V. Flux Compactifications

Inflaton can be an open-string or closed-string scalar.

Promising choice: in IIB string theory on O3/O7 orientifolds of CY threefolds, ϕ = position of a D3-brane.

We'll focus on type IIB string theory on CY orientifolds.

Specifically, we'll take fixed-point set of involution to be pts + 4-cycles

i.e. O3/O7-planes.

$$S_{IIB}^{D=10} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{G}\left\{R - \frac{\partial\tau\,\overline{\partial\tau}}{2(\mathrm{Im}\,\tau)^2} - \frac{G_3\cdot\overline{G_3}}{12\,\mathrm{Im}\,\tau} - \frac{1}{480}\tilde{F}_5^2\right\}$$

$$- \frac{i}{8\kappa_{10}^2}\int\frac{C_4\wedge G_3\wedge\overline{G_3}}{\mathrm{Im}\,\tau} + S_{\mathrm{local}}$$

$$\text{with}\begin{cases} G_3 = F_3 - \tau H_3 = dC_2 - \tau dB_2 \\ \tilde{F}_5 = dC_4 - \frac{1}{2}C_2\wedge H_3 + \frac{1}{2}B_2\wedge F_3 \\ \tau = C_0 + ie^{-\phi} \end{cases}$$

This is leading sugra, of course $\exists$ g_s and α' corrections.

We consider the warped ansatz (GKP)

$$ds^2 = e^{-6u(x)} e^{2A(y)} g_{\mu\nu} dx^\mu dx^\nu + e^{2u(x)} e^{-2A(y)} \tilde{g}_{ij} dy^i dy^j$$

with $\tilde{g}$ a CY metric
u 'breathing mode'
A warp factor.

Note: for real dynamics, a better ansatz is required, eg to solve $G_{\mu i}$ E.E. (STUD).
For our purposes, the above does suffice.

also, take $\tilde{F}_5 = \partial_i \alpha(y) [1+*] dy^i \wedge dx^0 \wedge \dots dx^3$

In general, compact warped solutions not easily found.

But suppose we insist that all sources obey the BPS-like condition

$$\frac{1}{4}\left(T^m{}_m - T^\mu{}_\mu\right)^{loc} \geq T_3 \rho_3^{loc}$$

$\hookrightarrow$ D3-brane charge

This <u>is</u> obeyed by D3, O3; D5 on collapsed cycles;
indeed, saturated   D7-branes BPS wrt D3.

Then, one finds solutions obeying:

$$*G_3 = i\,G_3 \quad \text{(ISD)}$$

$$e^{4A} = \alpha,$$

In these solutions, u is unconstrained.

Moreover, D3-branes have $\mathcal{M} = CY$, as we'll now see.

$$S_{DBI+CS} = -T_3 \int \sqrt{\det g_{ind}(-1)}\, d^4\xi + T_3 \int C_4$$

$$T_3 = \frac{1}{(2\pi)^3 g_s \alpha'^2}$$

$$(g_{ind})_{\alpha\beta} = \frac{\partial X^M}{\partial \xi^\alpha}\frac{\partial X^N}{\partial \xi^\beta} G_{MN} \qquad M,N: 0,..9 \quad \alpha,\beta: 0,..3$$

for homogeneous configs, $y^m = y^m(t)$,
we have

$$(g_{ind})_{ij} = g_{ij}\, e^{2A}\, e^{-6u}$$

$$(g_{ind})_{00} = \left[g_{00}\, e^{2A-6u} + G_{mn}\,\dot{y}^m \dot{y}^n\right]$$

Taking $g_{\mu\nu} = \eta_{\mu\nu}$ for simplicity,

$$-\det g_{ind} = - e^{8A} \underbrace{\det g}_{-1} e^{-24U} \Big[1 - e^{-4A+8U} \tilde{g}_{mn} \dot{y}^m \dot{y}$$

So

$$\mathcal{L}_{DBI+CS} = - T_3 e^{4A-12U} \sqrt{1 - e^{8U-4A} \tilde{g}_{mn} \dot{y}^m \dot{y}^n}$$
$$+ T_3 \alpha e^{-12U}$$

for low velocities,

$$\mathcal{L}_{DBI+CS} = - T_3 (e^{4A} - \alpha) e^{-12U} + \frac{1}{2} T_3 \tilde{g}_{mn} \dot{y}^m \dot{y}^n e^{-4U}$$

we define $\Phi_{\pm} = e^{4A} \pm \alpha$

$\phi^m = \sqrt{T_3}\, y^m$

for $\Phi_- = 0$ ($\Leftrightarrow$ ISD soln)

we have $V \equiv 0$.

One can 'derive' really packages all this in $\mathcal{N}=1$ data K, W, as we'll now see.

~~So,~~ One can show that compactifications of IIB on O3/O7 CY orientifolds,

containing D3 + D7-branes

and G-fluxes $*G = iG$ $G = F - \tau H$

and F_5 flux $F_5 = (1+*) d\alpha \wedge Vol_4$

and warping

$$e^{2A} g_{\mu\nu} dx^\mu dx^\nu + e^{-2A} \tilde{g}_{mn} dy^m dy^n$$

obeying

$$e^{4A} = \alpha$$

which we'll call ISD $\mathcal{O}$,

are described by $\mathcal{N}=1$ sugra in 4D,

with $W = \int G \wedge \Omega$

and $K = -2 \ln \mathcal{V} - \ln\left(-i \int \Omega \wedge \bar{\Omega}\right) - \ln\left(-i(\tau - \bar{\tau})\right)$

coords:
v^α t^α
$\mathcal{S}_2$
τ
ϕ_i

$$\left[\mathcal{V} = \tfrac{1}{6} C_{\alpha\beta\gamma} t^\alpha t^\beta t^\gamma\right].$$

2-cycle volumes

for $h^{1,1} = 1$, $\mathcal{V} = (\rho + \bar{\rho})^{3/2}$ $\rho = \frac{\partial \mathcal{V}}{\partial t}$?

and $K_{\text{Kähler}} = -3 \log(\rho + \bar{\rho})$.

