Holographic methods for condensed matter physics

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Plan of lectures

A: Motivation – introduction to the cuprates

B: Essentials of applied holography

C: Charged bosons

D: Charged fermions

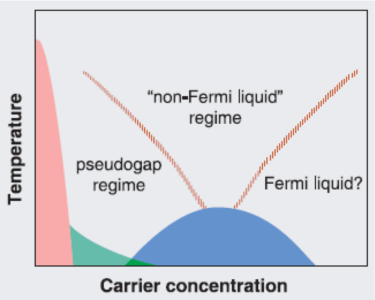
E: D-branes

A: Motivation – introduction to the cuprates

1. Anomalous scalings.
2. Critical temperature.
3. Absence of quasiparticles.
4. Fermi surface reconstruction.

Invitation to the cuprates

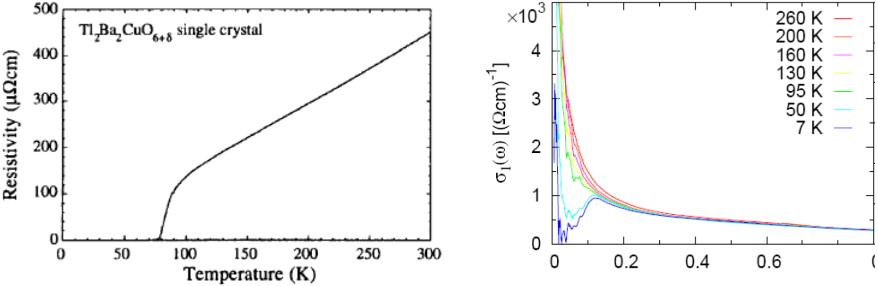
- The most glamorous non-Fermi liquids are the cuprate high-T_c superconductors.



- There is no consistent theory for the 'strange metal' (non Fermi liquid) regime. Many anomalous properties.

Anomalous scalings

- The 'strange metal' regime is characterised by unconventional scaling laws. Eg.



- DC resistivity: $\rho \sim T$, optical conductivity $\sigma(\omega) \sim \omega^{-0.65}$.

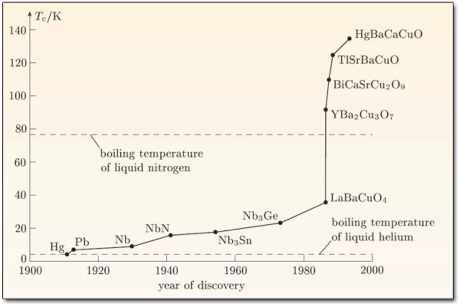
[Plots from McKenzie et al. '97 and van der Marel et al. '03.]

The critical temperature

- In BCS theory the critical temperature

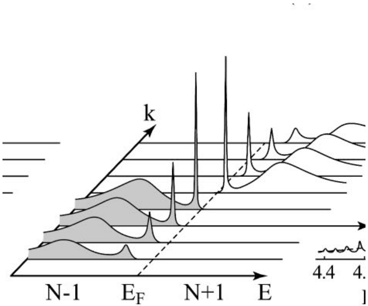
$$T_c \sim E_D e^{-1/[N(0)g^2]} .$$

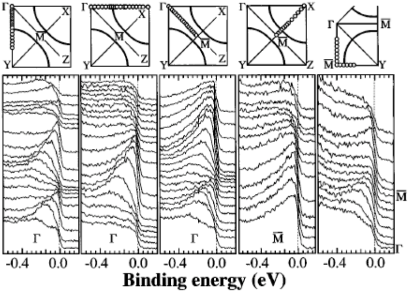
- Electron-phonon coupling g too large gives lattice instabilities.
$\Rightarrow$ Max. $T_c \sim 30$ K.



Absence of quasiparticles

- Weakly interacting Fermi liquid has sharp quasiparticle excitations at the Fermi surface (left below).
- The strange metallic region of the cuprates does not (right below).



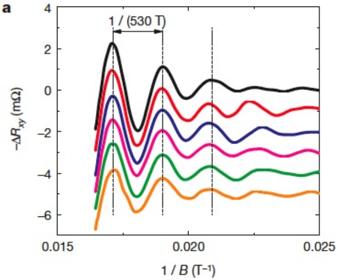


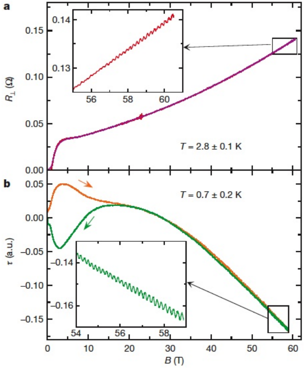
- Some structure in ARPES, but broad.

[Plot from Ding et al. '96]

Fermi surface reconstruction I

- de Haas - van Alphen oscillations detect the size of Fermi surface.

