The critical point of QCD: lattice and experiment

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Outline

1. The context
2. On lattice
3. In experiment

A part of the standard model

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5. Supersymmetry in nuclear energy levels. Why?

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3. What are phases of compressed baryonic matter? Are there phase transitions? Can some of these phases be found in compact stellar objects? What is the equation of state of such matter? What are the transport coefficients in such matter: do they transport momentum and energy efficiently?

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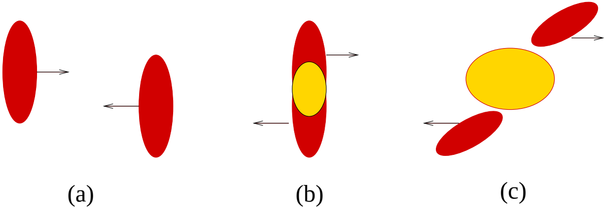
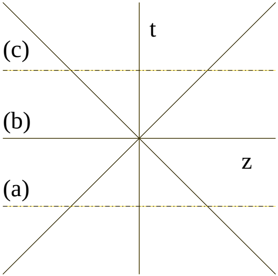
Counting variables

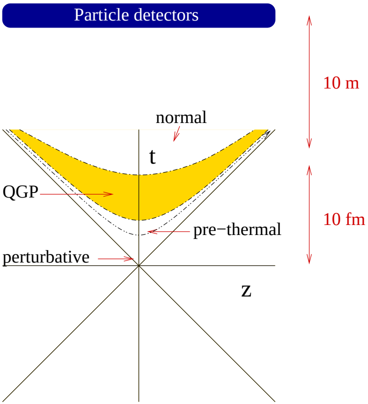
3 conserved charges: B, Q and S. Hence 3 chemical potentials; and T: 4 knobs to turn differently in different corners of the universe. In heavy-ion collisions change 2 quantities T and μ_B. But only one knob to tune: $\sqrt{S}$.

Heavy-ion collisions: kinematics



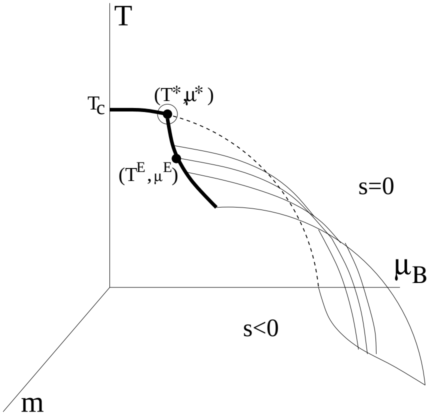


Heavy-ion collisions: evolution



- AGS (Brookhaven): Au, 10 GeV, ??– 1990
- SPS (CERN): S, Au, 20 GeV, 1987 – 1995
- RHIC (Brookhaven), d, Au, 5–200 GeV, 2000 – 2020
- LHC (CERN), Pb, 5.4 TeV, 2012 – ??
- FAIR (GSI Darmstadt), U, 10–40 GeV, 2015 –

The phase diagram: topology through symmetry



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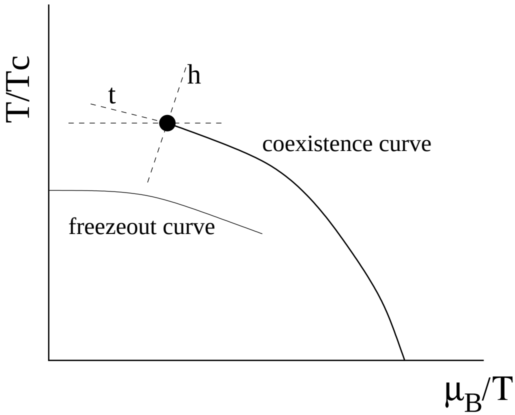

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Outline

The sign problem

QCD is solved on (super) computers by evaluating the path integral

$$Z = \int \prod_{\mathbf{x}\nu} dU_{\mathbf{x}\nu} \det D \exp[-S_E(U_{\mathbf{x}\nu})] \qquad U_{\mathbf{x}\nu} = \exp\left[iaA_\nu(\mathbf{x})\right]$$

x: spacetime point, ν: directions. Continuum limit: number of points goes to infinity, therefore infinite dimensional integral. Lattice cutoff, $a \to 0$ using QCD beta function. But even at small a, many points; need to use Monte Carlo techniques.

