Inflationary Cosmology

and

Inflation in String Theory

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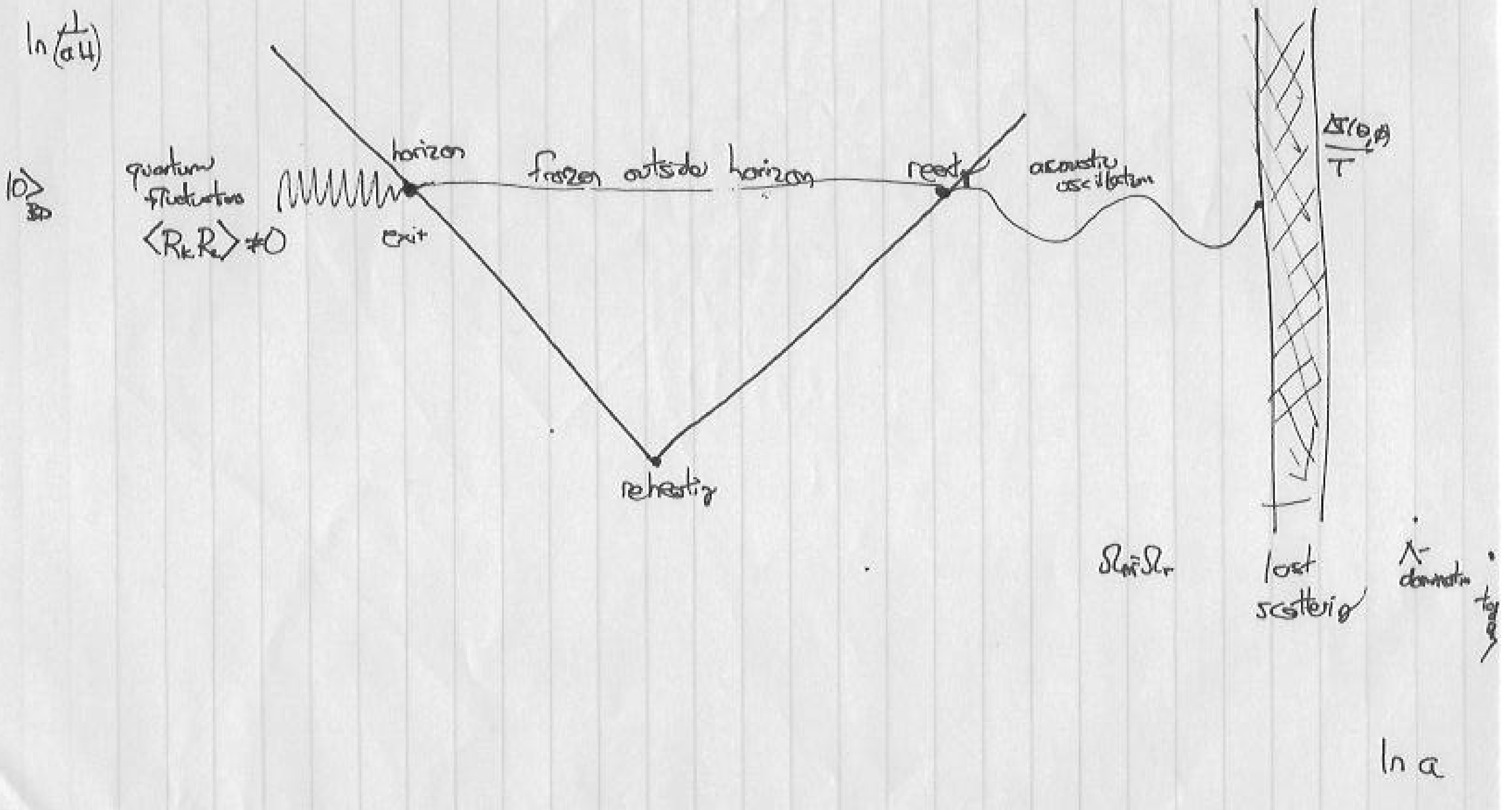
for further reading, I recommend

{ S. Weinberg, Cosmology
D. Baumann, TASI Lectures on Inflation }

So far: inflation as a solution to the horizon problem

quantum fluctuations during inflation
exit the horizon,
freeze,
reenter the horizon,
undergo acoustic oscillations,
and leave imprints $\Delta T(\theta,\phi)$ in the CMB,
with a characteristic spectrum of
acoustic peaks
that gives definitive evidence for
phase correlations on superhorizon
distances.