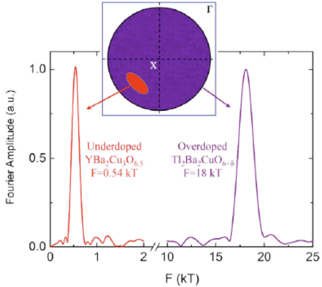




[Plots from Doiron-Leyraud et al. 2007, Vignolle et al. 2008]

Fermi surface reconstruction II

- Fourier transform the oscillations



- **Underdoped:** Fermi pockets.
  Overdoped: conventional Fermi surface.

Possible scenario

- Summary:
  1. Scaling laws
  2. Change in shape of Fermi surface
  3. Absence of well defined quasiparticles

- Consistent with a quantum critical point at $T = 0$ at a critical doping controlling the strange metal region.

- Layered structure of cuprates suggests $2+1$ dimensional critical theory.

- Such theories generically strongly coupled. Traditional perturbative methods not controlled.

$\Rightarrow$ Turn to the holographic correspondence....

B: Essentials of applied holography

1. Holography and renormalisation
2. Finite temperature and black holes
3. Finite chemical potential and charged black holes
4. Relevant operators
5. Green's functions

Holographic correspondence and renormalisation group

- A beautiful and nontrivial property of quantum theories:

  Once defined at an energy Λ, defined for all energies $E < \Lambda$.

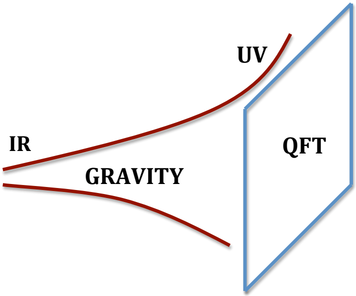
- Renormalisation shows low energy/long distance physics is not sensitive to high energy and short distance details.

- Physics is local in energy (Wilson):

$$E \frac{dg(E)}{dE} = \beta(g(E)) \,.$$

- The essential structure of the holographic correspondence is to make this locality geometrically manifest.

- Holographic correspondence: add the energy scale as an extra curved spacetime dimension.



- Curvature of the ‘holographic direction’ contains the RG flow information.

- Einstein’s equations are the RG equations.

- Locality in spacetime and in energy on the same footing.

Scale invariance and z

- Field theories can be defined with a cutoff or at a UV fixed point.
- At a fixed point, theory is invariant under space and time scaling

$$t \to \lambda^z t \,, \qquad \vec{x} \to \lambda \vec{x} \,.$$

- z is the dynamical critical exponent. There is no reason for $z = 1$.
- Minimal algebra has $\{M_{ij}, P_k, H, D\}$. Dilatations act

$$[D, M_{ij}] = 0 \,, \qquad [D, P_i] = i P_i \,, \qquad [D, H] = i z H \,.$$

- Sometimes called the Lifshitz algebra.

Scale invariant geometries

- Can we realise the Lifshitz algebra geometrically?
- Kachru et al. (2008):

$$ds^2 = L^2 \left(-\frac{dt^2}{r^{2z}} + \frac{dx^i dx^i}{r^2} + \frac{dr^2}{r^2} \right) .$$

- $z = 1$ is AdS_{d+1}, ehancement to Lorentzian conformal algebra.
- The case $z = 1$ is a solution to Einstein gravity

$$S = \frac{1}{2\kappa^2} \int d^{d+1}x \sqrt{-g} \left(R + \frac{d(d-1)}{L^2} \right) .$$

Other cases need additional matter.

Finite temperature

- The zero temperature background is AdS_{d+1}

$$ds^2 = L^2 \left(-\frac{dt^2}{r^2} + \frac{dr^2}{r^2} + \frac{dx^i dx^i}{r^2} \right) .$$

- Which is a solution to the theory

$$S = \frac{1}{2\kappa^2} \int d^{d+1}x \sqrt{-g} \left(R + \frac{d(d-1)}{L^2} \right) .$$

- Want relevant deformations, break scale invariance in the IR.
- Expect geometry of the form

$$ds^2 = L^2 \left(-\frac{f(r)dt^2}{r^2} + \frac{g(r)dr^2}{r^2} + \frac{h(r)dx^i dx^i}{r^2} \right) .$$

- Most universal deformation: temperature.