and any D3-branes enter as

$$K_{\text{Kahler}+D3} = -3\log\left(\rho+\bar{\rho}-\sum_{i=1}^{N_{D3}} k_i(\phi_i,\bar{\phi}_i)\right)$$

with $\nabla_\alpha \overline{\nabla}_{\beta} k = g_{\alpha\bar{\beta}} \underset{\mathbb{C}\ \ \mathbb{R}}{\Leftrightarrow} g_{mn}$

Then, this system enjoys the very important <u>no-scale</u> property:

$$V = V_F[K, W]$$

$$= e^K\left(K^{A\bar{B}} D_A W \overline{D_B W} - 3W\overline{W}\right)$$

A: $\rho, \tau, \phi_i,$ ζ_λ

$\lambda = 1 \ldots h^{21}$

$$\left[D_A W = \partial_A W + K_{,A} W\right]$$

$$= e^K\Bigg(\underbrace{\left[K^{\rho\bar{\rho}} D_\rho W \overline{D_\rho W} + \cdot\ K^{\phi_i\bar{\phi}_j} D_{\phi_i} W \overline{D_{\phi_j} W}\right]}_{} \Big\}\ \rho, \phi_1 \ldots \phi_{N}$$

$$+ K^{\tau\bar{\tau}} D_\tau W \overline{D_\tau W} + K^{\zeta_\lambda \bar{\zeta}_\nu} D_{\zeta_\lambda} W \overline{D_{\zeta_\nu} W}$$

$$- 3W\overline{W}\Bigg)$$

and $\partial_\rho W = \partial_\phi W \equiv 0$, but even better,

$$K^{X\bar{Y}} D_X W \overline{D_Y W} = K^{X\bar{Y}} K_X \overline{K_Y} W\bar{W} \overset{!}{=} 3W\bar{W}$$

if X, Y: $\rho, \phi_1, \ldots \phi_N$.

easy to check for no D3:

$$K_{\rho\bar{\rho}} = \frac{3}{(\rho+\bar{\rho})^2} \quad K_\rho = -\frac{3}{\rho+\bar{\rho}} \Big\} \; 3. \checkmark$$

So?

Well, the s_α, and τ_1 appear in W.

By eqn counting, in general $\exists$ sol. where

$$D_{s_\alpha} W = D_\tau W = 0 \quad \forall \alpha$$

(F-flat).

Then, $V = V_F[s,\tau] + 0 = 0$.

$V = 0$ at tree level

but $W \neq 0$

$\Rightarrow m_{3/2} = e^{K/2} |W| \neq 0$.

$\cancel{\text{SUSY}}$ $\forall$ $\langle\rho\rangle$ $(\rho < \infty)$.

So: turning on fluxes creates a scalar potential for the cx. str M ζ_λ,

via $$\mathcal{L} \supset G \wedge \underbrace{*}_{\hookrightarrow \text{ metric!}} \overline{G}$$

or equiv. $$W = \int G \wedge \Omega \curvearrowright \zeta_\lambda!$$

if the fluxes are ISD

ISD $\Leftrightarrow$ (2,1) primitive $\qquad J \wedge G^{(p)}_{2,1} = 0$

$+$

(0,3)

$+$

(1,2) nonprimitive $\qquad$ (of form $\underbrace{\omega_1}_{\text{hol. 1-form}} \wedge J$ $\nexists$ in cpt. CY)

then solutions $\overset{\text{in this class}}{\wedge}$ generally exist;

if $G = G^p_{(2,1)}$ then $W = 0$, $m_{3/2} = 0$;

if $G = G^p_{(2,1)} + G_{0,3}$ then $W = \int G \wedge \Omega \neq 0$, $m_{3/2} \neq 0$

Now since (generically) $\exists$ vacua where $D_{S_A}W \neq 0$, the S_A (and τ) receive a potential!

determine scale V_M for "isotropic" CY with radius L.

well, quantiz. $\Rightarrow$ $\int F = (2\pi)^2\alpha'$ H similar

$$V = \int G \wedge * \bar{G} \supset \int_{\Sigma_A^{(3)}} F \int_{\Sigma_B^{(3)}} H$$

$$F \sim \frac{\alpha'}{L^3} \quad V \sim \left(\frac{\alpha'}{L^3}\right)^2.$$

But ρ does not.

Good reason: $\rho = \int_{\Sigma_4} \sqrt{g} + i \int_{\Sigma_4} C_4$

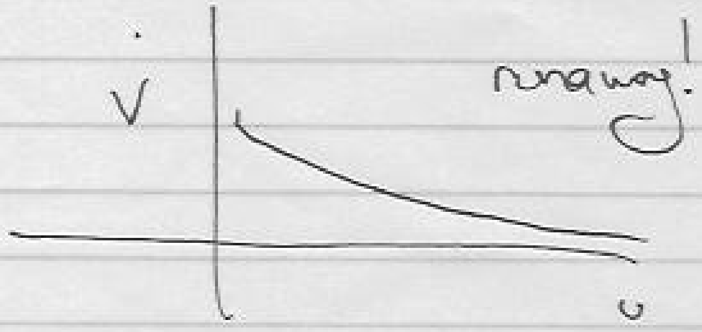
and $C_4 \to C_4 +$ const. is a symmetry

This is a flat dir. in no-scale $\mathcal{C}$, but adding any $\cancel{\text{SUSY}}$ source (IASD flux, $\overline{\text{D3}}$..) creates a (critical) decompactification instability

e.g. $V_{\overline{D3}} = T_3 e^{-12u} \Phi_+ \neq 0$

$V_{D3} = T_3 e^{-12u} \Phi_-$ can be $\neq 0$ once $\mathcal{C}$ is not ISD.

$V = 0$ only at $u = \infty$.



Hard to control this!

How can ρ be stabilized?

$\rho \to \rho + \overset{2\pi}{i}(\mathrm{const})$ should be a good symmetry to all orders in g_s + α'!

So, $W \supset \rho^{\alpha}$ is forbidden.

But:

i) $W = e^{-a\rho}$ is allowed : NP.

ii) K can receive corrections, e.g. $\Delta K \propto \frac{\alpha'^2}{\rho+\bar{\rho}}$

indeed,

The leading $\alpha'^3 \zeta(3)$ Riemann^4 term gives upon dim red (BBHL)

$$K = -2\log(\mathcal{V} + \zeta(3)\chi(M)).$$

This is not no-scale any more!