Now: inflation in string theory



$\ln\left(\frac{1}{aH}\right)$
$|0\rangle_{BD}$
quantum fluctuations
$\langle R_k R_k \rangle \neq 0$
exit
horizon
frozen outside horizon
reentry
acoustic oscillations
$\frac{\Delta T(\theta,\phi)}{T}$
reheating
$\Omega_m \approx \Omega_r$
last scattering
Λ domination today
$\ln a$

III Inflation and Planck-Scale Physics.

Condition for inflation:

$$\frac{\ddot{a}}{a} > 0 \Leftrightarrow \dot{H} + H^2 > 0$$

$$\Leftrightarrow -\frac{\dot{H}}{H^2} < 1$$

$$-\frac{\dot{H}}{H^2} \equiv \varepsilon_H,$$

$$\boxed{\varepsilon_H < 1}$$ inflation occurs

define $\eta_H = \frac{\dot{\varepsilon}_H}{H \varepsilon_H}$ $\boxed{\eta_H \ll 1}$ inflation lasts.

These simplify in 1-field SR to

$$\varepsilon_V \equiv \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1$$

$$\eta_V \equiv M_p^2 \frac{V''}{V} \ll 1.$$

for convenience, use $\begin{pmatrix} \varepsilon_H \to \varepsilon_H \\ \eta_H \to \eta_H \end{pmatrix}$

$$\eta_V \to \eta.$$

Let's examine $\eta \ll 1$.

$$V'' = m^2 \quad \text{if} \quad \mathcal{L} \supset \tfrac{1}{2} m^2 \varphi^2$$

$$V = 3H^2 M_\varphi^2$$

$$\Rightarrow \eta = \frac{m^2}{3H^2}. \qquad \eta \ll 1 \text{ if } m \ll H.$$

φ is a scalar. Why is it <u>light</u>?

<u>NB</u> SUSY useless.

de Sitter is $\cancel{SUSY}$ with $m_{3/2} \sim H$.

so, $m_\varphi \to H$.

More generally,

i) we know some new d.o.f. must appear at $M \lesssim M_{pl}$ to give a UV-completion of GR.

in ST, $M \lesssim M_s < M_p$.
(or even $M \sim M_{KK} \ll M_p$.)

If φ has $\mathcal{O}(1)$ couplings to these 'Planck slop' fields ξ, then $\int$ out ξ yields

$$\Delta \mathcal{L}_\varphi = \frac{\mathcal{O}^{[\Delta]}(\varphi)}{M^{\Delta-4}}$$ with $\mathcal{O}$ some allowed operator in the φ QFT.

so if before considering ξ we have

$$\mathcal{L}_\varphi = \frac{1}{2}(\partial\varphi)^2 - V_0(\varphi)$$

we should ask whether $\int$ out ξ yields

$$\Delta \mathcal{L}_\varphi = c V_0(\varphi) \frac{\varphi^2}{M^2}.$$

if this term arises, then

$$\Delta\eta = \frac{1}{V_0}\left[\frac{\partial^2}{\partial\varphi^2}\Delta V\right]M_p^2$$

$$= 2c\left(\frac{M_p}{M}\right)^2 + \mathcal{O}\left[\varphi V_0', V_0''\right]$$

now $M \overset{!}{\lesssim} M_p$

and Wilson $\Rightarrow$ $c \sim \mathcal{O}(1)$

so $\Delta\eta \gtrsim 1$.

In a very interesting subclass, this problem is *much stronger*.

$$\text{Since } \frac{k^3}{2\pi^2} P_{\mathcal{R}} = \left(\frac{H}{\dot\phi}\right)^2 \left(\frac{H}{2\pi}\right)^2$$

$$\text{and } \frac{k^3}{2\pi^2} P_T = \frac{8}{M_p^2} \left(\frac{H}{2\pi}\right)^2$$

$$\text{we have } r \equiv \frac{P_t}{P_s} = \frac{8}{M_p^2} \left(\frac{\dot\phi}{H}\right)^2$$

$$H\,dt = dN$$

$$r = \frac{8}{M_p^2} \left(\frac{d\phi}{dN}\right)^2$$

$$\frac{\Delta\phi}{M_p} = \int \sqrt{\frac{r(N)}{8}}\, dN$$

$\frac{d}{dN} \log r = -\left[(n_s - 1) + \frac{r}{8}\right]$ in 1-field SR

now also r cannot change quickly while ε, η remain small (1) (except in rather contrived cases),

so

$$\int \frac{\sqrt{r(N)}}{\sqrt{8}}\, dN \approx \Delta N \sqrt{\frac{r_{CMB}}{8}}.$$