Finite temperature

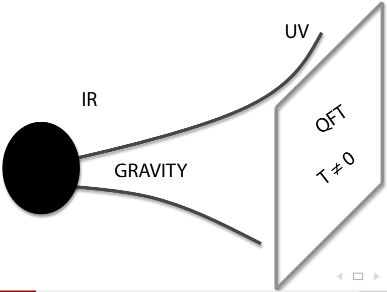
- Only one nontrivial solution to Einstein equations of this form:

$$ds^2 = \frac{L^2}{r^2}\left(-f(r)dt^2 + \frac{dr^2}{f(r)} + dx^i dx^i\right) ,$$

where

$$f(r) = 1 - \left(\frac{r}{r_+}\right)^d .$$

- Asymptotically AdS as $r \to 0$. (UV)
- Horizon at $r = r_+$. (IR)



Finite temperature

- Corresponds to a temperature (from e.g. Euclidean solution)

$$T = \frac{d}{4\pi r_+} \, .$$

- All $T \neq 0$ equivalent: $(r, t, x^i) \to r_+(r, t, x^i)$ eliminates r_+.
- By computing the action of the Euclidean solution

$$F = -T \log Z = T S_E[g_\star] = -\frac{(4\pi)^d L^{d-1}}{2\kappa^2 d^d} V_{d-1} T^d \, .$$

- Characterised by one number ('central charge'):

$$\frac{(4\pi)^d L^{d-1}}{2\kappa^2 d^d} \sim N^\# \, .$$

Finite chemical potential

- Want physics of a $U(1)$ symmetry. E.g. electricity!
- In nature $U(1)$ is gauged. In many condensed matter setups, sufficient to work with global symmetry.
  - Photons are screened in a charged medium.
  - Sufficient to consider external sources (no virtual photons).
- What is the dual to a global $U(1)$ in field theory?
- Take cue from global Lorentz invariance. Dual to part of the diffeomorphism invariance of the bulk. Suggests:

$$\begin{array}{c} \text{Global symmetry (field theory)} \\ d \text{ spacetime dimensions} \end{array} \quad \leftrightsquigarrow \quad \begin{array}{c} \text{Gauged symmetry (gravity)} \\ d+1 \text{ spacetime dimensions.} \end{array}$$

- Natural: QFT global symmetry are 'large' gauge symmetries in bulk.

- Therefore: Need bulk Maxwell field. Minimal action

$$S = \int d^{d+1}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R + \frac{d(d-1)}{L^2}\right) - \frac{1}{4g^2}F^2\right] .$$

- Symmetries allow (magnetic term in $d = 2+1$ only)

$$A = A_t(r)dt + B(r)x\,dy \,.$$

- Put $B = 0$ for the moment. Metric solution

$$ds^2 = \frac{L^2}{r^2}\left(-f(r)dt^2 + \frac{dr^2}{f(r)} + dx^i dx^i\right) ,$$

where

$$f(r) = 1 - \left(1 + \frac{r_+^2\mu^2}{\gamma^2}\right)\left(\frac{r}{r_+}\right)^d + \frac{r_+^2\mu^2}{\gamma^2}\left(\frac{r}{r_+}\right)^{2(d-1)} .$$

- Scalar potential is ($A_t(r_+) = 0$ for regularity)

$$A_t = \mu\left[1 - \left(\frac{r}{r_+}\right)^{d-2}\right] .$$

- Dimensionless constant

$$\gamma^2 = \frac{(d-1)g^2 L^2}{(d-2)\kappa^2}\,.$$

- Temperature

$$T = \frac{1}{4\pi r_+}\left(d - \frac{(d-2)r_+^2\mu^2}{\gamma^2}\right)\,.$$

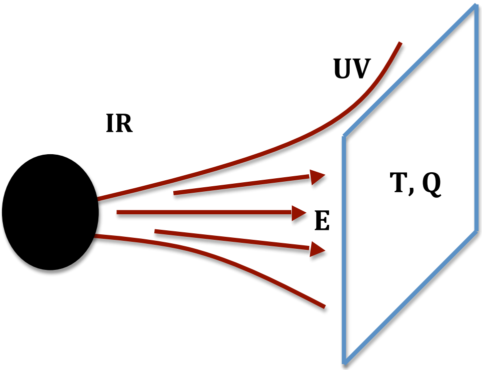
- Near the boundary

$$A_\mu(r) = A_{(0)\mu} + \cdots \quad \text{as} \quad r \to 0\,.$$

$$(\text{cf. } g_{\mu\nu}(r) = \tfrac{L^2}{r^2} g_{(0)\mu\nu} + \cdots \quad \text{as} \quad r \to 0\,.)$$

- $A_{(0)\mu}$ is background gauge field. $A_{(0)t} = \mu$ is the chemical potential.

Charged black hole



- There is now a physical dimensionless temperature T/μ.
- Limit $T/\mu \to 0$ can be taken continuously. Extremal black hole.
- Free energy

$$\Omega = -T \log Z = \mathcal{F}\left(\frac{T}{\mu}\right) V_{d-1} T^d \,.$$

- $\mathcal{F}\left(\frac{T}{\mu}\right)$ is a nontrivial function that is an output of AdS/CFT.
- Entropy: $S = -\frac{\partial\Omega}{\partial T}$.
- Discomforting fact: $S \neq 0$ and $T = 0$.
  - Large N effect?
  - Weak gravity conjecture $\to$ should be unstable?
  - Prediction?