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Sign problems everywhere: QCD at finite μ_B, Chern-Simmons theory, high temperature superconductors, many nano-systems $\cdots$. First solution in QCD.

The method

Taylor expansion of the pressure in μ_B

$$P(T,\mu_B) = \sum_n \frac{1}{n!}\chi^{(n)}(T)\mu_B^n$$

has Taylor coefficients that need to be evaluated only at $\mu_B = 0$ where there is no sign problem. The baryon number susceptibility (second derivative of P) has a related Taylor expansion

$$\chi_B(T,\mu_B) = \sum_n \frac{1}{n!}\chi^{(n+2)}(T)\mu_B^n.$$

χ_B diverges at the critical point. Series expansion can show signs of divergence. If all the coefficients are positive, then the divergence is at real μ_B.

The method is perfectly general and can be applied to any theory.

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- Simulation algorithm is R-algorithm. MD time step has been changed by factor of 10 without any change in results.

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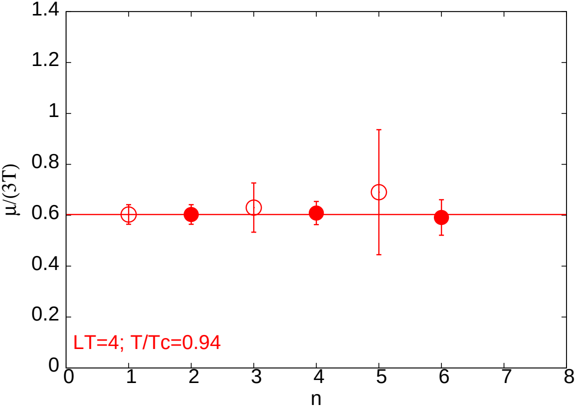
- Can the phase diagram be more complicated? Yes, we only find the nearest critical point to $\mu_B = 0$.

$N_t = 6$: Radius of convergence



Filled symbols: $(c_0/c_n)^{1/n}$. Open symbols: $\sqrt{c_{n-1}/c_{n+1}}$.

Critical end point

- Multiple criteria agree:
  - Small window in T where all the coefficients are positive.
  - Stability of radius of convergence with order and estimator
  - Finite size effects follow correct trend; more planned for the future.
  - Pinching of the radius of convergence with T.
- This gives $T^E/T_c = 0.94 \pm 0.01$ and μ_B^E/T^E as below

N_t	$V = (4/T)^3$	$V \to \infty$
4	1.3 ± 0.3	1.1 ± 0.1
6	1.8 ± 0.1	?

- Very naively: extrapolate to $V \to \infty$ by same factor, extrapolate to $a \to 0$ as a^2 (staggered quarks), then $\mu_B^E \simeq 325$ MeV. Somewhat lower at $m_\pi = 140$ MeV. Many assumptions, many caveats. May be in the range $\mu_B^E = 250$–**400 MeV with** $T^E = 165$–**175 MeV**.

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Gaussian Fluctuations

Fluctuations are Gaussian

Suggestion by Stephanov, Rajagopal, Shuryak: measure the width of momentum distributions. Better idea, use conserved charges, because at any normal (non-critical) point in the phase diagram:

$$P(\Delta B) = \exp\left(-\frac{(\Delta B)^2}{2VT\chi_B}\right). \qquad \Delta B = B - \langle B \rangle.$$

Bias-free measurement possible: Asakawa, Heinz, Muller; Jeon, Koch.

Why Gaussian?

At any non-critical point the appropriate correlation length (ξ) is finite. If the number of independently fluctuating volumes ($N = V/\xi^3$) is large enough, then net B has Gaussian distribution: **central limit theorem** (CLT).

Is the current RHIC point non-critical?

