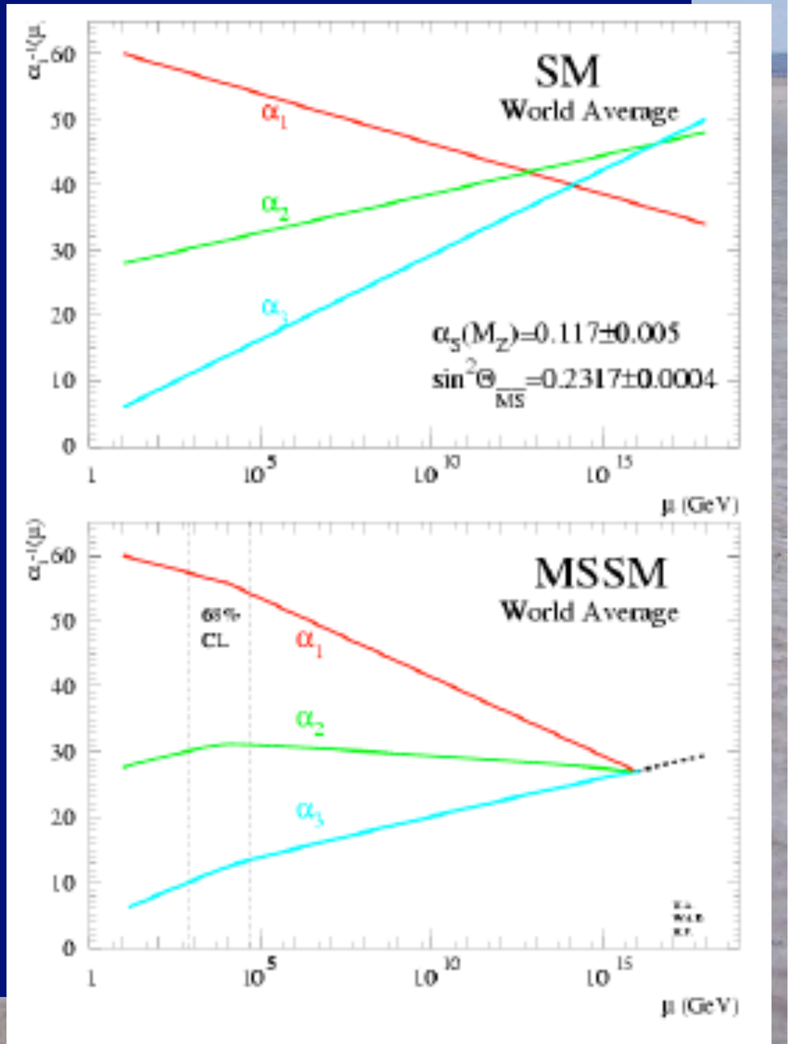
- X, Y exchanges OK
- Beware GUT Higgsinos:

$$G_X \to \mathcal{O}\left(\frac{\lambda^2 g^2}{16\pi^2}\right)\ \frac{1}{m_{\tilde{H}_3}\tilde{m}}$$

- Preferred decay modes:

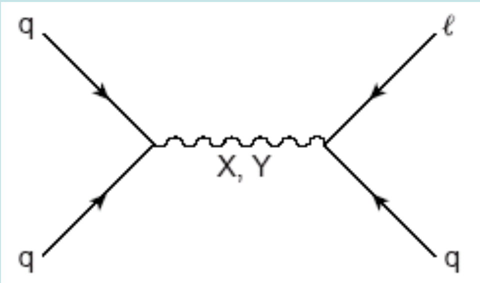
$$p \to \bar{\nu} K^+\ ,\quad n \to \bar{\nu} K^0\ ,\quad \ldots$$

- Lifetime too short?
- Suppressed in some models



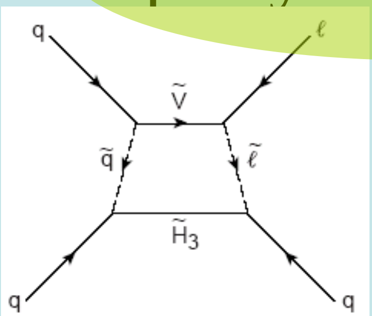
Proton Decays in GUTs

- Decay diagram in non-supersymmetric SU(5)



