I. Cosmology in 2010

Tremendously exciting time in theoretical + observational cosmology.

- diverse, rapidly-improving experiments (Planck, WMAP, Fermi; balloons, ground-based) provide a flood of data

- we seem to be on the cusp of a new understanding of the early universe

10 years from now, could well
- know nature of DM by direct detection on earth
- exclude all but a tiny fraction of theoretical models for the early universe
- detect primordial GW or NG in CMB experiments

Plan of these lectures:

I. The Concordance Cosmology (ΛCDM)

II. Inflation: mechanism
CMB anisotropies
models + phenomenology

III. Inflation in String Theory: preliminaries
- inflation and Planck-scale physics
- the moduli problem
- flux compactifications

IV. Inflation in String Theory: examples
D3-brane inflation
Axion monodromy inflation

First we'll describe the Concordance Cosmology, then explore its foundations.

Observations show that on scales $\gg 300$ Mpc (1pc = 3.26 ly)

the U. is approximately homogeneous + isotropic.

Most general spacetime with these properties:

FRW,

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right)$$

for $k=+1$ S^3

$k=0$ $\mathbb{R}^3$

$k=-1$ $\mathbb{H}^3$

as spatial $(t=t_0)$ slices.

or, $$\left(d\chi^2 + \begin{Bmatrix} \sin^2\chi \\ \chi^2 \\ \sinh^2\chi \end{Bmatrix}(d\theta^2 + \sin^2\theta\, d\phi^2)\right).$$

The coordinates inside the $(\;)$ are <u>comoving</u>

(r, θ, ϕ).

whilst the <u>physical</u> distances are

$$dp = a(t)\,dr \quad \text{etc.}$$

Useful to define conformal time τ

via $\quad dt = a\,d\tau \qquad \tau = \int \frac{dt}{a(t)}$

so $\quad ds^2 = a^2(\tau)\left[-d\tau^2 + \underbrace{ds^2_{\text{spatial}}}_{d\chi^2 + \{\}d\Omega^2}\right]$

radial null geodesics: $d\tau = d\chi$

Then, between t_1 and $t_2 > t_1$ a particle can travel at most

$$\Delta\chi = \tau_2 - \tau_1 = \int_{t_1}^{t_2} \frac{dt}{a(t)}$$

'comoving horizon'

Taking $t_1 = 0$ and $a(t=0) = 0$ to be the initial singularity,

we have $\chi_{max.}(t) = \int_0^t \frac{dt}{a(t)}$

whence we obtain the <u>physical</u> horizon,

$$d_{max}(t) = a(t)\,\chi_{max}(t).$$

Evolution of FRW.

U. is pervaded by dark energy,
dark matter,
dust of baryonic matter,
radiation.
(neutrinos, GW,..)

Treat all these as $\overset{\text{perfect}}{\underline{\text{fluids}}}$, so

$$T_{\mu\nu} = (\rho+P)\, u_\mu u_\nu + P\, g_{\mu\nu}$$

where $u_\mu = \frac{dx_\mu}{d\tau_p}$ $\quad \tau_p$ observer's proper time

is the fluid 4-velocity

and ρ, P measured in fluid rest frame.

$\underrightarrow{\text{Comoving}}$ observer sees $U^\mu = (1,0,0,0)$.

Let's work out $G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$

from the metric we get

$\cdot : \frac{d}{dt}$

$$R_{00} = -3\frac{\ddot{a}}{a}$$

$$R_{ij} = \left[2\dot{a}^2 + a\ddot{a} + 2\frac{k}{a^2}\right] g_{ij} \cdot \frac{1}{a^2}$$

So $R = +3\frac{\ddot{a}}{a} + 3\frac{\ddot{a}}{a} + 6\left(\frac{\dot{a}}{a}\right)^2 + 6\frac{k}{a^2}$

$$= 6\left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right]$$

So $R_{00} - \frac{1}{2} g_{00} R$

$$= 3\left(\frac{\dot{a}}{a}\right)^2 + \frac{3k}{a^2}$$

$$\Rightarrow \boxed{\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3}\rho - \frac{k}{a^2}}$$

often use $H = \frac{\dot{a}}{a}$, $M_p^2 = (8\pi G_N)^{-1} = 2.4\times10^{18}$ GeV

$\boxed{3H^2 M_p^2 = \rho}$ (for $k=0$).

Tracing E.E., $-R = 8\pi G T$

$$\frac{\ddot{a}}{a} + \frac{8\pi}{3} G\rho = -\frac{4\pi}{3} G\left(-\rho + 3P\right)$$

used $U^\mu U_\mu = -1$.

$$\boxed{\frac{\ddot{a}}{a} = -\frac{4\pi}{3} G(\rho + 3P)}$$

take $P = w\rho$

for $w < -1/3$, $\ddot{a} > 0$

$\ddot{a} < 0$ otherwise.

[NB can derive

$\dot{\rho} = -3H(\rho + P)$ from FI, FII

or from $\nabla_\mu T^{\mu\nu} = 0$.

Setting $M_p \to 1$, we have

$$\boxed{\begin{aligned} H^2 &= \frac{\rho}{3} - \frac{k}{a^2} \\ \frac{\ddot{a}}{a} &= \dot{H} + H^2 = -\frac{1}{6}(\rho + 3P) \end{aligned}}$$

for a single fluid, $P = w\rho$, we find solutions

$$a(t) = \left(t/t_0\right)^{\frac{2}{3(1+w)}}$$

dust (matter, DM, ..) $w = 0$ $a \sim t^{2/3}$

radiation $w = 1/3$ $a \sim t^{1/2}$

homogeneous: scalar ϕ

$$P = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

so for $\frac{1}{2}\dot{\phi} \ll V(\phi)$,

$w = -1$ $a \sim e^{Ht}$

while for $\frac{1}{2}\dot{\phi}^2 \gg V(\phi)$,

$w = +1$ $a \sim t^{1/3}$.

Using $\dot{\rho} = -3H(\rho + P)$ one derives

$$\frac{d\rho}{da} = -3\rho(1+w)\cdot\frac{1}{a}$$

$$d\ln\rho = -3(1+w)\frac{da}{a} \qquad \boxed{\rho = a^{-3(1+w)}}$$

$\rho \sim \frac{1}{a^4}$ radiation

$\rho \sim \frac{1}{a^3}$ matter

$\rho \sim$ const scalar potential

$\rho \sim \frac{1}{a^6}$ scalar kinetic energy

Now we can take stock of our universe and begin to discuss the very early universe.

Still with $M_p = 1$, we define

$$\rho_{crit} = 3H_0^2$$

and $\Omega_x = \frac{\rho_x}{\rho_{crit}}$.

Then $3H^2 = \rho - \frac{3k}{a^2}$ evaluated today, becomes

$$\sum_x \Omega_x = 1 - \Omega_k$$

$$\Omega_k = \frac{-k}{a_0^2 H_0^2}.$$

Combining many observations (key ones: CMB, SNe Ia (WD).)

we obtain the concordance cosmology:

1) $\Omega_k \approx 0$ ($\lesssim 1\%$), so set $k = 0$.

2) $\Omega_b = .04$ ($\Omega_{stars} \approx .01$)

3) $\Omega_{DM} = .23$

4) $\Omega_\Lambda = .73$ $w \approx -1$.

with $\Omega_r, \Omega_{GW}, \Omega_\nu$ negligible.

And of course $\dot{a} > 0$, $\ddot{a} \overset{!}{>} 0$.

Hubble '30s , Supernova Cosmology Project 1997.

Finally, the radiation is extremely interesting. Starlight is a small fraction; far more numerous are the microwave γ's, $\lambda \sim$ mm, with

$$n \sim 411/\text{cm}^3 \qquad (\sim 10^{10} n_b)$$

$$T \sim 2.73\,\text{K}$$

The CMBR is isotropic at leading order. But famously it has 10^{-5} anisotropies, (COBE ca. '96)

$$T(\theta, \phi) = T_0 + \Delta T(\theta, \phi)$$

$$\frac{\Delta T}{T} \sim 10^{-5}. \qquad (1.91 \times 10^{-5} \text{ (COBE normalization)})$$

These fluctuations are Gaussian (as far as we know), i.e.

$$\Big\langle \underbrace{\frac{\Delta T}{T}(\theta,\phi) \cdots \frac{\Delta T}{T}}_{\text{odd \#}} \Big\rangle = 0$$

$$\Big\langle \underbrace{\frac{\Delta T}{T}(\theta,\phi) \cdots \frac{\Delta T}{T}}_{\text{even \#}} \Big\rangle \text{ determined by } \Big\langle \frac{\Delta T}{T} \frac{\Delta T}{T} \Big\rangle.$$

Clearly, running backward in time, first matter + then radiation dominates.

(a^{-3}, a^{-4}).

IF these are the only 'fluids' present and IF no new physics arises as $\rho \to M_p^4$, $T \to M_p^4$,
(quite an assumption!)

then we encounter the famous <u>horizon problem</u>.