α'^3 corrections in 10D sugra break no scale + give ρ a potential.

__Not__ much success using Kähler corrections to stabilize ρ in detail.

Reason:
- if 1st term matters, why not 2nd?
- not all α'^3 terms known, let alone next order

Clever idea: (KKLT 03).

incorporate NP contrib to W and work in a regime where these are controllable __and__ dominate over $V_{(\alpha')}(\rho,\bar{\rho})$.

Nonperturbative W in brief.

Consider N D7 on Σ_4. (holomorphic) deformations of $\Sigma_4 \Leftrightarrow$ adjoint matter fields in 4D $\mathcal{N}=1$ SYM on D7.

If Σ_4 is rigid, D7 thy. is pure glue)
$\mathcal{N}=1$ SYM.

At low E, confines & generates a $\langle\lambda\lambda\rangle$ W:

$$W = M_{UV}^3 \exp\left(-\frac{8\pi}{g_{YM}^2}\frac{1}{c_2(G)}\right)$$

$$= M_{UV}^3 e^{-\left(\frac{8\pi}{g^2 N}\right)}.$$

Can check carefully:

$$\frac{8\pi}{g_{YM}^2} = \underbrace{\frac{1}{(2\pi)^3 g_s \alpha'^2}}_{T_{D3}} \int_{\Sigma_4} \sqrt{\tilde{g}_4}\, d^4\xi\, e^{-4A} \equiv T_3 V_4^w$$

$$\Big(\text{since } F_{MN}F^{MN}\underbrace{\sqrt{g_{4D}}}\,\underbrace{\sqrt{g_4^{\Sigma_4}}}$$

$$e^{-4A}\tilde{g}^{\mu\rho}\tilde{g}^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma}\quad e^{4A}\sqrt{\tilde{g}_{4D}}\sqrt{\tilde{g}_4^{\Sigma_4}}\,e^{-4A}\Big).$$

$$\text{So } W = e^{-T_3 V_4^w \cdot \frac{1}{N_{D7}}}$$

Similar contrib. from ED3,

$$W = e^{-T_3 V_4^W}.$$

(again, require suitably rigid 4-cycle.)

Identifying $T_3 V_4^W \equiv 2\pi\rho$ (long story to show this is a good Kähler coordinate!)

we have $W = e^{-\frac{2\pi}{N}\rho} \equiv e^{-a\rho}$.

Now, $W = W_0 + e^{-a\rho}$

$$K = -3\log(\rho+\bar{\rho}) + K_{\phi_n} + K_{\tau}$$

$\partial_\rho W \overset{!}{\neq} 0$, so no-scale is spoiled.

In general one expects solutions with

$D_\rho W = 0$ at $\rho = \rho_*$

while for $|\rho - \rho_*| \ll 1$, V_F increases

i.e. SUSY minima.

Recap: special class of warped, conformally-CY O3/O7 orientifolds,

ISD G,

$$\begin{cases} G_\pm = (i \pm *)G_3 \\ \Phi_\pm = e^{4A} \pm \alpha \end{cases}$$

with $G_- = \Phi_- = 0$

i.e. $$\begin{cases} *G = iG \\ e^{4A} = \alpha \end{cases}$$

have: no-scale structure at l.o.,

$V = 0$ $\forall \langle \rho \rangle$ $\forall \langle \phi^m_{D3} \rangle$ flat directions

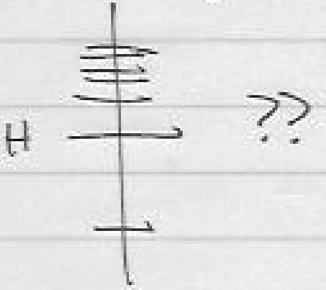
while (in general) the cx str mod z_α, and τ, find SUSY minima.

so tree-level flux G stabilizes cx str + τ via $W = \int G \wedge \Omega$
while ρ, ϕ^m_{D3} are unstabilized.

while nonperturbative effects on wrapped D7-branes can stabilize ρ.

Very natural to ask "can the D3-brane position ϕ be an inflaton?"
after all, $V=0$ at l.o.!

Now think back to our general worry
that lifting some $\mathcal{M}$ lifts the rest.



We've argued ρ has a de-€ instability
that is controlled in eg KKLT €,
by W_{NP} from D7-branes wrapping
a suitable rigid four-cycle.

In the l.o. no-scale €, D3 had $V=0$
$\forall \langle \phi^n \rangle$.

Can this <u>persist</u> upon stabilization of φ?

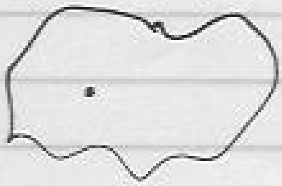
VI The D3-brane Potential in Stabilized Compactification

We will consider a D3-brane in an ISD ⊙
($V_\phi = 0$, but $\rho \to \infty$ upon adding $V \neq 0$)
supplemented by W_{NP} from wrapped D7-branes and study the D3-brane effective action.

- can't even get $\tilde{g}_{mn}$ explicitly for compact CY.

Need a simpler setting.

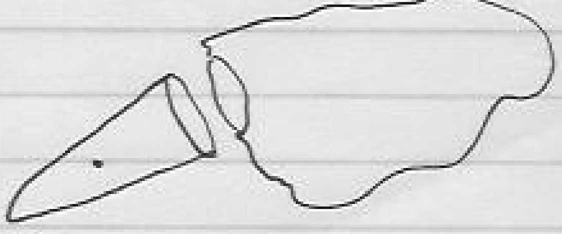
Idea: study D3 in noncompact CY cone and eventually (systematically) incorporate effects of compact bulk.



hopeless at present

case of interest

approximation



This is a *rich* arena for cosmology - diverse scenarios with, eg., NG, cosmic strings, firm bounds on r, unusual kinetic terms.