$A \sim 1/m_X^2$

- Decay diagram in supersymmetric SU(5)



$A \sim 1/m_X \, m_{squark}$

Experimental limits

p DECAY MODES	Partial mean life (10^{30} years)	Confidence level
Antilepton + meson		
$N \to e^+ \pi$	> 158 (n), > 1600 (p)	90%
$N \to \mu^+ \pi$	> 100 (n), > 473 (p)	90%
$N \to \nu \pi$	> 112 (n), > 25 (p)	90%
$p \to e^+ \eta$	> 313	90%
$p \to \mu^+ \eta$	> 126	90%
$n \to \nu \eta$	> 158	90%
$N \to e^+ \rho$	> 217 (n), > 75 (p)	90%
$N \to \mu^+ \rho$	> 228 (n), > 110 (p)	90%
$N \to \nu \rho$	> 19 (n), > 162 (p)	90%
$p \to e^+ \omega$	> 107	90%
$p \to \mu^+ \omega$	> 117	90%
$n \to \nu \omega$	> 108	90%
$N \to e^+ K$	> 17 (n), > 150 (p)	90%
$p \to e^+ K^0_S$	> 120	90%
$p \to e^+ K^0_L$	> 51	90%
$N \to \mu^+ K$	> 26 (n), > 120 (p)	90%
$p \to \mu^+ K^0_S$	> 150	90%
$p \to \mu^+ K^0_L$	> 83	90%
$N \to \nu K$	> 86 (n), > 670 (p)	90%
$n \to \nu K^0_S$	> 51	90%

Scenarios for Baryogenesis

- Out-of-equilibrium decays of GUT X, Y bosons?

  difficult to avoid dilution by 2 → 2 scattering

- Or GUT Higgs bosons?

  smaller couplings, lower mass (?) → less dilution

- Electroweak phase transition? Not in SM:

  second-order transition, not enough CP violation. MSSM?

- Leptogenesis?

  decays of heavy (s)neutrinos → lepton asymmetry

  converted to baryon asymmetry by non-perturbative EW effects

Neutrino Masses and Mixing

Why? Why not?

- There is no sacred symmetry to forbid m_ν
- The only sacred symmetries are EXACT gauge symmetries, e.g.,

  Q_{em} conserved

  $\leftrightarrow$ massless photon

  $\leftrightarrow$ U(1) gauge symmetry of SM
- No candidate gauge symmetry to forbid m_ν
- No massless gauge boson coupled to lepton # L
- Expect $m_\nu \neq 0$ in extensions of SM: GUTs, string

Models for Neutrino Masses

- Could be generated in Standard Model: using non-renormalizable interaction:
- $$\frac{1}{M}\nu H \cdot \nu H \quad \rightarrow \quad m_\nu \nu \cdot \nu : \; m_\nu = \frac{\langle 0|H|0\rangle^2}{M}$$
- Probably effective interaction due to exchange of massive fermion N = 'right-handed ν'
- Should then consider seesaw mass matrix:

$$(\nu_L, N)\begin{pmatrix} 0 & M_D \\ M_D^T & M \end{pmatrix}\begin{pmatrix} \nu_L \\ N \end{pmatrix}$$

- Does not need GUT, but $M \sim 10^{10} – 10^{15}$ GeV
- Add singlet N to SU(5)? automatic in SO(10)

Bigger GUT Models

- First look at groups of rank 5 with suitable complex representations
- Only suitable candidate is SO(10)
- Each generation in irreducible representation

$$16 = 10 + 5^* + 1 \text{ of } SU(5)$$

- Next step is rank 6: E_6 has suitable complex

$$27 = 16 + 10 + 1 \text{ of } SO(10)$$

Suitable for
right-handed neutrino

Neutrino Mixing

- Diagonalize neutrino mass matrix in flavour space:

$$U^T M_\nu U = M_\nu^d \quad \text{where} \quad M_\nu = Y_\nu^T \frac{1}{M} Y_\nu v^2$$

- Two ‘observable’ Majorana phases as well as Maki-Nakagawa-Sakata (MNS) mixing matrix:

$$U = U_\nu P_0 : \; P_0 \equiv \mathrm{Diag}\left(e^{i\phi_1}, e^{i\phi_2}, 1\right)$$

- MNS matrix has 3 real angles and 1 phase:

$$U_\nu = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \times \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13}e^{-i\delta} \end{pmatrix}$$

- But that is not all!