At worst, $\frac{\Delta\phi}{M_p} \approx 30\sqrt{\frac{r_{CMB}}{8}}$

so, rearranging,

$$\left(\frac{r_{CMB}}{.01}\right) \approx \left(\frac{\Delta\phi}{M_p}\right)^2$$

for $r_{CMB} \gtrsim .01$, $\Delta\phi \gtrsim M_p$.

[now, $r < .22$
within 5yo expect $r \ll .05$ or detection]

eg simple monomial 'chaotic inflation' models

$$V = \frac{1}{2}m^2\phi^2$$

$$\eta_v = 2\left(\frac{M_p}{\phi}\right)^2$$

$$\varepsilon_v = 2\left(\frac{M_p}{\phi}\right)^2$$

find $N_e = \frac{1}{M_p}\int \frac{d\phi}{\sqrt{2\varepsilon}} = \frac{1}{M_p}\frac{1}{2}\int \frac{\phi\, d\phi}{M_p} = \frac{1}{4}\left(\frac{\phi}{M_p}\right.$

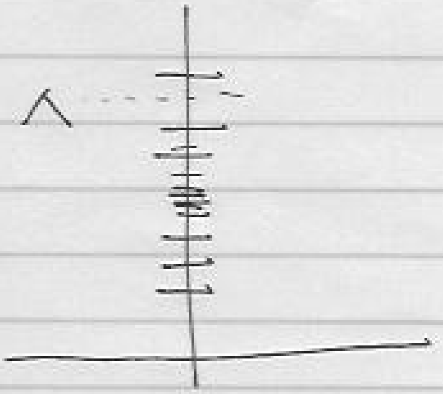
[used $dN = H\,dt = H\frac{d\phi}{\dot\phi} = H\,d\phi\left(\frac{-3H}{V'}\right) = -\frac{V}{V'}\frac{d\phi}{M_p^2}$]

so $\phi_{initial} \sim 15M_p$ for $N_e \sim 60$.

Implication: super-Planckian vevs $\Leftrightarrow$ r observable!
(Lyth).

Now in EFT, super-cutoff vevs are tricky.

take scalar fields φ, χ_i and cutoff Λ.



Wilson: $V \supset \Lambda^2 \chi_i^2 + \varphi^2 \chi_i^2 + \frac{\varphi^4 \chi_i^2}{\Lambda^2} + \ldots$

or, if φ, χ couple with strength g,

$$\Lambda^2 \chi_i^2 + g^2 \varphi^2 \chi_i^2 + g^4 \varphi^4 \frac{\chi_i^2}{\Lambda^2} + \ldots$$

so $V(\varphi, \chi_i) = U(\varphi) + \Lambda^2 \chi_i^2 f_i\left(\frac{g^2 \varphi^2}{\Lambda^2}\right)$.