Relevant operators

- What is dual to adding a relevant operator to the theory?
- Take inspiration from metric. If $g \to g_{(0)} + \delta g_{(0)}$:

$$\delta S = \int d^d x \sqrt{-g_{(0)}} \delta g_{(0)\mu\nu} T^{\mu\nu} \,.$$

- Equality of bulk and boundary partition functions implies:

$$Z_{\text{bulk}}[g \to g_{(0)} + \delta g_{(0)}] = \langle \exp \left(i \int d^d x \sqrt{-g_{(0)}} \delta g_{(0)\mu\nu} T^{\mu\nu} \right) \rangle_{\text{F.T.}} \,.$$

- Similarly for the gauge field. If $A \to \delta A_{(0)}$:

$$\delta S = \int d^d x \sqrt{-g_{(0)}} \delta A_{(0)\mu} J^{\mu} \,.$$

- Thus

$$Z_{\text{bulk}}[A \to \delta A_{(0)}] = \langle \exp \left(i \int d^d x \sqrt{-g_{(0)}} \delta A_{(0)\mu} J^{\mu} \right) \rangle_{\text{F.T.}} \,.$$

- Suggests a general correspondence

$$\begin{array}{c}\text{operator } \mathcal{O} \\ \text{(field theory)}\end{array} \quad \leftrightsquigarrow \quad \begin{array}{c}\text{dynamical field } \phi \\ \text{(bulk)}\,,\end{array}$$

such that

$$Z_{\text{bulk}}[\phi \to \delta\phi_{(0)}] = \langle \exp\left(i \int d^d x \sqrt{-g_{(0)}} \delta\phi_{(0)} \mathcal{O} \right) \rangle_{\text{F.T.}} \,.$$

where

$$\phi(r) = \left(\frac{r}{L}\right)^{d-\Delta} \phi_{(0)} + \cdots \quad \text{as} \quad r \to 0\,,$$

- I.e. Boundary value of field $\to$ source for dual operator.
- Δ is the scaling dimension of the operator $\mathcal{O}$.
- Can see that if $\mathcal{O}$ is relevant, $\Delta < d$, then $\phi \to 0$ near the boundary.

Expectation values

- From previous formula clear that

$$\langle \mathcal{O} \rangle = -i \frac{\delta Z_{\text{bulk}}[\phi_{(0)}]}{\delta \phi_{(0)}} = \frac{\delta S[\phi_{(0)}]}{\delta \phi_{(0)}} \,.$$

- Useful to make a Hamilton-Jacobi-esque identification

$$\frac{\delta S[\phi_{(0)}]}{\delta \phi_{(0)}} = - \lim_{r \to 0} \frac{\delta S[\phi_{(0)}]}{\delta \partial_r \phi_{(0)}} \equiv \lim_{r \to 0} \Pi[\phi_{(0)}] \,.$$

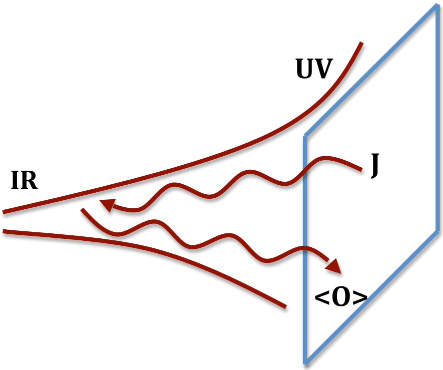
- Straightforward to check (adding appropriate counterterms) that if

$$\phi(r) = \left(\frac{r}{L} \right)^{d-\Delta} \phi_{(0)} + \left(\frac{r}{L} \right)^{\Delta} \phi_{(1)} + \cdots \quad \text{as} \quad r \to 0 \,.$$

- Then

$$\langle \mathcal{O} \rangle = \frac{2\Delta - d}{L} \phi_{(1)} \,.$$

Linear response



Retarded Green's functions

- Basic object describing perturbations away from equilibrium

$$\delta\langle\mathcal{O}_A\rangle(\omega,k) = G^R_{\mathcal{O}_A\mathcal{O}_B}(\omega,k)\delta\phi_{B(0)}(\omega,k)\,.$$

- From previous expression

$$G^R_{\mathcal{O}_A\mathcal{O}_B} = \left.\frac{\delta\langle\mathcal{O}_A\rangle}{\delta\phi_{B(0)}}\right|_{\delta\phi=0} = \lim_{r\to 0}\left.\frac{\delta\Pi_A}{\delta\phi_{B(0)}}\right|_{\delta\phi=0} = \frac{2\Delta_A - d}{L}\frac{\delta\phi_{A(1)}}{\delta\phi_{B(0)}}\,.$$