Parameters in Minimal Seesaw Model

- Effective light-neutrino theory

$$-\mathcal{L}_\nu \supset (Y_\nu)_{ij}\, H \bar{N}_i \begin{pmatrix} \nu \\ L \end{pmatrix}_j + \underbrace{\frac{1}{2} \bar{N}_i \mathcal{M}_{ij} \bar{N}_j}$$

- 3 masses, 3 mixing angles, 3 CP-violating phases
- Additional 9 parameters associated with heavy singlet ‘right-handed’ neutrinos:

  3 more masses, 3 more mixing angles,

  3 more CP-violating phases
- 12 contribute to leptogenesis, not MNS phase δ
- If supersymmetric, 16 parameters contribute to renormalization of soft susy-breaking m_0

Leptogenesis in Seesaw Model

- Asymmetry in decay of heavy neutrino, due to one-loop diagrams: $$\epsilon_{ij} = \frac{1}{8\pi}\frac{1}{\left(Y_\nu Y_\nu^\dagger\right)_{ii}}\mathrm{Im}\left(\left(Y_\nu Y_\nu^\dagger\right)_{ij}\right)^2 f\left(\frac{M_j}{M_i}\right)$$
- Possible even in 2-generation seesaw model, where there is no oscillation phase δ
- Scenario for determining baryon asymmetry:

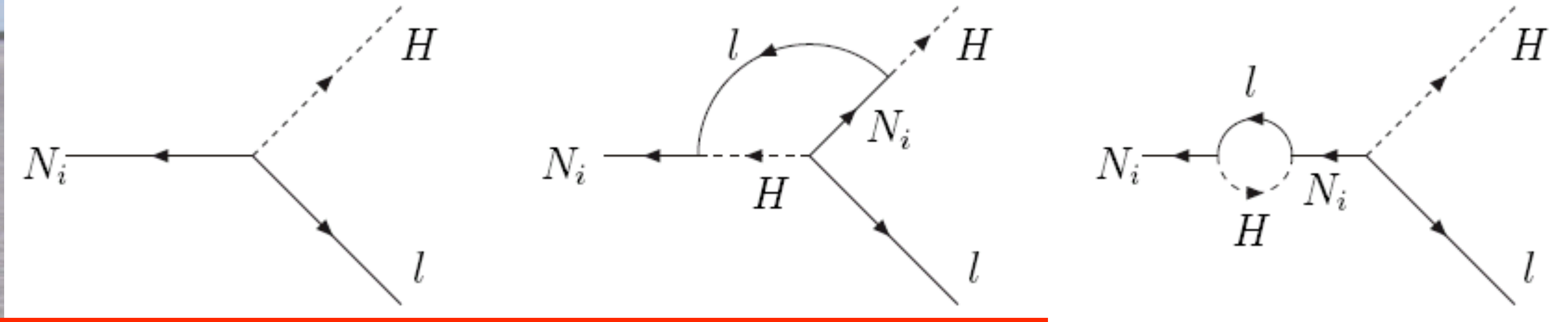
  Measure δ and low-E Majorana phases $\varphi_{1,2}$