1) $$\mathcal{L}_{kin} = -T_3 e^{4A-12u}\left(\sqrt{1-e^{-4A+8u}\,\tilde{g}_{mn}\dot{y}^m\dot{y}^n}-1\right)$$

fully general

$$\sim \sqrt{1-\dot{\phi}^2 e^{-4A}}-1$$

all orders in $\dot{\phi}$!

We'll study $\dot{\phi}^2 \ll 1$ and ignore these higher terms, because of time limitations.
But *really* interesting for NG!

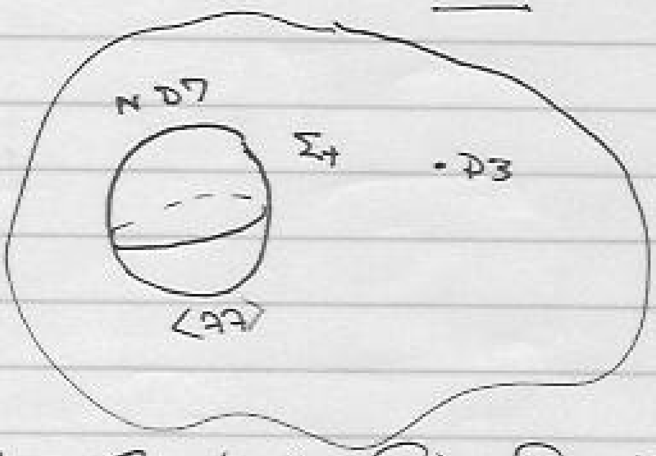
2) case we'll study:

$$\mathcal{L} = \frac{1}{2} T_3\, \tilde{g}_{mn}\dot{y}^m\dot{y}^n e^{-4u} - V(y)$$

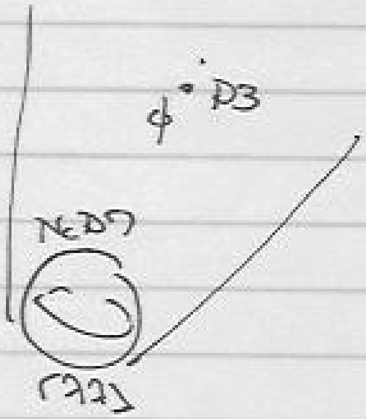
need to compute $V(y)$.

We know the l.o. result.

What is the effect of $\underline{W_{NP}}$ on V_{D3}?



take de-ℂ limit first for illustration,



4D: $G_{x}=0$

$\mathcal{N}=1$ SU(N) SYM

<u>not</u> pure glue, $N_f=1$.

mass $= \phi - \phi_{D7} \propto l_{3\text{-}7 \text{ string}}$

Using classic holomorphy arguments, one finds (cf lectures by Intriligator)

$$W = m^{1/N_c} \cdot \left(\begin{matrix}\text{indep}\\ \text{of}\\ m\end{matrix}\right) \qquad W_{low} = N_c (\det m)^{N_f/N_c} \times \times \Lambda^{3-N_f/N_c}$$

now $m \propto \phi$

so $W \propto \phi^{1/N_c}$ $\quad V_{4D}^{(\phi)} \neq 0!$ $\quad \rightarrow m^{1/N_c} \cdot (N_c \Lambda^{3-1/N_c})$

gauge thy

Many ^other ways to derive this:

open string 1-loop BHK '04

general topological considerations Ganor '96

closed string (ie. SUGRA) BDKMMM '05.

result: instead of $W_{NP} = A_0 e^{-a\rho}$

one has, for N_c D7-branes wrapping a 4-cycle

Σ_4 defined by $f(z) = 0$, $\quad z$: local $\mathbb{C}$ coordinates ($\leftrightarrow y^m \mathbb{R}$)

$$W_{NP} = A_0 f(z)^{1/N_c} e^{-a\rho} \qquad a = \frac{2\pi}{N_c}$$

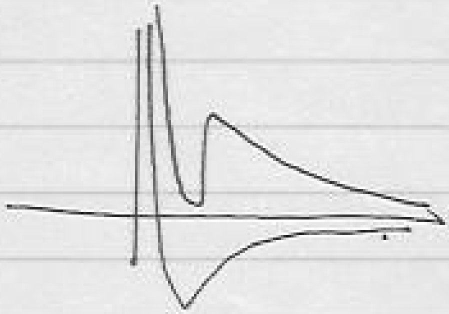
- A_0 a constant after stab. of δ_λ.

Implication: W_{NP} gives non-negligible contrib. to V_{D3}.

How to see?

instead of $V = V_F^{(M)}(\zeta_\alpha, \rho, \tau) + V^{(\phi)}(\phi)$

we have $V = V_F^{(M,\phi)}(\zeta_\alpha, \rho, \tau, \phi)$.



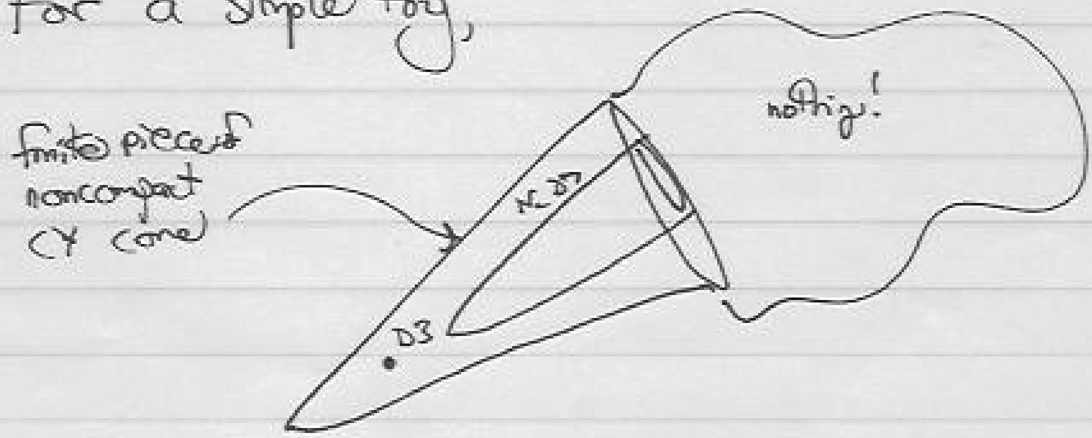
$V_F^M \overset{!}{\not\ll} V^{(\phi)}$ else de-$\mathcal{E}$ problem