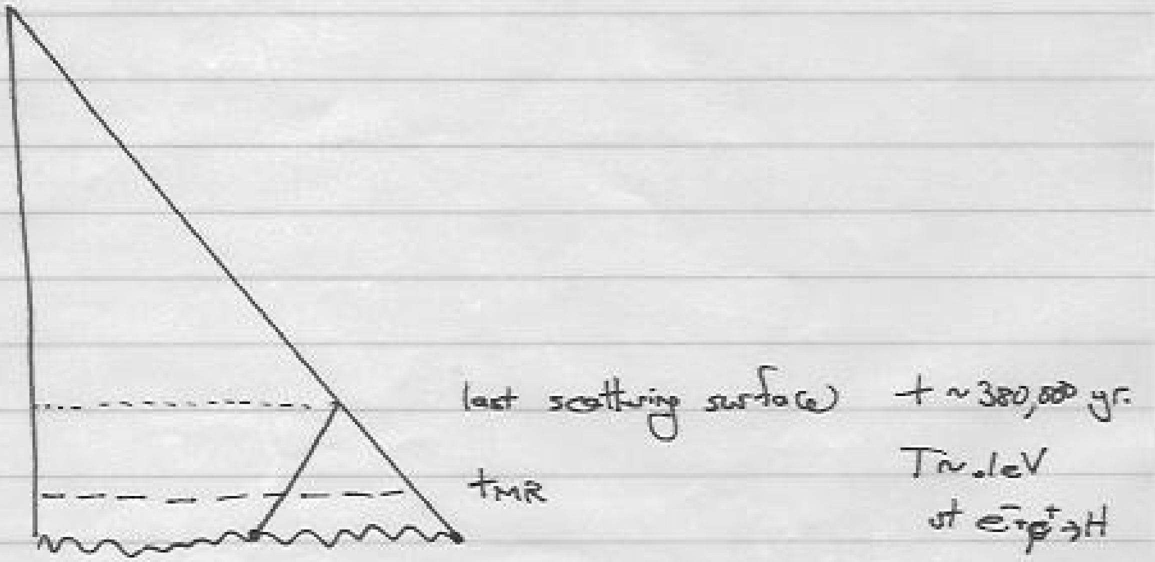
$t=0$	t_{MR}	t_0
0	~50,000yr	13.7 Gyr

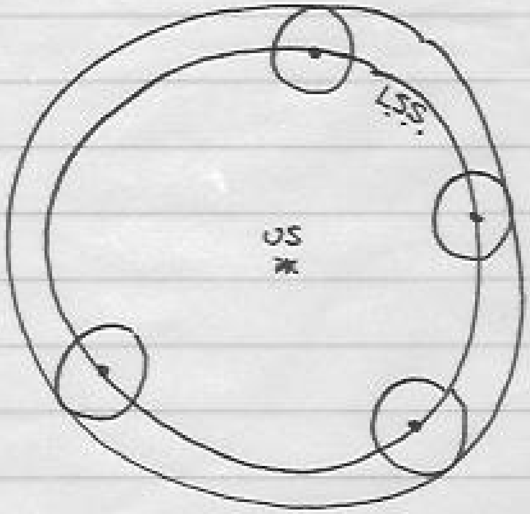
RD: $$x_{max}(t_r) = \int_{\underset{\text{NB!} \to}{0}}^{t_r} \frac{dt}{(t/t_r)^{1/2}} = 2t_r$$

$(t_r \text{ s.t. } a(t_r)=1)$

$$\text{so} \quad x_{max}(t_0) = \int_0^{t_{MR}} \frac{dt}{(t/t_{MR})^{1/2}} + \int_{t_{MR}}^{t_0} \frac{dt}{(t/t_{MR})^{2/3}}$$

We then find





The comoving particle horizon at LS is <u>finite</u>.

One finds it subtends $\sim 1^\circ$ on sky.

Why, then, is CMB so isotropic?

More generally, comoving horizon (for one fluid) is

$$\chi_{max}(t_2) = \int_{t_1}^{t_2} \frac{dt}{(t/t_2)^{\frac{2}{3(1+w)}}} = t_2^{\frac{2}{3(1+w)}} \, t^{1-\frac{2}{3(1+w)}} \cdot \frac{1}{1-\frac{2}{3(1+w)}} \Bigg|_{t_1}^{t_2}$$

$$= \left(\frac{1+w}{1+3w}\right)\left[t_2 - t_2^{\frac{2}{3(1+w)}} \, t_1^{\frac{1+3w}{1+w}}\right]$$

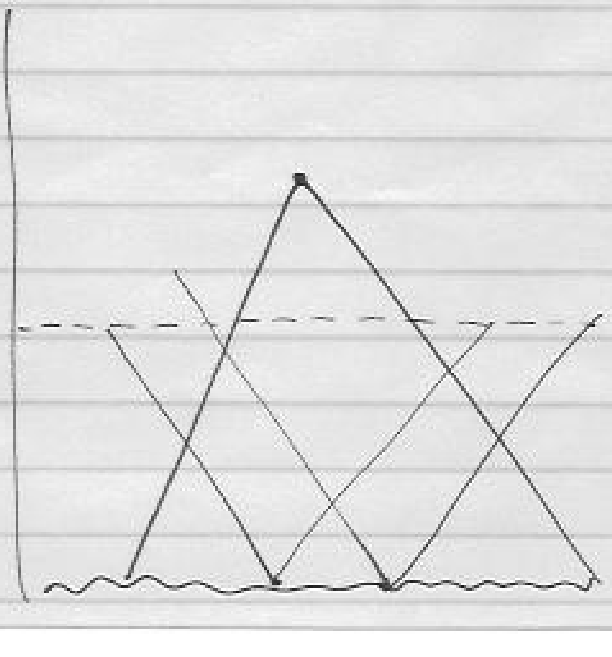
and for $w > -1/3$, this is finite; but

for $-1 < w < -1/3$,

$$\chi_{max}(t_2) \to \infty \quad \text{as } t_1 \to 0.$$

so $\Delta\tau \to \infty$.

This is seen as



[NB Harrison-Zeldovich-Peebles:
can just postulate homogeneous isotropic
initial conditions.]

Here we have a <u>causal</u> <u>mechanism</u>:

if a period of ($w < -1/3 \Leftrightarrow \ddot{a} > 0$)
accelerated expansion
intervenes between <u>us</u> + the initial
singularity,

This explains the observed isotropy of the CMB
by creating 'a lot of conformal time'
between $t=0$ and $t = 380,000$ yr.

II. Inflation

1. Homogeneous Evolution.

We've seen that a period of accelerated expansion ($\ddot{a}>0 \Leftrightarrow w<-1/3$) intervening between us and the initial sing., can explain the observed isotropy of the CMB.

This idea, inflation, (Guth '81; Linde; Albrecht + Steinhardt) turns out to be extremely powerful.

Key questions:

- what sort of field can drive inflation?
- when inflation occurs, what are the consequences?

In subsequent lectures we'll ask:
can (should) Lagrangians suitable for inflation be understood in the framework of a Planck-scale theory?

We'll begin in QFT + GR.

Consider a single scalar field ϕ with action

$$S = \int d^4x \sqrt{g} \left\{ \frac{1}{2} R - \frac{1}{2} \partial_\mu \phi \partial_\nu \phi g^{\mu\nu} \mp V(\phi) \right\}$$

where $g_{\mu\nu}$ is an FRW metric, and V is some fn.

- We've assumed minimal coupling, i.e. $\nexists \; R\phi^2$. Can achieve this by field redef. when $\nexists$ other 'matter' fields.

  [When $\exists$ matter,

  $$\frac{1}{2}R + \frac{1}{6}R\phi^2 + \frac{1}{2}(\partial\phi)^2 \mp V(\phi) + \mathcal{L}_{\text{matter}}$$

  $$\hookrightarrow \frac{1}{2}R + \frac{1}{2}(\partial\tilde{\phi})^2 \mp \tilde{V}(\phi) + \tilde{\phi}^\alpha \mathcal{L}_{\text{matter}}$$

  one gets ϕ–matter couplings.]

- We've not yet considered terms like

$$\sum_k \nu_k \frac{(\partial\phi)^{2k}}{\Lambda^{4k-4}}$$

but will do so after a first pass with only 2-deriv. kinetic terms.

At the 2-deriv. level, our proposal is fully general.

The EOM are [give P, ρ here]

$$\begin{cases} \ddot{\phi} + 3H\dot{\phi} = -V' \\ H^2 = \frac{\rho}{3} = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V\right) \\ \frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{1}{6}(\rho + 3P) \qquad \text{or} \quad \dot{\rho} = -3H(\rho + P) \\ \qquad = -\frac{1}{6}\left(2\dot{\phi}^2 - 2V\right) = -\frac{1}{3}\left(\dot{\phi}^2 - V\right) \end{cases}$$

$$\left(\Rightarrow \dot{H} = \frac{-1}{3}\dot{\phi}^2\right)$$

We would like to find solutions in which $\ddot{a} > 0$ for a prolonged period.
So, require $w < -1/3$.