  Measure susy renormalization effects

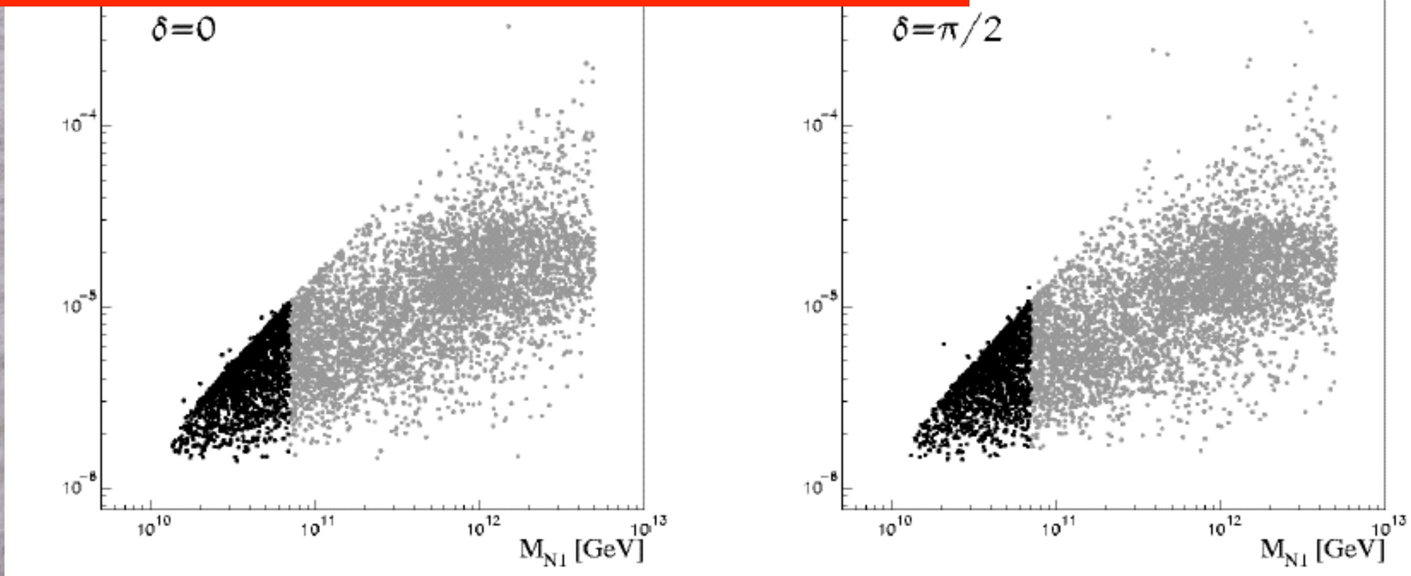
  Subtract contributions of δ, $\varphi_{1,2}$
- Remaining effect due to leptogenesis parameters

More on Leptogenesis

One-loop diagrams for N → H + lepton decay



Result does not depend on oscillation phase δ



Quantum Gravity & String

String Theory

- Point-like particles → extended objects
- Simplest possibility: lengths of string
  - Open and/or closed
- Quantum consistency fixes # dimensions:
  - Bosonic string: 26, superstring: 10
- Must compactify extra dimensions, scale ~ $1/m_P$?
- Perturbative string unification scale:

$$M_{GUT} = O(g) \times \frac{m_P}{\sqrt{8\pi}} \simeq \text{few} \times 10^{17}\,\text{GeV}$$

Close to GUT scale, but larger?

Bigger GUT Models

- First look at groups of rank 5 with suitable complex representations
- Only suitable candidate is SO(10)
- Each generation in irreducible representation

$$16 = 10 + 5^* + 1 \text{ of } SU(5)$$

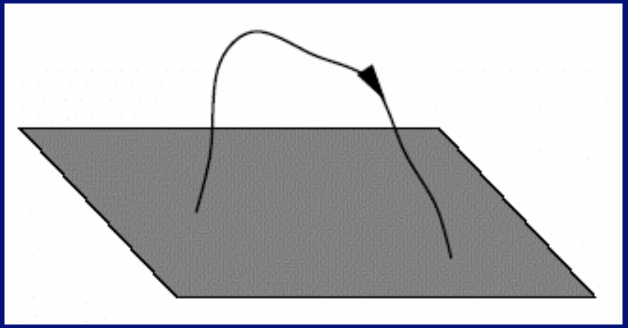
- Next step is rank 6: E_6 has suitable complex

$$27 = 16 + 10 + 1 \text{ of } SO(10)$$

Appears in string theory compactified on Calabi-Yau

Non-Perturbative String = M Theory

- Solitonic 'lumps' = balls of string: $m \propto \frac{1}{g_s}$
- Appear with various dimensions: 'D-branes'