- Near the boundary require: $\delta\phi_A(r) = r^{d-\Delta}\delta\phi_{A(0)} + \cdots$.
- Regularity on the future horizon $\rightarrow$ ingoing boundary conditions

$$\delta\phi_A(r) = C_A e^{-i4\pi\omega/T\log(r-r_+)} + \cdots \quad \text{as} \quad r \to r_+\,.$$

Example: Electrical and Thermal conductivity

- Want: zero momentum conductivity with a chemical potential.
- Chemical potential mixes thermal and electric conductivities

$$\left(\begin{array}{c} \langle J_x \rangle \\ \langle Q_x \rangle \end{array} \right) = \left(\begin{array}{cc} \sigma(\omega) & \alpha(\omega) T \\ \alpha(\omega) T & \bar{\kappa}(\omega) T \end{array} \right) \left(\begin{array}{c} E_x \\ -(\nabla_x T)/T \end{array} \right) ,$$

where

$$Q_x = T_{tx} - \mu J_x .$$

- Why the extra term in Q_x? Will see shortly that

$$\begin{aligned} \delta S &= \int d^{d-1}x dt \sqrt{-g_{(0)}} \left(T^{tx} \delta g_{tx(0)} + J^x A_{x(0)} \right) \\ &= \int d^{d-1}x dt \sqrt{-g_{(0)}} \left((T^{tx} - \mu J^x) \frac{-\nabla_x T}{i\omega T} + J^x \frac{E_x}{i\omega} \right) . \end{aligned}$$

- Background electric field:

$$i\omega\delta A_{x(0)} = E_x\,.$$

- Background thermal gradient:

$$i\omega\delta g_{tx(0)} = -\frac{\nabla_x T}{T} \qquad \& \qquad i\omega\delta A_{x(0)} = \mu\frac{\nabla_x T}{T}\,.$$

[To see this: rescale time so that the period of Euclidean time is fixed, then $g_{tt(0)} = -\frac{1}{T^2}$. A thermal gradient is then

$$\delta g_{tt(0)} = -\frac{2x\nabla_x T}{T^3}\,.$$

Now to a gauge transformation on the background field $\delta g_{ab(0)} = \partial_a\xi_b + \partial_b\xi_a$, with $\xi_t = x\nabla_x T/\omega T^3$. The Maxwell field also changes under this diffeomorphism.]

- Therefore:

$$\left(\begin{array}{c}\langle J_x\rangle \\ \langle Q_x\rangle\end{array}\right) = \left(\begin{array}{cc}\sigma & \alpha T \\ \alpha T & \bar{\kappa} T\end{array}\right)\left(\begin{array}{c}i\omega(\delta A_{x(0)} + \mu g_{tx(0)}) \\ i\omega\delta g_{tx(0)}\end{array}\right) .$$

- So conductivities are Green's functions! For instance

$$\sigma(\omega) = \frac{-iG^R_{J_x J_x}(\omega)}{\omega} .$$

- Need to solve bulk equations for A_x and g_{tx} such that

$$\begin{array}{l} A_x \to A_{x(0)} . \\ g_{tx} \to L^2/r^2\, g_{tx(0)} . \end{array}$$

- Get a decoupled equation for A_x:

$$(f\delta A_x')' + \frac{\omega^2}{f}\delta A_x - \frac{4\mu^2 r^2}{\gamma^2 r_+^2}\delta A_x = 0 .$$

- Near boundary:

$$\delta A_x = \delta A_{x(0)} + \frac{r}{L}\delta A_{x(1)} + \cdots \quad \text{as} \quad r \to 0 .$$

- Work out the 'momenta' ($\rho = -\partial\Omega/\partial\mu/V$)

$$\Pi_{g_{tx}} = -\frac{\delta S}{\delta \partial_r g_{tx(0)}} = -\rho\,\delta A_{x(0)} + \frac{2L^2}{\kappa^2 r^3}(1 - f^{-1/2})\delta g_{tx(0)}\,,$$

$$\Pi_{A_x} = -\frac{\delta S}{\delta \partial_r A_{x(0)}} = \frac{f\,\delta A'_{x(0)}}{g^2} - \rho\,\delta g_{tx(0)}\,.$$

- Taking the boundary limit $r \to 0$:

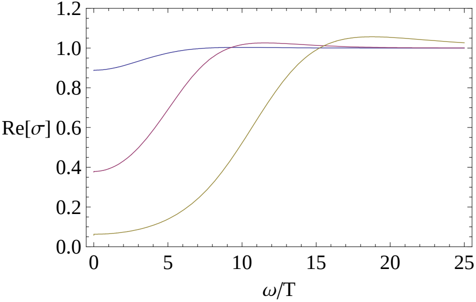
$$\begin{pmatrix} \langle J_x \rangle \\ \langle T_x \rangle \end{pmatrix} = \begin{pmatrix} \frac{1}{g^2}\frac{\delta A_{x(1)}}{L\delta A_{x(0)}} & -\rho \\ -\rho & -\epsilon \end{pmatrix} \begin{pmatrix} \delta A_{x(0)} \\ \delta g_{tx(0)} \end{pmatrix}\,,$$