For homogeneous configurations, $\nabla\phi = 0$ we have

$$\begin{Bmatrix} P = \frac{1}{2}\dot{\phi}^2 - V \\ \rho = \frac{1}{2}\dot{\phi}^2 + V \end{Bmatrix}$$

so that for $\frac{1}{2}\dot{\phi}^2 \ll V$, $w \approx -1$.

So if $\frac{1}{2}\dot{\phi}^2 \ll V$ persists, we'll find protracted expansion driven by the ϕ condensate.

Condition for inflation:

$$\frac{\ddot{a}}{a} > 0 \Leftrightarrow \dot{H} + H^2 > 0 \Leftrightarrow -\frac{\dot{H}}{H^2} < 1.$$

Define $\varepsilon_H \equiv -\frac{\dot{H}}{H^2}$. Inflation occurs if $\boxed{\varepsilon_H < 1}$.

Also, $\frac{\ddot{a}}{a} \simeq H^2\left[1 - \frac{3}{2}(1+w)\right]$ (upon using $H^2 = \frac{\rho}{3}$)

$= H^2 + \dot{H}$

So, $\varepsilon_H = \frac{3}{2}(1+w) = \frac{\frac{1}{2}\dot{\phi}^2}{H^2}$. $\left(\approx \frac{3}{2}\frac{\dot{\phi}^2}{V} \text{ when } \dot{\phi}^2 \ll V\right)$

Note also that if $a = e^{Ht} \equiv e^{N(t)}$

$dN = H dt = d\ln a$

then $\varepsilon_H = -\frac{d\ln H}{d\ln a}$.

So ε_H measures $\begin{cases} \bullet \text{ slow change of } H \text{ per efold} \\ \bullet \text{ smallness of KE wrt PE.} \end{cases}$

Most simple models have not just $\varepsilon_H < 1$, but $\varepsilon_H \ll 1$.

We've found conditions for inflation to occur, but will it persist?

We need $\frac{1}{2}\dot{\phi}^2 \ll V$ to persist.

So, $\ddot{\phi}$ should be small.

If $|\ddot{\phi}| \ll 3H|\dot{\phi}|, |V'|$,

then one finds solutions with prolonged inflation.

We define $\eta_H \equiv -\frac{\ddot{\phi}}{H\dot{\phi}}$ (dimless acceleration per Hubble time, measure of $\frac{\Delta\varepsilon}{\Delta N}$)

From the EOM,

$$\eta_H = -\frac{1}{2}\frac{d\ln\varepsilon_H}{d\ln a} + \varepsilon_H$$

(upon using $\frac{d\varepsilon_H}{dN} = \frac{\dot{\phi}\ddot{\phi}}{H^2} - \frac{\dot{H}}{H^2}\cdot\frac{\dot{\phi}^2}{H^2}$)

So, if $\eta_H \ll 1$, $\varepsilon_H \ll 1$, then H and ε_H both have small fractional changes per efold.

So far, we've not used any approximations (except restricting to actions 2nd order in derivatives).

We've just noted that in a regime where $\varepsilon_H \ll 1$, $\eta_H \ll 1$, inflation persists.

The simplest and most often used formalism imposes these conditions at the level of the EOM.

The 1st slow roll condition, $\boxed{\varepsilon_H \ll 1}$,

implies $\frac{1}{2}\dot{\phi}^2 \ll V$ so

$$\boxed{H^2 = \frac{1}{3}V}$$ 1st Friedman eqn. in slow roll approx.

The 2nd slow roll condition, $\boxed{\eta_H \ll 1}$,

implies $\ddot{\phi} \ll 3H|\dot{\phi}|$

$$\Rightarrow \quad \boxed{3H\dot{\phi} = -V'}$$ Klein-Gordon eq. in SR approx.

Now in this approximation, $\dot{\phi}^2 = \frac{(V')^2}{9H^2}$

so $\varepsilon_H = \frac{1}{2}\frac{\dot{\phi}^2}{H^2} \approx \frac{1}{2}\frac{V'^2}{V^2}$.

Also, $3\dot{H}\dot{\phi} + 3H\ddot{\phi} = -V''\dot{\phi}$

$$\text{so } \eta_H = -\frac{\ddot{\phi}}{H\dot{\phi}} = \frac{V''}{3H^2} + \frac{\dot{H}}{H^2} = \frac{V''}{V} - \varepsilon_H$$

So, when $\varepsilon_H \ll 1$ and $\eta_H \ll 1$, we have

$$\varepsilon_H \approx \varepsilon_V \equiv \frac{1}{2}\left(\frac{V'}{V}\right)^2 .$$

$$\eta_H \approx \eta_V - \epsilon_V \qquad \eta_V \equiv \frac{V''}{V}.$$

One therefore usefully assesses a potential V by computing

$$\left\{ \begin{array}{l} \eta_V \equiv \frac{V''}{V} \\ \epsilon_V \equiv \frac{1}{2}\left(\frac{V'}{V}\right)^2 \end{array} \right\} ,$$

the <u>potential</u> SR parameters.

When these are small, slow roll inflation can occur.

Example N_e in slow roll.

Assume SR, compute $N_e = \int_{t_{start}}^{t_{end}} H(t)\,dt$

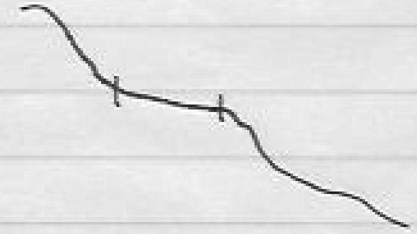
$$dN = H\,dt = \frac{H\,d\phi}{\dot{\phi}} = \left(-\frac{3H}{V'}\right)H\,d\phi = -\frac{V}{V'}\,d\phi$$

(sign not important, depends on whether ϕ ↑ or ↓ during inflation.)

So $N_e = -\int_{\phi_{start}}^{\phi_{end}} \frac{V}{V'}\,d\phi$.

Can take ϕ_{start}, ϕ_{end} to be boundaries of interval where $\varepsilon_V < 1$,

eg.



eg for $V = \frac{1}{2}m^2\phi^2$, $\varepsilon_V = \frac{2}{\phi^2}$ (secretly, $\frac{M_p^2}{2\phi^2}$)

So $N_e = -\int_{\phi_{start}}^{\phi=\sqrt{2}} \frac{1}{2}\phi\,d\phi = \frac{\phi_{start}^2}{4} - \frac{1}{2}$.

2. Inhomogeneities from Quantum Fluctuations

So far we've studied E.E. + K-G eqn, the classical eoms for a scalar field coupled to GR.

We've considered only homogeneous solutions, $\nabla\phi=0$, as a first step.

We will now study the EOM for small fluctuations around an inflationary bg.

So we need to linearize the fields,

$$\phi(t,\vec{x}) = \bar{\Phi}(t) + \delta\phi(t,\vec{x})$$

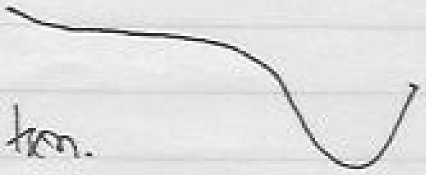
$$g_{\mu\nu}(t,\vec{x}) = \bar{g}_{\mu\nu}(t) + \delta g_{\mu\nu}(t,\vec{x})$$

and eventually also linearize the EOM, or expand the action.

We'll see that a perturbative treatment is well-justified: recall $\frac{\Delta T(\theta,\phi)}{T} \approx 10^{-5}$.

Why study fluctuations?

ϕ governs ρ,
and end of inflation.



$\bar{\phi} + \delta\phi(\vec{x}, t)$ gives local clock reading off amount of expansion remaining.

fluctuations $\delta\phi \Rightarrow$ different regions inflate by different amounts.

Intuitive picture: clocks are QM objects $\Rightarrow$ necessarily some variance.

Result: in quantum theory, there are necessarily local fluctuations in ρ, hence ultimately in T.

Efficient approach:

1) choose a good gauge
2) expand the action

For (1), we fix time + space reparametrizations by taking the comoving gauge,

$$\left\{ \begin{array}{l} \delta\phi = 0 \\ \delta g_{00} = \delta g_{0i} = 0 \\ \delta g_{ij} = a^2(t)\,\delta_{ij}(1-2\mathcal{R}) + a^2 h_{ij} \end{array} \right\}$$

Here h_{ij} is TT, $\nabla_i h^{ij} = h^i{}_i = 0$, and $\mathcal{R}$ is a scalar.

In fact, one can show that a slice of $\phi = \phi_0$ has $R_{(3)} = \frac{4}{a^2}\nabla^2\mathcal{R}$.