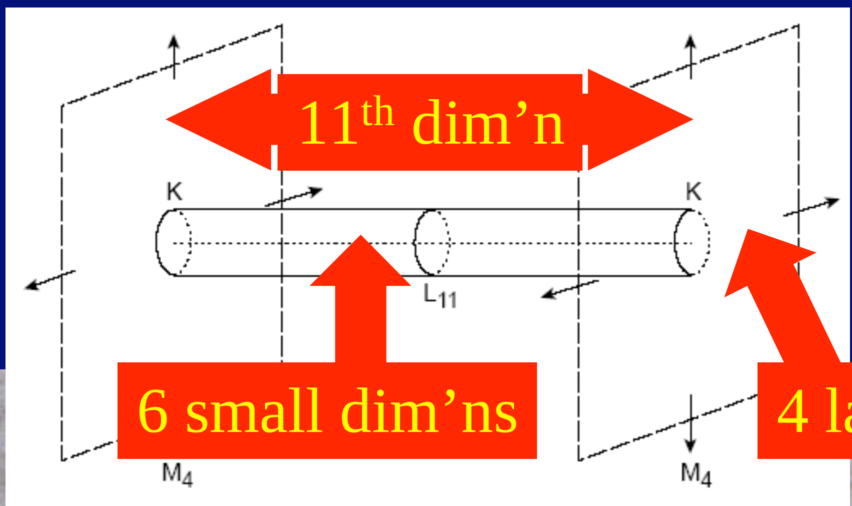


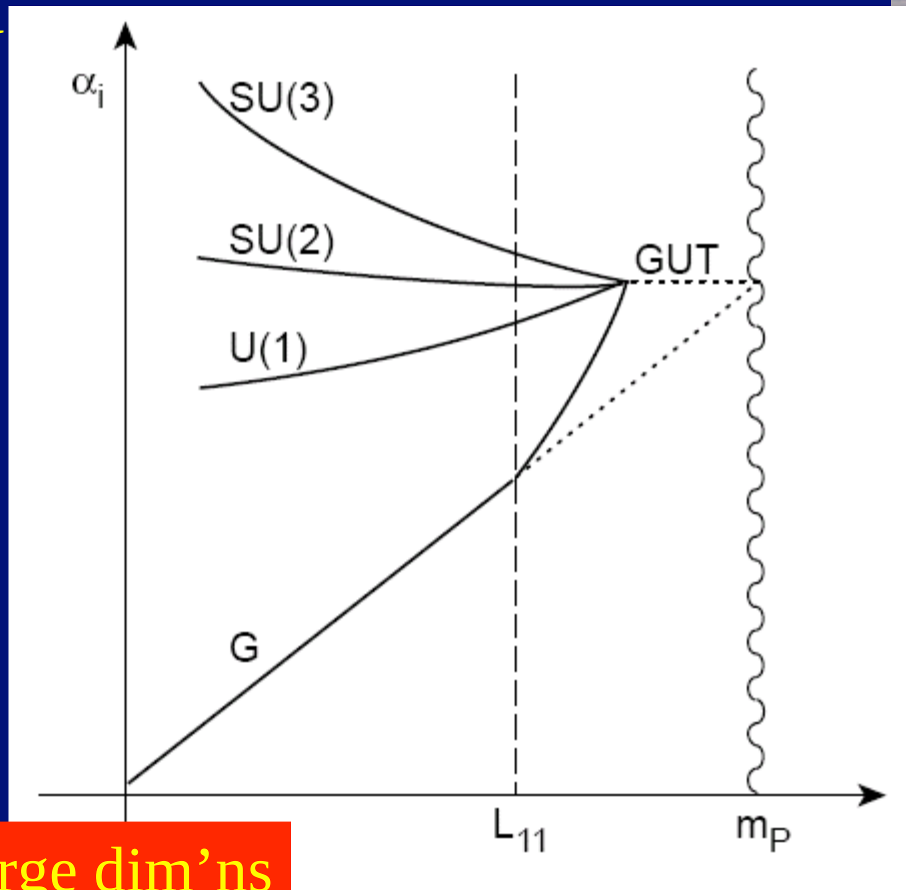
- Can regard string coupling as extra 'dimension'

  11-dimensional M theory
- Includes different string models in various limits
- New ways to get extra gauge symmetries

Scenario for String Unification

- E\If extra dimension below GUT scale: gravity grows faster with energy
- Unify at 10^{16} GeV?
- E.g., in M theory with large 11^{th} dimension





How large could extra Dimensions be?