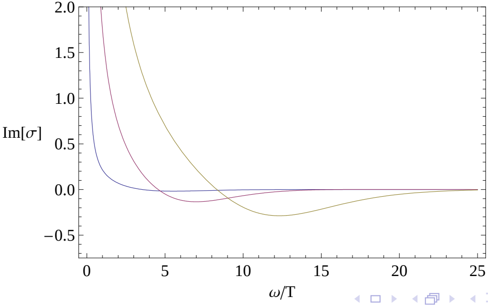
($\epsilon = -2\Omega/V$, energy density).

- Compare with above:

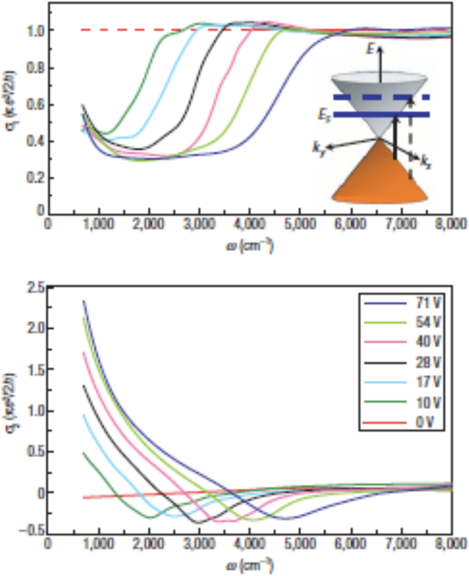
$$\sigma(\omega) = \frac{-i}{g^2\omega}\frac{\delta A_{x(1)}}{L\delta A_{x(0)}}\,;\; T\alpha(\omega) = \frac{i\rho}{\omega} - \mu\sigma(\omega)\,;\; T\bar{\kappa}(\omega) = \frac{i(Ts - \mu\rho)}{\omega} + \mu^2\sigma($$

- Solve the differential equation for A_x to get electrical conductivity





- For amusement, compare with experimental data on a 2+1 dimensional relativistic theory, graphene!



- Note that the real conductivity in the data goes up again at low frequencies, this is the Drude peak due to momentum relaxation from impurities, ions, etc.

- In the AdS/CFT theories (without impurities) there will be a delta function in the conductivity at $\omega = 0$.

- This is because a translation-invariant medium with a net charge cannot relax a DC ($\omega = 0$) current due to momentum conservation.

C: Charged bosons

1. Symmetry breaking and superconductivity
2. BCS theory
3. Holographic superconductors
4. Conductivity
5. String landscape of superconductors

Superconductivity, symmetry breaking and charge density

- Superconductivity $\sim$ spontaneous symmetry breaking.
- Contrast condensate with charge density
    - Charge density breaks Lorentz invariance, a condensate does not.
    - The charge operator $\rho = J^t$ is neutral, so does not break symmetry.
- Suppose we have a Lagrangian with $U(1)$ symmetry. Change variables so that the symmetry only acts by shifting a phase: $\theta \to \theta + \delta\theta$.
- By definition: kinetic term for the phase is: $J^t \partial_t \theta$.
- The charge density $\rho = J^t$ and the phase θ are conjugate:

$$[\theta, \rho] = i\hbar \,.$$

- Therefore a state with definite phase, breaking the symmetry, is maximally different from a state with a definite charge density.

Superconductivity and symmetry breaking

- Superconductivity follows from symmetry breaking.

- Free energy:

$$F = \int d^d x \sqrt{g_{(0)}} \mathcal{F} \left[A - d\theta \right] ,$$

- Current:

$$J_i = - \left. \frac{\delta F}{\delta A^i} \right|_{A = d\theta + \delta A} = -\mathcal{F}''[0] \delta A_i \, .$$

- Conductivity:

$$J_i = \frac{i \mathcal{F}''[0]}{\omega} \delta E_i \equiv \sigma(\omega) \delta E_i \, .$$

What does a typical theory of superconductivity look like?

- In a textbook on superconductivity one finds the 'BCS Hamiltonian'

$$H = \sum_{k,\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} - |g_{\text{eff}}|^2 \sum_{k,k'} c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} c_{-k'\downarrow} c_{k'\uparrow} + \sum_k A_k \cdot J_{-k} \,.$$

- Interaction term is generated by the exchange of a soft phonon between two effective electrons. Need $|\epsilon_k - \epsilon_F|, |\epsilon_{k'} - \epsilon_F| \ll \omega_D$.