So we call $\mathcal{R}$ the curvature perturbation.

h_{ij} is the tensor part, about which more later.

Let's study $\mathcal{R}$ first.

For (2), after some labor and tedium one finds that

$$S_{(0)} = \int d^4x \sqrt{g} \left\{ \frac{1}{2} R_{(4)} - \frac{1}{2} \partial_\mu \phi \partial_\nu \phi g^{\mu\nu} - V(\phi) \right\}$$

gives rise to

$$S_{(2)} = +\int d^4x \sqrt{g} \left\{ \dot{R}^2 - \frac{1}{a^2} \delta^{ij} \partial_i R \partial_j R \right\} \frac{\dot{\phi}^2}{2H^2}$$

$$= +\int d^4x \frac{a^3 \dot{\phi}^2}{2H^2} \left\{ \dot{R}^2 - \frac{1}{a^2} (\nabla R)^2 \right\}$$

So we have the 2nd-order action for our (classical variable) R.

Now define the canonically-normalized field

$$v \equiv \left(\frac{a \dot{\phi}}{H} \right) R \equiv z R \qquad \text{(Mukhanov)}$$

so that

$$S_{(2)} = +\frac{1}{2} \int d^4x \, a \left\{ z^2 \dot{R}^2 - \frac{z^2}{a^2} (\nabla R)^2 \right\}$$

$$=^{+}\frac{1}{2}\int d^3x\, d\tau \left\{ \cancel{a^2} z^2 \frac{R'^2}{\cancel{a^2}} - \cancel{z}(\nabla v)^2 \right\}$$

but $\frac{1}{2}\int d^3x\, d\tau \left\{ (v')^2 + \frac{z''}{z} v^2 \right\}$

$$= \frac{1}{2}\int d^3x\, d\tau \left\{ (v')^2 - z'\partial_\tau (v^2 z^{-1}) \right\}$$

$$= \frac{1}{2}\int d^3x\, d\tau \left\{ R^2 z'^2 + R'^2 z^2 + \underset{\cdot Rz}{2R'z'} - z'(z'R^2 + 2R\cancel{R}$$

$$= \frac{1}{2}\int d^3x\, d\tau \left\{ R'^2 z^2 \right\}$$

So,

$$S_{(2)} = \frac{1}{2}\int d\tau\, d^3x \left\{ +(v')^2 \overset{-}{\cancel{+}} (\nabla v)^2 \overset{+}{\cancel{+}} \frac{z''}{z} v^2 \right\}$$

Now $z = \frac{a\dot{\phi}}{H}$ is background-dependent.

So we have a scalar with a time-dependent mass,

$$\cancel{\left\{ S_{(2)} \sim \int d^4x \left\{ -\frac{1}{2}(\partial\phi)^2 + \ldots - \frac{1}{2} g_{\mu\nu}\partial^\mu\phi \right\} \right.}$$

$$S_{(2)} = \int d\tau\, d^3x \left\{ -\frac{1}{2}\eta^{\mu\nu}\partial_\mu v \partial_\nu v - \frac{1}{2}m^2(\tau) v^2 \right\}$$

with $m^2(\tau) = -\frac{z''}{z} = -\frac{H}{a\dot{\phi}}\frac{\partial^2}{\partial\tau^2}\left(\frac{a\dot{\phi}}{H}\right)$.

Now given a solution for the homog. bg,

$$\left\{ \begin{matrix} a(t) \\ \phi(t) \end{matrix} \right\} \Rightarrow \begin{matrix} \dot{\phi}(t) \\ H(t) \\ \tau(t) \end{matrix} \Rightarrow z(t),\ z(\tau)$$

one obtains $\dot{m}(\tau)$ directly.

Varying $S_{(2)}$ and using the Fourier rep'n

$$v(\tau, \vec{x}) = \int \frac{d^3k}{(2\pi)^3} v_k(\tau) e^{i\vec{k}\cdot\vec{x}}$$

we get the classical eom for fluctuations,

$$\boxed{v_k'' + \left(k^2 - \frac{z''}{z}\right)v_k = 0},$$

the Mukhanov-Sasaki eqn.

We can (and will) solve the MS eqn. for the Fourier coeff v_k, and thereby determine $\mathcal{R}_k$.

(Recall, $\mathcal{R}(\tau,\vec{x}) = \int \frac{d^3k}{(2\pi)^3} \mathcal{R}_k(\tau) e^{i\vec{k}\vec{x}}$)

and $\nabla^2 \mathcal{R} = \frac{a^2}{4} R_{(3)}$).

But first let's understand our goal in doing so.

Curvature perturbations (gauge-equiv to density pert.) ultimately source temperature perturbations.

The statistical properties of $\frac{\Delta T(\theta,\phi)}{T}$

are determined by

the statistical properties of $\mathcal{R}$.

Two-point function

In real space, consider

$$\xi_R(r) \equiv \langle R(x)\, R(x+r) \rangle .$$ (using isotropy)

We can relate this to

$$\langle R_k R_{k'} \rangle$$

as follows.

$$\langle R_k R_{k'} \rangle = \left\langle \int d^3x\, R(x)\, e^{-i\vec{k}\vec{x}} \int d^3x'\, R(x')\, e^{-i\vec{k}'\cdot\vec{x}'} \right\rangle$$

use $\vec{x}' = \vec{x} + \vec{r}$

$$= \left\langle \int d^3x\, e^{-i(\vec{k}+\vec{k}')\cdot\vec{x}} \int d^3r\, R(x)\, R(x+r)\, e^{-i\vec{k}'\vec{r}} \right\rangle$$

$$= \int d^3r\, \xi_R(r)\, e^{-i\vec{k}\vec{r}} \cdot \delta^3(\vec{k}+\vec{k}') \cdot (2\pi)^3$$

We define the <u>power</u> spectrum

$$P_R(k) \equiv \int d^3r\, \xi_R(r)\, e^{-i\vec{k}\cdot\vec{r}}$$ (i.e., Fourier transform of 2pt fn)

So that $\langle R_k R_{k'} \rangle = (2\pi)^3 P_R(k)\, \delta^3(\vec{k}+\vec{k}')$.

The scalar power spectrum is one of the key outputs of an inflationary model.

We'll compute $P_R(k)$ by solving the MS eqn.

The result gives (ensemble average) information about the sizes of primordial curvature perturbations

In the late universe, after processing through plasma physics, one obtains (ensemble average) information about the T anisotropies,

$$\left\langle \frac{\Delta T}{T}(\theta,\phi)\,\frac{\Delta T}{T}(\theta',\phi')\right\rangle. \qquad \text{“}\langle TT\rangle\text{”}$$

These are measured routinely and directly, with great precision.

Predictions + Measurements

Observers map the CMB sky, $\frac{\Delta T}{T}(\theta,\phi)$

and express this as

$$\frac{\Delta T}{T}(\theta,\phi) = \sum_{lm} a_{lm} Y_{lm}(\theta,\phi).$$

We're interested in the 2pt fn of $\frac{\Delta T}{T}$ — observation shows that $\langle(\frac{\Delta T}{T})^3\rangle$ etc are small, while $\langle \frac{\Delta T}{T} \rangle = 0$.

↑ avg over sky

i.e. we want to understand the variance governing $\frac{\Delta T}{T}$:

pick a point, then another,..

and ask: from what distrib is $\frac{\Delta T}{T}$ drawn?

Well, $\left(\frac{\Delta T}{T}(\theta,\phi)\right)^2 = \sum_{lm}\sum_{l'm'} a_{lm} a_{l'm'} \underbrace{Y_{lm} Y^*_{l'm'}}_{\text{[illegible]}}$

$$\text{Now } \frac{1}{4\pi}\int d\Omega \left(\frac{\Delta T}{T}(\theta,\phi)\right)^2$$

$$= \sum_{lm}\sum_{l'm'} a_{lm} a_{l'm'} \delta_{ll'}\delta_{mm'}$$

$$= \sum_{l}\sum_{m} |a_{lm}|^2 \cdot \text{[illegible]} \equiv \sum_{l} C_l (2l+1)$$

$$C_l = \sum_{m} |a_{lm}|^2 \cdot \frac{1}{2l+1}$$

Fourier transform of

The theory predicts $P_R(k)$. $\Leftrightarrow$ 2pt corrl fn.

With $\langle R(x)\, R(x) \rangle$

we've probed $\int d^3k\, P_R(k) \leftrightarrow \sum_\ell C_\ell (2\ell+1)$.

since
$$\left\{ \begin{aligned} P_R(k) &= \int d^3r\, \langle R(x)\, R(x+r) \rangle\, e^{-ikr} \\ \langle R(x)\, R(x+r) \rangle &= \frac{1}{(2\pi)^3} \int d^3k\, P_R(k)\, e^{ikr} \end{aligned} \right\}$$

By measuring $\langle R(x)\, R(x+r) \rangle$

NB!