- 1/TeV?

    could break supersymmetry, electroweak

- micron?

    can rewrite hierarchy problem

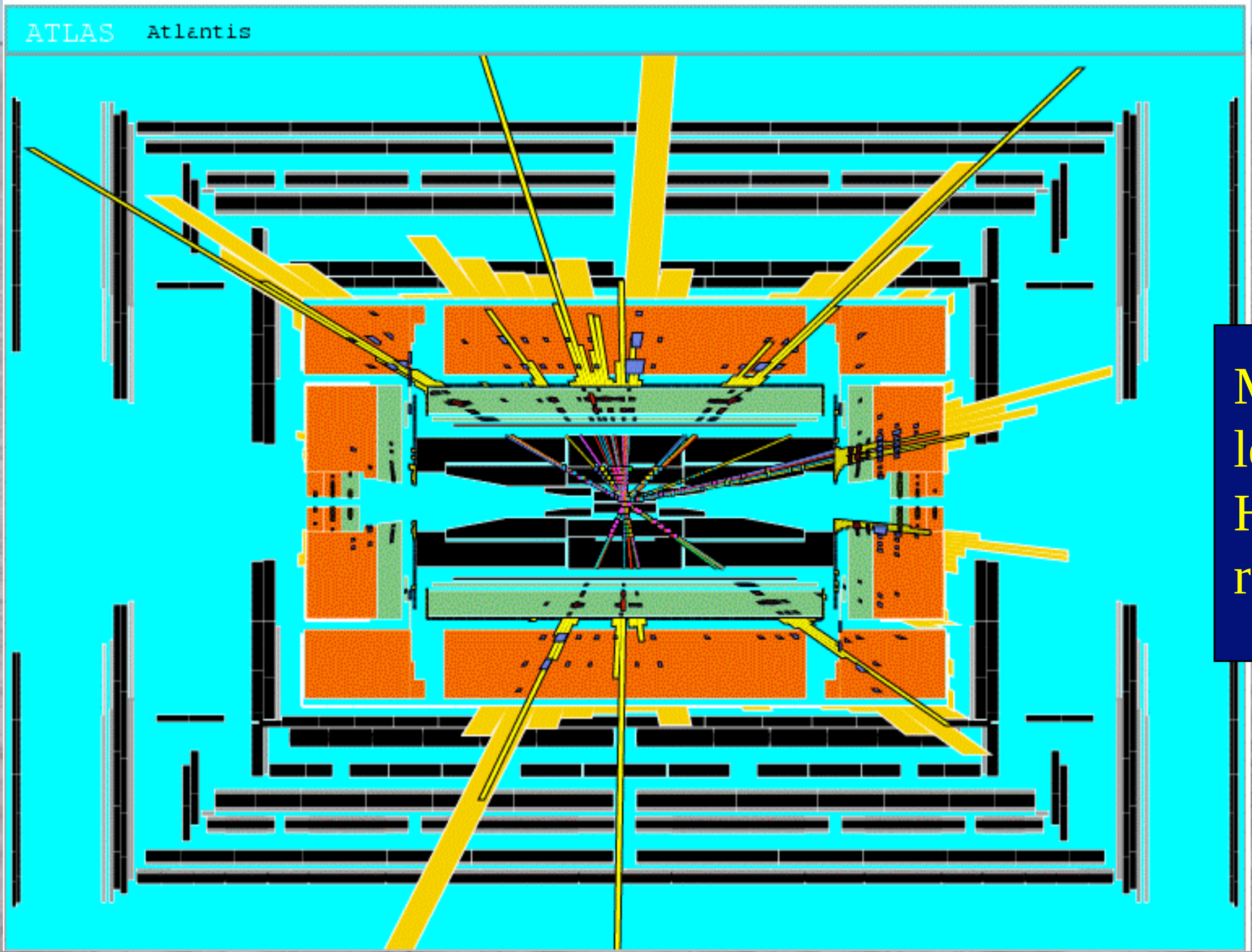
- Infinite?