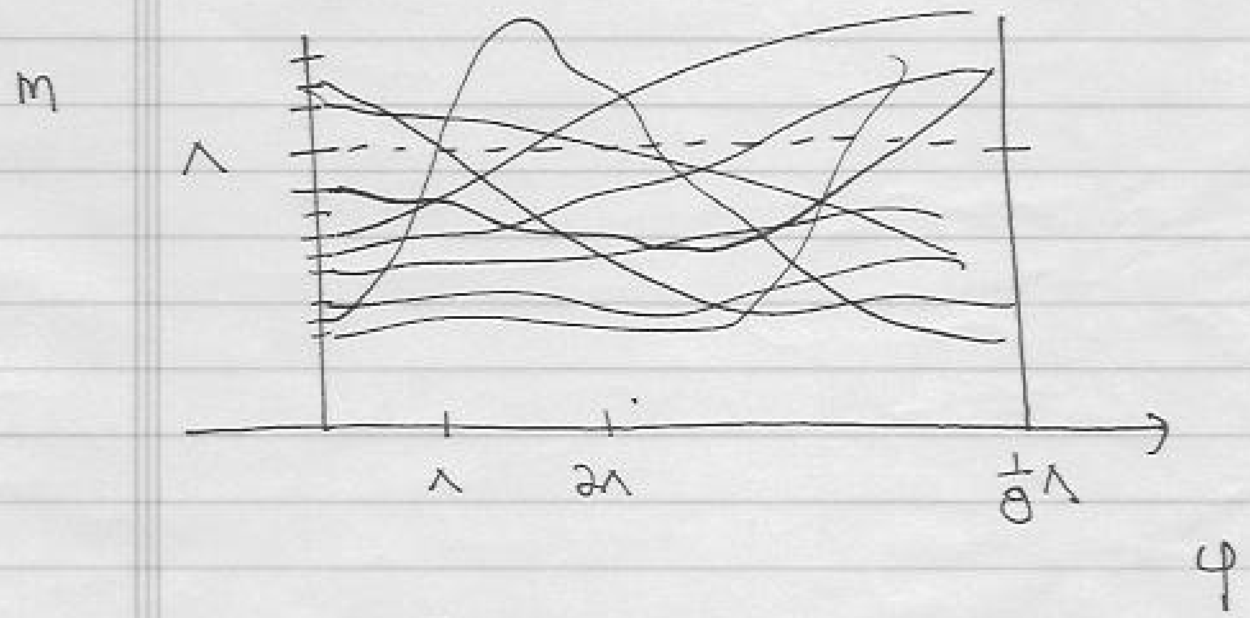
So if we try $\Delta\varphi \sim \Lambda$,

$$\Delta m_{x_i} \sim g_i \Delta\varphi \sim g_i \Lambda.$$

Conclude that for $\Delta\varphi \sim \frac{\Lambda}{g_i}$, we have

$$\Delta m_{x_i} \sim \Lambda.$$

Draw with all $g_i = \bar{g}$:



Some fields <u>leave</u> EFT

Some fields <u>join</u> EFT

$\Rightarrow$ although usually $\exists$ an EFT at $\langle\varphi\rangle = \frac{\Lambda}{g}$, in general it is <u>not</u> the EFT at $\langle\varphi\rangle = 0$.

So clearly, since $\Lambda \lesssim M_p$, we have quite a task to find an effective QFT + GR that

i) is consistent in the UV

ii) allows excursions $\Delta\varphi \gg M_p$.

for, (i) $\Rightarrow$ $\exists$ massive ($M \sim M_p$?) 'Planck slop' states

(in ST: moduli, KK, string modes may $\ll M_p$ in mass!)

while (ii) requires that coupling g to <u>all</u> these fields obeys

$$\frac{\Lambda}{g} \gg M_p$$

or

$$g \ll \cancel{\frac{M_p}{\Lambda}} \; \frac{\Lambda}{M_p}$$

(hard if $\Lambda \ll M_p$!).

Key question:

'Why does the inflaton couple so weakly (not even gravitationally!) to Planck-scale d.o.f.? ...'

A question for a Planck-scale theory.

Indeed, an unsurpassed opportunity to probe such a theory using CMB observations.

Idea: a symmetry forbids $\delta^2\varphi^2$ (etc.) couplings

Well, this symmetry must be (approx.) respected by PLANCK SCALE d.o.f.

Important to distinguish 2 cases:

i) $\Delta\phi \ll M_p$. 'small-field'

$$r \ll .01$$

$$\frac{\mathcal{O}_6}{M_p^2} \quad \left(\text{eg } V_0(\varphi)\frac{\varphi^2}{M_p^2}\right)$$

gives $\Delta\eta \sim 1$

while $\frac{\mathcal{O}_7}{M_p^3}$ gives $\Delta\eta \lesssim \frac{\Delta\phi}{M_p} \ll 1$.

important terms: $\Delta \lesssim 6$.

ii) $\Delta\phi \gg M_p$. 'large-field'

$$r \gg .01$$

terms $\frac{\mathcal{O}_\Delta}{M_p^{\Delta-4}}$ become larger for larger Δ.

important terms: all $(\Delta \to \infty)$.

NB detecting r directly probes scale of inflation

$$V^{1/4} = 3.6 \times 10^{16}\,\text{GeV} \cdot \left(\frac{r}{.01}\right)^{1/4}!$$

In case (i), we have a finite # of $\mathcal{O}$'s that contribute important corrections to the inflationary dynamics.

Can hope to enumerate all $\mathcal{O}_{(\Delta)}$ with $\Delta \leq 6$ and balance them against each other (fine-tuning).