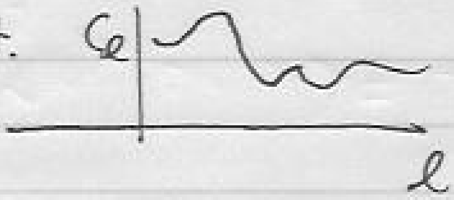
or equiv $\left\langle \frac{\Delta T}{T}(x)\; \frac{\Delta T}{T}(x+r) \right\rangle$

we probe C_ℓ individually.

Method:

Observer:

- measure $\frac{\Delta T}{T}(\theta,\phi)$.
- expand $\frac{\Delta T}{T}(\theta,\phi) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\theta,\phi)$
- compute $\frac{1}{2\ell+1}\sum_m |a_{\ell m}|^2 \equiv C_\ell$.
- plot it.



Theorist:

- compute $P_R(k) \Leftrightarrow$. Fourier transform of 2pt fn

  [in SR, we've done it]

  $$\left(P_R(k)\,\delta(\vec{k}+\vec{k}') = \frac{1}{(2\pi)^3}\underbrace{\langle R_k R_{k'}\rangle}_{\text{quantum average}} \right)$$

- compute $$C_\ell = \frac{2}{\pi}\int k^2 dk\, P_R(k)\, \underbrace{\Xi_\ell(k)}_{\text{transfer fn: projection effects + interactions}}$$

  [CAMB, CMBFAST]

- compare to observations.

  [likelihood analysis]

Note: $\langle TT \rangle$ is by no means the only useful observable.

Very important to constrain/detect $\langle TTT \rangle$,

i.e. late-universe consequence of

$$\langle R_{k_1} R_{k_2} R_{k_3} \rangle \neq 0. \qquad \text{(non-Gaussianity)}$$

Also, primordial tensor perturbations (h_{ij}, not R) leave traces in the CMB.

Plasma physics gives nonvanishing $\langle TE \rangle$, $\langle EE \rangle$,

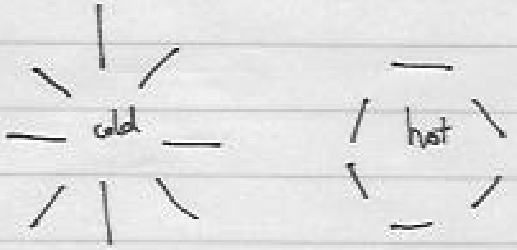
where

E-mode polarization is the curl-free component of the polariz. field,

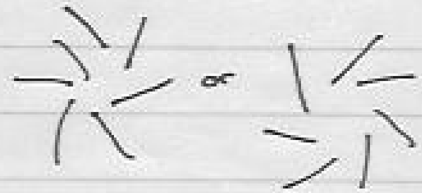
and

B-mode " " is the curl component,

E:



B:



We'll not study these in detail, but you should know:

$\mathcal{R} \Rightarrow E$
$\underline{\text{no}}\ B!$

$h_{ij} \Rightarrow E + B$

} detecting $\langle BB \rangle$
implies $\langle h_{ij} h_{ij} \rangle \neq 0$.
primordial GW.

$\langle TE \rangle$, $\langle EE \rangle$ have been measured + will improve) with Planck.

$\langle BB \rangle$ is a holy grail of CMB observations.

$$ds^2 = -dt^2 + a^2(t)\left[(1-2\mathcal{R})\delta_{ij} + h_{ij}\right]$$

Last time we found that the curvature perturbation,

$$\mathcal{R} = \frac{v}{z} \qquad z = \frac{a\dot{\phi}}{H}, \quad \text{obeys}$$

$$v_k'' + \left(k^2 - \frac{z''}{z}\right)v_k = 0$$

where $v(\tau, \vec{x}) = \int \frac{d^3k}{(2\pi)^3} v_k(\tau)\, e^{i\vec{k}\cdot\vec{x}}$.

We'll now solve this eqn. (MS eqn)

- first in de Sitter
- then quantize near de Sitter
- then more carefully, in SR approx.
  in detail

First step: a de Sitter background.

$$a = e^{Ht} \qquad H = \text{const.}$$

$$\tau = \int^{t} \frac{dt'}{a(t')} = -\frac{1}{H} e^{-Ht} + \text{const}$$

$$\text{so } \tau = -\frac{1}{aH}. \qquad \tau : -\infty \to 0.$$

Also, $z = \frac{a\dot{\phi}}{H}$

$$\frac{z''}{z} = \frac{a''}{a} = \frac{2}{\tau^2} = 2a^2H^2$$

$$\Rightarrow \quad u_k'' + \left(k^2 - 2a^2H^2\right)u_k = 0.$$

We can solve $u_k'' + \left(k^2 - \frac{2}{\tau^2}\right)u_k = 0$:

$$u_k = A\frac{e^{-ik\tau}}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right) + B\frac{e^{ik\tau}}{\sqrt{2k}}\left(1 + \frac{i}{k\tau}\right).$$

for $|k\tau| \gg 1$, oscillations, with fixed amplitude.

for $|k\tau| \ll 1$, 1st solution is

$$\approx \frac{A}{\sqrt{2k}}(1 - ik\tau)(1 + ik\tau)\frac{1}{ik\tau}$$

$$\approx \frac{A}{\sqrt{2k}}\frac{(1 + k^2\tau^2)}{ik\tau} \sim \frac{1}{\tau}.$$

$R = \frac{u}{z} \qquad z = \frac{a\dot{\phi}}{H} \sim \frac{1}{\tau} \quad$ so $R \sim \text{const}$ as $k\tau \to 0$.
(1st sol.).

Furthermore, for approx de Sitter, eg SR inflation,

$$\frac{z''}{z} \approx 2a^2H^2$$ up to corrections we'll compute in due course.

Therefore, in SR inflation at sufficiently early times ($|\tau| \gg 1$)

$$k \gg aH$$

so modes are well 'inside the horizon'.

They see $M^{3,1}$!

At late times, $k \ll aH$, and modes are 'outside the horizon'.

Now let's consider an approximate de Sitter solution

and quantize the [redacted] fluctuations.

For very early times, $k \gg aH$, we have

$$v_k'' + k^2 v_k = 0 \qquad \text{(SHO)}$$

with two independent solutions,

$$\varphi_k = C_k e^{-ik\tau}$$

and $\varphi_k^* = C_k e^{+ik\tau}$. (its c.c.)

So, we may write

$$v(\tau, \vec{x}) = \int \frac{d^3k}{(2\pi)^3} \Big[\varphi_k^{(t)} \alpha(k) e^{i\vec{k}\vec{x}} + \varphi_k^*(t) e^{-i\vec{k}\vec{x}} \underbrace{\alpha^*(k)}_{\text{by reality.}} \Big]$$

i.e. a combination of the two independent solutions.

Again for very early times, we just have $M^{3,1}$ (as seen by any given mode),

$S_{(2)} = \int d\tau\, d^3x \left(\frac{1}{2}\partial_\mu v \partial^\mu v - V(v,\tau)\right)$ so $\pi = \frac{\partial \mathcal{L}}{\partial v} = \dot{v}$

$\Rightarrow [v(\vec{x},t), \dot{v}(\vec{x}',t)] = i\delta^3(\vec{x}-\vec{x}')$

and we can simply impose canonical commutation relations,

$$\alpha(k) \to a(k)$$
$$\alpha^*(k) \to a^+(k)$$

$$[\hat{a}(k), a^+(k')] = \delta^3(\vec{k}-\vec{k}').$$

(this normalization requires $\varphi_k \varphi_k^{*\prime} - \varphi_k^* \varphi_k' = i$)

Now must choose a vacuum.
Since $m = m(\tau)$, no unique choice!

The most believable approach to this problem is to insist that at very early times, ~~and~~ when a given mode has $k \gg aH$,
i.e. when the mode with comoving wavenumber k is far inside the horizon and experiences $\mathcal{M}^{3,1}$,
a Minkowski observer sees no particles.

This vacuum is the Bunch-Davies vacuum.

It is defined by

$$\hat{a}_k |0\rangle_{BD} = 0 ,$$

i.e. $|0\rangle_{BD}$ is annihilated by the $\mathcal{O}$ whose coeff. is $\varphi_k(\tau)\, e^{+ik\vec{x}} \sim e^{-ik\tau + i\vec{k}\cdot\vec{x}}$

a priori, could have vacuum state

$$(c_a \hat{a}_k + c_{a^\dagger} \hat{a}^\dagger_k)|0\rangle_{other} = 0 .$$

We'll write $|0\rangle_{BD} \to |0\rangle$. henceforth.

The Bunch-Davies vacuum choice says that at very early times ($k \gg aH \Leftrightarrow |k\tau| \gg 1$), there should be no particles.