    warped compactifications

- **Look for black holes, Kaluza-Klein excitations @ colliders?**

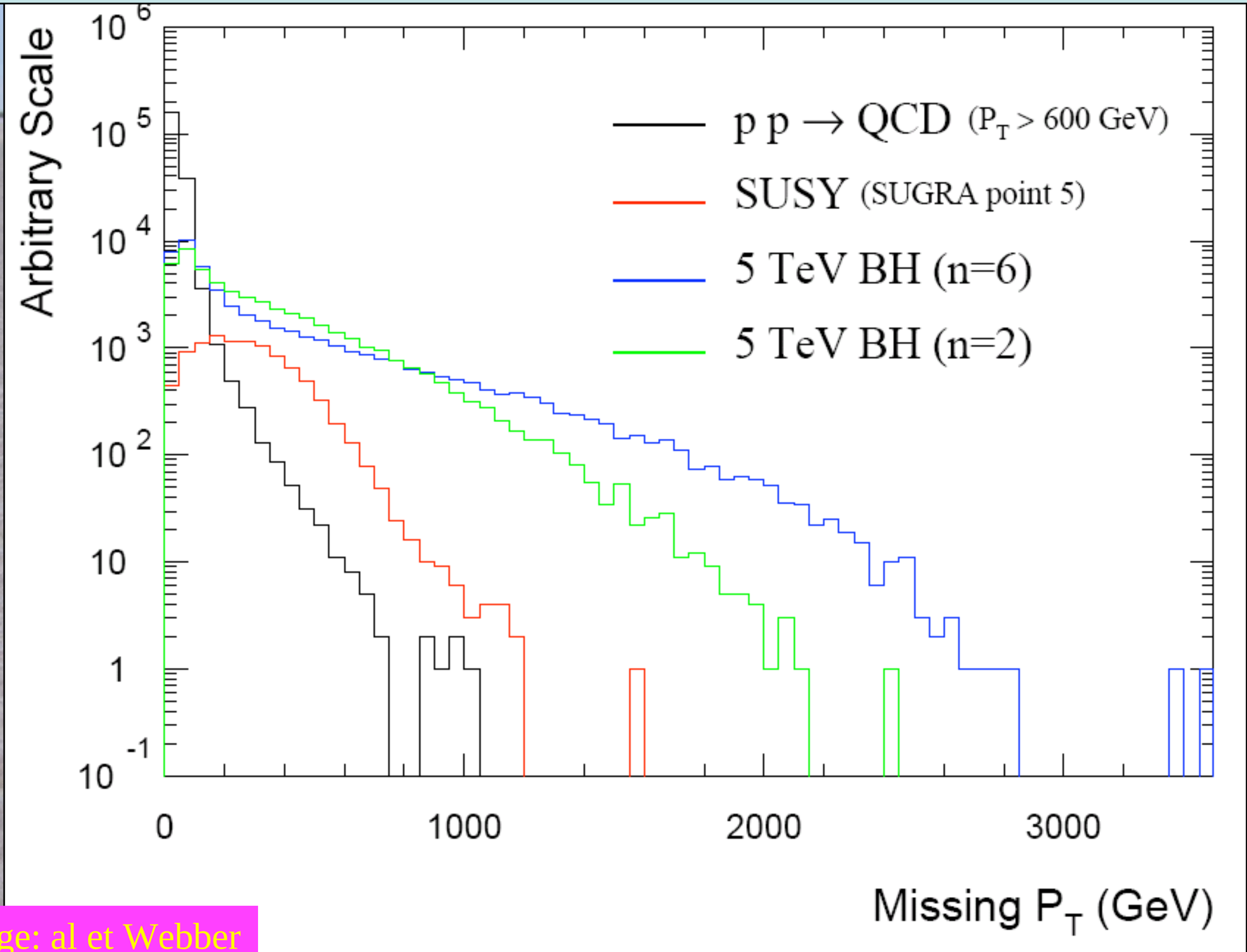
And if gravity becomes strong at the TeV scale …

Black Hole Production at LHC?