In case (ii), enumeration is hopeless. One needs a powerful symmetry.

I'll give examples of each case in these lectures.

(i): D3-brane inflation

(ii): axion monodromy inflation

IV Towards Inflation in String Theory

Natural to search for inflation in string theory.

We need a scalar which

- drives $\ddot{a} > 0$ phase for $\gtrsim 60$ efolds
- then reheats the universe.

Scalars are plentiful, too plentiful.

scalars with Gr-strength couplings and $m < 30$ TeV will decay $\gtrsim 1$s and spoil BBN
or overclose Ω.
or give 5th forces.

Moduli problem = presence of (numerous) gravitationally-coupled scalars in string compactifications.

In CY cpct ($\mathcal{N}=2$) one has many such.

$\begin{cases} \text{Kähler mod.} \\ \text{cx str mod} \\ \text{axiodilaton} \\ \text{D-brane scalars} \end{cases}$

Required: method of $\underline{M \text{ stab}}$.

i.e. method giving the M masses > 30 TeV so they do not ruin cosmology (late-time, $t \gtrsim 1s$)

Flux cpct. provide a class of vacua in which many/most/all moduli are stabilized; more on them later.

General issue:

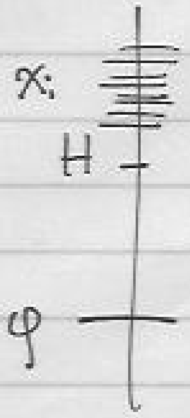
- inflaton is one of the M.

- giving other M masses $30\,\text{TeV} < m < H$ <u>does</u> solve the late-time M problem but the M are then dynamical during inflation!

  they fluctuate, $\delta X \sim \frac{H}{2\pi}$ and contribute to the curvature/entropy perturbations.

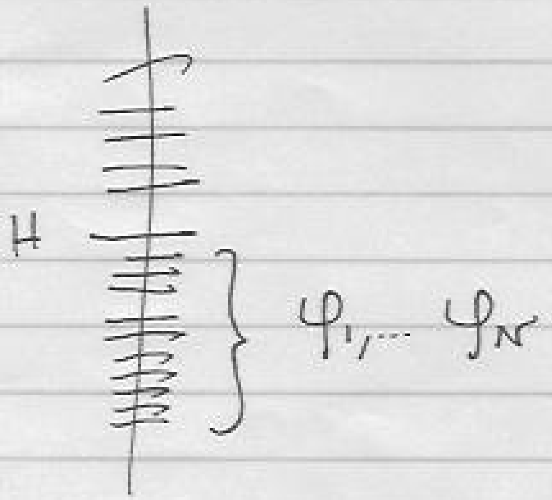
Simply put, unless all M have $m > H$, the EFT is not just that of an inflaton; it involves <u>many</u> dynamical scalars.

Options:



hard to arrange this disparity

signatures often straightforward.



easily arranged (complex + hard to study)

'Complex' signatures incl. NG are common

We'll focus on cases in which (after much work!) the low-E EFT contains only the inflaton. (+ SM fields etc; but no other light, Gr-coupled fields)

Basic scheme:

Find vacua in which the long-wavelength effective theory in 4D is

GR + 1 scalar field

with $V(\phi)$ obeying $\varepsilon_V, \eta_V \ll 1$.

and all moduli have $m > H$, and hence are non-dynamical.

Challenge: we've learned that $\int$ out even M_p fields gives substantial corrections to $\mathcal{L}$ (eg to η_V).

The $\mathcal{M}$ have $m_{\mathcal{M}} \ll M_p$ (indeed, usually $m_{\mathcal{M}} \ll M_{KK}$ if stabilized in 4D thy)

so $\int$ out the $\mathcal{M}$ $\Rightarrow$ large corrections to $\mathcal{L}$!

So one must very carefully $\int$ out all the M_4 incorporating their effects in $V(\varphi)$.

As we'll see, to know $V(\varphi)$ (more gen, $\mathcal{L}(\varphi)$) well enough to study inflation,

- one must always understand M stab. in detail
- g_s, α', backreaction, compactness corrections, often ignored, can be crucial

The name of the game is a complete + reliable dimensional reduction, almost always beyond leading order.

In context, this is one of several uses for the general program of

<u>determining the 4D effective action arising from stabilized string compactifications.</u>