- Theory predicts the symmetry breaking condensate

$$\Delta \equiv |g_{\text{eff}}|^2 \langle c_{-k\downarrow} c_{k\uparrow} \rangle = 2\omega_D e^{-1/|g_{\text{eff}}|^2 g(\epsilon_F)} \,.$$

ω_D is Debye frequency (energy scale of phonons) and $g(\epsilon_F)$ density of states at Fermi energy.

More on BCS theory

- Theory predicts the critical temperature in terms of the condensate

$$\frac{2\Delta}{T_C} = 3.52\,.$$

- The electrical conductivity in the superconducting phase is computed from the current two point function

$$\sigma(\omega) = \frac{-i\langle J_x J_x\rangle^R(\omega)}{\omega}\,.$$

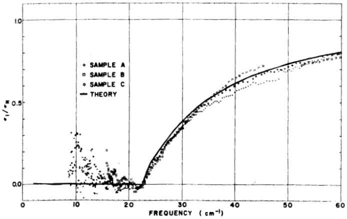


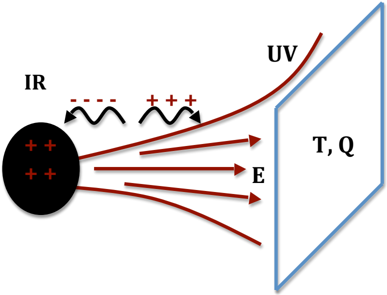
Fig. 2. Measured and calculated values of the conductivity ratio σ_1/σ_n of three superconducting lead films as a function of the photon frequency. [After Palmer (39).]

- The $U(1)$ symmetry is global in BCS theory – photons not important.

Superconductivity without glue

[Gubser; Hartnoll, Herzog, Horowitz '08]

- There is a natural instability of the black hole in the presence of charged bulk matter. Charged scalars can lead to superconductivity.



- Geometric instability. No 'glue' or weakly coupled 'pairing' required.

Minimal ingredients for a holographic superconductor

- Minimal ingredients
  - Continuum theory $\Rightarrow$ have $T^{\mu\nu}$ $\Rightarrow$ need bulk g_{ab}.
  - Conserved charge $\Rightarrow$ have J^{μ} $\Rightarrow$ need bulk A_a.
  - 'Cooper pair' operator $\Rightarrow$ have $\mathcal{O}$ $\Rightarrow$ need bulk ϕ.

- Write a minimal 'phenomenological' bulk Lagrangian

$$\mathcal{L}_{1+3} = \frac{1}{2\kappa^2} R + \frac{3}{L^2\kappa^2} - \frac{1}{4g^2} F_{ab}F^{ab} - \left|\nabla\phi - iqA\phi\right|^2 - m^2 \left|\phi\right|^2 .$$

  There are four dimensionless quantities in this action.
  - Newton's constant $\Rightarrow$ central charge of the CFT: $c = 192 L^2/\kappa^2$.
  - Maxwell coupling $\Rightarrow$ DC conductivity $\sigma_{xx} = \frac{1}{g^2}$.
  - Mass $\Rightarrow$ scaling dimension $\Delta(\Delta - 3) = (mL)^2$.
  - Charge q is the charge of the dual operator $\mathcal{O}$.

Two instabilities of a charged AdS black hole

- By dimensional analysis $T_c \propto \mu$.
- RN-AdS can be unstable against a (charged) scalar for two reasons.
- Reason 1 [Gubser '08]: Background charge shifts mass:

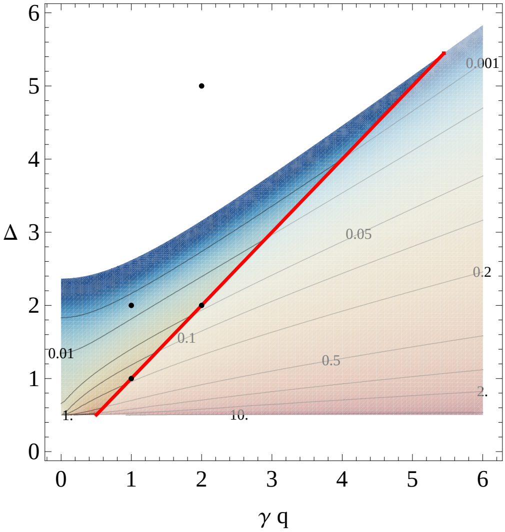
$$m^2_{\text{eff.}} \sim m^2 - q^2 A_t^2 \,.$$

- Reason 2 [SAH-Herzog-Horowitz '08]: Near extremality AdS_2 throat with

$$m^2_{\text{BF-2}} = -\frac{1}{4L_2^2} = -\frac{3}{2L^2} > -\frac{9}{4L^2} = m^2_{BF-4} \,.$$