Answer

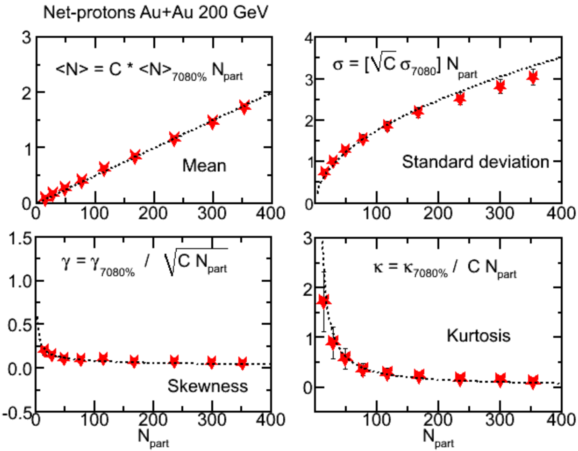
Check whether CLT holds.
Recall the scalings of extensive quantity such as B and its variance σ^2, skewness, $\mathcal{S}$, and Kurtosis, $\mathcal{K}$, given by

$$B(V) \propto V, \quad \sigma^2(V) \propto V, \quad \mathcal{S}(V) \propto \frac{1}{\sqrt{V}}, \quad \mathcal{K}(V) \propto \frac{1}{V}.$$

Caveat

Make sure that the nature of the physical system does not change while changing the volume. Perhaps best accomplished by changing rapidity acceptance while keeping centrality fixed. Alternative tried by STAR is to change the number of participants.

STAR measurements



STAR Collaboration: QM 2009, Knoxville.

QCD interpretation of STAR analysis

Can we compare STAR's measurements of $\sigma_{70-80\%}$ and $\mathcal{K}_{70-80\%}$ with lattice QCD?

Two questions to be answered before this is feasible:

1. N_{part} is a proxy for the volume. In changing this is the physics unchanged? Do the fluctuations give initial information or near-freezeout information? Need to develop a complete theory of diffusion+hydro (Son and Stephanov, hep-ph/0401052, Bower and Gavin, hep-ph/0106010, Bhalerao and SG, 0901.4677).
2. Have all other sources of non-Gaussianity have been subtracted out? What about jetty fluctuations, for example? Need studies of systematics.

What to compare with QCD

The cumulants of the distribution are related to Taylor coefficients—

$$[B^2] = T^3 V \left(\frac{\chi^{(2)}}{T^2}\right), \quad [B^3] = T^3 V \left(\frac{\chi^{(3)}}{T}\right), \quad [B^4] = T^3 V \chi^{(4)}.$$

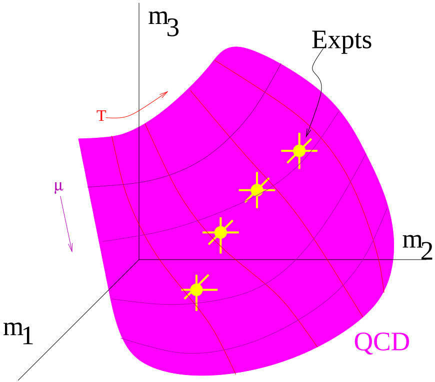
T and V are unknown, so direct measurement of QNS not possible (yet). Define variance $\sigma^2 = [B^2]$, skew $\mathcal{S} = [B^3]/\sigma^3$ and Kurtosis, $\mathcal{K} = [B^4]/\sigma^4$. Control all backgrounds in the measurements of $[B^n]$. Then construct the ratios

$$m_1 = \mathcal{S}\sigma = \frac{[B^3]}{[B^2]}, \qquad m_2 = \mathcal{K}\sigma^2 = \frac{[B^4]}{[B^2]}, \qquad m_3 = \frac{\mathcal{K}\sigma}{\mathcal{S}} = \frac{[B^4]}{[B^3]}.$$

These are comparable with QCD (Table III of Gavai, SG, 2008).