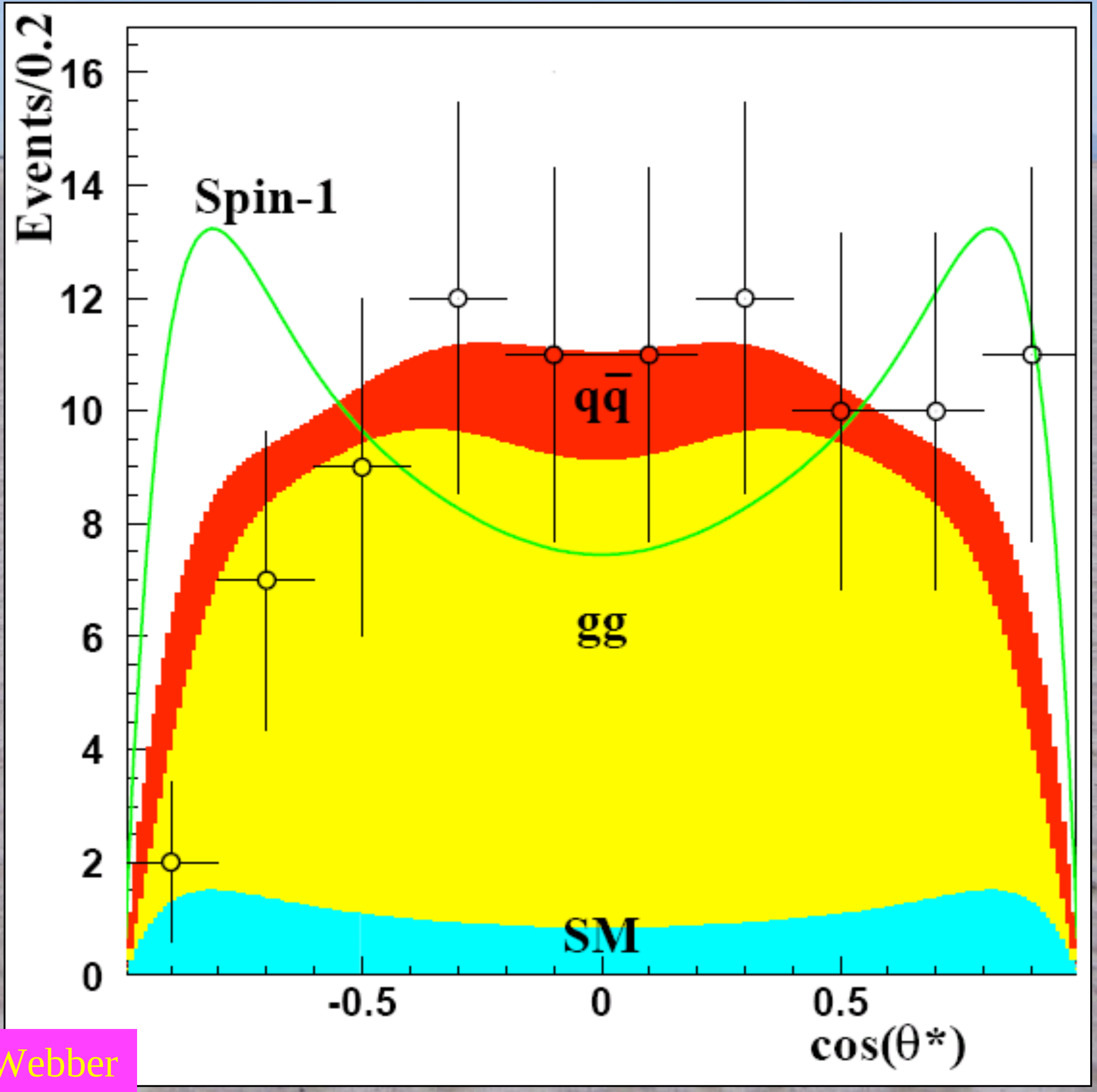
Multiple jets, leptons from Hawking radiation

Black Hole Production @ LHC



Cambridge: al et Webber

Identifying Graviton Resonance @ LHC



Cambridge: al et Webber

Summary

- The origin of mass is the most pressing in particle physics
- Needs a solution at energy < 1 TeV

  Higgs? Supersymmetry? Extra Dimensions?

  LHC will tell!
- Lots of speculative ideas for other physics beyond the Standard Model

  Grand unification, strings, branes, …
- Hints provided by neutrinos

  How else can one test these speculations?