Simple check: if $V = \left(\frac{\phi}{\Lambda}\right)^{1/r_c} \Lambda^{4-1/r_c}$

then $\eta = \left(\frac{1}{r_c}\right)\left(\frac{1}{r_c-1}\right)\left(\frac{M_p}{\phi}\right)^2$

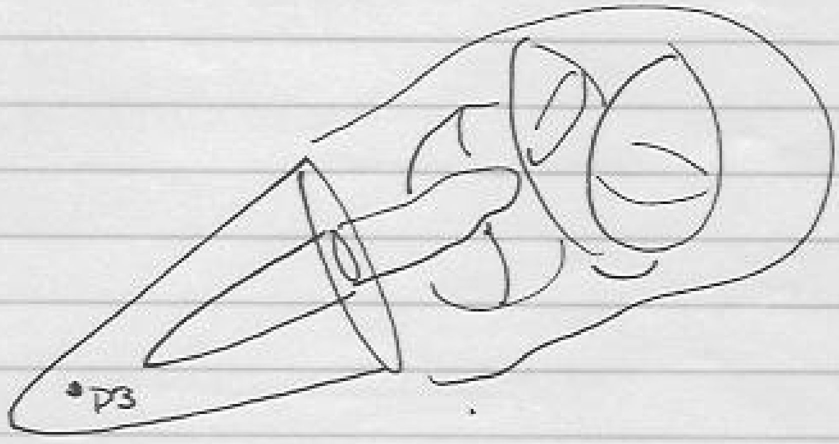
so unless $\phi r_c \gg M_p$, $\eta \not\ll 1$.

For a simple toy,



one can go ahead + compute $V_F(\rho,\phi)$ in full, explicitly. (BDKM '07).

But what about



$h^{1,1}_+ > 1$

<u>General point</u>: distant Σ_4 with $W_{D7}^{(NP)}$ give non-negligible contrib to $V^{(\phi)}$, i.e. $\Delta\eta \not\ll 1$.

Can we <u>decouple</u> these effects somehow?

idea: take

very slender, long cone so 3-7 strings have $M \gg M_p$ ⇒ effects suppressed by $\gg \frac{1}{M_p}$.

Does not work, as we'll show soon.

- Can we decouple W_{NP}?
- when we can't, what is $V(\phi)$?

To study these points,

we need to be more concrete about the noncompact CY cone.

We'll specialize to the conifold soon, but all methods generalize.

CY Cones

Let X_5 be a Sasaki-Einstein 5-manifold. The cone

$$ds_6^2 = dr^2 + r^2 ds_{X_5}^2$$

has a CY metric.

Taking N D3-branes at the tip of the cone, then taking the near-horizon limit, one obtains

$$ds_{10}^2 = \left(\frac{r}{R}\right)^2 \eta_{\mu\nu} dx^\mu dx^\nu + \left(\frac{R}{r}\right)^2 \left[dr^2 + r^2 ds_{X_5}^2\right]$$

$$\Leftrightarrow AdS_5 \times X_5$$

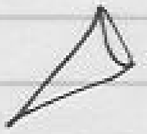
with $R^4 = \dfrac{4\pi^4 g_s N \alpha'^2}{\mathrm{vol}\, X_5}$.

This is dual to an $\mathcal{N}=1$ SCFT.

Simplest example: conifold.

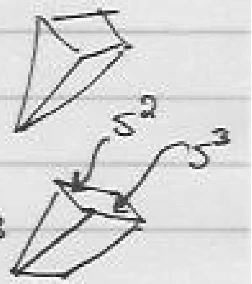
take z_i $i=1..4$ coords in $\mathbb{C}^4$

$\sum z_i^2 = 0$ singular conifold



$\sum z_i^2 = \epsilon^2$ deformed conifold

T^*S^3



cf. resolved conifold

topologically, cone over $S^2\times S^3$.

metrically, base is coset space

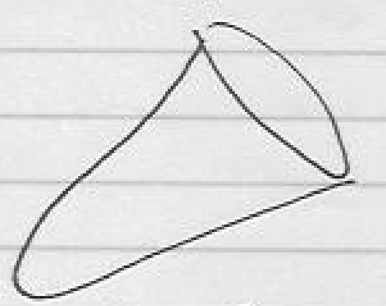
$$T^{1,1} = [SU(2)\times SU(2)]/U(1).$$

coords $\theta_1, \theta_2, \phi_1, \phi_2, \psi$.

we'll not need metric in detail.

The dual SCFT is the KW CFT, about which more soon.

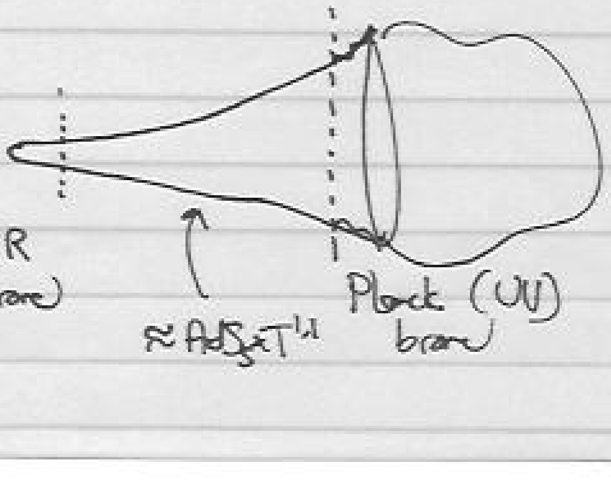
Finally, taking N D3 and M D5 wrapping the shrinking S^2, we find a supergravity solution that is everywhere <u>smooth</u>, the warped deformed conifold or Klebanov-Strassler throat.



near tip, $S^3 \times \mathbb{R}^3$ size of $S^3 \sim \sqrt{g_s M \alpha'}$.

far from tip, approximated by KW CFT $\Leftrightarrow$ warped conifold, up to 'log corrections.'

We'll systematically suppress mention of these log corrections during these lectures.