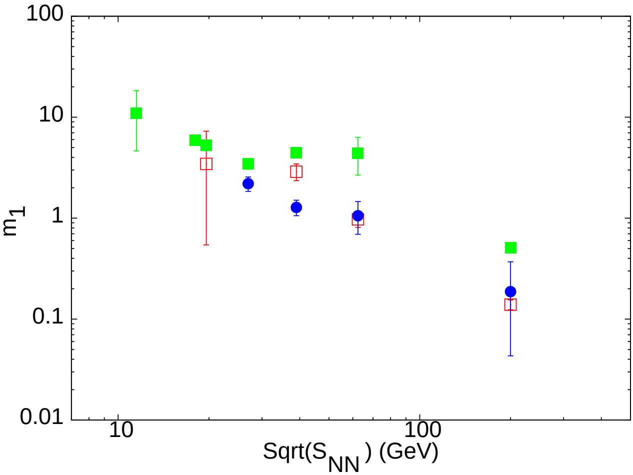
Is there an internally consistent check that all backgrounds and systematic effects are removed and comparison with lattice QCD possible?

How to compare with QCD



As T and μ_B are varied, the QCD predictions will lie on a surface in the space of measurements (m_1, m_2, m_3). If the data lies on this surface then all non-thermal backgrounds are removed. Then a comparison with QCD and a measurement of T and μ_B is immediate. Similarly for Q and S.

Lattice results along the freezeout curve



Open symbols: $T_c = 192$ GeV, filled symbols: $T_c = 175$ GeV. Boxes: $N_t = 4$, circles: $N_t = 6$.

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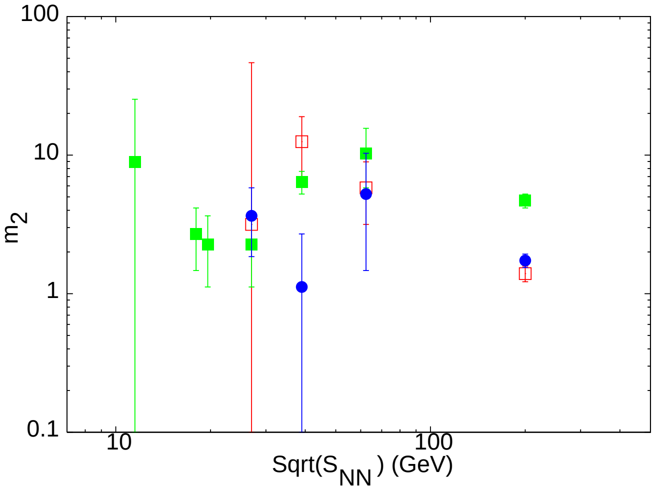

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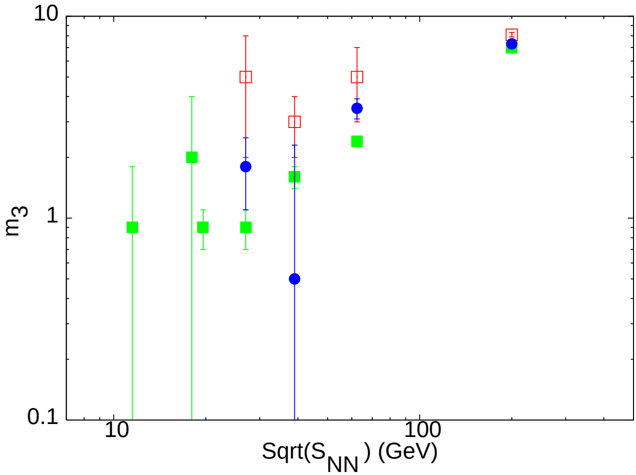

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One way to find the critical point

- Construct E-to-E distributions of B, Q and S. Since there are non-trivial linkages between them, comparison of the three distributions is important. Construct distributions in limited acceptance in order to simulate a grand canonical ensemble.
- Issues related to missed particles, in particular uncharged baryons and strange particles (neutrons and K^0). Require studies to see the effects of these.
- Observe the scaling of B, Q and S as a function of volume: if central limit theorem, then normal point. Otherwise close to critical point.
- Close to critical point the kurtosis does not scale with volume and may become very large due to critical exponent effects.
- Effect of hydrodynamic evolution needs to be included.