For $k \gg aH$, we have

$$u_k'' + k^2 u_k = 0. \qquad \text{(SHO)}.$$

The BD mode solution is then

$$u_k(\tau \to -\infty) = C_k e^{-ik\tau} \qquad \text{(+ve frequency)}.$$

imposing (B1), $|C_k|^2 \left(+ik - (-ik)\right) = i \quad (\hbar = 1)$

we have $C_k = \frac{1}{\sqrt{2k}}$.

So $\lim_{\tau \to -\infty} u_k = \dfrac{e^{-ik\tau}}{\sqrt{2k}}$.

(Comparing to our b.c. from vacuum choice and $[\hat{a}, \hat{a}^{+}]$ normalization, we have

$$B=0, \ A=1.$$

So we've <u>solved</u> for u_k in de Sitter space,

$$\boxed{u_k = \frac{e^{-ik\tau}}{\sqrt{2k}}\left[1-\frac{i}{k\tau}\right]} \ .$$

quantum average

Now let's compute. $\langle R_k R_{k'} \rangle$.

$$\left\langle \hat{u}_k(\tau)\, \hat{u}_{k'}(\tau) \right\rangle$$

$$= \langle 0 | \hat{a}_k\, \hat{a}^{+}_{-k'}\, u_k(\tau)\, u^{*}_{-k'}(\tau) | 0 \rangle$$

$+$ terms vanishing by $\hat{a}_k |0\rangle = 0 \quad \forall k.$

$$= (2\pi)^3 \delta^3(\vec{k}+\vec{k}')\, |u_k(\tau)|^2$$

$$= (2\pi)^3 \delta^3(\vec{k}+\vec{k}') \cdot \frac{1}{2k}\left[1+\frac{1}{k^2\tau^2}\right]$$

Now $R = \frac{v}{z}$ $z = \frac{a}{H}\dot{\phi}$ $(\to 0)$.

and $a = -\frac{1}{H\tau}$

$\Rightarrow \langle R_k(\tau) R_{k'}(\tau) \rangle$

$$= (2\pi)^3 \delta^3(\vec{k}+\vec{k}') \frac{1}{2k^3}\left[1+\frac{1}{k^2\tau^2}\right] k^2\tau^2 \frac{H^4}{\dot{\phi}^2}$$

on superhorizon scales, we get

$$\langle R_k(\tau) R_{k'}(\tau) \rangle = (2\pi)^3 \frac{H^2}{2k^3} \frac{H^2}{\dot{\phi}^2} \delta^3(\vec{k}+\vec{k}')$$

We've already defined

$$P_R(k) \equiv \int d^3r \, \xi_R(r) e^{-i\vec{k}\cdot\vec{r}}$$

so that $\langle R_k R_{k'} \rangle = (2\pi)^3 P_R(k) \delta^3(\vec{k}+\vec{k}')$.

We conclude that

$$\boxed{P_R(k) = \frac{H^2}{2k^3} \frac{H^2}{\dot{\phi}^2}.}$$

After our de Sitter warmup, let's do a more careful job: SR approximation.

Can rewrite $u_k'' + \left(k^2 - \frac{z''}{z}\right) u_k$

as $(u_k = z R_k)$

$$R_k'' + \frac{2}{z} z' R_k' + k^2 R_k = 0$$

Now $z = \frac{a\dot{\phi}}{H} = \frac{\phi'}{H}$ and $\dot{\phi}^2 = 2H^2 \varepsilon_H = -2\dot{H}$

$$\varepsilon = -\frac{\dot{H}}{H^2}$$

So $\frac{z'}{z} = \begin{pmatrix}\text{brief}\\ \text{algebra}\\ \text{using}\\ \dot{\phi}^2 = -2\dot{H}\end{pmatrix} aH(1 + \varepsilon_H + \eta_H)$

where $\eta_H = \frac{-\ddot{H}}{2H\dot{H}}$. $(= -\delta_{\text{weinberg}})$

Also, $\varepsilon_H - 1 = a \frac{d}{dt}(\dot{a})^{-1} = -\frac{\ddot{a}a}{\dot{a}^2}$

$$= \frac{d}{d\tau}\left(\frac{1}{aH}\right).$$

Exact so far!

Now we use the SR assumption: $\varepsilon_H, \delta_H \ll 1$
(and also $\delta_4 \ll 1$).

Then $\frac{d}{d\tau}\left(\frac{1}{aH}\right) = -1 + \varepsilon_H$

gives $\frac{1}{aH} \approx +\tau(-1+\varepsilon_H)$ or $aH \approx \frac{-1}{(1-\varepsilon_H)\tau}$

Use this in MS, giving

$$R_k'' + 2\left(-\frac{1}{(1-\varepsilon_H)\tau}\right)(1+\varepsilon_H-\eta_H)R_k' + k^2 R_k = 0$$

$$\approx: \boxed{R_k'' - \frac{2}{\tau}\left[1-\eta_H+2\varepsilon_H\right]R_k' + k^2 R_k = 0.}$$

MS eqn in SR approx.

For constant η_H, ε_H, this has solution

$$C_1 \tau^{\nu} H_\nu^{(1)}(-k\tau) + C_2 \tau^{\nu} H_\nu^{(2)}(-k\tau)$$

where $\nu = \frac{3}{2} + 2\varepsilon_H - \eta_H$.

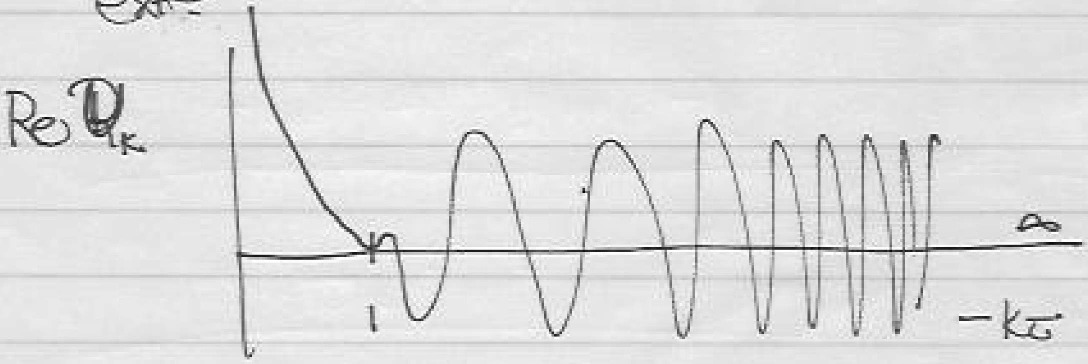
NB $z \sim \tau^{1/2-\nu}$
during SR.

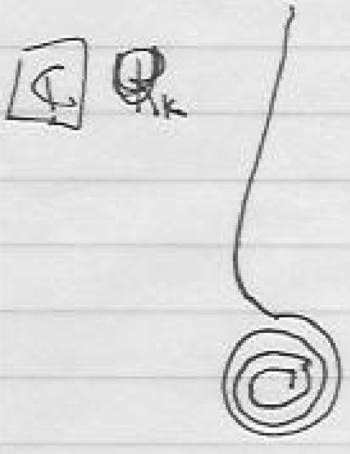
The BD initial condition fixes $c_2 = 0$.

Furthermore, $\tau^{\nu} H_{\nu}^{(1)}(-k\tau)$, for $|k\tau| \ll 1$,
has the asympt. behavior

$$(\tau^{\nu})(\tau^{-\nu}) = \underline{\text{constant}}.$$

So again we see modes $\underline{\text{freezing}}$ upon horizon exit.





$x \equiv -k\tau$.

as $x \to \infty$,

$$H_{\nu}^{(1)(2)}(x) \to \sqrt{\frac{2}{\pi x}} \exp\left[\pm i\left(x - \frac{\nu\pi}{2} - \frac{\pi}{4}\right)\right]$$

as $x \to 0$,

$$H_{\nu}^{(1)}(x) \to \frac{i}{\pi}\Gamma(\nu)\left(\frac{x}{2}\right)^{-\nu}$$

the bc fixes

$$u_k = \exp\left(\frac{i\pi}{4}(2\nu+1)\right)\left(\frac{\pi x}{4k}\right)^{1/2} H_{\nu}^{(1)}(x).$$

$\underline{\text{NB}}$ R_k freezes

$u_k \sim \frac{R_k}{\tau}$ grows as $\tau \to 0$.

$$u_k = z R_k \quad z \sim \tau^{1/2-\nu} \sim \tau^{-1}$$

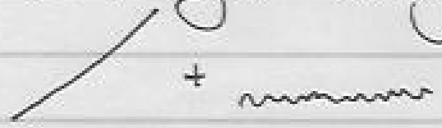
so $R_k \propto \tau^{-1/2+\nu}\,\tau^{1/2}\,\tau^{-\nu} \sim \text{const} \quad |k\tau| \to 0$

$\propto \underbrace{\tau^{1/2}\,\tau^{-1/2+\nu}}_{u \to R}\,\tau^{-1/2}\,e^{ix} \quad |k\tau| \to \infty$

$\sim e^{-ik\tau}|\tau|^{1}$

NB. Can certainly solve numerically without SR, or with interrupted SR.