KW CFT

gauge group $SU(N) \times SU(N)$ global sym. $SU(2) \times SU(2) \times U(1)_R$

matter fields	A_i	$N, \bar{N}$	$1/2$	0	1
	B_j	$\bar{N}, N$	0	$1/2$	1

chiral gauge field strength superfields $W_\alpha = \lambda_\alpha + \theta^\beta F_{\mu\nu} \sigma^{\mu\nu}_{\alpha\beta} + \dots$
$(W^{(1)}, W^{(2)})$.

$$W_{tree} = \lambda \epsilon^{ik} \epsilon^{jl} A_i B_j A_k B_l .$$

gauge-invt. comb. $Tr(A_i B_j)$. etc.

written $(AB)^k$.

now $A_1 = a_1 + \theta_\alpha \psi^\alpha_{(A_1)} + \theta^2 F_{A_1}$

then $\langle Tr\, a_i b_j \rangle$ characterizes position on $\mathcal{M}$

~~[illegible]~~

and for $a_i b_j = diag(a_i b_j^{(1)}, \dots a_i b_j^{(N)})$

~~[illegible]~~

~~[illegible]~~ we have the Coulomb branch of separated D3-branes probing the conifold.

To be clear about strategy:

noncompactification gives $G_N = 0$, so not helpful.

general SU(3) hol. is intractable (need metric data!)

Middle path: inclusion of $\mathcal{C}$ effects in a finite throat.

Approximate this finite throat by a slice of $AdS_5 \times T^{1,1}$. $r_{IR} < r < r_{UV}$ between IR and Planck branes.

Planck-scale effects come from bulk of $\mathcal{C}$, through the Planck brane.

We need to characterize these effects somehow.

- In SUGRA, find $V_{D3} = T_3 \, \Phi_- = T_3 (e^{4A} - \alpha)$ in bg of non-norm profile sourced by bulk objects + fields
- In CFT, find V on Coulomb branch given UV perturbations to $\mathcal{L}_{CFT}$.

Two views:

KW CFT	$AdS_5 \times T^{1,1}$
$\Delta \mathcal{L} = \sum_i \mathcal{O}_i^{(\Delta)} c_i$	$\chi = \chi_{UV} r^{\Delta-4} + \chi_{IR} r^{-\Delta}$ non-normalizable modes of sugra fields
vev describing location on Coulomb branch [ie branch ~~of~~ characterized by diagonal matrices for scalars]	ϕ_{D3}

Now in EFT, one could try to write

$$V = V_{renorm} + \sum_{(i \mid \Delta_i > 4)} c_i \mathcal{O}^{(\Delta)} \frac{1}{\Lambda^{\Delta_i - 4}}$$

for $\mathcal{O}^{(\Delta)}$ any allowed $\mathcal{O}$ in the theory;

This gives the '<u>structure</u>' of the potential, but not the Wilson coefficients c_i.

Here we cannot even get the structure $\{\Delta_i\}$ in QFT: the KW thy. is strongly-coupled.

We'll obtain the $\{\Delta_i\}$ in SUGRA, and learn which terms $\Leftrightarrow$ which sorts of physics (fluxes, branes, ..) in the bulk of the $\mathcal{C}$.

This is a <u>first</u> step; next one would try to understand typical values for the c_i and eventually study statistics of c_i across many vacua.

This is a worked example of incorporating the effects of <u>Planck-scale physics</u>, but it is still not fully explicit.

After considerable work one obtains

$$V = V_0 + c_1 j_1(\Psi)\phi^1 + a_{3/2} h_{3/2}(\Psi)\phi^{3/2}$$

$$+ \left[b_2 + c_2 j_2 + a_2 h_2(\Psi)\right]\phi^2$$

$$+ \quad c_{2.79}\, j_{2.79}(\Psi)\,\phi^{2.79}$$

2.79 is really $\sqrt{28} - 2$.

$$+ \dots$$

here the j, h fns can be determined explicitly, but will be suppressed here.

(j series)	c coeffs:	harmonic parts of Φ_-
(i series)	b coeffs:	effect of R_4
(h series)	a coeffs:	G_- fluxes

General scheme: try to derive general properties of $\mathcal{L}_{eff}$ ○ in terms of ⓔ data.

Much simpler than determining $V(\phi)$ is constraining r_{CMB}

Recall: 1) $\frac{r_{CMB}}{.01} \approx \left(\frac{\Delta\phi}{M_p}\right)^2.$

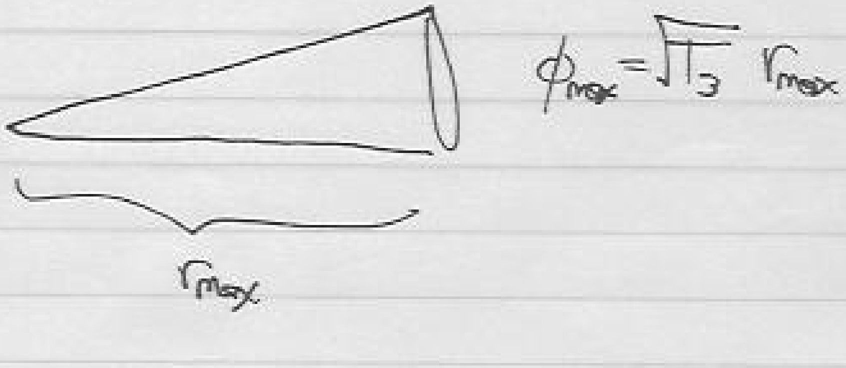
2) for D3 in cone, $\phi = \sqrt{T_3}\, r.$

Let's determine $r_{CMB}^{(max)}$ in these scenarios.