- Precise criterion for instability at $T = 0$ [Denef-SAH '09, Gubser '08]

$$q^2\gamma^2 \geq 3 + 2\Delta(\Delta - 3) \,.$$



[Denef-SAH '09]

Endpoint – hairy black holes

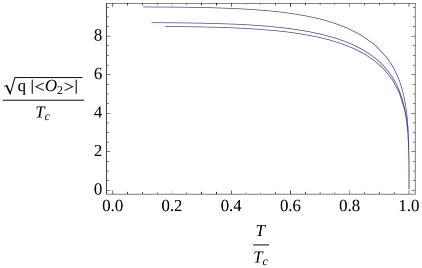
[SAH-Herzog-Horowitz '08]

- Endpoint of instability is a hairy black hole:

$$ds^2 = -g(r)e^{-\chi(r)}dt^2 + \frac{dr^2}{g(r)} + r^2\left(dx^2 + dy^2\right) ,$$

$$A_t = A_t(r) , \qquad \phi = \phi(r) .$$

- Solve numerically (take $\Delta = 2$). Can obtain $\langle \mathcal{O} \rangle$:

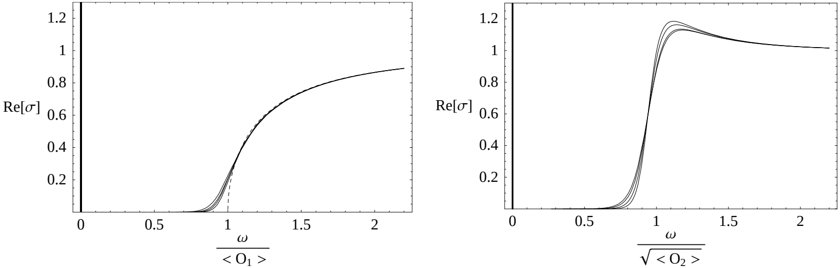


- Compare 8 to ~ 3.5 for BCS and $\sim 5 - 8$ for High-T_C.

Electrical conductivity

[SAH-Herzog-Horowitz '08]

- Computed the conductivity. At $T \sim 0$, typical curves



- If the gap is 2Δ then we found that

$$\mathrm{Re}\,\sigma(\omega \to 0) \sim e^{-\alpha\Delta/T}.$$

- Generally $\alpha \neq 1$, unlike BCS theory, no weakly coupled picture in terms of Cooper 'pairs'.

Landscape of superconducting membranes

- There appear to be many vacua of string theory.
- Potentially implies philosophical problems for stringy cosmology or stringy particle physics (only do experiments in one universe).
- Resonates well with atomic physics, which also has a landscape

	Atomic Landscape	String Landscape
Microscopic theory	Standard Model	M theory
Fundamental excitations	Leptons, quarks, photons, etc.	??
Typical vacuum	Atomic lattice	Compactification
Low energy excitations	Dressed electrons, phonons, spinons, triplons, etc.	Gravitons, gauge bosons, moduli, intersectons, etc.
Low energy theory	Various QFTs	Various supergravities

- Logic: Let's look at statistical properties of string AdS_4 vacua from the dual perspective as quantum critical points.

- Studied the stability of $AdS_4 \times X_7$ vacua for Sasaki-Einstein X_7.
- If X_7 has moduli: can build a 3-form mode that linearly decouples from all other modes, even in a background electric field.
- Calabi-Yau cone over X_7 has an anti-self dual closed (3,1)-form. Contracting with $r\partial_r$ get 3-form on Sasaki-Einstein. Mode is

$$\delta C = \phi Y_3 + \text{c.c.}\,.$$

- This mode is always unstable and leads to superconductivity.
- Examples supplied by Brieskorn-Pham cones:

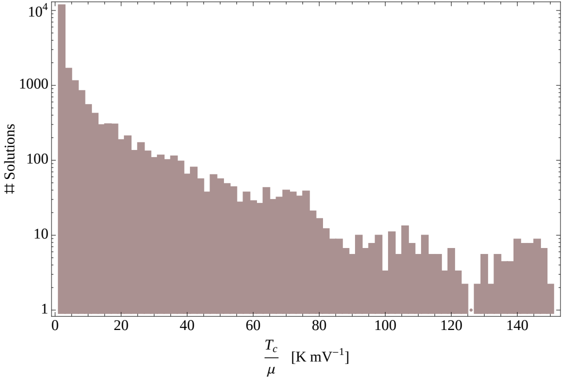
$$z_1^{m_1} + \cdots + z_5^{m_5} = 0\,.$$

- Computed a distribution of superconducting temperatures by scanning.

String landscape of superconductors

[Denef-SAH '09]

- This setup is realised in many concrete theories.
- Distribution of critical temperatures



C: Charged fermions

1. a