In some cases, can even get analytic solutions, eg if V=



(cf. final lecture).

Now we understand how to use the result

$$P_R(k) = \frac{H^2}{2k^3}\frac{H^2}{\dot\phi^2}.$$

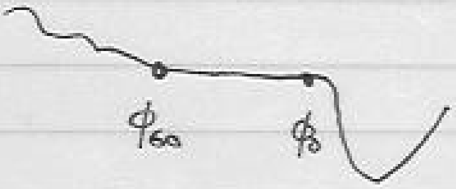
For SR case, compute $\dot\phi$ in SR ($\dot\phi \neq 0$)

and evaluate RHS when a given mode k_* exits the horizon:

$$P_R(k_*) = \frac{H^2}{2k_*^3}\frac{H^2}{\dot\phi^2}\Bigg|_{k_*=aH}.$$

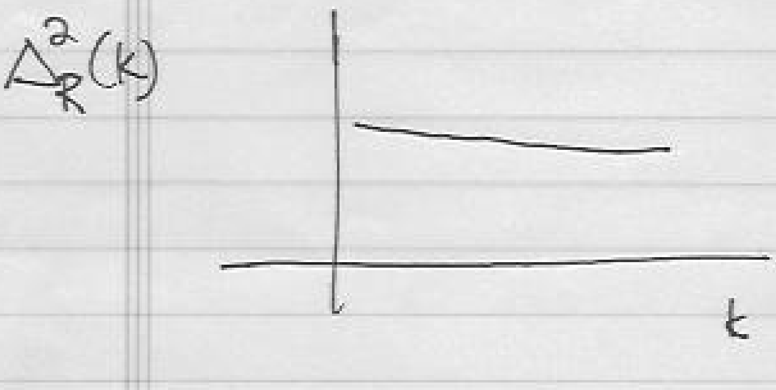
So, if inflationary potential changes over time, (i.e. over $\Delta\phi$)

This is written in $P_R(k)$.



early ($\phi \approx \phi_{60}$): small k $\approx$ present horizon

late ($\phi \approx \phi_0$): large k, angular size $\ll$ present horizon



<u>Remark:</u> Don't worry about $\dot{\phi} \to 0$ divergence.

For an ordinary V,

$$\delta\phi \Rightarrow \delta\rho \text{ (equiv, } \delta R\text{)}.$$

for a perfectly flat V, inflation never ends, and $\delta\phi$ is not an acceptable clock.

We'll work <u>near</u> but not <u>in</u> the de Sitter limit.

Conventional to define also the dimensionless power spectrum

$$\Delta^2_R(k) \equiv \frac{k^3}{2\pi^2} P_R(k)$$

$$= \frac{H^2}{(2\pi)^2} \frac{H^2}{\dot{\phi}^2}.$$

Using $\varepsilon_H = \frac{\dot{\phi}^2}{2H^2}$ and working in SR where $\varepsilon_H \approx \varepsilon_V$, we get

$$\Delta^2_R(k) = \frac{1}{24\pi^2} \frac{V}{\varepsilon_V} = \frac{1}{12\pi^2} \frac{V^3}{V'^2}.$$

Writing $\Delta^2_R(k) = A(k_0)\left(k/k_0\right)^{n_s-1}$ (1)

and comparing to $\Delta^2_R(k) = \frac{1}{12\pi^2}\frac{V^3}{V'^2}$ (2)

We compute $\frac{d}{d\ln k}\ln\Delta^2_R(k) = n_s - 1$ from (1)

while from (2), we use

$\ln k = \ln aH = \ln(e^N H) = N + \ln H$

and $\frac{d}{d\ln k} \approx \frac{d}{dN}$ $\quad dN \equiv H\,dt = \frac{H\,d\phi}{\dot\phi}$

$= \frac{\dot\phi}{H}\frac{d}{d\phi}$.

So from (2),

$$\frac{d}{d\ln k}\ln\Delta^2_R(k) = \frac{\dot\phi}{H}\left[3\frac{V'}{V} - 2\frac{V''}{V'}\right]$$

in SR, $\dot\phi = \frac{-V'}{3H}$ $\quad \frac{\dot\phi}{H} = \frac{-V'}{V}$

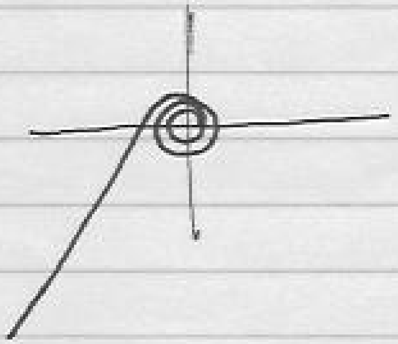
So $\quad n_s - 1 = -3\left(\frac{V'}{V}\right)^2 + 2\frac{V''}{V}$

$$n_s - 1 = 2\eta_V - 6\varepsilon_V. \quad ▨$$

As inflation proceeds, each mode is stretched to superhorizon size $(k < aH)$.
It exits the horizon.

As we have seen, for $k<aH$ the perturbations do not evolve: upon horizon exit, they freeze.

Sketch: u_k $\lfloor\mathbb{C}$



Essential process: aH increasing $(\ddot{a}>0)$.

But eventually, inflation will end. Then $\ddot{a}<0$, and aH decreases.
Eventually, $aH < k_*$. for any given k_*.

The modes awaken from their frozen sleep and begin their oscillations again!

Since $aH = \dot{a}$ increases iff inflation is occurring, useful to draw

$\frac{1}{aH}$ vs. a

or $\ln \frac{1}{aH}$ vs $\ln a$.

In inflation, $\ln \frac{1}{aH} = -\ln a + \text{const}$

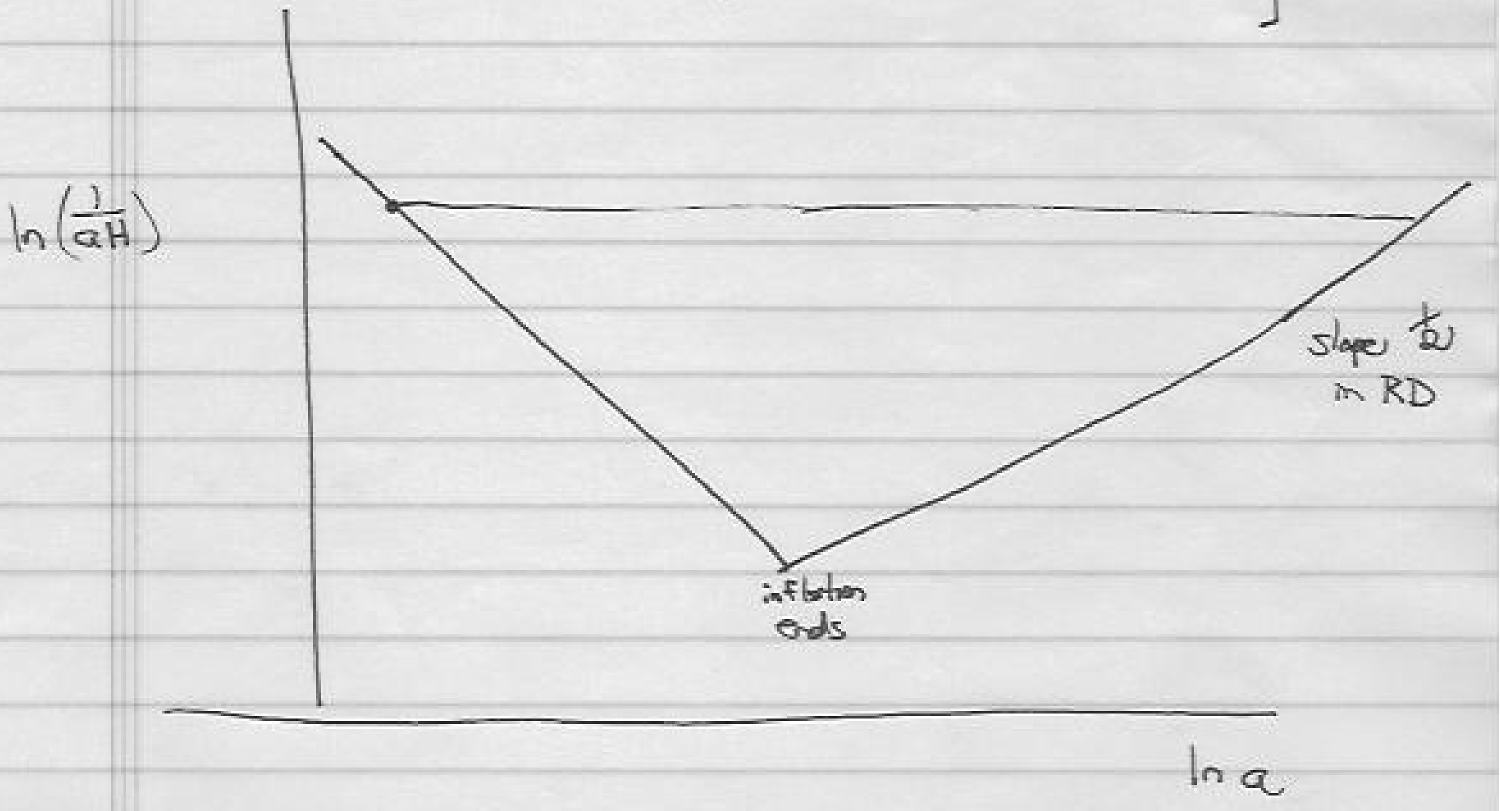
In regular FRW, $\ln \frac{1}{aH} = \ln \frac{1}{\dot{a}}$

$$\left[a = (t/t_0)^{\frac{2}{3(1+w)}} \qquad \dot{a} = \frac{2}{3(1+w)} \frac{a}{t} \right.$$