AdS approx,

$$ds^2 = \left(\frac{r}{R}\right)^2 \eta_{\mu\nu} dx^\mu dx^\nu + \left(\frac{R}{r}\right)^2 \left[dr^2 + r^2 ds_{X_5}^2\right]$$

$$R^4 = \frac{4\pi^4 g_s N \alpha'^2}{\mathrm{vol}\, X_5}$$



Now $\frac{M_p^2}{2} = \frac{V_{CY}^{(w)}}{2\kappa_{10}^2} = \frac{1}{2\kappa_{10}^2}\int d^6y\sqrt{\tilde{g}}\, e^{-4A(y)}$

$$\kappa_{10}^2 = \frac{1}{2}(2\pi)^7 g_s^2 \alpha'^4$$

now $V_{CY}^{w} = V_{throat}^{w} + V_{bulk}^{w} < V_{throat}^{w}$

$$V_{throat}^{w} = \int_0^{r_{max}} d\Omega_5\, dr\, r^5 \left(\frac{R}{r}\right)^4 = \frac{1}{2} r_{max}^2 R^4 \,\text{vol}\, X_5$$

So $M_p^2 > \frac{1}{2} r_{max}^2 R^4 \,\text{vol}\, X_5 \cdot \frac{1}{\kappa_{10}^2}$

and $\frac{\phi_{max}^2}{M_p^2} < \frac{r_{max}^2 T_3}{\frac{1}{2} r_{max}^2 R^4 \,\text{vol}\, X_5 \cdot \frac{1}{\kappa_{10}^2}}$

but $R^4 \,\text{vol}\, X_5 = 4\pi^4 g_s N \alpha'^2$

$\Rightarrow \left(\frac{\phi}{M_p}\right)^2 < \cancel{\frac{1}{(2\pi)^3}\frac{1}{g_s}\frac{1}{\alpha'^2}\cdot\frac{2}{(2\pi)^7}\frac{1}{g_s^2}\frac{1}{\alpha'^4}\cdot\frac{2}{4\pi^4}}$

$$\frac{(2\pi)^7 g_s^2 \alpha'^4}{(2\pi)^3 g_s \alpha'^2}\cdot\frac{1}{4\pi^4 g_s N \alpha'^2} = \frac{4}{N}.$$

So $\left(\frac{r_{CMB}}{.01}\right) < \frac{4}{N} \ll 1.$ indep of g_s X_5.

So for D3-brane inflation, we have learned:

- cannot get detectable r_{CMB}.
  These are small-field models

- kinetic term $\sim \sqrt{1 - \dot{\phi}^2 e^{-4A}}$ so for rapid motion one enters the 'DBI regime' in which $\dot{\phi}$ reaches a speed limit.

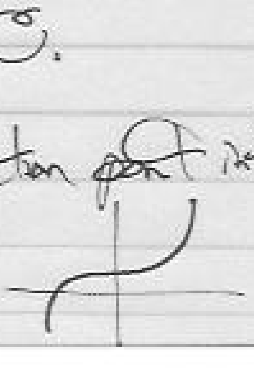
- scalar potential receives critical contributions from M stab, eg W_{NP} on D7-branes.

  One can determine the structure of this potential in supergravity,

$$V \sim V_0 + c_1 \phi^1 + c_{3/2} \phi^{3/2} + c_2 \phi^2 + c_{\sqrt{28}-2} \phi^{\sqrt{28}-2} + \ldots$$

  but the Wilson coeffs c_i depend on details of the $\mathcal{C}$.

Phenomenology: inflection point inflation



VII

Axion Monodromy Inflation

For a small-field model like D3-brane inflation one needs to know $\mathcal{L}$ up to $\Delta \sim 6$ M_p-suppressed terms.

\# of such terms is finite; one can determine all such terms explicitly, with a bit of work, in tractable examples like finite CY cones.

Still, this is a brute force approach.

Preferable to seek a symmetry argument that protects the inflaton potential.

Let's now consider a large-field model with a powerful all-orders shift symmetry.

As we've discussed, this symm. must prevent couplings to d.o.f. up to M_p.

Idea: axions (pNGBs) have shift symmetries, $a \to a + \text{const}$

broken by NP effects to $a \to a + 2\pi$.

Could the inflaton be an axion?

$$\mathcal{L} = \frac{1}{2} f^2 (\partial a)^2 - \Lambda^4 \cos a$$

$[f] = M$. 'decay constant'

define $\phi = af$, then

$$\mathcal{L} = \frac{1}{2} (\partial\phi)^2 - \Lambda^4 \cos(\phi/f)$$

$\eta = M_p^2 \frac{V''}{V} = -\frac{M_p^2}{f^2}$ so slow roll only if $f \gg M_p$.

Is it reasonable to take $f \gg M_p$?

In string theory, get axions from p-forms.

In IIB on CY M,

$$\Sigma_i = \text{basis of } H_2(M)$$
$$\omega^i = \text{" " } H^2(M)$$

then $B_2 = \sum_{i=1}^{h^{11}(M)} b_i(x)\omega^i$

and b_i is a 4D axion.

So we get $b_i, c_i^{(2)}, c_i^{(4)}, C_0$.

What is f? examine kinetic term.

$$S \supset \frac{1}{2(2\pi)^7 g_s^2 \alpha'^4}\int d^{10}x\sqrt{g}\,|dB|^2$$ use $B = \sum_i b_i(x)\omega^i$

then $|dB|^2 \supset \frac{1}{3!}\partial_\mu b_i \partial^\mu b_j \omega^i_{\alpha\beta}\omega^{j\alpha\beta}$ α,β internal indices

for a single axion $(i=1)$,

$$S = \left(\frac{1}{3!\,2(2\pi)^7 g_s^2 \alpha'^4}\right)\int d^4x\sqrt{g_4}\,\partial_\mu b\,\partial^\mu b \int d^6y\sqrt{g_6}\,\omega_{\alpha\beta}\omega^{\alpha\beta}$$
$$= \int d^4x\sqrt{g_4}\,\frac{1}{2}\partial_\mu b\,\partial^\mu b\, f^2$$

with $f^2 = \frac{1}{6(2\pi)^7 g_s^2 \alpha'^{(-)}} \int \omega \wedge * \omega$

now use $\alpha' M_p^2 = \frac{2}{(2\pi)^7} \frac{\mathcal{V}}{g_s^2}$ $\mathcal{V} = \frac{\mathrm{Vol\,CY}}{\alpha'^3}$

$$\Rightarrow \frac{f^2}{M_p^2} = \frac{1}{12\mathcal{V}} \int \omega \wedge * \omega$$

estimating $\int \omega \wedge * \omega \sim \sqrt{g_6}\, g_6^{\cdot\cdot} g_6^{\cdot\cdot} \sim L^2$

we have

$$\frac{f^2}{M_p^2} \propto \frac{g_s^{2\gamma}}{L^\delta} \qquad \delta > 0 \qquad \gamma = \begin{cases} 0 & \text{NSNS} \\ 1 & \text{RR} \end{cases}$$

So $f \ll M_p$ in computable limits of ST.

$\Rightarrow$ Natural Inflation
of Freese, Frieman, Olinto '90
seems hard to achieve
(BDFG).

But let's be careful about the $V(\phi)$ we assume.

In string theory, the origin of the shift symmetry is worth noting.

Wen-Witten '85
Dine-Seiberg

WS coupling $\frac{i}{2\pi\alpha'}\int B$

$$= \frac{i}{2\pi\alpha'}\int d^2\xi\, \epsilon^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu B_{\mu\nu}(X).$$

in sector with $p^\mu=0$, $\partial_{X^\mu} B_{\mu\nu}=0$

$$\Rightarrow S_{WS}^{p^\mu=0} = \frac{i}{2\pi\alpha'}\int d^2\xi\, \partial_\alpha\left(\epsilon^{\alpha\beta}X^\mu\partial_\beta X^\nu B_{\mu\nu}\right)$$

So zero-momentum coupling is a total derivative.

⇒ terms in 4D spacetime action w/o derivatives must vanish in absence of boundaries. (more generally in top. triv. sector.

$$\Rightarrow \mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \sum_k \lambda_k \frac{(\partial\phi)^{2k+2}}{\Lambda^{4k}} \qquad V=0.$$

So when $\nexists$ D-branes, V must vanish to all orders in α' (i.e. until topologically nontrivial WS aka WS instanton)
(σ-model pert.th)

and to all orders in g_s (since genus did not appear in argument.)

PQ symm broken by

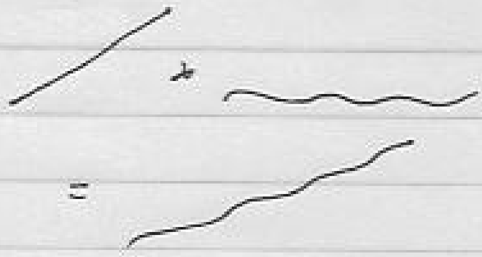
$$\begin{cases} \text{D-branes} \\ \text{NP effects} \begin{pmatrix} \text{WS instantons} \\ \text{D-brane instantons} \end{pmatrix} \end{cases}$$

Idea: take $\mathcal{C}$ w/o D-branes $(V=0)$,
then slightly lift the flat axion direction.

<u>Note</u>: <u>two kinds</u> of breaking!

1. explicit, $\overset{\text{monotonic}}{\text{but}}$ small : added D-brane)
2. 'periodic' from NP effects

we'll take $V_1 \gg V_2$.



Now let's be concrete.

IIB on CY_3 O3/O7.

involution Ω: $H^{1,1} \to H^{1,1}$. eigenvalues ±1.

So, $H^{1,1}(M) \to H^{1,1}_{(+)} + H^{1,1}_{(-)}$

$H_{1,1}(M) \to H^{(+)}_{1,1} + H^{(-)}_{1,1}$.

IIB $\mathcal{N}=2$ $D=4$
on CY

$C_4 = C^{\lambda}_{4,i}\,\omega_4^i$

$C_2 = c_{2,i}\,\omega_2^i$

$b_2 = b_{2,i}\,\omega_2^i$

Given $\omega_4 \in H^4(M)$

$\omega_2 \in H^2(M)$

$\Sigma_4 \in H_4(M)$

we form
$$\rho = \int_{\Sigma_4} \sqrt{g} + i \int_{\Sigma_4} C_4$$
$$G = \int_{\Sigma_2} C_2 + i \int_{\Sigma_2} B_2$$

(more details in Grimm + Louis.)

These are the proper Kähler coordinates.

One can check that
$$\begin{cases} \rho: \text{ is projected in if } \Sigma_4^{(i)} \text{ is \underline{even}} \\ \qquad \Sigma_4^{(i)} \in H_4^{(+)} \\ G: \text{ is projected in if } \Sigma_2^{(i)} \in H_2^{(-)} \end{cases}$$

So let us assume $h^{1,1}_{(-)} \geq 1$, so $\exists \geq 1$ G-field in the 4D theory.

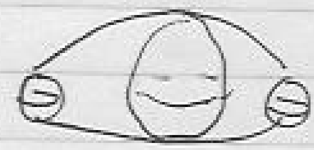
Now let $\Sigma_2^{(-)}$ be an odd 2-cycle, $G_{(-)} = c_- + i b_-$ the axion.

Wrap a D5-brane on $\Sigma_2^{(-)}$.

$$V(c_-, b_-) = V_{DBI+CS}$$
$$= T_5 \int_{\Sigma_2^{(-)}} d^2\xi \sqrt{\det(G+B)} + T_5 \int C_2$$

Wait, if M compact there is a Q_{D5} tadpole!

So wrap a $\overline{D5}$ on $\Sigma_2^{(-)\prime}$, homologous to $\Sigma_2^{(-)}$.



Then $V' = V_{DBI} - V_{CS}$, so in total

$$V = 2\int_{\Sigma_2^{(-)}} d^2\xi \sqrt{\det(G+B)} \cdot T_5$$

Now if $\Sigma_2^{(-)}$ has size) L,

$$G+B \sim \begin{pmatrix} g_1 & b_2 \\ -b_2 & g_{22} \end{pmatrix}$$

$$\text{so } \int \det(G+B) \sim L^4 + b_-^2 \qquad b_- = \int_{\Sigma_2^{(-)}} B_2 .$$

$$V = 2\sqrt{L^4 + b_-^2} \cdot T_5$$

and for $b_- \gg L^2$, we have $V \approx \underbrace{2T_5}_{\equiv \mu^3 f} b_- .$

so that, defining $\phi_b \equiv f b_-$,

$$V \approx \mu^3 \phi_b .$$

So $\mathcal{L} = \frac{1}{2}(\partial\phi_b)^2 - \mu^3 \phi_b .$

Very promising.