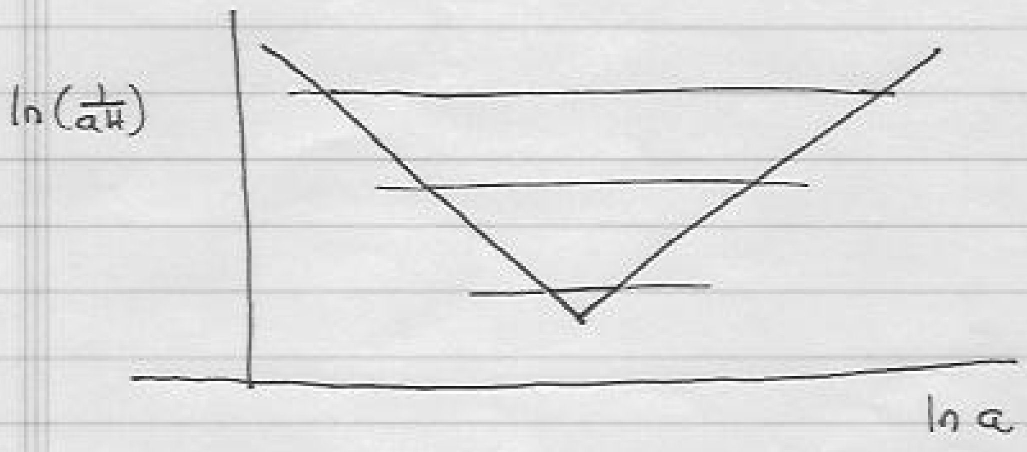
$$\ln \frac{1}{\dot{a}} = -\ln a + \ln t + \text{const}$$

$$= -\ln a + \frac{3(1+w)}{2} \ln a + \text{const}$$

$$\left. = \left(\frac{3}{2}w + \frac{1}{2}\right) \ln a \right]$$



The last modes to exit are the first to reenter:



Therefore, some modes reentered at the end of inflation and are very small today.

But other modes that ~~entered~~ exited + froze 60 efolds before the end are only now reentering the horizon.

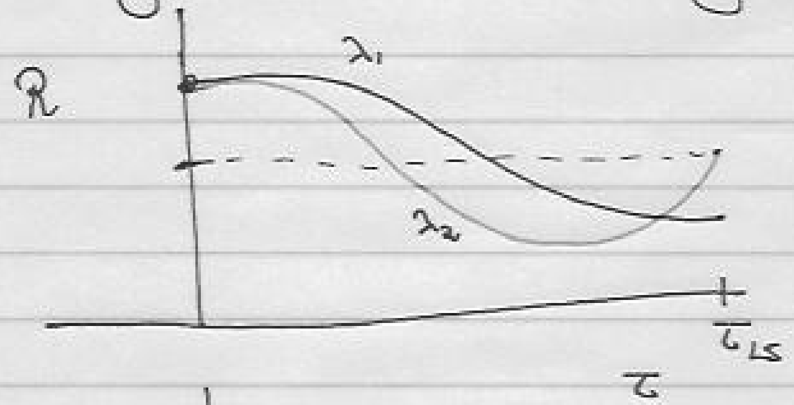
Inflation does something quite remarkable: it arranges that all the Fourier modes have the same phase.

An important topic we will not treat in detail is the evolution of the perturbations upon reentry.

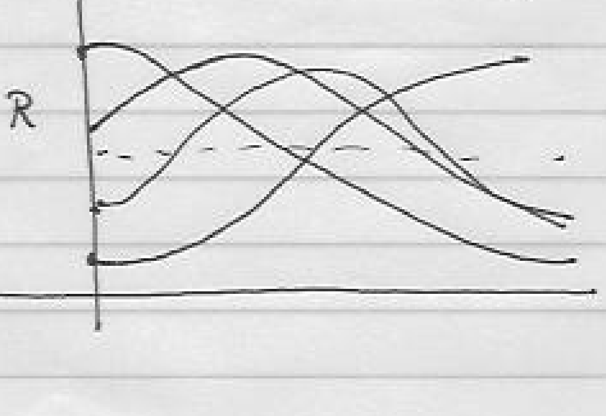
Brief version:
under/overdensities are driven to collapse by gravity, but baryon-photon fluid creates pressure opposing this.

At horizon entry we have $\mathcal{R}_k \neq 0$ but $\dot{\mathcal{R}}_k \approx 0$.
(all modes in phase)
So collapse sets in, then there is a rebound.

A $\mathcal{R}_{\vec{k}}$ for given mode (i.e. some given $\vec{k}$) will then:



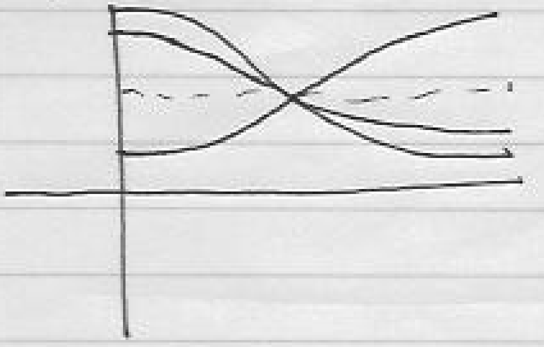
Now $\exists$ many modes $\vec{k}$ with same $|\vec{k}|$. If these have random phases,



destructive interference

But *if* all modes with $|\vec{k}| = \frac{1}{\lambda_1}$ have same phase,

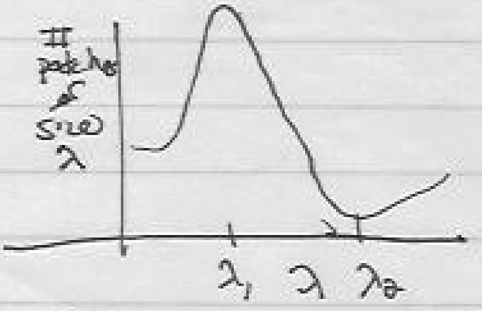
constructive interference



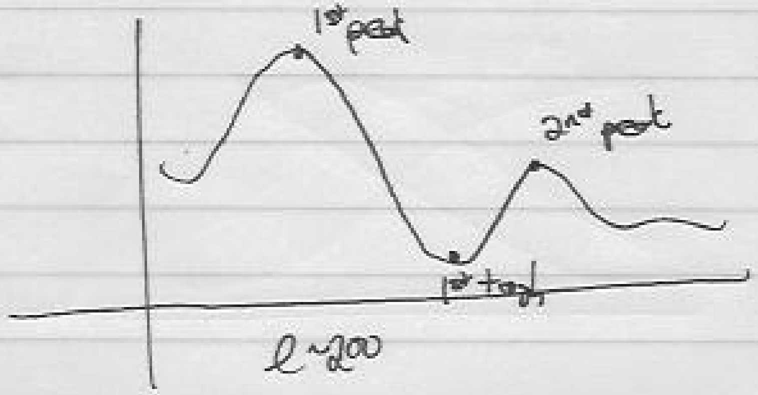
Patches of size $X \Leftrightarrow \lambda_1$ are enhanced on the surface of LS.

Patches of size $X \leftrightarrow \lambda_2$ are suppressed.

So a histogram



More precisely, C_ℓ has a pattern of acoustic peaks



The evolution giving the acoustic peaks is plasma physics, not inflationary physics.

But it is *crucial* that the initial conditions involve phase coherence among superhorizon modes.

Otherwise the peaks wash out!

Really need *all* modes to start (i.e. enter horizon) with $R \neq 0$ but $\dot{R} \approx 0$.

We have decisive exp'tl evidence for acoustic peaks,
which can only come from a spectrum of perts. involving phase correlations on superhorizon scales.

Inflation automatically provides a causal (semiclassical) mechanism preparing this initial condition!